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Some applications of Quantitative Techniques in the Asset Management Industry

A research report presented to

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by

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ABSTRACT

The aim of this thesis is to provide the reader with some practical applications of quantitative techniques in the area of portfolio management. The theme of the thesis is on the use of basic quantitative applications, with an emphasis on issues pertaining to optimisation, benchmarking and risk management.

Most of the contributions and analysis performed in this thesis has been borne out of actual applications in the financial market industry - thus the style of the thesis reflects an application level relevant to practitioners, and is not esoteric. A large element of the thesis consequently makes use of graphical aids in the discussions and dissemination of results. The level of presentation of the thesis is aimed at avoiding lengthy technical expositions in favour of an "easy-to-read" thesis.

The thesis thus consists a collection of studies, some of which contain a common theme; others which remain unavoidable disjointed studies.

A significant contribution in the thesis is on the demonstration of the innovative Black and Litterman (1991) technique of quantitative portfolio design based on managers' views in the local South African context. These views are incorporated into the optimisation process so that investment weights tilt away from a benchmark in an intuitive and practical way.

The area of benchmarking is also reviewed, with specific emphasis on the selection and construction of benchmarks for various client requirements. Issues in portfolio risk management for active managers are considered, with the objective of setting risk mandates to avoid specified underperformance of active managers. In this chapter optimisation techniques are also used to estimate unknown holdings of competitor groups and simple graphical aids are highlighted to assist with interpreting benchmarking objectives.

Lastly, two studies of multi-manager topics are reviewed, namely the impact of combining managers on active risk as well as the current findings regarding the persistence of manager performance.

The thesis concludes with recommendations for asset managers to apply and adapt Capital Market Theory in managing active portfolios efficiently and successfully.

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CHAPTER 1: BACKGROUND

1.1 Introduction

One of the first and most important lessons of investments remains the trade-off between risk and return – as proposed by Markowitz (1952). Although these concepts had been intuitive amongst practitioners long before Markowitz (1952), the proposed theory sought to quantify portfolio characteristics (as a whole) in terms of mean and variance. When portfolios began to be judged against the index using these criteria, it turned out that only a handful of managers were able to “beat the index”. An early school of thought led by a group of professors in the mid-1960s promoted the concept of investing in the market rather than actively managed funds, and herewith the concept of an underlying assumption market equilibrium. They concluded that the market index (as measured by a value-weighted index of all available assets) would be a mean-variance efficient portfolio, under the simplifying assumptions of CAPM¹.

Since the mid-1960's, much of Capital Market Theory was based on the premise that markets are efficient. Thus, academic literature frequently implied an underlying equilibrium assumption, favouring its application for passive fund management. As a consequence, there has been little in the way of direct guidance for active portfolio managers, since the theory of market equilibrium suggests that active management is futile. However, this says nothing about the *potential* for successful active portfolio management if disequilibrium or inefficiencies occur. In fact, active management would always have a place in capital markets, since the extensive research of analysts to identify undervalued stocks is the primary reason for market efficiency.

The theory of active portfolio management has thus emerged as an important building block in the study of portfolio theory. The increasing awareness of terms such as *tracking error*, *information ratio* and *beta* in the fund management industry is a clear indication of the popularity of active portfolio theory, as these basic concepts provide managers with

¹ See Sharpe (1964), Linter (1965) and Mossin (1966).

practical starting points for a framework which traditional modern portfolio theory could not offer.

Empirical analyses are performed for certain sections where deemed necessary. However, where existing academic theory is deemed inadequate for the purposes of an empirical analysis, practical application of the theory is used, while attempting not to compromise statistical rigour. The aim of this thesis is to provide the reader with some practical applications of quantitative techniques in the area of active portfolio management. The theme of the thesis is predominantly quantitative, with an emphasis on issues pertaining to optimisation, benchmarking and risk management.

1.2 Structure of thesis

The thesis is partitioned into distinct chapters, where major areas of study in active portfolio theory is reviewed. The thesis thus consists a collection of studies, some of which contain a common theme, others that remain unavoidably disjointed studies – yet still fall within the common theme of quantitative portfolio theory.

Chapter 2 reviews the groundwork of prior academic literature in the area of Modern Portfolio Theory (MPT), Capital Asset Pricing Models and the theory of Active Portfolio Management.

