THE INVESTIGATION OF STYLE INDICES
AND ACTIVE PORTFOLIO CONSTRUCTION
ON THE JSE

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Prepared under the supervision of Professor Paul van Rensburg and presented to the School of Management Studies in fulfilment of the requirements for the degree of

MASTER OF COMMERCE
(Special Field: Finance)

University of Cape Town ©
February 2008
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Abstract

This thesis investigates the construction and performance of style indices on the JSE. It then demonstrates how a 'toolkit' of style indices can be used, together with conventional passive indices, as a set of building blocks for efficient portfolio construction.

This study tests the performance of a variety of potential style indices representing 'size', 'value' and 'momentum' effects. Selected indices for each style together with JSE sector indices are subsequently utilised to replicate the returns obtained on actively managed domestic equity funds using Sharpe's (1988, 1992) style decomposition method. Finally, a 'toolkit' of selected style indices are employed as building blocks to construct mean-variance and mean-tracking error optimal portfolios at low cost.

The best performing and most robust indices for each investment style are identified to be: (1) the EW size 100 Index (EW(size)100) for the size investment style; (2) the Three-factor (Earnings, Book Value and Dividend) regression residual weighted value 100 Index (REW100(3)) for the value investment style, and (3) the Past 12 Month less Prior Month Return Weighted momentum 100 Index (MOM(12-1)W100) for the momentum investment style.

Adopting Sharpe's (1988) return-based style analysis, style portfolios are created using a passive mix of selected style and sector indices to replicate the performance of a sample of general equity unit trusts and hedge funds. In the case of unit trusts, the out-of-sample regressions of style returns over actual fund returns found that the synthesized portfolios are able to explain a large proportion of the variations in the actual fund returns. The very low t-values on selection returns indicate that the mean returns of unit trust synthesized style portfolio are not significantly different from the actual fund returns being replicated. However, on average, the replicated portfolios exhibited a tracking error of about 5% per annum in relation to the actual funds being mimicked.

With regard to active portfolio construction, it is found that in the case of a long-only mean-variance analysis, the value index would make a worthy long-only equity benchmark even in isolation. In a benchmark (SWIX) relative analysis, an illustrative enhanced index strategy with 3% tracking error would comprise 74% in the SWIX, 20% in the value index and 6% in the momentum index. A long-short equity hedge mean-variance analysis illustrates that the optimal portfolio is short about 20% in the ALSI40 and long 65% value and 10% momentum. This results in a strategic net exposure of 55%. Finally, the estimated composition of a market neutral hedge fund optimal portfolio is short 100% Top40, long 69% value and long 31% momentum.
Declaration

I, Xiao Yu, hereby declare that the work on which this thesis is based is my original work (except where acknowledgements indicate otherwise) and that neither the whole work nor any part of it has been, is being, or is to be submitted for another degree in this or any other University. I empower the University to reproduce for the purpose of research either the whole or any portion of the contents in any manner whatsoever.

February 2008
Acknowledgements

The author would like to acknowledge the supervision and guidance of Professor Paul van Rensburg of the University of Cape Town.

The author also acknowledges Dave Moore and Kudzi Zhou for editing the thesis.

The author also acknowledges the School of Management Studies at the University of Cape Town for permission to use the Finance Research Laboratory for data collection and analysis.

Finally, the author thanks the Postgraduate Funding Office of the University of Cape Town for the Entrance Merit Scholarship.
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1. Introduction

'A particular security's returns may have statistically significant sensitivities to a number of factors. Exposure to most factors can be eliminated through the process of diversification. The factors that cannot be (costlessly) diversified away result in investors requiring a risk premium in the form of higher expected returns.'

- Ross (1976)

1.1 Introduction

The Capital Asset Pricing Model (CAPM) postulates that in efficient markets, a share's beta to the market is able to explain all of the systematic variation in cross section of returns [Treynor and Sharpe (1964), Lintner (1965) and Mossin (1966)]. This implies that the market portfolio is mean-variance efficient and an average investor cannot consistently outperform a simple buy-and-hold strategy on the market portfolio in the long term. Empirical tests on the CAPM, however, have found numerous anomalous variables that have displayed the ability to predict security returns beyond that explained by the market portfolio. These anomalous variables identified, such as firm-specific attributes, have become known as 'CAPM anomalies', 'style factors' or 'style characteristics' [Haugen (1995) and Robertson (2002)].

The method of constructing portfolios which constitute assets with similar style characteristics or attributes is known as 'style investing'. A variety of methods are adopted internationally to construct style portfolios, however no exhaustive and systematic research has yet been conducted regarding the construction and operation of style indices on the Johannesburg Stock Exchange (JSE). The most comprehensive JSE style anomaly investigation to date is conducted by Van Rensburg (2001). It is proposed that 'three style-based risk factors can form a parsimonious representation of the style risk on the JSE. Earnings yield (EY) represents the value cluster, market capitalisation (MV) represents the size cluster and twelve-month past positive returns (MOM) represents the momentum cluster'.
This thesis reinvestigates appropriate representatives for these three clusters of style-based risk and, in particular, looks at how style indices (or ‘active indices’\(^1\) given their historic tendency to outperform on a risk-adjusted basis) representing these styles are best constructed. Together with the FTSE/JSE Africa Resource 20 Total Return Index (RESI), the FTSE/JSE Africa Financial 15 Total Return Index (FINI), the FTSE/JSE Africa Industrial 25 Total Return Index (INDI), the FTSE/JSE Africa Top 40 Total Return Index (Top 40) and the FTSE/JSE Africa Shareholder Weighted Top 40 Total Return Index (SWIX), these selected style indices are used as a ‘portfolio construction toolkit’. All of the above indices are listed as exchange traded funds (ETFs) on the JSE facilitating their easy use for this purpose. This thesis demonstrates how these ‘building blocks’ can be used both to replicate (Chapter Five) and, more interestingly, create active portfolios (Chapter Six).

In particular, the first objective of this thesis is to determine the most suitable style proxy representing the previously identified value, size and momentum clusters of style-based risk on the JSE (Van Rensburg, 2001). In conjunction, various style-index construction methods are explored.

Once the best performing and most representative indices are chosen for each investment style, this thesis attempts to use them, together with JSE sector indices, to replicate the returns of active South African (SA) domestic equity portfolios using Sharpe’s (1988) return-based style decomposition method. The second objective is to investigate whether an active equity fund manager investing on the JSE can produce significant excess returns after their investment styles are taken into account. Average selection returns, tracking errors, the mean and the Sharpe Ratios of style returns, and out of sample R\(^2\) values are the main statistics examined for this purpose.

The third objective is the active construction of mean-variance and mean-tracking error efficient portfolios using the selected ‘active indices’ and conventional passive indices (including SWIX and Top 40\(^2\)). The effect of various investment and leverage constraints on portfolio performance is also examined.

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1 The terms style indices, ‘active indices’ and active style indices are used interchangeably from hereon in this thesis.

2 No single liquid instrument currently trades representing the SWIX Index. Therefore the Top 40, on which very liquid futures contracts trade, is used in cases where shorting is required.
The remainder of this chapter is organised as follows: Section 1.2 discusses the contribution made by this thesis, and Section 1.3 provides a brief overview of each subsequent chapter.

1.2 Contribution

This thesis contributes to the body of research that investigates the performance and construction of style indices on the JSE. Quite different from that of the US market, the structure of the JSE is characterised by a resource and industrial-financial dichotomy (Van Rensburg and Slaney, 1997). While large volumes of research have been conducted on style indices overseas, there has not been any formal and systematic research done on the JSE. To the author's knowledge, this thesis examines the most comprehensive set of style-proxy and style-index-construction-method combinations on the JSE.

Some of the style proxies used in this paper are based on work previously done by Van Rensburg (2001) and Arnott (2004). This thesis, however, goes beyond the large volume of prior research on fundamental indices in the following ways: (1) it recognises that fundamental weightings merely reflect a tilt towards the value style and extends the analysis to other styles that are worthy of attention (i.e. momentum and size); (2) as a result, it can conceive of using a number of style indices together as a 'portfolio toolkit' for active optimal portfolio construction rather than merely providing an alternative benchmark to track and (3) extensive testing is conducted into the formulation of the value (and other style) indices. It would be highly coincidental if equal weighting is an optimized form of constructing the value index. The use of multi-factor regression residuals (RES) as a value measure is an innovative idea that has not been discussed in any previous literature.

Finally, this thesis also provides evidence on the areas regarding the performance of active domestic equity fund managers. With only one previous work in this context by a South African author (Scher and Muller, 2003), there exists ample opportunity for further knowledge accumulation.
1.3 Thesis organisation

Chapter Two opens with a detailed review of relevant prior research on style anomalies in overseas and SA markets. The chapter then outlines the literature that examines various methods of constructing style indices, with special focus on the development of fundamental indices. Research on the method and application of the return-based style decomposition is presented. Finally, a brief review is conducted on the characteristics of Exchange Traded Funds (ETFs) and the ETF products currently offered on the JSE.

Chapter Three introduces the dataset that are analysed in Chapters Five and Six, and from which the indices in Chapter Four are derived. The dataset consists of (1) share price and firm-specific attribute data of JSE listed shares, (2) total returns of JSE published indices, and (3) portfolio return data including total returns on SA Satrix ETFs, domestic equity unit trusts and hedge funds. Accuracy and consistency checks are conducted on the dataset.

Chapter Four constructs a set of style indices for each of the three investment styles mentioned above that have been identified to produce excess returns on the JSE (Van Rensburg, 2001). Style proxy, index construction method and a number of index constituents are combined in various permutations. For each investment style, the best performing indices are identified considering their returns, risk, liquidity, concentration and rebalancing frequency inter alia. The selected style indices are subsequently employed to compute the style decomposition on domestic equity fund returns and the active portfolio construction in Chapters Five and Six.

In Chapter Five, Sharpe’s (1988, 1992) returns-based style decomposition technique is used to replicate the returns of a sample of SA general equity unit trusts and hedge fund indices using the four active style indices constructed in Chapter Four and three passive sector indices representative of the JSE (Van Rensburg, 2001). A rolling window of data is used so as to avoid look ahead bias in the simulation. This chapter attempts to examine whether the SA domestic equity active fund managers can produce significant excess returns after their investment styles are taken into account.
Mean selection returns, tracking errors, means and the Sharpe Ratios of style returns, and out-of-sample $R^2$ values are the main statistics examined for this purpose.

Chapter Six looks at active portfolio construction based on full historical data and subject to various shorting and leverage constraints. Constrained mean-variance optimisation methodology is employed to create three types of efficient portfolios, namely: (i) long-only portfolios, (ii) long-short equity hedge funds and (iii) market neutral hedge funds. Lastly, the chapter investigates the risk-return trade-off of mean-tracking efficient portfolios constructed relative to the SWIX benchmark.

Chapter Seven provides a summary of the results obtained from Chapters Four to Seven. The thesis is concluded with several suggested areas of future research regarding style-index investigation on the JSE.
2. Literature Review

2.1 Introduction

The establishment of the Capital Asset Pricing Model (CAPM) by Treynor and Sharpe (1964), Lintner (1965), and Mossin (1966) sets the foundation for modern finance. The model postulates that in an efficient market, all of the variations in share returns can be explained by one single factor - the returns on the market portfolio. This gives rise to the notion of the 'efficient market hypothesis' which states that ‘...inefficient markets security prices fully reflect all available information on the value of an asset' (Fama, 1991). This implies that the market portfolio is mean-variance efficient and an average investor cannot consistently outperform a simple buy-and-hold strategy on the market portfolio in the long term. Empirical tests on the validity of the CAPM, however, have identified numerous 'anomalies', which are variables other than the market beta that have displayed evidence of the ability to predict security returns beyond that explained by the market portfolio.

Subsequently, Ross (1976) introduced the Arbitrage Pricing Theory (APT), stating that ‘a particular security’s returns may have statistically significant sensitivities to a number of factors. Exposure to most factors can be eliminated through the process of diversification. The factors that cannot be (costlessly) diversified away result in investors requiring a risk premium in the form of higher expected returns’. Since the number and nature of the systematic factors were not specified, the APT has stimulated numerous investigations attempting to identify potential variables that have predictive powers over share prices. These CAPM anomalies, such as firm-specific attributes or fundamental characteristics, have become known as 'style factors' or 'style characteristics' [Haugen and Baker (1996) and Robertson (2002)]. ‘Style investing’ refers to constructing portfolios that constitute assets (within a broad asset category, such as equities or fixed income) with similar ‘style attributes’ and hence a fund’s investment style depends on the
nature of its underlying assets. Sharpe (1988) first introduced the return-based style decomposition method to estimate a fund’s true investment style.

The remainder of this chapter is organised as follows: Section 2.2 documents the prior research on style anomalies in overseas and South African (SA) markets. Section 2.3 examines various methods of constructing style indices, with special focus on the development of fundamental indices. Section 2.4 reviews literature on the method and application of the return-based style decomposition. Section 2.5 outlines the characteristics of Exchange Traded Funds (ETFs) and the ETF products currently offered on the Johannesburg Stock Exchange (JSE). Section 3.7 summarises and concludes.

2.2 Style anomalies

Numerous studies have documented the effect of style anomalies when testing market efficiency using the CAPM. The early studies dated back to the 1980s and were mostly conducted on the US market. Style investing has also been recognised as a SA phenomenon, backed-up by extensive evidence indicating that the CAPM seriously misprices the JSE listed shares. Style anomalies that have been identified in both SA and International markets can be roughly grouped to represent three distinct investment styles, namely: size, value and momentum.

2.2.1. International evidence on style-anomalies


The value anomaly is also extensively investigated on the US market. Several firm-specific characteristics are identified as adequate proxies for the value style, including price to earnings ratio (PE) [Basu (1977) and Jaffe and Keim (1989)], dividend yield (DY) [Litzenberger and Ramaswamy (1979), Blume (1980), Keim (1983), Hodrick (1992)

Fama and French (1988, 1992, 1996) confirmed the results from these earlier studies and reported that the anomalies of size, PE, leverage and MTBV 'had strong individual relationships with the average returns realized on portfolios sorted according to these characteristics' over the period 1963 to 1990. Furthermore, they claimed that the CAPM does not hold since it is unable to explain cross-sectional variation in share returns over the period of investigation. They discovered that portfolio returns could be more accurately explained by a portfolio’s exposure to three factors: the market (i.e., beta), size (small capitalisation firms outperform large capitalisation ones) and value (value stocks outperform growth stocks).

Lo and MacKinlay (1988) showed that weekly share returns are positively correlated. Using one-year past returns as the style proxy, Jegadeesh and Titman's (1993) study revealed the existence of the momentum anomaly on US markets. Fama and French (2007) also suggested that 'momentum is the key challenge to market efficiency'. They found it difficult to explain 'why momentum exists in the market and why investors cannot profit from it; or if the momentum effect can be exploited, why it has not yet been arbitrated away'.

Some studies [Banz and Breen (1986), Kothari et al. (1995) and Davis (1994)] suggested that most of the CAPM anomalies identified above arise due to methodological bias, such as look-ahead bias, survivorship bias, data snooping and thin trading. The excess returns tend to become less or non-statistically significant after these biases are adjusted for. On the other hand, Fama and French (1993) stated that the anomalies observed are in fact consistent with the CAPM. They argued that the size and value anomalies act as proxies for some hidden risk factors, and hence the CAPM anomalies arise due to 'an inadequately specified asset pricing model rather than inefficient markets'.
This school of thought is supported by Arnott and Hsu (2006). Assuming a random walk model for share prices, they quantitatively showed that market capitalisation (MV) and value-related ratios such as MTBV and PE can explain the cross-sectional variance in share returns. Furthermore, they found that alphas of time series regressions can be largely eliminated after adjusting for the size and value factors.

Daniel and Titman (1997) proposed a three-factor model to measure performance of US unit trusts. They found that using three firm-specific characteristics (size, MTBV and momentum) as factor proxies, the model has greater explanatory power than the CAPM at predicting future share returns.

2.2.2. Evidence of style-anomalies on the JSE

Van Rensburg and Slaney (1997) highlighted the fact that CAPM’s beta coefficient is an incomplete measure of JSE’s market risk and does not adequately explain the common variation in JSE share returns due to the unique ‘sector dichotomy’ feature of the JSE. Using a factor analytic methodology they demonstrate that ‘a two factor APT model specified along the lines of the financial-industrial and resource dichotomy was more appropriate than the single factor CAPM on the JSE’. Van Rensburg (1998) suggested that the returns on the JSE Industrial and JSE Gold Indices may be employed as observable proxies in the application of the two factor APT model. Following the reclassification of the JSE indices in March 1999, Van Rensburg (2000) suggested that the FTSE/JSE Africa Financial-Industrial Index and the FTSE/JSE Africa Resources Index should be used together as APT factor risk proxies. Van Rensburg and Roberston (2002) also suggested that a two factor style-based model consisting of the size and PE factors has a higher explanatory power over the cross-section of share returns on the JSE.

Thereafter, Van Rensburg (2001) attempted to decompose style-based risk on the JSE. A small size effect was identified. Value anomalies identified are associated with earnings yield (EY), dividend yield (DY), price to book (P/NAV) and turnover; among which EY

3 FINDI. Coded as CI21X after the reclassification of the JSE Indices in March 1999, re-coded as J250.
4 RESI. Coded as CI11X after the reclassification of the JSE Indices in March 1999, re-coded as J210.
yielded the most significant abnormal returns over the sample period (February 1983 to March 1999). Companies with higher leverage (particularly, assets to debt and cashflow to debt ratios) also outperform their less geared counterparts. Past 12-month returns (MOM), as well as MOM3 and MOM6 to a lesser extent, were found to be significant measures of the momentum style. All these anomalies persisted after risk-adjustment using both the CAPM and the two factor APT model. A hierarchical agglomerative cluster analysis is applied to examine the interrelationships among these style factors and to group them into broad style clusters. The results suggested that three style factors, MV (representing the quality/size cluster), EY (representing the value cluster), and MOM (representing the momentum cluster), form a parsimonious representation of style-based risk on the JSE.

Many style anomalies studies on the JSE verified the conclusions of Van Rensburg (2001) and showed that there are particular styles that can provide an investor with persistent outperformance.


There is well-documented evidence on the value effect on the JSE. Page and Palmer (1991) confirmed Basu’s (1977 and 1983) finding that a PE effect persists. Returns on a value portfolio consisting of low PE (or high EY) stocks were 6.5% greater than a portfolio of high PE stocks over the period 1978 to 1987. Page (1996) illustrated that the PE effects persists after adjusting for risk using multi-factor models. In addition, Fraser and Page (2000) found evidence of a significant value effect. It is found that from July 1990 to June 2000, portfolios that contained low PE stocks earned higher returns in spite
of the fact that they had lower betas. The use of MTBV as a proxy was proposed by Plaistowe and Knight (1986). They found ‘discount’ firms (i.e., low MTBV firms) significantly outperformed the ‘premium’ firms on the JSE from 1973 - 1980.

Fraser and Page (2000) found evidence that apart from negative momentum shares, higher momentum was associated with higher returns. Van Rensburg (2001) also reported that shares with higher MOM tend to outperform those with lower past returns. Poterba and Summers (1988) and Exley et al (2004) suggested that a ‘mean reverting effect’ exists in share prices. As a result, the past one year’s returns excluding the most recent month’s return is a better proxy for a share’s past performance than MOM.

Lastly, Fraser and Page (2000) found that value strategies were not significant amongst momentum shares and vice versa. Their finding implies that style anomalies were independent on the JSE.

2.3 Style indices

Inspired by modern financial theory established by Markowitz (1952, 1959), Sharpe (1965) and many others, the traditional method for index construction is to weigh each index constituent by its MV. Arguing that capitalisation weighted indices (CWIs) are intrinsically inefficient due to the fact that they overweight over-valued shares and underweight under-valued shares, Arnott initiated the revolution of weighing companies by price insensitive fundamentals instead of by MV. Some studies, however, point out that fundamental indices are just value indices in disguise. Other index construction methods such as equally weighted (EW) indices and momentum-style indices are also explored in prior literature.

The style indices investigated in this thesis are not to be confused with so called ‘fundamental indices’, although a value style index may be formulated in the manner of a fundamental index.
2.3.1. Capitalisation weighted indices (CWIs)

The first CWI, S&P 500, was introduced in 1957. Nowadays, weightings of most major indices are based on the MV of the constituents. The CAPM implies that the capitalisation weighted market portfolio is mean-variance efficient, and thus investors cannot outperform a CWI without extraordinary skill, luck or information. The optimality of CWIs, however, has been subject to much debate during the last decade.

Estrada (2006) outlined two main advantages of the capitalisation weighting method. Firstly, a CWI properly represents the ‘investable universe’ of the equity market, and hence it can serve as the underlying benchmark for passive indexing on an immense scale by large institutional investors. Secondly, capitalisation weighting is by definition a low turnover strategy. Furthermore, large capitalisation firms are often traded more frequently, higher liquidity results in modest transaction costs and better tax efficiency. Other benefits of using capitalisation weighted indices as an investment strategy (e.g. through ETFs) include diversification, transparency and ease of index construction (Brandhorst, 2006).

2.3.2. Fundamental indexation

The CAPM is based on an array of simplified and restrictive assumptions. Roll (1977) and Markowitz (2005) emphasised that once real world conditions are taken into account, the market portfolio ceases to be mean-variance efficient. This is equivalent to rejecting the mean-variance efficiency of the CWIs.

Using a random walk model for share prices, Hsu (2006) rigorously showed that if prices do not fully reflect a share’s true value, a capitalisation weighted portfolio will underperform over time, across macroeconomic cycles and across countries. The degree of underperformance is in direct proportion to the noise in stock prices. Coyne (2006) explained that stock price and thus MV are heavily influenced by investor sentiment, and therefore the MV may grow rapidly without an increase in the true value of the stock. Capitalisation weighted indices overweight temporarily over-valued shares due to them giving greater weight to companies with larger MV.
2.3.2.1. Theoretical development

The earliest reference to ‘fundamental indices’ appears to be Wood and Evans (2003). They argue that the common market capitalisation weighting method ‘overweights every single stock that is trading above fair value and underweights every single stock that is trading below fair value’, leading to a drag on the return of capitalization weighted indices. Furthermore, the capitalisation weighted market portfolio may have a growth tilt instead of being style-neutral. The structural weighting errors and the growth tilt tend to result in severe underperformance, especially during equity market bubbles such as those happened in the late-1990s.

Developing on these ideas, Treynor (2005) proposed that if companies are weighted by valuation-indifferent measures instead of their MV, the weighting bias will be randomised and the return drag will be reduced.

The most popular price-insensitive weighting methodology is the approach proposed by Arnott et al (2005). They found that a portfolio of companies equally weighted by four fundamental measures of company size (i.e. revenue, earnings, book value and dividends) deliver higher returns and similar volatility to a portfolio weighted by companies’ MV (Brandhorst, 2005).

Arnott and West (2006) showed that ‘fundamental indexing is likely to keep pace with capitalisation indexing if the market is efficient’. Moreover, if the market is inefficient, the fundamental weighted indices are expected to outperform the capitalisation weighted indices while preserving many of the latter’s positive attributes for passive investors.

2.3.2.2. Index Construction

Fundamental indices are constructed in a similar manner to CWIs with the only difference being that the role played by MV is now replaced by a price-insensitive measure of company size. Such metrics are objective measures of company size and do not take account of market valuation.

Note that this would constitute ‘prior art’ in terms of Arnott’s August 2005 provisional US patent application for price insensitive indexes.
Two approaches are most commonly used to construct fundamental indices: (1) rank the companies, choose and weight the index constituents based on fundamental metrics; (2) re-weighting the constituent securities underlying a well-established broad-market index. For instance, VTL's Large Capitalisation Revenue Weighted Index consists of the same 500 stocks included in the S&P 500, but re-weights the index components with a fundamental factor. This branch of construction method is a result of Lowry's (2007) finding that 'capitalisation weighted indices underperform not because they own inferior stocks, but because they mis-weight the stocks in their portfolio'.

The fundamental metrics chosen should aim to accurately reflect the 'economic value' generated by the firms. Arnott et al (2005) considered cashflow, book value, revenue, sales, gross dividends and total employment as the price-insensitive fundamental factors of company size. However, Arnott (2005) mentioned that after extensive tests on potential valuation-indifferent measures, it is found that fundamental weighting added about 2% over CWIs no matter which fundamental factor is used.

In their article, Arnott et al (2005) also discussed the merits and drawbacks of each of the fundamental measures in detail. Although clear and transparent, the sales metric tends to be incomparable across different industries, such as trading companies and services/financial companies which do not book most deals as 'sales'. Revenue is less subject to sector bias and accounting manipulation in comparison to the sales figures. However, this metric may be very volatile for companies in cyclical industries and it excludes the potentially profitable new companies that have not yet managed a profit. MTBV has gained great popularity and support in literature; however, it is inherently biased against sectors with high intangible assets. Employment is the least favourable measured since it suffers from a lack of reliable data, high turnover, high beta and poor liquidity, as well as the serious flaw of encouraging low productivity and unduly

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6 trailing five-year average cashflow
7 trailing five-year average revenue
8 trailing five-year average gross sales
9 trailing five-year average gross dividends
overweight labour-intensive industries. Furthermore, Arnott and West (2006) pointed out that it is peculiar to 'value a McDonalds burger-flipper the same as a Genentech biochemist'.

Finally, Arnott et al (2005) found that 'dividend-weighting leads to the least value added, with the lowest excess returns relative to the CWIs and it is the only tested index that on average underperforms in bull markets'. The most severe problem of using this measure is that most growth companies do not pay dividends for an extensive period of time, and hence more than half of listed companies are excluded from the index. In contrast, WisdomTree Investments believes that dividends are independent of accounting principles and thus the most objective and transparent fundamental metric available. Consequently, the investment company weights its core indices by dividends alone.

It should be noted that the firm-specific attributes used in fundamental indexation are affected by changes in accounting standards. For instance, accounting policies regarding the recognition of sales have been altered in SA since the adoption of IFRS in January 2005.

A few studies have also investigated alternative fundamental measures to those introduced by Arnott et al (2005), and explored the possibility of using the fundamental indexation strategy in different permutations regarding sectors, styles, countries and regions.

### 2.3.2.3. Empirical evidence of excess return

There is extensive literature documenting the empirical evidence that fundamental indices significantly outperform their respective capitalisation benchmarks.

Using US data for the period 1962 to 2004, Arnott et al (2005) reported that indices weighted by price-insensitive fundamentals such as book value, sales, cashflow, revenue, dividends and total employment have higher returns and lower volatility than both the S&P500 Index (capitalisation weighted) and a custom-made 1000-stock CWI in all time
periods, under a variety of market conditions and throughout alternating phases of the business cycle. (The only exception being the IT technology bubble in during 1990s) The sales metric produces the highest excess returns and Sharpe ratio, but also the highest volatility. The dividends metric produces the lowest excess returns, highest tracking error, and lowest volatility, with the least statistical significance.

Arnott and Hsu (2006) construct the Research Affiliates Fundamental Index (RAFI) and subsequently found that the RAFI portfolio on average outperformed its respective capitalisation-weighted benchmark (S&P500) by 1.97% per annum over the 43-year period tested. In addition, they constructed a Small Composite Index based on the 2000 smaller companies ranked after those included in the RAFI. The Small Composite Index has produced significant excess returns to its corresponding capitalisation weighted small-capitalisation benchmark, the Russell 2000 Index, since 1979.

Likewise, Tamura and Shimizu (2005) showed that the fundamental indices have persistently outperformed the CWIs in Europe, Australia and the Far East (EAFE). Furthermore, Hsu et al (2006) reported that fundamental small- and mid-capitalisation indices yield higher annual returns than their respective capitalisation weighted small- and mid-cap indices.

Hsu and Campollo (2005) conducted a larger scale study by constructing composite fundamental indices for 23 countries over the period 1984 to 2004. They found an average excess return of 2.8% per annum relative to the corresponding CWIs, and a fundamental world index has beaten the capitalisation weighted world index (in this case, the MSCI) by 3.5%. They observed that the outperformance is statistically significant over different market environments, with the only exception being during the height of the technology bubble. They also reported that the fundamental indices are on average slightly less volatile than their respective capitalisation weighted benchmarks with betas averaging slightly lower than 1.
Estrada (2006) linked the concept of fundamental indexation and international diversification. He evaluated the relative effectiveness of using the capitalisation weighted strategy, the fundamental strategy and the traditional value strategy to achieve international diversification on 16 countries that cover more than 93% of the world MV over the period 1974 to 2005. He reported that ‘a dividend weighted fundamental index outperforms a CWI by a substantial margin of 1.9% a year’. The risk of the fundamental index is approximately in line with that of the CWI if measured by standard deviation or beta. However, the higher negative skewness and kurtosis seem to confirm that the dividend weighted fundamental index is riskier than the CWI. The most interesting finding, however, is that a simple DYs weighted value strategy produces an excess return of 1.7% per annum relative to that of the dividend weighted fundamental index. This implies that the fundamental indexation adds no value in addition to the traditional value strategy.

Lowry (2007) found that by re-weighting the capitalisation weighted S&P 500 according to the revenue metric, 2.7% excess average annual returns can be generated relative to the capitalisation weighted S&P 500 over the ten years starting from 1996. Furthermore, the revenue weighted fundamental index (RWI) enjoyed a lower risk measured by both standard deviation and beta over the period investigated. More importantly, they found that the RWI has more exposure to small capitalisation and value-oriented shares than the capitalisation weighted index. This is indicated by the smaller average MV and lower average PE of the RWI in comparison to that of the S&P 500.

Hsu (2006) showed that the turnover for the US Fundamental Index 1000 averaged a modest 10% to 12% per annum whereas turnovers of S&P 500 and Russell 1000 are estimated at 6% to 8%. He noted that, however, turnover for the CWI tends to concentrate in the companies with the smallest capitalisation. Meanwhile, the fundamental index turnover is largely due to rebalancing the larger companies. Therefore, Hsu concluded that ‘the true turnover cost associated with maintaining a fundamental index versus a capitalisation index may be comparable if not in favour of the fundamental
index’. Hsu further pointed out that the turnover costs are negligible if viewed in the context of the fundamental index’s alpha against its capitalisation weighted benchmark.

2.3.2.4. Criticisms of fundamental indices

Arnott’s fundamental indices have received as much criticism as support in academic circles. The main critics supported by empirical studies include hindsight bias, effect of transaction costs and ‘the value-tilt’.

Coyne (2006) pointed out that ‘Arnott has discovered a theoretically profitable anomaly with hindsight using ex-post returns, but there is no guarantee that the outperformance of fundamental indices to capitalisation weighted indices will continue in the future’. Since it is unreasonable to assume that after the weighting strategy has been publicised, the potential excess returns will not be arbitraged away, as the price of companies with high price-insensitive fundamentals are bided up by fundamental investors. Waid (2007) also argued that if Arnott’s fundamental indices had been calculated on ex-ante returns, the results regarding the excess returns could be different.

Many articles pointed out that differences in the expenses incurred in the design and maintenance of fundamental weighted portfolios may overwhelm the excess returns generated by them. Jackson (2005), Bogle et al (2006) and Bogle (2007) noted that fees, expenses and turnover of the fundamental weighted funds are significantly higher than the traditional capitalisation weighted funds. They criticised that the core results published by fundamental indexers are generally not adjusted for fees, transaction costs or taxes. Arnott et al (2005), however, countered this criticism by arguing that a few more basis points in fees are negligible ‘in comparison to the 200 basis points performance enhancements their strategies delivered over the capitalisation-weighting method’.

The most widely held criticism of fundamental weighting is that it is just ‘a value strategy in disguise’, and hence it simply captures the size and value premium. A heated
debate is subsequently initiated regarding the sources and nature of the excess returns generated by fundamental indices.

Arnott et al (2005) acknowledged that 'the fundamental indices sharply outpace the capitalisation weighted indices in bear markets but not bull markets, and hence the fundamental indices have a value bias relative to the capitalisation weighted indices'. They performed three-factor regressions on the RAFI and identified exposures to the value factor and, to a lesser degree, the size factor; however, they remained silent on the true driver of the excess returns generated by the fundamental indices over the CWIs.

Coyne (2006) suggested that uncertainty involved in the future growth rate of a company's dividend stream and earnings is a possible source of the systematic valuation errors eliminated by Arnott's fundamental indexation approach. Campbell and Vuolteenaho (2004) found that the value premium provides compensation for taking on higher risk, thus fundamental indexation is just a trendy application of value investing.

Jackson (2005) found that most of the excess returns of fundamental indices were achieved between 2000 and 2005 alone, which is 'one of the best periods in history for the relative returns of value stocks and small-capitalisation stocks'. In other words, the higher returns generated by fundamental indices are largely attributable to the superior performance of small-capitalisation stocks and value stocks, which is what previous researchers have identified as 'the value premium'.

Brandhorst (2005) stated that the fundamental metrics implicitly introduce a value bias into the fundamental indices. They are weighted more heavily towards low price-to-fundamental shares at the expense of high price-to-fundamental shares, leading to the formation of lower PE and higher DY portfolios.

Fama and French (2007) claimed that 'fundamentally weighted indices are a triumph of marketing, and not of new ideas'. Bogle and Malkiel (2006) called fundamental indexing 'little more than a fad made possible by the tremendous outperformance of value stocks
in the wake of the Internet bubble'. The latter pair further pointed out that the value style can not run forever since mean reversion will eventually come into play. For instance, from 1937 through 1967, growth funds consistently outperformed value funds, the position reverted since the late 1960s, but since 1977, the two has displayed very similar performance.

Bernstein (2006) regressed the Fama-French Large Value and Large Growth Indices on Arnott’s RAFI between 1962 and 2004. The empirical results showed that two-thirds of the excess return delivered by the fundamental indices relative to the S&P500 is attributable to factor exposure, while the remaining statistically insignificant one third is due to the fundamental indexation technique. He concluded that ‘the advantage of fundamental indexing over conventional value-strategies, to the extent that it exists at all, is relatively small and it could be mined from the data with hindsight’.

Schoenfeld and Ginis (2006) provided further empirical support for all of the above results. He reported that 90% of the return generated by the RAFI can be accounted by the size, value (represented by MTBV and PE value factors) and sector exposures. Finally, he showed that the RAFI returns have a very high correlation with those of the S&P500 Value Index and the Russell 1000 Value Index.

Hsu (2006), however, rejected the idea that fundamental indexing is simply ‘value investing’. He pointed out that, on the theoretical level, ‘traditional value indices select index constituents based on value factor and weight them by MV, therefore they are limited in capacity and do not provide broad market participation’. Empirically, he showed that the US Fundamental 1000 and 2000 Indices displayed reliable outperformance over their respective value index counterparts (i.e. the Russell 1000 and 2000 Value Indices.

Despite all the strong arguments and empirical evidence produced by both sides, the debate remains inconclusive. It is likely that the final answer can only be drawn when there is a longer and more reliable return history on the fundamental indices.
2.3.3. Other methods of constructing style indices

Fundamental indexation is just one alternative weighting scheme to reduce the structural weighting errors inherent in capitalisation weighted indices. Academics and practitioners have never ceased to explore other possible weighting schemes with the aim of enhancing index performance. These strategies are normally a variation of value investing, utilising different value anomalies identified; the most recent development is momentum investing exploiting the idea that strong performance tends to persist in the short run.

The most straightforward price-indifferent strategy to randomise the weighting errors is equal weighting, whereby some of the shares above true fair value are underweighted and some are overweighted. But Arnott (2005) pointed out that equally weighted indices give relatively more weight to smaller and less liquid companies. This consequently results in capacity constraint, higher index volatility and higher turnover costs. As a result, tracking EW indices on an institutional scale is not possible.

2.3.4. JSE evidence of style indices

While large volumes of research have been conducted on style indices overseas, no formal and systematic research has been done on the JSE.

The first formal style index on the JSE is the RAFI SA Index. It is introduced by the Plexus Asset Management in partnership with Research Affiliates (US), and serves as the underlying index of a recently launched fundamental indexing ETF - the Plexus RAFI Enhanced SA Strategy Fund. Plexus back-tested Arnott’s fundamental indexation strategy on the JSE for the period December 1993 to June 2007 and found that the RAFI SA Index delivered average excess returns of 6.8% per annum relative to the FTSE/JSE Africa All Share Total Return Index (the ALSI). August 2007 saw the launching of the second SA style-index tracking ETF - the Satrix DIVI ETF based on the FTSE/JSE Africa Dividend Plus Index (the Dividend Plus Index).

2.4 Style-based return decomposition
Investment style is defined as a portfolio's actively managed characteristic. It is equivalent to the long-term asset allocation policy in a balanced fund. Sharpe (1988) proposed that three interrelated decisions have a dominant influence on the performance of a portfolio, namely: security selection, market timing and long-term investment style. Sharpe (1995) termed the returns due to a fund's investment style as its 'style return' and the excess returns due to active security selection and market timing as a fund's 'selection return'.

It is widely agreed that investment style is the primary determinant of a portfolio's performance, with security selection and market timing playing minor roles. Brinson et al (1986) analysed quarterly return data of 91 large pension funds over the 1974 to 1983 period. It was found that style returns on average account for 93.6% of the fund's quarterly return variation. Ibbotson and Kaplan (2000) found that 81.4% of the monthly volatility in balanced fund returns is explained by the funds' investment style. US research conducted by Forge (2003) using a 40-year database of US balanced funds\(^\text{10}\) produced results supportive of the earlier findings that return variability of a broadly diversified portfolio is largely explained by its asset allocation policy. On average, 77% of the variability of a fund's returns was explained by its investment styles, while market-timing and security selection not only added risk but was also unable to overcome the higher costs of active management, such as research expenses and trading costs. The conclusion held in all time periods incorporating both bull and bear markets.

2.4.1. Theoretical development of return-based style analysis

Since investment style is the dominant influence on fund returns, a procedure to measure a fund's exposures to different investment styles and to determine the value-added through active management is most necessary.

There are two approaches to estimate a fund's investment style. The straightforward method is to analyse the individual securities held by the fund and add up each security's exposure to each style and sector from bottom up. However, this fundamental analysis

\(^{10}\) 420 US balanced mutual funds from the CRSP Survivorship-Bias Free US Mutual Fund Database, over the period 1962 to 2001.
approach is hardly feasible since while fund returns are readily available, timely fund holdings can be difficult to obtain, especially if the fund is actively managed. A Morningstar’s OnDiscTM survey (December 1995) showed that only 20% of the funds investigated had updated their reported portfolio holdings within the past three months. The Value Line’s Fund AnalyzerTM software showed that, as on October, 1995, less than half of the mutual funds registered had reported any portfolio holdings at all. Moreover, individual investors generally do not have resources to pay expensive consultants to perform elaborate fundamental analysis nor to obtain the required detailed information such as a fund manager’s decision-making process, turnover ratio and current prospectus.

In contrast, the second approach, the return-based style analysis, circumvents the previously mentioned problems since the only information required is a sufficient long history of total fund returns, which is not only easily obtainable, but also objective and comparable across funds. Forge (2003) showed that if such an analysis is properly performed, results produced would match what is known about the fund from a fundamental analysis.

The return-based style analysis technique was first introduced in two of Sharpe’s articles (1988, 1992)\(^\text{11}\). It is a statistical technique that identifies what combination of positions in passive indices would have most closely replicated the actual performance of a fund over a specified time period. This is accomplished using an asset class-factor model, along with historical monthly returns on a fund as the dependent variable and comparable returns on a selected set of passive indices as the independent variables. The passive indices selected typically represent distinct investment styles and equity sectors (or different asset classes if analysing a balanced fund).

The fund returns are regressed upon the selected style-index returns utilising a multi-factor regression. The resulting set of regression coefficients represents the fund’s average exposure to the corresponding style factors, and thus the fund’s historical

\(^{11}\) Sharpe originally used the terms “effective asset mix” and “attribution analysis” describing his work. In recent years the term “returns-based style analysis” has frequently been used to describe the Sharpe method.
investment style over the regression period (Sharpe, 1988). The overall objective was to select a set of coefficients that minimised the unexplained variation in returns and maximised the associated $R^2$ value. This is equivalent to minimising the variance of the error term during a least square regression (Sharpe, 1992). To remove negative coefficients when short and leverage positions are not allowed, Sharpe uses quadratic programming to constrain the coefficients (i.e., the style weights) to be greater than zero and sum to one.

Sharpe (1988) also set out the procedure for carrying out attribution analysis using the return-based style analysis. He suggested that style, selection, and market timing are the three main sources of a fund's performance. The market timing return is calculated by taking the arithmetic difference between the returns on a fund’s short-term rolling style and its long-term average style portfolios. It measures the value created (or removed) via active market timing or sector rotation. The selection return indicates the value added through the manager's security selection skills which is not accounted for by the fund’s short-term investment styles.

In a later simplified version of style-decomposition analysis, Sharpe (1995) defined the selection return as the measure of the value added by the fund relative to its average style return. It is computed by deducting the return on a fund’s long-term style portfolio from the actual observed fund returns. A positive selection return implies that fund returns in excess of the passive style exposure are present.

### 2.4.2. International application

Sharpe (1988) outlined two interrelated uses of the return-based style analysis, namely: style estimation and attribution analysis. He showed that the technique is more accurate than other alternative methods at identifying the fundamental asset allocation and true style of an actively-managed mutual fund. He further suggested that a customised benchmark index can be created using the combination of passive indices that best replicate a fund’s investment style (the style portfolio) for the purpose of attribution analysis. The return obtained by a fund in each month can be compared with the return on
a mix of passive indices with the same estimated style as the fund, where the style is estimated prior to the month in question. Such a benchmark is appropriate for measuring managerial performance due to it being identifiable in advance and easily constructed. Moreover, it produces a viable investment alternative which has low cost and not easily beaten.

Sharpe (1992) used a portfolio’s return sensitivities to selected style indices to infer a fund’s investment style. The method employed a weighted least square regression on prior 60-month returns. Sharpe showed that most of the differences in returns of US equity mutual funds can be attributed to the differences in their exposures to the size and value investment styles. Similarly, in Sharpe (1995), no significant selection returns are observed for the returns of a sample of US mutual funds from 1985 through 1994.

Using Sharpe’s (1992) method to infer a fund’s style, Saez and Izquierdo (2000) analysed the performance of Spanish unit trusts. They found that the funds can not persistently outperform their corresponding style portfolios (a passively managed portfolio of the same style as the evaluated fund) over the period of investigation. Quigley and Sinquefield (2000) investigated the performance of UK unit trusts, and concluded that the fund managers on average were not able to display any additional levels of skill to outperform a style benchmark.

In his analysis of the relationship between equity mutual fund performance and manager style, Davis (2001) found that no investment style displayed persistent positive excess returns during the 1965 to 1998 period. When funds are grouped by style, the value funds showed negative risk-adjusted returns of about 2.75% per annum. This implies that ‘once returns have been adjusted for style risk, fund managers were not able to add any significant additional value’. There was some evidence of short-run performance persistence among the best-performing growth funds and the worst-performing small-capitalisation funds.

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12 Each year, 100 largest, seasoned U.S. funds are chosen from among those categorized as bond funds, stock funds, balanced funds, global and international funds. To be included in a given year, a fund must have been in existence for at least five prior years.
Forge (2003) performed the return-based style analysis on all US diversified equity and taxable fixed income mutual funds with greater than three years of performance history as on December 1995 release per the Associates Fund Strategist Database. It is found that the funds showed an $R^2$ of 65% when compared to broad benchmarks such as the S&P 500. However, the average $R^2$ rises to 86% when using return-based style analysis.

2.4.3. SA application

There has only been one study on style-related analysis of SA domestic equity funds. Scher and Muller (2003) evaluated the performance of SA unit trusts for the 13-year period between 1990 and 2002 against a style-related benchmark. The funds are classified into four broad investment style groups using Sharpe’s (1992) asset class-factor model based on the prior 18-month returns. An attribution analysis is subsequently conducted on each fund using a multi-factor model to adjust for two style risks, size and value, measured by MV and MTBV.

Their findings were much in line with those of the previous overseas studies [Saez and Izquierdo (2000), Quigley and Sinquefield (2000), Davis (2001) and Forge (2003)]. They reported that ‘...for the most part South African funds were unable to outperform the market, once exposure to market, value and size anomalies were taken into account’. In particular, unit trusts focusing on investing in small-capitalisation companies have consistently earned the lowest returns; while large capitalisation funds realised some persistent positive style-adjusted returns. Negative performance of value funds seemed to extend for at least two years.

2.5 Exchange traded funds

The poor past performances of mutual funds has been well documented. Fama (1970) showed that mutual fund managers earn significantly negative abnormal returns in the long term, while Sharpe (1964), Treynor (1965), and Jensen (1978) concluded that a

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13 Large, small, value and growth.
portfolio constructed by naive buy-and-hold strategy on average outperforms an actively managed portfolio. Frino and Gallagher (2001) found that, in the period 1997 to 2001, the S&P index funds delivered higher risk- and expense-adjusted returns than actively managed funds. Consequently, more and more researches started to focus on passive investment strategies, in particular utilising index mutual funds and ETFs.

2.5.1. Introduction and theoretical overview

An ETF is a listed security that replicates the composition of an index or a pre-selected basket of shares. It trades in the same way as an ordinary listed share and invests in the same way as a passive index fund. Thus by investing in a single listed security, an investor effectively gains the same return as purchasing shares in each of the companies constituting the particular index underlying the ETF, without the additional costs of buying shares in several companies. Any changes to the composition of the index will trigger a change in the underlying assets of an ETF, to ensure it is exactly aligned with the index. Dividend and interest income on the underlying assets are paid to ETF investors on a quarterly basis.

There is a fair volume of recent literature on various aspects of ETFs such as the index tracking ability of the ETFs and the advantages and risks of investing in the ETFs.

2.5.1.1. Tracking ability

ETFs have full price transparency, since the prices of ETFs are quoted throughout the exchange's normal trading hours. An investor can also purchase or realise ETF shares at any time during a trading day. In theory, the performance associated with an ETF should be in line with, if not identical to, the index it tracks. Gallagher and Segara (2004) found that Australian ETFs' returns do follow the respective underlying index closely. Frino and Gallagher (2001), however, pointed out that all index funds (including ETFs) do not perfectly replicate the performance of the underlying index. The difference between the performance of the benchmark index and the index fund is known as the tracking error. The tracking error in ETFs is a result of two things: firstly transaction costs, and secondly the lag between ex-dividend date and the date cash is received by the fund (Kostovetsky, 2003).
Dellva (2001) found that, although ETFs can trade at a premium or a discount to their net asset value, the gap (i.e., the tracking error) is often arbitraged away very fast in efficient markets. Tracking errors are shown to be higher for less liquid shares such as out-of-favour sectors and emerging markets. Gastineau (2004) confirmed that ETF tracking errors have been modest and in general negative, but lower than most funds’ expense ratios.

2.5.1.2. Advantages of ETFs
Index investing has become increasingly popular among individual investors around the world for four main reasons:

1. Long-term outperformance
Dellva (2001) showed that index investments have displayed higher excess returns relative to inflation over the medium to long term. Furthermore, very few active managers can outperform an index over time. This is in line with Kostovetsky’s (2003) findings that ETFs tend to be more viable as long-term investments.

2. Low cost
ETFs provide a cost effective way of trading a basket of shares through a single transaction. Expenses in trading ETFs involve brokerage fees and bid-ask spreads. Overall, the costs are kept low because as a form of passive index fund, ETFs simply track an index without engaging in costly investment research. Moreover, they trade less often than actively managed funds, and hence incur lower transaction costs. ETFs are allowed to lend scrip for a fee (mainly in the derivative markets) which contributes to fund managers’ profit and reduces the charges passed onto the investors.

Kostovetsky (2003), Dellva (2001) and Poterba and Shoven (2002) found the expense ratios of ETFs, measured by management fees as a percentage of total managed assets, to be lower than those of other collective investments, including index unit trust funds. This is because index funds need to keep track of shareholder transactions, whereas an
ETF fund manager is mostly concerned with rebalancing the portfolio to reflect the correct weights in the index.

3. Tax efficiency
ETFs carry tax advantages over index funds due to the fact that ETF managers are not forced to purchase or liquidate underlying assets to meet the buying and selling demands of individual investors. And thus ETFs crystallise fewer short and long term capital gains than actively managed funds [Dellva (2001) and Kostovetsky (2003)]. When traded, ETF securities are taxed in the same way as any other share when you sell or trade the instrument. Holders of ETF securities pay capital gains tax (CGT) on any capital gain realised at the time of sale.

Additionally, an ETF may from time to time realise a capital gain due to a rebalancing of the underlying index. If the fund were to pay this CGT, it would result in a mis-tracking of the index. For this reason, it is standard practice for these capital gains to be paid by investors. If ETFs are made subject to the collective investment schemes legislation, the potential liability for CGT will fall away, as collective investment schemes do not incur CGT on portfolio rebalancing trades.

4. Diversification
Dellva (2001) confirmed the lower risk of ETFs when compared to other shareholding strategies due to higher level of diversification inherent in the underlying index. An ETF is also a convenient vehicle for small investors to secure exposure to the big blue chip companies with the sector ETFs able to facilitate sector rotation for more experienced investors.

2.5.1.3. Risks of investing in ETFs
An ETF investor is subject to the same basic risks as those investing in the underlying shares. These include, but not limited to, liquidity risk, market risk of general stock market fluctuations, and currency risk and political risk if the ETF invests in foreign countries.
2.5.2. International ETF industry

ETFs have been around for quite some time in the United States. Gastineau (2001) traced the development of ETFs in the US. The first ETF introduced was the S&P Index Participation Shares (IPSs). IPS were like S&P 500 futures contracts, but fully margined and collateralised like shares. They started trading on the American Stock Exchange in 1989. A Federal Court, however, soon found IPSs to be illegal futures and investors were forced to liquidate their positions. A legal replacement for IPSs was listed on the Toronto Stock Exchange in Canada. Other products were introduced in the US, such as Supershares in 1987. However, these were complex and high-cost products, and thus never traded actively. Subsequently, the modern-day ETFs with simple charging structure were introduced in the US, such as the Standard and Poor’s Depositary Index Receipts (SPDRs) and the Nasdaq 100 Index Tracking Stock. The US ETF market has shown phenomenal growth thereon and is currently extremely large.

There are a few ways to get exposure to the JSE through ETFs. One can purchase shares in an international ETF that invests exclusively on the JSE, or hold an international ETF that invests in emerging markets including SA, or invest in a SA ETF that focuses exclusively on the JSE.

There are a number of international ETF investing exclusively on the JSE, the most popular one is iShares MSCI South Africa Index Fund (EZA) created and managed by Barclays Global Fund Advisors. EZA seeks to provide investment results that in the aggregate correspond to the price and yield performance of publicly traded securities on the JSE. The Fund invests in a representative sample of securities in the MSCI South Africa Index, which is a capitalization weighted index that aims to capture 85% of the free float total market capitalisation of the JSE.

Non-SA ETFs that invest broadly in emerging markets provide broader diversification and serve to better minimize country, industry-specific, market, and currency risk. For example, as of 30th April 2007, Shares MSCI Emerging Markets Index Fund’s top country allocations include Korea (16.8%), South Africa (12.5%), Brazil (11.0%),
Taiwan (10.6%), China (7.8%), India (5.3%) and Thailand (2.9%). Vanguard Emerging Markets VIPERS tracks the MSCI Select Emerging Markets Free Index, and counts Korea (19.7%), Taiwan (16%), Brazil (12.2%), South Africa (11.9%), and Mexico (6.9%) as its largest country holdings on 30th April 2007. The fund objective of the SPDR S&P Emerging Middle East and Africa ETFs is to closely match the returns and characteristics of the total return performance of the S&P/Citigroup BMI Middle East and Africa Index before expenses.

2.5.3. SA ETF Industry

2.5.3.1. Introduction

In comparison to the international market, the SA ETF market is relatively new. It currently consists of ten ETFs. The Satrix 40 is the first ETF in SA, launched by the JSE and Gensec Bank in November 2000. The Satrix FINI ETF and the Satrix INDI ETF were both introduced on 16th October 2002, the Satrix RESI fund was traded since 12th April 2006. The New Gold and New Rand ETFs created by ABSA Capital were listed on 2nd November 2004 and 26th June 2006 respectively. The Satrix SWIX Top 40 started trading on 12th April 2006. The Itrix FTSE 100 and Itrix DJ Euro Stoxx 50 both commenced trading on 10th October 2005. The most recent entrant into the market is the Satrix Dividend Plus (DIVI) launched on 30th August 2007. Appendix A.1 provides a list of traded SA ETFs as well as their major characteristics as on 31st August 2007.

2.5.3.2. SATRIX ETFs

Satrix is a partnership between the JSE, Gensec Bank and financial services group CorpCapital. Satrix securities are listed as Collective Investment Schemes that replicate total returns of a particular index where all dividends received from companies in the underlying index are paid to Satrix shareholders at the end of each quarter. By holding a portfolio of listed securities that exactly replicate the index constituents, they provide the same returns as would be received if the investor had directly bought shares in each company constituting the index, but without incurring the duplicated transaction costs.
Satrix ETFs do not charge any management fees or other advisory and ongoing costs, and hence investors incur the same brokerage and other JSE transaction costs as with any other listed securities. The income Satrix earns from scrip lending activities cover the costs of running the funds.

The price of Satrix securities can be obtained from any media publication that reports the JSE prices on a daily basis. All settlement, registration, recording and guarantee of trade are done through the normal JSE market systems. Satrix Funds are subject to the same regulations, reporting and compliance requirements as those of any listed company on the JSE. Satrix Funds are also registered as Collective Investment Schemes and is therefore subject to the regulation of the Financial Services Board.

The most popular Satrix product is the Satrix 40 ETF\textsuperscript{14}. It tracks the FTSE/JSE Africa Top 40 Total Return Index (the Top 40 Index) and hence replicates the total performance (capital plus DY) of the top 40 companies listed on the JSE. The top 40 companies account for 95% of the trading on the JSE and spread over a wide spectrum of industries including resources, industrial, retail, telecommunication and financials. The other Satrix products that track JSE sector sub-indices are Satrix INDI, Satrix FINI and Satrix RESI. These three ETFs are described in details in Chapter Three. A relatively new Satrix ETF is the Satrix SWIX Top 40. This product has only been listed on the JSE in early April 2006. It provides a low cost way to invest in the FTSE/JSE Africa Shareholder Weighted Top 40 Total Return Index (the SWIX Index), which is a less volatile basket of Top 40 Shares. The latest Satrix DIVI fund tracks a SA value-style index - the Dividend Plus Index.

