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Transmit Antenna Selection in Fading Wireless Communication Systems

Mzabalazo Lupupa

This thesis is submitted in fulfilment of the academic requirements
for the degree of
Master of Science in Electrical Engineering
in the Faculty of Engineering and The Built Environment
University of Cape Town
2009
As the candidate’s supervisor, I have approved this dissertation for submission.

Name: Mqhele E. Dlodlo, PhD, Associate Professor

Signed: ______________________

Date: ________________________
Declaration

I declare that this thesis is my own work. Where collaboration with other people has taken place, or material generated by other researchers is included, the parties and/or materials are indicated in the acknowledgements or are explicitly stated with references as appropriate.

This work is being submitted for the Master of Science in Electrical Engineering at the University of Cape Town. It has not been submitted to any other university for any other degree or examination.

Mzabalazo Lupupa

________________________________________
Name                                                  Date
Dedication

To my family, relatives and friends.
Abstract

The use of multiple transmitting and multiple receiving antennas in wireless communication systems plays a great role in improving system performance. Nevertheless, such an improvement comes at the cost of increased system complexity. Due to the large number of radio frequency (RF) chains to be decoded at the receiver end and the high price in system hardware design.

To address the drawbacks associated with multiple-input multiple-output (MIMO) systems, we propose the use of the transmit antenna selection technique. In transmit antenna selection, the best performing antenna(s) is selected from all the available transmit antennas for transmission. Transmit antenna selection reduces the number of radio frequency chains, and the system complexity while still achieving the goals of multiple antenna systems. In this thesis the performance of a MIMO system employing transmit antenna selection and maximal-ratio-combining is studied. The proposed system model selects the single best performing transmit antenna from all the available antennas and uses this for communication. The unselected transmit antennas are put on sleep mode until the next selection cycle. The selected transmit antenna is the one that maximises the SNR at the receiver end. Two antennas are used at the receiver end to receive the signal from the selected antenna. The number of the receiving antennas is restricted to two, due to the power and size limitations of hand-held wireless communication devices. Communication through the selected transmit antenna continues until the signal level falls to a set threshold. When this happens all the other transmit antennas are reactivated for transmission and the selection criteria starts all over. The main objective of this study is to investigate the signal-to-noise ratio, amount of fading (AoF), average channel capacity, average bit-error probability and outage probability performance of the proposed system under Weibull fading channels in the downlink. The obtained results are then compared with those of the full complexity MIMO system and the single-input single-output (SISO) system.

In investigating the performance of the transmit antenna selective MIMO system, graphs of the Weibull fading parameter against amount of fading, average signal-to-noise ratio (SNR) against average channel capacity, average SNR against average bit-error probability and average
SNR against outage probability were plotted for different correlation coefficients and fading parameters. According to the plots, system performance improves as the number of transmit antennas increases. The SISO system, which is the simplest, is observed to be performing poorly compared to the full complexity MIMO and transmit antenna selective MIMO systems.

Regardless of the total number of available transmit antennas, the proposed transmit antenna selective MIMO system always has two branches to combine for decoding compared to the full complexity MIMO system. In the full complexity MIMO system, the number of RF branches increase with an increase in the number of available transmit antennas. It can then be concluded that the transmit antenna selective MIMO system helps bridge the gap between SISO and MIMO systems by providing inexpensive reliable communication with reduced complexity in wireless systems.
Acknowledgements

My sincere appreciation goes to my supervisor A/Prof. M. E. Dlodlo for his guidance, support and encouragement during the course of my study at the University of Cape Town.

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Most of all, thank you Dear Lord for making me see this work through.
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<th>Description</th>
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<tbody>
<tr>
<td>MIMO</td>
<td>Multiple-input multiple-output</td>
</tr>
<tr>
<td>SIMO</td>
<td>Single-input multiple-output</td>
</tr>
<tr>
<td>MISO</td>
<td>Multiple-input single-output</td>
</tr>
<tr>
<td>SISO</td>
<td>Single-input single-output</td>
</tr>
<tr>
<td>MRC</td>
<td>Maximal-ratio-combining</td>
</tr>
<tr>
<td>SC</td>
<td>Selective combining</td>
</tr>
<tr>
<td>EGC</td>
<td>Equal-gain-combining</td>
</tr>
<tr>
<td>BER</td>
<td>Bit-error-rate</td>
</tr>
<tr>
<td>BEP</td>
<td>Bit-error-probability</td>
</tr>
<tr>
<td>SER</td>
<td>Symbol-error-rate</td>
</tr>
<tr>
<td>STBC</td>
<td>Space-time block codes</td>
</tr>
<tr>
<td>STTC</td>
<td>Space-time trellis codes</td>
</tr>
<tr>
<td>BLAST</td>
<td>Bell Labs Layered Space-Time Architecture</td>
</tr>
<tr>
<td>VBLAST</td>
<td>Vertical Bell Labs Layered Space-Time Architecture</td>
</tr>
<tr>
<td>DBLAST</td>
<td>Diagonal Bell Labs Layered Space-Time Architecture</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-noise ratio</td>
</tr>
<tr>
<td>AoF</td>
<td>Amount of fading</td>
</tr>
<tr>
<td>BW</td>
<td>Bandwidth</td>
</tr>
<tr>
<td>GSM</td>
<td>Global System for Mobile Communications</td>
</tr>
<tr>
<td>GPRS</td>
<td>General Packet Radio Service</td>
</tr>
<tr>
<td>UMTS</td>
<td>Universal Mobile Telecommunications System</td>
</tr>
<tr>
<td>EDGE</td>
<td>Enhanced Data Rates for GSM Evolution</td>
</tr>
<tr>
<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-----------------------------------</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel state information</td>
</tr>
<tr>
<td>ML</td>
<td>Maximum-likelihood</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability density function</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative distribution function</td>
</tr>
<tr>
<td>MGF</td>
<td>Moment generating function</td>
</tr>
<tr>
<td>RF</td>
<td>Radio frequency</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>IID</td>
<td>Independent Identically Distributed</td>
</tr>
<tr>
<td>RV</td>
<td>Random variable</td>
</tr>
</tbody>
</table>
## Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$\gamma$</td>
<td>Instantaneous SNR</td>
</tr>
<tr>
<td>$\bar{\gamma}$</td>
<td>Average SNR</td>
</tr>
<tr>
<td>$K$</td>
<td>Rician fading parameter</td>
</tr>
<tr>
<td>$m$</td>
<td>Nakagami fading parameter</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Weibull fading parameter</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Average power</td>
</tr>
<tr>
<td>$I_0$</td>
<td>Modified Bessel function of the first kind and zero order</td>
</tr>
<tr>
<td>$\Gamma(\cdot)$</td>
<td>Gamma function</td>
</tr>
<tr>
<td>$E(\cdot)$</td>
<td>Statistical expectation</td>
</tr>
<tr>
<td>$H$</td>
<td>MIMO channel matrix</td>
</tr>
<tr>
<td>$y$</td>
<td>Received signal vector</td>
</tr>
<tr>
<td>$x$</td>
<td>Transmitted signal vector</td>
</tr>
<tr>
<td>$n$</td>
<td>Additive White Gaussian noise vector</td>
</tr>
<tr>
<td>$n_r$</td>
<td>Number of receive antennas</td>
</tr>
<tr>
<td>$n_t$</td>
<td>Number of transmit antennas</td>
</tr>
<tr>
<td>$h_{ij}$</td>
<td>Channel coefficient between $i_{th}$ receiving antenna and $j_{th}$ transmitting antenna</td>
</tr>
<tr>
<td>$Z$</td>
<td>Weibull random variable</td>
</tr>
<tr>
<td>$f(\cdot)$</td>
<td>Probability density function</td>
</tr>
<tr>
<td>$F(\cdot)$</td>
<td>Cumulative distribution function</td>
</tr>
<tr>
<td>$\mathcal{F}_2(\cdot,\cdot,\cdot)$</td>
<td>Gauss Hypergeometric function</td>
</tr>
</tbody>
</table>
\[ Q_i(z) \] Marcum-Q function
CHAPTER 1

1 Introduction

1.1 Motivation

There is need for reliable and affordable services in wireless communication; providing services without any extension in the bandwidth usage. However, the achievement of such is greatly affected by fading and multipath propagation. These conditions make it difficult to decode the received signals at the receiver end due to the reduced signal-to-noise ratio (SNR) and the increased probability of error.

Previous research has shown that using techniques such as multiple transmitting and multiple receiving antennas can significantly improve the system’s performance as this helps in mitigating the effects of fading and multipath propagation. This led to the development of systems referred to as multiple-input multiple-output (MIMO) systems. Here, the signals are transmitted and received using several antennas. The problem associated with MIMO systems is increased systems complexity and cost [1]. An increase in the number of transmitting and receiving antennas used results in an increase in the number of radio frequency (RF) chains to be decoded at the receiver end, thus leading to decoder complexity. This problem of system complexity can be addressed by the use of simplified coding techniques, namely the space-time block (STB) codes. In addition to simplified coding techniques, antenna selection can also be introduced to the system [2, 3]. The idea with antenna selection is to select the best performing antenna(s) from all the available antennas and only activate this selected antenna for transmission and/or reception of signals. Antenna selection may be applicable at the transmitter and/or receiver end. In this study transmit antenna selection is considered. This is due to the practicality in having antenna diversity at the transmitter side (base station) as compared to the receiver side of hand-held wireless communication devices, which only allows for a limited number of antennas to be used.
In studying the performance of wireless communication systems the Rayleigh, Nakagami and Rician fading channels have been widely considered. In this work, the Weibull fading channel is considered and its effects on the system's performance investigated. Using the Weibull distribution as a statistical model for modelling wireless indoor and outdoor channels has been proposed for a long time. Recent work by researchers through experimental data has proved the accuracy of using the Weibull distribution in modelling multipath fading [4]. The Weibull distribution also provides flexibility in describing the fading severity of the channel and considers special cases such as the well-known Rayleigh fading for a certain value of the fading parameter [5]. A study conducted in Rio de Janeiro, Brazil has also shown that the variation of the signals on small areas generally follows a Weibull distribution thus its correctness in modelling fading [6]. Moreover, the IEEE Vehicular Technology Society Committee on Radio Propagation recommended the use of a Weibull or a Nakagami model to compensate for shortcomings of the Rayleigh model [6].

Having considered transmit antenna selection, the signals are then combined using maximal-ratio-combining (MRC) at the receiver end. Due to the small size and power limitations of hand-held wireless communication devices, the number of receiving antennas is limited, and in this study is restricted to a maximum of two antennas. The reason behind using MRC is that it has been observed to outperform all the other combining techniques, namely selective combining (SC) and equal-gain-combining (EGC), but it is a bit complex [7, 8]. Though more complex than the other combining techniques, MRC would help compensate for any losses in performance due to transmit antenna selection. Even then, the complexity is not so pronounced due to the reduced number of diversity branches to be combined. To be specific, only two diversity branches need to be combined in our proposed transmit antenna selective MIMO system. Having applied transmit antenna selection and MRC, the performance of the proposed system will then be investigated. Performance parameters to be investigated are the signal-to-noise ratio, amount of fading, average channel capacity, average bit-error probability and outage probability.
1.2 Background Information

Enhanced communication in wireless systems has been of great interest over the past years. This means providing multimedia and data services at improved rates and/ or quality. Currently, Global System for Mobile Communications (GSM) operators provide data rates of 9.6 Kbps, General Packet Radio Service (GPRS) and Enhanced Data Rates for GSM Evolution (EDGE) provide 92 Kbps and 384 Kbps respectively, whereas the target in Universal Mobile Telecommunications System (UMTS) is 2Mbps and above [1]. References [1, 9] agree that the major drawback in achieving reliable communication in wireless systems is fading due to multipath propagation. There are two major effects of such fading. 1) Signal strength is attenuated, leading to a simple loss in $E_b/N_0$ (degraded error performance). 2) Channel-induced intersymbol interference (ISI) causes distortion and possibly an irreducible BER. The first effect can be mitigated with more $E_b/N_0$. But more $E_b/N_0$ will not necessarily help remove the degrading effects of ISI. Here, mitigation usually calls for an equalizer to remove the distortion.

To address these drawbacks, diversity and coding can be introduced to the system as discussed in [1]. Diversity can be divided into frequency, time and antenna diversity amongst others. The focus in [1, 9] was to improve the system’s performance with the use of antenna diversity. In antenna diversity, several antennas are used at the transmitter and/or the receiver end. Early work on multiple antennas at the transmitter was done by Guey et al., and Tarokh et al. introduced space-time codes (STC) for multiple transmit antennas. They were able to illustrate that space-time codes were characterised by coding and diversity gains. Diversity gain being based on the fact that it is most unlikely that several antennas are in a fading dip simultaneously [10]. They were also able to demonstrate that the performance of space-time code depended on the number of transmitters and receivers in the system. Within the space-time codes, Tarokh et al. proposed space-time trellis (STT) codes. STT codes performed extremely well but at the cost of high complexity [9]. To address the issue of complexity, Alamouti introduced a scheme that used two transmit antennas and a generalised number of receive antennas [9]. Although this is a simple rate 1 scheme, it only provides diversity gain (not coding gain), and thus operates at compromised performance compared to the STT codes. Tarokh et al.
then generalised the Alamouti scheme to any number of transmitting antennas and this gave birth to other space-time block (STB) codes. Foschini later developed another scheme called Bell Labs Layered Space-Time Architecture (BLAST). The performance of different space-time codes is studied in [11].

Higher diversity can be achieved if more transmit antennas are used in the wireless communication system. The signals from these antennas can then be combined at the receiver end using diversity combining techniques, namely selective combining (SC), maximum-ratio-combining (MRC) and equal-gain-combining (EGC), where MRC is observed to outperform all the other combining techniques [12]. In hand-held wireless communication devices the number of receive antennas that can be used at the receiver end is limited due to the power limitations and size of such devices. Thus, in this case antenna diversity is best applicable at the transmitter end where the base station is able to accommodate several transmitting antennas [2, 9]. Another advantage of transmit diversity is the simplicity of its implementation as compared to receive antenna diversity [13]. The base station also provides several services to the mobile stations, thus it would be economically viable to use antenna diversity at the base station [9].

The problem with antenna diversity is that it increases the system’s complexity and cost [1, 2]. An increase in the number of antennas used results in an increase in the number of radio frequency (RF) chains that the receiver has to decode, leading to decoder complexity. There is a general view between [1-3] that antenna selection can be introduced to the system to help solve the complexity problem. The idea with antenna selection is to select the best performing antenna(s) from all the available antennas. Antenna selection can be applicable at the transmitter and/or receiver end.