Chapter 3 reviews the standard optimisation framework put forward by Markowitz (1952), and demonstrates an innovative practical refinement using the Black - Litterman methodology for adjusting return inputs.

In *Chapter 4*, the broad area of benchmarking is reviewed, with specific emphasis on the practical application of selecting, constructing and estimating benchmarks in the South African context.

Chapter 5 focuses on some of the important aspects of risk management for active managers, including expected tracking error, setting active risk mandates as well as reviewing the risk taking of consistent top performers.

Chapter 6 considers two multi managers topics of interest, namely the effect of combining managers on active risk, as well as the persistence of fund performance in the South African Unit Trust Industry.

Lastly, *Chapter 7* concludes with recommendations for practitioners.

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CHAPTER 2: LITERATURE REVIEW

In this chapter, significant contributions to the areas of modern portfolio theory, the Capital Asset Pricing Model and theory relating to active portfolio theory will be discussed. This literature forms the groundwork for the areas of quantitative finance to be investigated in ensuing chapters. The contributions to the area of Modern Portfolio theory follows in section 2.1, with developments of the Capital Asset Pricing Model discussed in section 2.2, followed by a brief outline of developments in the literature of active portfolio theory.

2.1 Modern Portfolio Theory

In the March 1952 edition of Journal of Finance, Harry Markowitz established a framework, which was heralded as a new paradigm in portfolio theory. This article represented one of the most significant movements towards a quantitative understanding of portfolio diversification, risk and return, and subsequently laid down the foundation for modern portfolio theory (MPT). A brief overview of this development as well as related extensions in the area of MPT follow in the ensuing section, after which a more detailed emphasis is placed on the major practical areas of portfolio theory, namely the lessons of portfolio diversification, choosing optimal risky portfolios, and lastly the allocation amongst the bundle of risky assets and the risk-free asset.

2.1.1 Overview

When Markowitz began his work in the area of portfolio theory, he was a University of Chicago graduate looking for a dissertation topic. In his Ph.D., Markowitz argued that investors are mean – variance optimisers, i.e. looking for the highest expected return from their portfolios, yet at the same time wishing to reduce the portfolio risk. These two measures, of expected return and risk were deemed sufficient to serve as a starting point for developing a theory of portfolio construction. The underpinnings were based on premise that: given investors are rational and have the same expectations and time horizons, they will wish to hold portfolios that are efficient (highest return for the same level of risk). The important criterion of these efficient portfolios however is the trade-off between expected return and risk, with the portfolio satisfying the highest trade-off

(Sharpe Ratio), deemed the optimal. Thus, by exploring the risk-return relationship between assets, the benefits of a diversified portfolio can be attained and quantified. As a consequence, portfolio selection models based on mean-variance optimisation techniques were generated in order to select an optimal mix of risky assets, with further algorithms proposed by Wolfe (1959), Houthakker (1960) and Sharpe (1963).

Tobin (1958) extended the work done by Markowitz through suggesting that the investment decision be broken up into two separate decisions. The first decision concerns the choice of optimal proportions of risky assets held which is independent of the individual's asset preferences. Tobin suggested the choice would naturally depend on the investors' expectations on each asset's return, risk and covariance with all other assets, yet assumed that expectations are homogenous across all investors. Thus, Tobin found that all investors would hold the same proportions of risky assets as their optimal portfolio of risky assets irrespective of the preferences. This optimal portfolio would by construction be the market portfolio, i.e. the proportion market capitalisation of the asset in the market would be identical to the proportion in each investor's optimal portfolio.

Given the investor is allowed to borrow (or lend) at the risk free rate, Tobin suggested that the second decision concerns the proportion of the market portfolio the investor holds and combines with cash. This decision is based on the individual's risk preference, giving the investor the ability to alter his portfolio risk simply by adjusting the proportion of cash he held on the portfolio. Thus, if investors were risk-averse, they would hold more cash. Conversely, if they were risk-taking, they would borrow at the risk-free rate (assuming the borrowing and lending rates were equal). However, regardless of the proportion held in cash, the bundle of risky assets held would remain proportional to the initial asset proportions of the risky portfolio. This separation of the investment decision is thus known as the separation theorem.