2.5.3.3. ITRIX ETFs

The Itrix ETFs are launched by the JSE and Deutsche Bank. They provide a convenient way to gain offshore diversification by investing through a local broker and receive settlement guarantees of the JSE. The current two Itrix ETF products on offer are Itrix DJ EURO STOXX 50 and Itrix FTSE100.

\textsuperscript{14} http://www.satrix.co.za/satrix_40/40/index.jsp.
One of the most widely-used European Blue Chip Indices, the Dow Jones Euro STOXX 50 Index, underlies the Itrix DJ EURO STOXX 50 ETF. It is a free float MV weighted index, containing 50 liquid blue chip stocks from countries within the Eurozone.

The Itrix FTSE100 ETF tracks the FTSE100 Index, which is a free float MV weighted Index, containing the 100 largest highly liquid United Kingdom blue chip stocks listed on the London Stock Exchange.

2.5.3.4. ABSA ETFs

The third SA ETF provider is ABSA Capital. The ABSA NewRand ETF aims to replicate the New Rand index. It is an ABSA-compiled index\(^\text{15}\) consisting of the 10 SA stocks selected from the Top 40 Index that have the closest correlation to the rand/dollar exchange rate over the previous two-and-a-half years. This rand-hedge ETF protects its investors against rand’s volatility and possible depreciation against the dollar.

The New Gold ETF is first listed in November 2004. It gives an investor exposure to gold prices without undue exposure to company-specific risks inherent in investing in individual gold shares. In addition, investors can gain exposure to a US dollar-denominated commodity with no exchange control implications.

2.6 Summary and conclusion

In conclusion, despite the implication of the CAPM and the efficient market hypothesis that it is impossible for an average investor to outperform the market portfolio, practitioners and academics have never ceased to seek ways to achieve higher returns. Style anomalies, style investing and portfolio optimisations are active strategies exploring the flaws of CAPM to beat the market. On the other hand, ETFs are investment vehicles that facilitate passive outperformance over the market by minimising costs.

\(^{15}\) This index is created by ABSA Capital and provided and calculated by the FTSE and JSE.
Sharpe’s return-based style decomposition has provided an effective way to blend the two. Since a large proportion of an active fund’s returns is due to its investment style, the return-based style decomposition enables investors to replicate the superior performance of active funds using passive and low cost indices (including ETFs) by providing an easy way to estimate a fund’s ‘true’ investment style.
3. Data

3.1 Introduction

This chapter introduces the dataset that is analysed in Chapters Five and Six, and from which the indices in Chapter Four are derived. The dataset used in this thesis consists of (1) share price and firm-specific attribute data of JSE listed shares, (2) total returns of JSE published indices, and (3) portfolio return data including total returns on SA Satrix Exchange Traded Funds (ETFs), domestic equity unit trusts and hedge funds. These three datasets are described separately in this chapter.

Most of the analysis in this thesis is performed in Microsoft Excel, while the Econometrics Views (E-Views) statistical software package is used for more detailed regression analysis.

The remainder of the chapter is set out as follows: Section 3.2 discusses the stock price and firm data with details provided of how they are used to compute the total share returns. Section 3.3 presents the Johannesburg Stock Exchange (JSE) indices that are employed in the later chapters for CAPM, APT and active portfolio optimisations. Section 3.4 describes the portfolio returns dataset which is utilised to perform the style-return analysis. Finally, Section 3.5 summarises and concludes.

3.2 Stock-returns and firm-attribute data

Monthly price history and firm attribute data are collected from the DataStream International (DataStream). A set of records is updated every month and available from the Finance Research Laboratory at the University of Cape Town. This dataset is subsequently used to construct the JSE style indices in Chapter Four and perform the return-based style decomposition and active portfolio optimisation analysis in Chapters Five and Six.
DataStream has conducted accuracy checks on their data, for example for outliers. Random spot checks are applied to ensure correct downloading of the DataStream dataset. In addition, book value and earnings related data are compared against those from I-Net Bridge (I-Net) and ShareData to ensure consistency.

3.2.1. Data description

The population under consideration comprises 162 shares listed on the JSE during the period 1st January 1998 to 31st December 2006. From this share population, a sample constituting the largest 100 shares (as ranked by market capitalisation) of the FTSE/JSE Africa All Share Total Return Index (the ALSI) is used for index construction. These securities are almost exclusively members of the JSE Large and Mid-Cap indices, thus all of the indices constructed consist of reasonably liquid shares. To calculate the past 12-month return (MOM) and the 1-month forward returns, the total return series is required over the period 1st January 1997 to 1st February 2007. Therefore, monthly price history and firm attribute data are collected over this period for constructing the style indices.

As on 1st March 2007, the DataStream database explored had 162 SA companies. Not all companies listed at the beginning are still trading at the end of the period examined. Companies that delisted during the period of investigation are assigned weights of 0 when constructing the style indices immediately after their delisting. New firms are added on their respective listing dates.

The first month of the sample has returns data on only 120 companies. Thereafter, the sample size increases until reaching a maximum of 162 companies. Appendix B.1 shows the number of companies in the sample in each month over the period 1st January 1997 to 1st February 2007.

Figure 3.1 graphically compares the total market capitalisation (MV) of the JSE against that of all the companies collected for this thesis in each month over the period under consideration. The total MV of the 100 largest JSE listed companies is also plotted. The MV of the collected sample on average totals around 64% of the JSE total MV in each month, which indicates that our sample is fairly comparable with the
market. Furthermore, the top 100 shares are very representative of the collected sample, since on average they make up 96% of the total sample MV.

**Figure 3.1: Total market capitalisation (MV) of the JSE, collected sample and top 100 listed firms**

The (overlapping) area chart displays the total MV over the period January 1998 to December 2006 of the JSE, the sample of 162 companies collected for this thesis and the top 100 largest firms. The data are extracted from DataStream International, available at the University of Cape Town.

3.2.1.1. **Share return data**

The monthly total return indices computed by DataStream International are utilised throughout this thesis. A return index (RI) takes into account both changes in share prices (i.e., capital gains) and any distributions (i.e., dividends). Whenever a company announces a distribution, the dividend declared is assumed to be reinvested in the share in question on the ex-dividend date. In other words, the RI is equivalent to share price adjusted for dividends.

The ‘returns’ entry obtained from DataStream is 1-month trailing return. For instance, share return obtained on 1st January 1998 is the total return earned by investing over the period 1st December 1997 to 31st December 1997.
3.2.1.2. Firm-specific data

In addition, to construct the style indices nine firm-attribute data entries are required, namely: MV, earnings yield (EY), earnings per share (EPS), total earnings (EAR), total book value (BV), total cashflow (CF), dividend (DIV), sales (SALE) and book to market value ratio (BTMV). Appendix B.2 defines all the above accounting items used by DataStream. On a particular date, all of these entries apply to the previous one month period. For instance, MV on 31st December 2006 applies to the period 1st December 2006 to 31st December 2006.

The MV, EAR, price earnings ratio (PE), EPS, CF, SALE and DIV and the market to book value ratio (MTBV) figures of each share are obtained directly from DataStream. The EY entry is calculated as the reciprocal of PE, and BTMV is computed as 1/MTBV.

There are three earnings per share entries in the DataStream database, namely: EPS, EPS1 and EPS2. This thesis uses EPS, which is derived from the latest published accounts regarding the last financial year and updated for interim results. EPS1 is the average of all of the earnings per share forecasts supplied by analysts for the current (not yet reported) financial year of the company. EPS2 is a mean of all the earnings per share forecasts for the next financial year of the company. The denominator used is the weighted average number of shares in issue.

The DataStream EAR figures are headline earnings for the JSE listed companies, which have been checked for accuracy against those obtained from I-Net and ShareData. For dual listed companies16, however, the earnings are basic earnings which are earnings before extraordinary items. All EAR values reflect the published results for the last financial year and are updated for announced interim period earnings.

The entry labelled BV is equivalent to net tangible asset (NTA) per the DataStream definition, which is equivalent to the net asset value (NAV) obtained from I-Net17. It

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16 Including BHP Billiton (JSE), OM, Richemont, SAB, Brait SA, (JSE), Dimension Data Holdings (JSE), Liberty International (JSE), Anglo American (JSE) and Investec (JSE).
17 The accounting entries of I-Net do not come from I-Net it comes from BFA McGregors.
is calculated as shareholder's capital (excluding minority interests, exclude preference shares) minus intangible assets. DataStream obtains a company's equity values (i.e., shareholder's capital, minority interests and preference shares) from its published and consolidated financial statements. Random spot checks have been conducted by the author to ensure data accuracy.

DataStream values a company's non-investment tangible assets (e.g. equipments) at their book values, and records its financial investments, such as bonds and shares held, at their market values. Due to the fact that financial companies (including banks, investment banks, financial intermediaries and insurance companies) normally have a large proportion of their assets laid down in financial investments, there is an inherited inconsistency in the NTAs of financial companies (for which most assets are valued at market value) and non-financial companies (for which most assets are valued at adjusted book value).

Furthermore, the value reporting mechanism can differ substantially for different insurance companies, such as the value placed on current in force business and the treatment of policyholders' reasonably expected future benefit enhancement. In addition, short-term general insurers (e.g. Sanlam) and long-term life insurers (e.g. Liberty) can again adopt fairly different accounting and actuarial valuation techniques. DataStream has made an attempt to adjust the asset values of insurance companies to ensure consistency and comparability of results where possible. As a result, the post-adjustment BVs obtained from DataStream differ from those obtained from the ShareData database, which are unadjusted values.

It is noted that there are two similar entries in the DataStream database, namely, the MTBV ratio and the price to book value ratio (PTBV). MTBV divides a company's market value by its net book value (i.e., MTBV= MV/NTA). The PTBV is calculated as the price over the per share book value (i.e., PTBV=P/APSH). The numbers of shares used are those in issue at the end of the appropriate financial year. This thesis uses the MTBV ratio.

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18 DataStream does not have BV for certain financial companies, including: BHP Billiton Old Mutual, Richemont, Liberty International, Investec, Hosken consolidated investments, Tradehold, Avgold, Amalgamated Beverages, African Life Assurance, Softline, Randgold Exploration, Chemical Services and Coronation, Whereas I-Net have excluded insurance companies in their dataset completely.
3.2.2. Adjustments to the stock-returns and firm-specific dataset

Survivorship bias is ignored as a problem in this thesis, as investors are only able to invest in listed companies. Look-ahead bias occurs when predictor variables are used which were unknown to market participants at the time they are dated in the dataset. DataStream is free of look-ahead bias, since its EAR and BV related data entries change only on the announcement date. For instance, the EPS and BV entries normally stay the same till the next financial statement publication date, but are updated according to the interim reports as far as possible. In contrast, MV, EY and BTMV figures are updated on a daily basis.

3.2.2.1. Incomplete entries

All the shares with relevant entries missing in a particular month are ignored when constructing the style portfolio for that month. For instance, if EY data are missing for a particular month, the share will be excluded for the month when calculating value-style indices; while it will be included when calculating size-style indices, since EY data are irrelevant in the latter calculation. MV and forward return, however, are relevant in all style-index constructions, therefore shares with incomplete entries in these two fields are excluded from the sample for that month for the calculation of all style indices.

3.2.2.2. Thin trading

The ‘thin trading’ problem was originally observed when trading has not occurred for a while before month close. In which case, not only the share price at the end of the month, but also the share’s total return calculated for the month may be at an inaccurate level\(^{19}\).

Numerous studies have pointed out that the ‘thin trading’ problem may give rise to distorted results. Fama (1965) and Fisher (1966) reported that ‘indices constructed from the prices of thinly traded shares show spurious positive serial correlation in the index returns and downwardly-biased estimated variances of index returns’. Dimson (1979) noted that the thin trading problem results in the underestimation of the systematic risk of non-traded shares. This is because on a particular day, the prices of

\(^{19}\) Since the share price represents the outcome of a transaction which occurred prior to the month end, so the information content of the price does not relate to the current period but is rather a carry-over from a prior event, according to Janari (2004).
non-traded shares will not change which gives the false appearance that these shares do not move with the market (Van Rensburg and Robertson, 2002).

The less frequently traded a share is, the greater the mismatching between the true value of the share and the monthly closing price. Therefore the ‘thin trading problem’ has a more severe effect on analysis performed on smaller stock exchanges, such as the JSE.

Two methods have been primarily adopted in prior literature to deal with thin trading: to filter shares by (1) their trading volume, or (2) their MV. When trading volume data are used, a turnover ratio is often calculated and only shares with turnover ratio of greater than, say 0.01%, are included in the final dataset (Van Rensburg and Robertson, 2002).

The second method is employed in this thesis to solve the thin trading problem, and hence MV figures are adopted as the filtering criterion. All of the available shares are first ranked by their cross-sectional MV; as thin trading is likely to be more prevalent among smaller companies, the 100 companies with the highest MV (and with forward return entries available) are selected to form the ‘non-thinly-traded’ data sample on a monthly basis. This filtered sample comprising 100 shares is subsequently used in Chapter Four to construct the style indices of each month.

It is noted that simply filtering the sample using MV and excluding the smaller companies would be especially problematic for studies that test the relationship between share returns and the size-style anomaly (Van Rensburg and Robertson, 2002). This, however, is not a concern for this thesis. After removing the smaller companies on the JSE, the FTSE/JSE Africa Small Cap Total Return Index (the Small Cap Index) is included in the set of size-style indices for the portfolio replication and creation analysis to capture the performance of small companies and the small size effect.

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20 Through a private discussion with Professor van Rensburg (Head of Finance & Research, UCT.),
3.2.2.3. Data mining

Finally, data mining is a common problem suffered by most empirical investigation on share returns (Lo and MacKinlay, 1990). It refers to the fact that 'if enough methods are tested, one can invariably find some that would seem to be able to produce excess returns in at least some periods; but there is hardly any guarantee that they will work in the future' (Sharpe, 1995). To prevent this study from being subject to the 'data mining' problem, the author deliberately avoids testing an exhaustive set of style anomaly proxies or style-index construction methods. Instead, only those anomalies confirmed and checked against the results of other studies previously conducted in related fields are used. Moreover, the style metrics are not selected ex post on the basis of results, but consist of the most widely used and commonly available firm attributes.

3.2.3. Share returns computation

Two pieces of share price information are necessary to compute the style portfolio returns in Chapter Four. These are the 1-month forward total share returns and the prior 12-month momentum (MOM), both are constructed using the return indices obtained from DataStream.

The returns utilised throughout this thesis are 1-month forward returns, and hence share return on 1st January 1998 applies to the period 1st January to 31st January 1998. The monthly forward total return\(^{21}\) at the beginning of month \(t\) is calculated as:

\[
TR_t = \frac{RI_{t+1} - RI_t}{RI_t}
\]  

(3.1)

Where:

- \(TR_t\) represents the total monthly forward return in month \(t\)
- \(RI_t\) represents the DataStream return index at the beginning of month \(t\)
- \(RI_{t+1}\) represents the DataStream return index at the beginning of month \(t+1\).

MOM stands for the prior 12-month total return, and is computed as follows:

\(^{21}\) Which is in fact the RETURNS entry obtained from DataStream shifted backwards for one month.
\[ \text{MOM12}_t = \frac{R_I_t - R_{I,-12}}{R_{I,-12}} \]  

(3.2)

Where:
\( \text{MOM12}_t \) represents the prior 12 month total return at the beginning of month \( t \)
\( R_I_t \) represents the DataStream return index at the beginning of month \( t \)
\( R_{I,-12} \) represents the DataStream return index at the beginning of month \( t-12 \).

3.3 JSE indices data

Eight published JSE indices are collected over the period 1\textsuperscript{st} January 1998 to 31\textsuperscript{st} December 2006 from I-Net via the Finance Research Laboratory at the University of Cape Town (School of Management Studies). The monthly total return indices are used where available, while capital indices with dividend yield (DY) adjustments are used to compute the total returns if the corresponding total return indices are not available.

The FTSE/JSE Africa Top 40 Total Return Index (J200T, the Top 40 Index) and the Small Cap Index (J202T) are included as size indices. The SA 90-day Banker’s Acceptance Discount Rate (RBAS) series is used as the proxy for the risk-free rate.

3.3.1. Overview of JSE trading system and index series

The JSE is the largest stock exchange in Africa. It was initially established in 1887 to raise financing for the mining industry. In June 1996, an electronic trading platform, the JSE Equities Trading (JET) electronic system, was introduced, which was subsequently replaced with SETS in May 2002. Electronic clearing and settlement is conducted through the STRATE (Share Transactions totally Electronic) system.

On 24 June 2002 the JSE launched the FTSE/JSE Africa Index Series (new indices) to replace the JSE Actuaries Index Series (old indices). One of the main advanced design features of the new indices in comparison to the old indices is the use of free float adjusted constituent weightings instead of full MV.
Moreover, the new indices reclassify securities into sectors following the FTSE Global Classification System. The FTSE/JSE new indices are chain-linked to the old indices to achieve a smooth time series of index values\textsuperscript{22}.

Although the new indices were only introduced in June 2002, PeregrineQuant backward calculated their historical index values for the period July 1995 to December 2001. In this thesis, these backward calculated new index values are obtained from I-Net and are used as far back as possible. Then the chain-linked values of the old indices that are the closest logical counterparts of the new indices are used. Total return indices are available for the new FTSE classification. On the other hand, only capital indices are available for the old indices, therefore D\textsubscript{Y}s are combined with capital index values when calculating total returns.

Since the majority of data used in this thesis are the new indices, Section 3.3.2 below focuses on the construction of the new indices.

3.3.2. Index construction

To determine an index's constituents, liquidity screening is first applied to all the companies listed on the JSE to determine their eligibility in an index. Liquidity reviews occur annually each December, therefore shares suspended on liquidity basis will be excluded from the index calculations for a full year\textsuperscript{23}. A stock must be sufficiently liquid in order to be included in the indices. That is, it has to be traded at least 0.5\% of its free float per month in 10 out of the 12 months prior to an annual review. After becoming an index constituent, a security must turn over at least 0.5\% of their free float shares per month in at least eight out of the 12 months prior to an annual review to stay as a constituent. Therefore, the JSE indices have already been adjusted for the thin trading problem.

Secondly, if the companies are ranked, constituents are selected by the ranking of their full MV. The rationale is to ensure that the indices remain representative of the

\textsuperscript{22} For example, J200 (FTSE/JSE Africa Top 40) was rebased to the value of F101 (ALSI 40) so that the start of business value for the Top 40 on Monday the 24\textsuperscript{th} of June, 2002 was equal to the close of business value of the ALSI 40 on Friday the 21\textsuperscript{st} of June. For those indices where no counterpart existed in the JSE Actuaries Indices the figure has been rebased at 1000 as at the close of business on 21 June, 2002.

\textsuperscript{23} Until the next review in the following December.
market; large companies with low free float are still more representative of the market as a whole and hence cannot be excluded.

Thirdly, once the constituents of an index have been determined, free float weightings are applied to calculate the index values. Free-float market capitalisation refers to that portion of a listed company's share capital commonly accepted as being in general circulation (i.e., the amount of shares freely available to investors) and not firmly held as part of the corporate control or strategic alliance structure. It is obtained from the full market capitalisation by applying the free float banding percentage\textsuperscript{24}. Dual listed stocks are still eligible for inclusion in the new indices. The use of free float weightings benefits investors by presenting a more representative view of what is available in the market. It allows investors to track an index more closely and invest accordingly.

The following general formula applies to the calculation of all of the JSE indices collected:

\[
I_{i,t} = \sum_{k=1}^{n_i} \frac{MV_{k,t}}{B(t)}
\]  

(3.3)

Where:

- \( I_{i,t} \) represents the value of index \( i \) at time \( t \)
- \( MV_{k,t} \) represents the free float market capitalisation of index constituent \( k \) at time \( t \), calculated by multiplying the most recent share price by the number of shares after the free float weighting has been applied
- \( n_i \) represents the number of index constituents at time \( t \)
- \( B(t) \) represents the latest index divisor, an arbitrary number chosen at the starting point of the index, which is adjusted when capitalisation amendments are made to the constituents of the index, allowing the index value to remain smooth over time.

\textsuperscript{24} Because the number of free float shares varies regularly, FTSE and the JSE use a banding structure to round off the free float figure upwards. It serves the dual purpose of insulating the market from minor fluctuations in available free float and reflecting the change only when it has a significant impact on share availability. E.g. A constituent's free float will only be changed if its actual free float is more than 5 percentage points above the minimum or 5 percentage points below the maximum of an adjacent band.
The constituents of the new indices are reviewed quarterly, in March, June, September, and December. An existing constituent will be deleted if it fails to trade at least 0.5% of its shares in issue per month for more than eight of the 12 months. A 'one percent rule' states that the number of shares in issue for each company is only amended when the total shares in issue changes by more than 1% on a cumulative basis. This prevents a large number of insignificant weighting changes upon index review.

Where a constituent is the subject of a merger, restructuring, complex take-over or split, FTSE and the JSE will re-determine the industry sector classification of the resulting constituent(s) and the eligibility for all indices will be reviewed. The rules for inserting and deleting companies are designed to provide stability and ensure representativeness of the indices; therefore insertion and deletion of constituents of the indices may be done at any time that a significant corporate event occurs.

3.3.3. Computation of index total returns

Finally, the capital index $I_{i,t}$ calculated above is adjusted for DY to give total return indices (TRIs). TRIs reflect the total return on the underlying portfolio by combining both capital performance and the reinvested income on the ex-dividend date.

Where total return index is available (indicated by an index code with suffix T, which is the case for most new indices) the monthly total return used for further analysis in Chapters Four to Six are calculated as:

$$TR_t = \frac{TRI_{t+1} - TRI_t}{TRI_t}$$

(3.4)

Where:
- $TR_t$ represents the monthly forward total return of the index in month $t$
- $TRI_t$ represents the I-Net total return index at the beginning of month $t$
- $TRI_{t+1}$ represents the I-Net total return index at the beginning of month $t+1$.

On the other hand, the adjustment for DY is only an approximate figure for all of the indices which only have the capital index values (most old indices). DY given by I-
Net is calculated using current price and 12-month trailing dividend. Therefore total return in month $t$ is computed as:

$$TR_t = \frac{I_{t+1} - I_t + DY_{t+1}}{I_t} \times 12$$  \hspace{1cm} (3.5)

Where:
- $TR_t$ represents the total return in month $t$
- $I_t$ represents the I-Net capital index at the beginning of month $t$
- $I_{t+1}$ represents the I-Net capital index at the beginning of month $t+1$
- $DY_{t+1}$ represents the I-Net annual trailing DY adjustment at the beginning of month $t+1$.

3.3.4. Description of selected indices

The eight selected indices each have its own specific characteristics. The Small Cap Index and the Top 40 Index are examined along with the constructed portfolio return series in Section 4.2 of Chapter Four as potential size style index candidates; while the FTSE/JSE Africa Dividend Plus Index (the Dividend Plus Index) and the SA RAFI Index are explored as benchmarks in Section 4.3 of Chapter Four to assess the performance of the style indices constructed. The ALSI is used throughout Chapters Four and Six as a market proxy to carry out the single factor CAPM regressions. Similarly, the FTSE/JSE Africa Financial-Industrial Index (FINDI) and the FTSE/JSE Africa Resource 20 Total Return Index (RESI) are adopted as factor proxies for the two factor APT model. The FTSE/JSE Africa Financial 15 Total Return Index (FINI), the FTSE/JSE Africa Industrial 25 Total Return Index (INDI) and RESI are integrated with the ETF return dataset in Chapter Five to serve as the independent variables for the return-based style decomposition. The FTSE/JSE Africa Shareholder Weighted Top 40 Total Return Index (the SWIX Index) serves as the core portfolio for the computation of tracking errors in Chapter Six.

The constructions of the Top 40 Index and the Small Cap Index are described in detail in Chapter Four. The other six JSE indices are described below.
3.3.4.1. ALSI

The ALSI is coded J203T (T for the Total Return Index) on I-Net. This is an equity index intended to reflect the performance of the SA ordinary share market as a whole. It comprises the top 99% of all eligible listed companies on the JSE ranked by full MV, whereas the index constituents are weighted by free float MV.

3.3.4.2. FINDI

FINDI is short for the FTSE/JSE Africa Financials and Industrial Index. It is coded J250 after the adoption of the FTSE industrial classification system on June 2002; before that, the code was CI21X (Financial-Industrial) under the JSE Actuarial Classification. It comprises all of the companies which are constituents of the ALSI excluding those classified in the resources economic group.

3.3.4.3. RESI

Similarly, RESI is short for the FTSE/JSE Africa Resources 20 Total Return Index. It is coded CI11X (Resources) and J210 respectively before and after the adoption of the FTSE industrial classification system on June 2002. RESI consists of the 20 major resource companies listed on the JSE, based on free float MV. The resources based stocks include mining companies, mining holding companies, and mining finance and exploration companies.

3.3.4.4. FINI

FINI stands for the FTSE/JSE Africa Financial 15. Its index code was CI24X, which is later replaced by J212T under the new FTSE classification. Total return index is available for the new classification; whereas only capital index is available for the old code. This index comprises the top fifteen companies which are constituents of the Financial economic group ranked by full MV.

3.3.4.5. INDI

INDI is short for FTSE/JSE Africa Industrial 25 (J211T). It consists of the twenty-five largest companies by full MV which are constituents of either the Basic Industrial or General Industrial economic groups.
3.3.4.6. SWIX

The SWIX Index contains the top 40 companies on the JSE after liquidity screening and weighted by their free float adjusted MV. But weighting of the 40 constituent companies are adjusted downwards for foreign shareholdings relative to the normal Top 40 Index. In addition, the SWIX is adjusted for cross-holdings and strategic holdings. The impact is to reduce the net weightings of resources and dual listed stocks by approximately half and increasing the weightings of financial, industrial and telecommunication shares relative to the Top 40 Index.

The offshore investors react to changes in the rand exchange rate and exogenous factors affecting global stock markets, which can result in sharp price fluctuations from time to time. The SWIX Index, to a significant extent, eliminates the impact of such fluctuations on the local index. Therefore, the major benefit of re-weighting of the SWIX Index is the reduced volatility in index returns and more smoothed performance over time.

This index is regarded by SA asset managers and institutional investment advisers as the primary benchmark for the performance of the SA equity market.

3.3.4.7. RBAS

RBAS is a time series of per annual effective interest rates, reflecting the rate of return earned on cash equivalents (i.e., risk-free assets) with less than three months to maturity. It is not a JSE index but is employed to represent the risk-free rates when performing the CAPM and APT risk-adjusted regressions [Capaul et al. (1993), and Gilbertson and Vermaak (1982)].

RBAS rates obtained from I-Net are per annual forward rates, therefore the rate on 31st January 1998 applies to the period 1st February 1998 to 31st January 1999. This per annual rate is divided by 12 to obtain the monthly forward risk-free rates used in all later calculations.

3.4 Portfolios data (Indices and funds)
Portfolio data refers to the return series on (1) the JSE ETFs, and (2) representative domestic equity funds and broad indices (including unit trusts and hedge funds). The operation and development of ETFs are described in Chapter Two. The style indices constructed in Chapter Four, combined with the ETF returns dataset (merged with their corresponding underlying JSE sector sub-indices described in Section 3.3.4), are used to carry out return-based style analysis on domestic equity funds and broad indices in Chapter Five.

3.4.1. ETF

As described in Chapter Two, ten ETFs currently exist on the JSE. Appendix A.1 summarises the details of these ETFs, such as their names, issue companies and starting dates. The ETF price histories (capital index) and the DY figures since their inception are downloaded from I-Net, the total returns are subsequently computed.

Three ETFs are chosen to serve as the independent variables for the return-based style decomposition, each of them represents a distinct sector on the JSE. They are Satrix INDI, Satrix FINI and Satrix RESI.

3.4.1.1. Satrix INDI

Satrix INDI25 tracks the price performance and DY of INDI, which comprises the top 25 industrial companies listed on the JSE. The companies included have their major operations in industrial materials and construction, industrial goods and services, personal and household goods, consumer goods, healthcare, retail goods, media and telecommunications. Some of the firms that comprise the largest portion of the index are Richemont, SAB Miller, MTN Group and Naspers to name a few. It also contains a number of middle-capitalisation companies which do not qualify for the Top 40 Index. This has the benefit of spreading investors' exposure to some of the better performing smaller firms on the JSE.

3.4.1.2. Satrix FINI

Satrix FINI replicates the FTSE/JSE Financial 15 index (capital plus DY) by holding the exact weighting and number of shares that constitute this index. Any dividends that are paid by the top 15 financial companies are paid out to Satrix FINI shareholders at the end of each quarter.
FINI provides a focused portfolio of shares in the financial sector listed on the JSE. The components of FINI are a mixture of banking, general financial services and both life and non-life insurance. The top holdings of the index are Old Mutual, Standard Bank, First Rand Bank and Remgro.

3.4.1.3. Satrix RESI

The Satrix RESI provides investors with a tradable means of investing in the top 20 resource shares listed on the JSE. The underlying index is the FTSE/JSE Resources 20 Index. The Satrix RESI provides a focused exposure to the resources sector of the JSE, which includes major locally listed global mining holding companies, gold mines, platinum, uranium and base metal mines, mining resource, mining finance and exploration companies. This ETF enables investors to participate in trends in global commodity cycles and provides a rand hedge facility.

This product has only been listed on the JSE in early April 2006 and available on the Satrix Investment Plan platform from 1st June 2006.

3.4.2. Domestic equity funds and indices

This section introduces the domestic equity dataset utilised in Chapter Five to carry out the return-based style decomposition. The dataset includes total returns of unit trusts, unit trust indices and hedge fund indices that invest exclusively on the JSE. The list describing the indices and funds used, their inception dates, as well as the shorthand notations, are displayed in Appendix D.1.

3.4.2.1. Unit trust funds and indices

For unit trusts, three broad sector indices are examined, namely domestic equity general (DOEQ), domestic equity growth (DOEQGR) and domestic equity value (DOEQVL). In addition, 11 domestic equity unit trust funds are selected to form a parsimonious representation of the SA unit trust industry. This is not to say that their performance is necessarily representative of that of the remainder of the unit trust industry. In fact, the performance of the unit trusts chosen is likely to be at least as good as that of the remaining funds, and may well be superior. If the latter is the case, such funds would be of particular interest, since an astute investor could identify and invest in such funds.
The total returns on domestic equity unit trust indices and funds are obtained from the I-Net database for the period 1st January 1998 (or since fund inception) to 31st December 2006. The total return indices are available from I-Net till 30th November, 2005; from thereon, the close price (CL) and DY are used together to approximate the total returns. The total returns obtained and computed are per month effective. I-Net reports the trailing total returns, therefore the total return figure on 31st July 1999 applies to the prior one month, which is 1st July 1999 to 31st July 1999.

The indices and funds chosen operated at any time between 1998 and 2006. To ensure reliability, the availability of historic returns is checked and only funds with at least 36 months of return history are retained.

### 3.4.2.2. Hedge fund indices

Four hedge fund sector indices are obtained and analysed, they are Single Manager Composite (COMP) Index, Long Short Equity Index (LSE), Market Neutral and Quantitative Strategies Index (MKN), and Fund of Funds Index (FOFs). The hedge fund dataset is obtained from HedgeFund Intelligent for the period January 2004 to December 2006.

### 3.4.3. Total return computation of portfolio data

Whenever a fund provides a dividend distribution, the money is reinvested in the fund in question. Therefore total returns of the fund are calculated in the same way as those for the capital index with DY adjustments in Section 3.3.3.

Where total return index is available, the monthly total returns are calculated as:

\[ TR_t = \frac{TR_{t+1} - TRI_t}{TRI_t} \]  

(3.6)

Where:
- \( TR_t \) represents the monthly forward total return of the index in month \( t \)
- \( TRI_t \) represents the I-Net total return index at the beginning of month \( t \)
- \( TRI_{t+1} \) represents the I-Net total return index at the beginning of month \( t+1 \).
On the other hand, for all of the indices where total return index are not available, the monthly close price (CL) and DY are used to calculate the capital index, which is subsequently adjusted (approximately) for DY. The total return in month $t$ is computed in the same way as that of Equation (3.5).

### 3.5 Summary and conclusion

The chapter introduces the three dataset used in later chapters, namely (1) the share returns and firm-specific attribute dataset, (2) the JSE sub-indices and (3) the portfolio returns data on JSE ETFs and domestic equity funds. The period under investigation is 1st January 1998 to 31st December 2006. Total returns are computed to take into account of both capital gains and dividend distributions.

The returns and firm attributes obtained fromDataStream are checked for consistency and accuracy against those from the 1-Net and ShareData database. The returns on JSE indices, ETFs and domestic equity unit trusts are retrieved from 1-Net, whereas the hedge fund indices are provided by HedgeFund Intelligent.

Shares and indices, with missing relevant entries, or which do not have long enough return histories are excluded from relevant analysis, though not removed from the sample population. Thin trading is adjusted for by including only the top 100 shares ranked by MV when constructing the style indices.
4. Investigating Candidate Style Indices

'Style index solutions address two distinct needs. The first is for exhaustive style indices that can effectively form the basis for index funds and derivatives, providing broad, cost-efficient exposure to a certain style segment. The second need is for narrow, style-pure indices that provide a pure style return series, and serve as the basis for style-concentrated investment vehicles or “style spread” products.'

- Blitzer and Dash (2006)

4.1 Introduction

This chapter aims to construct a set of style indices for each of the three investment styles (namely: size, value and momentum) that have been identified to produce excess returns on the Johannesburg Stock Exchange (JSE) (Van Rensburg, 2001). For each investment style, the best performing indices are selected and subsequently employed to compute the style decomposition on domestic equity fund returns and form the basis for active portfolio construction in Chapters Five and Six.

The international literature discussed in Chapter Two has outlined several methods for constructing style indices in different countries. However, no exhaustive and systematic research has yet been conducted regarding the construction and operation of style indices on the JSE. The findings of Van Rensburg (2001) are consulted as a guiding philosophy to develop the style-index construction methods used in this chapter.

Van Rensburg (2001) suggested that ‘three style-based risk factors can form a parsimonious representation of the style-based risk on the JSE’. Cluster and principle component analysis were further conducted to identify firm-specific attributes that can serve as the most suitable proxies for these style factors. He stated that ‘earnings yield (EY) represents the value cluster, market capitalisation (MV) represents the size cluster and twelve-month past positive returns (MOM) represent the momentum cluster’. All three style-anomalies persist after risk-adjustment using the two factor APT model developed by Van Rensburg and Slaney (1997).
Other studies, however, have suggested some alternative proxies for the style factors. For instance, Arnott (2005) proposed total earnings (EAR), total book value (BV) and book to market value ratio (BTMV) for the value style; prior 12-month returns excluding the latest month’s return was recommended to proxy for the momentum style. As a result, this thesis adopts a comprehensive set of firm-specific attributes as the style proxies in the index construction for each of the three investment styles respectively. The aim therefore, is to examine the effectiveness of these alternative proxies and to identify the best performing ones. All results are rebalanced monthly while some quarterly rebalanced portfolios are explored in the appendices. The average turnover percentages are calculated to gauge the impact of transaction costs on the net style-portfolio performance. A list of all of the style indices constructed in this chapter together with brief index descriptions is presented in Appendix C.1.

The remainder of this chapter is set out as follows: Section 4.2 describes the data and methodology used to construct the size-style indices. In conducting the comparison between the constructed style indices, a number of conventional risk and return measures are calculated, such as mean, standard deviation and the Sharpe Ratio. CAPM and two factor APT regressions are conducted to obtain the Treynor Ratio and excess returns (i.e., alphas) for each index. Two best performing size-style indices are recommended based on a balanced consideration of average cumulative returns, excess returns, the Sharpe Ratios and portfolio diversifications. Sections 4.3 and 4.4 perform the same calculations and analysis for the value- and momentum-style indices respectively. Section 4.5 compares the performance of constructed and existing JSE style indices. Section 4.6 investigates the correlations among some of the selected style indices. Finally, Section 4.7 summarises and concludes.

4.2 Style indices: size

4.2.1. Data and methodology

While most studies on the US stock markets recognise a prominent ‘small firm effect’ [Fama and French (1992) and Banz (1981)], no conclusive evidence is established as to whether the ‘large capitalisation’ or ‘small capitalisation’ investment style
consistently yields higher returns. Therefore, both large and small capitalisation size-indices are investigated to identify the potential best performers.

4.2.1.1. Index construction

Van Rensburg (2001) proposed the use of MV as a proxy to construct the size-style portfolios. If a size-style investor believes that companies with larger MV generate higher returns on the JSE, his size-style portfolio in a particular month should only consist of those shares with relatively high MV at the beginning of that month. On the other hand, if a size-style investor believes in the ‘small firm effect’, the FTSE/JSE Africa Small Cap Total Return Index (the Small Cap Index) should provide a reasonable approximation of his size-style portfolio returns.

In total, five monthly rebalanced size-style indices are compared over the period 1st January 1998 to 31st December 2006. Among these five indices, three are constructed from the monthly share return data described in Chapter Three, while two are published JSE sub-indices, namely the Small Cap Index and the FTSE/JSE Africa Top 40 Total Return Index (the Top 40 Index). Portfolio weightings are calculated using monthly rebalancing frequency.

All the shares with relevant entries missing in a particular month are excluded when constructing the style portfolios of that month. For instance, if EY data are not available for a particular month, the share will be excluded for the month when calculating the value-style indices. It will, however, be included when calculating the size-style indices, since the EY data entry is irrelevant in the latter calculation. It should be noted that MV and forward returns are relevant in the constructions of all style indices, therefore shares with incomplete entries in these two fields in a particular month are always removed from the sample of that month for the purpose of index constructions.

A set of equally weighted (EW) indices are calculated to represent the (small) size investment style comprising the largest 100, 50 and 30 shares respectively.
**Monthly rebalanced equally weighted size index constituting top \( N \) shares by MV (EW(size)N)**

In order to calculate the EW(size)N Index, all of the shares obtained are first ranked by MV in a descending order. The equally weighted (EW) arithmetic average of the total returns of the top \( N \) shares by MV in each month gives the monthly total return of the size-style portfolio with \( N \) index constituents in month \( t \). The computation is as follows:

\[
R_{EW(size)N,t} = \frac{1}{N} \sum_{i=1}^{N} R_{i,t}
\]

(4.1)

Where:
- \( R_{EW(size)N,t} \) represents the monthly return on the EW size-style portfolio in month \( t \)
- \( N \) represents the number of constituents in the EW size indices, \( N \) takes on values of 100, 50 and 30
- \( R_{i,t} \) represents the monthly forward return of the \( i \)th index-constituent in month \( t \).

**Quarterly rebalanced EW size index constituting top \( N \) shares by MV (EW(size)NQ)**

Style portfolio returns are also calculated under a quarterly rebalancing strategy, where the index components are updated every quarter. In other words, a style portfolio is formed at the beginning of each quarter using the shares' MV values at that time. Each constituent’s position is then held through the following quarter. The total returns of a given size-style portfolio are still computed at the end of each month. The quarterly rebalanced indices are indicated by a suffix ‘Q’ attached to the index names of the corresponding monthly rebalanced portfolios, and the relevant results are presented in the appendices.

**The Top 40 Index and the Small Cap Index**

As described in Chapter Three, both indices are obtained from I-Net Bridge (I-Net) over the period 1\textsuperscript{st} January 1998 to 31\textsuperscript{st} December 2006. Both indices are published as
Investigating Candidate Style Indices 4: 5

arithmetic mean total return indices with each constituent weighted by its free float adjusted MV.

The Top 40 Index constitutes the forty\textsuperscript{25} largest companies on the JSE, measured by their full MV. These companies generally have sound financial standing and highly marketable shares\textsuperscript{26}, with operations spread over a variety of sectors, including resources, industrial, retail, telecommunication and financial services\textsuperscript{27}.

In contrast, the Small Cap Index contains the companies in the FTSE/JSE Africa All Share Total Return Index (the ALSI) ranked after the 100 largest companies\textsuperscript{28}. Free-float market capitalisation values are used when weighting the index constituents. In addition, small firms that are too illiquid are excluded from the Small Cap Index completely by the liquidity screening process. This index is used primarily for the purposes of comparison as it is recognised that due to liquidity constraints it is not well suited to being a member of the portfolio construction toolkit.

4.2.1.2. Rebalancing percentages and cost-adjusted returns

A measure of portfolio turnover is computed for each style-portfolio to quantify its rebalancing intensity and to adjust for the impact of transaction costs. The rebalancing indicator calculated is a two-way measure, summarising the amount of rebalancing required in terms of both selling and purchasing. As a result, the maximum value of the turnover indicator is 200\%. The computation formula is as follows:

\[
\text{Rebalancing at the beginning of month } t \text{ (\%)} = \sum_{i=1}^{N} |\Delta w_{i,t}| \quad (4.2)
\]

Where:

\(N\), represents the total number of shares available in the dataset, at the beginning of month \(t\)

\(\Delta w_{i,t} = w_{i,t \text{ new}} - w_{i,t \text{ performance}}\), represents the amount of rebalancing required for share \(i\) at the beginning of month \(t\). \(\Delta w_{i,t} > 0\) indicates that additional purchase of share \(i\) is

\textsuperscript{25} In fact, a firm will be included in the Top 40 Index if it is ranked 35th or above when the eligible firms are ranked by full market capitalisation, and it will be removed from the index if it is ranked 46th or below. A constant number of constituents will be maintained for the Top 40 Index where the deleted constituents are replaced by the highest-ranking company on the reserve list. (http://www.satrix.co.za/satrix_40/40/index.jsp).

\textsuperscript{26} 95\% of the trading on the JSE is accounted for by the top 40 companies.

\textsuperscript{27} http://www.satrix.co.za.

\textsuperscript{28} Ranked by full market cap before free float adjustment is applied.
required and vice versa. $w_{i,t \_new}$ represents the required weight of share $i$ in the index, at the beginning of month $t$.

$w_{i,t \_performance}$ represents the weight of share $i$, at the end of month $t-1$, after its share price movement in month $t-1$, and just before the rebalancing at the beginning of month $t$.

$$w_{i,t \_performance} = w_{i,t-1} \times \frac{1 + R_{i,t-1}}{1 + R_{p,t-1}}$$

(4.3)

Where:

$R_{i,t-1}$ represents the monthly price return on share $i$ in month $t-1$.

$R_{p,t-1}$ represents the monthly price return of the respective style portfolio in month $t-1$.

In estimating the cost-adjusted style index returns a realistic brokerage fee of 10 basis points (bps) per trade is assumed. A higher cost of 20 bps is also assessed to adjust for other smaller fees and the more nebulous, but potentially larger, influence of the market. Thus two cost-adjusted average returns are calculated for each index, using 10 bps and 20 bps per trade respectively:

$$R^*_p = R_p - C \times \text{mean rebalancing percentage of index } i$$

(4.4)

Where:

$C$ represents the transaction cost per trade and takes on the value of 0.1% and 0.2%.

$R^*_p$ represents the cost-adjusted geometric mean return of index $p$.

$R_p$ represents the gross geometric mean return of index $p$.

4.2.1.3. Effective number of shares

As a measure of portfolio concentration, following Strongin et al (2002), for each style index the effective number of equally weighted shares in each month ($n_{p,i}^*$) is estimated as follows:

---

29 According to the Online Share Trading system of Standard Bank, SA, brokerage is charged at a flat rate of 10 bps with a R70 minimum plus VAT. However, brokerage is negotiable for investors wishing to trade volumes of R500 000 or more per month.

30 This assumption is obtained by taking into account all the other trading costs, including: Uncertificated Securities Tax (UST) at 0.25% for purchases of shares only, a STRATE charged fee of 0.005459% based on the value of the share transaction to enable electronic settlement, a Financial Service Board levied investor protection levy of 0.02 bps, and the monthly account maintenance fee.
\[ n_{p,t} = \frac{1}{\sum_{i=1}^{N} w_{p,i,t}} \]  

(4.5)

Where:

- \( w_{p,i,t} \) represents the weight of company \( i \) in month \( t \) in index \( p \)
- \( N \) represents the number of companies in index \( p \).

The mean value of \( n_{p,t} \) relative to the average actual number of shares in each month is an indication of portfolio concentration. The smaller the ratio, the more concentrated the portfolio is.

### 4.2.1.4. Regression analysis

The basic regression analysis is performed in Excel, while advanced analysis and further investigations are performed in E-view. The abnormal return (measured by the constant term, alpha, of the regression), beta coefficients, t-statistic and p-value of each coefficient, as well as the adjusted \( R^2 \) values are produced and compared for each of the style indices. Risk adjusted alphas are estimated using both the Capital Asset Pricing Model (CAPM) and two factor APT model suggested by Van Rensburg and Slaney (1997).

#### The single factor CAPM model

For the CAPM model, each index’s time series of excess monthly returns is regressed against the excess market returns using the ordinary least squares (OLS) method. The ALSI is used as a proxy for the market portfolio, and the risk-free rate of return is proxied by the South African (SA) 90-day Banker’s Acceptance discount rate (RBAS). The regression model takes the form:

\[ R_{p,t} - R_{f,t} = \alpha_p + \beta_p (R_{M,t} - R_{f,t}) + \epsilon_{p,t} \]  

(4.6)

Where:

- \( R_{p,t} \) is the total return of style portfolio \( p \) in month \( t \)
- \( R_{M,t} \) represents the return on the ALSI in month \( t \)
- \( R_{f,t} \) represents the risk-free rate of interest in month \( t \) (as proxied by the South African 90-day Banker’s Acceptance discount rate)
\( \beta_p \) represents the estimated beta coefficient of portfolio \( p \)

\( \alpha_{p,t} \) represents the time-series regression intercept (abnormal return)

\( \varepsilon_{p,t} \) represents the error term for style index \( p \) in month \( t \).

The two factor APT model

Van Rensburg (2000) suggested that a two factor APT model specified using the JSE Financial-Industrial\(^{31}\) and Resources\(^{32}\) Indices as factor proxies is more appropriate than the single factor CAPM on the JSE. The regression equation is thus:

\[
R_{p,t} - R_{f,t} = \alpha_p + \beta_{p,FINDI} (R_{FINDI} - R_f) + \beta_{p,RESI} (R_{RESI} - R_f) + \varepsilon_{p,t} \tag{4.7}
\]

Where:

- \( R_{p,t} \) represents monthly return on style portfolio \( p \) in month \( t \)
- \( R_{f,t} \) represents the risk-free rate of interest in month \( t \)
- \( \alpha_{p,t} \) represents the intercept (excessive return) on style portfolio \( p \)
- \( \beta_{p,FINDI} \) represents the regression coefficient for the financial-industrial factor proxy
- \( \beta_{p,RESI} \) represents the regression coefficient for the resource factor proxy
- \( \varepsilon_{p,t} \) represents the random error term of style portfolio \( p \) in month \( t \).

The \( \beta_{p,factor} \) terms indicate the relationship between the variation in the size-style index returns and that in the factor-index returns.

4.2.1.5. Summary Statistics

A number of other conventional risk and return measures are computed to illustrate the characteristics of each style index constructed. The basic measures are geometric mean and standard deviation of monthly returns, the Sharpe Ratio and the Treynor Ratio.

The Sharpe Ratio is a risk-adjusted performance measure developed by Sharpe. It indicates how much excess return a portfolio is able to generate for an investor while

\(^{31}\) FINDI. Coded as C121X after the reclassification of the JSE Indices in March 1999, re-coded as J250 after the introduction of the FTSE/JSE Africa Index Series in June 2002.

\(^{32}\) RESI. Coded as C111X after the reclassification of the JSE Indices in March 1999, re-coded as J250 after the introduction of the FTSE/JSE Africa Index Series in June 2002.
taking into consideration the total variability of the portfolio returns. The higher the ratio, the better the historical risk-adjusted performance of the index. Therefore:

\[ S = \frac{R_p - R_f}{\sigma_p} \]  

(4.8)

Where:

- \( S \) represents the Sharpe Ratio
- \( R_p \) represents the mean monthly return on the \( p \)th style-index constructed
- \( R_f \) represents the risk-free rate, typically measured by the SA 90-day Banker’s Acceptance discount rate (RBAS)
- \( \sigma_p \) represents the standard deviation of the monthly returns on the \( p \)th style-index.

The Treynor Ratio is similar to the Sharpe ratio, except that the portfolio’s CAPM beta is utilised as the measure of risk as opposed to the standard deviation of the portfolio returns. Therefore, the Treynor Ratio summarises the excess return per unit of systematic risk, where excess return is measured by the difference between the observed returns and the risk-free investment. The Treynor Ratio is computed as follows:

\[ T = \frac{R_p - R_f}{\beta_p} \]  

(4.9)

Where:

- \( T \) represents the Treynor Ratio
- \( R_p \) represents the mean monthly return on the \( p \)th style-index constructed
- \( R_f \) represents the risk-free rate, typically measured by the SA 90-day Banker’s Acceptance discount rate (RBAS)
- \( \beta_p \) represents the beta coefficient from the CAPM regression on the \( p \)th style index.

It should be noted that all of the descriptive statistics, including the Sharpe Ratio and the Treynor Ratio, are calculated using gross geometric mean returns; whereas the
cost-adjusted geometric means give an indication of the net returns that an investor is able to achieve by investing in the indices.

4.2.2. Empirical results

4.2.2.1. Constructed indices

The monthly returns and rebalancing percentages (if available) of three monthly and three corresponding quarterly size-style indices, together with the Top 40 Index and the Small Cap Index, are computed over the period 1st January 1998 to 31st December 2006.

4.2.2.2. Summary statistics and regression analysis

The comprehensive set of descriptive and regression statistics computed for the five monthly rebalanced size-style indices is displayed in Table 4.1. Section A of Table 4.1 presents the summary statistics, whereas Sections B and C respectively show the relevant single-index CAPM and two factor APT regression outputs. The full list of CAPM and APT regression outcomes, including excess returns (alphas), regression coefficients (betas), t-statistics (Student, 1908) and p-values of the coefficients and adjusted $R^2$ values are attached in Appendix C.2 for both the monthly and quarterly rebalanced indices.

Table 4.1: Candidate size-style indices (monthly data)

<table>
<thead>
<tr>
<th>Index description</th>
<th>Returns analysis</th>
<th>Summary statistics</th>
<th>Single-index CAPM regression</th>
<th>Two factor APT regression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top 40 Index</td>
<td>Monthly returns</td>
<td>Sharpe Ratio</td>
<td>0.23</td>
<td>0.29</td>
</tr>
<tr>
<td>Small Cap Index</td>
<td>Monthly returns</td>
<td>Sharpe Ratio</td>
<td>0.20</td>
<td>0.23</td>
</tr>
</tbody>
</table>

The R2-adjusted figure adjusts for the degrees of freedom lost when more explanatory variables are used in the analysis.
Investigating Candidate Style Indices 4:11

among all of the indices is followed by ** and the second lowest value followed by *. The selected ‘best performing’ index is highlighted in grey.

<table>
<thead>
<tr>
<th>Style Indices</th>
<th>EW(size)100</th>
<th>EW(size)50</th>
<th>EW(size)30</th>
<th>Top 40</th>
<th>Small Cap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic mean (%)</td>
<td>1.95**</td>
<td>1.86</td>
<td>1.91*</td>
<td>1.90</td>
<td>1.35</td>
</tr>
<tr>
<td>Geometric mean (%)</td>
<td>1.73**</td>
<td>1.61</td>
<td>1.66</td>
<td>1.66</td>
<td>1.67*</td>
</tr>
<tr>
<td>Mean monthly rebalancing (%)</td>
<td>11.3</td>
<td>13.0</td>
<td>12.7</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Cost-adjusted geometric mean (10 bpt) (%)</td>
<td>1.71**</td>
<td>1.60</td>
<td>1.64*</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Cost-adjusted geometric mean (20 bpt) (%)</td>
<td>1.70**</td>
<td>1.59</td>
<td>1.63*</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Standard deviation (%)</td>
<td>6.56*</td>
<td>6.90</td>
<td>6.95</td>
<td>6.81</td>
<td>5.91**</td>
</tr>
<tr>
<td>Return/standard deviation ratio</td>
<td>0.26*</td>
<td>0.23</td>
<td>0.24</td>
<td>0.24</td>
<td>0.28**</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.13*</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0.13**</td>
</tr>
<tr>
<td>Treynor ratio</td>
<td>0.01*</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01**</td>
</tr>
<tr>
<td>Average no. of constituents</td>
<td>100</td>
<td>50</td>
<td>30</td>
<td>40</td>
<td>-</td>
</tr>
<tr>
<td>Average effective no. of constituents</td>
<td>100</td>
<td>50</td>
<td>30</td>
<td>40</td>
<td>-</td>
</tr>
<tr>
<td>maximum constituent holding (%)</td>
<td>1.00</td>
<td>2.00</td>
<td>3.33</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Section B: Single-index CAPM model results

| Alpha CAPM (%) | 0.18* | 0.02 | 0.03 | -0.03 | 0.30** |
| t-alpha CAPM | 0.59* | 0.07 | 0.13 | -0.38 | 0.76** |
| p-alpha CAPM | 0.55 | 0.95 | 0.90 | 0.70 | 0.45 |
| Beta CAPM | 0.89* | 0.96 | 1.00 | 1.04 | 0.67** |
| Adjusted R square | 0.78 | 0.82 | 0.87* | 0.99** | 0.55 |

Section C: Two-factor APT model results

| Alpha APT (%) | 0.35* | 0.18 | 0.11 | -0.17 | 0.49** |
| t-alpha APT | 1.63 | 0.87 | 0.51 | -2.46** | 1.50* |
| p-alpha APT | 0.11 | 0.39 | 0.61 | 0.02 | 0.14 |
| Adjusted R square | 0.89 | 0.91* | 0.90 | 0.99** | 0.70 |

In terms of gross mean returns, the EW(size)100 Index (monthly rebalanced) displays notably better performance than all the other size indices over the period 1st January 1998 to 31st December 2006 (1.73% per month). It exhibits the second highest CAPM- and APT-excess return (0.18% and 0.35%), Sharpe Ratio (0.129) and Treynor Ratio (0.010). The standard deviation of 6.56% and CAPM beta of 0.891 demonstrate that EW(size)100 has the second lowest total and systematic return volatilities.