Antenna selection considered in [1] is applied only at the receiver end where the receive antenna(s) with the best signal-to-noise ratio (SNR) is selected. In this correspondence, the effect of antenna selection on the pairwise error probability is investigated. Reference [7] studied the effect of antenna selection in terms of the channel capacity. They were able to show that only a small loss in capacity is experienced when only the receivers with the best SNR are selected from all the available receiving antennas.
In [14] antenna selection for STB codes is considered at the transmitter end. Through computer simulations, they were able to demonstrate that the system's performance improved by increasing the number of available transmit antenna whilst keeping the number of selected antennas fixed. By comparison, [11] investigates the performance of a wireless system with antenna selection applied at both the transmitter and the receiver ends. The receiver(s) with the best SNR is selected and the transmitter with the largest transmit power (as determined by the feedback information) is selected. Here the system’s performance was studied in terms of the bit-error-rate.

The focus of this study will be on the downlink performance analysis of a transmit antenna selective MIMO system that selects a single best performing transmit antenna, and uses two antennas at the receiver end. The main reason in selecting the single best transmitting antenna is to reduce the number of RF chains at the decoder end thus leading to a less complex and cheaper system design. We are also avoiding selecting two transmit antennas as this would represent the Alamouti scheme which has been extensively studied in literature. The receiving antennas are restricted to two due to battery life and size of hand-held wireless communication devices. To compensate for any losses due to transmit antenna selection, maximal-ratio-combining technique is used as the best combining technique [12] at the receiver end.

1.3 Objectives

By using MIMO systems, the goals of next generation networks of providing improved capacity, data rates and reliable communication can be achieved. The project hypothesis is thus stated.

Transmit antenna selection is a technique that can be used in wireless communication to help bridge the gap between SISO and MIMO systems by providing inexpensive, reliable communication, in which inexpensive is associated with SISO systems and reliable with MIMO systems.
In verifying the above stated hypothesis, the following project objectives will be considered.

1. Study the performance a transmit antenna selective MIMO system in terms of the average SNR, amount of fading, average channel capacity, average bit-error probability, and outage probability in a Weibull fading channel.
2. Study the performance a full complexity MIMO in terms of the average SNR, amount of fading, average channel capacity, average bit-error probability, and outage probability in a Weibull fading channel.
3. Compare the performance parameters of the full complexity MIMO, transmit antenna selective MIMO and SISO systems.

1.4 List of Publications

Below is a list of conference papers that have resulted from this thesis.


1.5 Organisation of Thesis

The rest of this thesis is organised as follows.

Chapter 2 introduces wireless fading, and the different fading channels are discussed with much emphasis on the Weibull fading channel, which is our channel of interest. In Chapter 3, an overview of the different types of diversity techniques is given. This is extended to antenna diversity, which then leads to the discussion on MIMO systems. The chapter goes on to explain the different types of diversity combining techniques, namely selection combining, equal-gain-combining and maximal-ratio-combining. Chapter 4 focuses on the transmit antenna selective MIMO system. This is the proposed system model. The five performance parameters: the average signal-to-noise ratio, amount of fading, average channel capacity, average bit-error probability and outage probability are presented. The different performance equations are derived for both independent and correlated Weibull fading channels. The results and the analysis of the five performance metrics are presented in Chapter 5, where the transmit antenna selective MIMO system performance results are then compared with those of the SISO and MIMO systems. Chapter 6 concludes the dissertation and highlights future.
CHAPTER 2

2 Wireless Fading and Fading Channels

2.1 Introduction

Fading which is a major concern in wireless communication is introduced. The different fading scenarios are explained in relation to time and frequency. Having discussed fading, the various types of fading channels: Rayleigh, Rician and Nakagami, are then presented, paying particular attention to the Weibull fading channels. Equations for the Weibull random variable, PDF and CDF are then illustrated.

2.2 Wireless Fading

As the signal propagates through the channel, the effects of fading can lead to a loss in SNR, or even worse, signal distortion. This distortion may vary in time, frequency and location [15]. The chief contributors of fading are multipath propagation and absorption. In multipath propagation the sent signals get diffracted or reflected on building, mountains, trees and water before reaching the receiver. At the receiver end, the signals that are received in-phase reinforce with each other to produce a stronger signal, whilst those that are received out of phase add up destructively. In absorption, there are energy losses at the receiver end. This makes it challenging to analyse the received signals and as a result fading forms a major cause for concern in wireless communication [16].

2.2.1 Slow and Fast Fading

Wireless fading channels can be classified into slow or fast fading channels, depending on the coherence time of the channel. Slow fading is when the coherence time is larger than the symbol time duration. Fast fading is when the coherence time is smaller than the symbol time
duration. In fast fading, the channel changes frequently over time. Slow fading helps in simplifying the system’s complexity as repeated channel estimation is not necessary. This is because in slow fading the channel is assumed not to be frequently changing with time compared to fast fading [17].

2.2.2 Flat and Frequency Selective Fading

Flat fading occurs when the delay spread is strictly less than the symbol time duration. On the other hand, frequency selective fading occurs when the delay spread is greater than the symbol time duration. In this, the received signal has multiple echoes of the transmitted signal that are attenuated and delayed in time. These might introduce inter-symbol interference in the system [18, 19].

Slow and flat fading is considered in this thesis. Having discussed some key generalities of wireless fading, we next proceed to discuss some particular fading channels in a wireless communication system.

2.3 Fading Channels

Fading channels are communication channels that experience fading. Transmitted signals propagate through these channels. They can be modelled using different fading models, namely the Rayleigh, Rician, Nakagami and Weibull.

2.3.1 Rayleigh Fading Channel

Rayleigh fading is a reasonable model when there are many objects in the environment that scatter the radio signal before it arrives at the receiver. It is mostly used when there is no direct line-of-sight (LOS) path between the transmitter and the receiver, especially in densely populated cities [20].
2.3.2 Rician Fading Channel

This is similar to Rayleigh fading except that there exists a line-of-sight signal between the transmitter and the receiver, which happens to be stronger than the other signals [21].

2.3.3 Nakagami Fading Channel

This is used when the fading channel does not fit any of the previously discussed fading channels, Nakagami fading may give an improved fit to the data. The advantage of the Nakagami fading is that it can be reduced to Rayleigh fading and can model fading conditions more severe or less severe than those in the Rayleigh case.

The statistics of the symbol SNR for the above fading channel models are summarised in Table 2.1 [22].
Table 2.1: Statistics of the symbol SNR for various fading channel models

<table>
<thead>
<tr>
<th>Type of fading</th>
<th>Fading Parameter</th>
<th>PDF, $f_\gamma(\gamma)$</th>
<th>CDF, $F_\gamma(s)$</th>
<th>MGF, $M_\gamma(s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rayleigh</td>
<td></td>
<td>$\frac{1}{\bar{\gamma}} \exp \left( -\frac{\gamma}{\bar{\gamma}} \right)$</td>
<td>$1 - \exp \left( -\frac{\gamma}{\bar{\gamma}} \right)$</td>
<td>$(1 - s\bar{\gamma})^{-1}$</td>
</tr>
<tr>
<td>Rician</td>
<td>$K \geq 0$</td>
<td>$\frac{(1+K)e^{-K}}{\bar{\gamma}} \times \exp \left( -\frac{(1+K)\gamma}{\bar{\gamma}} \right)$</td>
<td>$1 - Q_1 \left[ \sqrt{2K}, \sqrt{\frac{2(1+K)\gamma}{\bar{\gamma}}} \right]$</td>
<td>$\frac{1+K}{1+K-s\bar{\gamma}} \times \exp \left( \frac{s\bar{\gamma}K}{1+K-s\bar{\gamma}} \right)$</td>
</tr>
<tr>
<td>Nakagami</td>
<td>$m \geq \frac{1}{2}$</td>
<td>$\frac{\left( \frac{m}{\bar{\gamma}} \right)^m \gamma^{m-1}}{\Gamma(m)} \times \exp \left( -\frac{m\gamma}{\bar{\gamma}} \right)$</td>
<td>$1 - \frac{\Gamma \left( m, \frac{m\gamma}{\bar{\gamma}} \right)}{\Gamma(m)}$</td>
<td>$\left( 1 - \frac{s\bar{\gamma}}{m} \right)^{-m}$</td>
</tr>
</tbody>
</table>

2.3.4 Weibull Fading Channel

This considers a signal composed of clusters of one multipath wave, each propagating in a non-homogeneous environment. Within any one cluster, the phases of the scattered waves are random and have similar delay times with delay time spreads of different clusters being relatively large [23].

Since the focus of this study is the system performance under the Weibull fading channels, we will proceed to give their detailed analysis.
In a MIMO system with a flat fading wireless channel, the received signal can be modelled as [24]

\[ y = Hx + n \]  \hspace{1cm} (2.1)

where \( x \) is an \((n_r \times 1)\) transmitted signal vector, with \( n_r \) being the number of transmitters. \( y \) is an \((n_r \times 1)\) received signal vector, with \( n_r \) being the number of receivers. \( H \) is the \((n_r \times n_r)\) channel matrix and \( n \) is an \((n_r \times 1)\) additive white Gaussian noise vector. The channel matrix \( H \) is given as

\[ H = \begin{bmatrix} h_{ij} \\ \vdots \\ h_{i_{n_r}j_{n_r}} \end{bmatrix} \]  \hspace{1cm} (2.2)

where \( h_{ij} \) describes the channel gain between the \( i_{th} \) receiving antenna and the \( j_{th} \) transmitting antenna.

In the Weibull fading model, the complex envelope, \( h_{ij} \) may be written as a function of the Gaussian in-phase \( X_{ij} \) and quadrature \( Y_{ij} \) elements of the multipath components [25]

\[ h_{ij} = (X_{ij} + jY_{ij})^{2/\beta_{ij}} \]  \hspace{1cm} (2.3)

where \( j = \sqrt{-1} \) and \( \beta_{ij} \) is the fading parameter expressing the fading severity.

Let \( Z_{ij} \), the Weibull distributed random variable (RV) be the magnitude of \( h_{ij} \), i.e. \( Z_{ij} = |h_{ij}| \). Since the Weibull distributed random variable is a transformation of the Rayleigh distributed random variable, \( R_{ij} = |X_{ij} + jY_{ij}| \), the Weibull random variable can thus be written as [26]

\[ Z_{ij} = R_{ij}^{2/\beta_{ij}} \]  \hspace{1cm} (2.4)

Considering (2.4), the PDF of \( Z_{ij} \) is given as [27]
\[ f_{Z_{ij}}(r) = \frac{\beta_{ij}}{\Omega_{ij}} r^{\beta_{ij}-1} \exp\left(-\frac{r^{\beta_{ij}}}{\Omega_{ij}}\right) \]  
\hspace{1cm} (2.5)

where \( \Omega_{ij} \) is the expectation and \( \beta_{ij} \) is the fading parameter expressing the fading severity, where \( \beta_{ij} > 0 \). As \( \beta_{ij} \) increases, the effect of fading decreases. For the special case of \( \beta_{ij} = 2 \), the Weibull PDF of \( Z_{ij} \) reduces to the Rayleigh PDF. Whilst for \( \beta_{ij} = 1 \) the Weibull PDF of \( Z_{ij} \) reduces to negative exponential PDF. By integrating (2.5), the corresponding CDF of \( Z_{ij} \) can be expressed [26]

\[ F_{Z_{ij}}(r) = 1 - \exp\left(-\frac{r^{\beta_{ij}}}{\Omega_{ij}}\right) \]  
\hspace{1cm} (2.6)

The \( n \)th order moment of \( Z_{ij} \) which is defined as the expected value of \( Z_{ij}^n \) is derived from \( E\left(Z_{ij}^n\right) = \int_{-\infty}^{\infty} Z_{ij}^n f_{Z_{ij}}(r) dz \), where \( f_{Z_{ij}}(r) \) is the PDF of \( Z_{ij} \). and can be written as [28]

\[ E\left(Z_{ij}^n\right) = \Omega_{ij}^{n/\beta_{ij}} \Gamma\left(d_{n,ij}\right) = \Omega_{ij}^{n/\beta_{ij}} \Gamma\left(1 + \frac{n}{\beta_{ij}}\right) \]  
\hspace{1cm} (2.7)

where \( \Gamma(\cdot) \) is the Gamma function and \( n \) is a positive integer.

We then present the theoretical analysis for the bivariate Weibull distribution which is necessary in studying the performance of the transmit antenna selective MIMO system with dual receivers. When studying the performance of diversity receivers, two cases can be considered. The first case being when the two branches are independent of each other and the second case being when the branches are correlated. We let \( Z_{1j} \) and \( Z_{2j} \) be the Weibull distributed random variables.

According to [29] the general expressions for the CDF and PDF for the bivariate independent random variables can be written as
\[ F_{z_1, z_2} (r_{ij}, r_{kj}) = F_{z_1} (r_{ij}) F_{z_2} (r_{kj}) \]  

(2.8)

and

\[ f_{z_1, z_2} (r_{ij}, r_{kj}) = f_{z_1} (r_{ij}) f_{z_2} (r_{kj}) \]  

(2.9)

respectively, where \( j \) denotes the selected transmit antenna. Using (2.5) and (2.8), (2.5) and (2.9) the joint CDF and joint PDF of the Weibull distributed independent random variables, \( Z_{1j} \) and \( Z_{2j} \) is given as [30]

\[
F_{z_1, z_2} (r_{ij}, r_{kj}) = \left[ 1 - \exp \left( -\frac{r_{ij}^{\beta_{1j}}}{\Omega_{1j}} \right) \right] \times \left[ 1 - \exp \left( -\frac{r_{kj}^{\beta_{2j}}}{\Omega_{2j}} \right) \right] 
\]  

(2.10)

and

\[
f_{z_1, z_2} (r_{ij}, r_{kj}) = \frac{\beta_{1j} \beta_{2j} r_{ij}^{\beta_{1j} - 1} r_{kj}^{\beta_{1j} - 1}}{\Omega_{1j} \Omega_{2j}} \exp \left( -\frac{r_{ij}^{\beta_{1j}}}{\Omega_{1j}} - \frac{r_{kj}^{\beta_{2j}}}{\Omega_{2j}} \right) \]

(2.11)

respectively.