However, it was only years later when anyone related the risk of assets to their price. Sharpe, another PhD candidate in search of a research topic, was working at the Rand Corporation, where Markowitz had taken up residence. Sharpe (1964) made the initial breakthrough in the theory of asset pricing, by proposing that the risk of a security can be decomposed into two parts. Sharpe suggested the one aspect risk could be diversified away, whereas the other could not. Sharpe also remarked that diversifiable risk would be

borne by investors if they held a sub-set of the market portfolio, and since this risk could be avoided at no additional cost, it is not compensated for. Non-diversifiable risk, Sharpe stated, is unavoidably borne by investors and is thus rewarded. Sharpe referred to non-diversifiable risk as “systematic risk” and to diversifiable risk as “specific risk”. Given the common factor of every marketable asset is “systematic risk” (as reflected by the beta of the asset to the market), beta became the determining factor in the asset pricing model. This model was simple and elegant and became known as the Capital Asset Pricing Model (CAPM). Ironically, at around the same time Litner (1965) and Mossin (1966) were also deriving the CAPM. A more detailed discussion of the development of the CAPM will be examined in section 2.2.

The CAPM is essentially an expectations model of equilibrium asset prices, and implies that the risk premium on any individual asset or portfolio is the product of the risk premium on the market portfolio and the beta co-efficient of the asset. The beta co-efficient is estimated by covariance of the asset with the market portfolio as a fraction of the variance of the market portfolio. The most important implication of the CAPM however is that individuals will only be compensated for bearing market risk, and not unique risk. Subsequently, many researchers have suggested extensions of the CAPM and its underlying assumptions, for example Meyers (1972), Merton (1973), Brennan (1970) and Litzenberger and Ramaswamy (1979).

2.1.2 Portfolio Diversification

The principle of diversification was explored in the seminal work of Markowitz (1959) titled: “Portfolio Selection: Efficient Diversification of Investments.” This work on portfolio diversification was later extended by John and Archer (1968), Wagner and Lau (1971) and Merton (1973). The primary lesson of the literature on portfolio diversification asserts that the investor can eliminate some of the risk of the portfolio through spreading an investment across many assets will. These central concepts on the subject of portfolio diversification emerging from the literature will briefly be outlined in the ensuing paragraphs.

2.1.2.1 Two-asset case

Following Bodie, Kane and Marcus (1997), the case for a portfolio of two assets where the expected returns of each asset being $E(r_1)$ and $E(r_2)$, and the weights of each asset being w_1 and w_2 , with the expected return on the portfolio defined as:

$$E(r_p) = w_1E(r_1) + w_2E(r_2) \quad \dots(2.1)$$

The variance of each asset's returns is

$$\sigma_u^2 = \text{var}(r_i) = E[r_i - E(r_i)]^2 \quad \dots(2.2)$$

with the covariance between returns being

$$\sigma_{12}^2 = \text{Cov}(r_1, r_2) = E(r_1 - E(r_1))(r_2 - E(r_2)) \quad \dots(2.3)$$

We additionally assume that the correlation coefficient between the two asset's returns is ρ , ($-1 < \rho < 1$) where ρ is defined as:

$$\rho = \frac{\sigma_{12}}{\sigma_1\sigma_2} \quad \dots(2.4)$$

If $\rho = 1$, the two asset's returns are perfectly positively (linearly) correlated to each other, and always move in the same direction. For $\rho = -1$, the converse applies, and for $\rho = 0$, the asset returns are not related.

Given that the portfolio variance is defined as

$$\begin{aligned} \sigma_p^2 &= \text{var}(r_p) \\ &= E[r_p - E(r_p)]^2 \\ &= E[w_1(r_1 - E(r_1)) + w_2(r_2 - E(r_2))]^2 \\ &= w_1^2 E(r_1 - E(r_1))^2 + w_2^2 E(r_2 - E(r_2))^2 + 2w_1w_2 [E(r_1 - E(r_1))(r_2 - E(r_2))] \\ &= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1w_2 \sigma_{12}^2 \\ &= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1w_2 \rho \sigma_1 \sigma_2, \quad \dots(2.5) \end{aligned}$$