The Small Cap Index follows the EW(size)100 Index closely in respect of gross geometric mean (1.67% per month). It also generates the highest risk-adjusted excess return (0.30% per month for CAPM and 0.49% per month for APT). Furthermore, the Small Cap Index displays the lowest risk in the case of both total return volatility represented by standard deviation (5.91% per month) and the systematic risk measured by CAPM beta (0.673). As a direct consequence of relatively high return and low risk, the Small Cap Index enjoys the highest Sharpe (0.135) and Treynor Ratio (0.012) among all of the indices examined.
Section B of Table 4.1, however, shows that none of the CAPM-phas’s p-values are smaller than 0.1, therefore none of the excess returns generated by the size-style indices are significantly different from 0 even at 10% significance level. Comparing to the CAPM results, the APT-alpha’s p-values are improved (i.e., p-values have decreased under the APT model); but still none is significant at 10% level. This confirms the findings of numerous SA literature stating that, tested using both the CAPM and APT models, there is no paramount ‘size-effect’ on the JSE.

Among the three monthly rebalanced size-style indices constructed, EW(size)100 has lowest average monthly turnover (11.35% counting both buys and sells). Comparing each of the monthly rebalanced indices with the corresponding quarterly rebalanced numbers in Section A of Appendix C.2, it is noted that although the quarterly rebalanced indices have lower turnover percentages, they also have lower after-cost geometric means and Sharpe Ratios. Therefore, less frequent rebalancing seems to have a negative impact on the cost-adjusted performance achieved.

Although the average rebalancing percentage of the Small Cap Index is not available, for the index that ranked the 3rd in geometric mean (i.e. EW(size)30) to overtake the Small Cap Index in its cost-adjusted geometric mean, the average rebalancing percentage of the Small Cap Index needs to be at least 17.66% per month. This amount of turnover is unlikely given that the maximum rebalancing amount calculated in this section is 12.99%. Therefore, it is appropriate to assume that the EW(size)100 Index did the best job in generating net average returns (1.71% per month) assuming transaction costs of 10 bps per trade or 1.70% per month assuming transaction costs of 20 bps per trade), followed by the Small Cap Index.

The adjusted R^2 values show that the CAPM model explains 54.7% of the variation in excess returns on the JSE in the case of the Small Cap Index, and 78.2% of the variation in the case of the EW(size)100 Index. The adjusted R^2 of each of the indices

---

34 Solve for the minimum rebalancing percentage that will equate the cost-adjusted geometric means of the EW size 30 and that of the Small Cap Index. Assume transaction costs of 10 bps per trade, \([1.67\%- (1.66\%-12.66\% \times 0.1\%)/0.1\%] = 22.66\%\), therefore the EW size 30 index will out-perform the Small Cap Index in terms of its cost-adjusted geometric mean if the rebalancing % of the Small Cap Index is greater than 22.66%.

Similarly, assume transaction costs of 20 bps per trade, \([1.67\%- (1.66\%-12.66\% \times 0.2\%)/0.2\%] = 17.66\%\)

35 The gross mean of the Small Cap Index is less than the net return of the EW(size)100 Index, thus the net means of the Small Cap Index cannot exceed those of the EW(size)100 Index.
tested is higher under the APT model, indicating that the two factor APT model has
the ability to explain a greater proportion of variation in the excess returns on the JSE
than the CAPM model, confirming findings from Van Rensburg and Slaney (1997).

4.2.2.3. Cumulative returns and relative returns

The performance of the five monthly rebalanced size-style indices over the period 1st
January 1998 to 31st December 2006 is plotted in Figure 4.1. The horizontal axis
displays the dates and the vertical axis shows the log cumulative returns of each
index. The relative returns\(^{36}\) of each index are plotted in Appendix C.3, which when
studied together with Figure 4.1, provides additional insight as to the performance of
the style indices relative to that of the equity market as a whole.

The Small Cap Index started with strong cumulative performance which persisted
until the end of 1999. Thereafter, however, it experienced the most significant
underperformance among all indices over the period 2000 to 2003. A growth trend
much steeper than that of the other indices was displayed over the years 2003 to 2004.
This suggests that the high geometric mean of the Small Cap Index is primarily a
result of strong performance from 2003 onwards. Furthermore, the Small Cap Index’s
performance in the early years appears to be less volatile than that of the ‘large
capitalisation’ indices.

From Appendix C.3, it is noted that while all the other size-style indices exhibited a
similar trend in their cumulative returns, the Top 40 Index showed a distinctive
pattern over the period of investigation. This is further illustrated by its low
correlation with the other indices as discussed in Section 4.5.

The cumulative relative returns of the small size indices demonstrate evidence of clear
cyclicality of performance. After the Asian crisis (August 1998) all proxies, except for
the Top 40 Index, performed poorly relative to the ALSI until May 2002 from where
they outperformed for the rest of the sample period. The Top 40 Index remained the
best performer over this period. After 2003, however, the Top 40 Index’s performance
deteriorated. It produced an overall cumulative return approximately in line with that
of the ALSI over the entire period of investigation.

\(^{36}\) Relative to ALSI
On the other hand, the EW(size)100 Index lagged in performance during the early period (1st January 1998 to 1st January 2003), but its cumulative returns climbed above those of the ALSI from 1st January 2004 onwards. Figure 4.1 illustrates that over the period analysed the final cumulative returns were the highest in the case of the EW(size)100 Index.
Figure 4.1: Log cumulative returns of the size-style indices

The graph displays the log cumulative returns for the five monthly rebalanced size-style indices over the period 1st January 1998 to 31st December 2006. EW stands for equally weighted indices. The data are obtained from DataStream International at the University of Cape Town.
4.3 Style indices: value

4.3.1. Data and methodology

For each value-factor proxy, value-style indices are constructed using two types of weighting schemes: (1) an equal weighting (EW) portfolio construction approach applied to the top ranked (by the attributed concerned) 50 and 30 stocks from the sample of the largest 100 shares in each month and (2) a characteristic based weighting approach (referred to as value-proxy weighting from hereon) applied to the top 100, 50 and 30 shares ranked by market capitalisation. Eleven value-factor proxies are examined through these two weighting methods. With different number of index constituents used for each method, a set of five monthly rebalanced value indices are produced using each value proxy. The aim of this section is to identify the value proxy that delivers the highest style returns on the JSE.

4.3.1.1. Value-style proxy

Eight firm-specific characteristics are tested respectively as value proxies. In addition, a regression based composite valuation measure, RES(N), is introduced as a further value proxy. Firm attributes are regressed on observed share returns. The resulting residuals (RES) are effectively a measure of the relative cheapness of each share.

4.3.1.1.1. Firm attributes as value proxy

There exists well-documented evidence on the value effect on the JSE, where returns are higher for a strategy based on a portfolio of low PE (therefore high EY) stocks than those for a portfolio of high PE [Page (1996), Page and Palmer (1991) and Basu (1977, 1983)]. The evidence of EY as a proxy for the value-style cluster is proposed by Van Rensburg (2001). The value effect indicates that shares with higher EY outperform shares with lower EY.

Other than EY, the following seven additional firm attributes are tested as value-factor proxies: BTMV, total cashflow (CF), dividend (DIV), sales (SALE), earnings per share (EPS), EAR and BV. These alternative proxy factors are inspired by Arnott’s (2006) fundamental metrics as described in Chapter Two.

37 Which is the same as that used for the size-style indices
38 Including eight firm-specific attributes: earnings yield (EY), earnings per share (EPS), total earnings (EAR), total book values (BV), total cashflow (CF), total sales (SALE), dividends (DIV) and the book to market value ratio (BTMV); and three Residuals: RES(4), RES(3), RES(2).
39 RES(N) refers to the residuals obtained from the N-factor regression.
4.3.1.1.2. Residual (RES) as value proxy

Academics and practitioners have been persistently seeking a reliable measure of a company’s true value which may be compared to the company’s share price so as to assist in formulating buy-and-sell decisions. The most popular and simplest measure is PE, and thus:

\[ \text{Company value} = \frac{P}{E} \quad (4.10) \]

Equation (4.10) can be expanded to include a range of measures of company values in addition to the PE ratio. A generalised version of the above formula becomes:

\[ \text{Company value} = \frac{P}{(c_1 \text{EAR} + c_2 \text{BV} + c_3 \text{DIV} + c_4 \text{SALE})} \quad (4.11) \]

Where:
- \( P \) represents the company’s share price
- \( \text{EAR} \) represents total earnings
- \( \text{BV} \) represents book value
- \( \text{DIV} \) represents dividends
- \( \text{SALE} \) represents sales
- \( c_i \)’s indicates the extent that a company’s fair value can be reflected by the \( i \)th fundamental measure of firm values.

Making \( c_1 = 1 \) and \( c_2 \) to \( c_4 \) to be 0 transforms Equation (4.11) into Equation (4.10). The items in the brackets of Equation (4.11) represent common measures of a firm’s fundamental value. Performing a log transformation on Equation (4.11) not only separates company price (\( P \)) on one side of the equation but also removes extreme values. Put numerically:

\[ \ln(\text{Company value}) = \ln P - (c_1 \ln E + c_2 \ln BV + c_3 \ln Div + c_4 \ln Sales) \quad (4.12) \]

Re-arrange Equation (4.12), the following formula is obtained:

\[ \ln P = c_1 \ln E + c_2 \ln BV + c_3 \ln Div + c_4 \ln Sales + \epsilon \quad (4.13) \]
By comparing Equations (4.12) and (4.13), \( \varepsilon \) is equivalent to \( \ln(\text{company value}) \) and in direct proportion to \( \text{company value} \). Stated in general from, the following cross-sectional ordinary least squares regression is conducted in each month to produce residual values:

\[
\ln P_{i,t} = c_0 + \sum_{k=1}^{m} c_{k,t} \ln F_{k,i,t} + \varepsilon_{i,t}
\]  \hspace{1cm} (4.14)

Where:

- \( P_{i,t} \) represents the share price of company \( i \) in month \( t \)
- \( F_{k,i,t} \) represents the \( k \)th firm-specific attribute that can serve as a fundamental measure of company values. In this chapter, \( F_{1,i,t} = \text{EAR} \) of share \( i \) in month \( t \). \( F_{2,i,t} = \text{BV} \) of share \( i \) in month \( t \). \( F_{3,i,t} = \text{DIV} \) of share \( i \) in month \( t \). \( F_{4,i,t} = \text{SALE} \) of share \( i \) in month \( t \)
- \( m \) represents the number of fundamental factors utilised. In this chapter, \( m \) takes on the value 2, 3 and 4 respectively
- \( c_{k,t} \) represents the coefficient of the \( k \)th factor in month \( t \)
- \( \varepsilon_{i,t} \) represents the residual of share \( i \) in month \( t \), it is a measure of the cheapness of a company's shares.

The estimated fair value of the share is represented by the summation term in Equation (4.14). Positive \( \varepsilon_{i,t} \) indicates the share's actual price is greater than its predicted price and hence the share is over-priced. The higher the absolute value of the negative residuals, the more the share is under-priced, and hence the 'cheaper' the share is. Only the shares with negative residual values (i.e. cheap shares) are used in the construction of the composite value indices. Weightings are calculated in proportion to the absolute values of these residuals in each month. Therefore it is important to note that \textbf{the value proxy utilised is not} \( \varepsilon_{i,t} \), \textbf{but the} -\( \varepsilon_{i,t} \) values. Putting into formula:

\[
RES_{i,t} = -\varepsilon_{i,t}
\]  \hspace{1cm} (4.15)
When $m$ in Equation (4.14) takes on the value of 2, the residuals are denoted by RES(2), indicating that a two factor regression is to be conducted where the fundamental measures, EAR and BV, are utilised. If $m$ is 3, DIV is also included in a three-factor regression. RES(4) indicates that the residual values are obtained from a 4-factor regression, whereby the SALES metric is also included.

Logging the price and fundamental metrics reduces the extreme residual values, and thus result in lower portfolio concentration. The resulting ‘fair value’ may also be interpreted as price divided by a weighted list of fundamental characteristics. It is noted that some firms do not pay DIV which is problematic as logging a DIV value of zero will result in the share being completely excluded from the index. To lessen the problem of having too few shares in the style portfolio, an adjustment is made where if DIV value is 0, the $\ln(DIV)$ variable is also assigned a value of zero. Thus if the share’s other fundamental metric values are available, the share will be included in the sample.

4.3.1.2. Index construction methods

In a particular month, a value investor would construct his portfolio in such a way that either more weights are assigned to the shares with higher ‘values’ (indicated by higher value-proxy values) at the beginning of that month; or he would only invest in shares with relatively high fundamentals whilst maintaining an acceptable level of diversification in his EW portfolio. These two approaches are the underlying principles utilised to construct the value-factor weighted and the EW value-style indices.

Three value-proxy weighted (with 100, 50 and 30 constituents respectively) and two EW value-style indices (with 50 and 30 constituents respectively) are constructed for each value-factor proxy. Some corresponding quarterly rebalanced indices are also calculated, with the subsequent results being appended. The detailed construction method for the style portfolios are as follows.
4.3.1.2.1. Equally weighted indices

The EW method constructs equally weighted value-style portfolios out of the 100 shares with the largest MV in each month\(^4\). These 100 shares are sorted by the value-factor proxy (such as EY, BTMV, RES *etc*) in a descending order. The EW(value-proxy)50 Index comprises the 50 shares with the highest respective value proxy in each month. Similarly, the EW(value-proxy)30 Index constitutes the top 30 shares ranked by the value-factor proxy in each month. Within each portfolio, each of the \(N\) constituents is given an equal weight; hence the total value-style portfolio return in a month is derived as an arithmetic mean of the total returns of the \(N\) index constituents in that month. The computation formula is as follows:

\[
R_{EW(\text{value-proxy})N, t} = \frac{1}{N} \sum_{i=1}^{N} R_{i,t} \tag{4.16}
\]

Where:
- \(R_{EW(\text{value-proxy})N, t}\) represents the monthly return of the EW value index in month \(t\)
- \(N\) represents the number of constituents in the index, takes on the value of 50 or 30
- \(R_{i,t}\) represents the forward monthly return of the \(i\)th index constituent, in month \(t\).

4.3.1.2.2. Value-proxy weighted indices

The characteristic or value-proxy \((F)\) based weighting for each share in month \(t\) \((w_{i,t})\) is calculated as follows:

\[
w_{i,t} = \frac{F_{i,t}}{\sum_{j=1}^{N} F_{j,t}} \tag{4.17}
\]

Where:
- \(w_{i,t}\) represents the weight of the \(i\)th constituent in the index in month \(t\)
- \(F_{i,t}\) represents the value of the selected value proxy (or characteristic) of share \(i\) at the beginning of month \(t\). It is either one of the eight firm attributes or one of the multi-regression residuals computed.

Negative characteristic values are ignored as it is assumed there is an absence of short-selling in portfolio construction. Intuitively explained, a share with double the

\(^{40}\) Only top 100 shares in each month are used from the sample to take care of the “thin-trading” problem.
value of $F$ of another share in a particular month will have double the weighting of the latter.

Take EY as a value-proxy example (i.e. let $F$ takes on the EY values). The earnings yield weighted value $N$ (EYWN) portfolios are constructed using the firm-attribute data available at the beginning of each month. All the shares from the DataStream International (DataStream) that have returns available for the following month are ranked on the basis of MV, then the top $N$ shares by MV are collected to form the respective value-style portfolios. Within each of the resulting portfolios, constituents are weighted in accordance with their EY values at the beginning of that month. Both the portfolio returns and the rebalancing amount are calculated on a monthly basis. The entire construction process is repeated for $N$ taking on the values of 100, 50 and 30.

If MV, forward return or EY is missing, or the EY entry is negative for share $i$ in month $t$, share $i$ will be excluded from the sample of month $t$. Consequently, the EYWN indices normally have less than $N$ constituents in each month.

In summary, let us consider the EYW indices for example. The value of an EYWN index point in month $t$ is calculated as the earnings yield weighted arithmetic average of the total monthly returns of the index constituents, as follows:

$$R_{EYWN,t} = \sum_{i=1}^{n_t} w_{i,t} \times R_{i,t}$$  \hspace{1cm} (4.18)

Where:

- $R_{EYWN,t}$ represents the monthly return on the EYW portfolio consisting the top $N$ shares by MV in month $t$.
- $N$ takes on values of 100, 50 or 30.
- $n_t$ represents the number of index constituents in month $t$ (number of shares that ranked top $N$ by MV, and have non-negative EY).
- $R_{i,t}$ represents the forward monthly return of the $i$th index constituent in month $t$.
- $w_{i,t} = \frac{EY_{i,t}}{\sum_{j=1}^{n_t} EY_{j,t}}$, represents the weight of the $i$th constituent in the index, in month $t$. 

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$EY_{it}$ represents the EY of share $i$ at the beginning of month $t$.

The method of constructing value indices using the other value proxies is the same as that adopted for the EYWN indices, except that $F$ takes on the values of the selected alternative firm-specific attribute. In other words, instead of its EY, the weighting of each corresponding index constituent is derived from its $BTMV, CF, DIV, SALE, EPS, EAR, BV, RES(1), RES(2)$ or $RES(3)$. The computation formula for each set of indices takes the same form as that of Equation (4.18).

The formula for the RESWN $(K)$ Index is given below for illustration purpose. Turnover percentages, summary statistics and regression analysis are computed in the same way as described in Sections 4.2.1.2 to 4.2.1.4.

$$R_{RESWN(K),t} = \sum_{i=1}^{n_t} w_{i,t} \times R_{i,t}$$

(4.19)

Where:

$R_{RESWN(K),t}$ represents the monthly return on the RES$(K)$ weighted portfolio, in month $t$. $K$ takes on values of 2, 3 and 4

$N$ takes on values of 100, 50 or 30

$n_t$ represents the number index constituents in month $t$ (number of shares that ranked top $N$ by MV, and have non-negative RES values. In other words, non-positive residuals)

$R_{i,t}$ represents the forward monthly return of the $i$th index constituent, in month $t$

$w_{i,t} = \frac{RES(K)_{i,t}}{\sum_{j=1}^{k} RES(K)_{j,t}}$, represents the weight of the $i$th index constituent in month $t$

$RES(K)_{i,t}$ represents the negative of the residual values of share $i$ at the beginning of month $t$, obtained from carrying out a $K$-factor regression.
4.3.2. Empirical results

4.3.2.1. Constructed indices

The monthly returns, number of shares per month and rebalancing percentages of the 55 monthly rebalanced value-style indices are computed over the period 1st January 1998 to 31st December 2006.

It should be noted that the value indices are particularly exposed to 'data problems'. Unlike the size- and momentum-style indices which utilise objective share return data, accounting metrics are used in the construction of the value-style indices. This problem becomes more acute for insurance and financial service companies. A detailed discussion relating to the firm-specific accounting entries is presented in Section 3.2 of Chapter Three. All of the value-index results displayed below need to be viewed with caution, bearing in mind the lack of reliability and stability of the accounting information.

4.3.2.2. Summary statistics and regression analysis

Section A of Table 4.2a and Table 4.2b displays the summary statistics calculated for the 55 value-style indices constructed over the period 1st January 1998 to 31st December 2006. Each of the 55 value-style indices is tested using the CAPM and the APT model, the results including excess returns, regression coefficients, t-statistics and p-values of the excess returns and adjusted $R^2$ are summarised in Sections B and C of Table 4.2a and Table 4.2b. Table 4.2a presents the 40 indices constructed using single firm attributes as value proxies, whereas indices in Table 4.2b employ the residual (RES) weighting approach as described above. Regression results on the quarterly rebalanced portfolios using EY as value proxy are attached in Appendix C.4

Among the RES weighted indices, using residuals from a four-factor regression as the value proxy has delivered the highest returns. The monthly rebalanced RESW100(4) Index outperformed all the other value-indices in terms of their return generating ability. It does not only have the highest geometric mean (2.69% per month) and the CAPM- and APT-risk-adjusted excess returns (alphas of 1.15% and 1.12% per month respectively), but also the most favourable Treynor Ratios (0.02). Both of its excess returns are significant at 5% level. These findings persist after trading cost adjustments.
It is noted that as the number of fundamental firm attributes used in the multi-factor regression increases, the returns generated appear to increase, and therefore RES(4) is found to be a better value proxy than RES(3). The number of index constituents, however, decreases sharply as the number of independent variables used in the regression picks up. For instance, the number of constituents is 32 for the RESW100(4) Index in comparison to 40 for the corresponding RESW100(3) Index.

The second highest gross return (2.48% per month) and risk-adjusted returns (2.46% per month for CAPM and 2.43% for APT) are achieved on the RESW100(3) portfolios, which also display the second highest Sharpe (0.23) and Treynor Ratios (0.02). More importantly, portfolio concentration of this index has reduced in comparison to the indices utilising RES(4) as the factor proxy (indicated by maximum holding of 12.19% vs. 19.96% for the RES(4) Index). As a result, RESW100(3) is chosen for the analysis in the following sections to strike a balance between high return, acceptable portfolio diversification and rebalancing frequency.

Among all of the value indices, EW(RES)50(4) yields the lowest standard deviation of 6.11% per month. DIV weighted indices have also produced steady returns, indicated by its second lowest standard deviation (6.15%). This may be a result of the fact that companies tend to keep the size of dividends stable over time. EW(RES)50(4) and EW(BTMV)30 are the two indices associated with minimum risk, measured by CAPM beta. EW(RES)50(4) yields the lowest CAPM beta (0.79) while the EW(BTMV)30 Index yields the second lowest CAPM beta (0.82).

The turnover amounts are comparatively low for the indices constructed using the less volatile individual firm attributes: EAR, EPS, BV, CF and DIV. BVW100 has the lowest rebalancing percentage (10.3%) among all 55 of the value indices. In contrast, the minimum average turnover required by the RESW100(4) Index and the EW(BTMV)30 Index is 33.3% and 28% respectively.

EW(BTMV)30 experiences the third highest mean returns (2.43% gross geometric mean, 2.41% and 2.38% after adjusting for 10bps and 20 bps transaction costs) among all of the value indices constructed. It has delivered a high Sharpe Ratio of 0.24.
In summary, it is clear that RES, particularly RES(4) and RES(3), and BTMV are the best performing value proxies over the period 1st January 1998 to 31st December 2006. In general, RES appears to be superior to all the other value proxies examined, whereas BTMV is the top proxy among all of the eight single factor firm attributes investigated. Furthermore, although the results provide no support for the hypothesis that EAR, EPS, BV, CF and DIV are accurate proxies for the value-style factor, these value proxies appear to enable the forming of more stable portfolios that require significantly less rebalancing at the beginning of each month.

According to Appendix C.4, the row of rebalancing figures calculated suggests that the average turnover amounts of the quarterly rebalanced indices are approximately 55-58% of those of the corresponding monthly rebalanced indices during the periods analysed. Despite of the higher turnover and thus the higher transaction costs, indices constructed based on a monthly rebalancing strategy once again produced both cost-adjusted returns and Sharpe Ratios that outperformed the corresponding quarterly-rebalanced indices. Therefore it seems that less frequent rebalancing depresses overall index performance.

Due to the exclusion of the shares with negative or incomplete firm attribute data entries when constructing the value indices, the average number of constituents of the value-proxy weighted value $N$ index is less than $N$. For instance, on average, a portfolio underlying the EYW100 Index has 95 constituents, one underlying the BTMVW50 Index has 45 constituents, and one underlying the SALEW30 Index has only 17 constituents.

The adjusted $R^2$ values are shown in the last rows of Sections B and C of Table 4.2a and Table 4.2b. It is observed that a substantial portion (from 62% for CFW100 to 91% for EARW100) of the monthly variation in the indices' excess returns is explained by the CAPM model. The slight increase in adjusted $R^2$ values in Section C in comparison to those in Section B illustrates that although both models provide good fit to the time series data, the APT model is able to explain more variation in the excess returns on the JSE than the CAPM model. The alphas' p-values of all the indices examined have also improved under the two factor APT model in comparison to those under the CAPM model.
The table displays the descriptive and regression statistics of the 40 value-style indices constructed over the period 1st January 1998 to 31st December 2006. In total 108 time-series returns are calculated for each index. Returns are monthly. All portfolios are rebalanced monthly. The first three indices are constructed based on earnings yield weighted (EYW) portfolios containing the 100, 50 and 30 shares with the highest market capitalisation. The next two indices are constructed based on equally weighted (EW) portfolios using the top 50 and 30 shares ranked by EY out of the 100 shares with the highest MV in each month. The next 35 indices can be divided into seven groups with five indices in each group, each group uses an alternative value-style proxy other than EY, namely: book to market value (BTMV), total cashflow (CF), dividend (DIV), total sales (SALE), earnings per share (EPS), total earnings (EAR) and book value (BV). The South African 90-day Banker’s Acceptance discount rate (RBAS) is used as the risk-free rate. P-values are calculated using two-tailed tests. The data are obtained from DataStream International at the University of Cape Town. Section A shows the descriptive and summary statistics of the total returns for each of the 40 value-style indices. Section B shows the single-index CAPM regression statistics, using the ALSI as the market proxy. The results are obtained by regressing the excess monthly returns of the market index on the excess monthly returns of each of the 40 value-style indices. Section C shows the two-factor APT regression statistics, using FINDI and RESI as the APT-factor proxies. The results are obtained by regressing the excess monthly returns of the APT factors on the excess monthly returns of each of the 40 value-style indices. All of the descriptive statistics, including the Sharpe Ratio and the Treynor Ratio, are calculated using gross geometric mean returns; whereas the cost-adjusted geometric means give an indication of the net returns that an investor is able to achieve by investing in the indices. In a particular row, if a higher value indicates better performance (e.g. the Sharpe Ratio), the maximum value among of all the indices is indicated by **. The second highest value is marked by *. Similarly, if a lower value indicates better performance (e.g. standard deviation), the minimum value among all of the indices is followed by ** and the second lowest value followed by *. The selected ‘best performing’ index is highlighted in grey.
Table 4.2a: Candidate value-style indices (monthly data) (Continued)

<table>
<thead>
<tr>
<th>Style indices</th>
<th>SALEV100</th>
<th>SALEV150</th>
<th>Ew/mL</th>
<th>Ew/SALEV130</th>
<th>EPSW100</th>
<th>EPSPQ100</th>
<th>EPSPQ10</th>
<th>Ew/EPSPQ100</th>
<th>Ew/EPSPQ10</th>
<th>EARR100</th>
<th>EARQ100</th>
<th>Fw/EARR100</th>
<th>Ew/EARQ100</th>
<th>BvW100</th>
<th>BvW50</th>
<th>BvW30</th>
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<td>0.59</td>
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<td>0.82</td>
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<td>0.65</td>
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<td>0.09</td>
<td>0.06</td>
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<td>0.34</td>
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<td>0.88</td>
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<td>0.91</td>
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<td>0.92</td>
<td>0.92</td>
<td>0.92</td>
<td>0.91</td>
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</table>
Table 4.2b: Candidate composite value-style indices (monthly data)

The table displays the descriptive and regression statistics of the 15 value-style indices constructed over the period 1st January 1998 to 31st December 2006. In total, 108 time-series returns are calculated for each index. Returns are monthly effective. All portfolios are rebalanced monthly. There are three groups of indices each containing five indices. The first group uses residuals of a four-factor regression (earnings, book value, dividend, sales) as the value proxy, the second group uses residuals of a three-factor regression (earnings, book value, dividend) as the value proxy, the third group uses residuals of a two-factor regression (earnings, book value) as the value proxy. In each group, the first three indices contain the shares with negative regression residuals (i.e., under-priced shares) among the top 100, 50 and 30 shares by market capitalisation respectively, the constituent shares are weighted by the absolute value of the negative residuals. The next two indices are constructed based on equally weighted (EW) portfolios using all the shares with negative residuals among the top 50 and 30 shares ranked by the absolute value of RES(- residual values) out of the 100 shares with the highest MV in each month. The South African 90-day Banker’s Acceptance discount rate (RBAS) is used as the risk-free rate.

Section A shows the single-index CAPM regression statistics, using the ALSI as the market proxy. The results are obtained by regressing the excess monthly returns of the market index on the excess monthly returns of each of the 15 value-style indices. Section B shows the single-index APT-factor proxies. The results are obtained by regressing the excess monthly returns of the APT factors on the excess monthly returns of each of the 15 value-style indices. All of the descriptive statistics, including the Sharpe Ratio and the Treynor Ratio, are calculated using gross geometric mean returns; whereas the cost-adjusted geometric means give an indication of the net returns that an investor is able to achieve by investing in the indices. In a particular row, if a higher value indicates better performance (e.g. the Sharpe Ratio), the maximum value among of all the indices is followed by ** and the second lowest value followed by *. The selected ‘best performing’ index is highlighted in grey.

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<th>Style Indices</th>
<th>RESW100(4)</th>
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<th>RESW30(4)</th>
<th>EW(RES)100(4)</th>
<th>EW(RES)50(4)</th>
<th>EW(RES)30(4)</th>
<th>RESW100(3)</th>
<th>RESW50(3)</th>
<th>RESW30(3)</th>
<th>EW(RES)50(3)</th>
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<tr>
<td>Arithmetic mean (%)</td>
<td>2.98**</td>
<td>2.56</td>
<td>2.74</td>
<td>2.25</td>
<td>2.50</td>
<td>2.73</td>
<td>2.49</td>
<td>2.82*</td>
<td>2.29</td>
<td>2.36</td>
<td>2.38</td>
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<td>Geometric mean (%)</td>
<td>2.69**</td>
<td>2.25</td>
<td>2.43</td>
<td>2.06</td>
<td>2.28</td>
<td>2.48*</td>
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<td>2.42</td>
<td>2.06</td>
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<td>2.14</td>
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<td>Mean monthly rebalancing (%)</td>
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<td>38.3</td>
<td>45.6</td>
<td>19.5</td>
<td>25.2</td>
<td>25.8</td>
<td>28.9</td>
<td>23.6</td>
<td>21.0</td>
<td>23.5</td>
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<td>Cost-adjusted geometric mean (10 bpt) (%)</td>
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<td>2.22</td>
<td>2.39</td>
<td>2.04</td>
<td>2.26</td>
<td>2.46*</td>
<td>2.14</td>
<td>2.39</td>
<td>2.04</td>
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<td>2.12</td>
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<td>2.34</td>
<td>2.02</td>
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<td>2.10</td>
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<td>1.69</td>
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<td>1.94</td>
<td>1.77</td>
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<td>1.75</td>
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<td>Sharpe ratio</td>
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<td>0.20</td>
<td>0.19</td>
<td>0.21</td>
<td>0.23*</td>
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<td>0.17</td>
<td>0.18</td>
<td>0.19</td>
<td>0.18</td>
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<td>Treynor ratio</td>
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<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.03*</td>
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<td>0.01</td>
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<td>0.01</td>
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<td>9</td>
<td>34</td>
<td>40</td>
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<td>12</td>
<td>60</td>
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<td>44</td>
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<td>Average effective no. of constituents</td>
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<td>5</td>
<td>49</td>
<td>30</td>
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<td>23</td>
<td>12</td>
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<td>8</td>
<td>30</td>
</tr>
<tr>
<td>Maximum constituent holding (%)</td>
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<td>38.74</td>
<td>88.12</td>
<td>2.05</td>
<td>3.33</td>
<td>12.19</td>
<td>25.91</td>
<td>36.81</td>
<td>2.00</td>
<td>3.33</td>
<td>11.05</td>
</tr>
</tbody>
</table>

Section B: Single-index CAPM model results

| Alpha CAPM (%) | 1.15** | 073 | 090 | 058 | 078 | 095* | 061 | 087 | 055 | 062 | 066 | 035 | 066 | 043 | 058 |
| t-alpha CAPM | 2.40* | 1.49 | 1.89 | 1.79 | 2.05 | 2.56* | 1.29 | 1.48 | 1.60 | 1.78 | 1.63 | 1.75 | 1.09 | 1.16 | 1.43 |
| p-alpha CAPM | 0.02 | 0.14 | 0.06 | 0.08 | 0.04 | 0.06 | 0.01 | 0.20 | 0.14 | 0.11 | 0.08 | 0.11 | 0.45 | 0.28 | 0.15 |
| Beta CAPM | 0.95 | 0.95 | 0.95 | 0.79** | 0.84* | 0.90 | 1.00 | 1.07 | 0.86 | 0.86 | 0.85 | 0.94 | 1.04 | 0.86 | 0.86 |
| Adjusted R square | 0.62 | 0.60 | 0.63 | 0.71* | 0.66 | 0.71 | 0.65 | 0.58 | 0.72** | 0.71 | 0.64 | 0.63 | 0.55 | 0.69 | 0.65 |

Section C: Two-factor APT model results

| Alpha APT (%) | 1.22** | 071 | 086 | 070 | 090 | 1.09* | 0.71 | 1.01 | 0.71 | 0.90 | 0.89 | 0.57 | 0.96 | 0.66 | 0.82 |
| t-alpha APT | 2.67 | 1.49 | 1.82 | 2.62 | 2.68 | 2.74* | 1.65 | 1.84 | 2.74 | 2.97* | 2.93 | 1.53 | 1.92 | 2.50 | 2.77 |
| p-alpha APT | 0.01 | 0.14 | 0.07 | 0.01 | 0.01 | 0.00 | 0.07 | 0.01 | 0.00 | 0.00 | 0.00 | 0.13 | 0.06 | 0.01 | 0.01 |
| Adjusted R square | 0.66 | 0.64 | 0.65 | 0.81 | 0.75 | 0.79 | 0.72 | 0.64 | 0.84* | 0.83 | 0.80 | 0.77 | 0.70 | 0.85** | 0.82 |
4.3.2.3. Cumulative returns and relative returns

The log cumulative returns obtained on the 11 value-proxy weighted indices over the period 1\textsuperscript{st} January 1998 to 31\textsuperscript{st} December 2006 are illustrated in Figure 4.2, whereas the corresponding relative returns are depicted in Appendix C.5. The horizontal axis indicates date, while the vertical axis shows the log cumulative return on the value-style indices.

As Figure 4.2 shows, RESW100(4) remains the top performer in terms of total cumulative returns over time. The next strongest cumulative growth is displayed by RESW100(3), which has consistently produced cumulative returns above those of the ALSI from January 1998.

The cumulative returns earned on the BTMVW100 portfolios are ranked the 3\textsuperscript{rd} highest and are slightly better than those earned on the CFW100 portfolios. In fact, CFW100 has significantly underperformed the earlier period and only caught up with the BTMVW100 Index from around July 2004.

A distinguishable trend in the cumulative returns can be identified for all of the value indices no matter which proxy is used. After the market crash around September 1998, the indices recovered and demonstrated close to exponential growth from July 2003 to December 2006. It should be noted that the EAR weighted, EPS weighted, BV weighted and DIV weighted indices have displayed relatively more stable cumulative performance than the other value indices. Unlike the size-style indices, even the worst performing value index, EARW100, has yielded total cumulative returns above those of the ALSI over the period 1\textsuperscript{st} January 1998 to 31\textsuperscript{st} December 2006.
Figure 4.2: Log cumulative returns of the value-style indices
The graph displays the log cumulative returns for the 11 value-style indices that use different value-factor proxies and consist of the top 100 shares by MV over the period 1st January 1998 to 31st December 2006. The data are obtained from DataStream International at the University of Cape Town.
4.4 Style indices: momentum

4.4.1. Data and methodology
On the JSE, higher momentum was found to be associated with higher returns (Fraser and Page, 2000). This is generally referred to as the ‘momentum effect’. In a particular month, a momentum investor would either (1) assign more weights to the shares with superior past performance at the beginning of that month (which gives rise to the momentum-proxy weighted indices); or (2) construct an EW portfolio which includes only those shares with relatively high past returns (which is the principle underlying the construction of the EW momentum indices).

Two candidate proxies for the momentum-style factor are investigated. Van Rensburg (2001) stated that shares with higher past 12-month returns (MOM) are the best performers in the momentum cluster of style effects. Therefore, this thesis uses prior 12-month returns as a proxy to represent the momentum investment style. Furthermore, Kim et al (1991) and Exley et al (2004) also found that the most recent month’s share return shows a ‘reversal effect’. As a result, the past one year’s return excluding the latest month’s return (MOM(12-1)) is also investigated as a possibly better performing proxy. The calculation of MOM and MOM(12-1) is described in detail in Chapter Three.

The same principles that guided the construction of the individual firm characteristic-based value indices are applied to compute the momentum indices. Again, to cater for the absence of short selling, negative values are excluded from the constructed portfolios. To investigate the influence of this exclusion, a monthly rebalanced equally weighted portfolio of negative past 12 month return shares is also constructed.

The results in this chapter generally provide support for the observation that less frequent rebalancing tends to destroy value, reduce average returns and the Sharpe Ratio. Consequently, only monthly rebalanced MOM(12-1) portfolios are investigated. The detailed construction methods for the style portfolios are as follows:

Monthly rebalanced momentum-proxy weighted momentum index constituting top N shares by MV (MOMWN or MOM(12-1)WN)
The momentum-proxy weighted $N$ portfolios (where $N$ takes on the value of 100, 50 or 30) consist of the $N$ shares that have the largest MV on the JSE in each month. Portfolio components are updated at the beginning of each month for monthly rebalanced indices. Shares with MV and forward return entries missing are excluded from the sample of that month when constructing the momentum-style portfolio. If MOM is negative or not available for index constituent $i$, this constituent will be excluded from the sample of that month. Consequently, the momentum-proxy weighted $N$ indices have significantly less than $N$ constituents in some months.

The total index return in month $t$ is derived as a weighted mean of the total returns of the index constituents in that month, where the component weightings are in accordance to the corresponding momentum-proxy values. For instance, the formula to determine the value of the MOMW momentum $N$ index in month $t$ is thus:

$$R_{MOMWN,t} = \sum_{i=1}^{n_t} w_{i,t} \times R_{i,t}$$

(4.20)

Where:

- $R_{MOMWN,t}$ represents the monthly return of the MOM weighted portfolio in month $t$
- $N$ takes on values of 100, 50 and 30
- $n_t$ represents the number of index constituents in month $t$ (number of shares that ranked top $N$ by MV, and have non-negative MOM values)
- $R_{i,t}$ represents the forward monthly return of the $i$th index constituent in month $t$
- $w_{i,t} = \frac{MOM^{12},i}{\sum_{j=1}^{n_t} MOM^{12},i}$, represents the weight of the $i$th index constituent in month $t$.

The MOM(12-1)$WN$ indices are specified in the same way as the MOMWN indices, except that the role played by MOM is now undertaken by MOM(12-1).

**Monthly rebalanced EW momentum index constituting top $N$ shares by MV**

(EW$(\text{momentum-proxy})N$)

To construct an EW momentum portfolio with $N$ constituents, the 100 shares with the largest MV are sorted by momentum proxy in a descending order. For instance, the EW$(\text{momentum-proxy})50$ Index is based on the 50 shares that have attained the highest momentum proxy, being either MOM or MOM(12-1), out of the 100 shares
with the largest MV in each month respectively. The computation formula takes the same form as that of Equation (4.16) for the value indices, except that the role played by a value proxy is now undertaken by a momentum proxy. The following formula applies to the indices using MOM as the momentum-style proxy:

\[ R_{EW(MOM)N,t} = \frac{1}{N} \sum_{i=1}^{N} R_{i,t} \]  

(4.21)

Where:

- \( R_{EW(MOM)N,t} \) represents the monthly return of the EW momentum index in month \( t \)
- \( N \) represents the number of constituents in the index, takes on the value of 50 or 30
- \( R_{i,t} \) represents the forward monthly return of the \( i \)th index constituent in month \( t \).

**EW negative momentum index (EW(MOM)Negative)**

Fraser and Page (1999) pointed out that for shares that earned negative returns over the previous year, there was a reversal effect. Therefore the EW(MOM)Negative Index is constructed to inspect the performance of ‘past losers’. The returns in month \( t \) are obtained on an EW portfolio that consists of all the shares (out of the 100 largest capitalisation shares) with negative MOM figures at the beginning of month \( t \).

### 4.4.2. Empirical results

#### 4.4.2.1. Constructed indices

The monthly returns, number of shares per month and rebalancing percentages are computed for the 17 momentum-style indices, including five quarterly rebalanced indices, over the period 1\(^{st}\) January 1998 to 31\(^{st}\) December 2006. A more complete version of regression results are displayed in Appendix C.6.

#### 4.4.2.2. Summary statistics and regression analysis

Section A of Table 4.3 displays the summary statistics of the 12 momentum-style indices constructed by employing MOM and MOM(12-1) as the style-factor proxies over the period 1\(^{st}\) January 1998 to 31\(^{st}\) December 2006. CAPM and two factor APT regressions are performed on the 12 momentum-style indices, the results including
regression coefficients (betas), t-statistics and p-values of the excess returns and adjusted R² values are summarised in Sections B and C.

It is evident that all of the five indices computed utilising MOM(12-1) figures have outperformed their corresponding MOM12 versions. This seems to confirm the short term ‘reversal’ effect [Kim et al (1991), Fraser and Page (1999), Van Rensburg (2001) and Exley et al (2004)]. This indicates that MOM(12-1) may serve as an improved proxy for the momentum-style factor. In contrast, it is clear that the negative-momentum portfolios have significantly underperformed the other indices. They produce the lowest geometric means, Sharpe Ratios, Treynor Ratios and negative risk-adjusted returns under both the CAPM and the APT models. Therefore, there is no evidence supporting the premise that ‘past losers tend to become future winners’ on the JSE. Another general finding is that the equally weighted (EW) indices appear to underperform the characteristic weighted indices.

In particular, MOM(12-1)W30 shows the strongest performance in terms of gross (2.66%) and net (2.62% and 2.57%) average returns, which evidently exceed those produced by the best return-generating index constructed using MOM as the momentum proxy. However, its relatively volatile index returns caused by the smaller number of index constituents and high portfolio concentration in each month have resulted in a high standard deviation and, hence, a low Sharpe Ratio which ranked only 6th among all of the momentum indices analysed.

MOM(12-1)W50 is the next best performer in the case of geometric mean return (2.65% of gross return, 2.61% and 2.57% of net returns), slightly trailing the performance of the MOM(12-1)W30 Index. MOM(12-1)W50 also generates the leading CAPM-risk-adjusted excess return (1.12%, statistically significant at 5% level), closely followed by MOM(12-1)W30 (1.11%, statistically significant at 5% level) and MOM(12-1)W100 (1.02%, statistically significant at 10% level).

The MOM(12-1)W50 and MOM(12-1)W100 indices ranked first and second in APT-risk-adjusted excess returns (1.28% and 1.25% respectively). The alphas obtained are

---

41 MOMW50, with gross return of 2.51% and net returns of 2.48% and 2.44%.
significant at 5% level, demonstrating that both indices produced significant superior returns even after adjusting for risks using the APT model.

It should be noted that although the MOM(12-1)W30 Index has generated the highest average returns, it is not a suitable style index for the purpose of performance evaluation and index tracking due to severe lack of diversification. This is because in the worst month the number of index constituents is as low as two shares, and the maximum constituent holding is 72.65%. This clearly does not produce an acceptably diversified portfolio. A similar problem exists for the MOM(12-1)W50 Index, which has only six shares in the most depressed month over the period under consideration.

A lowest beta (0.867) and standard deviation (6.58%) among all of the 12 indices reflect that EW(MOM)50 has not only the lowest total return volatility but also the lowest risk relative to the market represented by the ALSI. The EW(MOM(12-1))50 Index has the second lowest beta (0.89) and standard deviation (6.76%). In general, the EW momentum indices have significantly lower standard deviation than their MOM and MOM(12-1) weighted counterparts.

Among all of the 12 momentum indices tested, EW(MOM(12-1))30 generates the most impressive Sharpe (0.22) and Treynor Ratio (0.017), followed by MOM(12-1)W50 with Sharpe Ratio of 0.20 and Treynor Ratio of 0.017. EW(MOM)50 yields the 4th highest Sharpe Ratio (0.197) among all indices examined, proceeded by the EW(MOM(12-1))50 Index (0.202). However, given that both EW(MOM)50 and EW(MOM(12-1))50 have displayed relatively low standard deviations, the outperformance in their Sharpe Ratios is more likely to be caused by low risks rather than high returns.

A momentum-proxy weighted N portfolio typically constitutes less than N number of constituents in each month. For instance, MOMW50 has on average 36 constituents, while the minimum number of constituents in one month is 12 for MOMW50 and 3 for MOMW30. This is caused by excluding the shares with negative (or incomplete) MOM and MOM(12-1) entries during the index constructions, which is likely to occur as a result of market depression over the last 12 months. The consideration of portfolio diversifications renders the MOMW30 and MOM(12-1)W30 indices, and to
a lesser extent the MOMW50 and MOM(12-1)W50 indices, unsuitable for performance measurement and index tracking.

The portfolios underlying the MOMW100 Index require minimum average rebalancing per month (32.95%). However, the turnovers experienced by all of the monthly rebalanced momentum indices are not significantly different. EW(MOM)50Q has the lowest turnover amount (17.51%) among the quarterly rebalanced portfolios. The average turnovers of the quarterly rebalanced indices are approximately 53%-60% of those of the corresponding monthly rebalanced indices. The rebalancing costs, however, are insignificant in comparison to the returns achieved on the indices, and hence the ranking of the indices' average returns is the same prior and post adjustment for transaction costs. Furthermore, all of the quarterly indices have lower cost-adjusted average returns and Sharpe Ratios than the corresponding monthly rebalanced indices. Therefore, it is safe to conclude that the decrease in transaction costs due to less frequent (e.g. quarterly) rebalancing is not enough to compensate for the decrease in cumulative returns.

The adjusted $R^2$ values are presented in the last rows of Sections B and C of Table 4.3. These figures reveal that the CAPM model explains from 56.4% (MOMW100) to 73.8% (EW(MOM)50) of the variation in excess returns on the JSE. The two factor APT model has higher adjusted $R^2$ for all of the indices examined, which implies that the APT model has the ability to explain a greater proportion of variation in the excess returns on the JSE than the CAPM model. Furthermore, the alphas' p-values of all indices have improved under the two factor APT model in comparison to the CAPM model.
Table 4.3: Candidate momentum-style indices (monthly data)

The table presents the descriptive and regression statistics of the 11 momentum (mom) style indices constructed over the period 1st January 1998 to 31st December 2006. Returns are monthly. The first three indices are constructed based on MOM weighted (MOMW) portfolios containing the 100, 50 and 30 shares with the highest market capitalisation (MV). The next two indices are constructed based on equally weighted (EW) portfolios using the top 50 and 30 shares ranked by MOM out of the 100 shares with the highest market capitalisation (MV) in each month. The next index is based on equally weighted portfolios consisting of all the shares with negative MOM values (out of the top 100 shares ranked by MV) in each month. The last five indices are based on monthly rebalanced EW (i.e. EW(MOM(12-1)) and MOM(12-1) weighted (MOM(12-1)W) portfolios using MOM(12-1) as the momentum-style proxy. The South African 90-day Banker’s Acceptance discount rate (RBAS) is used as the risk-free rate. P-values are calculated using two-tailed tests. The data are obtained from DataStream International at the University of Cape Town. Section A shows the descriptive and summary statistics for each of the 11 momentum style indices. Section B shows the single-index CAPM regression statistics, using the ALSI as the market proxy. The results are obtained by regressing the excess monthly returns of the market index on the excess monthly returns of each of the 11 momentum-style indices. Section C shows the two factor APT regression statistics, using FIND and RESI as the APT-factor proxies. The results are obtained by regressing the excess monthly returns of the APT factors on the excess monthly returns of each of the 11 momentum-style indices. All of the descriptive statistics, including the Sharpe Ratio and the Treynor Ratio, are calculated using gross geometric mean returns; whereas the cost-adjusted geometric means give an indication of the net returns that an investor is able to achieve by investing in the indices. In a particular row, if a higher value indicates better performance (e.g. the Sharpe Ratio), the maximum value among all the indices is indicated by **. The second highest value is marked by *. Similarly, if a lower value indicates better performance (e.g. standard deviation), the minimum value among all of the indices is followed by ** and the second lowest value followed by *. The selected ‘best performing’ index is highlighted in grey.

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<tr>
<th>Style Indices</th>
<th>MOMW100</th>
<th>MOMW50</th>
<th>MOMW30</th>
<th>EW(MOM)50</th>
<th>EW(MOM)30</th>
<th>EW(MOM)Negative</th>
<th>MOM(12-1)W100</th>
<th>MOM(12-1)W50</th>
<th>MOM(12-1)W30</th>
<th>EW(MOM(12-1))50</th>
<th>EW(MOM(12-1))30</th>
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<td><strong>Section A: Summary Statistics</strong></td>
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<td>Arithmetic mean (%)</td>
<td>2.90 2.92 2.96</td>
<td>2.41 2.54</td>
<td>1.46</td>
<td>2.95</td>
<td>3.07</td>
<td>3.12</td>
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<td>Geometric mean (%)</td>
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<td>2.17 2.25</td>
<td>1.15</td>
<td>2.51</td>
<td>2.65</td>
<td>2.66</td>
<td>2.24</td>
<td>2.51</td>
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<tr>
<td>Mean monthly rebalancing (%)</td>
<td>33.0 35.8 37.6</td>
<td>33.0 44.6</td>
<td>58.5</td>
<td>40.1</td>
<td>42.3</td>
<td>43.9</td>
<td>36.0</td>
<td>47.8</td>
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<td>Cost-adjusted geometric mean (10 bpt) (%)</td>
<td>2.43 2.48 2.47</td>
<td>2.14 2.21</td>
<td>1.09</td>
<td>2.47</td>
<td>2.61</td>
<td>2.62</td>
<td>2.21</td>
<td>2.46</td>
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<td>Cost-adjusted geometric mean (20 bpt) (%)</td>
<td>2.40 2.44 2.43</td>
<td>2.11 2.17</td>
<td>1.03</td>
<td>2.43</td>
<td>2.57</td>
<td>2.57</td>
<td>2.17</td>
<td>2.41</td>
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<td>Return/standard deviation ratio</td>
<td>0.28 0.29 0.27</td>
<td>0.33 0.32</td>
<td>0.14</td>
<td>0.28</td>
<td>0.30</td>
<td>0.29</td>
<td>0.33*</td>
<td>0.35**</td>
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<td>Sharpe ratio</td>
<td>0.18 0.19 0.18</td>
<td>0.20 0.19</td>
<td>0.03</td>
<td>0.18</td>
<td>0.20</td>
<td>0.19</td>
<td>0.20</td>
<td>0.22**</td>
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<td>Treynor ratio</td>
<td>0.02 0.02 0.01</td>
<td>0.02 0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02**</td>
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<td>No. of constituents</td>
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<td>30 50</td>
<td>29</td>
<td>68</td>
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<td>Average effective no. of constituents</td>
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<td>30 50</td>
<td>29</td>
<td>37</td>
<td>21</td>
<td>12</td>
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<td>30</td>
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<td>Maximum constituent holding (%)</td>
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<td>100.00</td>
<td>28.87</td>
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<td>72.65</td>
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<td>Alpha CAPM (%)</td>
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<td>-0.35</td>
<td>1.02</td>
<td>1.12**</td>
<td>1.11*</td>
<td>0.72</td>
<td>0.98</td>
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<td>t-alpha CAPM</td>
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<td>2.00 1.93</td>
<td>-0.69</td>
<td>1.79</td>
<td>2.19*</td>
<td>2.02</td>
<td>2.11</td>
<td>2.58**</td>
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<td>p-alpha CAPM</td>
<td>0.09 0.05 0.08</td>
<td>0.05 0.06</td>
<td>-</td>
<td>0.03</td>
<td>0.03</td>
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<td>0.04</td>
<td>0.01</td>
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<td>Beta CAPM</td>
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<td>0.99</td>
<td>1.05</td>
<td>1.06</td>
<td>1.13</td>
<td>0.89*</td>
<td>0.94</td>
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<td>Adjusted R square</td>
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<td>0.74** 0.70</td>
<td>0.58</td>
<td>0.58</td>
<td>0.64</td>
<td>0.64</td>
<td>0.73*</td>
<td>0.72</td>
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<td><strong>Section C: Two-factor APT model results</strong></td>
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<tr>
<td>Alpha APT (%)</td>
<td>1.20 1.12 1.00</td>
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<td>1.28**</td>
<td>1.18</td>
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<td>t-alpha APT</td>
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<td>3.50*</td>
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<tr>
<td>p-alpha APT</td>
<td>0.03 0.02 0.08</td>
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<td>-</td>
<td>0.02</td>
<td>0.01</td>
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<td>0.00</td>
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<tr>
<td>Adjusted R square</td>
<td>0.62 0.67 0.63</td>
<td>0.84* 0.76</td>
<td>0.62</td>
<td>0.64</td>
<td>0.68</td>
<td>0.64</td>
<td>0.85**</td>
<td>0.79</td>
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</table>
4.4.2.3. Cumulative returns and relative returns

The log cumulative returns of all the monthly rebalanced momentum indices over the period 1\textsuperscript{st} January 1998 to 31\textsuperscript{st} December 2006 are illustrated in Figure 4.3 below. The relative returns of MOMW100, MOM(12-1)W100 and EW(MOM)Negative are also depicted in Appendix C.7.

From Figure 4.3, MOM(12-1)W30 appears to produce the highest cumulative returns over time, followed very closely by MOM(12-1)W50. In the next cluster, MOMW50, MOM(12-1)W100, EW(MOM(12-1))30 and MOMW30, in the order of ranking, have all generated similar cumulative returns. MOM(12-1)W30 and, to a lesser extent, MOM(12-1)W50, have shown stable outperformance over the entire period of investigation. Some remarkable performance (which even exceeded that of MOM(12-1)W50) was experienced by the MOMW30 Index until 1\textsuperscript{st} June 2002.

The cumulative returns of six out of the 12 momentum indices are observed to constantly outperform the ALSI in all the month over the period of investigation. These indices are: MOM(12-1)W50, MOM(12-1)W30, EW(MOM(12-1))50, EW(MOM(12-1))30, MOMW50 and MOMW30. The other indices whose relative cumulative returns oscillated around 1 in the early years all surpassed and remained above the ALSI at the beginning of 2002. The only exceptions are the two negative momentum indices, which have constantly underperformed the ALSI and produced approximately half of the ALSI cumulative returns over the period of investigation.