Next, we consider the correlated case, and we start by defining correlation. Correlation is defined as the degree of relationship between two random variables. This relationship is expressed as the correlation coefficient. The joint PDF of the Weibull distributed random variables \( Z_{1j} \) and \( Z_{2j} \) under correlated conditions can be written as [31]

\[
f_{z_1, z_2} (r_{ij}, r_{kj}) = \frac{\beta_{1j} \beta_{2j} r_{ij}^{\beta_{1j} - 1} r_{kj}^{\beta_{1j} - 1}}{\Omega_{1j} \Omega_{2j} (1 - \rho)} \exp \left( -\frac{1}{1 - \rho} \left[ \frac{r_{ij}^{\beta_{1j}}}{\Omega_{1j}} + \frac{r_{kj}^{\beta_{2j}}}{\Omega_{2j}} \right] \right) \times 
\]

(2.12)

\[
I_0 \left( 2\sqrt{\rho r_{ij}^{\beta_{1j}/2} r_{kj}^{\beta_{2j}/2}} \right) \]

where \( I_0 (.) \) is the modified Bessel function of the first kind and zero order. The Bessel function can be expressed as [32]
\[ I_0(u) = \sum_{k=0}^{\infty} \frac{1}{(k!)^2} \left( \frac{u}{2} \right)^{2k} \]  \hspace{1cm} (2.13)

Substituting (2.13) into (2.12), the joint PDF of \( Z_{1j} \) and \( Z_{2j} \) can be expressed as

\[
f_{z_{1j},z_{2j}}(r_{1j},r_{2j}) = \beta_{1j} \beta_{2j} \times \exp \left( -\frac{1}{(1 - \rho)} \left[ \frac{r_{1j}^{\beta_{1j}} + r_{2j}^{\beta_{2j}}}{\Omega_{1j} + \Omega_{2j}} \right] \right) \times \sum_{k=0}^{\infty} \frac{1}{(k!)^2} \frac{\rho^k r_{1j}^{-1+(k+1)\beta_{1j}} r_{2j}^{-1+(k+1)\beta_{2j}}}{(\Omega_{1j} \Omega_{2j})^{k+1}} \]  \hspace{1cm} (2.14)

where \( \rho \) is the correlation coefficient. The general expression for the correlation coefficient is [33]

\[ \rho \equiv \frac{\text{cov}(Z_{1j}^2, Z_{2j}^2)}{\sqrt{\text{var}(Z_{1j}^2) \text{var}(Z_{2j}^2)}} = \frac{E(Z_{1j}^2 Z_{2j}^2) - E(Z_{1j}^2) E(Z_{2j}^2)}{\sqrt{E(Z_{1j}^4) - E^2(Z_{1j}^2)}} \]  \hspace{1cm} (2.15)

According to [34] the correlation coefficient is related to a parameter called the dependence factor, \( \delta \) whose value is \((0 < \delta \leq 1)\). Considering this relationship, and making the necessary substitutions using (2.7), (2.15) can be written as [34]

\[ \rho = \frac{\Gamma^2(d_\delta) \Gamma(d_{1j}) - \Gamma^2(d_{1j}) \Gamma(d_\delta)}{\Gamma(d_\delta) \Gamma(d_{1j}) - \Gamma^2(d_{1j})} = \frac{\Gamma^2(1 + \delta / \beta_{1j}) - \Gamma(1 + \beta_{1j}) \Gamma(1 + \delta / \beta_{1j})}{\Gamma(1 + 2\delta / \beta_{1j}) [1 - a \Gamma^2(1 + \beta_{1j})]} \]  \hspace{1cm} (2.16)

Figure 2.1 plots the correlation coefficient as a function of the dependence factor for several values of the fading parameter. This plot shows the relation between the two factors as illustrated in (2.16). For \( \delta \to 0, \rho \to 1 \), whilst for \( \delta = 1, \rho = 0 \) [35].
Having discussed the joint PDF of $Z_{1j}$ and $Z_{2j}$ we then proceed to discuss the joint moments of $Z_{1j}$ and $Z_{2j}$. Using (2.4), the joint moments of $Z_{1j}$ and $Z_{2j}$ can be expressed as [33]

$$E(Z_{1j}^{n} Z_{1j}^{m}) = E(R_{1j}^{2n/\beta_{1}} R_{2j}^{2m/\beta_{2}})$$

(2.17)

By using the joint moments of $R_{1j}$ and $R_{2j}$ which are written as [36]

$$E(R_{1j}^{n} R_{2j}^{m}) = \Omega_{1j}^{n/2} \Omega_{2j}^{m/2} \Gamma\left(1+\frac{n}{2}\right) \times \Gamma\left(1+\frac{m}{2}\right)$$

(2.18)

we can be able to write the joint moments of $Z_{1j}$ and $Z_{2j}$ over independent Weibull fading channel as

$$E(Z_{1j}^{n} Z_{2j}^{m}) = \Omega_{1j}^{n/\beta_{1}} \Omega_{2j}^{m/\beta_{2}} \Gamma\left(1+\frac{n}{\beta_{1j}}\right) \times \Gamma\left(1+\frac{m}{\beta_{2j}}\right)$$

(2.19)
where $n$ is replaced by $\frac{2n}{\beta_{1j}}$ and $m$ is replaced by $\frac{2m}{\beta_{2j}}$ in (2.18).

When considering correlated channels the joint moments of $Z_{1j}$ and $Z_{2j}$ can be derived from [8] as

$$E\left( R_{1j}^n R_{2j}^m \right) = (1 - \rho)^{1+(m+n)/2} \frac{\Omega_{1j}^{n/2} \Omega_{2j}^{m/2}}{\Gamma \left( 1 + \frac{n}{2} \right) \times \Gamma \left( 1 + \frac{m}{2} \right)} \times F_1 \left( 1 + \frac{n}{2}, 1 + \frac{m}{2}; 1; \rho \right)$$

(2.20)

whereby $n$ is replaced by $\frac{2n}{\beta_{1j}}$ and $m$ is replaced by $\frac{2m}{\beta_{2j}}$ as

$$E\left( Z_{1j}^n Z_{2j}^m \right) = (1 - \rho)^{1+(n/\beta_{1j})+(m/\beta_{2j})} \frac{\Omega_{1j}^{n/\beta_{1j}} \Omega_{2j}^{m/\beta_{2j}}}{\Gamma \left( 1 + \frac{n}{\beta_{1j}} \right) \times \Gamma \left( 1 + \frac{m}{\beta_{2j}} \right)} \times F_1 \left( 1 + \frac{n}{\beta_{1j}}, 1 + \frac{m}{\beta_{2j}}; 1; \rho \right)$$

(2.21)

### 2.4 Chapter Summary

In this chapter, slow and fast, flat and frequency selective fading has been presented. The discussion on fading has also been extended to fading channels where the Rayleigh, Rician and Nakagami fading channels have been briefly considered. The Weibull fading channels together with their channel statistics have been illustrated. These channel statistics will play a great role in Chapter 4 where the performance parameters are presented.

We have seen in Chapter 2 that fading presents a significant problem in the performance of wireless communication systems, thus there is need to discuss techniques that can be used to mitigate fading. This leads us to Chapter 3, where different diversity techniques are discussed and this is extended to MIMO systems.
CHAPTER 3

3 Diversity Techniques and MIMO Systems

3.1 Introduction

Diversity is one of the techniques used to overcome fading in wireless communication systems. With diversity, the receiver is meant to receive multiple replicas of the transmitted signal, thus increasing the SNR and decreasing the probability of error. In this way, the performance of the wireless communication system is improved. Diversity is divided into frequency, time and antenna diversity. Antenna diversity has led to the development of MIMO systems, where multiple antennas are used at the transmitter and receiver ends. The MIMO system will be discussed in terms of its impact on diversity gain and spatial multiplexing. This discussion will extend to space-time coding, which is a coding technique used in conjunction with multiple transmit antenna systems. Within space-time codes, much emphasis will be put on the space-time block codes. With multiple receivers employed at the receiver end, the signals are then combined using several combining techniques, namely selection combining, equal-gain-combining and maximal-ratio-combining. These combining techniques will be discussed in section 3.7.

3.2 Frequency Diversity

Frequency diversity is whereby the message signal is sent over multiple carriers that are separated by more than the coherence bandwidth of the channel. This is not efficient as more bandwidth is required with an increase in the number of carriers in the systems.
3.3 Time Diversity

In time diversity, the message signal is sent over different time slots whose separation is equal to or greater than the coherence time of the channel. With this, the effective data rate is less than the real data rate that can be achieved by the system.

3.4 Antenna Diversity

In antenna diversity, the signal is transmitted and/or received using several antennas. These antennas are sufficiently spaced apart to achieve independence between signals and to avoid the linkage of signals. This is the diversity technique considered when studying MIMO
systems and is what we will consider when studying the transmit antenna selective MIMO system.

3.5 MIMO Systems

These are communication systems that use several antennas at both the transmitter and receiver ends. The use of multiple antennas provides diversity gain or spatial multiplexing gain. Diversity gain helps in improving the quality of the signal, as many replicas of the signal are sent using different antennas. Sending many replicas of the signal increases the probability of detecting the signal at the receiver end. Signal detection is also improved by the availability of several receive antennas at the receiver end. This makes it unlikely that all the signals fall under deep fade simultaneously [37].

Spatial multiplexing gain improves system performance through the increase in data rates. In spatial multiplexing the transmitted data is split into several data streams and transmitted simultaneously across the channel through the different transmit antennas via distinct paths. The signals are then received by the several receivers at the receiver end as a combination of the transmitted data streams [37, 38].

The achievement of diversity gain or spatial multiplexing gain depends on the encoding techniques used to encode the signals before they are transmitted through the different antennas. These different encoding schemes will be discussed in section 3.6. Figure 3.3 shows a 4×4 MIMO system, i.e. a MIMO system with 4 transmit and 4 receive antenna. This MIMO system has a diversity gain of 16, meaning that there are 16 individual branches/radio frequency chains between all the transmit and receive antennas. Table 3.1 shows the diversity gains for 2 x 2, 3 x 2 and 4 x 2 MIMO systems.
Table 3.1: Diversity gains for various MIMO system configurations

<table>
<thead>
<tr>
<th>Total number of available transmit antennas</th>
<th>Number of receive antennas</th>
<th>Number of radio frequency chains</th>
<th>Diversity gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 3.1 shows the improvement in diversity gain as the number of transmit antennas is increased. However, such an improvement is associated with an increase in the number of radio frequency chains to be decoded, and this leads to decoder complexity.
In antenna diversity, the signals are encoded using space-time coding techniques, namely space-time-trellis coding, space-time-block coding and Bell Labs Layered Space-Time Architecture.

### 3.6 Space-Time Codes

Space-time coding is a technique employed when the communication system uses several antennas to transmit information. Space-time codes do not need any bandwidth expansion, thus they are a good candidate for wireless systems. They also provide full system diversity. Space-time-block codes (STBC) and space-time-trellis codes (STTC) are examples of space-time codes and are used to provide high quality communication by improving the BER and the SNR performance of the communication link, whereas BLAST is used to provide high data rates, thus improving the systems throughput.

#### 3.6.1 Space-Time Block Codes, STBC

These are the simplest of the space-time codes. In studying the proposed system performance, STBC will be considered as the encoding technique used to encode the signals before they are transmitted through the different antennas. As a result there is need to present a detailed analysis of STBC.

In STBC every \(k\) input bit symbols are mapped into a \(p \times N\) coded matrix. Where \(N\) represents the symbols transmitted from each transmit antenna during \(p\) different time slots. The Alamouti code is a special case where two transmit antennas are used. STBC provide transmit diversity gain but no coding gain [9]. The space-time block code where two transmit antennas are used is defined by [13] as

\[
G_2 = \begin{pmatrix}
    x_1 & x_2 \\
    -x_2^* & x_1^*
\end{pmatrix}
\]  

(3.1)
where * represents the complex conjugates.

Considering the transmission matrix, (3.1), during the first time slot signals $x_1$ and $x_2$ are transmitted by the first and second transmitters, i.e. $Tx_1$ and $Tx_2$ respectively. Then on the second time slot signals $-x_2^*$ and $x_1^*$ are transmitted by $Tx_1$ and $Tx_2$ respectively. These transmitted signals propagate through the fading channel that can be presented as $h_{ij}$, where $i$ is the receiver index and $j$ is the transmitter index.

In [39] the STBC were extended to more than two transmit antennas. According to [9, 39] the transmission matrices for a three and four transmitter system are defined as

$$G_3 = \begin{pmatrix}
  x_1 & x_2 & x_3 \\
  -x_2 & x_1 & -x_4 \\
  -x_3 & x_4 & x_1 \\
  -x_4 & -x_3 & x_2 \\
  x_1^* & x_2^* & x_3^* \\
  -x_2^* & x_1^* & -x_4^* \\
  -x_3^* & x_4^* & x_1^* \\
  -x_4^* & -x_3^* & x_2^*
\end{pmatrix} \quad (3.2)$$

and

$$G_4 = \begin{pmatrix}
  x_1 & x_2 & x_3 & x_4 \\
  -x_2 & x_1 & -x_4 & x_3 \\
  -x_3 & x_4 & x_1 & -x_2 \\
  -x_4 & -x_3 & x_2 & x_1 \\
  x_1^* & x_2^* & x_3^* & x_4^* \\
  -x_2^* & x_1^* & -x_4^* & x_3^* \\
  -x_3^* & x_4^* & x_1^* & -x_2^* \\
  -x_4^* & -x_3^* & x_2^* & x_1^*
\end{pmatrix} \quad (3.3)$$

respectively.
These transmission matrices were found to have a rate of a half, thus to try to improve on this Tarokh et al. proposed the three-quarter rate transmission matrices. In the context of STB codes the rate of a code is defined as the total number of input symbols in the transmit antennas divided by the total number of time slots in which these symbols are transmitted. The corresponding three-quarter rate transmission matrices for a three and four transmitter system were defined in [9, 39] as

$$H_3 = \begin{bmatrix}
    x_1 & x_2 & \frac{x_3}{\sqrt{2}} \\
    -x_2^* & x_1^* & \frac{x_3}{\sqrt{2}} \\
    \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} & (-x_1 - x_1^* + x_2 - x_2^*) \\
    \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{2} \\
    \frac{x_3^*}{\sqrt{2}} & -x_3^* & (x_2 - x_2^* + x_1 - x_1^*) \\
    \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{2}
\end{bmatrix}$$

(3.4)

and

$$H_4 = \begin{bmatrix}
    x_1 & x_2 & \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} \\
    x_2^* & x_1^* & \frac{x_3}{\sqrt{2}} & -\frac{x_3}{\sqrt{2}} \\
    \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} & (-x_1 - x_1^* + x_2 - x_2^*) & (-x_2 - x_2^* + x_1 - x_1^*) \\
    \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \\
    \frac{x_3^*}{\sqrt{2}} & -x_3^* & (x_2 + x_2^* + x_1 - x_1^*) & (x_1 - x_1^* + x_2 + x_2^*) \\
    \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}$$

(3.5)

respectively, where the labels $H_3$ and $H_4$ are introduced to try to differentiate between the half rate and three-quarter rate codes.

The same idea as that discussed for the $G_2$ STB codes is used to transmit the signals in the $G_3$, $G_4$, $H_3$ and $H_4$ STB codes. The respective signals are transmitted by the different transmit antennas, namely $T_{x_1}$, $T_{x_2}$ and $T_{x_3}$ for $G_3$ and $H_3$ and $T_{x_1}$, $T_{x_2}$, $T_{x_3}$ and $T_{x_4}$
for $G_i$ and $H_i$. Again, the transmission is done at different time slots and the signals propagate through the fading channel $h_{ij}$.