Markowitz (1959) showed that that the portfolio variance is influenced by, amongst other factors, the sign and size of ρ . If $\rho = -1$, portfolio risk can be eliminated. Even with $0 < \rho < 1$, the portfolio can be reduced (although not to zero) through diversification. Given the assessment that the portfolio risk can be reduced through diversification, Merton (1972) showed that that there exists a set of weights that would be optimal for an investor to hold in each asset to achieve the least possible portfolio risk. By differentiating the portfolio risk with respect to w_1 , Merton showed that:

$$\frac{\partial(\sigma_p^2)}{\partial w_1} = 2w_1\sigma_1^2 - 2(1-w_1)\sigma_2^2 + 2(1-2w_1)\rho\sigma_1\sigma_2 = 0 \quad \dots(2.6)$$

Solving for w_1 in 2.6,

$$w_1 = \frac{\sigma_2^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} \quad \dots(2.7)$$

or,

$$w_1 = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{21}} \quad \dots(2.8)$$

2.1.2.2 For many assets:

It thus follows that for a portfolio of n assets, we have

$$E(r_p) = \sum_n w_i E(r_i) \quad \dots(2.9)$$

and

$$\sigma_p^2 = \sum_{j=1}^n \sum_{i=1}^n w_j w_i \text{Cov}(r_i, r_j) \quad \dots(2.10)$$

Merton showed that the proportion to hold in each stock w^* in a portfolio of n stocks in order to have to minimum variance is given as:

$$w_j^* = \frac{\sum_{k=1}^n v_{kj}}{C}, \quad \text{for } k = 1 \text{ to } m, \quad \dots(2.11)$$

where:

v_{kj} is defined as the elements of the inverse of the variance covariance matrix, and

$$C = \sum_{j=1}^n \sum_{k=1}^n v_{kj}$$

2.1.3 Optimal Risky Portfolio

The previous section highlighted how investors are able to construct portfolios in order to lower the portfolio risk (to the extent of constructing portfolios with the least possible risk i.e. minimum variance portfolio). Markowitz however argued that investors are mean – variance efficient, i.e. expecting the highest return for each level of risk. The portfolios which satisfy the mean-variance criterion are known as the efficient portfolio and subsequently lie on the efficient frontier. The quantification of these proportions were initially proposed by Markowitz in the following formulation:

$$\text{Min}Z = W' \Sigma W - \lambda W' \mu$$

Subject to $0 \leq w_i \leq 1$, $i = 1, \dots, n$, which requires that the portfolio have no short sales.

Thus varying λ will yield the efficient frontier.