Except for EW(MOM)Negative, all of the momentum indices displayed in Figure 4.3 follow a distinctive trend. Following the market crash around August 1998, the indices gradually recovered and profited from a notable market boom over the period October 2001 to April 2003. From thereon, the indices demonstrated exponential growth till the end of the period analysed.
Figure 4.3: Log cumulative returns of the momentum-style indices

The graph displays the log cumulative returns for the 11 monthly rebalanced momentum-style indices over the period 1st January 1998 to 31st December 2006. EW stands for equally weighted indices. The momentum-factor proxies used are MOM and MOM(12-1). The EW(MOM)Negative index constitutes all the shares among the top 100 shares by MV that have negative MOM values. The data are obtained from DataStream International at the University of Cape Town.
4.5 Relative performance in comparison to existing JSE style indices

The FTSE/JSE Africa Dividend Plus Index (the Dividend Plus Index) and the SA RAFI Index are the only two investable JSE style indices.

The Satrix DIVI fund tracks the FTSE/JSE Dividend Plus Index, which equally weights the 30 companies that have the highest one-year forecast cash DY among the companies constituting the FTSE/JSE Top 40 and FTSE/JSE Mid-Cap indices (excluding property companies)\(^42\).

The SA RAFI Index has been backward calculated by Plexus Asset Management, who offers the RAFI Enhanced SA Strategy tracking this fundamental index. The SA RAFI Index is constructed by applying Arnott’s (2005) fundamental indexation technique on the ALSI40 constituents. In constructing RAFI, four fundamental metrics are collected for each company, namely: sales, book value, gross dividends and cashflow. Subsequently, a fundamental index is constructed using each metric, where index constituents are selected and weighted according to the value of the metric. For instance, the Sale1000 Index is computed by ranking all companies by sales, then using the 1000 companies with the largest sales figures as index constituents and assigning weight to each constituent in proportion to their sales. Finally, the weights of each company in the four fundamental indices are combined in equal proportions to yield the composite weights. The RAFI is rebalanced annually on January 1\(^st\). Arnott pointed out that ‘the blend of multiple measures along with the use of multi-year smoothing’ mitigates exposure to extreme values of any of the indices by providing greater diversification across industries and sharply reduces rebalancing and taxation costs.

Being effectively two value-style indices, the JSE Dividend Plus Index and the SA RAFI Index are suitable comparison benchmarks to assess the performance of the fifty-five value indices that have been constructed in this thesis. At the time of writing,

\(^{42}\) In other words, the FTSE/JSE Africa Dividend Plus Index is effectively an EW top 30 value-style index that uses DY as its value-factor proxy to select eligible shares (i.e., EW(DY)30 in the notation adopted in this thesis).
only annual data was available to the author on the SA RAFI Index. Hence, the comparison is conducted using on annualised basis.

As displayed in Table 4.4, the JSE DIVI and SA RAFI indices have, indeed, outperformed the three broad capitalisation weighted JSE indices, namely the ALSI, the Top 40 Index and the FTSE/JSE Africa Shareholder Weighted Top 40 Total Return Index (the SWIX Index). Both of these two value indices, however, are in turn significantly outperformed by some of the constructed style indices, such as the RESW100(3) and MOM(12-1)W100 indices. The line graph of the cumulative returns on the above mentioned indices are plotted in Appendix C.8.

### Table 4.4: Comparing annualised statistics on constructed and benchmark indices

The table compares the major statistics of the constructed style indices to those of the five existing JSE indices over the period 1st July 2002 to 31st December 2006. All results are annualised except that the rebalancing percentages are calculated as monthly effective. The first index is the value-style index constructed using the residuals of a 3-factor regression based on earnings, book values and dividends as the value-style proxy. The second index is the momentum-style index constructed using the past 12-month but excluding the latest month’s return (MOM(12-1)) as the momentum-style proxy. JSE DIVI stands for the FTSE/JSE Africa Dividend Plus Index, RAFI SA stands for the Plexus South African/JSE RAFI Index. The other three benchmark indices explored are the ALSI, the Top 40 Index and the SWIX Index. The South African 90-day Banker’s Acceptance discount rate (RBAS) is used as the risk-free rate.

<table>
<thead>
<tr>
<th>Style indices (Annualised results)</th>
<th>RESW100 (3)</th>
<th>MOM(12-1)W100</th>
<th>JSE DIVI</th>
<th>RAFI SA (PLEXUS)</th>
<th>JSE ALSI</th>
<th>JSE Top 40</th>
<th>JSE SWIX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric mean (%)</td>
<td>36.98</td>
<td>41.21</td>
<td>26.94</td>
<td>30.80</td>
<td>24.55</td>
<td>22.85</td>
<td>27.37</td>
</tr>
<tr>
<td>Mean monthly rebalancing (%)</td>
<td>26.8</td>
<td>40.1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Cost-adjusted geometric mean (10 bpt)</td>
<td>36.66</td>
<td>40.73</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Cost-adjusted geometric mean (20 bpt)</td>
<td>36.34</td>
<td>40.25</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Standard deviation (%)</td>
<td>16.42</td>
<td>17.29</td>
<td>16.42</td>
<td>18.60</td>
<td>17.75</td>
<td>18.80</td>
<td>16.41</td>
</tr>
<tr>
<td>Return/standard deviation ratio</td>
<td>2.25</td>
<td>2.38</td>
<td>1.64</td>
<td>1.66</td>
<td>1.38</td>
<td>1.22</td>
<td>1.67</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>1.54</td>
<td>1.70</td>
<td>0.92</td>
<td>0.70</td>
<td>0.72</td>
<td>0.59</td>
<td>0.95</td>
</tr>
<tr>
<td>Average no. of constituents</td>
<td>40</td>
<td>68</td>
<td>30</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>-</td>
</tr>
</tbody>
</table>

#### 4.6 Correlation of the style indices

The most suitable index of each investment style is selected to serve as the explanatory variable in the return-based style analysis in Chapter Five. Given that EW(size)100 and MOM(12-1)W100 both produce superior returns and are representative of their own style category, the decision regarding the most appropriate size- and momentum-style indices is relatively easy to make.

The value indices, however, are subject to potential unreliability due to the use of accounting data in their construction. Although RES weighted and BTMV weighted indices have both produced similar returns, the RES weighted indices have delivered
lower standard deviation. Furthermore, the RESW indices are more reliable and smoothed as a result of the use of multiple firm attributes. Therefore, RES seems to be the most suitable value proxy. As discussed previously, the RES(3) appears to be the best performing index after a balanced consideration of returns achieved and the benefits of better portfolio diversification.

Sharpe (1992) suggested that it is desirable that the variables utilised in the return-based style regressions have relatively low correlations with each other. Therefore a correlation matrix of the potential candidate value-style indices is constructed to aid the selection. Table 4.5 shows the correlation between the excess returns on the candidate style and sector indices after deducting market returns\(^43\) from each index over the period 1\(^{st}\) January 1998 to 31\(^{st}\) December 2006. It is observed that the RESW indices in general have lower correlation to the chosen size and momentum indices than the other value indices. Therefore, choosing a RESW value index will result in a more exhaustive and less correlated set of explanatory variables for the return-based style decomposition analysis conducted in Chapter Five. Returns obtained on the EPS weighted portfolios seem to move the least with those obtained on the other selected portfolios.

\(^{43}\) JSE All Share Index (ALSI) is used as the market proxy before 1\(^{st}\) January 2002, and JSE Shareholder Weighted All Share Index (SWIX) is used as the market proxy thereafter.
Table 4.5: Correlation matrix of candidate style indices

The table displays the correlations between the style- and sector-indices which are potential candidates for independent variables of the return-based style analysis in Chapter Five. Correlations are calculated between the excess returns on the style indices after deducting market returns from each index. The FTSE/JSE Africa All Share Total Return Index (the ALSI) is used as the market proxy. Correlations between the style and sector indices are computed over the period 1st January 1998 to 31st December 2006, whereas correlations between the style/sector indices and SWIX are computed over the period 1st January 2002 to 31st December 2006. The JSE index data are obtained from I-Net Bridge at the University of Cape Town.

<table>
<thead>
<tr>
<th></th>
<th>EW100</th>
<th>EWY100</th>
<th>EWY30</th>
<th>EYW30</th>
<th>EWY100</th>
<th>EYW30</th>
<th>EYW100</th>
<th>EPSW50</th>
<th>EPSW100</th>
<th>EASW100</th>
<th>BWV100</th>
<th>BWV100</th>
<th>ESW100</th>
<th>ESW100</th>
<th>FIN115</th>
<th>INDI2S</th>
<th>RESI</th>
<th>SWIX</th>
<th>ALSI</th>
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<td>0.067</td>
<td>0.911</td>
<td>0.664</td>
<td>0.758</td>
<td>0.284</td>
<td>0.551</td>
<td>0.592</td>
<td>0.615</td>
<td>0.759</td>
<td>0.597</td>
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<td>0.807</td>
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<td>0.600</td>
<td>0.613</td>
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<td>0.292</td>
<td>0.574</td>
<td>0.278</td>
<td>1.000</td>
<td>0.882</td>
<td>0.506</td>
<td>0.454</td>
<td>0.283</td>
<td>0.044</td>
<td>0.218</td>
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<td>0.652</td>
<td>1.000</td>
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<td>0.513</td>
<td>0.406</td>
<td>0.492</td>
<td>0.211</td>
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<td>0.753</td>
<td>0.680</td>
<td>0.454</td>
<td>0.602</td>
<td>0.777</td>
<td>1.000</td>
<td>0.814</td>
<td>0.599</td>
<td>0.601</td>
<td>0.211</td>
<td>0.259</td>
<td>0.241</td>
<td></td>
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<tr>
<td>BTVMV100</td>
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<td>0.717</td>
<td>0.482</td>
<td>0.786</td>
<td>0.283</td>
<td>0.500</td>
<td>0.513</td>
<td>0.818</td>
<td>1.000</td>
<td>0.677</td>
<td>0.697</td>
<td>0.245</td>
<td>0.217</td>
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<td>0.807</td>
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<td>0.344</td>
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<td>1.000</td>
<td>0.372</td>
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<tr>
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<td>0.813</td>
<td>0.565</td>
<td>0.787</td>
<td>0.218</td>
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<td>0.492</td>
<td>0.601</td>
<td>0.697</td>
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<td>0.259</td>
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<td>-0.609</td>
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<td>SWIX</td>
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<td>-0.336</td>
<td>0.058</td>
<td>-0.147</td>
<td>-0.166</td>
<td>0.070</td>
<td>0.050</td>
<td>-0.369</td>
<td>-0.240</td>
<td>-0.272</td>
<td>-0.215</td>
<td>0.045</td>
<td>0.239</td>
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</tr>
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</tr>
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</table>
4.7 Summary and conclusion

This chapter investigates the performance and characteristics of the candidate style indices of the three investment styles identified on the JSE. Index constructions are conducted using the monthly total share return data obtained from the DataStream and the JSE index values obtained from the I-Net over the period 1st January 1998 to 31st December 2006.

EW and style-proxy weighted portfolios are constructed. Assumptions regarding transaction costs are applied to the gross portfolio returns to generate cost-adjusted results. Thereafter, CAPM and APT regressions are performed to assess each index’s ability to generate risk-adjusted excess returns. The Sharpe and the Treynor ratios are adopted to summarise the indices’ risk-return tradeoffs. Finally, the maximum turnover percentage and the effective number of shares give an indication of the concentration of the constructed portfolios.

4.7.1. Size-style indices

Market capitalisation is adopted as a proxy for the size style. A comparison among the five size-style indices computed reveals that both the EW(size)100 Index and the Small Cap Index have produced overall cumulative returns in excess of the ALSI return over the period of investigation.

The EW(size)100 Index showed the most outstanding performance in terms of gross and cost-adjusted average returns over the period analysed.

The Small Cap Index is the next best performer, characterised by the top Sharpe and Treynor Ratios, and the lowest standard deviation and beta. Furthermore, the Small Cap portfolios are also able to generate the maximum CAPM- and APT-risk-adjusted excess returns. However, the outstanding statistics regarding the Small Cap Index are primarily a result of strong performance from 2003 onwards. The Small Cap Index has, in fact, considerably underperformed the other indices as well as the ALSI over the period January 2000 to July 2003.
Furthermore, it is noted that the Top 40 Index showed a distinctive pattern over the period of investigation. This is further illustrated by its low correlation with the other indices. The Top 40 Index remained the best performer over the period January 2000 to December 2003 while all the other indices performed poorly relative to the ALSI.

The high p-values of all regressions conducted, however, suggest that none of the abnormal returns generated by any of the size-style indices are significantly different from zero at the 10% significance level. Thus there is no significant relationship between the size style and portfolio performance over the period of investigation. This agrees with the findings of most prior studies, stating that there is no evidence of a size effect on the JSE [Bradfield et al (1988) and Page and Palmer (1991)].

It is concluded that given their return-generating ability, both the monthly rebalanced EW(size)100 Index and the Small Cap Index qualify as the ‘best performing’ size-style index, representing the ‘small capitalisation’ and ‘large capitalisation’ size investment style respectively. In Chapters Five and Six, however, the EW(size)100 Index is selected as the preferred size index. In contrast, the Small Cap Index is used primarily for the purposes of comparison as it is recognised that due to liquidity constraints this index is not well suited to being a member of the portfolio construction toolkit.

4.7.2. Value-style indices

The chapter then undertakes to assess and compare the performance of the value-style indices. The ability of a single firm attribute as a proxy for the value-style factor is investigated separately for eight firm attributes, including $EY$, $BTMV$, $CF$, $DIV$, $SALE$, $EPS$, $EAR$ and $BV$. Furthermore, $RES(K)$ from three types of multi-factor regressions are employed as further value proxies, indicating the relative cheapness of each share.

It is found that all of the value indices have outperformed the ALSI over the period investigated. This confirms the value effect identified on the JSE by most literature [Basu (1977, 1983), Plaistowe and Knight (1987), Page and Palmer (1991), Page (1996) and Fraser and Page (2000)].
RES, particularly RES(4) and RES(3), and BTMV are the best performing value proxy over the period 1st January 1998 to 31st December 2006. In general, RES appears to be superior to all the other value proxies examined, whereas BTMV is the top proxy among all eight of the single factor firm attributes investigated. On the other hand, although EAR, EPS, BV, CF and DIV seem to be inadequate value proxies when generating superior returns, they appear to produce more stable portfolios with considerably lower turnover amounts.

The RESW100(4) Index is clearly the overall best performer, characterised by the highest geometric mean, cost- and risk-adjusted returns and the Treynor Ratio. It also displays relatively low risk: the second lowest standard deviation and the lowest CAPM beta. The next best performing index following RESW100(4) in all aspects of performance is RESW100(3). EW(BTMV)30 has exhibited the third fastest cumulative growth over the period of investigation.

In summary, it is clear that the residual weighted indices, particularly RES(4) and RES(3), are the best performing value proxies over the period 1st January 1998 to 31st December 2006. As the number of fundamental firm attributes used in the multi-factor regression increases, the returns generated appear to increase whereas the number of index constituents seems to decrease sharply. As a result, RESW100(3) is chosen for the analysis in the following chapters to strike a balance between high return, acceptable portfolio diversification and rebalancing frequency.

Finally, it should be noted that all of the value-index results need to be viewed with caution, bearing in mind the lack of reliability, stability and availability of the accounting information. However, this is less of a concern for the RESW indices, which are more reliable as a result of using logged multiple firm attributes when deriving the RES values.

4.7.3. Momentum style indices

MOM and MOM(12-1) are employed as momentum proxy are to compute ten monthly rebalanced momentum-style indices. All of the momentum-style indices constructed, except for the EW(MOM)Negative Index, have generated significant excess returns relative to the ALSI.
The five indices constructed using MOM(12-1) appear to outperform those corresponding indices constructed using MOM in all aspects. Therefore, it appears that *past 1-year share returns excluding the return in the latest month (MOM(12-1)) is a superior proxy for the momentum style-factor than the return of the past 12 months*. In contrast, the negative-momentum portfolios have significantly underperformed the other indices, and hence, there is no evidence supporting the premise that past losers tend to become future winners on the JSE. Another general finding is that the equally weighted (EW) indices appear to underperform the characteristic weighted indices.

MOM(12-1)W30 has produced the leading gross and cost-adjusted average returns among all of the momentum indices constructed. It has also generated CAPM- and APT-aldhas (2\textsuperscript{nd} and 3\textsuperscript{rd} highest respectively) significant at 5% level, suggesting that the index is able to generate statistically significant risk-adjusted excess returns. The 2\textsuperscript{nd} highest average returns (both prior and post adjustment for transaction costs) and the highest CAPM- and APT-risk-adjusted excess returns (both significant at 5% level) are generated by the MOM(12-1)W50 Index.

Both of the above two indices have consistently outperformed the ALSI over the entire period of investigation. However, it should be noted that these two indices are unsuitable candidates for the purpose of performance analysis due to their small number of index constituents in certain months and the highly concentrated portfolios resulting. For instance, the minimum number of index constituents in a particular month is two for the MOM(12-1)W30 Index and six for the MOM(12-1)W50 Index.

Although slightly trailing, MOM(12-1)W100 mirrors the above two indices quite closely in all aspects of performance (average returns, the Sharpe and the Treynor Ratios and risk-adjusted excess returns). Even though its overall cumulative return only ranked the 3\textsuperscript{rd} among the ten momentum-style indices, the MOM(12-1)W100 Index is characterised by more stable and diversified style portfolios in each month.

Lastly, according to results in Appendices C.9, turnover percentages between the monthly and quarterly rebalanced indices do differ. Due to the small amount of
transaction costs, however, there is no difference on each index's performance ranking prior and post adjustment for the transaction costs.

4.7.4. Selected indices for return-based style analysis

The 'best performing' and most representative indices identified for each investment style are as follows:
- the Small Cap Index and the EW(size)100 Index for the size investment style. These two indices have generated the highest, though not significant, excess returns among all size-style indices constructed
- RESW100(3) for the value investment style. This index has produced the second highest return among all of the value indices, but is more representative than RESW100(4). It also has relatively low correlation to the other selected indices
- MOM(12-1)W100 for the momentum investment style. It ranked the third best performer, and is the most suitable candidate momentum index after taking portfolio diversification into consideration.

These four indices are employed for the style-return decomposition in Chapter Five and the portfolio optimisation analysis in Chapter Six.
5. Replicating Active Equity Portfolios

"...for the most part South African funds were unable to outperform the market, once exposure to market, value and size anomalies were taken into account".

- Scher and Muller (2003)

5.1 Introduction

It is widely claimed that the dominant source of an equity fund’s performance is its investment style and strategic sector allocation. In other words, a prominent portion of the returns achieved by an equity fund can be attributed to the joint effect of its chosen exposures to different investment styles and equity sectors.

Consequently, Sharpe (1995) suggested that a relatively limited number of key factors representative of investment styles and equity sectors may be identified to explain the majority of the return variations on a typical equity fund. These key factors collectively capture the fund manager’s style. Therefore one can replicate a fund’s style by passively investing in these key factors in appropriate proportions. Since its introduction by Sharpe in 1988, the returns-based style analysis has provided a standard methodology to establish a fund’s investment style. It uses past returns over a specified time period to determine the weights to the selected key style and sector factors which most closely replicate the actual performance of a fund. The returns obtained on portfolios thus constructed are termed ‘style return’ in this chapter\textsuperscript{44}.

The remaining portion of the fund’s returns in excess or deficit of the style return is accounted for by the choice of specific securities. It indicates the manager’s stock selection and market timing ability, and is therefore termed ‘selection return’. Overall, a fund’s returns can be attributed to the return due to the fund’s specific investment styles (style returns) and the returns as a result of the manager’s active stock selection not captured by his investment style (selection returns). A passive manager provides

\textsuperscript{44} The definition of style return in Chapter Five refers to the return as a result of both a fund’s style exposure (as defined in Chapter Four) and its sector exposure.
investors with style, whereas a successful active manager provides both style returns and selection returns.

The four ‘active/style indices’ selected in Section 4.7.4 of Chapter Four and the three broad sector indices representative of the Johannesburg Stock Exchange (JSE) (Van Rensburg, 2001) are used as a ‘portfolio toolkit’ to replicate a sample of South African unit trusts and hedge funds over the years 2002 through 2006. The equity indices selected to act as the independent variables in the return-based style decomposition include the three selected style indices, namely EW(size)100 for the size investment style, RESW100(3) for the value investment style and MOM(12-1)W100 for the momentum investment style. In addition three sector indices are selected, the FINI 15 representing the financial sector, the INDI 25 representing the industrial sector and the RESI 20 representing the resources sector. Each of the sector indices currently has an ETF representing them.

This chapter examines whether the South African (SA) active domestic equity fund managers can produce significant excess returns after their investment styles are taken into account. This is equivalent to test whether the returns of the active fund can be replicated by simply investing in a weighted portfolio of representative passive indices. Mean selection returns, tracking errors, means and the Sharpe Ratios of style returns, and out-of-sample R² values are the main statistics examined for this purpose.

The rest of this chapter is set out as follows: Section 5.2 describes the data and methodology used for the return-based style decomposition and the selection return calculations. Section 5.3 displays the empirical output on some of the most representative SA unit trust and hedge fund indices as well as on a selected set of top performing domestic equity unit trusts. Section 5.4 summarises and concludes.

5.2 Data and methodology

5.2.1 Overview

A fund’s total return is attributable to (1) a fund’s investment style (style return) and (2) the fund manager’s stock picking ability (selection return). Strong evidence shows that style return is the primary source of fund performance and selection return is
often negligible over the long run. Therefore this chapter sets out to replicate an active fund’s *style return*\(^{45}\) by adopting Sharpe’s return-based style decomposition technique. A fund’s actual returns are subsequently tested for outperformance against its style returns by assessing the t- and p-values of the *selection returns*. In the empirical analysis the following steps are repeated for each active equity fund on a monthly rolling basis to replicate and assess the fund’s performance:

**Step 1. Return-based style analysis:** the first step is to estimate the style of each fund over a *certain* time period. Sharpe (1988) showed that a fund’s investment style can be represented by the fund’s combined exposure to a few key factors. In other words, the return on a portfolio comprising selected style and sector indices held in appropriate proportions can replicate a fund’s style return. The composition of this replicating portfolio can be estimated using Sharpe’s (1988) return-based style analysis. *Selected* style- and sector-index returns are regressed upon historical fund returns utilising a multi-factor regression. The resulting beta coefficients give the constituent weights of the portfolio that replicates the fund’s historical investment style. Since a fund’s investment style changes over time, estimated fund style represented by weights of the selected indices is the *average historical* investment style of the fund *over the period of regression*.

**Step 2. Out-of-sample prediction model:** assuming that there is no style shift between the period used to estimate a fund’s investment style (e.g. previous 36 months, i.e., month t-36 to t-1) and the subsequent evaluation period (month t), the fund’s style return in month t can simply be *forecasted* by the return on the style *portfolio* comprising the selected style and sector indices held in proportions inferred using the return-based style analysis in Step 1. A fund’s selection return in month t is calculated as the difference between the observed fund return and the predicted style return in that month. It can be interpreted as the value added by the fund manager’s active security selection activities that are independent of *his* investment style.

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\(^{45}\) This paper mainly examines the broad sector *indices* of SA domestic equity unit trusts, hedge funds and pension funds. However, from hereon, these indices will be referred to as *funds* to prevent the potential confusion with the selected key style- and sector-index used as independent variables for the return-based style regressions.
Step 3. Summary statistics: rolling regressions are conducted by repeating Steps 1 and 2 for each successive 36\(^{46}\) month-period to obtain a time series of investment styles, style returns and selection returns for each fund. Related summary statistics such as mean, standard deviations and t- and p-values of the selection returns are subsequently computed to investigate if the selection returns are significantly different from 0. Furthermore, regressions of the fund's actual returns against its style returns are also conducted to derive the out-of-sample R\(^2\) values which indicate how closely the style portfolios are able to track the performance of the actual fund.

5.2.2. Dataset

The equity indices selected to act as the independent variables in the return-based style decomposition include (1) the three selected style indices, namely EW(size)\(^{100}\) for the size investment style, RESW100(3) for the value investment style and MOM(12-1)W100 for the momentum investment style, and (2) three sector indices with FINI 15 representing the financial sector, INDI 25 representing the industrial sector and RESI 20 representing the resources sector. Each of the sector indices currently has an ETF representing them.

The unit trust and hedge fund data under consideration is described in detail in Section 3.4 of Chapter Three. For unit trusts, the results of three broad sector indices, namely domestic general equity (DOEQ), domestic equity growth (DOEQGR) and domestic equity value (DOEQVL), are discussed in detail. In addition, eleven domestic equity unit trust funds are examined which form a parsimony representation of the SA unit trust industry. A minimum of four years' data is required for a unit trust to be included in the analysis\(^{47}\). Finally, four hedge fund sector indices are obtained and analysed, namely Single Manager Composite (COMP), Long Short Equity Index (LSE), Market Neutral and Quantitative Strategies Index (MKN), and Fund of Funds Index (FOFs). The list describing the indices and funds used, their inception dates, as well as the short-hand notations, are presented in Appendix D.1.

\(^{46}\) For unit trusts. 24-month for hedge funds.

\(^{47}\) For the rolling-style regressions, a rolling period of three years is chosen to infer the investment style of unit trusts and pension funds\(^{2}\) (E.g. returns used are from month t-36 to t-1), therefore minimum four years' data are needed if a unit trust fund is to be included in the analysis.
5.2.3. Return-based style analysis

Sharpe's return-based style analysis methodology is adopted to infer the funds' investment styles. Intuitively speaking, the ultimate aim of this method is to find a set of weights representative of the 'behaviour' of the actual fund based solely on the co-movements of the fund’s returns with those of the selected style and sector indices. The resulting style portfolio reflects the 'behaviour' of the fund, but does not necessarily reproduce the precise asset and sector composition of the fund. Moreover, style analysis only provides an estimate of a fund's average style over a period of several months, not a fund’s exact style on a particular day.

5.2.3.1. Methodology: multi-factor regressions

Sharpe (1988) employed a multiple-factor regression model to identify the appropriate weights of the selected indices that move most closely with the fund in terms of monthly performance. According to Sharpe, the ordinary least square (OLS) multiple-regression model is as follows:

\[ R_{FUND,t} = \sum_{p=1}^{P} w_p R_{p,t} + e_{FUND,t} \]  

(5.1)

Where:

- \( R_{FUND,t} \) represents the actual monthly return on fund \( i \) in month \( t \)
- \( R_{p,t} \) represents the monthly return of the \( p \)th equity index in month \( t \)
- \( P \) represents the number of key factors (i.e., style and sector indices) to be used to replicate the fund’s investment style.
- \( w_p \) represents the weightings of the fund’s return of the \( p \)th equity index
- \( e_{FUND,t} \) represents the fund’s residual return after adjustment for its investment style, i.e., the in sample selection return in month \( t \) for fund \( i \).

Historical fund returns are used as the dependent variable and the key factor returns serve as the independent variables. The resulting slope coefficients (i.e., the \( w \)s) can be interpreted as the fund’s historical exposures to the selected key factors. A fund’s collective exposures to the key factors constitute its investment style. Therefore, the
summation term in Equation (5.1) gives the in-sample style returns and $e_{\text{FUND},i}$ represents the in-sample selection returns.

The standard deviation of $e_{\text{FUND},i}$ is termed the fund's tracking error, and gives an indication of how well the style returns have tracked the actual fund returns during the in-sample period. The objective of the style-decomposition is to select a set of coefficients (i.e., $w_k$) that minimise the unexplained variation in returns (the variance of $e_{\text{FUND},i}$) and maximised the associated $R^2$ value.\footnote{Minimisation subject to stated constraints is described in detail in Section 5.2.3.4.}

A fund's investment styles through time are estimated by performing a series of 'rolling' regressions on a fixed number of months with the starting time moving forward by one month for each analysis. A fixed period of three years is chosen to infer the investment style of unit trusts. A 36 month period is short enough to capture considerable style movements, while long enough to avoid excessive noise in the data. A minimum of four years' data is required for a unit trust to be included in the analysis. The SA hedge fund indices, however, are newly established and only have a very short history going back to January, 2004. Consequently, a two-year fixed period is used for hedge funds, leaving one year's data for conducting the out-of-sample predictions. The results of the rolling regression are plotted in the exposure distribution area graphs which illustrate the changes in a fund's investment style over time. Since the style portfolio is rebalanced monthly, the style returns do cover the value added by a manager's style rotation insights.

Finally, the regression over the maximum available period of the fund returns (from 1\textsuperscript{st} January 1998 or fund inception to 31\textsuperscript{st} December 2006) gives the average style of a fund over the period covered (Sharpe, 1988). The resultant constituent weights are plotted in the average style bar charts. It should be noted that the 'average style' regression tend to mask changes in a fund's style and hence it only produces meaningful results when the fund's style is relatively consistent over time.
5.2.3.2. Choice of independent variables

Sharpe (1992) suggested that ‘one of the most essential elements in performing return-based style analysis is to select the appropriate independent variables that are able to emulate the actual funds’ investment styles as closely as possible’. This requires the variables as a whole to be relatively exhaustive so that they fully capture the style and sector exposure of the fund. Otherwise, the regression will have trouble pinning down an explanatory index that consistently explains the fund’s behaviour from period to period, and will flip-flop between those that temporarily provide a best fit. This generally results in a poorly fitted model characterised by low $R^2$ values.

The independent variables are typically proxied by representative style and sector indices. The seven indices adopted in this thesis (four constructed style indices and three JSE sector indices) meet the requirement of comprehensiveness by capturing most risks on the JSE.

To be specific, the four style indices used in Equation (5.1), as summarised in Section 4.7.4 of Chapter Four, are: (1) EW(size)100 and the FTSE/JSE Africa Small Cap Total Return Index (the Small Cap Index) for size style, (2) RESW100(3) for the value style and (3) MOM(12-1)W100 for the momentum style. They are selected based on a balanced consideration of performance, representativeness, portfolio diversification and ease of construction.

The value index chosen is one with low correlations with the other style indices. Using differentiated style indices reduces the unnecessary fluctuation of the style weights calculated. Otherwise, the model may struggle to distinguish between two or more highly correlated indices, and therefore the regression coefficients may oscillate among different indices with similar returns (Sharpe, 1995).

After selecting the style indices, a set of sector indices are also added to make the list of explanatory variables as exhaustive as possible. Van Rensburg (2000) concluded that the returns generated from the resources, financial and industrial sectors can explain approximately 80% of the returns obtained on the JSE; hence a set of indices

49 Furthermore, an ‘investment style’ not explicitly included in the explanatory variable set can be represented by the ones that are included.
broadly representing these three sectors is utilised. Moreover, it is desirable that the factor proxies chosen can be traded directly. This is achieved by using returns on relevant Satrix Exchanged Traded Funds (ETFs). However, even the longest established ETF, SATRIX FINI, was only launched in February 2002. Thus while the ETF data are used as far back as possible, the corresponding JSE indices underlying the ETFs are used to ‘fill up’ the earlier total return history from January 1998 till the relevant ETF comes into existence. For instance, the FTSE/JSE Africa Financial 15 Total Return Index (FINI) is integrated with STXFIN\textsuperscript{50}, the FTSE/JSE Africa Industrial 25 Total Return Index (INDI) with STXIND\textsuperscript{51} and the FTSE/JSE Africa Resources 20 Total Return Index (RESI) with STXRESI\textsuperscript{52} to produce the series of total returns data that serve as the independent variables for the style-return constrained regression analysis\textsuperscript{53}.

Finally, it should be noted that although adding the Small Cap Index results in a more complete list of independent variables and therefore theoretically increases the explanatory power and performance of the predicted style returns; the Small Cap Index is not tradable for most of the unit trusts due to the illiquidity of the shares constituting the index. As a result, for unit trusts, despite the fact that seven-factor regressions are also conducted to gauge the impact of the inclusion of the Small Cap Index on the explanatory power of the model, only the strategies excluding the Small Cap Index can be realistically implemented. On the other hand, hedge funds have more flexible mandates and in general can invest in less liquid and smaller companies. Thus regressions with the Small Cap Index more realistically reflect hedge funds’ true investment positions. The number of independent variables ($n$ in Equation (5.1)) is six when the Small Cap Index is excluded and seven when the Small Cap Index is included.

\textsuperscript{50} JSE FINI 15 is used for the period 1\textsuperscript{st} January, 1998 to 1\textsuperscript{st} February 2002. STXFIN is used for the period 1\textsuperscript{st} March, 2002 to 31\textsuperscript{st} December 2006. There is no chain-linking.

\textsuperscript{51} JSE INDI 25 is used for the period 1\textsuperscript{st} January, 1998 to 1\textsuperscript{st} February 2002. STXIND is used for the period 1\textsuperscript{st} March, 2002 to 31\textsuperscript{st} December 2006. There is no chain-linking.

\textsuperscript{52} JSE RESI 20 is used for the period 1\textsuperscript{st} January, 1998 to 1\textsuperscript{st} April 2006. STXRES is used for the period 1\textsuperscript{st} May, 2006 to 31\textsuperscript{st} December 2006. There is no chain-linking.

\textsuperscript{53} These data series, shorthand notations and their meanings are described in detail in Sections 3.3 and 3.4.1 of Chapter Three: Data.
5.2.3.3. OLS and WLS

With no further constraints on the coefficients, Equation (5.1) effectively conducts a zero-constant-term OLS regression to minimise the tracking error.

A practical modification suggested by Sharpe (1992) is applied in this thesis which places greater emphasis on returns achieved in more recent months than those achieved in more distant months when inferring a fund’s style decomposition. This entails minimising the fund’s weighted tracking error over the regression period. If a period of \( T \) months is used to estimate a fund’s investment style, the returns in each month are assigned a weight equal to \( 2^{1/T} \) times the weight assigned to its predecessor in the prior month resulting in a ‘half-life’ of \( T \) periods i.e. the weighting assigned to period 1 is half that of period \( T \). The following series illustrates the resultant weights used on the historical returns if the regression period is 36 months.

<table>
<thead>
<tr>
<th>Month(s)</th>
<th>( t-36 )</th>
<th>( t-35 )</th>
<th>( t-34 )</th>
<th>...</th>
<th>( t-1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression Weights (Vs)</td>
<td>1</td>
<td>( \frac{1}{2^{36}} )</td>
<td>( \frac{1}{2^{35}} )</td>
<td>...</td>
<td>( \frac{1}{2^{36}} )</td>
</tr>
</tbody>
</table>

The resulting model is equivalent to using a weighted least square (WLS) regression. Therefore Equation (5.1) for OLS regressions is modified as follows:

\[
R^*_{Y/NDJ} = \sum_{p=1}^{P} w_p R^*_{p,j} + e^*_{Y/NDJ} \tag{5.2}
\]

Where:

- all symbols are defined in the same way as in Equation (5.1), only that 
  \( R^*_{p,j} = R_{p,j} \times v_t \), \( v_t \) represents the regression weights in month \( t \).

Both OLS and WLS regressions are conducted in this chapter, and their outputs are compared to evaluate which method is superior for the purpose of replicating funds’ investment styles.

5.2.3.4. Constraints of regressions

Several constraints are imposed on Equations (5.1) and (5.2) so that the style weights compare more closely to the actual investment policy of the funds under consideration.
Regressions with appropriate constraints are more likely to produce meaningful coefficients with out-of-sample data.

**Unit Trusts**

Unit trust managers are prevented by their mandates from taking short or leverage positions. Therefore a fund’s exposure to each key factor \( w_p \) has to lie between 0 (long only) and 1 and the weights have to sum to unity (no leverage). Given these constraints, a quadratic programming (QP) algorithm similar to that proposed by Sharpe (1992) is followed to solve for the style weights that minimise the variance of the in-sample residuals subject to all of the constraints mentioned above.

Put numerically, Equation (5.1) becomes the following:

\[
R_{\text{FUND}, t} = \sum_{p=1}^{P} w_p R_{p,t} + e_{\text{FUND}, t}
\]

(5.3)

Subject to the constraints:

\[
\sum_{p=1}^{P} w_p = 1 \text{ and } w_p \geq 0 \text{ for all } p=1 \ldots P.
\]

In each month, the above equation is estimated using quadratic programming.

**Hedge Funds**

Firstly, hedge funds can employ both net long and net short positions, therefore the constraint of style weights to be between 0 and 1 is removed. Secondly, they can use proceeds from their short sales to fund further long positions, therefore the constraint of weights sum to one can be discarded. One additional hedge fund constraint is that the leverage (calculated as the sum of absolute index weights) does not exceed 12\textsuperscript{54}.

A table summarising the relevant constraints for each fund type and the shorthand codes for the corresponding regression types are presented in Appendix D.2.

\textsuperscript{54} Private discussion with Professor van Rensburg.
5.2.4. Out-of-sample prediction model

Once a fund’s style is estimated based on a weighted rolling three years\(^{55}\) of historical returns, the out-of-sample predicted style return for the coming month \(t\) is calculated using the following formula:

\[
R_{\text{style},t} = \sum_{p=1}^{P} w_{p,t} R_{p,t}
\]

(5.4)

Where:

- \(R_{\text{style},t}\) represents the style return of the fund under investigation in month \(t\)
- \(R_{p,t}\) represents the monthly return of key factor \(p\) in month \(t\)
- \(P\) represents the number of key factors used to replicate the fund’s investment style.
  - i.e., number of independent variables in the return-based regression. \(P = 6\) when the Small Cap Index is excluded from the regression and \(P = 7\) when the Small Cap Index is included
- \(w_{p,t}\) represents the style weights of the fund’s return to the \(k\)th key factor, estimated from past period of data, i.e., month \(t-36\) or \(t-24\) to month \(t-1\).

Assuming that the fund’s style does not change in the coming month, the out-of-sample selection return gives the proportion of a fund’s return that cannot be replicated from its historical style decomposition but is produced by the manager’s unique stock-picking skill. In each month, it is calculated by taking the arithmetic difference between a fund’s observed actual return \(R_{\text{FUND},t}\) and its predicted style return \(R_{\text{style},t}\). The computation formula for the out-of-sample selection return \(\varepsilon_{i,t}\) is thus:

\[
\varepsilon_{\text{Selection},i,t} = R_{\text{FUND},i,t} - R_{\text{style},i,t}
\]

(5.5)

Where:

- \(R_{\text{FUND},i,t}\) represents fund \(i\)'s return in month \(t\)

\(^{55}\) 36 months for unit trusts and 24 months for hedge funds.
$e_{Selection,i}$ represents the fund's residual return after adjustment for its investment style, i.e., the out-of-sample selection return in month $t$ for fund $i$.

It should be noted that the out-of-sample $e_{i,t}$ in Equation (5.5) differs from the in-sample $e_{i,t}$ obtained as a byproduct of a style analysis using Equation (5.1). A positive selection return indicates that the fund managers have added value through security selection or other factors which is not explained by the fund's long-term investment style represented by a passive mix of selected key factors.

5.2.5. Summary statistics

A set of summary statistics is computed to test if the estimated style returns can closely track the actual fund returns. One important measure is the out-of-sample $R^2$ from regressing the fund’s actual returns against its predicted style returns using the following equation:

$$R_{FUND,i} = \alpha_{FUND} + \beta_{FUND} \times R_{style,i} + \epsilon_{FUND}$$

And $R^2 = 1 - \frac{Var(\epsilon_{FUND})}{Var(R_{FUND})}$ (5.6)

Where:

$R_{FUND,i}$ represents the actual return on fund $i$ in month $t$

$R_{style,i}$ represents out-of-sample predicted style return on fund $i$ in month $t$

$\alpha_{FUND}$ represents the regression constant, indicating the excess return on the fund that cannot be attributed to style returns (i.e., $R_{style,i}$)

$\beta_{FUND}$ represents the regression coefficient, indicating sensitivity of changes in fund returns to changes in style returns

$\epsilon_{FUND}$ represents the regression residual that cannot be explained by style returns.

$Var(\epsilon_{FUND})$ represents the variance of $\epsilon_{FUND}$ over the period of regression

$Var(R_{FUND})$ represents the variance of the observed return on fund $i$ ($R_{FUND}$) over the period of regression
A high out-of-sample $R^2$ indicates that variations in the style returns explain a high proportion of the total variations in the actual fund return, and therefore returns on the style portfolios can quite closely track those on the actual funds.

Furthermore, to investigate if the fund’s monthly out-of-sample selection returns are significantly different from 0, the standard deviations and t- and p-statistics, as well as the Sharpe Ratios of the selection returns are computed. A t-statistic is used to measure the statistical significance of selection returns, it is calculated as follows:

$$ t = \frac{(r_{selection} - \mu) \times \sqrt{n}}{\sigma_{selection}} $$

(5.7)

Where:

$ t $ represents the t-statistic

$r_{selection}$ represents the monthly selection return

$\mu = 0$ for the null hypothesis: selection return is 0

$\sigma_{selection}$ represents the monthly standard deviation of the selection return

$n$ represents the number of months over which out-of-sample selection returns are calculated.

The p-value of the t-statistics is calculated from a two-tailed t-distribution with $(n-1)$ degrees of freedom. Approximately, a t-value of selection returns greater than 2 corresponds to a significant p-value at 5% level, which leads to the rejection of the null hypothesis and the conclusion that the fund selection returns are significantly different from 0. On the other hand, non-significant p-values indicate that the fund manager cannot add significant value by active stock selection after his investment style is taken into account.

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56 Since the hypothesis is selection return is 0, deviation can be either positive or negative.
5.3 Empirical results

5.3.1. Unit trusts

Three unit trusts sector indices and eleven unit trust funds are analysed. To illustrate the style-decomposition approach, the results of the three unit trusts sector indices are discussed in relative more detail below. Interpretations of the results on the individual unit trust funds are similar and, therefore, only the summarised numerical outputs are presented. The related graphical results on individual unit trust funds are presented in Appendix D.3-D.18.

For unit trusts (both funds and indices), a regression period of 36 months is used to infer a fund’s investment style. QP is used to incorporate the constraint that (1) each of the style weights lies between 0 and 1, and (2) style weights sum to unity. The table entries entitled ‘OLS’ show the results obtained applying Equation (5.1), whereas ‘WLS’ indicates that Equation (5.2) is implemented. If six independent variables are used for the regression, then the Small Cap Index is excluded, else it is included.

5.3.1.1. Domestic equity general (DOEQ)

Table 5.1 and Column 2 of Appendix D.6 summarise the style-decomposition results of the DOEQ Index obtained by conducting four different types of regressions\(^{57}\). The average style weights illustrating DOEQ’s average style from a long-term perspective (where the style weights are obtained by performing regressions over the entire period of investigation) are included in Appendix D.3.

The DOEQ Index is available from 1\(^{st}\) January 1998 to 31\(^{st}\) December 2006, which gives a maximum available period of 108 months, and thus a total out-of-sample period of 72\(^{58}\) months. The average return on DOEQ over the out-of-sample period (January 2001 to December 2006) is 1.86% per month, with a monthly standard deviation of 4.33%.

According to Table 5.1 and Column 2 of Appendix D.6, the selection returns range from -0.06% to 0.02% per month, but none is significantly different from 0. Therefore,

\(^{57}\) Using four types of regressions: OLS CS, WLS QP, OLS QP.

\(^{58}\) Out-of-sample periods over which predictions are carried out = maximum available period - rolling period to infer fund styles (36 months for unit trusts).
if an investor has estimated the style of the DOEQ Index on a monthly basis using 3-year return history by employing Sharpe's style decomposition method, and has invested in a portfolio of six style and sector indices, combined in an appropriate proportion so as to replicate the inferred style of DOEQ; he would have achieved an overall return not significantly different from that of the DOEQ Index over the period January 2001 to December 2006. Furthermore, if he has included the Small Cap Index in his style portfolio, he would have earned a return that is slightly higher than the DOEQ Index that he was tracking.

In addition, the out-of-sample regressions of style returns over actual fund returns using Equation (5.6) have produced very high $R^2$ values (0.91 - 0.93). This provides strong support for the premise that the style portfolio returns are able to explain a large proportion of the variations in the actual returns obtained on an actively managed equity fund.

Figure 5.1 below compares the performances of the DOEQ Index and the portfolio of passive indices designed to reproduce DOEQ's investment style (the style portfolio). The horizontal axis indicates the estimated style returns, while the vertical axis shows the actual fund returns. The style is predicted using WLS regressions which, as observed in Table 5.1 and Appendix D.6, produce higher style returns and $R^2$ values than the OLS regressions. The style portfolio excludes the Small Cap Index since it is too illiquid to be traded by unit trusts.

From Figure 5.1, the similarity between the style returns and the observed returns is relatively clear. However, a monthly out-of sample tracking error of 1.33% is observed. This equates to an annualised figure of about 4.6% which is moderately high and the kind of level usually associated with an active fund. Thus, while the mean return is very similar over the out-of sample period there would be a moderate amount of benchmark relative risk within period. In other words, though the overall selection return is close to 0, the style portfolio returns may not track the actual fund returns closely in every individual month.

**Figure 5.1: Scatter plot of style returns and observed returns on the DOEQ Index**
The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis over the period 1st January 1998 to 31st December 2006. The monthly total returns are computed from the closing price and dividend yields obtained from 1-Net Bridge at the University of Cape Town. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with six independent regression variables excluding the Small Cap Index. For unit trusts (funds and indices), 36-month rolling periods are used to infer a fund's investment style. Quadratic programming (QP) is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.

Figure 5.2 presents the monthly selection returns of the DOEQ Index from January 2001 to December 2006. The monthly selection returns are found to reveal no discernable trend.

**Figure 5.2: Histogram of monthly selection returns on the DOEQ Index**

The graph displays the monthly selection returns on the domestic equity unit trust index (DOEQ) over the period 1st January 2001 to 31st December 2006. The monthly total returns are computed from the closing price and dividend yields obtained from 1-Net Bridge at the University of Cape Town. Investment style is estimated from weighted least square regressions (WLS using Equation (5.2)), with six independent regression variables excluding the Small Cap Index. For unit trusts (funds and indices), 36-month rolling periods are used to infer a fund's investment style. Quadratic programming (QP) is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.
The exposure distribution area graph, also known as the rolling style graph, or the rolling portfolio decomposition graph, shows the changes in a fund’s style over time as obtained from the series of monthly rolling optimisations. The point at the left end of the diagram represents the style weights estimated when analysing the 36 months ending in December 2000. Every other point moving to the right represents a set of style weights obtained using a different period of 36-month returns, where the starting month of the period moves forward by one month each time. Putting this into the context of the DOEQ Index, the first regression finds the style weights that best match DOEQ’s actual returns from January 1998 through December 2000; the second regression is for the period February 1998 to January 2001 and so on.

As shown in Figure 5.3, around 50% of DOEQ’s overall style exposure is captured by the size index. Over time, the momentum index has consistently added value on top of the size index, whereas contribution of the value index is only visible in some of the early periods where its weight percentages are not zero.

Figure 5.3 indicates that the DOEQ Index’s style appeared to have changed gradually over time. In broad terms, the index’s exposure to the financial sector (FINI 15) seems to have decreased over time, giving way mainly to the industrial sector (INDI 25).
Whereas the investment in the resource sector (RESI 20) has first risen consistently and then oscillated and declined. As the index's allocation to the resource sector increases, the allocation to the industrial sector (INDI 25) decreases accordingly. Industrial sector exposure reaches a minimum of zero when the allocation to the resource sector peaked. The exposure to the momentum style (MOM(12-1)W100) stayed fairly constant until 1st January 2006, but jumped up toward the end. While the weight of the size style index seems to have increased steadily until May 2005, mainly at the expense of the three sector indices, it did decline steeply over 2006.

As mentioned in Section 5.2.3.2, using style-indices with low correlation reduces the unnecessary fluctuation of the style weights calculated. However, since the 100 companies with the largest market capitalisation (MV) are always used to create the style portfolios, the correlation among the style indices are high by construction. As a result, the percentages of weights in this chapter should not be interpreted directly as the fund's exposure to the investment styles under consideration. A fund's exposures to the value and momentum indices are better interpreted as the extra value added by these two indices which are not captured by the movement of the size index.

59 Such interpretation is only appropriate when the indices are mutually exclusive
Figure 5.3: Exposure distribution area graph of the DOEQ Index

The graph displays the monthly exposure of the Domestic Equity Unit Trust Index (DOEQ) to the six selected explanatory indices over the period 1st January 1998 to 31st December 2006. The monthly total returns are computed from the closing price and dividend yields obtained from 1-Net Bridge at the University of Cape Town. The indices used to construct the style portfolios are: three style indices selected from Chapter Four, namely EW(size)100 for the size investment style, RESW100(3) for the value investment style and MOM(12-1)W100 for the momentum investment style; and three sector indices constituting the FTSE/JSE Africa Financial and Industrial Index (FINI, J212T) merged with the Satrix FINI15 for the financial sector (FINI 15), the FTSE/JSE Africa Industrial 25 (INDI, J211T) merged with the Satrix INDI25 for the industrial sector (INDI 25), and the FTSE/JSE Africa Resources 20 Index (RESI, J210T) merged with the Satrix RESI for the resources sector(RESI 20). Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with six independent regression variables excluding the Small Cap Index. For unit trusts (funds and indices), 36-month rolling periods are used to infer a fund’s investment style. Quadratic programming (QP) is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.
5.3.1.2. Domestic equity growth index (DOEQGR)

The style-decomposition outputs of the DOEQGR index over the period 1st January 2001 to 31st December 2006 is shown in Table 5.1 and Column 3 of Appendix D.6. The table displaying DOEQGR’s average style over the entire period of investigation is attached in Section A of Appendix D.4. Very high out-of-sample $R^2$ values (0.92 - 0.93) demonstrate that the style portfolio returns are able to explain a large proportion of the actual fund returns.

The mean and standard deviation of returns on DOEQGR over January 2001 to December 2006 is 1.69% and 4.43% per month respectively. The selection returns lie between -0.18% and -0.11% per annum, suggesting that the style portfolios have outperformed the actual fund although the outperformance is not statistically significant even at 10% level. In addition, the histogram of selection returns in Section B of Appendix D.4, confirms that the monthly selection returns follow a relatively random display of positive and negative values with no remarkable trends.

Figure 5.4 below shows the scatter plot of the DOEQGR Index, the conclusion of the analysis looks very similar to that of the DOEQ Index in Figure 5.2. The scatter plot reveals a very roughly linear line sloping up from the origin at 45 degrees, demonstrating that in each individual month the actual fund return is not followed very closely by the style return predicted from a 3-year rolling WLS regression. The tracking error is approximately 1.34% per month as shown in Table 5.1.

Figure 5.4: Scatter plot of style returns and observed returns on the DOEQGR Index

The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis over the period 1st January 1998 to 31st December 2006. The monthly total returns are computed from the closing price and dividend yields obtained from 1-Net Bridge at the University of Cape Town. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with six independent regression variables excluding the Small Cap Index. For unit trusts (funds and indices), 36-month rolling periods are used to infer a fund’s investment style. Quadratic programming (QP) is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.
Figure 5.5 depicts a gradual change in the style of the DOEQGR Index over time, broken down into six component indices. From the chart it appears that DOEQGR’s exposures to various styles are fairly similar to those of the DOEQ Index.

Overall, the DOEQGR Index acted as if it has invested in a portfolio constituting all indices, with around 50% in the size index and minimal allocation to the value index. DOEQGR’s exposure to the financial sector (FINI 15) dropped in 2006 after having remained fairly stable in earlier years. In contrast, DOEQGR’s allocation to the resource sector (RESI 20) is much smaller than that of DOEQ, the difference is made up by higher allocation to the industrial sector (INDI 25) in earlier periods. This may reflect the fact that growth managers have invested more heavily in industrial stocks over the period January 2001 to December 2003 in order to take advantage of their higher growth potential during the economic boom. The trend of DOEQGR’s exposure to the industrial sector, however, is similar to that of DOEQ: the exposure declined at first and then gradually accelerated.

As mentioned previously, the three style indices used in this thesis overlap with each other. This is caused by the fact that all three of them constitute the top 100 shares by MV in each month; the only difference is the weighting of each stock in the style indices. Consequently, the style weights of the value and momentum indices must be
interpreted as their contribution to the style return in excess of that captured by the size index.

An interesting observation is that the momentum index has taken up a higher exposure than in the case of DOEQ, especially towards the end of the period investigated, which may indicate that more growth managers have become aware of the momentum investment approach. On the other hand, the value index played little role in replicating the DOEQGR Index. This is expected since the value index signals investing in value stocks with high BTMVs, while the growth funds constituting the DOEQGR Index are suppose to invest mainly in low BTMV growth shares.
Figure 5.5: Exposure distribution area graph of the DOEQGR Index

The graph displays the monthly exposure of the Domestic Equity Growth Unit Trust Index (DOEQGR) to the six selected explanatory indices over the period 1st January 1998 to 31st December 2006. The monthly total returns are computed from the closing price and dividend yields obtained from 1-Net Bridge at the University of Cape Town. The indices used to construct the style portfolios are: three style indices selected from Chapter Four, namely EW(size)100 for the size investment style, RESW100(3) for the value investment style and MOM(12-1)W100 for the momentum investment style; and three sector indices constituting the FTSE/JSE Africa Financial and Industrial Index (FINI, J212T) merged with the Satrix FINI15 for the financial sector (FINI 15), the FTSE/JSE Africa Industrial 25 (INDI, J211T) merged with the Satrix INDI25 for the industrial sector (INDI 25), and the FTSE/JSE Africa Resources 20 Index (RESI, J210T) merged with the Satrix RESI for the resources sector (RESI 20). Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with six independent regression variables excluding the Small Cap Index. For unit trusts (funds and indices), 36-month rolling periods are used to infer a fund's investment style. Quadratic programming (QP) is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.
5.3.1.3. Domestic equity value index (DOEQVL)

Table 5.1 and Column 4 of Appendix D.6 depict the style-decomposition results of the DOEQVL Index. The returns on DOEQVL over the 72 out-of-sample months (January 2001 to December 2006) have a mean of 2.36% and a standard error of 4.06% per month. DOEQVL has the highest Sharpe Ratio (0.36) among all of the three domestic equity unit trust indices investigated.

Although slightly lower than those of the other two unit trust indices, the out-of-sample $R^2$ values obtained from regressions on DOEQVL are satisfactory at (0.79 - 0.84). This suggests that by investing in an appropriately weighted and monthly rebalanced portfolio of selected passive indices, an investor can successfully duplicate the returns achieved by the DOEQVL Index over the period January 2001 to December 2006.

Furthermore, if WLS regression with the Small Cap Index is utilised to construct the style portfolio, an overall return slightly higher (0.02%) than that of the DOEQVL Index could have been earned. If other types of regressions are used, the mean style return will slightly fall short of the average observed return (by 0.01% to 0.05% per month). According to the p-values of the selection returns, however, none of the underperformance is statistically significant.