The information for a two, three and four transmitter antenna matrix is summarised in Table 3.2, [9]

Table 3.2: Space-time block codes with their different rates and other properties

<table>
<thead>
<tr>
<th>Space-time block code</th>
<th>Rate</th>
<th>Number of transmitters</th>
<th>Number of input symbols</th>
<th>Time slots</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_2$</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$G_3$</td>
<td>$\frac{1}{2}$</td>
<td>3</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>$G_4$</td>
<td>$\frac{1}{2}$</td>
<td>4</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>$H_3$</td>
<td>$\frac{3}{4}$</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>$H_4$</td>
<td>$\frac{3}{4}$</td>
<td>4</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 3.2 shows that when the transmit antennas are greater than two the STBC have a rate of less than one. STBC can therefore be used to provide improved BER performance as a trade-off for higher data rates. At the receiver end, the signals can be decoded using the simple maximum-likelihood technique based on the Euclidean distances between the received signal and all the possible transmitted symbols, thus making STBC less complex than STTC. It must be
noted that for the maximum-likelihood technique to be applicable the STBC must be of orthogonal design [39].

3.6.2 Space-Time Trellis Codes, STTC

Their design is based on the trellis structure as presented by Tarokh et al., where each input symbol is mapped into $N$ symbols, each of which is then transmitted through each of the transmit antennas. This structure has complex decoding as compared to space-time block codes. Just like space-time block codes, space-time trellis codes provide transmit diversity gain. They also provide coding gain in addition. STTC are used to improve signal quality in wireless communication systems. STTC are decoded using the viterbi technique, owing to their complexity [40].

3.6.3 BLAST Bell Labs Layered Space-Time Architecture, BLAST

This was developed by Foschini, and here independent bit streams are transmitted through each antenna. Unlike the other two coding techniques, this does not use space encoding. There are two types of BLAST, namely Vertical and Diagonal BLAST. In V-BLAST, the bit streams are encoded independently from each other using channel coding without introducing any redundancy. D-BLAST introduces redundancy through coding between all the bit streams, thus it is more efficient than V-BLAST but requires high computational complexity. BLAST is used in wireless communication systems to provide improved data rates [41].

3.7 Diversity Combining Techniques

These are methods used to combine all the signals received by the individual receiving antennas to help improve the system performance, especially in conditions affected by fading and multipath propagation.
3.7.1 Selective Combining

The receiving antenna giving the highest SNR is always selected from all the available receiving antennas [42]. The instantaneous output SNR is given as

$$\gamma_{SC} = \max \{\gamma_1, \gamma_2, \ldots, \gamma_L\}$$

where \(L\) is the number of branches.

The drawback with selective combining is that all the received signals need to be constantly monitored so that the best of them is then selected.

3.7.2 Maximal-Ratio-Combining

In selective combining, the output of the combiner is equal to one of the branches. Whereas in MRC the output of the combiner is the weighted sum of all the branches. The instantaneous SNR in MRC is given as [43]

$$\gamma_{MRC} = \sum_{i=1}^{L} \gamma_i$$

where \(\gamma_i\) is the instantaneous SNR in the \(i_{th}\) branch.

3.7.3 Equal-Gain-Combining

This is similar to MRC, except that the received signals are not weighted implying that it is less complex as compared to MRC [44]. The instantaneous SNR in EGC is given by

$$\gamma_{EGC} = \frac{1}{N_0L} \left(\sum_{i=1}^{L} r_i\right)^2$$

where \(r_i\) is the received signal from the \(i_{th}\) branch and \(N_0\) is the noise power.
Figure 3.4 shows the plot of the number of antennas used in a wireless system against the signal-to-noise ratio for different combining techniques [45]. The MRC combining technique is shown to outperform all the other combining techniques. For this reason, it is considered as the optimal combining technique in this project.

![Figure 3.4: Number of antennas against signal-to-noise ratio for SC, EGC and MRC](image)

### 3.8 Chapter Summary

We have discussed different diversity techniques, and this has been extended to MIMO systems. In MIMO systems it has been observed that with an increase in the number of antennas used there is an increase in the diversity gain and in the number of radio frequency chains. The increase in diversity gain implies the unlikelihood of all the signals simultaneously falling into a region of deep fade. The different types of space-time codes associated with multiple antenna
systems have also been discussed, with space-time block codes being discussed at length. Space-time block codes provide improved quality of communication at reduced decoder complexity, especially if the codes are of orthogonal design. A table summarising the space-time block codes transmission matrices shows that the highest data rate that can be achieved is unity. STBC can therefore be used to provide improved BER performance as a trade-off for higher data rates. The different combining techniques have also been discussed, where selection combining has been found to be the simplest of all the other combining techniques but its performance compromised. A performance graph of the three combining techniques shows that maximal-ratio-combining is superior in performance compared to the other techniques but complex, especially with increasing radio frequency chains.

To try to solve the problem of complexity associated with MIMO systems, an antenna selective approach is proposed. Chapter 4 gives a detailed discussion on the transmit antenna selective MIMO system.
CHAPTER 4

4 Transmit Antenna Selective MIMO System

4.1 Introduction

The transmit antenna selective MIMO system model is discussed. Its analysis and the assumptions we will make when studying the system performance parameters are presented. The parameters of interest include the average signal-to-noise ratio, amount of fading, average channel capacity, average bit-error probability and outage probability. Equations for all these parameters will be derived under independent and correlated channels.

A wireless communication system is referred to as a full complexity MIMO system if it uses all the available transmit and receive antennas to send and receive signals. On the other hand, a transmit antenna selective MIMO system is one that selects the best performing antenna(s) from all the available transmit antennas, and uses this together with all the receive antennas for communication. If \( T_n \) and \( R_n \) represent the number of transmit and receive antennas respectively then the full complexity system is referred to as a \((T_n;R_n)\) system, in which all the \( T_n \) transmit and \( R_n \) receive antennas are used. On the other hand, \((n_r;1;R_n)\) denotes the transmit antenna selective system, whereby one out of \( n_r \) antennas is used.

4.2 System Model

In the transmit antenna selective MIMO system the available number of antennas to be used before selection is \( 2 \leq n_r \leq 4 \) and \( R_n = 2 \). The downlink system performance will be investigated as the number of transmit antennas is increased from two to four. In all these cases the single best transmit antenna will be selected. This is the antenna that gives the maximum combined SNR at the receiver end. Having selected the best antenna, all the other antennas are then put on sleep mode and the transmit power concentrated on the selected antenna.
Communication continues until the signal in the selected antenna falls below a set threshold at the receiver end. When this occurs all the transmit antennas are reactivated and the selection cycle starts all over again. This process continues until the end of communication, Figure 4.1. [46]. The receive antennas will be fixed at two antennas. All antennas at both ends are assumed to be sufficiently spaced to avoid any interference between neighbouring antennas. In carrying out system performance analysis, identically independent distributed Weibull fading channels will be considered, and will be assumed to vary slowly and the channel gain, $h_i$, kept constant within one frame and change independently from one frame to another, i.e. quasi-static fading channels.

The channel state information (CSI) will be assumed available at the receiver end, and partially available at the transmitter end through a feedback channel from the receiver end. In this case the transmitter only needs to know which single best transmit antenna is activated for transmission based on the information it receives via the feedback channel. Before the signals are transmitted by the different antennas, information bits will be added to the signals to help identify which signal has been sent by a particular transmit antenna. At the receiver end, the signals received from the various transmit antennas will then be compared in terms of their respective SNR. The transmit antenna which results in a high SNR compared to the other antennas will then be identified using the extra information added to the signals. Having identified the best antenna this information is then sent back to the transmitter end through the feedback channel. The feedback information has $\lceil \log_2(n_r) \rceil$ bits, where $\lfloor x \rfloor$ denotes the minimum integer not smaller than $x$ [11]. This means that when $n_r = 3$ or 4, only 2 bits of feedback are required and when $n_r = 2$, only 1 bit of feedback is required. Since the system will be assumed to be working in a quasi-static flat fading channel at a high data rate, then the feedback transmission rate is comparatively low. Thus, it will not cause much delay in the system performance.

The details of modulation and demodulation are ignored as we are interested in the performance analysis of the transmit antenna selective MIMO system. It is worth noting that before transmission the modulated signals will be encoded using STBC of orthogonal design. The use of orthogonal space-time block codes is due to their minimised complexity compared to
space-time trellis codes. Binary phase shift keying (BPSK) modulation is assumed, but any form of modulation can be used to modulate the signals before encoding. BPSK modulation is selected due to its simplicity in analysis when dealing with system performance.

We will start by considering the case when two transmitting antennas are available. The single best performing transmit antenna is selected from all the available antennas. This is the antenna that gives the maximum combined SNR at the two receivers. The single selected transmit antenna, denoted as $Tx$, and is determined by [47]

$$Tx = \arg \max_{1 \leq j \leq n} \left\{ T_{x_i} = \sum_{i=1}^{2} h_{ij}^2 \right\}$$  \hspace{1cm} (4.1)
Having selected the best transmit antenna, the signals in the two receivers are then combined using MRC. The idea with transmit antenna selection is to reduce the number of terms to be combined at the receiver end and then sent to the maximum likelihood decoder for decoding. This in turn helps reduce the system complexity but still attains the goals of reliable communication as the best signals will be sent for decoding.

Let the single best selected transmit antenna be denoted as $Tx$ and the two receive antennas be $Rx_1$ and $Rx_2$, Figure 4.2 [48]. The transmitted signal, $x$ propagates through two different channels, namely $h_1$ and $h_2$, where $h_1$ is the channel between $Tx$ and $Rx_1$ and $h_2$ the channel between $Tx$ and $Rx_2$. The received signals are [9]

$$y_1 = h_1 x + n_1$$

$$y_2 = h_2 x + n_2$$

where $n_1$ and $n_2$ represent complex noise samples in $Rx_1$ and $Rx_2$ respectively. The different received signals $y_1$ and $y_2$ are then combined using MRC to extract the transmitted signal, $x$ as follows.

$$\tilde{x} = h_1^* y_1 + h_2^* y_2$$

where $h_1^*$ and $h_2^*$ are the complex conjugates of channels $h_1$ and $h_2$ respectively.

Substituting (4.2) and (4.3) into (4.4) gives

$$\tilde{x} = (h_1^2 + h_2^2) \cdot x + h_1^* n_1 + h_2^* n_2$$

The combined signal $\tilde{x}$ is then sent to the maximum likelihood detector. In this, the most likely transmitted signal is determined using the Euclidean distance between the combined signal and all possible transmitted signals. According to [48] the simplified decision rule is based on choosing $x_i$ if and only if
where \( \text{dist}(A,B) \) is the Euclidean distance between signals A and B and \( j \) extends to all possible transmitted signals. The maximum likelihood transmitted signal is the one with the minimum Euclidean distance from the combined signal \( \tilde{x} \).

![Diagram of 1 x 2 Transmit antenna selective MIMO system](image)

**Figure 4.2: 1 x 2 Transmit antenna selective MIMO system**

When three or four transmit antennas are used in the system, the analysis is the same as discussed above, since in all cases the single best performing antenna is selected for transmission. Thus regardless of the number of available transmit antennas the transmit antenna selective system always has signals \( y_1 \) and \( y_2 \) to combine. This means that the system complexity in this scheme does not increase with an increase in the number of available transmitters. Thus, we can always think of the transmit antenna selective MIMO system as a one transmitter and two receiver system, i.e. \((1;2)\) improved SIMO system. The transmit antenna selective MIMO system is summarised as follows.
Table 4.1: Diversity gains for various transmit antenna selective MIMO system configurations

<table>
<thead>
<tr>
<th>Total number of available transmit antennas</th>
<th>Number of selected antennas</th>
<th>Number of receive antennas</th>
<th>Number of radio frequency chains</th>
<th>Diversity gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 4.1 summarises the diversity gain improvement of the proposed transmit antenna selective MIMO system. It can be observed that the diversity gain increases with an increase in the number of available transmit antennas as the case with the full complexity MIMO systems. Another advantage of the transmit antenna selective MIMO system is the reduction in radio frequency chains to be decoded. Regardless of the total number of available transmit antennas there are always two radio frequency chains to be decoded at the receiver end. These radio frequency chains are a result of the signal transmitted between the selected transmit antenna and the first receive antenna and also the selected transmit antenna and the second receive antenna.

### 4.3 Performance Parameters

Performance parameters are system metrics used to determine the performance of a system in a given environment. The performance parameters we will discuss include the average signal-to-noise ratio, amount of fading, average channel capacity, average bit-error probability and the outage probability.
4.3.1 Average Signal-to-Noise Ratio

This is one of the parameters used to study the performance of a communication system. It is the ratio of the signal power to that of noise power. For better performance, this ratio must be a high value, meaning that the signal power should be much greater than the noise power. If this is the case then the system is less prone to errors. In Weibull fading the instantaneous signal-to-noise ratio at the input of the receiver is given by [49]

\[ \gamma_j = Z_j^2 \frac{E_s}{N_o} \]  

(4.7)

and the average SNR can be expressed as [23]

\[ \overline{\gamma}_j = E(Z_j^2) \frac{E_s}{N_o} \]  

(4.8)

Substituting (2.7) into (4.8) for \( n = 2 \), the average SNR is written as

\[ \overline{\gamma}_j = \Omega_j^{2/3} \Gamma \left(1 + \frac{2}{\beta_j} \right) \frac{E_s}{N_o} \]  

(4.9)

Using the Weibull distribution property that the \( n \)-th power of a Weibull distributed random variable with parameters \((\beta_j, \Omega_j)\) is another Weibull distributed random variable with parameters \((\beta_j / n, \Omega_j)\) [49]. It can then be said that \( \gamma_j \) is also a Weibull random variable with parameters \((\beta_j / 2, (a_j \overline{\gamma}_j)^{\beta_j/2})\) where \( a_j = 1 / \Gamma(1 + 2 / \beta_j) \). The PDF of \( \gamma_j \) can then be derived from (2.5) by replacing \( \beta_j \) with \( \beta_j / 2 \) and \( \Omega_j \) with \((a_j \overline{\gamma}_j)^{\beta_j/2}\) as [50]

\[ p_{\gamma_j}(\gamma_j) = \frac{\beta_j}{2(a_j \overline{\gamma}_j)^{\beta_j/2}} \gamma_j^{(\beta_j/2) - 1} \exp \left[ - \left( \frac{\gamma_j}{a_j \overline{\gamma}_j} \right)^{\beta_j/2} \right] \]  

(4.10)

The corresponding CDF can be obtained by integrating (4.10) and is written as [23]
\[
p_{\gamma_i} (\gamma_j) = 1 - \exp \left[ - \left( \frac{\gamma_j}{a_{ij} \bar{\gamma}_{ij}} \right)^{\beta_j/2} \right] \tag{4.11}
\]

For the dual branch diversity receiver, the PDF of the SNR can be generally expressed as \( p_{\gamma_i,\gamma_j} (\gamma_i, \gamma_j) \). Over independent Weibull fading channels, the joint PDF of \( \gamma_i \) and \( \gamma_j \) can be written as [29]

\[
p_{\gamma_i,\gamma_j} (\gamma_i, \gamma_j) = p_{\gamma_i} (\gamma_i) \times p_{\gamma_j} (\gamma_j) \tag{4.12}
\]