The histogram in Section B of Appendix D.5 portrays the monthly selection returns on the DOEQVL Index. In the case of DOEQVL, there seems to be a greater prevalence of more positive selection returns as the actual returns marginally outperform the style returns. There are three relatively large selection return outliers. However, no clusters of positive or negative values are observed in any specific period. As in the case of the two indices previously discussed, Figure 5.6 reveals that though the style returns stayed fairly close to the actual fund returns on average, there are still fluctuations in the selection returns from month to month.
Figure 5.6: Scatter plot of style returns and observed returns on DOEQVL Index

The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis over the period 1st January 1998 to 31st December 2006. The monthly total returns are computed from the closing price and dividend yields obtained from 1-Net Bridge at the University of Cape Town. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with six independent regression variables excluding the Small Cap Index. For unit trusts (funds and indices), 36-month rolling periods are used to infer a fund’s investment style. Quadratic programming (QP) is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.

The exposure distribution area graph in Figure 5.7 provides a dynamic view of the DOEQVL Index’s investment style over the period January 2001 to December 2006. In contrast to Figures 5.5 and 5.3, the investment style of DOEQVL is relatively inconsistent.

In general, DOEQVL puts greater emphasis on the value index. This is expected since the unit trust funds in the DOEQVL category is supposed to invest more heavily in value companies. The most remarkable observation is that the index has periodically shown large exposure to the value style index from January 1998 to October 2003. The value exposure, however, reduced to zero thereafter and was replaced by increasing weights in the size index. This does not mean that DOEQVL’s exposure to the value style has vanished post 2003. Due to the correlation of the value and size
indices, it is more likely that in the later period, the value index does not add extra value on top of the size index.

The percentages and trends of the exposure to the financial sector are relatively consistent with those of the DOEQ Index and the DOEQGR Index. The exposure to both the resources and more specifically the industrial sectors are however small in comparison to the previous two indices. This reduction in exposure may be caused by unit trust managers perceiving industrial companies as more growth orientated. DOEQVL has progressively increased its emphasis on the momentum style and decreased its exposure to the resources sector. Although still dominating the index's exposure to the selected indices, the size index gave way to the momentum index from May 2005 onward.
The graph displays the monthly exposure of the Domestic Equity Value Unit Trust Index (DOEQVL) to the six selected explanatory indices over the period 1st January 1998 to 31st December 2006. The monthly total returns are computed from the closing price and dividend yields obtained from i-Net Bridge at the University of Cape Town. The indices used to construct the style portfolios are: three style indices selected from Chapter Four, namely EW(size)100 for the size investment style, RESW100(3) for the value investment style and MOM(12-1)W100 for the momentum investment style; and three sector indices constituting the FTSE/JSE Africa Financial and Industrial Index (FINI, J212T) merged with the Satrix FINI15 for the financial sector (FINI 15), the FTSE/JSE Africa Industrial 25 (INDI, J211T) merged with the Satrix INDI25 for the industrial sector (INDI 25), and the FTSE/JSE Africa Resources 20 Index (RESI, J210T) merged with the Satrix RESI for the resources sector (RESI 20). Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with six independent regression variables excluding the Small Cap Index. For unit trusts (funds and indices), 36-month rolling periods are used to infer a fund’s investment style. Quadratic programming (QP) is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.
5.3.1.4. Summarised results of SA unit trust funds

Table 5.1 displays the WLS regression outputs on the three unit trust indices and 11 unit trust funds over the period 1st January 1998 to 31st December 2006. The corresponding OLS results are displayed in Appendix D.6. The histograms, scatter plots and exposure graphs obtained by performing WLS regressions with six independent variables are presented in Appendix D.7 - D.18. In addition, the WLS results are inserted for the Sanlam General Equity and Sanlam Small Cap Unit Trusts in Appendix D.16 - D.18 to illustrate the effectiveness of the style decomposition method: unsurprisingly, the Sanlam Small Cap Fund has a much greater exposure to the Small Cap Index.

In Table 5.1, negative selection return indicates that the style portfolios have displayed superior performance. Similarly, higher Sharpe Ratio of the style returns relative to that of the observed fund returns signals that the style portfolio outperforms the actual fund.

Selection returns, p- and t-statistics

Most importantly, the results in Table 5.1 provide little support for the hypothesis that the performance of a typical actively managed unit trust is able to beat that of a passive alternative with the same style composition. Across the entire set of funds, the maximum outperformance achieved by a fund over its style portfolio is 0.24% (24 basis points) per month and the maximum underperformance achieved is -0.39% per month. The t-statistics associated with the mean selection returns are small in absolute value, and consequently none of the selection returns of the 14 indices and funds analysed is significantly different from zero at 10% level. This provides strong evidence of the difficulty for funds to outperform their style portfolios. However, though the style returns and the observed returns are roughly equal in statistical sense, the scatter plots in Appendices D.7 - D.17 confirmed that the tracking is not precise on a month-to-month basis.

WLS vs. OLS

The time-weighted optimisation procedure (WLS) is compared to equally weighting the prior 36 months (OLS). The former is found to be pervasively superior to the latter.
in terms of performance. However, the non-significant selection returns are evidence that both regression methods are able to reasonably replicate the actual fund returns.

With the Small Cap Index vs. without the Small Cap Index
It is also found that, including the Small Cap Index generally produces slightly higher style returns over the period investigated. Such a strategy, however, may be too expensive to implement due to the illiquidity of the Small Cap Index constituents.
Table 5.1: Synthesising South African unit trust indices and representative funds

The table shows the summary statistics and regression results on analysis of the South Africa Unit Trust indices and funds over the period 1st January 1998 to 31st December 2006. The three unit trust indices examined are the Domestic Equity Index (DOEQ), the Domestic Equity Growth Index (DOEGR) and the Domestic Equity Value Index (DOEVL). The 11 unit trust funds examined are Allan Gray equity fund - A (AGEF), Coronation equity fund - R (CORG), Investec equity fund - R (METF), Nedbank rainmaker fund - A (AHVE), Old Mutual investors fund (OMTL), Prudential equity fund (PRUO), PSG Alphen Growth fund - A (RMEF), Sanlam General equity fund (SNTR), and Sanlam Small Cap Fund (SNST). The monthly total returns are computed from the closing price and dividend yields obtained from I-Net Bridge at the University of Cape Town. All results are monthly. Investment style is estimated using the weighted least square using Equation (5.2). The return-based style decompositions are conducted using Sharpe's (1988) multi-factor regression with EW(size)100, RESW100(3), MOM(l2-I)W100, REST 20, FINI 15 (QP) is used to incorporate the constraints that (1) style weights lie between -1 and 1, (2) style weights sum to unity. The South African 90-day Banker's Acceptance discount rate (RBAS) is used as the risk-free rate when calculating the Sharpe Ratios. P-values are calculated using two-tailed tests. Those significant at 10% level are indicated with *, p-values significant at 5% level are indicated with **. If six independent regression variables are used, then the Small Cap Index is not included, else it is included.

<table>
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<tr>
<th>Unit trust index/fund code</th>
<th>DOEQ</th>
<th>DOEGR</th>
<th>DOEVL</th>
<th>AGEF</th>
<th>CORG</th>
<th>METF</th>
<th>AHVE</th>
<th>OMTL</th>
<th>PRUO</th>
<th>PSGG</th>
<th>RMEF</th>
<th>SNTR</th>
<th>SNST</th>
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<tr>
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<td>1.14</td>
<td>1.88</td>
<td>2.70</td>
<td>1.65</td>
<td>3.10</td>
<td>2.40</td>
<td>2.39</td>
<td>1.53</td>
<td>1.95</td>
<td>1.41</td>
<td>1.59</td>
<td>1.40</td>
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<tr>
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<td>4.88</td>
<td>4.95</td>
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<td>5.82</td>
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<td>0.37</td>
<td>0.16</td>
<td>0.14</td>
<td>0.36</td>
<td>0.40</td>
<td>0.10</td>
<td>0.22</td>
<td>0.09</td>
<td>0.12</td>
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<td>2.50</td>
<td>1.85</td>
<td>2.27</td>
<td>3.04</td>
<td>2.88</td>
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<td>2.58</td>
<td>1.83</td>
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<td>4.06</td>
<td>4.32</td>
<td>4.23</td>
<td>4.54</td>
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<td>4.49</td>
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<td>Mean style return (%)</td>
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<td>2.49</td>
<td>1.84</td>
<td>2.97</td>
<td>2.40</td>
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<td>-0.03</td>
<td>-0.03</td>
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<td>0.03</td>
<td>-0.24</td>
<td>0.03</td>
<td>0.24</td>
<td>0.14</td>
<td>0.16</td>
<td>0.01</td>
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<td>1.33</td>
<td>1.34</td>
<td>1.94</td>
<td>2.55</td>
<td>1.71</td>
<td>1.99</td>
<td>1.82</td>
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<td>1.63</td>
<td>1.76</td>
<td>1.68</td>
<td>1.37</td>
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<td>0.02</td>
<td>0.02</td>
<td>1.02</td>
<td>0.11</td>
<td>0.83</td>
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<td>0.66</td>
<td>0.65</td>
<td>0.12</td>
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<td>0.91</td>
<td>0.99</td>
<td>0.98</td>
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<td>0.41</td>
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<td>0.67</td>
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<td>0.52</td>
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<td>0.92</td>
<td>0.79</td>
<td>0.87</td>
<td>0.89</td>
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<td>0.89</td>
<td>0.82</td>
<td>0.92</td>
<td>0.85</td>
<td>0.85</td>
<td>0.78</td>
<td>0.87</td>
</tr>
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</table>

| WLS QP 7 (incl-small cap) | Mean style return (%) | 1.90 | 1.85 | 2.44 | 2.59 | 1.89 | 2.08 | 3.14 | 3.24 | 1.82 | 2.35 | 1.73 | 1.96 | 1.73 | 2.40 |
| Standard deviation of style return (%) | 4.65 | 4.58 | 4.52 | 4.42 | 4.76 | 4.83 | 3.82 | 3.75 | 4.97 | 4.39 | 4.60 | 4.78 | 4.95 | 4.05 |
| Sharpe ratio of style return | 0.22 | 0.21 | 0.32 | 0.40 | 0.21 | 0.25 | 0.59 | 0.63 | 0.19 | 0.34 | 0.19 | 0.22 | 0.17 | 0.38 |
| Mean selection return (%) | -0.06 | -0.18 | -0.11 | -0.12 | -0.07 | -0.16 | -0.14 | -0.38 | 0.00 | 0.21 | 0.10 | 0.13 | -0.05 | 0.10 |
| Standard deviation of selection return (%) | 1.33 | 1.26 | 1.76 | 2.49 | 1.57 | 1.99 | 1.87 | 1.48 | 1.65 | 1.65 | 1.80 | 1.90 | 1.35 | 1.97 |
| t-selection return | 0.49 | 1.20 | 0.11 | 0.58 | 0.37 | 0.70 | 0.47 | 1.30 | 0.16 | 0.35 | 0.48 | 0.50 | 0.30 | 0.43 |
| p-selection return | 0.63 | 0.32 | 0.91 | 0.57 | 0.71 | 0.49 | 0.64 | 0.20 | 0.88 | 0.73 | 0.63 | 0.62 | 0.77 | 0.67 |
| R-square (out-of-sample) | 0.93 | 0.93 | 0.84 | 0.89 | 0.91 | 0.85 | 0.89 | 0.84 | 0.93 | 0.86 | 0.80 | 0.89 | 0.86 | 0.78 |
5.3.2. Hedge funds

Four SA hedge fund sector indices are analysed to examine if the style-decomposition technique can satisfactorily replicate hedge fund returns. The full set of summary statistics, scatter plots and rolling style charts is discussed below for each of the indices in turn. Relevant complementary graphs on each index can be found in Appendix D.19 - D.23.

In the case of hedge fund indices, rolling periods of 24-months are used to infer a fund’s investment style. Two types of constraints are used in conjunction with the WLS and OLS regression methods outlined in Equations (5.1) and (5.2). The first type is called constrained sum regressions (CS), where the only constraint is that the sums of the absolute values of the style weights must be smaller than 12, this constraint complies with the actual investment mandates of most hedge funds. Secondly, QP regressions incorporating the constraints that, (1) style weights lie between 0 and 1, and (2) sum to unity are also performed. It is noted that QP may not produce an accurate picture since in general hedge funds can short and leverage.

In contrast to the unit trust strategies where small capitalisation investment is often not feasible, investing in small firms is relatively easier and indeed common for hedge funds. As a result, regressions with seven independent variables tend to give a more realistic reflection of the hedge funds’ investment styles.\(^{61}\)

5.3.2.1. Summarised results of four SA hedge fund indices

Table 5.2 below shows the summary statistics and WLS regression results of the four SA Hedge Fund Indices similar to those of the unit trusts analysed above. The corresponding OLS results are attached in Appendix D.18. The indices are available from 1\(^{st}\) January 2004 to 31\(^{st}\) December 2006, resulting in a maximum available period of 36 months, and thus a total out-of-sample period of 12 months. It is clear that LSE is the best performing index in terms of mean returns and the Sharpe ratio, whereas MKN is the worst.

\(^{61}\) If six independent regression variables are used, then the Small Cap Index is not included; else it is included.
Overall, two aspects of the regression results are of interest. Firstly, out-of-sample $R^2$ values are obtained to indicate how closely the style portfolio can track the actual fund's performance. Secondly, selection returns and the Sharpe Ratios are computed to test if the style returns can outperform the observed returns. These two aspects are investigated for each of the four indices in turn, with special attention given to compare the effect of the inclusion of the Small Cap Index and the relaxation of the QP constraints.

**With the Small Cap Index vs. without the Small Cap Index**

All regressions with the Small Cap Index have invariably produced higher $R^2$ values. This is expected since adding the Small Cap Index results in a more comprehensive set of independent variables which more realistically represents the hedge funds' investment sphere.

**CS vs. QP**

Under CS, all of the indices have frequently produced positive selection returns as displayed in the histograms in Appendices D.18 - D.21. The MKN index has yielded positive selection returns significant at 5% level. However, accompanied by the lowest $R^2$ values among all of the indices, it is dubious to conclude that the positive selection returns imply underperformance of the style portfolios.

The most striking observation is that the overall tracking power measured by the out-of-sample $R^2$ values is much lower than those of the unit trusts. This may be a result of the more active management style of hedge funds. MKN has experienced the lowest out-of-sample explanatory power, the next lowest is the COMP Index. The style portfolios seem to track the other two indices, LSE and FOFs, reasonably well, indicated by their high $R^2$ values. Surprisingly, QP has produced higher $R^2$ for three out of four indices though the CS method is believed to more realistically reflect a hedge fund's investment style.

**WLS vs. OLS**

It seems that the OLS method occasionally produces higher $R^2$ values and thus has higher explanatory power. However, it is unlikely that a definite conclusion can be drawn with out-of-sample regressions basing on only 12 data points.
Table 5.2: Synthesising South African hedge fund indices

The table shows the summary statistics and regression results on analysis of the South African Hedge Fund indices over the period 1st January 2004 to 31st December 2006. The four hedge fund indices examined are Single Manager Composite (COMP), Long Short Equity Index (LSE), Market Neutral & Quantitative Strategies Index (MKN) and Fund of Funds Index (FOFs). The monthly total returns are computed from the closing price and dividend yields obtained from HedgeFund Intelligence Database. All results are monthly effective. Investment style is estimated using the weighted least square regressions (WLS) using Equation (5.2). The return-based style decompositions are conducted using Sharpe’s (1988) multi-factor regression with EW(size)100, RESW100(3), MOM(12-1)W100, RES10, FINI 15 and INDI 25. R² values are obtained from the out-sample regressions of predicted style returns on actual fund returns. For hedge funds, 24-month rolling periods are used to infer a fund’s investment style. Quadratic programming (QP) is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity. If six independent regression variables are used, then the Small Cap Index is not included, else it is included. The South African 90-day Banker’s Acceptance discount rate (RBAS) is used as the risk-free rate when calculating the Sharpe Ratios. P-values are calculated using two-tailed tests. P-values significant at 10% level are indicated with *, p-values significant at 5% level are indicated with **.

<table>
<thead>
<tr>
<th>Hedge fund index code</th>
<th>COMP</th>
<th>LSE</th>
<th>MKN</th>
<th>FOFs</th>
<th>COMP</th>
<th>LSE</th>
<th>MKN</th>
<th>FOFs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean index return</td>
<td>1.36</td>
<td>2.30</td>
<td>0.86</td>
<td>1.37</td>
<td>1.21</td>
<td>2.31</td>
<td>0.87</td>
<td>1.14</td>
</tr>
<tr>
<td>Standard deviation of Index return</td>
<td>0.92</td>
<td>2.59</td>
<td>0.43</td>
<td>1.15</td>
<td>0.81</td>
<td>2.65</td>
<td>0.49</td>
<td>1.27</td>
</tr>
<tr>
<td>Sharpe ratio of index return</td>
<td>0.52</td>
<td>0.55</td>
<td>-0.04</td>
<td>0.42</td>
<td>0.40</td>
<td>0.54</td>
<td>-0.02</td>
<td>0.26</td>
</tr>
<tr>
<td>Mean style return (%)</td>
<td>0.73</td>
<td>1.42</td>
<td>0.33</td>
<td>0.96</td>
<td>3.17</td>
<td>3.15</td>
<td>3.11</td>
<td>3.21</td>
</tr>
<tr>
<td>Standard deviation of style return (%)</td>
<td>1.39</td>
<td>2.85</td>
<td>0.69</td>
<td>1.44</td>
<td>4.25</td>
<td>4.07</td>
<td>4.30</td>
<td>4.25</td>
</tr>
<tr>
<td>Sharpe ratio of style return (%)</td>
<td>-0.11</td>
<td>0.33</td>
<td>-0.80</td>
<td>0.06</td>
<td>0.54</td>
<td>0.56</td>
<td>0.52</td>
<td>0.55</td>
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<tr>
<td>Mean selection return (%)</td>
<td>0.47</td>
<td>0.47</td>
<td>0.54</td>
<td>0.17</td>
<td>-2.11</td>
<td>-0.90</td>
<td>-2.40</td>
<td>-2.20</td>
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<tr>
<td>Standard deviation of selection return (%)</td>
<td>0.94</td>
<td>1.42</td>
<td>0.67</td>
<td>0.58</td>
<td>3.64</td>
<td>2.09</td>
<td>3.96</td>
<td>3.27</td>
</tr>
<tr>
<td>t-selection return (%)</td>
<td>1.73</td>
<td>1.15</td>
<td>2.77</td>
<td>1.03</td>
<td>-2.00</td>
<td>-1.49</td>
<td>-2.10</td>
<td>-2.33</td>
</tr>
<tr>
<td>p-selection return (%)</td>
<td>0.11</td>
<td>0.28</td>
<td>0.02</td>
<td>0.33</td>
<td>0.07*</td>
<td>0.17**</td>
<td>0.06*</td>
<td>0.04**</td>
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<tr>
<td>R square (out-of-sample)</td>
<td>0.43</td>
<td>0.75</td>
<td>0.11</td>
<td>0.81</td>
<td>0.65</td>
<td>0.77</td>
<td>0.49</td>
<td>0.77</td>
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</table>

<table>
<thead>
<tr>
<th>Hedge fund index code</th>
<th>COMP</th>
<th>LSE</th>
<th>MKN</th>
<th>FOFs</th>
<th>COMP</th>
<th>LSE</th>
<th>MKN</th>
<th>FOFs</th>
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</thead>
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<tr>
<td>Mean index return</td>
<td>0.81</td>
<td>1.78</td>
<td>0.35</td>
<td>1.07</td>
<td>3.12</td>
<td>3.10</td>
<td>3.08</td>
<td>3.24</td>
</tr>
<tr>
<td>Standard deviation of style return (%)</td>
<td>1.43</td>
<td>2.85</td>
<td>0.69</td>
<td>1.46</td>
<td>4.12</td>
<td>4.08</td>
<td>4.12</td>
<td>4.14</td>
</tr>
<tr>
<td>Sharpe ratio of style return (%)</td>
<td>-0.04</td>
<td>0.32</td>
<td>-0.76</td>
<td>0.13</td>
<td>0.55</td>
<td>0.54</td>
<td>0.53</td>
<td>0.57</td>
</tr>
<tr>
<td>Mean selection return (%)</td>
<td>0.38</td>
<td>0.51</td>
<td>0.51</td>
<td>0.07</td>
<td>-2.05</td>
<td>-0.85</td>
<td>-2.36</td>
<td>-2.22</td>
</tr>
<tr>
<td>Standard deviation of selection return (%)</td>
<td>1.08</td>
<td>1.40</td>
<td>0.70</td>
<td>0.64</td>
<td>3.49</td>
<td>2.14</td>
<td>3.79</td>
<td>3.08</td>
</tr>
<tr>
<td>t-selection return (%)</td>
<td>1.22</td>
<td>1.26</td>
<td>2.55</td>
<td>0.36</td>
<td>-2.03</td>
<td>-1.38</td>
<td>-2.15</td>
<td>-2.50</td>
</tr>
<tr>
<td>p-selection return (%)</td>
<td>0.25</td>
<td>0.24</td>
<td>0.03**</td>
<td>0.72</td>
<td>0.07*</td>
<td>0.20</td>
<td>0.06*</td>
<td>0.03**</td>
</tr>
<tr>
<td>R square (out-of-sample)</td>
<td>0.44</td>
<td>0.76</td>
<td>0.12</td>
<td>0.81</td>
<td>0.66</td>
<td>0.78</td>
<td>0.50</td>
<td>0.78</td>
</tr>
</tbody>
</table>

Sections 5.3.2.2 to 5.3.2.5 are devoted to each of the four hedge fund indices respectively to facilitate more detailed index-specific discussions.

5.3.2.2. Single Manager Composite Index (COMP)

The returns of COMP over the 12 out-of-sample period (January 2006 to December 2006) averaged 1.21% per month with a standard error of 0.81%. The out-of-sample regressions of style returns over actual fund returns have produced moderate R² values (about 43% under CS and 66% under QP), implying that the style portfolios may not
be able to explain the majority of the variations in the actual fund returns. This may be caused by the fact that hedge funds tend to focus more on active stock picking.

As shown in Table 5.2 and the histograms of selection returns in Section D of Appendix D.19, the COMP Index has provided positive but statistically insignificant outperformance when compared with its style portfolio. Figure 5.8 portrays the relationship between the observed returns and style returns of the COMP Index, where the style is inferred from WLS CS regressions using seven selected indices, including the Small Cap Index.

The scatter plot contrasts sharply with those of the unit trusts. There are two evident outliers among the 12 data points; so given the data on hand, the tracking is extremely rough. The major problem, however, is that the number of out-of-sample periods is too few to draw any meaningful conclusion.

The graphs obtained on QP regressions on COMP are attached in Sections A to C of Appendix D.19. The selection returns are negative and statistically significant at 10% under the QP method.

Figure 5.8: Scatter plot of style return and observed return on the COMP Index
The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis over the period 1st January 2004 to 31st December 2006. The Single Manager Composite Index (COMP) total returns are obtained from HedgeFund Intelligence Database. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with seven independent regression variables including the Small Cap Index. For hedge funds, 24-month rolling periods are used to infer a fund's investment style. Constrained Sum (CS) regression is used to incorporate the constraint that the sum of the absolute values of the style weights < 12.
A series of rolling style regressions are performed using a fixed number of months (24) for each analysis through time. The 24-month rolling style chart shows the characteristics and changes in COMP's style over time, with the leftmost set of weights derived from returns over the 24 months ending in December 2005.

In sharp contrast to those of the unit trusts, the Hedge Fund COMP Index's investment style is fairly unstable over time, under both CS regressions in Figure 5.9 and to a lesser extent under QP regressions in Section C of Appendix D.19. The major instability under the CS regressions is caused by the changes of exposure to the size and value-style indices. The former has vacillated from positive to negative in June 2006; while the later from negative to positive. The exposure to the momentum and resources indices also swayed from negative to positive in June; however, the change is of a much smaller magnitude. Weights to the Small Cap Index and financial sector remained positive throughout 2006. The industrial index maintained a short position in the style portfolio over the entire out-of-sample period.
5.3.2.3. Long Short Equity Index (LSE)

The LSE Index displays an average return and standard deviation of 2.31% and 2.65% per month respectively over January 2006 to December 2006. It is the hedge fund index with the highest Sharpe Ratio, mean return and standard deviation.

Again, the selection returns are all positive under CS conditions while negative if the QP method is used. However, despite their signs, none of the selection returns are significantly different from 0. The histogram of the CS selection returns in Section D of Appendix D.20 displays no distinguishable trends.

The CS scatter plot is presented in Figure 5.10 below, whereas the relevant QP results can be found in Appendix D.20. This graph resembles those of the unit trusts. It is more or less a straight line with fewer outliers than the COMP Index. In fact, LSE is one of the two hedge fund indices that are reasonably tracked judging from the out-of-sample $R^2$ values. During the period covered, approximately 78% (for both CS and QP regressions with the Small Cap Index) of the monthly variation in LSE returns could be attributed to the concurrent returns on its passive style portfolio.
Figure 5.10: Scatter plot of style return and observed return on the LSE Index
The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis over the period 1st January 2004 to 31st December 2006. The Long Short Equity Index (LSE) total returns are obtained from HedgeFund Intelligence Database. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with seven independent regression variables including the Small Cap Index. For hedge funds, 24-month rolling periods are used to infer a fund’s investment style. Constrained Sum (CS) regression is used to incorporate the constraint that the sum of the absolute values of the style weights < 12.

Figure 5.11 and Section C of Appendix D.20 present the 24-month rolling-style chart depicting weights from the CS and QP regressions on the LSE Index respectively. For the CS regressions, the most volatile index seems to be EW(size)100, whose exposures have swung from highly positive to slightly negative in June 2006. The momentum and value indices are the other two indices that have taken up significant weights (either positive or negative) in the style portfolio. Weights of the former have stayed positive the whole time, whereas weights of the later have fluctuated between negative and positive. The allocations of LSE’s style portfolio to the other four indices are of relatively insignificant amounts, and have remained reasonably level over time. Exposures to the Small Cap Index, industrial and resources sectors continue to be negative, while that to the financial sector is consistently positive.
Figure 5.11: Exposure distribution area graph of the LSE Index
The graph displays the monthly exposure of the Long Short Equity Index (LSE) to the seven selected explanatory indices over the period 1st January 2004 to 31st December 2006. The monthly total returns are computed from the closing price and dividend yields obtained from HedgeFund Intelligence Database. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with seven independent regression variables including the Small Cap Index. For hedge funds, 24-month rolling periods are used to infer a fund’s investment style. Constrained Sum (CS) regression is used to incorporate the constraint that the sum of the absolute values of the style weights < 12.

5.3.2.4. Market Neutral and Quantitative Strategies Index (MKN)
The MKN Index has generated the worst performance from January 2006 to December 2006 among the four hedge fund indices investigated, characterised by a mean return of 0.87% per month, which even underperforms the risk-free rate, resulting in a negative Sharpe Ratio of -0.02. From Figure 5.12 and the histogram of selection returns in Appendix D.19, it is clear that the observed returns systematically and significantly outperform the style returns under CS regressions, while systematically underperform the style returns under QP regressions.

The out-of-sample regressions of style returns over actual fund returns only have produced the lowest R² values among the four indices tested. Here, style accounts for about 11% under CS and 50% under QP (with the Small Cap Index) of the monthly variation in actual fund returns. The inability of the style returns to track the observed
returns are further illustrated and verified by Figure 5.12 and Section A of Appendix D.21. Both scatter plots show that there are large numbers of outliers. However, a low \( R^2 \) does not necessarily mean that the style analysis is a 'failure'; \( R^2 \) is most informative when viewed in context with other statistics especially the exposure distribution area graph. In the case of MKN, the index has underperformed the risk-free rate and most of the seven selected indices, consequently some index weights are forced to be negative to match the very low MKN returns. This results in highly volatile style weights, and thus renders the style decomposition descriptive and regression statistics relatively unreliable.

Figure 5.12: Scatter plot of style return and observed return on the MKN Index

The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis over the period 1st January 2004 to 31st December 2006. The Market Neutral & Quantitative Strategies Index (COMP) total returns are obtained from HedgeFund Intelligence Database. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with seven independent regression variables including the Small Cap Index. For hedge funds, 24-month rolling periods are used to infer a fund’s investment style. Constrained Sum (CS) regression is used to incorporate the constraint that the sum of the absolute values of the style weights < 12.

The 24-month rolling style chart under CS regressions on the MKN Index is shown in Figure 5.13, the QP graph is displayed in Appendix D.21. Under the CS regressions, the indices that displayed vast volatile exposures are the size, value and momentum indices. The size index has its weights slipped from positive to negative in June 2006, while weights of the value and momentum indices jumped from negative to positive.
The fourth index that has a relatively significant negative presentation in the style portfolio is INDI 25. The exposures to other indices seem to be negligible and have vacillated around 0.

**Figure 5.13: Exposure distribution area graph of the MKN Index**
The graph displays the monthly exposure of the Market Neutral & Quantitative Strategies Index (MKN) to the seven selected explanatory indices over the period 1st January 2004 to 31st December 2006. The monthly total returns are computed from the closing price and dividend yields obtained from HedgeFund Intelligence Database. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with seven independent regression variables including the Small Cap Index. For hedge funds, 24-month rolling periods are used to infer a fund's investment style. Constrained Sum (CS) regression is used to incorporate the constraint that the sum of the absolute values of the style weights < 12.

5.3.2.5. **Fund of Funds Index (FOFs)**
As summarised in Table 5.2, FOFs' average return is 1.14% per month over January 2006 to December 2006, with a standard error of 1.27%. The histograms of selection returns in Appendix D.22 show that the selection returns are distributed in a fairly random fashion. Although being predominantly positive under CS regressions and negative under QP regressions, the t-values of the selection returns are not statistically significant.
The out-of-sample $R^2$ identifies how well the style portfolio has tracked FOFs’ actual performance over the 1998-2006 period. In this case, the style allocation is associated with about 81% (with the Small Cap Index, for both CS and QP) of the variability of the manager’s actual performance, which is higher than all the other hedge fund indices. The remaining 19% is due to the manager’s selection of securities that behaved differently to the passive indices selected. The implication is that this category of hedge funds may have adopted the least active stock picking and other exotic trading strategies; or more likely, the funds’ investment mix is more diversified and thus smooths out the effect of individual stock betting. Figure 5.14 below visually illustrate the tracking ability of the FOFs style portfolio. Inspecting the 12 out-of-sample data points, there seems to be no major outliers, confirming the high out-of-sample $R^2$ values.

**Figure 5.14: Scatter plot of style return and observed return on the FOFs Index**
The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis over the period 1st January 2004 to 31st December 2006. The Fund of Funds Index (FOFs) total returns are obtained from HedgeFund Intelligence Database.
Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with seven independent regression variables including the Small Cap Index. For hedge funds, 24-month rolling periods are used to infer a fund’s investment style. Constrained Sum (CS) regression is used to incorporate the constraint that the sum of the absolute values of the style weights < 12.

Figure 5.15 and Section C of Appendix D.22 show that investment styles under both CS and QP (to a lesser extent) regressions on the Hedge Fund FOFs Index are
unstable over the out-of-sample period. The weights obtained from CS regressions display a similar trend to those of the other three hedge fund indices: the prominent portion of actual returns is again attributable to the size, value and momentum indices. The only notable difference is that there is an increased and consistently positive allocation to the Small Cap Index.

Figure 5.15: Exposure distribution area graph of the FOFs Index
The graph displays the monthly exposure of the Fund of Funds Index (FOFs) to the seven selected explanatory indices over the period 1st January 2004 to 31st December 2006. The monthly total returns are computed from the closing price and dividend yields obtained from HedgeFund Intelligence Database. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with seven independent regression variables including the Small Cap Index. For hedge funds, 24-month rolling periods are used to infer a fund’s investment style. Constrained Sum (CS) regression is used to incorporate the constraint that the sum of the absolute values of the style weights < 12.

5.3.2.6. Sources for instability of styles
As noted in Sections 5.3.2.2 to 5.3.2.5, there is some instability in the exposure distribution area graph of the hedge fund indices investigated and the \( R^2 \) values are low. Suppose the regression constraints are appropriate and the selected independent variables are adequate in all of the aspects discussed in Sections 5.2.3.2, then the low \( R^2 \) are likely to be due to three potential sources: (1) fund management strategies, (2) nature of the investments, and (3) incorrect or insufficient data.
A volatile exposure distribution area graph accompanied by high annual investment turnover tends to imply an active investment style incorporating market timing or sector rotation. This may account for the low $R^2$ and the vacillations in the style weights.

If the fund turnover is low and the fund investment mix is highly concentrated and the exposure distribution area graph is consistent, the low $R^2$ values could be caused by changes in the nature of the investments (e.g. how much a company behaves like a value or growth stock) or by the fact that the fund is holding some exotic securities (e.g. derivatives) whose performance is not well captured by the explanatory variables used in the regression analysis.

Lastly, if there are any errors in the underlying return data that permeate through to the regressions or if the data period available is too short, a low $R^2$ and volatile style weights may be produced. For instance, even if the in-sample fit is good, regressing on 24 data points hardly yields convincing coefficients for the out-of-sample hedge fund analysis. Regressions with rolling period of 18 and 12 months are also conducted for the found hedge fund indices, it is found that the out-of-sample $R^2$ values progressively decline as fewer months of past returns are used to infer the styles. Therefore, the limited history of hedge fund returns may render the results less reliable.

### 5.4 Summary and conclusions

This chapter examines how well the style indices selected in Chapter Four (size, value and momentum), together with the directly tradable sector indices (Satrix FINI, Satrix RESI and Satrix INDI), are able to replicate the returns generated by various unit trust and hedge fund managers investing on the JSE. The return-based style decomposition introduced by Sharpe (1988) is utilised to estimate a fund's investment style over the period January 1998 to December 2006. A note of caution is that given the relatively strong performance of the JSE over the period of investigation, it may be beneficial to test if the conclusions still hold in prolonged bear markets in the future.
5.4.1. Unit trusts

When comparing the returns on an actual unit trust fund to those of a passively managed portfolio with the same style as the evaluated fund, it is found that even the best performing domestic equity unit trusts could not provide style-adjusted excess returns. Across the entire set of funds, the maximum outperformance achieved by a fund over its style portfolio is 0.24% (24 basis points) per month and the maximum underperformance achieved is -0.39% per month. The very low t-values (in absolute value) of the selection returns obtained from both the WLS and the OLS regressions indicate that none of the selection returns of the 14 indices and funds analysed are significantly different from zero.

It must be noted, however, the month-on-month tracking is not precise. The average tracking error of selection returns is approximately 1.34% per month. This equates to an annualised figure of about 4.6% which is moderately high and the kind of level usually associated with an active fund. Thus, while the mean return is very similar over the out-of-sample period there would be a moderate amount of benchmark relative risk within period.

The out-of-sample $R^2$ values are in general above 0.8, therefore the domestic equity fund returns can be closely tracked by investing in a portfolio of low cost, tradable style and sector indices combined in appropriate proportions. In practical sense, one can adopt the methodology described in this chapter and achieve overall returns similar to those of a chosen active domestic equity fund without incurring the expensive active fund management charges.

In further details, it is found that in general WLS regressions are pervasively superior to the OLS regressions for the purpose of producing style returns that outperform actual fund returns. However, similar and very satisfactory out-of-sample $R^2$ values are evidence that both regression methods are able to replicate the actual fund returns reasonably well.

It is evident that the regressions with the Small Cap Index generally produced higher style returns over the period investigated, although implementation of such a strategy may be too expensive due to the illiquidity of the Small Cap Index constituents.
constraints adopted when regressing on unit trusts are: (1) style weights sum to unity (no leverage), and (2) each of the style weight lies between 0 and 1 (no shorting).

5.4.2. Hedge funds

Two methods, namely CS and QP regressions, are performed on four SA hedge fund indices over the period January 2004 to December 2006. It is noted that the overall tracking power measured by the out-of-sample $R^2$ values is lower than those of the unit trusts. This may be because hedge funds tend to adopt more active investment strategies, invest in more exotic and unusual securities and make more bets on the performance of individual stocks. Or, equally likely, this may be a result of the fact that only a very short period of return history is available. It can be argued that no definite conclusions can be drawn basing on a sample as small as 12 data points.

All regressions with the Small Cap Index have invariably produced higher $R^2$ values. This is expected since adding the Small Cap Index results in a more comprehensive set of independent variables which more realistically represents the hedge funds' investment sphere.

Although the CS method is believed to reflect a hedge fund's investment style more realistically, QP has produced higher $R^2$ values and style returns for most indices examined. Under CS, all of the indices have frequently produced positive selection returns. However, only when inadequacy of the independent variables, inconsistency of the fund's style, and insufficiency of the historical return data have been ruled out, should one assume that low $R^2$ and high positive selection returns signal the extent of value creation contributed by the managers' stock selection abilities. In other words, the positive selection returns for hedge fund indices under CS regressions need to be interpreted with caution.

In summary, in the case of hedge fund, given the current available data, a return-based style regression methodology is not able to generate style returns that closely replicate actual hedge fund returns. Moreover, it is inconclusive whether the CS or the QP method is superior nor is it clear whether the style portfolio significantly outperforms or underperforms the actual hedge fund. Given that the major concern for the hedge fund analysis is the lack of sufficient return history, it is recommended that the
analysis to be conducted again after at least four years' historical data are accumulated.
6. Portfolio Optimisation Using Style Indices

"Persistent bear market conditions have led to a shift of focus in the tracking error literature. Until recently the portfolio allocation literature focused on tracking error minimization as a consequence of passive benchmark management under portfolio weights, transaction costs and short selling constraints. Abysmal benchmark performance shifted the literature's focus towards active portfolio strategies that aim at beating the benchmark while keeping tracking error within acceptable bounds."


6.1 Introduction

In Chapter Five, a set of style indices have been used in portfolios constructed to minimise the historical prevalence of tracking errors relative to existing representative SA active equity funds. However, up until now no attention has explicitly been given to improving returns. In this chapter, the ex-post optimal portfolios are created utilising selected style and appropriate benchmark indices.

The concepts of mean-variance optimisation, mean-tracking error optimisation and the efficient frontier are considered when constructing optimal portfolios subject to various constraints based on the entire history of past return data. The portfolio building blocks are the three tradable style indices formulated in Chapter Four (namely EW(size)100 for the size style, RESW100(3) for the value style and MOM(12-1)W100 for the momentum style), the FTSE/JSE Africa Shareholder Weighted Top 40 Total Return Index (the SWIX Index, if shorting is forbidden) and the FTSE/JSE Africa Top 40 Total Return Index (the Top 40 Index, if shorting is allowed). The main objective of this chapter is to assess the risk-return trade-off of the actively constructed optimal portfolios by comparing its performance to that of the SWIX Index.

The remainder of the chapter is set out as follows: Section 6.2 describes the portfolio construction methodology adopted, Section 6.3 reports the empirical results of the optimisation using historical data, and Section 6.4 summarises and concludes.
6.2 Methodology

Two sets of optimal portfolios are constructed in this chapter: (1) the mean-variance efficient portfolios, and (2) the mean-tracking error efficient portfolios. The optimisation is conducted on the entire history of total index returns over the period of investigation (January 1998 through December 2006). The Top 40 Index is used if short positions are permitted; else the SWIX Index serves as an appropriate benchmark.

6.2.1. Dataset

The optimal portfolios are constructed by investing in all or a subset of the following five indices: the three style indices selected in Chapter Four, the SWIX Index and the Top 40 Index. The SWIX is deemed to be an appropriate choice as it has emerged as a popular long-only equity benchmark. The Top 40 is used when shorting is allowed as unlike the SWIX it has a liquid futures contract that can be used for this purpose. The portfolios do not comprise the FTSE/JSE Africa Small Cap Total Return Index (the Small Cap Index) since this index is not readily tradable due to liquidity concerns.

The monthly total returns over the period 1\textsuperscript{st} January 1998 to 31\textsuperscript{st} December 2006 are computed for each index. Since the SWIX Index is only available from January 2002, the FTSE/JSE Africa All Share Total Return Index (the ALSI) is merged with it from 1\textsuperscript{st} January 1998 to 31\textsuperscript{st} December 2006 to fill the gap in historical returns.

6.2.2. Mean-variance optimisation

In the case of mean-variance optimisation, the term ‘mean’ refers to the mean or the expected return of the portfolio and ‘variance’ is a measure of the risk associated with the portfolio. The optimisation problem can be mathematically formulated in many ways with various linear or nonlinear constraints, but the underlying principle remains the same as those proposed by Markowitz (1952, 1959 and 1987). For instance, to minimise risk for a specified expected return or maximise the expected return for a specified risk level are essentially mathematically equivalent, and the solutions for both approaches yield mean-variance efficient portfolios. The efficient points in the risk-return diagram together form the efficient frontier.
In this thesis, the mean-variance optimisation is conducted by solving for the minimum total risk (measured by variance) portfolio at each level of mean portfolio returns. As previously mentioned, the same constituent weightings would be obtained if the maximum return portfolio had been sought for at each level of total risk. The optimal portfolios resulting are effectively *mean-variance efficient* portfolios.

Three strategies are investigated where the underlying optimisation procedure remains similar, the only difference being that each strategy specifies a different set of constraints in terms of allowable levels of short and leverage positions. The optimisation method is described in detail for the long-only strategy while the constraints for the other two strategies are emphasised.

### 6.2.2.1. Long-only strategy

The portfolios with minimum standard deviation ($S_p$) at each level of total portfolio returns ($R_p$) are constructed based on historical index returns over the period 1\textsuperscript{st} January 1998 to 31\textsuperscript{st} December 2006. The long-only strategy permits no shorting or leveraged positions to be undertaken by the investor. Consequently, the weights of all indices constituting the portfolio must be non-negative and sum to unity. The detailed construction process is as follows.

Firstly, a maximum and a minimum value are chosen for $R_p$ to serve as the constraint used for the optimisation process. The natural range of $R_p$ is to take the lowest and highest mean returns of its constituting indices. In this case, they are returns on the EW(size)100 (1.73% per month) and the MOM(12-1)W100 Index (2.51% per month).

Secondly, the range of $R_p$ is annualised and then divided into sub-intervals of, for instance, 1% per annum. This gives a series of specified $R_p$ values, each being 1% higher than the previous value. Thirdly, the mean-variance optimisation technique is used to solve for the set of constituent weights that produces a portfolio with the lowest risk measured by $S_p$ at each specified level of $R_p$. 
Summarised numerically, the aim is to find the set of \( w_i \)s that minimise \( S_p \) subject to specified \( R_p \) values over the period of investigation, where:

\[
R_p = \prod_{t=1}^{n} (1 + R_{p,t})^{\frac{1}{n}}
\]  

(6.1)

And

\[
S_p = \sqrt{\frac{1}{n-1} \sum_{t=1}^{n} (\bar{R}_p - R_{p,t})^2}
\]

(6.2)

Where:

\[
R_{p,t} = \sum_{i=1}^{n} w_i R_{i,t} + \epsilon_{p,t}
\]

(6.3)

\[
\bar{R}_p = \sum_{t=1}^{n} R_{p,t}
\]

\( R_p \) represents the geometric mean monthly return on the optimal portfolio \( p \)

\( \bar{R}_p \) represents the arithmetic mean monthly return on the optimal portfolio \( p \)

\( S_p \) represents the monthly standard deviation on the optimal portfolio \( p \) over the entire period of investigation

\( R_{p,t} \) represents the monthly return on the optimal portfolio \( p \) in month \( t \)

\( R_{i,t} \) represents the monthly return on constituent \( i \) in month \( t \)

\( w_i \) represents the weight of constituent \( i \) over the entire period of investigation

\( m \) represents the number of constituents in the optimal portfolio, here \( m=4 \), the constituents are SWIX (the SWIX Index merged with the ALSI), EW(size)100, RESW100(3), and MOM(12-t)W100

\( n \) represents the number of monthly returns over the entire period of investigation, here \( n=108 \).

6.2.2.2. Long-short-equity strategy

This strategy implies that both long and short positions are permissible. As mentioned the SWIX Index is deemed not ‘shortable’ since it consists of almost all JSE All-Share constituents and there is no existing equivalent Exchange Traded Fund (ETF).
A ‘shortable’ market proxy is the Top 40 Index, which consists of only the 40 largest companies on the JSE and hence can be shorted without incurring excessive transaction costs. Therefore, the Top 40 Index is adopted as the substitute for the SWIX Index whenever the strategy involved allows shorting and leverage. The weights of the style indices must remain non-negative since shorting style indices is currently inappropriate as discussed previously for the SWIX Index.

Furthermore, the leverage (calculated as the sum of absolute weights of the portfolio constituents) is capped at 200%. In other words, the investor can borrow to invest up to a maximum of 200% of his original capital.

The mean-variance portfolios are obtained in the same way as in the previous section subject to the changes and additional constraints described above.

### 6.2.2.3. Market neutral strategy

This strategy imposes the constraint that the weights of all the constituting indices must sum to zero, and hence the shorting proceeds are utilised to finance the long position and portfolio does not take a position in the market. The Top 40 Index is used since shorting is involved. As described for the long-short-equity strategy, the weights of the style indices must be non-negative and a 200% cap on leverage is imposed.

### 6.2.3. Mean-tracking error optimisation

Tracking a widely recognised benchmark index as closely as possible is normally sufficient during bull market conditions. On the other hand, during persistent bear markets (such as those over the late 1990s), most fund managers aim to persistently generate excess returns against their benchmark while accepting some active risk normally capped in the form of a specified maximum portfolio tracking error. Therefore there are two uses of index tracking errors when formulating investment strategies: (1) minimising tracking error to ensure close tracking of passive indices, and (2) actively seeking to beat the benchmark index while keeping tracking error within acceptable bounds (Fernando, 2000). This thesis adopts the second approach.

In this section, tracking error is calculated with respect to the SWIX Index. This resembles the application of a core-satellite strategy using SWIX as the ‘core’
Portfolio and all or some of the three selected style indices as the 'satellites' to enhance performance. The portfolios with the maximum $R_p$ at each level of tracking error ($S_e$) are constructed based on the entire history of total index returns over the period January 1998 to December 2006. No shorting or leverage is permitted. The remainder of this section describes the optimisation process in detail.

Firstly, an acceptable range is chosen for the magnitude of $S_e$. The values of $S_e$ used in this thesis rise from 0% to 20% per annum, with increments of 0.5%. Secondly, the monthly excess return ($R_{x,t}$) is computed by comparing the returns on the optimal portfolio to those of the 'core' portfolio (in this case, it only contains the SWIX Index). Therefore:

$$R_{x,t} = R_{p,t} - R_{c,t}$$ (6.4)

Where:

- $R_{x,t}$ represents the monthly excess return in month $t$
- $R_{c,t}$ is the monthly observed return on the SWIX Index in month $t$
- $R_{p,t}$ represents the overall monthly return on the optimal portfolio $p$ in month $t$, it is calculated in the same way as in Equation (6.3).

Lastly, the standard deviation of this excess return over the period investigated is defined as $S_e$. Thus:

$$S_e = \sqrt{\frac{\sum_{t=1}^{n} \left( \frac{R_{x,t} - R_{x,c,t}}{n-1} \right)^2}{n-1}}$$ (6.5)

Where:

$$\bar{R}_x = \frac{\sum_{t=1}^{n} R_{x,t}}{n}$$
The optimal portfolios are obtained by solving the weights of the three style indices and the SWIX Index (i.e., the $w_i$'s) that produce a portfolio with the highest $R_p$ (as defined in Equation (6.1)) at each specified level of $S_x$.

6.3 Empirical results

The optimal portfolio structures obtained over the entire period of January 1998 to December 2006 are reported for each optimisation strategy. The main graphical results are presented in the following sections, whereas the corresponding numerical results, including means and standard deviations of portfolio returns and weights of the constituents, can be found in Appendix E.2- E.7.

6.3.1. Correlation matrix of indices

Index correlation is an important factor to consider since it influences the total risk of the constructed portfolio. Due to the fact that all the indices move in line with the market at least to a certain extent, one can expect the total return correlation matrix to be very highly correlated. Therefore, excess returns relative to SWIX (the market proxy used in this chapter) are investigated to give a clearer indication of the relative performance of the indices. The indices included in the matrix are: SWIX, EW(size)100, RESW100, MOM(12-1)W100 and Top 40.

It should be noted that the value and size index are quite highly correlated. The momentum index has a relatively low correlation not only with the size index but also with the value index. The low correlation between momentum and value is expected since they employ different performance measures to formulate the style portfolios. It is noted that the Top 40 index has negative correlation with all the other indices, therefore including it significantly reduces the variance of the optimal portfolios. Section A of Appendix E.1 shows the correlation matrix based on indices’ excess returns relative to the ALSI, where as Section B of Appendix E.1 displays the correlation matrix computed on the total returns of the five indices employed in this chapter. The conclusions drawn are very similar to those mentioned above.

Table 6.1: Correlation matrix of style-index excess returns relative to SWIX
The table displays the correlation matrix of the excess monthly returns relative to SWIX of the three style indices constructed and the Top 40 Index over the period 1st January 1998 to 31st December 2006.

<table>
<thead>
<tr>
<th></th>
<th>EW 100</th>
<th>MOM(12-1)W 100</th>
<th>RESW 100</th>
<th>Top 40</th>
</tr>
</thead>
<tbody>
<tr>
<td>EW 100</td>
<td>1.000</td>
<td>0.570</td>
<td>0.688</td>
<td>-0.575</td>
</tr>
<tr>
<td>MOM(12-1)W 100</td>
<td>0.570</td>
<td>1.000</td>
<td>0.309</td>
<td>-0.372</td>
</tr>
<tr>
<td>RESW 100</td>
<td>0.688</td>
<td>0.309</td>
<td>1.000</td>
<td>-0.487</td>
</tr>
<tr>
<td>Top 40</td>
<td>-0.575</td>
<td>-0.372</td>
<td>-0.487</td>
<td>1.000</td>
</tr>
</tbody>
</table>

6.3.2. Mean-variance optimisation

The mean-variance efficient portfolio refers to the portfolio that has the lowest variance at a specified level of portfolio return, or equivalently, the highest return at a specified level of risk.

6.3.2.1. Long-only strategy

The scatter plot in Figure 6.1 plots resulting estimated efficient frontier over the period January 1998 to December 2006. The vertical axis depicts the annualised mean portfolio returns, and the horizontal axis features the annual total portfolio standard deviations.

The lowest point on Figure 6.1 is the result from investing entirely in the EW(size)100 Index. Being the lowest risk-return combination, this portfolio is not mean-variance efficient. In other words, a higher return can be achieved by taking on the same level of risk. In this case, a portfolio with a similar annual standard deviation of 22.7% can be constructed by allocating 35% of the overall portfolio to the SWIX Index and 65% to the RESW100(3) Index. This mean-variance efficient portfolio delivers a return of 31% per annum in comparison to the 22.8% generated by the inefficient portfolio with the same risk level. Similarly, the other portfolios represented by triangular dots also do not form part of the efficient frontier. The efficient frontier comprises all of the portfolios represented by the square dots.

The round dot on Figure 6.1 depicts the benchmark SWIX Index, with mean annualised return of 23.5% and annualised standard deviation of 22%. The SWIX Index is dominated by those portfolios constructed using the style indices. To achieve a level of $R_p$ in excess of that of the SWIX Index, it is necessary to take on a higher level of risk. Based on ex-post returns the maximum mean return achievable is 34.8%. 
this optimal portfolio produces an annualised standard deviation of around 26.7%. This indicates that by taking on extra risk of 4.7% per annum, one is able to earn an additional annual return of 11.3% relative to the SWIX Index over the period of investigation.

Figure 6.1: Efficient frontier of the long-only mean-variance efficient portfolios (SWIX benchmark, no shorting, no leverage)
The graph displays efficient frontier (in squared dots) of the minimum variance portfolios subject to specified total portfolio returns. EW stands for equally weighted indices. The optimisation process is conducted using the historical return data over the period January 1998 to December 2006. The portfolio building blocks are the three tradable style indices formulated in Chapter Four (namely EW(size)100 for the size style, RESW100(3) for the value style and MOM(12-1)W100 for the momentum style), and the SWIX Index. The optimisation is conducted subject to the constraint that there is no shorting or leverage positions and the weights of all constituents are positive.

Figure 6.2 shows the weights change of different indices making up the optimal portfolios as the values of portfolio return constraint ($R_p$) changes.

It is interesting to note that the weights of the RESW100(3) and MOM(12-1)W100 Index have increased progressively in order to achieve higher portfolio returns. The worst performing style index, EW(size)100, quickly dropped out of the optimal portfolio as an investor aiming for higher levels of returns is willing to take on increased risk. The minimal influence of the EW(size)100 Index is also evident in the portfolio construction scenarios to be discussed later in Sections 6.3.2.2 and 6.3.2.3.
As an investor’s risk tolerance increases, the allocation to the SWIX Index first increases to fill up the gap left by the size index and then gradually gives way to the stronger-performing value and momentum indices. Eventually, the high-risk high-return optimal portfolio consists of 42% of the value index and 58% of the momentum index.

Tilting away for the concentrated passive equity indices (SWIX and Top 40) towards the value and momentum indices tends to also have a strong effect on reducing the concentration of the entire portfolio, which may nullify the unique benefit conferred by introducing the equally weighted index. In terms of return-risk ratio it seems that a portfolio almost entirely composed of the value index would perform very well (although the optimiser does suggest that about 10% should be allocated to momentum). This suggests that the value index could serve as a worthy long-only benchmark in its own right.

Figure 6.2: Weights of the long-only mean-variance efficient portfolios (SWIX benchmark, no shorting, no leverage)

The graph shows the change in the weights of different indices to form the minimum variance portfolios as the specified constraint value of total portfolio return changes. The optimisation process is conducted using the historical return data over the period January 1998 to December 2006. The portfolio building blocks are the EW(size)100 for the size style, RESW100(3) for the value style and MOM(12-1)W100 for the momentum style, and the SWIX Index. The optimisation is conducted subject to the constraint that there is no shorting or leverage positions and the weights of all constituents are positive.
6.3.2.2. Long-short-equity strategy

Figure 6.3 portrays the efficient frontier of the mean-variance efficient portfolios obtained over the period January 1998 to December 2006. The annual mean portfolio returns are displayed on the vertical axis and the annual total portfolio standard deviations plotted on the horizontal axis.

As opposed to the long-only strategy, allowing a short position results in the minimum risk level being significantly lower than that under a long-only strategy. In addition, unlike in Figure 6.1 where portfolios located towards the bottom of the graph are inefficient portfolios, all portfolios in Figure 6.3 are mean-variance efficient. This communicates the fact that the more risk averse the investor may be, the lower returns achievable as a result of bearing minimal risk.

Comparing the long-only and the long-short-equity optimal portfolios, the latter strategy has delivered higher return for the same level of risk. Simply put, the long-short-equity optimal portfolio with 22.7% of risk has yielded 33.2% average annualised return, whereas the long-only optimal portfolio only generated an average return of 23% for the same level of risk.
Furthermore, the highest return achievable is higher under the long-short equity scenario (64.8%). This is largely a result of being able to take on a leverage position greater than 100%. The proceeds from shorting the historically underperforming index (Top 40) can be invested in the better performing indices (value and momentum) and hence boost overall portfolio return.

**Figure 6.3: Efficient frontier of the long-short-equity mean-variance efficient portfolios (with shorting on Top 40 allowed, leverage up to 200%)**

The graph displays efficient frontier of the minimum variance portfolios subject to specified total portfolio returns. EW stands for equally weighted indices. The optimisation process is conducted using the historical return data over the period January 1998 to December 2006. The portfolio building blocks are the EW(size)100 for the size style, RESW100(3) for the value style and MOM(12-1)W100 for the momentum style, and the Top 40 Index. The optimisation is conducted where shorting and leverage are allowed, therefore the constraints are the sum of the absolute weights of the indices (leverage) to be not more than 200% and the weights of the style indices to be positive.

Figure 6.4 depicts the composition of the mean-variance efficient portfolios at different level of risk, return and leverage. It is noted that the proportion of style indices compromising the long-short-equity optimal portfolio stayed relatively constant before the 200% leverage cap comes into effect. Figure 6.3 shows an upward sloping straight line with slight concavity until the leverage cap is evoked. Therefore, almost the same optimal portfolio is leveraged up to achieve higher returns at the expense of higher risks. The optimal position is approximately 70% in the value index, 10% in the momentum index and -20% in the market (proxied by the Top 40 Index).
The optimal portfolio has a fair-sized net exposure of 55%. It should be noted however that the sample period has on average been a good performing period for equity markets. The (unreported) efficient frontier increases linearly from the origin at the return-standard deviation ratio of 1.4 until the leverage cap is reach. The introduction of shorting allows an improvement on the highest ratio achieved (1.25) in the long-only space.

When the portfolio leverage reaches the 200% cap specified, the risk-return trade-off flattens out. Therefore although higher risk is still accompanied by higher returns, the returns increase at a decreasing rate. Since shorting reduces both risk and return of a portfolio, when the leverage cap is reached, the negative weights in the Top 40 Index start to decline as more long position needs to be undertaken to secure higher returns; meanwhile, the weights in the value index increases exponentially.

**Figure 6.4: Weights of the long-short-equity mean-variance efficient portfolios (with shorting on Top 40 allowed, leverage up to 200%)**

The graph shows the change in the weights of different indices to form the minimum variance portfolios as the specified constraint value of total portfolio return changes. The optimisation process is conducted using the historical return data over the period January 1998 to December 2006. The portfolio building blocks are the EW(size)100 for the size style, RESW100(3) for the value style and MOM(12-1)W100 for the momentum style, and the Top 40 Index. The optimisation is conducted where shorting and leverage are allowed, therefore the constraints are the sum of the absolute weights of the indices (leverage) to be not more than 200% and the weights of the style indices to be positive.
6.3.2.3. Market-neutral strategy

The market-neutral strategy implies that the net investment in the market sums to 0, as a result, the long and short positions must cancel out each other. This tends to reduce the overall portfolio risk. It is noted; however, that the procedure of constructing market neutral portfolios can result in unnecessarily low expected returns for a certain level of risk, and therefore generate inefficient risk-return points. This is clear if one compares the market neutral results to the long-short-equity results, where for the same level of risk, the latter is able to deliver higher returns.

In Figure 6.5, the portfolio with highest return (10.8%) has experienced an annual standard deviation of 19.5%. Although this portfolio is plotted in Figure 6.5, it is mean-variance inefficient. The numerical outputs of portfolio structure are attached in Appendix E.4. The efficient frontier increases linearly from the origin at the return-standard deviation ratio of 0.6 before the 200% leverage cap comes into effect.

Figure 6.5: Efficient frontier of the market-neutral minimum-variance efficient portfolios (Top 40 benchmark, market neutral strategy, leverage capped at 200%)

The graph displays efficient frontier of the minimum variance portfolios subject to specified total portfolio returns. EW stands for equally weighted indices. The optimisation process is conducted using
the historical return data over the period January 1998 to December 2006. The portfolio building blocks are the EW(size)100 for the size style, RESW100(3) for the value style and MOM(12-1)W100 for the momentum style, and the Top 40 Index. The optimisation is conducted subject to the constraint that the leverage position is not greater than 200%, the weights of all constituents sum to 0, and short position only allowed on the Top 40 Index.

Figure 6.6 displays the composition of the market-neutral optimal portfolios at different levels of risk, return and leverage, which follows a very theme pattern to that for the long-short-equity strategy. At a leverage of unity the optimal portfolio's constituents are as follows: short 100% Top 40 and long 69% value and 31% momentum. This same ratio is scaled up or down as leverage changes until the cap of 200% is reached. Over this period of relatively strong performing equity markets, the impact of the market neutral constraint is clearly stronger than of disallowing shorting. This return-risk ratio may prove more sustainable in periods of equity weakness. It should be noted that despite being rand neutral this strategy may exhibit a negative beta due to the high betas observed in Top 40 shares.

Exposure to the momentum index increases at the expense of the value index. Since the momentum index has slightly higher return and much greater risk and than the value index, increasing the weighting to the former index has the effect of producing inefficient high-risk-high-return portfolios.
Figure 6.6: Weights of the market-neutral mean-variance efficient portfolios (Top 40 benchmark, market neutral strategy, leverage capped 200%)

The graph shows the change in the weights of different indices to form the minimum variance portfolios as the specified constraint value of total portfolio return changes. The optimisation process is conducted using the historical return data over the period January 1998 to December 2006. The portfolio building blocks are the EW(size)100 for the size style, RESW100(3) for the value style, and MOM(12-1)W100 for the momentum style, and the Top 40 Index. The optimisation is conducted subject to the constraint that the leverage position is not greater than 200%, the weights of all constituents sum to 0, and short position only allowed on the Top 40 Index.

Two sets of complementary market neutral results are displayed in Appendices E.6 and E.7. They each utilise an alternative leverage constraint: leverage fixed at 100% for E.5 and at 200% for E.6.

6.3.3. Mean-tracking error optimisation

The full set of optimal portfolio results is presented in Appendix E.7 and summarised graphically in Figure 6.7. The bar graph on the primary axis portrays the composition of the maximum return portfolio at each specified level of $S_r$. The line graph on the secondary axis illustrates the relationship among size of the tracking error, the $R_p$ and the total portfolio standard deviation. The primary vertical axis is on the right hand side of Figure 6.7, showing the weights of different indices constituting the maximum return portfolios.
As expected, with 0% tracking error, the optimal portfolio starts off by investing entirely in the SWIX Index (the core benchmark), yielding an average return of 23.5% per annum over the period of investigation. The weights of the MOM(12-1)W100 Indices have increased consistently as investors are willing to take on higher risks in pursuing higher returns; while allocation to the RESW100(3) has also increased but in a somewhat larger yet consistent proportion. Once the entire SWIX is displaced by the value and momentum indices (at a tracking error of 11.5%) the overall efficiency of the stalls with increased tracking error and total risk making little impact on returns. The EW(size)100 has no weighting in any of the optimal portfolio for the reasons mentioned in the previous section.

The mean portfolio return increases at a faster rate than portfolio standard deviation until the tracking error reaches the 11.5% level; from thereon the risk-return trade-off is reversed. As a further illustration, by taking on extra risk measured in tracking errors of 10% per annum and thus investing in a portfolio constituting 13% in SWIX, 66% in the value index and 21% in the momentum index, one is able to earn a total annual return of 32.9%.

In other words, despite the increase in tracking error, there is not a meaningful appreciation on the portfolio’s total risk up to the 11.5% tracking error level. This portfolio (about 74% value, 26% momentum) is an alternative optimal long-only style allocation that pays relatively little heed to the SWIX benchmark.

For an enhanced index fund (3% tracking error) the strategic fund weightings would be about 74% SWIX, 20% value and 6% momentum. A typical benchmark cognisant active fund (5% tracking error) would be constructed as about 56% SWIX, 33% value and 11% momentum.

Figure 6.7: Total returns, standard deviations and weights of the mean-tracking error efficient portfolios (SWIX benchmark, no shorting, no leverage)
The primary axis shows the change in the weights of different indices to form the maximum return portfolios as the specified value of tracking errors changes. The line graph on the secondary axis illustrates the relationship among size of the tracking error, the total portfolio returns and the total portfolio standard deviation. The vertical axis depicts the total portfolio returns and standard deviations per annum, and the horizontal axis features the annual tracking error. The optimisation process is conducted using the historical return data over the period January 1998 to December 2006. The portfolio building blocks are the EW(size)100 for the size style, RESW100(3) for the value style and MOM(12-1)W100 for the momentum investment style, and the SWIX Index. The optimisation is conducted subject to the constraint that there is no shorting or leverage positions.

6.4 Summary and conclusion

The constrained mean-variance optimisation technique is utilised in this chapter to produce the best-performing portfolios based on historical returns of the three style indices selected from Chapter Four and the SWIX Index (or Top 40 if shorting is allowed). The two sets of optimal portfolios constructed are (1) the minimum variance portfolios subject to specified portfolio returns (mean-variance efficient portfolios), and (2) the maximum return portfolios subject to specified tracking errors (mean-tracking error efficient portfolios). For the mean-variance efficient portfolios, three constraints are investigated, namely: long-only, long-short-equity and market neutral.

Overall, the long-short-equity strategy with leverage capped at 200% has produced portfolios that deliver higher returns for the same level of risk as compared to long-
only and market neutral strategies. The optimal portfolios for all strategies have large exposure to the value index and no exposure to the size index.

6.4.1. Mean-variance optimisation

Under the long-only strategy, it is necessary to take on a higher level of risk in order to achieve a level of $R_p$ in excess to that of the SWIX Index. The maximum mean return achievable is 34.8% accompanied by an annualised standard deviation of around 26.7%. This indicates that by taking on extra risk of 4.7% per annum, one is able to earn an additional annual return of 11.3% relative to the SWIX Index over the period of investigation. It is noted that weights of the RESW100(3) Index and the MOM(12-1)W100 Index have increased progressively in order to achieve higher portfolio returns while the worst performing index, EW(size)100, quickly dropped out of the optimal portfolio. The lower influence of the equally weighted top 100 index is also evident in the other portfolio construction scenarios discussed in this chapter.

In comparison to the long-only strategy, allowing short positions results in higher returns for the same level of risk. Due to the possibility of leverage, much higher potential returns (64.8% at the maximum leverage) are available when compared to the long only option. The proceeds from shorting the underperforming index (Top 40) can be invested in the better performing indices (value and momentum). Furthermore, the proportion of style indices compromising the optimal portfolios stayed relatively constant before the 200% leverage cap comes into effect. The optimal position is approximately 70% in the value index, 10% in the momentum index and -20% in the market (proxied by the Top 40 Index). When the portfolio leverage reaches the 200% cap specified, the risk-return trade-off flattens out.

The market-neutral strategy implies that the portfolio does not take a net position in the market. It is noted that, relative to the other strategies employed, the market-neutral construction method can generate inefficient risk-return portfolios. For instance, for the same level of risk, the long-short-equity strategy is able to deliver higher returns than the market neutral strategy. The composition of the market-neutral optimal portfolios at different levels of risk, return and leverage follows a very similar pattern to that for the long-short-equity strategy. The optimal portfolio has the following approximate composition: short 100% Top 40 and long 69% value and 31%
momentum. This same ratio is scaled up or down as leverage changes until the cap of 200% is reached. Over this period of relatively strong performing equity markets, the impact of the market neutral constraint is clearly stronger than that of disallowing shorting. This return-risk ratio may prove more sustainable in periods of equity weakness. It should be noted that despite being rand neutral this strategy may exhibit a negative beta due to the high betas observed in Top 40 shares.

6.4.2. Mean-tracking error optimisation

In the case of the return-tracking error efficient portfolios, despite the increase in tracking error, there is not a meaningful appreciation on the portfolio’s total risk up to the 11.5% tracking error level. The portfolio with 34.3% return and 24.6% standard deviation is achieved by allocating 74% of the portfolio to the value index and 26% to the momentum index. This portfolio is an alternative optimal long-only style allocation that pays relatively little heed to the SWIX benchmark. For an enhanced index fund (3% tracking error) the strategic fund weightings would be about 74% SWIX, 20% value and 6% momentum. A typical benchmark cognisant active fund (5% tracking error) would be constructed as about 56% SWIX, 33% value and 11% momentum.

As an investor is willing to make more risky deviations from the SWIX Index in pursuing higher returns, weights of the value index first increase at the expense of the SWIX Index and then give way to the momentum index. As an illustration, by taking on the extra risk measured in tracking errors of 10% per annum, one is able to earn an excess return of 9.4% per annum relative to SWIX.
7. Summary and Conclusion

7.1 Introduction

There exist three distinct investment styles (namely size, value and momentum) identified to produce excess returns on the Johannesburg Stock Exchange (JSE) (Van Rensburg, 2001). This thesis attempts to determine the most suitable style proxy and style-index construction method for each of the three investment styles. The best performing and most robust indices identified for each investment style are identified as: (1) the Equally Weighted size 100 Index (EW(size)100) for the size investment style; (2) the Three-factor (Earnings, Book Value and Dividend) Regression Residual Weighted value 100 Index(RESW100(3)) for the value investment style, and (3) the Past 12 Month less Prior Month Return Weighted momentum 100 Index(MOM(12-1)W100) for the momentum investment style.

Subsequently, adopting Sharpe’s (1988) return-based style analysis, style portfolios are created using a passive mix of selected style and sector indices to replicate the performance of a sample of SA domestic equity portfolios. Empirical evidence shows that in general, active equity fund manager investing on the JSE cannot consistently outperform their respective style portfolio benchmark. The tracking errors of the style portfolios, however, are relatively significant (on average 1.3% per month), indicating that precise return replication cannot be achieved on a monthly basis.

The behaviour of the mean-variance and mean-tracking error optimal portfolios constructed using selected style indices, the FTSE/JSE Africa Shareholder Weighted Top 40 Total Return Index (the SWIX Index) and the FTSE/JSE Africa Top 40 Total Return Index (the Top 40 Index) differ according to the shorting and leverage constraints imposed. The long-only mean-variance analysis shows that the value index would make a worthy long-only equity benchmark even in isolation. The long-short equity mean-variance optimal portfolio is short about 20% in Top 40 and long 65% value and 10% momentum. This results in a strategic net exposure of 55%. The market neutral optimal portfolio estimated is short 100% Top 40, long 69% value and
long 31% momentum. Overall, the highest annual return is achieved by the long-short-equity optimal portfolio allowing 200% leverage. The optimal portfolios for all mean-variance efficient portfolios have large exposure in the low-risk-high-return value index and no exposure in the inefficient size index.

In the case of the return-tracking error efficient portfolios, as an investor is willing to make more risky deviations from the benchmark portfolio (the SWIX Index) in pursuing higher returns, the weights of the value index first increases at the expense of the SWIX Index and then decreases to give way to the momentum index. An illustrative enhanced index strategy with 3% tracking error would comprise 74% in the SWIX, 20% in the value index and 6% in the momentum index.

The remainder of this chapter is set out as follows: Section 7.2 summarises the findings of the analysis performed in Chapters Four to Seven, and Section 8.3 suggests potential extensions to the research conducted in this thesis.

7.2 Summary of results

7.2.1. Investigating candidate style indices

7.2.1.1. Size indices

Various style indices are tested for each of the three noted investment styles on the JSE (Van Rensburg, 2001). For the size indices, the EW(size)100 Index shows the most outstanding performance in terms of returns. The monthly rebalanced Small Cap Index is the next best performer, characterised by the highest Sharpe and Treynor Ratios, and the lowest standard deviation and beta. Furthermore, a distinctive return pattern has been displayed by the Top 40 Index over the period investigated, evidenced by its low correlation with the other size-style indices.

The high p-values of regression alphas, however, suggest that none of the abnormal returns generated by any of the size indices are significantly different from zero at the 10% significance level. Thus the size anomaly is not significant on the JSE. This finding concurs with the work of [Bradfield et al (1988) and Page and Palmer (1991)] who refute the prevalence of a size effect on the JSE.
In summary, the EW(size)100 Index is selected as the preferred size index. The Small Cap Index is used primarily for the purposes of comparison as it is recognised that due to liquidity constraints it is not well suited to being a member of the portfolio construction toolkit.

7.2.1.2. Value indices

There is well-documented evidence on the value effect on the JSE, where the major value proxy identified by most literature is PE (or EY in this thesis) [Basu (1977) and Jaffe and Keim (1989)] and, to a lesser extent, earnings (Lamont, 1998) and market to book value (MTBV) [Statman (1980) and Rosenberg, Reid, Lanstein (1985). Kothari and Shanken (1997), and Loughran (1997)]. This thesis builds on this school of study and examines the suitability of some additional firm attributes as value proxies. The eight single factor firm attributes investigated are: earnings yield (EY), the book to market value ratio (BTMV), total cashflow (CF), dividend (DIV), sales (SALE), earnings per share (EPS), total earnings (EAR) and total book value (BV). In addition, the residuals (RES) from three types of multi-factor regressions are employed as potential value proxies, measuring the relative cheapness of the shares. It is found that all of the value indices have outperformed the FTSE/JSE Africa All Share Total Return Index (the ALSI) over the period of investigation.

It is clear that the residual weighted indices, particularly RES(4) and RES(3), are the best performing value proxies over the period 1 January 1998 to 31 December 2006; while BTMV is the most satisfactory proxy among all of the eight single factor firm attributes investigated. It is also noted that as the number of fundamental firm attributes used in the multi-factor regression increases; the returns generated by the corresponding RES(N)-weighted index appear to increase, whereas the number of index constituents seems to fall sharply. This renders RESW100(3) the ‘best performer’ after a balanced consideration of return generating ability, representatives of the index and portfolio concentration. On the other hand, although EAR, EPS, BV, CF and DIV seem to be inadequate value proxies in terms of generating superior returns, they appear to produce more stable portfolios with considerably lower turnover costs.
Finally, most of the value-index results need to be viewed with caution, bearing in mind the lack of reliability, stability and availability of accounting information. However, this is less of a concern for the residual-weighted (RESW) indices. The RESW indices are not only more reliable as a result of being derived by blending multiple firm attributes, but also less exposed to extreme firm-attribute values due to the use of log on all regression factors.

7.2.1.3. Momentum indices
Empirical tests of the momentum indices confirm the momentum effect on the JSE previously reported by Fraser and Page (1999) and Van Rensburg (2001), but show no support for the premise that ‘past losers tend to become future winners’ on the JSE. All of the momentum-style indices constructed, except for the EW(MOM)Negative Index, have generated significant excess returns relative to the ALSI.

Furthermore, indices computed utilising MOM(12-1) as the momentum proxy remarkably outperform their respective MOM-proxied version. This seems to confirm the ‘mean reverting effect’ [Kim et al (1991) and Exley et al (2004)] and indicate that past one year’s returns excluding the most recent month’s return is a more satisfactory proxy for the momentum-style factor than MOM. Another general finding is that the equally weighted (EW) indices appear to underperform the characteristic weighted indices.

MOM(12-1)W30 produces the leading gross and cost-adjusted average returns among all of the momentum indices constructed, followed by MOM(12-1)W50. Even though its overall cumulative return only ranked the 3rd among the ten momentum-style indices, MOM(12-1)W100 entails a more stable and diversified style portfolio. It is decided to select the monthly rebalanced MOM(12-1)W100 Index to represent the momentum style.

In summary, the best performing and most representative indices identified for each investment style are: (1) the Small Cap Index and the EW(size)100 Index for the size investment style; (2) the RESW100(3) Index for the value investment style; and (3) the MOM(12-1)W100 Index for the momentum investment style.
7.2.2. Replicating Active Equity Portfolios

Adopting Sharpe's (1988) return-based style analysis, style portfolios are created using a passive mix of selected style and sector indices to replicate the performance of a sample of SA general equity unit trusts and hedge funds over the period January 1998 to December 2006. Style portfolios are created using a passive mix of the four selected style indices mentioned above and three sector indices, representing the resource, financial and industrial sectors on the JSE respectively.

7.2.2.1. Unit trusts

The empirical results provide little support for the hypothesis that the performance of a typical actively managed unit trust is able to beat that of a passive alternative with the same style composition. The major observation is consistent with that of prior international [Saez and Izquierdo (2000), Quigley and Sinquefield (2000) and Davis (2001)] and local studies (Scher and Muller, 2003) who find that SA unit trusts do not provide excess returns after adjustment for their investment styles. The practical implication is that one can obtain the returns of top active unit trusts without incurring the high management fees by investing in appropriate style portfolios constituting passive low-cost style- and sector-Exchange Traded Funds (ETFs).

Unit trusts on average outperform or underperform their style portfolios by a maximum of 0.24% (24 basis points) or 0.39% per month. None of the selection returns of the 14 indices and funds analysed is significantly different from 0 even at 10% level. The out-of-sample regressions of style returns over actual fund returns have produced very high R² values (in general above 0.8). Thus the synthesized portfolio is able to explain a large proportion of the variation in the actual fund returns.

The very low t-values on selection returns obtained from both the weighted least square (WLS) and the ordinary least square (OLS) regressions indicate that the mean returns of synthesized style portfolio are not significantly different from the actual fund returns being replicated. However, on average, the replicated portfolios exhibited a tracking error of about 5% per annum in relation to the actual funds being mimicked. This indicates that the style portfolio cannot seamlessly track the active fund returns from month to month.
The time-weighted optimisation procedure (WLS) is compared to equally weighting the prior 36 months (OLS). The former is found to be pervasively superior to the latter in terms of performance. However, similar and very high out-of-sample $R^2$ values indicate that both methods are able to replicate the actual fund returns reasonably well. It is also found that, including the Small Cap Index generally produced slightly higher style returns and $R^2$ over the period investigated. However, such a strategy may be too expensive to implement due to the illiquidity of the Small Cap Index constituents. The other constraints adopted when regressing on unit trusts are: (1) style weights sum to unity, and (2) each of the style weight lies between 0 and 1.

### 7.2.2.2. Hedge funds

In the case of hedge funds, given the data currently available, using a return-based style regression methodology is not able to generate style returns that closely replicate returns of the actual funds. The overall tracking power measured by out-of-sample $R^2$ values is much lower than that of the unit trusts. This may be because hedge funds tend to adopt more active investment strategies, invest in more exotic and unusual securities and make more bets on the performance of individual stocks. Or, equally likely, this may be a result of the low number of out-of-sample data points currently available (only 12). Moreover, it is inconclusive whether Constrained Sum (CS) or Quadratic Programming (QP) method should be used for future investigation, and it is unclear whether the style portfolio significantly outperforms or underperforms the actual hedge fund.

### 7.2.3. Portfolio optimisation using style indices

Mean-variance and mean-tracking error optimal portfolios are constructed using three selected style indices, EW(size)100, RESW100(3) and MOM(12-1)W100, and the SWIX Index (or the Top 40 Index if shorting is allowed). The mean-variance efficient portfolio refers to the minimum total risk (measured by variance) portfolio at each level of mean portfolio returns, or equivalently, the maximum return portfolios at each level of total risk. The mean-variance optimisations are constructed under the long-only, long-short-equity and market neutral strategies. The mean-tracking error optimisation is constructed by seeking the maximum $R_p$ at each level of tracking errors constructed based on the entire history of total index returns over the period.
January 1998 to December 2006. Tracking error is calculated with respect to the SWIX Index. No shorting or leverage is permitted.

7.2.3.1. Mean-variance optimisation

Overall, the long-short-equity strategy with leverage capped at 200% has delivered higher returns for the same level of risk than the long-only and market neutral strategies. Before the 200% leverage cap comes into effect, the optimal position is approximately 65% in the value index, 10% in the momentum index and -20% in the market (proxied by the Top 40 Index) under the long-short-equity strategy. This results in a strategic net exposure of 55%. The market neutral optimal portfolio estimated is short 100% Top 40, long 69% value and long 31% momentum. The optimal portfolios for all strategies investigated have large exposure in the value index and no exposure in the size index.

The long-only mean-variance analysis shows that the value index would make a worthy long-only equity benchmark even in isolation. The long-short equity mean-variance optimal portfolio estimated is short 20% Top 40, long 65% value and long 10% momentum, resulting in a strategic net exposure of 55%. The market neutral optimal portfolio is obtained by shorting 100% Top 40, and longing 69% value and 31% momentum.

It is noted that the procedure of constructing market neutral portfolios can result in unnecessarily low expected returns for a certain level of risk, and therefore generate inefficient risk-return portfolios. This is clear if one compares the market neutral results to the long-short-equity results, where for the same level of risk the latter is able to deliver higher returns.

7.2.3.2. Mean-tracking error optimisation

In the case of the return-tracking error efficient portfolios, as an investor is willing to make more risky deviations from the benchmark portfolio (the SWIX Index) in pursuing higher returns, weights of the value index first increases at the expense of the SWIX Index and then decreases to give way to the momentum index. An illustrative enhanced index strategy with 3% tracking error would comprise 74% in the SWIX, 20% in the value index and 6% in the momentum index. A typical
benchmark cognisant active fund (5% tracking error) would be constructed as about 56% SWIX, 33% value and 11% momentum.

Despite the increase in tracking error, there is not a meaningful appreciation on the portfolio’s total risk up to the 11.5% tracking error level. As an illustration, by taking on extra risk of 10% per annum (measured in tracking errors), one is able to earn an excess return of 9.4% per annum relative to SWIX.

7.3 Suggested extensions

This thesis introduces several areas of further research on the topic of style analysis on the JSE. Five of the more prominent areas are: (1) alternative rebalancing frequency, residual weighting and calculation period of style indices, (2) return-based style analysis on hedge funds, (3) optimal WLS half-life period, (4) out-of-sample portfolio optimisation, and (5) practicality.

The majority of the style indices constructed in this thesis are rebalanced monthly. Although a brief examination on the use of quarter rebalancing frequency indicates that less frequent rebalance tends to destroy value despite the lower turnover costs, a more detailed investigation into the alternative rebalancing frequency of style indices may be valuable. For instance, bi-annual or annual rebalancing is adopted by many style-index studies (Arnott et al., 2005). Different RES-computation formula for constructing the value-style indices can be explored in future studies, such as using per-share attributes in the calculation (e.g. EPS instead of EAR). In this thesis, all of the style indices are constructed over 1st January 1997 to 31st December 2007. It will also be interesting to compare the results by constructing the same style indices over alternative periods; for instance, over 5 years of bull market and 5 years of bear market respectively.

Secondly, the prominent problem with the return-based style analysis on hedge funds is the very short period of historical returns available. Given the currently available data (three years’ return history and 12 out-of-sample data points), the estimated style portfolios are not able to generate style returns that closely replicate actual hedge fund returns. In general, the minimum return history required to perform a proper style-
decomposition analysis is eight to ten years and it is argued that no definite conclusions can be drawn basing on a shorter period of observed returns (Sharpe, 1992). Therefore, it is recommended that the analysis on hedge funds to be conducted again after at least eight years’ historical data are accumulated.

Thirdly, the half-life of the WLS regressions employed for the return-based style decomposition is fixed at 2 in accordance with that of Sharpe’s method (1992). However, the author realises that this is a random choice, and further investigations into the optimal or the best-fit distribution of the half-life period may yield more satisfactory style-replicating portfolios.

Fourthly, all of the optimal portfolios created in Chapter Seven are based entirely on ex-post returns and not tested for their prediction power. An out-of-sample optimisation method adopting the concept of style timing may be looked at as a future extension. It will also be interesting to explore the effect of other types of optimisation constraints.

Finally, many practitioners question the practicality and profitability of exploiting empirical style anomalies, especially when the results are obtained from ex-post analysis. It is argued that the reported abnormal returns maybe mitigated due to methodology failures, liquidity constraints, transaction costs and other potential market frictions [Stoll and Whaley (1983), Amihud and Mendelson (1986), Roll (1994), Roll and Ross (1980) and Roll and Ross (1994)]. The statement put forward by Bogle and Malkiel (2006) and Fama and French (2007) criticising Arnott’s fundamental indices also applies to any other style-index studies including this thesis. They commented that: ‘While it is clear that Bob Arnott, with hindsight, has discovered a theoretically profitable anomaly, what basis is there for assuming it will continue in the future, now that it has been publicized?... you have to believe in two assumptions: (a) that whoever has been making the valuation errors that gave rise to the superior historical returns will continue making them, and (b) other investors will not arbitrage away the potential excess returns by bidding up the price of larger companies’ stocks. While we have often stated that we don’t believe financial markets are perfectly efficient, we believe they are strongly attracted to efficiency. For this
reason, we do not believe that, in the future, Bob Arnott’s Fundamental Index will deliver the superior performance it has in the past.'
References


(http://ssrn.com/abstract=928167)


(http://www.efficientfrontier.com/ef/0adhoc/fi.htm)


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Appendix A

Appendices contained in Appendix A refer to Chapter Two, Literature Review.

Appendix A.1. Name and brief description of the Exchange Traded Funds (ETFs) on the Johannesburg Stock Exchange (JSE)

The table lists all of the ten Exchange Traded Funds (ETFs) that exist on the JSE as at 31 August 2007. For each ETF, the name, issue company/administrator, brief description of the index tracked and fund starting date are displayed. The table is sourced from the JSE website.

<table>
<thead>
<tr>
<th>ETF Name</th>
<th>Index Tracked</th>
<th>Features</th>
<th>ETF Issuer</th>
<th>Issue Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Satrix 40</td>
<td>FTSE/JSE Africa Top 40 (1200)</td>
<td>A portfolio of the top 40 companies on the JSE, representative of the JSE as a whole.</td>
<td>Index Co Ltd (The JSE and Gensec Bank)</td>
<td>November, 2000</td>
</tr>
<tr>
<td>Satrix FINI</td>
<td>FTSE/JSE Africa Financial 15 (1212)</td>
<td>A portfolio with exposure to major JSE banks and insurance companies.</td>
<td>Index Co Ltd</td>
<td>February, 2002</td>
</tr>
<tr>
<td>Satrix INDI</td>
<td>FTSE/JSE Industrial 25 (613)</td>
<td>A portfolio of industrial companies who are major beneficiaries of strong domestic economic growth.</td>
<td>Index Co Ltd</td>
<td>February, 2002</td>
</tr>
<tr>
<td>Satrix RESI</td>
<td>FTSE/JSE Resources 20 (210)</td>
<td>A portfolio with focused exposure to the resource companies, enables participation in the global commodity price boom and has strong rand hedge qualities.</td>
<td>Index Co Ltd</td>
<td>April, 2006</td>
</tr>
<tr>
<td>Satrix Swix Top 40</td>
<td>FTSE/JSE Shareholder Weighted Top 40 (400)</td>
<td>A less volatile basket of JSE top 40 shares, down-weighting the shares in the Top 40 Index held by non-South African shareholders, thereby reducing the weightings of resource and dual listed stocks and increasing the weightings of financial, industrial and telecommunications shares.</td>
<td>Index Co Ltd</td>
<td>April, 2006</td>
</tr>
<tr>
<td>Satrix Divi</td>
<td>FTSE/JSE Dividend Plus (159)</td>
<td>A portfolio comprises the top 30 companies that have the highest one-year forecast cash dividend yield among all the companies constitute the FTSE/JSE Top 40 and FTSE/JSE Mid-Cap indices.</td>
<td>Index Co Ltd</td>
<td>August, 2007</td>
</tr>
<tr>
<td>NewRand</td>
<td>FTSE 100</td>
<td>A portfolio containing the 100 largest highly liquid United Kingdom each constituent's free float market capitalisation. Provide international diversification into the UK market.</td>
<td>Itrix (Deutsche Bank)</td>
<td>October, 2005</td>
</tr>
<tr>
<td>NewGold</td>
<td>Dow Jones Euro STOXX 50</td>
<td>A portfolio containing 50 well-established and highly liquid blue chip stocks from countries within the Eurozone. Provide international diversification into the European markets.</td>
<td>Itrix</td>
<td>October, 2005</td>
</tr>
<tr>
<td>NewGold</td>
<td>ABSA-compiled Rand Index</td>
<td>A new equity portfolio comprising 10 rand hedge shares selected from ABSA Capital (200), which has the maximum long-term correlation with the Rand/USD exchange rate, provides protection against possible depreciation of the Rand. The index is created by ACMB and calculated by the FTSE and JSE.</td>
<td>ACMB</td>
<td>June, 2003</td>
</tr>
</tbody>
</table>
Appendix B

Appendices contained in Appendix B refer to Chapter Three, Data Description.

Appendix B.1. Number of companies in the thesis sample

The following table shows the number of companies in each month in the sample utilised in this thesis over the period 1st January 1997 to 1st February 2006. The company data are obtained from DataStream International at the University of Cape Town.

<table>
<thead>
<tr>
<th>Date</th>
<th>No. of Shares</th>
<th>Date</th>
<th>No. of Shares</th>
<th>Date</th>
<th>No. of Shares</th>
</tr>
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<tbody>
<tr>
<td>1997-1-1</td>
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<td>2003-11-1</td>
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<tr>
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</table>
Appendix B.2. Definition and brief description of the DataStream accounting entries

The table briefly describes the definition of all the firm-specific accounting items used in this thesis. These are based on DataStream International Online Definition available at the University of Cape Town.

**Assets per share (APSH)**
This is also referred to as the book value or net tangible assets per share. This is defined as net tangible assets (shareholder’s equity minus intangibles) divided by the year-end number of shares. It is calculated from company account items 305 (shareholder’s equity) and 344 (intangible assets).

**Book Value per Share (BVPS)**
This is calculated on an issue basis, using that portion of share capital and reserves (excluding preference capital) minus intangibles attributable to the issue, divided by the year-end number of shares in that issue. It is adjusted for subsequent rights and scrip issues.

**Dividends Paid (DIV)**
This refers to the ordinary and preference dividends paid during the period, often representing the previous year’s final and current year’s interim dividends.

**Dividends per Share (DPS)**
This refers to the dividend per share on a twelve-month rolling basis, taking interim dividends into account.

**Earnings per share, current rate (EPS)**
This is the latest annualised rate that may reflect the last financial year or be derived from an aggregation of interim period earnings. For certain countries, for which interim announcements are irregular or lacking in detail, the current earnings per share (EPS) may be a forecast provided by local sources. The countries for which EPS forecasts are used are: UK, Austria, Germany, Belgium, Italy, Spain, Finland, France and Switzerland.

**Earnings per share, current financial year; consensus forecast (EPS1)**
This is a mean of all the EPS forecasts supplied by analysts for the current financial year of the company, that is, the financial year not yet reported.

**Earnings per share, next financial year; consensus forecast (EPS2)**
This is a mean of all the EPS forecasts supplied by analysts for the next financial year of the company. The next financial year is defined as that following the current year.
Appendix B.2. Definition and brief description of the DataStream accounting entries (Continued)

Market Value to Book Value (MTBV)
The market value to book value (also called discount to net asset value) divides the market value by the net book value. The calculation is as follows:

\[ MTBV = \frac{MV}{NTA} \]

Where NTA = Net tangible assets
MV = Market value as at financial year end date

For companies which have more than one class of equity capital, both MV and NTA are expressed according to the individual issue.

Net cashflow (CF)
This refers to the changes in net cash before the impact of exchange adjustments and reflects cash inflow after financing.

Net Tangible Assets (NTA)
Net tangible assets (also referred to as net book value) is defined as total assets, excluding intangible assets less total liabilities, minority interest and preference stock. It can also be defined as ordinary shareholder’s equity less tangible assets. The value is calculated using Datastream’s company account items 305 and 344 in the following expression:

\[ NTA = 305 - 344 \]

Where 305 = Ordinary shareholder’s equity and intangible assets
344 = Total intangible assets

For companies which have issued more than one class of shares, net tangible assets is expressed according to the individual issue.

Price (P)
This refers to the latest price available to DataStream International from the appropriate market in primary units of currency. It is the previous day’s closing price from the default exchange. The ‘current’ prices taken at the close of market are stored each day. These stored prices are adjusted for subsequent capital actions, and this adjusted figure then becomes the default price. Prices are generally based on ‘last trade’ or an official price fixing. For stocks which are listed on more than one exchange within the country, default prices are taken from the primary exchange of that country (note that this is not necessarily the ‘home’ exchange of the stock).

Price/earnings ratio (PER, PE)
This is the price divided by the earnings rate per share at the required date.
Appendix B.2. Definition and brief description of the DataStream accounting entries (Continued)

**Price to Book Value (PTBV)**
This is the price dividend by the book value or net tangible assets per share for the appropriate financial year end, adjusted for capital changes. It is calculated as:

\[ PTBV = \frac{P}{APSH} \]

Where \( APSH \) = Assets per share

**Return Index (RI)**
This refers to the theoretical growth in value of a share holding over a specified period, assuming that dividends are re-invested to purchase additional units of an equity at the closing price applicable on the ex-dividend date.

**Total Assets (392)**
This refers to the sum of tangible fixed assets, intangible assets, investments (including associates), other assets, total stocks & work in progress, total debtors & equivalent and cash & cash equivalents.

Common adjustments:
- deferred tax, if shown as an asset, is offset against deferred tax liability
- goodwill carried in reserves is transferred to intangible assets
- advances on work in progress, if disclosed as a liability by the company, has been offset against
- stocks and work in progress.

**Total Cash and Equivalent (375)**
For industrials, this includes cash, bank balances, short-term loans and deposits, and investments shown under current assets. For banks and finance companies, it includes cash and balances with other banks, money at call and short notice, treasury bills and term deposits maturing under one month. Placements with banks are excluded.

**Total Current Assets (376)**
This includes stocks, work in progress, trade and other debtors, cash and equivalent, and any other current assets. Trade accounts receivable after one year are included.

**Total Current Liabilities (389)**
This includes current provisions, trade and other creditors, borrowings repayable within one year, and any other current liabilities. Trade accounts payable after one year are included. Where the balance sheet is stated before profit appropriation, the as reported figure for current liabilities is increased by the amount of proposed dividends outstanding at balance sheet date.
Appendix B.2. Definition and brief description of the DataStream accounting entries (Continued)

**Total Debt (1301)**
This refers to the total of all long and short term borrowings, including any subordinate debt and ‘debt-like’ hybrid finance instruments.

**Total Debtors and Equivalent (370)**
This refers to the total of balances outstanding due to the organisation in the normal course of trading. Accounts receivable after one year are included in this item.

**Total Intangibles (344)**
This includes research and development, goodwill, patents, trade marks, deferred charges, formation expenses and concessions. The figure may differ from that reported due to the fact that deferred charges may have been shown as part of ‘other assets’ and goodwill on acquisition may have been deducted from share capital and reserves.

**Total Loan Capital (321)**
This refers to the total loan capital repayable after one year. It includes debentures, bonds, convertibles, notes, leasing finance, and ‘debt-like’ hybrid financial instruments.

**Total Number of Employees (219)**
This refers to the average number of employees as disclosed by the company. The year end number is used if the average number is not disclosed.

**Total Sales (104)**
This refers to the amount of sales of goods and services to third parties relating to the normal industrial activities of the company. It is net of sales related taxes and excludes any royalty income, rental income, and other operating income.

**Total Stock and Work In Progress (364)**
This includes finished goods, raw materials, work in progress less any advances paid, and any other stocks. It is stated net of any provisions for obsolete stocks. The most common adjustment applied to the as disclosed figure is the inclusion of advances on work in progress if shown as a liability.

**Turnover by Volume (VO)**
This refers to the number of shares traded for a stock for a particular month. The figure is always expressed in thousands. For stocks which are traded on more than one exchange within the country, default volumes are taken from the primary exchange of that country (note that this is not necessarily the ‘home’ exchange of the stock).
Appendix B.2. Definition and brief description of the DataStream accounting entries (Continued)

**Working Capital Ratio (741)**
This refers to the total current assets divided by total current liabilities.
Appendix C

Appendices contained in Appendix C refer to Chapter Four, Investigating Candidate Style Indices.

Appendix C.1. Definition and brief description of the constructed style indices

The table lists all of the style indices constructed in Chapter Four, grouped into three sub sections: size indices, value indices and momentum indices. For the size indices, both monthly and quarterly rebalanced indices are included for illustration purpose. The short-hand index code, full index name and a brief description of the construction method of each index are displayed.
### Size-style indices (8)

<table>
<thead>
<tr>
<th>Index Code</th>
<th>Index Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>EW(size)100</td>
<td>Equally weighted size 100 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the top 100 shares ranked by MV, constituents updated at the beginning of each month.</td>
</tr>
<tr>
<td>EW(size)100Q</td>
<td>Equally weighted size 100 quarterly rebalanced</td>
<td>Equally weighted total return index, consists of the top 100 shares ranked by MV, constituents updated at the beginning of each quarter.</td>
</tr>
<tr>
<td>EW(size)50</td>
<td>Equally weighted size 50 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the top 50 shares ranked by MV, constituents updated at the beginning of each month.</td>
</tr>
<tr>
<td>EW(size)50Q</td>
<td>Equally weighted size 50 quarterly rebalanced</td>
<td>Equally weighted total return index, consists of the top 50 shares ranked by MV, constituents updated at the beginning of each quarter.</td>
</tr>
<tr>
<td>EW(size)30</td>
<td>Equally weighted size 30 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the top 30 shares ranked by MV, constituents updated at the beginning of each month.</td>
</tr>
<tr>
<td>EW(size)30Q</td>
<td>Equally weighted size 30 quarterly rebalanced</td>
<td>Equally weighted total return index, consists of the top 30 shares ranked by MV, constituents updated at the beginning of each quarter.</td>
</tr>
<tr>
<td>Top 40</td>
<td>FTSE/JSE Africa Top 40 Total Return Index (12/20T)</td>
<td>MV weighted total return indices, consists of the top 40 shares ranked by MV, constituents updated at the beginning of each quarter.</td>
</tr>
<tr>
<td>Small Cap</td>
<td>FTSE/JSE Africa Small Cap Total Return Index (12/20T)</td>
<td>MV weighted total return indices, consists of the top 100 shares ranked in the FTSE/JSE Africa All Share Index, constituents updated at the beginning of each quarter.</td>
</tr>
</tbody>
</table>

### Value-style indices (55)

<table>
<thead>
<tr>
<th>Index Code</th>
<th>Index Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>EYW100</td>
<td>Earnings yield weighted value 100 monthly rebalanced</td>
<td>Total return index, consists of the top 100 shares ranked by MV, returns of the constituents are weighted by their EY, constituents are updated at the beginning of each month. Shares with negative EY are excluded.</td>
</tr>
<tr>
<td>EYW50</td>
<td>Earnings yield weighted value 50 monthly rebalanced</td>
<td>Total return index, consists of the top 50 shares ranked by MV, returns of the constituents are weighted by their EY, constituents are updated at the beginning of each month. Shares with negative EY are excluded.</td>
</tr>
<tr>
<td>EYW30</td>
<td>Earnings yield weighted value 30 monthly rebalanced</td>
<td>Total return index, consists of the top 30 shares ranked by MV, returns of the constituents are weighted by their EY, constituents are updated at the beginning of each month. Shares with negative EY are excluded.</td>
</tr>
<tr>
<td>EW(EY)50</td>
<td>Equally weighted earnings yield 50 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the 50 shares with the highest EY out of the top 100 shares ranked by MV, constituents updated at the beginning of each month.</td>
</tr>
<tr>
<td>EW(EY)30</td>
<td>Equally weighted earnings yield 30 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the 30 shares with the highest EY out of the top 100 shares ranked by MV, constituents updated at the beginning of each month.</td>
</tr>
<tr>
<td>BTMVW100</td>
<td>Book to market value weighted value 100 Monthly rebalanced</td>
<td>Total return index, consists of the top 100 shares ranked by MV, returns of the constituents are weighted by their BTMV, constituents are updated at the beginning of each month. Shares with negative BTMV are excluded.</td>
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<tr>
<td>BTMVW50</td>
<td>Book to market value weighted value 50 monthly rebalanced</td>
<td>Total return index, consists of the top 50 shares ranked by MV, returns of the constituents are weighted by their BTMV, constituents are updated at the beginning of each month. Shares with negative BTMV are excluded.</td>
</tr>
<tr>
<td>BTMVW30</td>
<td>Book to market value weighted value 30 monthly rebalanced</td>
<td>Total return index, consists of the top 30 shares ranked by MV, returns of the constituents are weighted by their BTMV, constituents are updated at the beginning of each month. Shares with negative BTMV are excluded.</td>
</tr>
<tr>
<td>EW(BTMV)50</td>
<td>Equally weighted book to market value 50 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the 50 shares with the highest BTMV out of the top 100 shares ranked by MV, constituents updated at the beginning of each month.</td>
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<tr>
<td>EW(BTMV)30</td>
<td>Equally weighted book to market value 30 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the 30 shares with the highest BTMV out of the top 100 shares ranked by MV, constituents updated at the beginning of each month.</td>
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### Appendix C.1. Definition and brief description of the constructed style indices (Continued)

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<thead>
<tr>
<th>Index Code</th>
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<th>Description</th>
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<tr>
<td><strong>Value-style indices (55) - continued</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CFW100</td>
<td>Cashflow weighted value 100 monthly rebalanced</td>
<td>Total return index, consists of the top 100 shares ranked by MV, returns of the constituents are weighted by their total cashflows (CF), constituents are updated at the beginning of each month. Shares with negative CF are excluded.</td>
</tr>
<tr>
<td>CFW50</td>
<td>Cashflow weighted value 50 monthly rebalanced</td>
<td>Total return index, consists of the top 50 shares ranked by MV, returns of the constituents are weighted by their CF, constituents are updated at the beginning of each month. Shares with negative CF are excluded.</td>
</tr>
<tr>
<td>CFW30</td>
<td>Cashflow weighted value 30 monthly rebalanced</td>
<td>Total return index, consists of the top 30 shares ranked by MV, returns of the constituents are weighted by their CF, constituents are updated at the beginning of each month. Shares with negative CF are excluded.</td>
</tr>
<tr>
<td>EW(CF)30</td>
<td>Equally weighted cashflow 30 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the top 30 shares ranked by MV, constituents updated at the beginning of each month.</td>
</tr>
<tr>
<td>EW(CF)50</td>
<td>Equally weighted cashflow 50 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the top 50 shares ranked by MV, constituents updated at the beginning of each month.</td>
</tr>
<tr>
<td>DIVW100</td>
<td>Dividend weighted value 100 monthly rebalanced</td>
<td>Total return index, consists of the top 100 shares ranked by MV, returns of the constituents are weighted by their dividends (DIV), constituents are updated at the beginning of each month. Shares with negative DIV are excluded.</td>
</tr>
<tr>
<td>DIVW50</td>
<td>Dividend weighted value 50 monthly rebalanced</td>
<td>Total return index, consists of the top 50 shares ranked by MV, returns of the constituents are weighted by their DIV, constituents are updated at the beginning of each month. Shares with negative DIV are excluded.</td>
</tr>
<tr>
<td>DIVW30</td>
<td>Dividend weighted value 30 monthly rebalanced</td>
<td>Total return index, consists of the top 30 shares ranked by MV, returns of the constituents are weighted by their DIV, constituents are updated at the beginning of each month. Shares with negative DIV are excluded.</td>
</tr>
<tr>
<td>EW(DIV)30</td>
<td>Equally weighted dividend 30 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the top 30 shares ranked by MV, constituents updated at the beginning of each month.</td>
</tr>
<tr>
<td>EW(DIV)50</td>
<td>Equally weighted dividend 50 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the top 50 shares ranked by MV, constituents updated at the beginning of each month.</td>
</tr>
<tr>
<td>SALEW100</td>
<td>Sale weighted value 100 monthly rebalanced</td>
<td>Total return index, consists of the top 100 shares ranked by MV, returns of the constituents are weighted by their total sales (SALE), constituents are updated at the beginning of each month. Shares with negative SALE are excluded.</td>
</tr>
<tr>
<td>SALEW50</td>
<td>Sale weighted value 50 monthly rebalanced</td>
<td>Total return index, consists of the top 50 shares ranked by MV, returns of the constituents are weighted by their SALE, constituents are updated at the beginning of each month. Shares with negative SALE are excluded.</td>
</tr>
<tr>
<td>SALEW30</td>
<td>Sale weighted value 30 monthly rebalanced</td>
<td>Total return index, consists of the top 30 shares ranked by MV, returns of the constituents are weighted by their SALE, constituents are updated at the beginning of each month. Shares with negative SALE are excluded.</td>
</tr>
<tr>
<td>EW(SALE)30</td>
<td>Equally weighted sale 30 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the top 30 shares ranked by MV, constituents updated at the beginning of each month.</td>
</tr>
<tr>
<td>EW(SALE)50</td>
<td>Equally weighted sale 50 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the top 50 shares ranked by MV, constituents updated at the beginning of each month.</td>
</tr>
<tr>
<td>EPSW100</td>
<td>Earnings per share weighted value 100 monthly rebalanced</td>
<td>Total return index, consists of the top 100 shares ranked by MV, returns of the constituents are weighted by their earnings per share (EPS), constituents are updated at the beginning of each month. Shares with negative EPS are excluded.</td>
</tr>
<tr>
<td>EPSW50</td>
<td>Earnings per share weighted value 50 monthly rebalanced</td>
<td>Total return index, consists of the top 50 shares ranked by MV, returns of the constituents are weighted by their EPS, constituents are updated at the beginning of each month. Shares with negative EPS are excluded.</td>
</tr>
<tr>
<td>EPSW30</td>
<td>Earnings per share weighted value 30 monthly rebalanced</td>
<td>Total return index, consists of the top 30 shares ranked by MV, returns of the constituents are weighted by their EPS, constituents are updated at the beginning of each month. Shares with negative EPS are excluded.</td>
</tr>
<tr>
<td>EW(EPS)50</td>
<td>Equally weighted EPS 50 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the top 50 shares ranked by MV, constituents updated at the beginning of each month.</td>
</tr>
</tbody>
</table>
# Appendix C.1. Definition and brief description of the constructed style indices (Continued)