Using (4.10) and (4.12) the joint PDF of \( \gamma_i \) and \( \gamma_j \) can be expressed as

\[
p_{\gamma_i,\gamma_j} (\gamma_i, \gamma_j) = \frac{\beta_j \beta_j}{4(a_{ij} \bar{\gamma}_{ij})^{\beta_j/2}} \left( \frac{\gamma_j}{a_{ij} \bar{\gamma}_{ij}} \right)^{(\beta_j/2)-1} \times \exp \left[ - \left( \frac{\gamma_j}{a_{ij} \bar{\gamma}_{ij}} \right)^{\beta_j/2} \right] \exp \left[ - \left( \frac{\gamma_j}{a_{ij} \bar{\gamma}_{ij}} \right)^{\beta_j/2} \right] \tag{4.13}
\]

Under correlated conditions the joint PDF of \( \gamma_i \) and \( \gamma_j \) can be obtained from (2.14) by using the Weibull transformation property of replacing \( \beta_j \) with \( \beta_j / 2 \) and \( \Omega_j \) with \( (a_{ij} \bar{\gamma}_{ij})^{\beta_j/2} \) as

\[
p_{\gamma_i,\gamma_j} (\gamma_i, \gamma_j) = \frac{\beta_j \beta_j}{4} \exp \left[ - \frac{1}{(1 - \rho)} \left( \frac{\gamma_j}{(a_{ij} \bar{\gamma}_{ij})^{\beta_j/2}} + \frac{\gamma_j}{(a_{ij} \bar{\gamma}_{ij})^{\beta_j/2}} \right) \right] \times \sum_{k=0}^{\infty} \frac{\rho^k}{(k!)^2} (1 - \rho)^{2k+1} \gamma_j \left( \frac{\gamma_j}{(a_{ij} \bar{\gamma}_{ij})^{\beta_j/2}} \right)^{\beta_j/2} \gamma_j \left( \frac{\gamma_j}{(a_{ij} \bar{\gamma}_{ij})^{\beta_j/2}} \right)^{\beta_j/2} \tag{4.14}
\]

For the transmit antenna selective MIMO system the total instantaneous SNR, \( \gamma_{MRC} \) is given as the sum of the instantaneous SNR in the two receiving antennas and is written as [51]

\[
\gamma_{MRC} = \gamma_i + \gamma_j \tag{4.15}
\]

The total average SNR, \( \gamma_{MRC} \) is given as [51]

\[
\gamma_{MRC} = \bar{\gamma}_i + \bar{\gamma}_j \tag{4.16}
\]
where $j$ denotes the selected transmit antenna. The discussion on the amount of fading is then considered.

### 4.3.2 Amount of Fading

Amount of fading (AoF) is the measure of how severe fading is in any given channel. It is generally expressed as the variance of the received signal divided by the squared average of the received signal \[36\]

\[
A_f = \frac{\text{var}(\gamma)}{\bar{\gamma}^2} = \frac{E(\gamma^2) - \bar{\gamma}^2}{\bar{\gamma}^2} = \frac{E(\gamma^2)}{\bar{\gamma}^2} - 1 \tag{4.17}
\]

In calculating the amount of fading there is need to find the moments of the SNR. According to [52] the $n_{th}$ order moment, $\mu_n$ of the SNR for the dual branch diversity receiver is written as

\[
\mu_n = E\left(\gamma_{MRC}^n\right) = E\left(\left(\gamma_{1j} + \gamma_{2j}\right)^n\right) = \sum_{k=0}^{n} \binom{n}{k} E\left(\gamma_{1j}^k \gamma_{2j}^{n-k}\right) \tag{4.18}
\]

The $n_{th}$ order moment of the SNR under independent Weibull fading channels can be obtained by substituting (2.19) into (4.18) and replacing $\beta_{ij}$ with $\beta_{ij}/2$ and $\Omega_{ij}$ with $(a_{ij}\bar{\gamma}_{ij})^{\beta_{ij}/2}$ as

\[
\mu_n = \sum_{k=0}^{n} \binom{n}{k}(a_{ij}\bar{\gamma}_{ij})^k (a_{2j}\bar{\gamma}_{2j})^{(n-k)} \Gamma\left[1 + \frac{2k}{\beta_{ij}}\right] \times \Gamma\left[1 + \frac{2(n-k)}{\beta_{2j}}\right] \tag{4.19}
\]

Substituting (2.21) into (4.18) and replacing $\beta_{ij}$ with $\beta_{ij}/2$ and $\Omega_{ij}$ with $(a_{ij}\bar{\gamma}_{ij})^{\beta_{ij}/2}$, the $n_{th}$ order moment of the SNR under correlated Weibull fading channels can be written as

\[
\mu_n = \sum_{k=0}^{n} \binom{n}{k}(1-\rho)^{1+(2k/\beta_{ij})+2(n-k)/\beta_{ij}} (a_{ij}\bar{\gamma}_{ij})^k (a_{2j}\bar{\gamma}_{2j})^{(n-k)} \times \\
\Gamma\left[1 + \frac{2k}{\beta_{ij}}\right] \times \Gamma\left[1 + \frac{2(n-k)}{\beta_{2j}}\right] \text{$_2F_1$}\left[1 + \frac{2k}{\beta_{ij}},1 + \frac{2(n-k)}{\beta_{2j}};1;\rho\right] \tag{4.20}
\]
The amount of fading for the dual branch diversity receiver can then be expressed in terms of moments of the SNR as [8]

\[ A_f = \frac{\mu_2}{\mu_1^2} - 1 \]  \hspace{1cm} (4.21)

where \( \mu_1 \) and \( \mu_2 \) are the first and second order moments of the SNR respectively. Next, we consider the channel capacity.

### 4.3.3 Average Channel Capacity

Channel capacity is defined as the maximum possible mutual information between the input \((x)\) and output \((y)\). The maximisation is over the probability distribution of the input \(f_x(x)\) [53]

\[ C = \max_{f_X(x)} \left[ I(X;Y) \right] = \max_{f_X(x)} \left[ h(Y) - h(Y/X) \right] \]  \hspace{1cm} (4.22)

The standard formula for the Shannon capacity is [54]

\[ C = BW \log_2 (1 + \gamma_g) \]  \hspace{1cm} (4.23)

where \( BW \) is the bandwidth.

We then present the channel capacity analysis for different system arrangements, namely SISO, MISO, SIMO, full complexity MIMO and antenna selective MIMO systems.

#### 4.3.3.1 SISO System

This is a wireless communication system with one transmitting and one receiving antenna, i.e. \((1;1)\), thus no antenna diversity. The channel matrix \(H\) is given as \(H = [h_{11}]\) and the instantaneous channel capacity is [55]
\[ C = BW \log_2 \left( 1 + Z_{ii}^2 \frac{E_s}{N_o} \right) = BW \log_2 (1 + \gamma_{11}) \] (4.24)

### 4.3.3.2 MISO System

This is a system that only has transmit antenna diversity, i.e. multiple transmitting antennas and one receiving antenna, \((n_T;1)\). The channel matrix \(\mathbf{H}\) is given as \(\mathbf{H} = \left[ h_{ij} \right]_{i \leq j \leq n_T} \). In this the total transmit power is divided equally between all the transmit antennas, i.e. \(P/n_T\), where \(P\) is the total transmit power. The instantaneous channel capacity is [56]

\[ C = BW \log_2 \left( 1 + E_s / n_T \sum_{j=1}^{n_T} Z_{ij}^2 \right) \] (4.25)

### 4.3.3.3 SIMO System

In contrast to the MISO system, this system configuration only has receive antenna diversity, and is referred to as a \((1; n_R)\) system. The channel matrix \(\mathbf{H}\) is \(\mathbf{H} = \left[ h_{ij} \right]_{i \leq j \leq n_R}^T \). Its instantaneous channel capacity is [56]

\[ C = BW \log_2 \left( 1 + \frac{E_s}{N_o} \sum_{i=1}^{n_R} Z_{ii}^2 \right) \] (4.26)

### 4.3.3.4 Full Complexity MIMO System

In a full complexity MIMO system, all the antennas are used to transmit and receive information. The channel matrix \(\mathbf{H}\) is given in (2.2). Just like in the system configurations already discussed, the capacity of a MIMO system depends on the distribution of \(h_{ij}\) in the channel matrix \(\mathbf{H}\). Considering the case whereby the transmitter has no channel state information. The best distribution scheme is to spread the energy evenly between all antennas.
transmitters. The instantaneous capacity of a MIMO system with no CSI at the transmitter is given by [38, 57, 58]

\[
C = BW \log_2 \left[ \det \left( I_{n_R} + \frac{\rho}{n_T} HH^\dagger \right) \right]
\]  \hspace{1cm} (4.27)

where \( \rho = E_s / N_o \) is the average SNR per receiving antenna, with \( E_s \) as the average power at the output of each of the receiving antennas and \( N_o \) the corresponding noise power. \( I_{n_R} \) is the \((n_R \times n_R)\) identity matrix and \( HH^\dagger \) denotes the conjugate and transposition and \( BW \) is the bandwidth.

### 4.3.3.5 Transmit Antenna Selective MIMO System

This system has the same channel matrix as the full complexity MIMO system, i.e. a total of \( n_T \) transmit antennas and \( n_R \) receive antennas are used. The difference with this system arrangement is that only one transmit antenna is used out of all the available transmit antennas at a time, and it is referred to as an \((n_T, 1; n_R)\) system. The selected transmit antenna is the one that maximises the SNR at the receive antennas. We cycle through all \( n_T \) transmitters periodically with period \( n_T \). The advantage with this technique is that there is no interference from the other transmit antennas and there is reduced decoder complexity. The instantaneous channel capacity is given as [38]

\[
C = BW \left( \frac{1}{n_T} \right) \cdot \sum_{j=1}^{n_T} \log_2 \left[ 1 + \frac{E_s}{N_o} \cdot \sum_{i=1}^{n_R} Z_{ij}^2 \right]
\]  \hspace{1cm} (4.28)

In this system arrangement the transmit power is not shared amongst all the transmit antennas but it is entirely dedicated to the single selected transmit antenna. This tends to improve the strength of the signals sent through this antenna, thus increasing the probability of overcoming fading. We then consider the average channel capacity, which in Shannon’s sense is given as [59]
\[
\overline{C} = BW \log_2 \left( 1 + \gamma_{ij} \right) p_{\gamma_{ij}}(\gamma_{ij}) d\gamma_{ij}
\]  
(4.29)

where \( p_{\gamma_{ij}}(\gamma_{ij}) \) is the PDF of the signal-to-noise ratio and \( BW \) the bandwidth. Substituting (4.10) into (4.29), for Weibull fading the average channel capacity can be written as

\[
\overline{C} = \frac{BW \beta_{ij}}{2(a_{ij}\overline{\gamma}_{ij})^{\beta_{ij}/2} \ln(2)} \int_0^\infty \gamma_{ij}^{\beta_{ij}/2 - 1} \ln(1 + \gamma_{ij}) \times \exp \left[ - \left( \frac{\gamma_{ij}}{a_{ij}\overline{\gamma}_{ij}} \right)^{(\beta_{ij}/2)} \right] d\gamma_{ij}
\]  
(4.30)

The above integral can be evaluated in closed-form by expressing the logarithmic and exponential integrands as Meijer’s G-functions as [60, 61]

\[
\ln(1 + \gamma_{ij}) = G_{2,2}^{1,2} \left[ \gamma_{ij} \left| \begin{array}{c} 1,1 \\ 1,0 \end{array} \right. \right]
\]  
(4.31)

and

\[
\exp \left[ - \left( \frac{\gamma_{ij}}{a_{ij}\overline{\gamma}_{ij}} \right)^{(\beta_{ij}/2)} \right] = G_{0,1}^{1,0} \left[ \frac{\gamma_{ij}}{a_{ij}\overline{\gamma}_{ij}} \left| \begin{array}{c} \beta_{ij}/2 \\ - \end{array} \right. \right]
\]  
(4.32)

Using (4.31) and (4.32) in (4.30), the average channel capacity can then be written in closed-form as [60]

\[
\overline{C} = \frac{\beta_{ij}}{2(a_{ij}\overline{\gamma}_{ij})^{\beta_{ij}/2} \ln(2) \sqrt{2\pi}} \times G_{k,l}^{k+l} \left[ \left. I(l, \frac{\beta_{ij}}{2}) I(l, \frac{\beta_{ij}}{2}) \right| \frac{(a_{ij}\overline{\gamma}_{ij})^{-\beta_{ij}/2}}{k^k} \right]
\]  
(4.33)
where \( I(n, \xi) = \frac{\xi}{n}, \left(\frac{\xi+1}{n}\right), \ldots, \left(\frac{\xi+n-1}{n}\right) \) with \( \xi \) as an arbitrary real value and \( n \) as a positive integer. Furthermore, \( \frac{k}{l} = \frac{\beta}{2} \) where \( k \) and \( l \) are positive integers, depending on the value of \( \beta \), e.g. for \( \beta = 1.5 \) we choose \( k = 4 \) and \( l = 3 \) and for \( \beta = 2 \) we choose \( k = 2 \) and \( l = 2 \).

In studying the channel capacity of the dual branch diversity receiver (4.29) is used whereby the PDF of \( \gamma_{ij} \) is replaced by the joint PDF of \( \gamma_{i_1} \) and \( \gamma_{j_2} \) as

\[
C = BW \int_0^{\infty} \int_0^{\infty} \log_2(1 + \gamma_{i_1}, \gamma_{j_2}) p_{\gamma_{i_1}, \gamma_{j_2}}(\gamma_{i_1}, \gamma_{j_2}) d\gamma_{i_1} d\gamma_{j_2} \quad (4.34)
\]

For independent bivariate Weibull distribution, the average channel capacity is obtained by substituting (4.13) into (4.34) as

\[
C = BW \int_0^{\infty} \int_0^{\infty} \log_2(1 + \gamma_{i_1}, \gamma_{j_2}) \frac{\beta_{i_1} \beta_{j_2}}{4(a_{i_1, j_1} \gamma_{i_1})^{\beta_{i_1}/2} (a_{j_2, j_1} \gamma_{j_2})^{\beta_{j_2}/2}} \times \exp\left[-\left(\frac{\gamma_{i_1}}{a_{i_1, j_1}}\right)^{\beta_{i_1}/2}\right] \exp\left[-\left(\frac{\gamma_{j_2}}{a_{j_2, j_1}}\right)^{\beta_{j_2}/2}\right] d\gamma_{i_1} d\gamma_{j_2} \quad (4.35a)
\]

\[
C = \frac{BW \beta_{i_1} \beta_{j_2}}{4(a_{i_1, j_1})^{\beta_{i_1}/2} (a_{j_2, j_1})^{\beta_{j_2}/2} \ln(2)} \int_0^{\infty} \int_0^{\infty} \gamma_{i_1}^{\beta_{i_1}/2} \gamma_{j_2}^{\beta_{j_2}/2} \ln(1 + \gamma_{i_1}, \gamma_{j_2}) \times \exp\left[-\left(\frac{\gamma_{i_1}}{a_{i_1, j_1}}\right)^{\beta_{i_1}/2}\right] \exp\left[-\left(\frac{\gamma_{j_2}}{a_{j_2, j_1}}\right)^{\beta_{j_2}/2}\right] d\gamma_{i_1} d\gamma_{j_2} \quad (4.35b)
\]

Again by expressing the logarithmic and exponential integrands as Meijer’s G-functions, (4.35b) can be written in closed-form as
\[ C = \frac{\text{BW} \beta_{i,j} \beta_{2,j} \left( \sqrt{k} l^{-1} \right)^2}{4 \left( a_{i,j} \bar{\gamma}_{i,j} \right)^{\beta_i/2} \left( a_{2,j} \bar{\gamma}_{2,j} \right)^{\beta_j/2} \left( \sqrt{2\pi} \right)^{k+2l-3} \ln(2)} \]

\[ \times G_{2l,k+2l}^{l+2l} \left[ \left( a_{i,j} \bar{\gamma}_{i,j} \right)^{-\beta_i,k/2} \left( a_{2,j} \bar{\gamma}_{2,j} \right)^{-\beta_j,k/2} \right] \left[ \left( l, \frac{\beta_{1,j}}{2} \right), \left( l, \frac{\beta_{2,j}}{2} \right) \right] \left[ I(k,0), I \left( l, \frac{\beta_{1,j}}{2} \right), I \left( l, \frac{\beta_{2,j}}{2} \right) \right] \]

(4.36)

where \( I(n, \xi) \equiv \frac{\xi}{n}, \left( \frac{\xi+1}{n} \right), \ldots, \left( \frac{\xi+n-1}{n} \right) \) with \( \xi \) as an arbitrary real value and \( n \) as a positive integer.

Under correlated conditions, the average channel capacity can be obtained from (4.34) by substituting (4.14). Having made this substitution, the average channel capacity is written as

\[ \bar{C} = \text{BW} \int_0^\infty \log_2 (1 + \gamma_{i,j}, \gamma_{2,j}) \beta_{1,j} \beta_{2,j} 4 \times \exp \left( -\frac{1}{(1-\rho)} \left[ \frac{\gamma_{i,j}^{\beta_i/2}}{(a_{i,j} \bar{\gamma}_{i,j})^{\beta_i/2}} + \frac{\gamma_{2,j}^{\beta_j/2}}{(a_{2,j} \bar{\gamma}_{2,j})^{\beta_j/2}} \right] \right) \times \]

\[ \sum_{k=0}^\infty \frac{1}{(k!)^2} \frac{\rho^k}{(1-\rho)^{2k+1}} \left( a_{i,j} \bar{\gamma}_{i,j} \right)^{\beta_i,k/2} \left( a_{2,j} \bar{\gamma}_{2,j} \right)^{\beta_j,k/2} d\gamma_{1,j} d\gamma_{2,j} \]  

(4.37)

According to [8], by expressing the logarithmic and exponential integrands as Meijer’s G-functions, (4.37) can be written in closed-form as
\[
\tilde{C} = \frac{BW \beta_1 \beta_2 j (\sqrt{k} t^{-1})^2}{4\left((\sqrt{2\pi})^{k+2l-3} \ln(2)\right)^2} \sum_{h=0}^{\infty} \frac{\rho^h}{(h!)^2 (1 - \rho)^{2h+1}} \left( (a_{1j} g_{1j})^{\beta_1/2} \right)^{h+1} \left( (a_{2j} g_{2j})^{\beta_2/2} \right)^{h+1}
\]

\times C_{2l,k+2l}^{k+2l/j} \left[ \frac{(a_{1j} g_{1j})^{-\beta_1/2}}{(1 - \rho)^k k^j} \left( I(l, -\frac{\beta_1 j}{2}), I(l, 1 - \frac{\beta_1 j}{2}) \right) \right]

\times C_{2l,k+2l}^{k+2l/j} \left[ \frac{(a_{2j} g_{2j})^{-\beta_2/2}}{(1 - \rho)^k k^j} \left( I(l, -\frac{\beta_2 j}{2}), I(l, 1 - \frac{\beta_2 j}{2}) \right) \right]

(4.38)

where \( I(n, \xi) \equiv \frac{\xi}{n}, \left( \frac{\xi + 1}{n} \right), \left( \frac{\xi + n - 1}{n} \right) \) with \( \xi \) as an arbitrary real value and \( n \) as a positive integer and the \( k \) appearing in (4.37) is replaced by \( h \) (4.38) to distinguish between the variables.

Another parameter to consider when studying the performance of wireless communication systems is the average bit-error probability.

### 4.3.4 Average Bit-Error Probability

The bit-error-rate refers to the ratio of the number of bits, symbols or elements incorrectly received to the total number of bits, symbols or elements sent during a specified time interval. Ideally this ratio is suppose to be zero but in reality there are errors experienced in communication channels especially those affected by fading, thus this can never be zero. A best performing communication system is one that is able to reduce the bit-error-rate (BER) to a smallest possible value. It is worth noting that the terms BER and BEP are used interchangeably in this thesis.
In studying the performance of the full complexity MIMO system and the transmit antenna selective MIMO system in terms of the BER, the moment-generating function (MGF) based approach is considered. This is because the MGF based approach is a bit easier to work with as it avoids the need to find the joint PDF of the output SNR which can be a bit tedious for high-order systems [22]. In general, the MGF of a random variable $Z_{ij}$ is given as [62, 63]

$$M_{Z_{ij}}(s) = \mathbb{E}(\exp(-sZ_{ij})) \tag{4.39}$$

and the joint MGF of $Z_{1j}$ and $Z_{2j}$ can be expressed as [6]

$$M_{Z_{1j}Z_{2j}}(s_1, s_2) = \mathbb{E}(\exp(-s_1Z_{1j} - s_2Z_{2j})) \tag{4.40}$$

If $p_{\gamma_j}(\gamma)$ denotes the PDF of $\gamma_j$, then based on (4.39) the MGF of the SNR can be written as [22]

$$M_{\gamma_j}(s) = \int_0^\infty p_{\gamma_j}(\gamma) \exp(s\gamma_j) d\gamma_j \tag{4.41}$$

By substituting (4.10) into (4.41), the MGF of the output SNR of a MRC receiver operating over independent Weibull fading channels, $M_{\gamma_{ij}}(s) = \prod_{r=1}^{l}M_{\gamma_j}(s)$ where $M_{\gamma_j}(s)$ is the MGF of the SNR of the $ij$th transmission path can be obtained as [34]

$$M_{\gamma_{ij}}(s) = \prod_{r=1}^{l} \beta_{ij} \left( \frac{\beta_{ij}/2}{\beta_{ij}} \right) \mathbb{E}(\exp(-s^{\gamma_j} - \beta_{ij}/2)) \tag{4.42}$$

The above integral can be evaluated in closed-form by expressing the exponential function as a Meijer’s G-function given as [34]

$$\exp\left[-s\gamma_j - \left( \frac{\gamma_j}{a_j\overline{\gamma}_j} \right)^{\beta_{ij}/2} \right] = G_{1,1}^{1,0}\left[s\gamma_j \middle| 0 \right] G_{1,1}^{1,0}\left[\frac{\gamma_j}{a_j\overline{\gamma}_j} \left| \beta_{ij}/2 \right. \right] \tag{4.43}$$

Substituting (4.43) into (4.42) the MGF expression can be written as
\[
M_{\gamma_{\text{MRC}}} (s) = \prod_{i=1}^{l} \frac{\beta_{ij}}{2 (a_{ij} \overline{\gamma}_{ij})^{\frac{\beta_{ij}}{2}}} x \int_{0}^{\infty} G^{(\beta_{ij}/2-1)}_{0,1} \left[ s \gamma_{ij} \right] \left[ a_{ij} \overline{\gamma}_{ij} \right] d\gamma_{ij} \tag{4.44}
\]

In closed form (4.44) can be written as [61]

\[
M_{\gamma_{\text{MRC}}} (s) = \prod_{i=1}^{l} \frac{\beta_{ij}}{2 (a_{ij} \overline{\gamma}_{ij})^{\frac{\beta_{ij}}{2}}} \left( \frac{k}{2} \cdot \frac{k+1}{2} \right)^{\frac{\beta_{ij}}{2}} x G^{k,l}_{0,1} \left[ \frac{1}{s^l \left( \frac{k}{2} \cdot \frac{k+1}{2} \right)^{\frac{\beta_{ij}}{2}}} \right] \tag{4.45}
\]

where \(a_{ij} = 1/\Gamma(1 + 2/\beta_{ij})\), \(\overline{\gamma}_{ij}\) is the average SNR in the \(ij\)th input branch and \(\frac{l}{k} = \beta_{ij}/2\) where \(k\) and \(l\) are positive integers, depending on the value of \(\beta\), e.g. for \(\beta = 1.5\) we choose \(k = 4\) and for the transmit antenna selective MIMO system \(L = 2\).

When considering correlation, the MGF of the output SNR of a MRC dual receiver, \(M_{\gamma_{\text{MRC}}} (s) = M_{\gamma_{1,2}} (s, s)\) can be written as [8]

\[
M_{\gamma_{\text{MRC}}} (s) = \frac{\beta_{1,2} \beta_{2,1}}{4} \sum_{h=0}^{\infty} \frac{\rho^{h}}{(h!)^2 (1 - \rho)^{2h+1}} \prod_{i=1}^{L} \frac{1}{\left( s, a_{ij} \overline{\gamma}_{ij} \right)^{\frac{\beta_{ij}}{2}}} \tag{4.46}
\]

where
\[
\gamma \left[ \frac{1}{(1 - \rho)\left(s, \alpha_j, \gamma_j\right)^{\beta_j/2}} \right]^{(h+1)\beta_j/2} = \frac{l^{-(h+1)\beta_j/2}}{(\sqrt{2\pi})^{k+1/2}} \times \\
\frac{1}{(1 - \rho)\left(s, \alpha_j, \gamma_j\right)^{\beta_j/2}} \cdot \frac{l^{-(h+1)\beta_j/2}}{(\sqrt{2\pi})^{k+1/2}} \times \\
G_{t,k}^{k} \left[ \left( \frac{1}{(1 - \rho)\left(s, \alpha_j, \gamma_j\right)^{\beta_j/2}} \right)^{l'} \cdot \frac{1}{k^k} \right]^{l'(h+1)\beta_j/2 - \frac{2-(h+1)\beta_j}{2} - \frac{l-(h+1)\beta_j}{2}} \cdot \frac{0}{k^{-k-1/k}}
\]

\[
\rho = \frac{\Gamma^2(d_d)\Gamma(d_z) - \Gamma^2(d_z)}{\Gamma(d_z)} [\Gamma(d_z) - \Gamma^2(d_z)]
\]

is the correlation coefficient, with \((0 < \delta \leq 1)\) denoting the dependence factor. Equation (4.46) can then be written as

\[
M_{\gamma_{\text{MRC}}} (s) = \frac{\beta_{ij} \beta_{zz}}{4} \sum_{h=0}^{\infty} \frac{\rho^h}{(h!)^2 (1 - \rho)^{2h+1}} \cdot \prod_{i=1}^{l} \frac{1}{\left(\left(s, \alpha_j, \gamma_j\right)^{\beta_j/2}\right)^{h+1} \left(\left(s, \alpha_j, \gamma_j\right)^{\beta_j/2}\right)^{h+1}} \times \\
\frac{1}{\left(\left(s, \alpha_j, \gamma_j\right)^{\beta_j/2}\right)^{h+1}} \cdot \frac{l^{-(h+1)\beta_j/2}}{(\sqrt{2\pi})^{k+1/2}} \times \\
G_{t,k}^{k} \left[ \left( \frac{1}{(1 - \rho)\left(s, \alpha_j, \gamma_j\right)^{\beta_j/2}} \right)^{l'} \cdot \frac{1}{k^k} \right]^{l'(h+1)\beta_j/2 - \frac{2-(h+1)\beta_j}{2} - \frac{l-(h+1)\beta_j}{2}} \cdot \frac{0}{k^{-k-1/k}}
\]

For the transmit antenna selective MIMO system, \(L = 2\).

In analysing the average error probability performance for M-ary phase shift keying (M-PSK), we need to evaluate integrals of the form \([64]\)

\[
\overline{P_e} = \frac{1}{\pi} \int_0^{(M-1)/M} \int_0^{\infty} \left[ \exp \left( - \frac{a^2 \gamma_j}{2\sin^2 \phi} \right) \right]^{p_{\gamma_j} \phi} d\gamma_j d\phi
\]

(4.48)

where \(a^2 = 2 \sin^2 \pi / M\).

After integrating (4.48) with respect to \(\gamma_j\), the equation simplifies to \([6]\)
\begin{equation}
\overline{P_e} = \frac{1}{\pi} \int_0^{(M-1)\pi/M} M_{\text{Ray}} \left( \frac{g_{\text{PSK}}}{\sin^2 \phi} \right) d\phi
\end{equation}

where \( g_{\text{PSK}} = \sin^2 \left( \pi / M \right) \)

For binary phase-shift keying (BPSK), where \( M = 2 \) in (4.49), the equation simplifies to

\begin{equation}
\overline{P_e} = \frac{1}{\pi} \int_0^{\pi/2} M_{\text{Ray}} \left( \frac{1}{\sin^2 \phi} \right) d\phi
\end{equation}

Substituting (4.45) into (4.50), the average bit-error probability under independent Weibull channel can be expressed as

\begin{equation}
\overline{P_e} = \frac{1}{\pi} \int_0^{\pi/2} M_{\text{Ray}} \left( \frac{1}{\sin^2 \phi} \right) d\phi
\end{equation}

Substituting (4.47) into (4.50), the average bit-error probability under correlated Weibull channels can be written as

\begin{equation}
\overline{P_e} = \frac{1}{\pi} \int_0^{\pi/2} M_{\text{Ray}} \left( \frac{1}{\sin^2 \phi} \right) d\phi
\end{equation}

where for the dual branch transmit antenna selective MIMO system \( L = 2 \).

Substituting (4.47) into (4.50), the average bit-error probability under correlated Weibull channels can be written as

\begin{equation}
\overline{P_e} = \frac{1}{\pi} \int_0^{\pi/2} M_{\text{Ray}} \left( \frac{1}{\sin^2 \phi} \right) d\phi
\end{equation}

where for the dual branch transmit antenna selective MIMO system \( L = 2 \).
where again for the dual branch transmit antenna selective MIMO system $L = 2$. Lastly, we consider the outage probability performance.

### 4.3.5 Outage Probability

This is the last performance parameter to consider for discussion. Outage probability is defined as the probability that the system’s output SNR, $\gamma_{MRC}$, falls below a set threshold level, $\gamma_{th}$ [64]. According to [65], using the MGF approach, the outage probability can be expressed as

$$P_{out} = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} \frac{M_{\gamma_{MRC}}(s)}{s} e^{s\gamma_{th}} ds$$

where $c$ is a constant selected in the region of convergence in the complex $s$-plane.