<table>
<thead>
<tr>
<th>Index Code</th>
<th>Index Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>EW(EPS)30</td>
<td>Equally weighted EPS 30 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the 30 shares with the highest EPS out of the top 100 shares ranked by MV, constituents updated at the beginning of each month.</td>
</tr>
<tr>
<td>EARW100</td>
<td>Earnings weighted value 100 monthly rebalanced</td>
<td>Total return index, consists of the top 100 shares ranked by MV, returns of the constituents are weighted by their total earnings (EAR), constituents are updated at the beginning of each month. Shares with negative EY are excluded.</td>
</tr>
<tr>
<td>EARW50</td>
<td>Earnings weighted value 50 monthly rebalanced</td>
<td>Total return index, consists of the top 50 shares ranked by MV, returns of the constituents are weighted by their total earnings (EAR), constituents are updated at the beginning of each month. Shares with negative EY are excluded.</td>
</tr>
<tr>
<td>EARW100</td>
<td>Earnings weighted value 100 monthly rebalanced</td>
<td>Total return index, consists of the top 100 shares ranked by MV, returns of the constituents are weighted by their total earnings (EAR), constituents are updated at the beginning of each month. Shares with negative EY are excluded.</td>
</tr>
<tr>
<td>EW(EAR)50</td>
<td>Equally weighted earnings 50 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the 50 shares with the highest EAR out of the top 100 shares ranked by MV, constituents updated at the beginning of each month.</td>
</tr>
<tr>
<td>EW(EAR)30</td>
<td>Equally weighted earnings 30 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the 30 shares with the highest EAR out of the top 100 shares ranked by MV, constituents updated at the beginning of each month.</td>
</tr>
<tr>
<td>BVW100</td>
<td>Book value weighted value 100 monthly rebalanced</td>
<td>Total return index, consists of the top 100 shares ranked by MV, returns of the constituents are weighted by their total book value (BV), constituents are updated at the beginning of each month. Shares with negative EY are excluded.</td>
</tr>
<tr>
<td>BVW50</td>
<td>Book value weighted value 50 monthly rebalanced</td>
<td>Total return index, consists of the top 50 shares ranked by MV, returns of the constituents are weighted by their total book value (BV), constituents are updated at the beginning of each month. Shares with negative EY are excluded.</td>
</tr>
<tr>
<td>BVW100</td>
<td>Book value weighted value 100 monthly rebalanced</td>
<td>Total return index, consists of the top 100 shares ranked by MV, returns of the constituents are weighted by their total book value (BV), constituents are updated at the beginning of each month. Shares with negative EY are excluded.</td>
</tr>
<tr>
<td>BVW50</td>
<td>Book value weighted value 50 monthly rebalanced</td>
<td>Total return index, consists of the top 50 shares ranked by MV, returns of the constituents are weighted by their total book value (BV), constituents are updated at the beginning of each month. Shares with negative EY are excluded.</td>
</tr>
<tr>
<td>BVW30</td>
<td>Book value weighted value 30 monthly rebalanced</td>
<td>Total return index, consists of the top 30 shares ranked by MV, returns of the constituents are weighted by their total book value (BV), constituents are updated at the beginning of each month. Shares with negative EY are excluded.</td>
</tr>
<tr>
<td>EW(BV)50</td>
<td>Equally weighted book value 50 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the 50 shares with the highest BV out of the top 100 shares ranked by BV, constituents updated at the beginning of each month.</td>
</tr>
<tr>
<td>EW(BV)30</td>
<td>Equally weighted book value 30 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the 30 shares with the highest BV out of the top 100 shares ranked by BV, constituents updated at the beginning of each month.</td>
</tr>
<tr>
<td>RESW100(4)</td>
<td>4-factor-regression residual weighted value 100 monthly rebalanced</td>
<td>Total return index, consists of the top 100 shares ranked by MV, returns of the constituents are weighted by their 4-factor-regression residuals (RES(4)), constituents are updated at the beginning of each month. Shares with negative RES(4) are excluded.</td>
</tr>
<tr>
<td>RESW50(4)</td>
<td>4-factor-regression residual weighted value 50 monthly rebalanced</td>
<td>Total return index, consists of the top 50 shares ranked by MV, returns of the constituents are weighted by their 4-factor-regression residuals (RES(4)), constituents are updated at the beginning of each month. Shares with negative RES(4) are excluded.</td>
</tr>
<tr>
<td>RESW100(4)</td>
<td>4-factor-regression residual weighted value 100 monthly rebalanced</td>
<td>Total return index, consists of the top 100 shares ranked by MV, returns of the constituents are weighted by their 4-factor-regression residuals (RES(4)), constituents are updated at the beginning of each month. Shares with negative RES(4) are excluded.</td>
</tr>
<tr>
<td>RESW50(4)</td>
<td>4-factor-regression residual weighted value 50 monthly rebalanced</td>
<td>Total return index, consists of the top 50 shares ranked by MV, returns of the constituents are weighted by their 4-factor-regression residuals (RES(4)), constituents are updated at the beginning of each month. Shares with negative RES(4) are excluded.</td>
</tr>
<tr>
<td>RESW100(3)</td>
<td>3-factor-regression residual weighted value 100 monthly rebalanced</td>
<td>Total return index, consists of the top 100 shares ranked by MV, returns of the constituents are weighted by their 3-factor-regression residuals (RES(3)), constituents are updated at the beginning of each month. Shares with negative RES(3) are excluded.</td>
</tr>
<tr>
<td>RESW50(3)</td>
<td>3-factor-regression residual weighted value 50 monthly rebalanced</td>
<td>Total return index, consists of the top 50 shares ranked by MV, returns of the constituents are weighted by their 3-factor-regression residuals (RES(3)), constituents are updated at the beginning of each month. Shares with negative RES(3) are excluded.</td>
</tr>
<tr>
<td>RESW100(3)</td>
<td>3-factor-regression residual weighted value 100 monthly rebalanced</td>
<td>Total return index, consists of the top 100 shares ranked by MV, returns of the constituents are weighted by their 3-factor-regression residuals (RES(3)), constituents are updated at the beginning of each month.</td>
</tr>
<tr>
<td>RESW50(3)</td>
<td>3-factor-regression residual weighted value 50 monthly rebalanced</td>
<td>Total return index, consists of the top 50 shares ranked by MV, returns of the constituents are weighted by their 3-factor-regression residuals (RES(3)), constituents are updated at the beginning of each month.</td>
</tr>
</tbody>
</table>

constituents are weighted by their 3-factor-regression residuals (RES(3)), constituents are updated at the beginning of each month. Shares with negative RES(3) are excluded. |
| RESW100(3) | 3-factor-regression residual weighted value 100 monthly rebalanced | Total return index, consists of the top 100 shares ranked by MV, returns of the constituents are weighted by their 3-factor-regression residuals (RES(3)), constituents are updated at the beginning of each month. Shares with negative RES(3) are excluded. |
| RESW50(3)  | 3-factor-regression residual weighted value 50 monthly rebalanced | Total return index, consists of the top 50 shares ranked by MV, returns of the constituents are weighted by their 3-factor-regression residuals (RES(3)), constituents are updated at the beginning of each month. Shares with negative RES(3) are excluded. |
| RESW100(3) | 3-factor-regression residual weighted value 100 monthly rebalanced | Total return index, consists of the top 100 shares ranked by MV, returns of the constituents are weighted by their 3-factor-regression residuals (RES(3)), constituents are updated at the beginning of each month. Shares with negative RES(3) are excluded. |
| RESW50(3)  | 3-factor-regression residual weighted value 50 monthly rebalanced | Total return index, consists of the top 50 shares ranked by MV, returns of the constituents are weighted by their 3-factor-regression residuals (RES(3)), constituents are updated at the beginning of each month. Shares with negative RES(3) are excluded. |
### Appendix C.1. Definition and brief description of the constructed style indices (Continued)

<table>
<thead>
<tr>
<th>Index Code</th>
<th>Index Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Value-style indices (55) - continued</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EW(RES)30(3)</td>
<td>EW weighted 3-factor-regression residual 30 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the 30 shares with the highest RES(3) out of the top 10 shares ranked by RES(3), constituents updated at the beginning of each month.</td>
</tr>
<tr>
<td>RESW100(2)</td>
<td>2-factor-regression residual weighted value 100 monthly rebalanced</td>
<td>Total return index, consists of the 100 shares ranked by MV, returns of the constituents are weighted by their 2-factor-regression residuals (RES(2)), constituents are updated at the beginning of each month. Shares with negative RES(2) are excluded.</td>
</tr>
<tr>
<td>RESW50(2)</td>
<td>2-factor-regression residual weighted value 50 monthly rebalanced</td>
<td>Total return index, consists of the 50 shares ranked by MV, returns of the constituents are weighted by their RES(2), constituents are updated at the beginning of each month. Shares with negative RES(2) are excluded.</td>
</tr>
<tr>
<td>RESW30(2)</td>
<td>2-factor-regression residual weighted value 30 monthly rebalanced</td>
<td>Total return index, consists of the 30 shares ranked by MV, returns of the constituents are weighted by their RES(2), constituents are updated at the beginning of each month.</td>
</tr>
<tr>
<td>EW(RES)50(2)</td>
<td>EW weighted 2-factor-regression residual 50 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the 50 shares with the highest RES(2) out of the top 10 shares ranked by RES(2), constituents updated at the beginning of each month.</td>
</tr>
<tr>
<td>EW(RES)30(2)</td>
<td>EW weighted 2-factor-regression residual 30 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the 30 shares with the highest RES(2) out of the top 10 shares ranked by RES(2), constituents updated at the beginning of each month.</td>
</tr>
<tr>
<td><strong>Momentum-style indices (12)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MOMW100</td>
<td>MOM12 weighted momentum 100 monthly rebalanced</td>
<td>Total return index, consists of the top 100 shares ranked by MV, returns of the constituents are weighted by their prior 12-month returns (MOM12), constituents are updated at the beginning of each month. Shares with negative MOM12 are excluded.</td>
</tr>
<tr>
<td>MOMW50</td>
<td>MOM12 weighted momentum 50 monthly rebalanced</td>
<td>Total return index, consists of the top 50 shares ranked by MV, returns of the constituents are weighted by their MOM12, constituents are updated at the beginning of each month. Shares with negative MOM12 are excluded.</td>
</tr>
<tr>
<td>MOMW30</td>
<td>MOM12 weighted momentum 30 monthly rebalanced</td>
<td>Total return index, consists of the top 30 shares ranked by MV, returns of the constituents are weighted by their MOM12, constituents are updated at the beginning of each month. Shares with negative MOM12 are excluded.</td>
</tr>
<tr>
<td>EW(MOM)50</td>
<td>Equally weighted MOM12 50 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the 50 shares with the highest MOM12 out of the top 100 shares ranked by MV, constituents updated at the beginning of each month.</td>
</tr>
<tr>
<td>EW(MOM)30</td>
<td>Equally weighted MOM12 30 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the 30 shares with the highest MOM12 out of the top 100 shares ranked by MV, constituents updated at the beginning of each month.</td>
</tr>
<tr>
<td>EW(MOM)negative</td>
<td>Equally weighted MOM12 negative monthly rebalanced</td>
<td>Equally weighted total return index, consists of all the shares with negative MOM12 figures out of the top 100 shares ranked by MV, constituents updated at the beginning of each month.</td>
</tr>
<tr>
<td>MOM(12-1)W100</td>
<td>MOM[12-1] weighted/momentum 100 monthly rebalanced</td>
<td>Total return index, consists of the 100 shares ranked by MV, returns of the constituents are weighted by their prior 12-month return excluding the latest month (MOM[12-1]), constituents are updated at the beginning of each month. Shares with negative MOM[12-1] are excluded.</td>
</tr>
<tr>
<td>MOM(12-1)W50</td>
<td>MOM[12-1] weighted momentum 50 monthly rebalanced</td>
<td>Total return index, consists of the top 50 shares ranked by MV, returns of the constituents are weighted by their MOM[12-1], constituents are updated at the beginning of each month. Shares with negative MOM[12-1] are excluded.</td>
</tr>
<tr>
<td>MOM(12-1)W30</td>
<td>MOM[12-1] weighted momentum 30 monthly rebalanced</td>
<td>Total return index, consists of the top 30 shares ranked by MV, returns of the constituents are weighted by their MOM[12-1], constituents are updated at the beginning of each month. Shares with negative MOM[12-1] are excluded.</td>
</tr>
<tr>
<td>EW(MOM)(12-1)50</td>
<td>Equally weighted MOM[12-1] 50 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the 50 shares with the highest MOM[12-1] out of the top 100 shares ranked by MV, constituents updated at the beginning of each month.</td>
</tr>
<tr>
<td>EW(MOM)(12-1)30</td>
<td>Equally weighted MOM[12-1] 30 monthly rebalanced</td>
<td>Equally weighted total return index, consists of the 30 shares with the highest MOM[12-1] out of the top 200 shares ranked by MV, constituents updated at the beginning of each month.</td>
</tr>
</tbody>
</table>
Appendix C. Full regression results of the size-style indices

The table displays the results from the time series CAPM and APT regressions on the eight size-style indices constructed in Chapter Four over the period 1st January 1998 to 31st December 2006. The South African 90-day Banker’s Acceptance discount rate (RBAS) is used as the risk-free rate. P-values are calculated using two-tailed tests. The data are obtained from DataStream International at the University of Cape Town. In Section A, the single-index CAPM model utilises the ALSI as the market proxy. The results are obtained by regressing the excess monthly returns of the market index on the excess monthly returns of each of the eight size-style indices. In Section B, the two-factor APT model utilises FINDI and RESI as the APT-factor proxies. The results are obtained by regressing the excess monthly returns of the APT factors on the excess monthly returns of each of the 8 size-style indices.

<table>
<thead>
<tr>
<th>Style indices</th>
<th>EW(size100)</th>
<th>EW(size100Q)</th>
<th>EW(size50)</th>
<th>EW50(size100Q)</th>
<th>EW100Q</th>
<th>EW250Q</th>
<th>EW500Q</th>
<th>Top 40</th>
<th>Small Cap</th>
</tr>
</thead>
</table>

**Section A: Summary Statistics**

<table>
<thead>
<tr>
<th></th>
<th>EW(size100)</th>
<th>EW(size100Q)</th>
<th>EW(size50)</th>
<th>EW50(size100Q)</th>
<th>EW100Q</th>
<th>EW250Q</th>
<th>EW500Q</th>
<th>Top 40</th>
<th>Small Cap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic mean (%)</td>
<td>1.95%</td>
<td>1.84%</td>
<td>1.88%</td>
<td>1.79%</td>
<td>1.51%</td>
<td>1.65%</td>
<td>1.90%</td>
<td>1.85%</td>
<td></td>
</tr>
<tr>
<td>Geometric mean (%)</td>
<td>1.73%</td>
<td>1.61%</td>
<td>1.61%</td>
<td>1.53%</td>
<td>1.64%</td>
<td>1.66%</td>
<td>1.66%</td>
<td>1.67%</td>
<td></td>
</tr>
<tr>
<td>Mean monthly rebalancing (%)</td>
<td>1.13%</td>
<td>6.51%</td>
<td>12.99%</td>
<td>7.74%</td>
<td>12.64%</td>
<td>7.46%</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Cost-adjusted geometric mean (10 bpt) (%)</td>
<td>1.71%</td>
<td>1.60%</td>
<td>1.60%</td>
<td>1.52%</td>
<td>1.69%</td>
<td>1.64%</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Cost-adjusted geometric mean (70 bpt) (%)</td>
<td>1.70%</td>
<td>1.69%</td>
<td>1.59%</td>
<td>1.51%</td>
<td>1.63%</td>
<td>1.69%</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Standard deviation (%)</td>
<td>6.56%</td>
<td>6.62%</td>
<td>6.90%</td>
<td>7.07%</td>
<td>6.95%</td>
<td>6.93%</td>
<td>6.81%</td>
<td>5.91%</td>
<td></td>
</tr>
<tr>
<td>Return/standard deviation ratio</td>
<td>0.26</td>
<td>0.24</td>
<td>0.23</td>
<td>0.22</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.13</td>
<td>0.11</td>
<td>0.11</td>
<td>0.09</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>Treynor ratio</td>
<td>0.02</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>No. of constituents</td>
<td>100</td>
<td>100</td>
<td>50</td>
<td>50</td>
<td>30</td>
<td>30</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

**Section B: Single-index CAPM results**

| Alpha CAPM | 0.18% | 0.06% | 0.02% | -0.07% | 0.03% | 0.02% | -0.03% | 0.02% |
| Beta CAPM | 0.55 | 0.84 | 0.95 | - | 0.90 | 0.95 | - | 0.45 |
| Beta FINDI | 0.89 | 0.90 | 0.96 | 0.99 | 1.00 | 1.00 | 1.04 | 0.67 |
| Beta RESI | 15.50 | 19.34 | 22.27 | 22.63 | 26.96 | 27.37 | 102.63 | 11.32 |
| p-alpha CAPM | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Adjusted R square CAPM | 0.78 | 0.78 | 0.62 | 0.83 | 0.87 | 0.88 | 0.99 | 0.55 |

**Section C: Two-factor APT model results**

| Alpha APT | 0.07% | 0.01% | 0.01% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% |
| Beta FINDI | 1.63 | 0.99 | 0.87 | 0.38 | 0.51 | 0.35 | -2.46 | 1.50 |
| Beta RESI | 0.11 | 0.32 | 0.39 | 0.71 | 0.61 | 0.73 | - | 0.14 |
| p-beta FINDI | 0.90 | 0.89 | 0.93 | 0.94 | 0.85 | 0.82 | 0.60 | 0.77 |
| p-beta RESI | 23.93 | 22.55 | 26.26 | 25.65 | 22.49 | 21.09 | 49.09 | 13.54 |
| p-beta FINDI | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| p-beta RESI | 0.08 | 0.08 | 0.11 | 0.12 | 0.20 | 0.22 | 0.43 | 0.01 |
| Adjusted R square APT | 2.82 | 2.93 | 4.39 | 4.55 | 7.31 | 7.72 | 48.64 | 0.19 |

| Adjusted R square APT | 0.89 | 0.88 | 0.91 | 0.91 | 0.90 | 0.90 | 0.99 | 0.70 |
Appendix C.3. Relative returns of the size-style indices

The graph displays the relative cumulative returns, calculated as index cumulative returns divided by the ALSI cumulative returns, for the eight size-style indices, over the period 1st January 1998 to 31st December 2006. Relative returns provide insight as to the performance of the style indices relative to that of the South African equity market as a whole, proxied by the ALSI. The horizontal line with a y-intercept of 1 represents the level of the ALSI cumulative returns. The data are obtained from DataStream International at the University of Cape Town.
Appendix C.4. Quarterly rebalanced portfolio results of the value-style indices using earnings yield (EY) as value proxy

The table displays the summary statistics and full regression results of the five quarterly rebalanced value-style indices constructed over the period 1st January 1998 to 31st December 2006. In total 108 time-series returns are calculated for each index. Returns are monthly effective. All portfolios are rebalanced quarterly. The first three indices are constructed based on earnings yield weighted (EYW) portfolios containing the 100, 50 and 30 shares with the highest market capitalisation (MV). Shares with negative EY in a month are excluded from the sample for that month. The last two indices are constructed based on equally weighted (EW) portfolios using the top 50 and 30 shares ranked by EY out of the 100 shares with the highest MV in each month. Shares with MV, forward return or EY entries missing in a month are excluded from the sample used for that month. The South African 90-day Banker’s Acceptance discount rate (RBAS) is used as the risk-free rate. P-values are calculated using two-tailed tests. The data are obtained from DataStream International at the University of Cape Town. In Section A, the summary statistics are displayed. In Section B, the single-index CAPM model utilises the ALSI as the market proxy. The results are obtained by regressing the excess monthly returns of the market index on the excess monthly returns of each of the five value-style indices. In Section C, the two factor APT model utilises FINDI and RESI as the APT-factor proxies. The results are obtained by regressing the excess monthly returns of the APT factors on the excess monthly returns of each of the five value-style indices.

<table>
<thead>
<tr>
<th>Style indices</th>
<th>EYW100Q</th>
<th>EYW50Q</th>
<th>EYW30Q</th>
<th>EW(EY)50Q</th>
<th>EW(EY)30Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section A: Summary Statistics</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arithmetic mean (%)</td>
<td>1.96%</td>
<td>1.99%</td>
<td>2.17%</td>
<td>2.07%</td>
<td>2.29%</td>
</tr>
<tr>
<td>Geometric mean (%)</td>
<td>1.73%</td>
<td>1.74%</td>
<td>1.93%</td>
<td>1.84%</td>
<td>2.06%</td>
</tr>
<tr>
<td>Mean monthly rebalancing (%)</td>
<td>11.70%</td>
<td>11.58%</td>
<td>11.15%</td>
<td>15.11%</td>
<td>19.87%</td>
</tr>
<tr>
<td>Cost-adjusted geometric mean (10 bpt) (%)</td>
<td>1.72%</td>
<td>1.72%</td>
<td>1.92%</td>
<td>1.83%</td>
<td>2.04%</td>
</tr>
<tr>
<td>Cost-adjusted geometric mean (20 bpt) (%)</td>
<td>1.71%</td>
<td>1.71%</td>
<td>1.91%</td>
<td>1.81%</td>
<td>2.02%</td>
</tr>
<tr>
<td>Standard deviation (%)</td>
<td>6.68%</td>
<td>7.09%</td>
<td>6.80%</td>
<td>6.70%</td>
<td>6.74%</td>
</tr>
<tr>
<td>Return/standard deviation ratio</td>
<td>0.26</td>
<td>0.24</td>
<td>0.28</td>
<td>0.28</td>
<td>0.31</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.13</td>
<td>0.12</td>
<td>0.16</td>
<td>0.14</td>
<td>0.18</td>
</tr>
<tr>
<td>Treynor ratio</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>No. of constituents</td>
<td>95</td>
<td>48</td>
<td>29</td>
<td>50</td>
<td>30</td>
</tr>
<tr>
<td>Section B: Single-index CAPM model results</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alpha CAPM</td>
<td>0.20%</td>
<td>0.14%</td>
<td>0.32%</td>
<td>0.33%</td>
<td>0.56%</td>
</tr>
<tr>
<td>t-alpha CAPM</td>
<td>0.58</td>
<td>0.44</td>
<td>1.25</td>
<td>0.91</td>
<td>1.47</td>
</tr>
<tr>
<td>p-alpha CAPM</td>
<td>0.56</td>
<td>0.66</td>
<td>0.21</td>
<td>0.37</td>
<td>0.14</td>
</tr>
<tr>
<td>Beta CAPM</td>
<td>0.88</td>
<td>0.97</td>
<td>0.96</td>
<td>0.86</td>
<td>0.85</td>
</tr>
<tr>
<td>t-beta CAPM</td>
<td>17.26</td>
<td>20.44</td>
<td>24.73</td>
<td>15.47</td>
<td>14.80</td>
</tr>
<tr>
<td>p-beta CAPM</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Adjusted R Square CAPM</td>
<td>0.74</td>
<td>0.80</td>
<td>0.85</td>
<td>0.69</td>
<td>0.67</td>
</tr>
<tr>
<td>Section C: Two-factor APT model results</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alpha APT</td>
<td>0.33%</td>
<td>0.25%</td>
<td>0.37%</td>
<td>0.47%</td>
<td>0.61%</td>
</tr>
<tr>
<td>t-alpha APT</td>
<td>1.21</td>
<td>1.00</td>
<td>1.58</td>
<td>1.55</td>
<td>1.78</td>
</tr>
<tr>
<td>p-alpha APT</td>
<td>0.23</td>
<td>0.32</td>
<td>0.12</td>
<td>0.12</td>
<td>0.08</td>
</tr>
<tr>
<td>Beta FINDI</td>
<td>0.86</td>
<td>0.90</td>
<td>0.79</td>
<td>0.85</td>
<td>0.75</td>
</tr>
<tr>
<td>t-beta FINDI</td>
<td>18.07</td>
<td>20.40</td>
<td>19.62</td>
<td>15.98</td>
<td>12.53</td>
</tr>
<tr>
<td>p-beta FINDI</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Beta RESI</td>
<td>0.11</td>
<td>0.15</td>
<td>0.22</td>
<td>0.09</td>
<td>0.17</td>
</tr>
<tr>
<td>t-beta RESI</td>
<td>3.05</td>
<td>4.74</td>
<td>7.58</td>
<td>2.41</td>
<td>3.87</td>
</tr>
<tr>
<td>p-beta RESI</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>Adjusted R square APT</td>
<td>0.83</td>
<td>0.87</td>
<td>0.88</td>
<td>0.79</td>
<td>0.74</td>
</tr>
</tbody>
</table>
Appendix C.5. Relative returns of the value-style indices

The graph displays the relative cumulative returns, calculated as index cumulative returns divided by the ALSI cumulative returns, for the 11 value-style indices that use different value-factor proxies and consist of the top 100 shares by MV over the period 1st January 1998 to 31st December 2006. Relative returns provide insight as to the performance of the style indices relative to that of the South African equity market as a whole, proxied by the ALSI. The horizontal line with a y-intercept of 1 represents the level of the ALSI cumulative returns. The data are obtained from DataStream International at the University of Cape Town.
Appendix C.6. Full regression results of the momentum-style indices
The table presents the descriptive and regression statistics of the 17 momentum style indices constructed over the period I 51 January 1998 to 31 51 December 2006. Returns are
monthly effective. In total I 08 time-series returns are calculated for each index. The South African 90-day Banker's Acceptance discount rate (RBAS) is used as the risk-free
rate. P-values are calculated using two-tailed tests. The data are obtained from DataStream International at the University of Cape Town. Section A displays the summary
statistics of all the indices. Section B shows the descriptive and summary statistics for each of the 17 momentum style indices. Section C shows the single-index CAPM
regression statistics, using the ALSI as the market proxy. The results are obtained by regressing the excess monthly returns of the market index on the excess monthly returns of
each of the II momentum-style indices. Section C shows the two factor APT regression statistics, using FIND! and RES! as the APT-factor proxies. The results are obtained by
regressing the excess monthly returns of the APT factors on the excess monthly returns of each of the 17 momentum-style indices.

Style Indices

MOMW1 MOMW1 MOMWS MOMWS MOMW3 MOMW3 EW(MOM) EW(MOM) EW(MOM) EW(MOM} EW(MOM) EW(MOM)
00
OOQ
0
OQ
0
OQ
50
SOQ
30
30Q _
Neuti=---!l!~iveQ

Section A: Summary Satistics
Arithmetic mean
Geometric mean
Mean monthly rebalancing
Cost-adjusted geometric mean (10 bpt)
Cost-adjusted geometric mean (20 bpt)
Standard deviation
Return/standard deviation ratio
Sharpe ratio
Treynor ratio
No. of constituents
Section B: Single-index CAPM model results
Alpha CAPM
t-alpha CAPM
p-alpha CAPM
Beta CAPM
t-beta CAPM
p-beta CAPM
Adjusted R square CAPM
Section C: Two-factor APT model results
Alpha APT
t-alpha APT
p-alpha APT
Beta FINDI
t-beta FINDI
p-beta FINDI
Beta RESI
t-beta RESI
p-beta RESI
Adjusted R square APT

MOM(121)W100

MOM(121)W50

MOM(12- EW(MOM( EW(MOM(
1)W30
12-1ll50
12-1ll30

2.90%
2.47%
32.95%
2.43%
2.40%
0.089
0.276
0.178
0.015
69

2.29%
1.91%
19.61%
1.89%
1.87%
0.083
0.230
0.124
0.010
68

2.92%
2.51%
35 .76%
2.48%
2.44%
0.086
0.292
0.190
0.016
36

2.76%
2.33%
20.51%
2.30%
2.28%
0.088
0.264
0.165
0.013
35

2.96%
2.50%
37.56%
2.47%
2.43%
0.092
0.273
0.177
0.015
21

2.83%
2.36%
21.22%
2.34%
2.32%
0.094
0.252
0.158
0.013
21

2.41%
2.17%
32 .98%
2.14%
2.11%
0.066
0.331
0.197
0.015
50

2.36%
2.11%
17.51%
2.10%
2.08%
0.067
0.314
0.184
0.014
50

2.54%
2.25%
44.59%
2.21%
2. 17%
0.071
0.316
0.193
O.D15
30

2.47%
2.19%
23.57%
2.16%
2.14%
0.071
0.307
0.184
0.014
30

1.46%
1.15%
58.51%
1.09%
1.03%
0.080
0.144
0.034
0.003
29

1.30%
0.96%
31.59%
0.93%
0.90%
0.083
0.116
0.010
0.001
30

2.95%
2.51%
40.06%
2.47%
2.43%
0.090
0.279
0.181
0.016
68

3.07%
2.65%
42.34%
2.61%
2.57%
0.087
0.305
0.204
0.017
35

3.12%
2.66%
43 .90%
2.62%
2.57%
0.093
0.286
0.192
0.016
21

2.49%
2.24%
36.03%
2.21%
2.17%
0.068
0.332
0.202
0.015
50

2.80%
2.51%
47.75%
2.46%
2.41%
0.073
0.345
0.225
0.017
30

0.99%
1.72
0.09
1.03
11.87
0.00
0.57

0.43%
0.83
0.41
0.99
12.65
0.00
0.60

0.98%
1.95
0.05
1.05
13.75
0.00
0.64

0.77%
1.56
0.12
1.10
14.71
0.00
0.67

0.95%
1.76
0.08
1.12
13.64
0.00
0.64

0.78%
1.47
0.14
1.16
14.43
0.00
0.66

0.66%
2.00
0.05
0.87
17.26
0.00
0.74

0.59%
1.75
0.08
0.88
17.18
0.00
0.74

0.75%
1.93
0.06
0.92
15.65
0.00
0.70

0.67%
1.77
0.08
0.92
16.07
0.00
0.71

-0.35%
-0.69

-0.54%
-1.00

0.93
12.00
0.00
0.58

0.96
11.72
0.00
0.56

1.02%
1.79
0.08
1.05
12.15
0.00
0.58

1.12%
2.19
0.03
1.06
13.73
0.00
0.64

1.11%
2.02
0.05
1.13
13 .59
0.00
0.64

0.72%
2.11
0.04
0.89
17.06
0.00
0.73

0.98%
2.58
0.01
0.94
16.41
0.00
0.72

1.20%
2.19
0.03
1.01
10.55
0.00
0.09
1.30
0.20
0.62

0.68%
1.44
0.15
1.02
12.36
0.00
0.04
0.70
0.48
0.68

1.12%
2.28
0.02
0.95
11.06
0.00
0.17
2.71
0.01
0.67

0.92%
1.96
0.05
1.01
12.22
0.00
0.16
2.76
0.01
0.71

1.00%
1.80
0.08
0.87
8.95
0.00
0.28
3.98
0.00
0.63

0.84%
1.54
0.13
0.91
9.53
0.00
0.28
4.05
0.00
0.65

0.86%
3.29
0.00
0.89
19.49
0.00
0.05
1.47
0.14
0.84

0.79%
2.89
0.00
0.90
18.80
0.00
0.06
1.63
0.11
0.83

0.94%
2.70
0.01
0.90
14 .85
0.00
0.08
1.77
0.08
0.76

0.85%
2.50
0.01
0.90
15.07
0.00
0.09
1.97
0.05
0.77

-0.30%
-0.62

-0.46%
-0.91

0.81
9.47
0.00
0.19
3.13
0.00
0.62

0.86
9.65
0.00
0.18
2.73
0.01
0.62

1.25%
2.34
0.02
1.05
11.21
0.00
0.08
1.16
0.2 5
0.64

1.28%
2.65
0.01
1.00
11 .71
0.00
0.14
2.25
0.03
0.68

1.18%
2.12
0.04
0.92
9.44
0.00
0.2 5
3.57
0.00
0.64

0.93%
3.52
0.00
0.92
19.94
0.00
0.04
1.33
0.19
0.85

1.17%
3.50
0.00
0.93
15.85
0.00
0.08
1.91
0.06
0.79

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Appendix C.7. Relative returns of the momentum-style indices

The graph displays the relative cumulative returns, calculated as index cumulative returns divided by the ALSI cumulative returns, for the MOMW100, MOM(12-1)W100 and EW(MOM)Negative Indices, over the period 1st January 1998 to 31st December 2006. Relative returns provide insight as to the performance of the style indices relative to that of the South African equity market as a whole, proxied by the ALSI. The horizontal line with a y-intercept of 1 represents the level of the ALSI cumulative returns. The data are obtained from DataStream International at the University of Cape Town.
Appendix C.8. Comparing annualised statistics on constructed style indices and the benchmark indices

The graph compares the major statistics of the constructed style indices to those of the five existing JSE style indices over the period 1st July 2002 to 31st December 2006. Returns are annualised using monthly effective returns, except for the RAFI SA Index where returns are per annum effective. The style indices plotted are: (1) the value-style index constructed using the residuals of a three-factor regression based on earnings, book values and dividends as the value-style proxy (RESW100(3)), and (2) the momentum-style index constructed using the past 12-month returns but excluding the latest month’s return as the momentum-style proxy (MOM(12-1)W100). The benchmark indices selected are: JSE DIVI standing for the JSE Dividned Plus Index, RAFI SA standing for the Plexus South African/JSE RAFI Index, the ALSI, the Top 40 Index and the SWIX Index.
Appendix D

Appendices contained in Appendix D relate to Chapter Five. Replicating Active Equity Portfolios.

**Appendix D.1. Name and code of selected domestic equity unit trust and hedge fund**

The table displays the code, name and starting date (1<sup>st</sup> January 1998 or inception) of the funds and indices investing on the JSE that are analysed in Chapter Five. Unit trusts (unit trust) and hedge funds (HF) total returns data are obtained from 1-Net Bridge and HedgeFundIntelligent via the finance research lab in UCT. Fund names in blue indicate that the maximum period of past returns available is too short (at least 36 months is required to conduct the return-based regressions to infer a fund’s investment style). Therefore funds in blue (KAQF and RCFB) are excluded from the rolling style regressions in Chapter Five.

<table>
<thead>
<tr>
<th>Code</th>
<th>Name</th>
<th>Starting Date</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SA Unit Trust Indices</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DOEQ</td>
<td>Domestic equity index - PLEXUS</td>
<td>1998-1-1</td>
</tr>
<tr>
<td>DOEQGR</td>
<td>DOMESTIC EQUITY GROWTH INDEX - PLEXUS</td>
<td>1998-1-1</td>
</tr>
<tr>
<td>DOEQVL</td>
<td>DOMESTIC EQUITY VALUE INDEX - PLEXUS</td>
<td>1998-1-1</td>
</tr>
<tr>
<td><strong>SA Unit Trust funds (Domestic Equity General)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AGEF</td>
<td>Allan Gray equity fund - A</td>
<td>1998-11-1</td>
</tr>
<tr>
<td>CORG</td>
<td>coronation equity fund - R</td>
<td>1998-1-1</td>
</tr>
<tr>
<td>METF</td>
<td>investec equity fund - R</td>
<td>1998-1-1</td>
</tr>
<tr>
<td>KAQF</td>
<td>kagiso active quants fund</td>
<td>2004-5-1</td>
</tr>
<tr>
<td>AHVE</td>
<td>Nedbank rainmaker fund - A</td>
<td>2000-8-1</td>
</tr>
<tr>
<td>OGEN</td>
<td>oasis general equity fund</td>
<td>2001-10-1</td>
</tr>
<tr>
<td>OMTL</td>
<td>old mutual investors fund</td>
<td>1998-1-1</td>
</tr>
<tr>
<td>PRUO</td>
<td>prudential equity fund</td>
<td>1999-9-1</td>
</tr>
<tr>
<td>PSGG</td>
<td>PSG Alphen Growth fund - A</td>
<td>1998-1-1</td>
</tr>
<tr>
<td>RCFB</td>
<td>RECM core equity fund B</td>
<td>2005-4-1</td>
</tr>
<tr>
<td>RMEF</td>
<td>RMB equity fund</td>
<td>1998-1-1</td>
</tr>
<tr>
<td>SNTR</td>
<td>Sanlam General equity fund</td>
<td>1998-1-1</td>
</tr>
<tr>
<td><strong>SA Unit Trust funds (Domestic Equity Growth)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SNST</td>
<td>Sanlam Small Cap Fund</td>
<td>1998-1-1</td>
</tr>
<tr>
<td><strong>SA Hedge fund indices</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>COMP</td>
<td>Single Manager Composite</td>
<td>2004-1-1</td>
</tr>
<tr>
<td>LSE</td>
<td>Long Short Equity Index</td>
<td>2004-1-1</td>
</tr>
<tr>
<td>MKN</td>
<td>Market Neutral &amp; Quantitative Strategies Index</td>
<td>2004-2-1</td>
</tr>
<tr>
<td>FOFs</td>
<td>Fund of Funds Index</td>
<td>2004-1-1</td>
</tr>
</tbody>
</table>
Appendix D.2. Code, definition and constraints of different types of regressions

The table contains the shorthand code, the relevant constraints and the corresponding fund types on which different types of regressions are conducted in Chapter Five.

<table>
<thead>
<tr>
<th>Type of Regression</th>
<th>Short-hand notation</th>
<th>Constrains</th>
<th>Applicable fund types</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconstrained</td>
<td>UC</td>
<td>None</td>
<td>-</td>
</tr>
<tr>
<td>Constrained Sum</td>
<td>CS</td>
<td>( \sum</td>
<td>w_i</td>
</tr>
</tbody>
</table>
| Quadratic Programming | QP | \( 0 < w_i < 1 \) \\
|                     |                     | \( \sum w_i = 1 \) | Unit trusts. Hedge funds for comparison (if short and leverage are not allowed). |
Appendix D.3. Regression results on the Domestic Equity Index (DOEQ)

The monthly total returns are computed from the closing price and dividend yields obtained from I-Net Bridge at the University of Cape Town. All results are monthly effective. For unit trusts (funds and indices), 36 months' rolling periods are used to infer a fund's investment style. If six independent regression variables are used, then the Small Cap Index is not included; else it is included. The beta coefficients are obtained for the explanatory variables in the following order: Small Cap, EW(size)100, RESW100(3), MOM(12-I)W100, FIN115-STXF, INDI25-STXIND, RESI20-STXRESI. The table shows the average whole-period regression coefficients (style weights) on the DOEQ Index over the period 1st January 1998 to 31st December 2006. Investment style is estimated using both ordinary least square regressions (OLS) and weighted least square regressions (WLS), using Equations (5.1) and (5.2) respectively. Unconstrained (UC), constrained sum (CS) and quadratic programming (QP) are used, each defined as in Appendix D.2. P-values are calculated using two-tailed tests.

<table>
<thead>
<tr>
<th>Index Code: DOEQ</th>
<th>OLS UC 7</th>
<th>OLS CS 7</th>
<th>OLS QP 7</th>
<th>OLS UC 6</th>
<th>OLS CS 6</th>
<th>OLS QP 6</th>
<th>WLS UC 7</th>
<th>WLS CS 7</th>
<th>WLS QP 7</th>
<th>WLS UC 6</th>
<th>WLS CS 6</th>
<th>WLS QP 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average beta1</td>
<td>0.1885</td>
<td>0.3066</td>
<td>0.2834</td>
<td></td>
<td></td>
<td></td>
<td>0.1460</td>
<td>0.2385</td>
<td>0.2176</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average beta2</td>
<td>0.3621</td>
<td>0.3947</td>
<td>0.1147</td>
<td>0.5092</td>
<td>0.7225</td>
<td>0.4703</td>
<td>0.3978</td>
<td>0.4523</td>
<td>0.2371</td>
<td>0.5159</td>
<td>0.6988</td>
<td>0.5080</td>
</tr>
<tr>
<td>Average beta3</td>
<td>-0.1931</td>
<td>-0.2412</td>
<td>0.0000</td>
<td>-0.1552</td>
<td>-0.1968</td>
<td>0.0000</td>
<td>-0.1482</td>
<td>-0.1889</td>
<td>0.0000</td>
<td>-0.1185</td>
<td>-0.1494</td>
<td>0.0000</td>
</tr>
<tr>
<td>Average beta4</td>
<td>0.0694</td>
<td>0.0265</td>
<td>0.0735</td>
<td>0.0721</td>
<td>0.0016</td>
<td>0.0421</td>
<td>0.0735</td>
<td>0.0356</td>
<td>0.0690</td>
<td>0.0752</td>
<td>0.0230</td>
<td>0.0508</td>
</tr>
<tr>
<td>Average beta5</td>
<td>0.1564</td>
<td>0.1569</td>
<td>0.1779</td>
<td>0.1482</td>
<td>0.1401</td>
<td>0.1586</td>
<td>0.1264</td>
<td>0.1357</td>
<td>0.1453</td>
<td>0.1223</td>
<td>0.1318</td>
<td>0.1398</td>
</tr>
<tr>
<td>Average beta6</td>
<td>0.1939</td>
<td>0.2258</td>
<td>0.2376</td>
<td>0.1801</td>
<td>0.2201</td>
<td>0.2302</td>
<td>0.1891</td>
<td>0.2012</td>
<td>0.2096</td>
<td>0.1804</td>
<td>0.1898</td>
<td>0.1974</td>
</tr>
<tr>
<td>Average beta7</td>
<td>0.1096</td>
<td>0.1308</td>
<td>0.1128</td>
<td>0.0935</td>
<td>0.1125</td>
<td>0.0988</td>
<td>0.1068</td>
<td>0.1256</td>
<td>0.1213</td>
<td>0.0924</td>
<td>0.1060</td>
<td>0.1040</td>
</tr>
<tr>
<td>R square (in-sample)</td>
<td>0.9560</td>
<td>0.9446</td>
<td>0.9359</td>
<td>0.9498</td>
<td>0.9235</td>
<td>0.9177</td>
<td>0.9523</td>
<td>0.9407</td>
<td>0.9355</td>
<td>0.9483</td>
<td>0.9283</td>
<td>0.9250</td>
</tr>
</tbody>
</table>
Appendix D.4. Regression results on the Domestic Equity Growth Index (DOEQGR)

The monthly total returns are computed from the closing price and dividend yields obtained from 1-Net Bridge at the University of Cape Town. All results are monthly. For unit trusts (funds and indices), 36-month rolling periods are used to infer a fund's investment style. If six independent regression variables are used, then the Small Cap Index is not included; else it is included. The beta coefficients are obtained for the explanatory variables in the following order: Small Cap, EW(size)100, RESW100(3), MOM(12-1)W100, FINI15-STXFIN, INDI25-STXIND, RESI20-STXRESI.

Section A: Average style regression results
The table shows the average whole-period regression coefficients (style weights) on the analysis of the DOEQ Index over the period 1st January 1998 to 31st December 2006. Investment style is estimated using both ordinary least square regressions (OLS) and weighted least square regressions (WLS), using Equations (5.1) and (5.2) respectively. Unconstrained (UC), constrained sum (CS) and quadratic programming (QP) are used, each defined as in Appendix D.2. P-values are calculated using two-tailed tests.

Section B: Histogram of monthly selection returns of WLS 6
The graph displays the monthly selection returns on DOEQGR over the period 1st January 2001 to 31st December 2006. Investment style is estimated from weighted least square regressions (WLS) using Equations 5.2, with six independent regression variables excluding the Small Cap Index. Quadratic programming (QP), as defined in Appendix D.2, is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.

Section A: Average style regression results

<table>
<thead>
<tr>
<th>Index Code: DOEQGR</th>
<th>OLS UC 7</th>
<th>OLS CS 7</th>
<th>OLS QP 7</th>
<th>OLS UC 6</th>
<th>OLS CS 6</th>
<th>OLS QP 6</th>
<th>WLS UC 7</th>
<th>WLS CS 7</th>
<th>WLS QP 7</th>
<th>WLS UC 6</th>
<th>WLS CS 6</th>
<th>WLS QP 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average beta1</td>
<td>0.2831</td>
<td>0.3475</td>
<td>0.3380</td>
<td>0.3229</td>
<td>0.3863</td>
<td>0.2611</td>
<td>0.3562</td>
<td>0.5000</td>
<td>0.5327</td>
<td>0.2630</td>
<td>0.6883</td>
<td>0.8306</td>
</tr>
<tr>
<td>Average beta2</td>
<td>0.4510</td>
<td>0.4683</td>
<td>0.1502</td>
<td>0.6718</td>
<td>0.8379</td>
<td>0.5362</td>
<td>0.5300</td>
<td>0.5327</td>
<td>0.2630</td>
<td>0.6883</td>
<td>0.8306</td>
<td>0.5647</td>
</tr>
<tr>
<td>Average beta3</td>
<td>-0.2493</td>
<td>-0.3768</td>
<td>0.0000</td>
<td>-0.1923</td>
<td>-0.2247</td>
<td>0.0000</td>
<td>-0.2113</td>
<td>-0.2367</td>
<td>0.0000</td>
<td>-0.1648</td>
<td>-0.1889</td>
<td>0.0000</td>
</tr>
<tr>
<td>Average beta4</td>
<td>0.0437</td>
<td>0.0206</td>
<td>0.0744</td>
<td>0.0477</td>
<td>0.0072</td>
<td>0.8411</td>
<td>0.0455</td>
<td>0.0228</td>
<td>0.0647</td>
<td>0.0462</td>
<td>0.0076</td>
<td>0.0432</td>
</tr>
<tr>
<td>Average beta5</td>
<td>0.1402</td>
<td>0.1404</td>
<td>0.1643</td>
<td>0.1278</td>
<td>0.1215</td>
<td>0.1482</td>
<td>0.1277</td>
<td>0.1392</td>
<td>0.1453</td>
<td>0.1212</td>
<td>0.1285</td>
<td>0.1403</td>
</tr>
<tr>
<td>Average beta6</td>
<td>0.2612</td>
<td>0.2761</td>
<td>0.2913</td>
<td>0.2405</td>
<td>0.2717</td>
<td>0.2746</td>
<td>0.2317</td>
<td>0.2389</td>
<td>0.2494</td>
<td>0.2178</td>
<td>0.2251</td>
<td>0.2319</td>
</tr>
<tr>
<td>Average beta7</td>
<td>0.0101</td>
<td>0.0213</td>
<td>0.0009</td>
<td>-0.0141</td>
<td>0.0008</td>
<td>0.0000</td>
<td>0.0096</td>
<td>0.0208</td>
<td>0.0155</td>
<td>-0.0135</td>
<td>-0.0029</td>
<td>0.0000</td>
</tr>
<tr>
<td>R square (in-sample)</td>
<td>0.9575</td>
<td>0.9546</td>
<td>0.9465</td>
<td>0.9448</td>
<td>0.9304</td>
<td>0.9232</td>
<td>0.9535</td>
<td>0.9496</td>
<td>0.9421</td>
<td>0.9460</td>
<td>0.9329</td>
<td>0.9279</td>
</tr>
</tbody>
</table>

Section B: Histogram of monthly selection returns of WLS 6
Appendix D.5. Regression results on the Domestic Equity Value Index (DOEQVL)

The monthly total returns are computed from the closing price and dividend yields obtained from I-Net Bridge at the University of Cape Town. All results are monthly. For unit trusts (funds and indices), 36-month rolling periods are used to infer a fund’s investment style. If six independent regression variables are used, then the Small Cap Index is not included; else it is included. The beta coefficients are obtained for the explanatory variables in the following order: Small Cap, EW(size)100, RESW100(3), MOM(12-1)W100, FINI15-STXFIN, INDI25-STXIND, RESI20-STXRESI.

Section A: Average style regression results

The table shows the average whole-period regression coefficients (style weights) on the analysis of the DOEQ Index over the period 1st January 1998 to 31st December 2006. Investment style is estimated using both ordinary least square regressions (OLS) and weighted least square regressions (WLS), using Equations (5.1) and (5.2) respectively. Unconstrained (UC), constrained sum (CS) and quadratic programming (QP) are used, each defined as in Appendix D.2. P-values are calculated using two-tailed tests.

Section B: Histogram of monthly selection returns of WLS 6

The graph displays the monthly selection returns on DOEQVL over the period 1st January 1998 to 31st December 2006. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with six independent regression variables excluding the Small Cap Index. Quadratic programming (QP), as defined in Appendix D.2, is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.

<table>
<thead>
<tr>
<th>Index Code: DOEQVL</th>
<th>OLS UC 7</th>
<th>OLS CS 7</th>
<th>OLS GP 7</th>
<th>OLS UC 6</th>
<th>OLS CS 6</th>
<th>OLS GP 6</th>
<th>WLS UC 7</th>
<th>WLS CS 7</th>
<th>WLS GP 7</th>
<th>WLS UC 6</th>
<th>WLS CS 6</th>
<th>WLS GP 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average beta1</td>
<td>0.2650</td>
<td>0.4780</td>
<td>0.4513</td>
<td>0.2669</td>
<td>0.4065</td>
<td>0.4081</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average beta2</td>
<td>-0.0762</td>
<td>-0.0174</td>
<td>0.0000</td>
<td>0.1304</td>
<td>0.0174</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average beta3</td>
<td>0.2786</td>
<td>0.1900</td>
<td>0.1790</td>
<td>0.3001</td>
<td>0.2593</td>
<td>0.2220</td>
<td>0.2730</td>
<td>0.2116</td>
<td>0.2194</td>
<td>0.2789</td>
<td>0.2991</td>
<td></td>
</tr>
<tr>
<td>Average beta4</td>
<td>0.0287</td>
<td>-0.0488</td>
<td>0.0000</td>
<td>0.0932</td>
<td>-0.0876</td>
<td>0.0000</td>
<td>0.0430</td>
<td>-0.0143</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>Average beta5</td>
<td>0.1192</td>
<td>0.1201</td>
<td>0.1156</td>
<td>0.1076</td>
<td>0.0993</td>
<td>0.1186</td>
<td>0.0914</td>
<td>0.1054</td>
<td>0.1081</td>
<td>0.0840</td>
<td>0.1095</td>
<td></td>
</tr>
<tr>
<td>Average beta6</td>
<td>0.0801</td>
<td>0.1252</td>
<td>0.1232</td>
<td>0.0644</td>
<td>0.0968</td>
<td>0.0966</td>
<td>0.0697</td>
<td>0.0980</td>
<td>0.0983</td>
<td>0.0433</td>
<td>0.0646</td>
<td>0.0650</td>
</tr>
<tr>
<td>R square (in-sample)</td>
<td>0.8903</td>
<td>0.8450</td>
<td>0.8422</td>
<td>0.8753</td>
<td>0.7827</td>
<td>0.7777</td>
<td>0.8842</td>
<td>0.8552</td>
<td>0.8551</td>
<td>0.8695</td>
<td>0.8158</td>
<td>0.8152</td>
</tr>
</tbody>
</table>

Section B: Histogram of monthly selection returns of WLS 6

![Monthly Selection Returns](image-url)
Appendix D.6. Synthesising South African unit trust indices and representative funds (OLS results)

The table shows the summary statistics and regression results on analysis of the South African Unit Trust indices and funds over the period 1\textsuperscript{st} January 1998 to 31\textsuperscript{st} December 2006. The three unit trust indices examined are the Domestic Equity Index (DOEQ), Domestic Equity Growth Index (DOEQGR) and Domestic Equity Value Index (DOEQVL). The 11 unit trust funds examined are Allan Gray Equity Fund-A (AGEF), Coronation Equity Fund-R (CORC), Investec Equity Fund-R (METF), Nedbank Rainmaker Fund-A (AHVE), Oasis General Equity Fund (OGEN), Old Mutual Investors Fund (OMTL), Prudential Equity Fund (PRUO), PSG Alphen Growth Fund-A (PSGG), RMB Equity Fund (RMEF), Sanlam General Equity Fund (SNTR), and Sanlam Small Cap Fund (SNST). The monthly total returns are computed from the closing price and dividend yields obtained from I-Net Bridge at the University of Cape Town. All results are monthly effective. Investment style is estimated using the OLS using Equation 5.1. The return-based style decompositions are conducted using Sharpe's (1988) multi-factor regression with EW(size)100, RESW100(3) (4), MOM(12-I)W100, FINI5-STXFIN, IND125-STXIND, RESI20-STXRESI. R\textsuperscript{2} values are obtained from the out-sample regressions of predicted style returns on actual fund returns. For unit trusts (funds and indices), 36-month rolling periods are used to infer a fund's investment style. Quadratic programming (QP), as defined in Appendix D.2, is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.

The South African 90-day Banker's Acceptance discount rate (RBAS) is used as the risk-free rate when calculating the Sharpe Ratios. P-values are calculated using two-tailed tests.

<table>
<thead>
<tr>
<th>Unit trust index/fund code</th>
<th>DOEQ</th>
<th>DOEQGR</th>
<th>DOEQVL</th>
<th>AGEF</th>
<th>CORC</th>
<th>METF</th>
<th>AHVE</th>
<th>OGEN</th>
<th>OMTL</th>
<th>PRUO</th>
<th>PSGG</th>
<th>RMEF</th>
<th>SNTR</th>
<th>SNST</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean return (whole period) (%)</td>
<td>1.38</td>
<td>1.14</td>
<td>1.88</td>
<td>2.70</td>
<td>1.65</td>
<td>1.70</td>
<td>2.40</td>
<td>2.39</td>
<td>1.53</td>
<td>1.95</td>
<td>1.41</td>
<td>1.59</td>
<td>1.40</td>
<td>1.11</td>
</tr>
<tr>
<td>Standard deviation (whole period) (%)</td>
<td>5.75</td>
<td>6.13</td>
<td>5.01</td>
<td>4.88</td>
<td>4.95</td>
<td>6.10</td>
<td>4.27</td>
<td>3.79</td>
<td>6.40</td>
<td>4.84</td>
<td>6.20</td>
<td>5.99</td>
<td>5.82</td>
<td>7.61</td>
</tr>
<tr>
<td>Sharpe ratio (whole period)</td>
<td>0.09</td>
<td>0.04</td>
<td>0.20</td>
<td>0.37</td>
<td>0.16</td>
<td>0.14</td>
<td>0.36</td>
<td>0.40</td>
<td>0.10</td>
<td>0.22</td>
<td>0.09</td>
<td>0.12</td>
<td>0.09</td>
<td>0.03</td>
</tr>
<tr>
<td>Mean return (out-of-sample period) (%)</td>
<td>1.85</td>
<td>1.69</td>
<td>2.36</td>
<td>2.50</td>
<td>1.85</td>
<td>2.27</td>
<td>3.04</td>
<td>2.88</td>
<td>1.86</td>
<td>2.58</td>
<td>1.83</td>
<td>2.12</td>
<td>1.71</td>
<td>2.53</td>
</tr>
<tr>
<td>Standard deviation (out-of-sample period) (%)</td>
<td>4.33</td>
<td>4.43</td>
<td>4.06</td>
<td>4.32</td>
<td>4.23</td>
<td>4.54</td>
<td>3.20</td>
<td>3.40</td>
<td>4.49</td>
<td>4.04</td>
<td>5.00</td>
<td>4.49</td>
<td>4.64</td>
<td>4.53</td>
</tr>
<tr>
<td>Sharpe ratio (out-of-sample period)</td>
<td>0.23</td>
<td>0.18</td>
<td>0.36</td>
<td>0.38</td>
<td>0.23</td>
<td>0.31</td>
<td>0.68</td>
<td>0.59</td>
<td>0.22</td>
<td>0.42</td>
<td>0.19</td>
<td>0.28</td>
<td>0.18</td>
<td>0.36</td>
</tr>
</tbody>
</table>

OLS OP 6 (ex-small cap)

| Mean style return (%) | 1.80 | 1.78 | 2.27 | 2.55 | 1.77 | 1.89 | 2.93 | 3.12 | 1.72 | 2.25 | 1.69 | 1.84 | 1.63 | 2.23 |
| Standard deviation of style return (%) | 4.87 | 4.82 | 4.93 | 4.81 | 5.15 | 4.98 | 3.79 | 3.79 | 4.99 | 4.40 | 4.86 | 4.82 | 5.15 | 4.35 |
| Sharpe ratio of style return | 0.19 | 0.19 | 0.26 | 0.35 | 0.17 | 0.20 | 0.54 | 0.59 | 0.17 | 0.31 | 0.20 | 0.15 | 0.15 | 0.31 |
| Mean selection return (%) | 0.02 | -0.11 | 0.03 | -0.11 | 0.03 | 0.34 | 0.07 | -0.26 | 0.10 | 0.31 | 0.13 | 0.25 | 0.05 | 0.24 |
| Standard deviation of selection return (%) | 1.39 | 1.33 | 2.06 | 2.80 | 1.76 | 2.00 | 1.87 | 1.41 | 1.66 | 1.69 | 1.80 | 1.90 | 1.41 | 2.50 |
| t-selection return | 0.12 | 0.72 | 0.17 | 0.43 | 0.27 | 1.46 | 0.25 | 0.93 | 0.22 | 0.68 | 0.59 | 1.01 | 0.28 | 0.84 |
| p-selection return | 0.90 | 0.47 | 0.86 | 0.67 | 0.79 | 0.15 | 0.81 | 0.36 | 0.82 | 0.50 | 0.55 | 0.32 | 0.78 | 0.41 |
| R Square (out-of-sample) | 0.90 | 0.92 | 0.78 | 0.86 | 0.87 | 0.85 | 0.88 | 0.82 | 0.90 | 0.85 | 0.77 | 0.85 | 0.85 | 0.69 |

OLS OP 7 (incl-small cap)

| Mean style return (%) | 1.87 | 1.86 | 2.38 | 2.68 | 1.86 | 1.99 | 3.12 | 3.26 | 1.75 | 2.29 | 1.77 | 1.86 | 1.71 | 2.43 |
| Standard deviation of style return (%) | 4.64 | 4.56 | 4.61 | 4.56 | 4.79 | 4.84 | 3.88 | 3.71 | 4.96 | 4.40 | 4.61 | 4.75 | 4.96 | 4.00 |
| Sharpe ratio of style return | 0.22 | 0.21 | 0.31 | 0.40 | 0.20 | 0.23 | 0.58 | 0.64 | 0.18 | 0.32 | 0.19 | 0.20 | 0.17 | 0.39 |
| Mean selection return (%) | -0.03 | -0.18 | -0.06 | -0.22 | -0.05 | 0.25 | -0.12 | -0.40 | 0.07 | 0.27 | 0.07 | 0.23 | -0.02 | 0.08 |
| Standard deviation of selection return (%) | 1.40 | 1.24 | 1.87 | 2.65 | 1.62 | 2.00 | 1.97 | 1.47 | 1.67 | 1.72 | 1.84 | 1.93 | 1.39 | 2.02 |
| t-selection return | 0.31 | 1.23 | 0.07 | 0.76 | 0.18 | 1.01 | 0.40 | 1.35 | 0.18 | 0.66 | 0.31 | 0.90 | 0.12 | 0.30 |
| p-selection return | 0.76 | 0.22 | 0.98 | 0.45 | 0.85 | 0.32 | 0.69 | 0.19 | 0.85 | 0.51 | 0.76 | 0.37 | 0.90 | 0.77 |
| R Square (out-of-sample) | 0.92 | 0.92 | 0.80 | 0.88 | 0.90 | 0.86 | 0.89 | 0.83 | 0.92 | 0.86 | 0.86 | 0.86 | 0.85 | 0.75 |
Appendix D.7. Regression results on the Allan Gray Equity Fund-A (AGEF)
The monthly total returns are computed from the closing price and dividend yields obtained from 1-Net Bridge at the University of Cape Town. All results are monthly. For unit trusts (funds and indices), 36-month rolling periods are used to infer a fund’s investment style. The analysis is for the period 1\textsuperscript{st} November 1998 to 31\textsuperscript{st} December 2006. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with six independent regression variables excluding the Small Cap Index. Quadratic programming (QP), as defined in Appendix D.2, is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.