In closed form, outage probability can be written as [66]

$$P_{out} = L^{-1} \left[ \frac{M_{\gamma_{MRC}}(s)}{s} ; \gamma_{MRC}, \gamma_{th} \right]_{\gamma_{MRC}=\gamma_{th}}$$

where $L^{-1}[\cdot]$ denotes the inverse Laplace transform.

By substitution (4.45) into (4.54), the outage probability under independent distributed Weibull channels can be obtained as
Under correlated Weibull distributed fading channels, the outage probability can be obtained by substituting (4.47) into (4.54), and in this case the outage probability can be expressed as

$$P_{\text{out}} = L^{-1} \left[ \prod_{\ell=1}^{L} \frac{\beta_{ij} \left( \frac{j}{\ell} \right)^{\beta_{ij}} \left( \frac{j}{\ell} \right)^{\frac{\beta_{ij}}{2}}}{2 \left( a_j \bar{\gamma}_j \right)^{\beta_{ij}} \left( 2\pi \right)^{\frac{k+1}{2}}} \times G_{i,k}^{k,l} \left( \frac{a_j \bar{\gamma}_j}{s^l} \right)^{\frac{\beta_{ij}}{2}} \frac{l^l}{k^k} \right] s \gamma_{\text{MRC}}$$

(4.55)

$$P_{\text{out}} = L^{-1} \left[ \prod_{\ell=1}^{L} \frac{\beta_{ij} \left( \frac{j}{\ell} \right)^{\beta_{ij}} \left( \frac{j}{\ell} \right)^{\frac{\beta_{ij}}{2}}}{2 \left( a_j \bar{\gamma}_j \right)^{\beta_{ij}} \left( 2\pi \right)^{\frac{k+1}{2}}} \times G_{i,k}^{k,l} \left( \frac{a_j \bar{\gamma}_j}{s^l} \right)^{\frac{\beta_{ij}}{2}} \frac{l^l}{k^k} \right] s \gamma_{\text{MRC}}$$

(4.56)
For both the independent and correlated conditions, $L = 2$ denoting the dual branch transmit antenna selective MIMO system.

### 4.4 Chapter Summary

The transmit antenna selective MIMO system model has been presented and analysed. Under system performance, the equation for the average SNR has been stated. This is a very important parameter as the other performance parameters discussed have been derived based on average SNR. Using the average SNR, we have been able to derive the equations for the amount of fading, average channel capacity, average bit-error probability and outage probability. These equations have been derived for both independent and correlated systems.

In the following chapter, results based on the derived equations are presented. These are further analysed to give a clear understanding of the different performance.
CHAPTER 5

5 Results and Analysis

5.1 Introduction

This chapter presents the performance results obtained using the equations derived in Chapter 4. Graphs showing the SNR improvement with an increase in the number of antennas are plotted. We also show plots of the fading parameter against the amount of fading, average SNR against average channel capacity, average SNR against average bit-error probability and average SNR against outage probability for the transmit antenna selective MIMO system. These results are then compared with those of the full complexity MIMO and SISO systems. In our analysis, perfect channel state information is assumed at the receiver and partial channel state information assumed at the transmitter through a feedback channel. The transmit power is maintained constant independent of the number of transmitters used. When a single transmit antenna is selected, all the transmit power is then concentrated on this selected antenna. At the receiver end, two sufficiently spaced receiving antennas are used and MRC is used to combine the two signals in dual receivers. (1;1) represents a SISO system, (4;2), (3;2) and (2;2) represent the full complexity MIMO systems with four, three and two transmitting antennas respectively and two receiving antennas. The transmit antenna selective MIMO system with four, three and two transmitting antennas is represented by (4,1;2), (3,1;2) and (2,1;2) respectively. Where in all the instances one transmit antenna is selected and the signal received by two receiving antennas.

5.2 Experimental Results and Analysis

In studying the performance of the transmit antenna selective MIMO, full complexity MIMO and SISO systems the equations that were derived in Chapter 4 were analysed using MATLAB and Mathematica.
5.2.1 Average Signal-to-Noise Ratio

The average SNR for the different transmit antenna selective MIMO systems was plotted using (4.16). Figures 5.1, 5.2 and 5.3 show the average SNR plots for the different transmit antenna selective MIMO systems for \( \beta = 1.5, 2 \) and 3 respectively, with \( \gamma_2 = \gamma_1 \). Figures 5.4, 5.5 and 5.6 also show the average SNR improvement for \( \gamma_2 = 0.5 \gamma_1 \) and \( \beta = 1.5, 2 \) and 3 respectively. In all the plots, the total average SNR improves with an increase in the number of available transmit antennas in the system. An improvement in the average SNR implies the system is better performing. This means that the signals are less prone to fading and they can be easily decoded at the receiver end. For higher values of the fading parameter, larger values of the total average SNR are obtained. This shows the impact of the fading parameter on the average SNR performance. The SISO system is shown to be outperformed by both the full complexity MIMO system and the transmit antenna selective MIMO system as expected. In all the plots the performance of the \( (2;2) \) system compares to that of the \( (4,1;2) \) system. This shows that the same performance standards as those of the full complexity MIMO system can be achieved by increasing the number of available transmit antennas in the transmit antenna selective MIMO system. Increasing the number of available transmit antennas does not affect the system complexity but only improves its performance.
Figure 5.1: Average SNR improvement for $\beta = 1.5$ and $\gamma_2 = \gamma_1$.

Figure 5.2: Average SNR improvement for $\beta = 2$ and $\gamma_2 = \gamma_1$. 
Figure 5.3: Average SNR improvement for $\beta = 3$ and $\gamma_2 = \gamma_1$

Figure 5.4: Average SNR improvement for $\beta = 1.5$ and $\gamma_2 = 0.5\gamma_1$
Figure 5.5: Average SNR improvement for $\beta = 2$ and $\gamma_2 = 0.5\gamma_1$

Figure 5.6: Average SNR improvement for $\beta = 3$ and $\gamma_2 = 0.5\gamma_1$
5.2.2 Amount of Fading

Using (4.21) the amount of fading for the transmit antenna selective MIMO system was plotted as a function of the fading parameter and different correlation coefficients. In Figure 5.7, the two receive antennas have equal average SNR, i.e. $\gamma_2 = \gamma_1$. Whereas in Figure 5.8 the average SNR in the two receive antennas is not equal, i.e. $\gamma_2 = 0.5\gamma_1$. The two plots show that as the correlation coefficient decreases, so does the amount of fading, implying better system performance. This is due to the reduced degree of interference between the two receive antennas. It is also shown that as the fading parameter increases, there is a decline in the amount of fading, meaning that at higher values of the fading parameter, the system performs better as it is least affected by fading. It is worth noting that the fading parameter is characteristic of the fading channel used.

![Amount of Fading](image)

Figure 5.7: Amount of fading (AoF) against the Weibull fading parameter for several values of the correlation coefficient, $\rho$ with $\gamma_2 = \gamma_1$
In studying the average channel capacity, average bit-error probability and outage probability under correlated conditions there is need to calculate the number of terms to be summed up in order for the different equations to converge. Equation (4.38) was used to construct Tables 5.1, 5.2 and 5.3, which show the number of terms to be summed for the average channel capacity to converge for different transmit antenna selective MIMO systems. Tables 5.4, 5.5 and 5.6 show the number of terms needed to be summed up in order for (4.52) to converge. These values are then considered when evaluating the average bit-error probability. All these tables show that as $\rho$ increases, so does the number of terms needed to be summed up to achieve certain accuracy in the average channel capacity and average bit-error probability. The average SNR also plays a role in determining the number of terms that need to be summed up. An increase in the average SNR results in fewer terms to be summed up. The same conclusions can be said for (4.56) which is used to evaluate the outage probability. Tables 5.7, 5.8 and 5.9 show the outage probability convergence properties.
Table 5.1: Number of terms for convergence of the average channel capacity for
(2,1;2) transmit antenna selective MIMO system, with $\beta = 1.5$, $\gamma = \gamma_1 = \gamma_2$

<table>
<thead>
<tr>
<th>$\bar{\gamma}$ (dB)</th>
<th>$\rho = 0.1$</th>
<th>$\rho = 0.3$</th>
<th>$\rho = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.7578</td>
<td>3</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>7.7578</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>14.758</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 5.2: Number of terms for convergence of the average channel capacity for
(3,1;2) transmit antenna selective MIMO system, with $\beta = 2$, $\gamma = \gamma_1 = \gamma_2$

<table>
<thead>
<tr>
<th>$\bar{\gamma}$ (dB)</th>
<th>$\rho = 0.1$</th>
<th>$\rho = 0.3$</th>
<th>$\rho = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5.3: Number of terms for convergence of the average channel capacity for
(4,1;2) transmit antenna selective MIMO system, with $\beta = 3$, $\gamma = \gamma_1 = \gamma_2$

<table>
<thead>
<tr>
<th>$\bar{\gamma}$ (dB)</th>
<th>$\rho = 0.1$</th>
<th>$\rho = 0.3$</th>
<th>$\rho = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.55565</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>6.5557</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>13.556</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 5.4: Number of terms for convergence of the average bit-error probability for
(2,1;2) transmit antenna selective MIMO system, with $\beta = 1.5$, $\bar{V} = \bar{V}_1 = \bar{V}_2$

<table>
<thead>
<tr>
<th>$\bar{V}$ (dB)</th>
<th>$\rho = 0.1$</th>
<th>$\rho = 0.3$</th>
<th>$\rho = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.7578</td>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>7.7578</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>14.758</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 5.5: Number of terms for convergence of the average bit-error probability for
(3,1;2) transmit antenna selective MIMO system, with $\beta = 2$, $\bar{V} = \bar{V}_1 = \bar{V}_2$

<table>
<thead>
<tr>
<th>$\bar{V}$ (dB)</th>
<th>$\rho = 0.1$</th>
<th>$\rho = 0.3$</th>
<th>$\rho = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 5.6: Number of terms for convergence of the average bit-error probability for
(4,1;2) transmit antenna selective MIMO system, with $\beta = 3$, $\bar{V} = \bar{V}_1 = \bar{V}_2$

<table>
<thead>
<tr>
<th>$\bar{V}$ (dB)</th>
<th>$\rho = 0.1$</th>
<th>$\rho = 0.3$</th>
<th>$\rho = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.55565</td>
<td>4</td>
<td>7</td>
<td>18</td>
</tr>
<tr>
<td>6.5557</td>
<td>3</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>13.556</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>
Table 5.7: Number of terms for convergence of the outage probability for
(2,1;2) transmit antenna selective MIMO system, with $\beta = 1.5$, $\bar{y} = \bar{y}_1 = \bar{y}_2$

<table>
<thead>
<tr>
<th>$\bar{y}$ (dB)</th>
<th>$\rho = 0.1$</th>
<th>$\rho = 0.3$</th>
<th>$\rho = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.7578</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>7.7578</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>14.758</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 5.8: Number of terms for convergence of the outage probability for
(3,1;2) transmit antenna selective MIMO system, with $\beta = 2$, $\bar{y} = \bar{y}_1 = \bar{y}_2$

<table>
<thead>
<tr>
<th>$\bar{y}$ (dB)</th>
<th>$\rho = 0.1$</th>
<th>$\rho = 0.3$</th>
<th>$\rho = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 5.9: Number of terms for convergence of the outage probability for
(4,1;2) transmit antenna selective MIMO system, with $\beta = 3$, $\bar{y} = \bar{y}_1 = \bar{y}_2$

<table>
<thead>
<tr>
<th>$\bar{y}$ (dB)</th>
<th>$\rho = 0.1$</th>
<th>$\rho = 0.3$</th>
<th>$\rho = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.55565</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>6.5557</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>13.556</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
5.2.3 Average Channel Capacity

The average channel capacity performance for the transmit antenna selective MIMO system under independent channel conditions was analysed using (4.36) and the results plotted in Figures 5.9 for $\beta = 1.5$, Figure 5.10 for $\beta = 2$ and Figure 5.11 for $\beta = 3$. In these figures, the average channel capacity is plotted as a function of the average SNR. Here the two receiver diversity branches are assumed to have equal average SNR, i.e. $\bar{\gamma}_2 = \bar{\gamma}_1$. The plots illustrate an improvement in performance with an increase in the average SNR. This shows that average SNR is an important parameter in wireless communication systems. Since the average SNR is dependent on the transmit power in the system, an increase in the transmit power results in an improvement in the average SNR. Nevertheless, there is a limit to the amount of power that can be used in wireless communication systems. There is therefore a need to consider another factor that can enhance system performance. This is the number of transmit antennas. An improvement in performance is observed as the number of available transmit antennas is increased in the system. The fading parameter is also shown to play a role in system performance, whereby the performance is enhanced with increases in fading parameter, implying that system is less prone to fading.
Figure 5.9: Normalised average channel capacity against average SNR with $\beta = 1.5$ and $\overline{\gamma}_2 = \overline{\gamma}_1$

Figure 5.10: Normalised average channel capacity against average SNR with $\beta = 2$ and $\overline{\gamma}_2 = \overline{\gamma}_1$
The average channel capacity performance under correlated channel conditions was analysed using (4.38). Based on this equation, Figures 5.12, 5.13 and 5.14 were plotted for $\beta = 1.5$, 2 and 3 respectively with $\rho = 0.1$, 0.3 and 0.5 and equal average SNR on each diversity branch. Just like in the independent conditions, the average channel capacity performance is enhanced by improving the average SNR and also by increasing the number of available transmit antennas. In this case, the correlation coefficient also plays a role in system performance, whereby for lower correlation coefficient the transmit antenna selective MIMO system performs better. This is because at low correlation coefficients the level of interference is minimum thus the system is better performing.