Section A: Scatter plot of style return and observed return of WLS 6
The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis.

Section B: Histogram of monthly selection returns of WLS 6
The graph displays the monthly selection returns on AGEF.

Section C: Exposure distribution area graph of WLS 6
The graph displays the monthly exposure of the AGEF to the six selected explanatory indices.
Appendix D.7. Regression results on the Allan Gray Equity Fund-A (AGEF) (Continued)

Section C: Exposure distribution area graph of WLS 6

![Exposure distribution area graph of WLS 6]

- RESI 20
- IND 25
- FINI 15
- MOM(12-1)W100
- RESW100(3)
- EW(size)100
Appendix D.8. Regression results on the Coronation Equity Fund-R (CORG)
The monthly total returns are computed from the closing price and dividend yields obtained from 1-Net Bridge at the University of Cape Town. All results are monthly. For unit trusts (funds and indices), 36-month rolling periods are used to infer a fund's investment style. The analysis is for the period 1st January 1998 to 31st December 2006. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with six independent regression variables excluding the Small Cap Index. Quadratic programming (QP), as defined in Appendix D.2, is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.

Section A: Scatter plot of style return and observed return of WLS 6
The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis.

Section B: Histogram of monthly selection returns of WLS 6
The graph displays the monthly selection returns of CORG.

Section C: Exposure distribution area graph of WLS 6
The graph displays the monthly exposure of the CORG to the six selected explanatory indices.
Appendix D.8. Regression results on the Coronation Equity Fund-R (Continued)

Section C: Exposure distribution area graph of WLS 6
Appendix D.9. Regression results on the Investec Equity Fund-R (METF)

The monthly total returns are computed from the closing price and dividend yields obtained from 1-Net Bridge at the University of Cape Town. All results are monthly. For unit trusts (funds and indices), 36-month rolling periods are used to infer a fund’s investment style. The analysis is for the period 1st January 1998 to 31st December 2006. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with six independent regression variables excluding the Small Cap Index. Quadratic programming (QP), as defined in Appendix D.2, is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.

Section A: Scatter plot of style return and observed return of WLS 6
The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis.

Section B: Histogram of monthly selection returns of WLS 6
The graph displays the monthly selection returns on METF.

Section C: Exposure distribution area graph of WLS 6
The graph displays the monthly exposure of the METF to the six selected explanatory indices.
Appendix D.9. Regression results on the Investec Equity Fund-R (METF) (Continued)

Section C: Exposure distribution area graph of WLS 6
Appendix D.10. Regression results on the Nedbank Rainmaker Fund-A (AHVE)

The monthly total returns are computed from the closing price and dividend yields obtained from I-Net Bridge at the University of Cape Town. All results are monthly. For unit trusts (funds and indices), 36-month rolling periods are used to infer a fund’s investment style. The analysis is for the period 1st August 2000 to 31st December 2006. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with six independent regression variables excluding the Small Cap Index. Quadratic programming (QP), as defined in Appendix D.2, is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.

Section A: Scatter plot of style return and observed return of WLS 6
The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis.

Section B: Histogram of monthly selection returns of WLS 6
The graph displays the monthly selection returns on AHVE.

Section C: Exposure distribution area graph of WLS 6
The graph displays the monthly exposure of the AHVE to the six selected explanatory indices.
Appendix D.10. Regression results on the Nedbank Rainmaker Fund-A (AHVE) (Continued)

Section C: Exposure distribution area graph of WLS 6

![Exposure distribution area graph of WLS 6](image-url)
Appendix D.11. Regression results on the Oasis General Equity Fund (OGEN)
The monthly total returns are computed from the closing price and dividend yields obtained from I-Net Bridge at the University of Cape Town. All results are monthly. For unit trusts (funds and indices), 36-month rolling periods are used to infer a fund’s investment style. The analysis is for the period 1\textsuperscript{st} October 2001 to 31\textsuperscript{st} December 2006. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with six independent regression variables excluding the Small Cap Index. Quadratic programming (QP), as defined in Appendix D.2, is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.

Section A: Scatter plot of style return and observed return of WLS 6
The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis.

Section B: Histogram of monthly selection returns of WLS 6
The graph displays the monthly selection returns on AHVE.

Section C: Exposure distribution area graph of WLS 6
The graph displays the monthly exposure of the AHVE to the six selected explanatory indices.
Appendix D.11. Regression results on the Oasis General Equity Fund (OGEN) (Continued)

Section C: Exposure distribution area graph of WLS 6
Appendix D.12. Regression results on the Old Mutual Investors Fund (OMTL)

The monthly total returns are computed from the closing price and dividend yields obtained from I-Net Bridge at the University of Cape Town. All results are monthly. For unit trusts (funds and indices), 36-month rolling periods are used to infer a fund's investment style. The analysis is for the period 1st January 1998 to 31st December 2006. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with six independent regression variables excluding the Small Cap Index. Quadratic programming (QP), as defined in Appendix D.2, is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.

Section A: Scatter plot of style return and observed return of WLS 6
The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis.

Section B: Histogram of monthly selection returns of WLS 6
The graph displays the monthly selection returns on AHVE.

Section C: Exposure distribution area graph of WLS 6
The graph displays the monthly exposure of the AHVE to the six selected explanatory indices.
Appendix D.12. Regression results on the Old Mutual Investors Fund (OMTL) (Continued)

Section C: Exposure distribution area graph of WLS 6

![Graph showing exposure distribution area graph of WLS 6 with various style weights and dates.]
Appendix D.13. Regression results on the Prudential Equity Fund (PRUO)

The monthly total returns are computed from the closing price and dividend yields obtained from I-Net Bridge at the University of Cape Town. All results are monthly. For unit trusts (funds and indices), 36-month rolling periods are used to infer a fund’s investment style. The analysis is for the period 1st September 1999 to 31st December 2006. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with six independent regression variables excluding the Small Cap Index. Quadratic programming (QP), as defined in Appendix D.2, is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.

Section A: Scatter plot of style return and observed return of WLS 6
The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis.

Section B: Histogram of monthly selection returns of WLS 6
The graph displays the monthly selection returns on PRUO.

Section C: Exposure distribution area graph of WLS 6
The graph displays the monthly exposure of the PRUO to the six selected explanatory indices.

---

Section A: Scatter plot of style return and observed return of WLS 6

![Scatter plot](image)

Section B: Histogram of monthly selection returns of WLS 6

![Histogram](image)
Appendix D.13. Regression results on the Prudential Equity Fund (PRUO) (Continued)

Section C: Exposure distribution area graph of WLS 6
Appendix D.14. Regression results on the PSG Alpha Growth fund -A (PSGG)

The monthly total returns are computed from the closing price and dividend yields obtained from I-Net Bridge at the University of Cape Town. All results are monthly. For unit trusts (funds and indices), 36-month rolling periods are used to infer a fund’s investment style. The analysis is for the period 1st January 1998 to 31st December 2006. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with six independent regression variables excluding the Small Cap Index. Quadratic programming (QP), as defined in Appendix D.2, is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.

Section A: Scatter plot of style return and observed return of WLS 6
The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis.

Section B: Histogram of monthly selection returns of WLS 6
The graph displays the monthly selection returns on PSGG.

Section C: Exposure distribution area graph of WLS 6
The graph displays the monthly exposure of the PSGG to the six selected explanatory indices.

Section A: Scatter plot of style return and observed return of WLS 6

Section B: Histogram of monthly selection returns of WLS 6
Appendix D.14. Regression results on the PSG Alpha Growth fund -A (PSGG) (Continued)

Section C: Exposure distribution area graph of WLS 6

![Exposure distribution area graph of WLS 6](image-url)
Appendix D.15. Regression results on the RMB Equity Fund (RMEF)

The monthly total returns are computed from the closing price and dividend yields obtained from I-Net Bridge at the University of Cape Town. All results are monthly. For unit trusts (funds and indices), 36-month rolling periods are used to infer a fund's investment style. The analysis is for the period 1st January 1998 to 31st December 2006. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with six independent regression variables excluding the Small Cap Index. Quadratic programming (QP), as defined in Appendix D.2, is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.

Section A: Scatter plot of style return and observed return of WLS 6
The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis.

Section B: Histogram of monthly selection returns of WLS 6
The graph displays the monthly selection returns on RMEF.

Section C: Exposure distribution area graph of WLS 6
The graph displays the monthly exposure of the RMEF to the six selected explanatory indices.
Appendix D.15. Regression results on the RMB equity fund (RMEF) (Continued)

Section C: Exposure distribution area graph of WLS 6

![Exposure distribution area graph of WLS 6]

- RESI 20
- INDI 25
- FINI 15
- MOM(12-1)/W100
- RESW100(3)
- EW(size)100
Appendix D.16. Regression results on the Sanlam General Equity Fund (SNTR)

The monthly total returns are computed from the closing price and dividend yields obtained from I-Net Bridge at the University of Cape Town. All results are monthly. For unit trusts (funds and indices), 36-month rolling periods are used to infer a fund's investment style. The analysis is for the period 1st January 1998 to 31st December 2006. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with six independent regression variables excluding the Small Cap Index or seven independent variables including the Small Cap Index. Quadratic programming (QP), as defined in Appendix D.2, is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.

Section A: Scatter plot of style return and observed return of WLS 6
The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis, without the Small Cap Index.

Section B: Histogram of monthly selection returns of WLS 6
The graph displays the monthly selection returns on SNTR, without the Small Cap Index.

Section C: Exposure distribution area graph of WLS 6
The graph displays the monthly exposure of the SNTR to the six selected explanatory indices, without the Small Cap Index.

Section D: Scatter plot of style return and observed return of WLS 7
The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis, with the Small Cap Index.

Section E: Histogram of monthly selection returns of WLS 7
The graph displays the monthly selection returns on SNTR, with the Small Cap Index.

Section F: Exposure distribution area graph of WLS 7
The graph displays the monthly exposure of the SNTR to the six selected explanatory indices, with the Small Cap Index.

Section A: Scatter plot of style return and observed return of WLS 6
Appendix D.16. Regression results on the Sanlam General Equity Fund (SNTR) (Continued)

Section B: Histogram of monthly selection returns of WLS 6

Section C: Exposure distribution area graph of WLS 6
Appendix D.16. Regression results on the Sanlam General Equity Fund (SNTR) (Continued)

Section D: Scatter plot of style return and observed return of WLS 7

![Scatter plot](image-url)

**Section E: Histogram of monthly selection returns of WLS 7**

![Histogram](image-url)
Appendix D.16. Regression results on the Sanlam General Equity Fund (SNTR) (Continued)

Section F: Exposure distribution area graph of WLS 7
Appendix D.17. Regression results on the Sanlam Small Cap Fund (SNST)

The monthly total returns are computed from the closing price and dividend yields obtained from I-Net Bridge at the University of Cape Town. All results are monthly. For unit trusts (funds and indices), 36-month rolling periods are used to infer a fund's investment style. The analysis is for the period 1st January 1998 to 31st December 2006. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with six independent regression variables excluding the Small Cap Index or seven independent variables including the Small Cap Index. Quadratic programming (QP), as defined in Appendix D.2, is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.

Section A: Scatter plot of style return and observed return of WLS 6
The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis, without the Small Cap Index.

Section B: Histogram of monthly selection returns of WLS 6
The graph displays the monthly selection returns on SNST, without the Small Cap Index.

Section C: Exposure distribution area graph of WLS 6
The graph displays the monthly exposure of the SNST to the six selected explanatory indices, without the Small Cap Index.

Section D: Scatter plot of style return and observed return of WLS 7
The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis, with the Small Cap Index.

Section E: Histogram of monthly selection returns of WLS 7
The graph displays the monthly selection returns on SNST, with the Small Cap Index.

Section F: Exposure distribution area graph of WLS 7
The graph displays the monthly exposure of the SNST to the six selected explanatory indices, with the Small Cap Index.

Section A: Scatter plot of style return and observed return of WLS 6

![Scatter plot](image)
Appendix D.17. Regression results on the Sanlam Small Cap Fund (SNST) (Continued)

Section B: Histogram of monthly selection returns of WLS 6

Section C: Exposure distribution area graph of WLS 6
Appendix D.17. Regression results on the Sanlam Small Cap Fund (SNST) (Continued)

Section D: Scatter plot of style return and observed return of WLS 7

Section E: Histogram of monthly selection returns of WLS 7
Appendix D.17. Regression results on the Sanlam Small Cap Fund (SNST) (Continued)

Section F: Exposure distribution area graph of WLS 7

![Exposure distribution area graph of WLS 7](image-url)
Appendix D.18. Synthesising South African hedge fund indices (OLS)

The table shows the summary statistics and regression results on analysis of the South African Hedge Fund indices over the period 1st January 2004 to 31st December 2006. The four hedge fund indices examined are Single Manager Composite (COMP), Long Short Equity Index (LSE), Market Neutral and Quantitative Strategies Index (MKN) and Fund of Funds Index (FOFs). The monthly total returns are computed from the closing price and dividend yields obtained from HedgeFund Intelligence Database. All results are based on monthly effective returns. Investment style is estimated using the ordinary least square regressions (OLS) using Equation (5.1). The return-based style decompositions are conducted using Sharpe’s (1988) multi-factor regression with EW(size)100, RESW100(3), MOM(12-1)W100, Satrix RESI20, Satrix FINI15 and Satrix INDI25. R² values are obtained from the out-sample regressions of predicted style returns on actual fund returns. For hedge funds, 24-month rolling periods are used to infer a fund’s investment style. Quadratic programming (QP), as defined in Appendix D.2, is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity. If six independent regression variables are used, then the Small Cap Index is not included, else it is included. The South African 90-day Banker’s Acceptance discount rate (RBAS) is used as the risk-free rate when calculating the Sharpe Ratios. P-values are calculated using two-tailed tests. P-values significant at 10% level are indicated with *, p-values significant at 5% level are indicated with **.

<table>
<thead>
<tr>
<th>Hedge fund index code</th>
<th>COMP</th>
<th>LSE</th>
<th>MKN</th>
<th>FOFs</th>
<th>COMP</th>
<th>LSE</th>
<th>MKN</th>
<th>FOFs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Whole period statistics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Mean index return</td>
<td>1.36</td>
<td>2.30</td>
<td>0.86</td>
<td>1.37</td>
<td>1.21</td>
<td>2.31</td>
<td>0.87</td>
<td>1.14</td>
</tr>
<tr>
<td>Standard deviation of index return</td>
<td>0.92</td>
<td>2.59</td>
<td>0.43</td>
<td>1.15</td>
<td>0.81</td>
<td>2.65</td>
<td>0.49</td>
<td>1.27</td>
</tr>
<tr>
<td>Sharpe ratio of index return</td>
<td>0.52</td>
<td>0.55</td>
<td>-0.04</td>
<td>0.42</td>
<td>0.40</td>
<td>0.54</td>
<td>-0.02</td>
<td>0.20</td>
</tr>
<tr>
<td><strong>OLS CS 6 (ex-small cap)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean style return (%)</td>
<td>0.76</td>
<td>1.91</td>
<td>0.32</td>
<td>0.99</td>
<td>3.02</td>
<td>3.09</td>
<td>2.93</td>
<td>3.03</td>
</tr>
<tr>
<td>Standard deviation of style return (%)</td>
<td>1.43</td>
<td>2.44</td>
<td>0.63</td>
<td>1.46</td>
<td>4.11</td>
<td>4.07</td>
<td>4.08</td>
<td>4.08</td>
</tr>
<tr>
<td>Sharpe ratio of style return</td>
<td>-0.08</td>
<td>0.36</td>
<td>-0.89</td>
<td>0.08</td>
<td>0.52</td>
<td>0.54</td>
<td>0.50</td>
<td>0.53</td>
</tr>
<tr>
<td>Mean selection return (%)</td>
<td>0.44</td>
<td>0.39</td>
<td>0.55</td>
<td>0.14</td>
<td>-1.94</td>
<td>-0.85</td>
<td>2.20</td>
<td>2.01</td>
</tr>
<tr>
<td>Standard deviation of selection return (%)</td>
<td>0.96</td>
<td>1.24</td>
<td>0.59</td>
<td>0.54</td>
<td>3.45</td>
<td>2.03</td>
<td>3.76</td>
<td>3.02</td>
</tr>
<tr>
<td>t-selection return</td>
<td>1.59</td>
<td>1.09</td>
<td>3.19</td>
<td>0.89</td>
<td>-1.95</td>
<td>-1.44</td>
<td>-2.03</td>
<td>-2.01</td>
</tr>
<tr>
<td>p-selection return</td>
<td>0.14</td>
<td>0.30</td>
<td>0.03**</td>
<td>0.39</td>
<td>0.08*</td>
<td>0.18</td>
<td>0.07*</td>
<td>0.04**</td>
</tr>
<tr>
<td>R² (out-of-sample)</td>
<td>0.43</td>
<td>0.79</td>
<td>0.1h</td>
<td>0.81</td>
<td>0.69</td>
<td>0.77</td>
<td>0.50</td>
<td>0.80</td>
</tr>
<tr>
<td><strong>OLS CS 7 (incl-small cap)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean style return (%)</td>
<td>0.85</td>
<td>1.88</td>
<td>0.35</td>
<td>1.09</td>
<td>3.03</td>
<td>3.05</td>
<td>2.99</td>
<td>3.07</td>
</tr>
<tr>
<td>Standard deviation of style return (%)</td>
<td>1.54</td>
<td>2.84</td>
<td>0.65</td>
<td>1.53</td>
<td>4.15</td>
<td>4.13</td>
<td>4.08</td>
<td>4.17</td>
</tr>
<tr>
<td>Sharpe ratio of style return</td>
<td>-0.02</td>
<td>0.35</td>
<td>-0.82</td>
<td>0.14</td>
<td>0.52</td>
<td>0.53</td>
<td>0.52</td>
<td>0.53</td>
</tr>
<tr>
<td>Mean selection return (%)</td>
<td>0.35</td>
<td>0.41</td>
<td>0.52</td>
<td>0.04</td>
<td>-1.96</td>
<td>-0.81</td>
<td>-2.26</td>
<td>-2.05</td>
</tr>
<tr>
<td>Standard deviation of selection return (%)</td>
<td>1.16</td>
<td>1.24</td>
<td>0.63</td>
<td>0.64</td>
<td>3.50</td>
<td>2.16</td>
<td>3.76</td>
<td>3.07</td>
</tr>
<tr>
<td>t-selection return</td>
<td>1.03</td>
<td>1.16</td>
<td>2.86</td>
<td>0.24</td>
<td>-1.97</td>
<td>-1.29</td>
<td>-2.08</td>
<td>-2.31</td>
</tr>
<tr>
<td>p-selection return</td>
<td>0.33</td>
<td>0.27</td>
<td>0.02**</td>
<td>0.81</td>
<td>0.08*</td>
<td>0.23</td>
<td>0.06*</td>
<td>0.04**</td>
</tr>
<tr>
<td>R² (out-of-sample)</td>
<td>0.45</td>
<td>0.81</td>
<td>0.17</td>
<td>0.83</td>
<td>0.69</td>
<td>0.78</td>
<td>0.50</td>
<td>0.81</td>
</tr>
</tbody>
</table>
Appendix D.19. Regression results on the Single Manager Composite Index (COMP)

The monthly total returns are computed from the closing price and dividend yields obtained from HedgeFund Intelligence Database. All results are monthly. For hedge funds, 24-month rolling periods are used to infer a fund’s investment style. The analysis is for the period 1st January, 2004 to 1st December, 2006. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with seven independent regression variables including the Small Cap Index. Constrained Sum (CS) regression is used to incorporate the constraint that sum of absolute values of style weights < 12. Whereas Quadratic programming (QP), as defined in Appendix D.2, is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.

Section A: Scatter plot of style return and observed return of WLS 7 QP
The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis, using QP regressions.

Section B: Histogram of monthly selection returns of WLS 7 QP
The graph displays the monthly selection returns on COMP, using QP regressions.

Section C: Exposure distribution area graph of WLS 7 QP
The graph displays the monthly exposure of the COMP to the six selected explanatory indices, using QP regressions.

Section D: Histogram of monthly selection returns of WLS 7 CS
The graph displays the monthly selection returns on COMP, using CS regressions.

Section A: Scatter plot of style return and observed return of WLS 7 QP

Section B: Histogram of monthly selection returns of WLS 7 QP
Appendix D.19. Regression results on the Single Manager Composite Index (COMP) (Continued)

Section C: Exposure distribution area graph of WLS 7 QP

Section E: Histogram of monthly selection returns of WLS 7 CS
Appendix D.20. Regression results on the Long Short Equity Index (LSE)

The monthly total returns are computed from the closing price and dividend yields obtained from HedgeFund Intelligence Database. All results are monthly. For hedge funds, 24-month rolling periods are used to infer a fund’s investment style. The analysis is for the period 1st January, 2004 to 1st December, 2006. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with seven independent regression variables including the Small Cap Index. Constrained Sum (CS) regression is used to incorporate the constraint that sum of absolute values of style weights <12. Whereas Quadratic programming (QP), as defined in Appendix D.2, is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.

Section A: Scatter plot of style return and observed return of WLS 7 QP

The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis, using QP regressions.

Section B: Histogram of monthly selection returns of WLS 7 QP

The graph displays the monthly selection returns on LSE, using QP regressions.

Section C: Exposure distribution area graph of WLS 7 QP

The graph displays the monthly exposure of the LSE to the six selected explanatory indices, using QP regressions.

Section D: Histogram of monthly selection returns of WLS 7 CS

The graph displays the monthly selection returns on LSE, using CS regressions.

Section A: Scatter plot of style return and observed return of WLS 7 QP

![Scatter plot of style return and observed return of WLS 7 QP](image1)

Section B: Histogram of monthly selection returns of WLS 7 QP

![Histogram of monthly selection returns of WLS 7 QP](image2)
Appendix D.20. Regression results on the Long Short Equity Index (LSE) (Continued)

Section C: Exposure distribution area graph of WLS 7 QP

Section D: Histogram of monthly selection returns of WLS 7 CS
Appendix D.21. Regression results on the Market Neutral and Quantitative Strategies Index (MKN)

The monthly total returns are computed from the closing price and dividend yields obtained from HedgeFund Intelligence Database. All results are monthly. For hedge funds, 24-month rolling periods are used to infer a fund’s investment style. The analysis is for the period 1st January, 2004 to 1st December, 2006. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with seven independent regression variables including the Small Cap Index. Constrained Sum (CS) regression is used to incorporate the constraint that sum of absolute values of style weights < 12. Whereas Quadratic programming (QP), as defined in Appendix D.2, is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.

Section A: Scatter plot of style return and observed return of WLS 7 QP
The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis, using QP regressions.

Section B: Histogram of monthly selection returns of WLS 7 QP
The graph displays the monthly selection returns on MKN, using QP regressions.

Section C: Exposure distribution area graph of WLS 7 QP
The graph displays the monthly exposure of the MKN to the six selected explanatory indices, using QP regressions.

Section D: Histogram of monthly selection returns of WLS 7 CS
The graph displays the monthly selection returns on MKN, using CS regressions.

Section A: Scatter plot of style return and observed return of WLS 7 QP

![Scatter plot of style return and observed return of WLS 7 QP](image)

Section B: Histogram of monthly selection returns of WLS 7 QP

![Histogram of monthly selection returns of WLS 7 QP](image)
Appendix D.21. Regression results on the Market Neutral and Quantitative Strategies Index (MKN) (Continued)

Section C: Exposure distribution area graph of WLS 7 QP

Section E: Histogram of monthly selection returns of WLS 7 CS
Appendix D.22. Regression results on the Fund of Funds Index (FOFs)

The monthly total returns are computed from the closing price and dividend yields obtained from HedgeFund Intelligence Database. All results are monthly. For hedge funds, 24-month rolling periods are used to infer a fund’s investment style. The analysis is for the period 1st January, 2004 to 31st December, 2006. Investment style is estimated from weighted least square regressions (WLS) using Equation (5.2), with seven independent regression variables including the Small Cap Index. Constrained Sum (CS) regression is used to incorporate the constraint that sum of absolute values of style weights < 12. Whereas Quadratic programming (QP), as defined in Appendix D.2, is used to incorporate the constraints that (1) style weights lie between 0 and 1, and (2) style weights sum to unity.

Section A: Scatter plot of style return and observed return of WLS 7 QP
The graph displays the monthly style returns (the ex-post synthesised returns) on the x-axis and the monthly observed index returns on the y-axis, using QP regressions.

Section B: Histogram of monthly selection returns of WLS 7 QP
The graph displays the monthly selection returns on FOFs, using QP regressions.

Section C: Exposure distribution area graph of WLS 7 QP
The graph displays the monthly exposure of the FOFs to the six selected explanatory indices, using QP regressions.

Section D: Histogram of monthly selection returns of WLS 7 CS
The graph displays the monthly selection returns on FOFs, using CS regressions.
Appendix D.22. Regression results on the Fund of Funds Index (FOFs) (Continued)

Section C: Exposure distribution area graph of WLS 7 QP

Section E: Histogram of monthly selection returns of WLS 7 CS
Appendix E

Appendices contained in Appendix E relate to Chapter Six, Portfolio Optimisation Using Style Indices.

Appendix E.1. Correlation matrix of style indices
The table in Section A displays the correlation matrix of the excess monthly returns relative to the ALSI of the three style indices constructed and the Top 40 Index over the period 1st January 1998 to 31st December 2006. The table in Section B displays the correlation matrix of the monthly returns of the three style indices constructed, the SWIX Index and the Top 40 Index over the period 1st January 1998 to 31st December 2006.

Section A: Correlation matrix of style-index excess returns relative to ALSI

<table>
<thead>
<tr>
<th></th>
<th>EW 100</th>
<th>MOM(12-1)W 100</th>
<th>RESW 100</th>
<th>Top 40</th>
<th>SWIX</th>
</tr>
</thead>
<tbody>
<tr>
<td>EW 100</td>
<td>1.000</td>
<td>0.570</td>
<td>0.689</td>
<td>-0.713</td>
<td>0.542</td>
</tr>
<tr>
<td>MOM(12-1)W 100</td>
<td>0.570</td>
<td>1.000</td>
<td>0.305</td>
<td>-0.552</td>
<td>0.261</td>
</tr>
<tr>
<td>RESW 100</td>
<td>0.689</td>
<td>0.305</td>
<td>1.000</td>
<td>-0.565</td>
<td>0.729</td>
</tr>
<tr>
<td>Top 40</td>
<td>-0.713</td>
<td>-0.552</td>
<td>-0.565</td>
<td>1.000</td>
<td>-0.825</td>
</tr>
<tr>
<td>SWIX</td>
<td>0.542</td>
<td>0.261</td>
<td>0.729</td>
<td>-0.825</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Section B: Correlation matrix of style-index total returns

<table>
<thead>
<tr>
<th></th>
<th>EW100</th>
<th>RESW100</th>
<th>MOM(12-1)W100</th>
<th>SWIX</th>
<th>Top 40</th>
</tr>
</thead>
<tbody>
<tr>
<td>EW100</td>
<td>1.000</td>
<td>0.932</td>
<td>0.864</td>
<td>0.906</td>
<td>0.844</td>
</tr>
<tr>
<td>RESW100</td>
<td>0.932</td>
<td>1.000</td>
<td>0.777</td>
<td>0.869</td>
<td>0.808</td>
</tr>
<tr>
<td>MOM(12-1)W100</td>
<td>0.864</td>
<td>0.777</td>
<td>1.000</td>
<td>0.779</td>
<td>0.715</td>
</tr>
<tr>
<td>SWIX</td>
<td>0.906</td>
<td>0.869</td>
<td>0.779</td>
<td>1.000</td>
<td>0.981</td>
</tr>
<tr>
<td>Top 40</td>
<td>0.844</td>
<td>0.808</td>
<td>0.715</td>
<td>0.981</td>
<td>1.000</td>
</tr>
</tbody>
</table>
Appendix E.2. Returns, risk and weights of the long-only mean-variance efficient portfolios (SWIX benchmark, no shorting, no leverage)

The table displays the annualised returns, standard deviations and component weightings of the mean-variance efficient portfolios. The optimisation process is conducted using the historical return data over the period January 1998 to December 2006. The portfolio building blocks are the SWIX Index, EW(size)100 for the size style, RESW100(3) for the value style and MOM(12-1)W100 for the momentum investment style. The optimisation is conducted subject to the constraint that there is no short or leverage positions and the weights of all constituents are positive.

<table>
<thead>
<tr>
<th>Rp</th>
<th>Sp</th>
<th>Weight(SWIX)</th>
<th>Weight(size)</th>
<th>Weight(value)</th>
<th>Weight(mom)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.8%</td>
<td>22.7%</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>23.1%</td>
<td>22.1%</td>
<td>0.312</td>
<td>0.688</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>24.4%</td>
<td>21.8%</td>
<td>0.659</td>
<td>0.257</td>
<td>0.084</td>
<td>0.000</td>
</tr>
<tr>
<td>25.6%</td>
<td>21.9%</td>
<td>0.647</td>
<td>0.161</td>
<td>0.192</td>
<td>0.000</td>
</tr>
<tr>
<td>26.8%</td>
<td>22.0%</td>
<td>0.635</td>
<td>0.063</td>
<td>0.302</td>
<td>0.000</td>
</tr>
<tr>
<td>28.1%</td>
<td>22.1%</td>
<td>0.586</td>
<td>0.000</td>
<td>0.414</td>
<td>0.000</td>
</tr>
<tr>
<td>29.3%</td>
<td>22.4%</td>
<td>0.469</td>
<td>0.000</td>
<td>0.531</td>
<td>0.000</td>
</tr>
<tr>
<td>30.6%</td>
<td>22.7%</td>
<td>0.349</td>
<td>0.000</td>
<td>0.651</td>
<td>0.000</td>
</tr>
<tr>
<td>31.9%</td>
<td>23.1%</td>
<td>0.230</td>
<td>0.000</td>
<td>0.761</td>
<td>0.009</td>
</tr>
<tr>
<td>33.2%</td>
<td>23.6%</td>
<td>0.115</td>
<td>0.000</td>
<td>0.845</td>
<td>0.040</td>
</tr>
<tr>
<td>34.5%</td>
<td>24.2%</td>
<td>0.000</td>
<td>0.000</td>
<td>0.918</td>
<td>0.082</td>
</tr>
<tr>
<td>34.8%</td>
<td>26.7%</td>
<td>0.000</td>
<td>0.000</td>
<td>0.425</td>
<td>0.575</td>
</tr>
</tbody>
</table>
Appendix E.3. Returns, risk and weights of the long-short-equity mean-variance efficient portfolios (with shorting on Top 40 allowed, leverage up to 200%)

The table displays the annualised returns, standard deviations and component weightings of the mean-variance efficient portfolios. The optimisation process is conducted using the historical return data over the period January 1998 to December 2006. The portfolio building blocks are the EW(size)100 for the size style, RESW100(3) for the value style and MOM(12-1)W100 for the momentum style, and the Top 40 Index. The optimisation is conducted where shorting and leverage are allowed, therefore the constraints are the sum of the absolute weights of the indices (leverage) to be not more than 2 and the weights of the style indices to be positive.

<table>
<thead>
<tr>
<th>Rp</th>
<th>Sp</th>
<th>Weight(Top 40)</th>
<th>Weight(size)</th>
<th>Weight(value)</th>
<th>Weight(mom)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0%</td>
<td>0.0%</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>1.0%</td>
<td>0.7%</td>
<td>-0.010</td>
<td>0.000</td>
<td>0.032</td>
<td>0.005</td>
</tr>
<tr>
<td>2.0%</td>
<td>1.4%</td>
<td>-0.020</td>
<td>0.000</td>
<td>0.064</td>
<td>0.010</td>
</tr>
<tr>
<td>3.0%</td>
<td>2.2%</td>
<td>-0.030</td>
<td>0.000</td>
<td>0.097</td>
<td>0.015</td>
</tr>
<tr>
<td>4.1%</td>
<td>2.9%</td>
<td>-0.041</td>
<td>0.000</td>
<td>0.130</td>
<td>0.020</td>
</tr>
<tr>
<td>5.2%</td>
<td>3.6%</td>
<td>-0.051</td>
<td>0.000</td>
<td>0.162</td>
<td>0.026</td>
</tr>
<tr>
<td>6.2%</td>
<td>4.4%</td>
<td>-0.061</td>
<td>0.000</td>
<td>0.195</td>
<td>0.031</td>
</tr>
<tr>
<td>7.2%</td>
<td>5.1%</td>
<td>-0.072</td>
<td>0.000</td>
<td>0.229</td>
<td>0.036</td>
</tr>
<tr>
<td>8.3%</td>
<td>5.9%</td>
<td>-0.082</td>
<td>0.000</td>
<td>0.262</td>
<td>0.041</td>
</tr>
<tr>
<td>9.4%</td>
<td>6.6%</td>
<td>-0.093</td>
<td>0.000</td>
<td>0.296</td>
<td>0.047</td>
</tr>
<tr>
<td>10.5%</td>
<td>7.4%</td>
<td>-0.103</td>
<td>0.000</td>
<td>0.330</td>
<td>0.052</td>
</tr>
<tr>
<td>11.6%</td>
<td>8.2%</td>
<td>-0.114</td>
<td>0.000</td>
<td>0.364</td>
<td>0.057</td>
</tr>
<tr>
<td>12.7%</td>
<td>8.9%</td>
<td>-0.125</td>
<td>0.000</td>
<td>0.398</td>
<td>0.062</td>
</tr>
<tr>
<td>13.8%</td>
<td>9.7%</td>
<td>-0.136</td>
<td>0.000</td>
<td>0.433</td>
<td>0.068</td>
</tr>
<tr>
<td>14.9%</td>
<td>10.5%</td>
<td>-0.147</td>
<td>0.000</td>
<td>0.468</td>
<td>0.073</td>
</tr>
<tr>
<td>16.1%</td>
<td>11.2%</td>
<td>-0.158</td>
<td>0.000</td>
<td>0.502</td>
<td>0.078</td>
</tr>
<tr>
<td>17.2%</td>
<td>12.0%</td>
<td>-0.169</td>
<td>0.000</td>
<td>0.538</td>
<td>0.084</td>
</tr>
<tr>
<td>18.4%</td>
<td>12.8%</td>
<td>-0.180</td>
<td>0.000</td>
<td>0.573</td>
<td>0.089</td>
</tr>
<tr>
<td>19.6%</td>
<td>13.6%</td>
<td>-0.192</td>
<td>0.000</td>
<td>0.609</td>
<td>0.095</td>
</tr>
<tr>
<td>20.7%</td>
<td>14.4%</td>
<td>-0.203</td>
<td>0.000</td>
<td>0.645</td>
<td>0.100</td>
</tr>
<tr>
<td>21.9%</td>
<td>15.2%</td>
<td>-0.215</td>
<td>0.000</td>
<td>0.682</td>
<td>0.105</td>
</tr>
<tr>
<td>23.1%</td>
<td>16.0%</td>
<td>-0.227</td>
<td>0.000</td>
<td>0.719</td>
<td>0.111</td>
</tr>
<tr>
<td>24.4%</td>
<td>16.9%</td>
<td>-0.239</td>
<td>0.000</td>
<td>0.756</td>
<td>0.116</td>
</tr>
<tr>
<td>25.6%</td>
<td>17.7%</td>
<td>-0.251</td>
<td>0.000</td>
<td>0.793</td>
<td>0.121</td>
</tr>
<tr>
<td>26.8%</td>
<td>18.5%</td>
<td>-0.263</td>
<td>0.000</td>
<td>0.831</td>
<td>0.127</td>
</tr>
<tr>
<td>28.1%</td>
<td>19.3%</td>
<td>-0.275</td>
<td>0.000</td>
<td>0.869</td>
<td>0.132</td>
</tr>
<tr>
<td>29.3%</td>
<td>20.2%</td>
<td>-0.288</td>
<td>0.000</td>
<td>0.908</td>
<td>0.137</td>
</tr>
<tr>
<td>30.6%</td>
<td>21.0%</td>
<td>-0.301</td>
<td>0.000</td>
<td>0.947</td>
<td>0.143</td>
</tr>
<tr>
<td>31.9%</td>
<td>21.9%</td>
<td>-0.314</td>
<td>0.000</td>
<td>0.987</td>
<td>0.148</td>
</tr>
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<td>33.2%</td>
<td>22.7%</td>
<td>-0.327</td>
<td>0.000</td>
<td>1.026</td>
<td>0.153</td>
</tr>
<tr>
<td>34.5%</td>
<td>23.6%</td>
<td>-0.341</td>
<td>0.000</td>
<td>1.067</td>
<td>0.158</td>
</tr>
<tr>
<td>35.8%</td>
<td>24.5%</td>
<td>-0.354</td>
<td>0.000</td>
<td>1.107</td>
<td>0.164</td>
</tr>
<tr>
<td>37.1%</td>
<td>25.3%</td>
<td>-0.368</td>
<td>0.000</td>
<td>1.149</td>
<td>0.169</td>
</tr>
<tr>
<td>38.5%</td>
<td>26.2%</td>
<td>-0.382</td>
<td>0.000</td>
<td>1.191</td>
<td>0.174</td>
</tr>
<tr>
<td>39.8%</td>
<td>27.1%</td>
<td>-0.397</td>
<td>0.000</td>
<td>1.234</td>
<td>0.179</td>
</tr>
<tr>
<td>41.2%</td>
<td>28.0%</td>
<td>-0.411</td>
<td>0.000</td>
<td>1.277</td>
<td>0.184</td>
</tr>
<tr>
<td>42.6%</td>
<td>28.9%</td>
<td>-0.426</td>
<td>0.000</td>
<td>1.320</td>
<td>0.188</td>
</tr>
<tr>
<td>44.0%</td>
<td>29.8%</td>
<td>-0.442</td>
<td>0.000</td>
<td>1.365</td>
<td>0.193</td>
</tr>
<tr>
<td>45.4%</td>
<td>30.8%</td>
<td>-0.458</td>
<td>0.000</td>
<td>1.390</td>
<td>0.192</td>
</tr>
<tr>
<td>46.8%</td>
<td>31.7%</td>
<td>-0.472</td>
<td>0.000</td>
<td>1.415</td>
<td>0.193</td>
</tr>
<tr>
<td>48.2%</td>
<td>32.7%</td>
<td>-0.486</td>
<td>0.000</td>
<td>1.441</td>
<td>0.188</td>
</tr>
<tr>
<td>49.7%</td>
<td>33.7%</td>
<td>-0.500</td>
<td>0.000</td>
<td>1.468</td>
<td>0.186</td>
</tr>
<tr>
<td>51.1%</td>
<td>34.7%</td>
<td>-0.514</td>
<td>0.000</td>
<td>1.496</td>
<td>0.183</td>
</tr>
<tr>
<td>52.6%</td>
<td>35.8%</td>
<td>-0.528</td>
<td>0.000</td>
<td>1.525</td>
<td>0.179</td>
</tr>
<tr>
<td>54.1%</td>
<td>36.8%</td>
<td>-0.542</td>
<td>0.000</td>
<td>1.555</td>
<td>0.175</td>
</tr>
<tr>
<td>55.5%</td>
<td>37.9%</td>
<td>-0.556</td>
<td>0.000</td>
<td>1.587</td>
<td>0.170</td>
</tr>
<tr>
<td>57.1%</td>
<td>39.0%</td>
<td>-0.571</td>
<td>0.000</td>
<td>1.620</td>
<td>0.164</td>
</tr>
<tr>
<td>58.6%</td>
<td>40.2%</td>
<td>-0.586</td>
<td>0.000</td>
<td>1.654</td>
<td>0.156</td>
</tr>
<tr>
<td>60.1%</td>
<td>41.4%</td>
<td>-0.601</td>
<td>0.000</td>
<td>1.690</td>
<td>0.149</td>
</tr>
<tr>
<td>61.6%</td>
<td>42.6%</td>
<td>-0.616</td>
<td>0.000</td>
<td>1.728</td>
<td>0.139</td>
</tr>
<tr>
<td>63.2%</td>
<td>43.8%</td>
<td>-0.631</td>
<td>0.000</td>
<td>1.768</td>
<td>0.129</td>
</tr>
<tr>
<td>64.8%</td>
<td>45.1%</td>
<td>-0.646</td>
<td>0.000</td>
<td>1.812</td>
<td>0.115</td>
</tr>
</tbody>
</table>
Appendix E.4. Returns, risk and weights of the market neutral mean-variance efficient portfolios (with shorting on Top 40 allowed, leverage up to 200%)

The table displays the annualised returns, standard deviations and component weightings of the mean-variance efficient portfolios. The optimisation process is conducted using the historical return data over the period January 1998 to December 2006. The portfolio building blocks are the EW(size)100 for the size style, RESW100(3) for the value style and MOM(12-1)W100 for the momentum style, and the Top 40 Index. The optimisation is conducted where shorting and leverage are allowed, therefore the constraints are the sum of the absolute weights of the indices (leverage) to be not more than 2 and the weights of the style indices to be positive.

<table>
<thead>
<tr>
<th>Rp</th>
<th>Sp</th>
<th>Weight(Top 40)</th>
<th>Weight(size)</th>
<th>Weight(value)</th>
<th>Weight(mom)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0%</td>
<td>0.0%</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>1.0%</td>
<td>1.4%</td>
<td>-0.093</td>
<td>0.000</td>
<td>0.065</td>
<td>0.029</td>
</tr>
<tr>
<td>2.0%</td>
<td>2.8%</td>
<td>-0.189</td>
<td>0.000</td>
<td>0.131</td>
<td>0.058</td>
</tr>
<tr>
<td>3.0%</td>
<td>4.2%</td>
<td>-0.286</td>
<td>0.000</td>
<td>0.198</td>
<td>0.088</td>
</tr>
<tr>
<td>4.1%</td>
<td>5.7%</td>
<td>-0.385</td>
<td>0.000</td>
<td>0.267</td>
<td>0.118</td>
</tr>
<tr>
<td>5.1%</td>
<td>7.2%</td>
<td>-0.487</td>
<td>0.000</td>
<td>0.338</td>
<td>0.149</td>
</tr>
<tr>
<td>6.2%</td>
<td>8.7%</td>
<td>-0.590</td>
<td>0.000</td>
<td>0.409</td>
<td>0.181</td>
</tr>
<tr>
<td>7.2%</td>
<td>10.2%</td>
<td>-0.696</td>
<td>0.000</td>
<td>0.484</td>
<td>0.213</td>
</tr>
<tr>
<td>8.3%</td>
<td>11.8%</td>
<td>-0.805</td>
<td>0.000</td>
<td>0.559</td>
<td>0.246</td>
</tr>
<tr>
<td>9.4%</td>
<td>13.5%</td>
<td>-0.916</td>
<td>0.000</td>
<td>0.637</td>
<td>0.279</td>
</tr>
<tr>
<td>10.4%</td>
<td>15.5%</td>
<td>-1.000</td>
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<td>0.447</td>
</tr>
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<td>16.4%</td>
<td>-1.000</td>
<td>0.000</td>
<td>0.441</td>
<td>0.559</td>
</tr>
<tr>
<td>10.8%</td>
<td>19.6%</td>
<td>-1.000</td>
<td>0.000</td>
<td>0.160</td>
<td>0.840</td>
</tr>
</tbody>
</table>
Appendix E.5. Returns, risk and weights of the market neutral mean-variance efficient portfolios (with shorting on Top 40 allowed, leverage fixed at 100%)

The table in Section A displays the annualised returns, standard deviations and component weightings of the mean-variance efficient portfolios. The optimisation process is conducted using the historical return data over the period January 1998 to December 2006. The portfolio building blocks are the EW(size)100 for the size style, RESW100(3) for the value style and MOM(12-1)W100 for the momentum style, and the Top 40 Index. The optimisation is conducted where shorting and leverage are allowed, therefore the constraints are the sum of the absolute weights of the indices to be fixed at 100% and the weights of the style indices to be positive. The graph in Section B displays efficient frontier of the minimum variance portfolios subject to specified total portfolio returns. The graph in Section C shows the change in the weights of different indices to form the minimum variance portfolios as the specified constraint value of total portfolio return changes.

Section A: Annualised returns, standard deviations and component weightings of the mean-variance efficient portfolios

<table>
<thead>
<tr>
<th>Rp</th>
<th>Sp</th>
<th>Weight(Top 40)</th>
<th>Weight(size)</th>
<th>Weight(value)</th>
<th>Weight(mom)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9%</td>
<td>6.5%</td>
<td>-0.500</td>
<td>0.425</td>
<td>0.069</td>
<td>0.000</td>
</tr>
<tr>
<td>1.0%</td>
<td>6.4%</td>
<td>-0.500</td>
<td>0.406</td>
<td>0.094</td>
<td>0.000</td>
</tr>
<tr>
<td>2.0%</td>
<td>6.5%</td>
<td>-0.500</td>
<td>0.301</td>
<td>0.187</td>
<td>0.012</td>
</tr>
<tr>
<td>3.0%</td>
<td>6.7%</td>
<td>-0.500</td>
<td>0.201</td>
<td>0.258</td>
<td>0.041</td>
</tr>
<tr>
<td>4.1%</td>
<td>6.9%</td>
<td>-0.500</td>
<td>0.101</td>
<td>0.329</td>
<td>0.070</td>
</tr>
<tr>
<td>5.1%</td>
<td>7.2%</td>
<td>-0.500</td>
<td>0.000</td>
<td>0.398</td>
<td>0.102</td>
</tr>
<tr>
<td>5.9%</td>
<td>10.9%</td>
<td>-0.500</td>
<td>0.000</td>
<td>0.000</td>
<td>0.500</td>
</tr>
</tbody>
</table>

Section B: Efficient frontier of the minimum-variance efficient portfolios (Top 40 benchmark, market neutral strategy, leverage at 100%)
Appendix E.5. Returns, risk and weights of the market neutral mean-variance efficient portfolios (with shorting on Top 40 allowed, leverage fixed at 100%) (Continued)

Section C: Weights of the mean-variance efficient portfolios (Top 40 benchmark, market neutral strategy, leverage at 100%)

![Diagram showing weights and portfolio returns]

Legend:
- Top 40
- EW(size)100
- RESW100(3)
- MOM(12-1)W100

Portfolio Return (Annualised)
Appendix E.6. Returns, risk and weights of the market neutral mean-variance efficient portfolios (with shorting on Top 40 allowed, leverage fixed at 200%)

The table in Section A displays the annualised returns, standard deviations and component weightings of the mean-variance efficient portfolios. The optimisation process is conducted using the historical return data over the period January 1998 to December 2006. The portfolio building blocks are the EW(size)100 for the size style, RESW100(3) for the value style and MOM(12-l)W100 for the momentum style, and the Top 40 Index. The optimisation is conducted where shorting and leverage are allowed, therefore the constraints are the sum of the absolute weights of the indices to be fixed at 200% and the weights of the style indices to be positive. The graph in Section B displays efficient frontier of the minimum variance portfolios subject to specified total portfolio returns. The graph in Section C shows the change in the weights of different indices to form the minimum variance portfolios as the specified constraint value of total portfolio return changes.

Section A: Annualised returns, standard deviations and component weightings of the mean-variance efficient portfolios

<table>
<thead>
<tr>
<th>Rp</th>
<th>Sp</th>
<th>Weight(Top 40)</th>
<th>Weight(size)</th>
<th>Weight(value)</th>
<th>Weight(mom)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0%</td>
<td>13.0%</td>
<td>-1.000</td>
<td>0.985</td>
<td>0.000</td>
<td>0.015</td>
</tr>
<tr>
<td>1.0%</td>
<td>13.2%</td>
<td>-1.000</td>
<td>0.899</td>
<td>0.000</td>
<td>0.101</td>
</tr>
<tr>
<td>2.0%</td>
<td>13.6%</td>
<td>-1.000</td>
<td>0.812</td>
<td>0.000</td>
<td>0.188</td>
</tr>
<tr>
<td>3.0%</td>
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<td>0.661</td>
<td>0.333</td>
<td>0.006</td>
</tr>
<tr>
<td>4.1%</td>
<td>13.1%</td>
<td>-1.000</td>
<td>0.561</td>
<td>0.404</td>
<td>0.036</td>
</tr>
<tr>
<td>5.1%</td>
<td>13.3%</td>
<td>-1.000</td>
<td>0.476</td>
<td>0.475</td>
<td>0.040</td>
</tr>
<tr>
<td>6.2%</td>
<td>13.5%</td>
<td>-1.000</td>
<td>0.358</td>
<td>0.547</td>
<td>0.095</td>
</tr>
<tr>
<td>7.2%</td>
<td>13.7%</td>
<td>-1.000</td>
<td>0.256</td>
<td>0.620</td>
<td>0.124</td>
</tr>
<tr>
<td>8.3%</td>
<td>14.0%</td>
<td>-1.000</td>
<td>0.153</td>
<td>0.693</td>
<td>0.154</td>
</tr>
<tr>
<td>9.4%</td>
<td>14.3%</td>
<td>-1.000</td>
<td>0.050</td>
<td>0.766</td>
<td>0.184</td>
</tr>
<tr>
<td>10.5%</td>
<td>21.4%</td>
<td>-1.000</td>
<td>0.029</td>
<td>0.777</td>
<td>0.192</td>
</tr>
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</table>

Section B: Efficient frontier of the minimum-variance efficient portfolios (Top 40 benchmark, market neutral strategy, leverage at 200%)
Appendix E.6. Returns, risk and weights of the market neutral mean-variance efficient portfolios (with shorting on Top 40 allowed, leverage fixed at 200%) (Continued)

Section C: Weights of the mean-variance efficient portfolios (Top 40 benchmark, market neutral strategy, leverage at 200%)

![Bar chart showing weights and portfolio returns](image-url)
Appendix E.7. Returns, risk and weights of the mean-tracking error efficient portfolios (SWIX benchmark, no shorting, no leverage)

The table displays the annualised returns, standard deviations and component weightings of the mean-variance efficient portfolios. The optimisation process is conducted using the historical return data over the period January 1998 to December 2006. The portfolio building blocks are the EW(size) for the size style, RESW100(3) for the value style and MOM(12-l)W100 for the momentum style, and the SWIX Index. The optimisation is conducted subject to the constraint that there is no shorting or leverage positions.

<table>
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<th>Sp</th>
<th>Weight(SWIX)</th>
<th>Weight(size)</th>
<th>Weight(value)</th>
<th>Weight(mom)</th>
</tr>
</thead>
<tbody>
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<td>0.0 %</td>
<td>23.5 %</td>
<td>22.0 %</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.5 %</td>
<td>24.0 %</td>
<td>22.0 %</td>
<td>0.956</td>
<td>0.000</td>
<td>0.033</td>
<td>0.011</td>
</tr>
<tr>
<td>1.0 %</td>
<td>24.4 %</td>
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<td>0.000</td>
<td>0.067</td>
<td>0.021</td>
</tr>
<tr>
<td>1.5 %</td>
<td>24.9 %</td>
<td>22.1 %</td>
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<td>0.000</td>
<td>0.099</td>
<td>0.032</td>
</tr>
<tr>
<td>2.0 %</td>
<td>25.4 %</td>
<td>22.1 %</td>
<td>0.826</td>
<td>0.000</td>
<td>0.132</td>
<td>0.043</td>
</tr>
<tr>
<td>2.5 %</td>
<td>25.8 %</td>
<td>22.1 %</td>
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<td>0.000</td>
<td>0.165</td>
<td>0.053</td>
</tr>
<tr>
<td>3.0 %</td>
<td>26.3 %</td>
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</tr>
<tr>
<td>3.5 %</td>
<td>26.7 %</td>
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<td>0.000</td>
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<tr>
<td>4.0 %</td>
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<tr>
<td>5.5 %</td>
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<td>0.363</td>
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</tr>
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<tr>
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<td>0.728</td>
<td>0.231</td>
</tr>
<tr>
<td>11.5 %</td>
<td>34.3 %</td>
<td>24.6 %</td>
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<td>0.256</td>
</tr>
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<td>12.0 %</td>
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<td>0.551</td>
</tr>
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<td>0.000</td>
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<td>0.597</td>
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<td>34.5 %</td>
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<td>0.000</td>
<td>0.361</td>
<td>0.639</td>
</tr>
<tr>
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<td>27.6 %</td>
<td>0.000</td>
<td>0.000</td>
<td>0.321</td>
<td>0.679</td>
</tr>
<tr>
<td>15.5 %</td>
<td>34.5 %</td>
<td>28.0 %</td>
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<td>0.000</td>
<td>0.282</td>
<td>0.718</td>
</tr>
<tr>
<td>16.0 %</td>
<td>34.5 %</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.245</td>
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</tr>
<tr>
<td>16.5 %</td>
<td>34.6 %</td>
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<td>0.000</td>
<td>0.209</td>
<td>0.791</td>
</tr>
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<td>0.000</td>
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<td>34.6 %</td>
<td>29.5 %</td>
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<td>0.141</td>
<td>0.859</td>
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<td>0.107</td>
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<td>0.011</td>
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