Figure 5.11: Normalised average channel capacity against average SNR with $\beta = 3$ and $\gamma_2 = \gamma_1$
Figure 5.12: Normalised average channel capacity against average SNR with $\beta = 1.5$, $\rho = 0.1, 0.3$ and $0.5$ and $\gamma_2 = \gamma_1$

Figure 5.13: Normalised average channel capacity against average SNR with $\beta = 2$, $\rho = 0.1, 0.3$ and $0.5$ and $\gamma_2 = \gamma_1$
5.2.4 Average Bit-Error Probability

We first consider the BEP performance of the transmit antenna selective MIMO system under independent channel conditions. Figures 5.15, 5.16 and 5.17 show plots of the average BEP plotted against the average SNR for $\beta = 1.5$, $2$ and $3$ respectively and equal average SNR in each diversity branch. In these figures, the average BEP results are based on (4.51). For the various fading parameters the system performance is enhanced by increasing average SNR and the number of available antennas in the transmit antenna selective MIMO system. This is shown by the improved performance in terms of the average BEP, where the SISO system is greatly outperformed by the proposed system.
Figure 5.15: Average BEP against average SNR with $\beta = 1.5$ and $\gamma_2 = \gamma_1$

Figure 5.16: Average BEP against average SNR with $\beta = 2$ and $\gamma_2 = \gamma_1$
The next set of plots show the average BEP performance of the transmit antenna selective MIMO system versus the average SNR under correlated channels. In this, (4.52) is used to plot Figures 5.18, 5.19 and 5.20 under different correlation coefficients with equal average SNR and for $\beta = 1.5, 2$ and $3$ respectively with $\rho = 0.1, 0.3$ and $0.5$. In all the figures, the performance evaluation shows that the average BEP improves with decrease in the correlation coefficient. The number of available transmitting antennas also plays a great role in improving the average BEP. An improved average BEP implies better system performance in terms of quality as the signals are received with minimal errors.
Figure 5.18: Average BEP against average SNR

with $\beta = 1.5$, $\rho = 0.1, 0.3$ and $0.5$ and $\bar{\gamma}_2 = \bar{\gamma}_1$

Figure 5.19: Average BEP against average SNR

with $\beta = 2$, $\rho = 0.1, 0.3$ and $0.5$ and $\bar{\gamma}_2 = \bar{\gamma}_1$
Figure 5.20: Average BEP against average SNR

with $\beta = 3, \rho = 0.1, 0.3$ and $0.5$ and $\gamma_2 = \gamma_1$

5.2.5 Outage Probability

The outage probability under independent channel conditions was plotted using (4.55). Figures 5.21, 5.22 and 5.23 show the plot of the outage probability versus the average SNR with a threshold SNR of 5dB, $\gamma_2 = \gamma_1$ and $\beta = 1.5, 2$ and $3$ respectively. Just like the average BEP performance, the outage probability performance is enhanced by increasing the number of available transmit antennas in the transmit antenna selective MIMO system. For an increase in the fading parameter there is also an improvement in the system performance, but as earlier stated, the fading parameter is characteristic of the fading channel.
Figure 5.21: Outage probability against average SNR

\[ \text{with } \beta = 1.5, \gamma_2 = \gamma_1 \text{ and } \gamma = 5dB \]

Figure 5.22: Outage probability against average SNR

\[ \text{with } \beta = 2, \gamma_2 = \gamma_1 \text{ and } \gamma = 5dB \]
Figure 5.23: Outage probability against average SNR

\[ \beta = 3, \gamma_2 = \gamma_1 \text{ and } \gamma_{th} = 5dB \]

Figures 5.24, 5.25 and 5.26 show the outage probability plotted under correlated fading channels using (4.56). These are the plots of outage probability versus average SNR with \( \gamma_2 = \gamma_1 \), \( \gamma_{th} = 5dB \) and \( \beta = 1.5, 2, 3 \) respectively for different correlation coefficients. The plots show deterioration in the outage probability performance with increase in the correlation coefficients. However, performance improves with increase in the number of available transmit antennas.
Figure 5.24: Outage probability versus average SNR
with $\beta = 1.5$, $\rho = 0.1, 0.3$ and $0.5$, $\bar{\gamma}_2 = \bar{\gamma}_1$ and $\gamma_{th} = 5dB$

Figure 5.25: Outage probability versus average SNR
with $\beta = 2$, $\rho = 0.1, 0.3$ and $0.5$, $\bar{\gamma}_2 = \bar{\gamma}_1$ and $\gamma_{th} = 5dB$
Figure 5.26: Outage probability versus average SNR

with $\beta = 3, \rho = 0.1, 0.3 \text{ and } 0.5, \gamma_2 = \gamma_1$ and $\gamma_{th} = 5dB$

5.3 Chapter Summary

The performance results for both independent and correlated transmit antenna selective MIMO systems were presented. Graphs of Weibull fading parameter against amount of fading, average SNR against average channel capacity, average BEP and outage probability were plotted. Some of these results were compared with those of the full complexity MIMO system and SISO system. In all the plots, the transmit antenna selective MIMO system was observed to outperform the SISO system but was low-grade in performance compared to the full complexity MIMO system, though less complex. These results confirm the advantages associated with using multiple antennas in wireless communication systems. Another factor shown to affect performance was the correlation coefficient. As the correlation coefficient was increased a decline in the performance of the system was observed. This is due to the interference that one diversity branch experiences with respect to the other.

The last chapter concludes the study and highlights issues for future studies.
CHAPTER 6

6 Conclusions and Future Work

6.1 Introduction

The conclusions and future work on the transmit antenna selective MIMO system are discussed. The project hypothesis and the summary of the entire work are also presented.

6.2 Conclusions

MIMO systems play a great role in improving the performance of wireless communication systems, especially those affected by fading. However, such an improvement comes at the cost of increased system complexity. By increasing the number of antennas used in the system one also increases the RF chains to be decoded at the receiver end. To solve the complexity problem, the transmit antenna selective MIMO system was proposed in the downlink. The single best transmit antenna is selected from all the available transmit antennas via a feedback channel which has the CSI from the receiver end. Having selected this single antenna, the transmit power is then concentrated on this one antenna whilst the other antennas are put into a sleep mode. Communication continues through this antenna until the signal level falls below a set threshold after which all the other antennas are reactivated. The system then goes through the same selection criteria again until the end of communication. In studying the performance of the proposed system, the Weibull fading channels were considered.

In Chapter 2, the Weibull RV’s PDF and CDF were derived. These parameters were used in Chapter 4 where the system performance metrics were presented. Equations for the average SNR, amount of fading, average channel capacity, average BEP and outage probability were derived for both independent and correlated systems. Using these derived equations, the performance results were presented in Chapter 5 in the form of performance graphs.
In all the plotted graphs the SISO system was outperformed by both the transmit antenna selective MIMO and the full complexity MIMO system as expected. The average channel capacity, average BEP and outage probability performance in the transmit antenna selective MIMO system was observed to improve with an increase in the number of available transmit antennas. This improvement in performance verified the merits of multiple antenna systems. We were also able to show that regardless of the number of available transmit antennas in the proposed system, there are always two branches to decode at the receiver end as compared to the full complexity system whereby the number of branches increases with the number of antennas. In all our results the (4,1;2) system’s performance parameters compare to those of the (2;2) system. The (4,1;2) system has two diversity branches to be decoded compared to the (2;2) system which has four diversity branches. This shows that the same performance standards may be achieved by using the less complex transmit antenna selective MIMO system. Hence, for a large number of available transmit antennas the transmit antenna selective MIMO system can outperform the lower level full complexity MIMO systems at minimised complexity.

Having analysed the performance of the transmit antenna selective MIMO, SISO and full complexity MIMO systems the project hypothesis may then be verified. The project hypothesis can be restated as transmit antenna selection is a technique that can be used in wireless communication to help bridge the gap between SISO and MIMO systems by providing inexpensive, reliable communication. In which inexpensive is associated with SISO systems and reliable with MIMO systems. From the results, we observe an improvement in the performance of the transmit antenna selective MIMO system with an increase in the number of available transmit antennas. This increase in the number of transmit antennas does not affect the number of RF chains to be decoded at the receiver end. In the (4,1;2), (3,1;2) and (2,1;2) systems there are always two diversity branches at the receiver end. Thus, the goal of providing improved communication without any expansion in the system complexity is attained with transmit antenna selection. The transmit antenna selective MIMO system does indeed bridge the gap between the SISO and MIMO systems by providing inexpensive, reliable communication.
6.3 Future Work

In studying the performance of the transmit antenna selective MIMO system the CSI was assumed to be available at the receiver end. However, in real life situations this is not the case. It would be interesting to observe how channel estimation would affect the performance of the proposed system. The transmitter and receiver ends were assumed to be in stationary positions but we know that wireless mobile communication devices are always in motion with respect to the transmitter station. Work could be done on the system performance whereby mobility is taken into consideration, thus the impact of the Doppler Effect on the performance the transmit antenna selective MIMO system. The performance of the transmit antenna selective MIMO system was studied under identically independent distributed Weibull fading channels. One can further extend this work to non-identically independent distributed Weibull fading channels. Slow, flat fading was considered when analysing the performance of the transmit antenna selective MIMO system. Research may be done on the system performance whereby fast fading or frequency selective fading is considered. Work can be done where the actual cost of implementing the transmit antenna selective MIMO system is compared to the cost of implementing the SISO and full complexity MIMO systems.

6.4 Chapter Summary

The summary of the project has been presented, where conclusions based on the performance of the proposed transmit antenna selective MIMO system compared to the SISO and full complexity MIMO systems where made. We were able to verify the project hypothesis which states that transmit antenna selection is a technique that can be used in wireless communication to help bridge the gap between SISO and MIMO systems by providing inexpensive, reliable communication. In which inexpensive is associated with SISO systems and reliable with MIMO systems. Based on our results and analysis transmit antenna selection is indeed a technique that can be employed in wireless communication to help provide cheap and reliable communication.
References


Summary of Corrections, LPPMZA001

N. B. All changes made are with reference to the original document.

Chapter 1,
Section 1.1
On page 2, paragraph 2, the words “it is” were added in sentence 3, and “more” was added in sentence 4.

Section 1.2
On page 3, paragraph 1, sentence 4, the word “and” was replaced with “due to”.

Still in paragraph 1, page 3, the fifth sentence which reads “Fading leads to the poor performance of a wireless system as it makes it difficult to decode the received signals due to reduced signal power at the receiver end. Furthermore, multipath propagation leads the signals to add up destructively at the receiver end.” was replaced with “There are two major effects of such fading. 1) Signal strength is attenuated, leading to a simple loss in $E_b / N_0$ (degraded error performance). 2) Channel-induced intersymbol interference (ISI) causes distortion and possibly an irreducible BER. The first effect can be mitigated with more $E_b / N_0$. But more $E_b / N_0$ will not necessarily help remove the degrading effects of ISI. Here, mitigation usually calls for an equalizer to remove the distortion.”

The first part of sentence 12 in paragraph 2, page 3, which reads “The problem with his scheme was that it operated” was replaced with “Although this is a simple rate 1 scheme, it only provides diversity gain (not coding gain), and thus operates”.

Still on the same paragraph, page 3, the word “other” was added in sentence 13.

The words “(as determined by the feedback information)” were added in the first sentence on page 5.

Reference [12] was added in the last sentence of paragraph 1 on page 5.

Section 1.3
The words “communication. In” were replaced with “communication, in” in paragraph 2 of page 5.

Section 1.5
On page 7, sentence 7, the words “Chapter 5. Where the” were replaced with “Chapter 5, where the”.

Chapter 2
Section 2.1
The word “Raleigh” was replaced with “Rayleigh” on page 8, sentence 3.
Section 2.2
On page 8, the sentence “Fading is the distortion that signals experience as they propagate through a channel.” was replaced with “As the signal propagates through the channel, the effects of fading can lead to a loss in SNR, or even worse, signal distortion.”

Still on the same page, the sentence “Fading may affect the quality of the sent signals and may cause multipath propagation, in which the signals may add up destructively at the receiver end.” was replaced with “The chief contributors of fading are multipath propagation and absorption. In multipath propagation the sent signals get diffracted or reflected on building, mountains, trees and water before reaching the receiver. At the receiver end, the signals that are received in-phase reinforce with each other to produce a stronger signal, whilst those that are received out of phase add up destructively. In absorption, there are energy losses at the receiver end.”


Section 2.2.1
On page 8, the sentence which reads “However, the system can suffer from reduced overall performance. By comparison, fast fading is complex but leads to improved system performance” was deleted and the words “compared to fast fading” were added at the end of “changing with time”.

Section 2.2.2
On page 9, second sentence, “on” was replaced with “some key generalities of”, “then” replaced with “next” and “on the various” replaced with “some particular”.

Section 2.3.4
On page 12, third paragraph, “βij is the fading parameter expressing the fading severity” was added.

Still on page 12, paragraph 4, “Rij = |Xij + jYij|, The Weibull” was replaced with “Rij = |Xij + jYij|, the Weibull”.

On page 15, paragraph 1, the second sentence was corrected from “Considering this relationship, (2.15) can be written as [33]” to “Considering this relationship, and making the necessary substitutions using (2.7), (2.15) can be written as [34]”.

Section 2.4
In the first sentence of the second paragraph on page 17, the word “great” was replaced with “significant”.

University of Cape Town
Chapter 3
Section 3.5

At the end of the second paragraph on page 20, reference [37] was added.

Section 3.6.1
On page 23, part of the first sentence on paragraph 1 was changed from “During the first” to “Considering the transmission matrix, (3.1), during the first”.

In the last paragraph of page 23, the sentence “In the context of STB codes the rate of a code is defined as the total number of input symbols in the transmit antennas divided by the total number of time slots in which these symbols are transmitted” was added after the first sentence.

The last sentence before the first paragraph on page 24 was modified to read as “respectively, where the labels $H_3$ and $H_4$ are introduced to try to differentiate between the half rate and three-quarter rate codes.”

The second sentence in the first paragraph on page 25 which reads “This implies that STBC are not ideal for providing increased system data rate, but can be used to provide reliable communication, in terms of improved signal quality.”, was replaced with “STBC can therefore be used to provide improved BER performance as a trade-off for higher data rates”.

Added reference [39] at the end of the last paragraph on page 25.

Section 3.7.3
Added a new reference [45], (M. Lupupa, and M. E. Dlodlo, “Channel capacity performance of transmit antenna selective MIMO system in Weibull fading,” in *Proceedings of the 12th Southern Africa Telecommunication Networks and Applications Conference (SATNAC)*, Sep. 2009.) on page 28 at the end of the first sentence in the first paragraph. This shifts all the references thereafter by one and the total number of references in the document increases to 66 references.

Section 3.8
Deleted the words “on the” on page 29 in the first sentence of paragraph 1.

The seventh sentence in the first paragraph on page 29 which reads “This means that space-time block codes are not ideal for improving data rates, but are good for improved
quality of communication.” was replaced with “STBC can therefore be used to provide improved BER performance as a trade-off for higher data rates.”

**Chapter 4**

Section 4.2

On page 30, the words “available” and “before selection” were added in the first sentence of the first paragraph.

The word “increase” was replaced with “increased” and the word “up” was deleted in the second sentence, first paragraph on page 30.

On page 31, the sentences “Before the signals are transmitted by the different antennas, information bits will be added to the signals to help identify which signal has been sent by a particular transmit antenna. At the receiver end, the signals received from the various transmit antennas will then be compared in terms of their respective SNR. The transmit antenna which results in a high SNR compared to the other antennas will then be identified using the extra information added to the signals. Having identified the best antenna this information is then sent back to the transmitter end through the feedback channel.” were added after the second sentence of paragraph 1.

**Section 4.3.1**

On page 36, paragraph 2, “n_th” was replaced with “n-th”.

**Section 4.3.3.4**

Added reference [38] to the list of references on top of page 41.

**Chapter 5**

Section 5.2

Added “In studying the performance of the transmit antenna selective MIMO, full complexity MIMO and SISO systems the equations that were derived in Chapter 4 were analysed using MATLAB and Mathematica.” on page 53.

**Chapter 6**

Section 6.2

On page 76, paragraph 1, sentence 6, the word “of” was replaced with “into a”.

Section 6.3

The sentence “Work can be done where the actual cost of implementing the transmit antenna selective MIMO system is compared to the cost of implementing the SISO and full complexity MIMO systems.” was added after the last sentence on page 78.

**References**