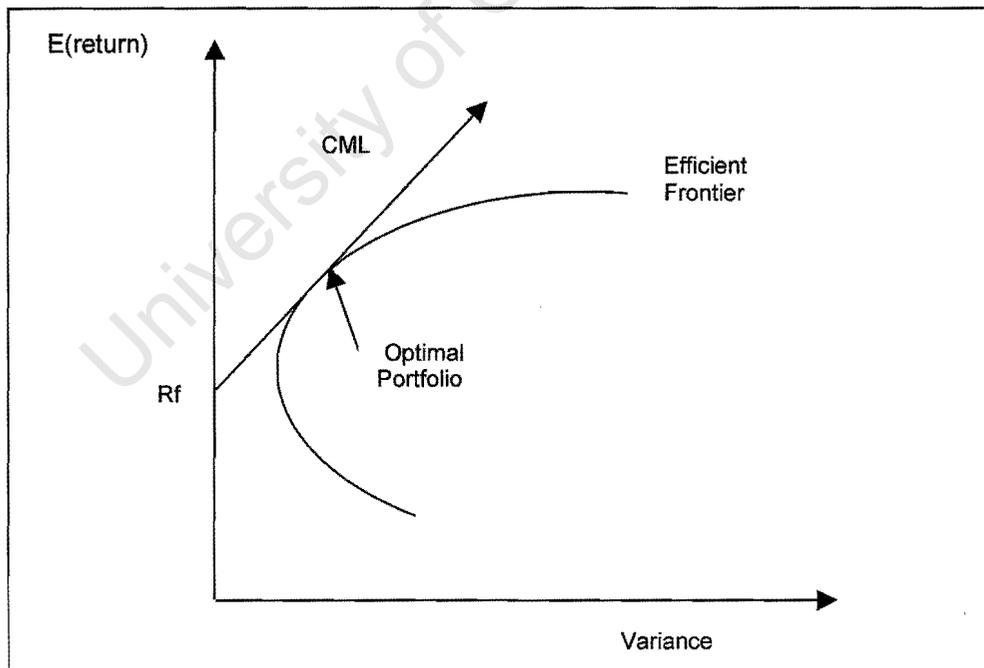


Figure 2.1: Efficient Frontier

The efficient frontier in Figure 2.1 thus shows all combinations of expected return and variance which satisfies the mean-variance criterion. As can be seen, the optimal portfolio lies at the position of tangency to the Capital Market Line (denoted as CML in Figure 2.1), emanating from the risk – free rate (denoted as R_f in Figure 2.1).

Another formulation used for generating the efficient frontier is given in (2.12).

$$\begin{aligned}
 &\text{Maximise return } \sum w_i E(r_i) = W' \mu \quad , \text{ for each level of variance} \\
 &\quad \sum_{i=1}^n \sum_{j=1}^n w_j w_i \text{Cov}(r_i, r_j) = W' \Sigma W \\
 &\text{Subject to } \sum_{i=1}^n w_i \equiv 1 \quad \dots(2.12) \\
 &\quad w_i \geq 0 \quad \text{for all } i
 \end{aligned}$$

Computing efficient frontiers using the above algorithms are often computationally cumbersome. Bodie, Kane and Marcus (1997) showed that the solution for the weights of the optimal portfolio for a portfolio of two assets is explicitly given by:

$$\begin{aligned}
 w_1 &= \frac{[E(r_1) - r_f] \sigma_2^2 - [E(r_2) - r_f] \text{Cov}(r_1, r_2)}{[E(r_1) - r_f] \sigma_2^2 + [E(r_2) - r_f] \sigma_1^2 - [E(r_1) - 2r_f + E(r_2)] \text{Cov}(r_1, r_2)} \\
 w_2 &= 1 - w_1 \quad \dots(2.13)
 \end{aligned}$$

where r_f is the risk-free rate.

A standard assumption accompanying the computation of the optimal portfolio is that investors are rational and mean – variance optimizers, i.e. seeking the highest excess return with lowest risk. This implies that they would seek to maximize the Sharpe Ratio of the portfolio. When maximizing the Sharpe Ratio function with respect to the weights of the portfolio, the formulation in (2.13) is generated.

The weights of the optimal portfolio, w_k given n assets was explored by Merton (1973), and given as:

$$w_k = \frac{\sum_{j=1}^m v_{kj} (E_j - r_f)}{(A - RC)}, \quad k = 1, \dots, m \quad \dots(2.14)$$

where:

v_{kj} is defined as the elements of the inverse of the variance covariance matrix, C

$$C = \sum_{j=1}^n \sum_{k=1}^n v_{kj} ,$$

$$A = \sum_{j=1}^n \sum_{k=1}^n v_{kj} E_j ,$$

E_j is the expected return on the j^{th} asset, and

R_f is the rate of return on the risk-free asset.

An underlying assumption in this formulation requires the covariance matrix to have a determinant which is non-zero, i.e. the matrix is invertible.

This formulation however does not restrict short sales, and consequently the weights of the optimal are not guaranteed to have positive proportions.

In the next section, Tobin's separation property is outlined. This property allows investors to combine the bundle of optimal risky assets (as discussed in this section) with a risk-free asset optimally.

2.1.4 Capital Allocation

The second stage of the portfolio selection process considers the optimal allocation of the investor's portfolio. This stage is concerned with the distribution of the overall portfolio between equity and cash, consequently affecting the total risk of the portfolio. Important to this process is the risk aversion or preference, let's say c of the investor. In attempting to maximise the investors utility, Tobin (1958) in his risk aversion model proposed the function involving only the expected return (μ_N) and variance (σ_N^2) to be maximised:

$$\mu_N - \frac{c}{2}\sigma_N^2. \quad \dots(2.15)$$

In this simple formulation Tobin considers only two assets, i.e. a single risky asset and a bundle of risky assets. The risk-less or risk-free asset has a known return, r_f , and the risky asset has an expected return of μ_r . It thus follows from 2.15 that:

$$\mu_N = yr_f + (1-y)\mu_R \quad \dots(2.15)$$

$$\sigma_N = (1-y)\sigma_R \quad \dots(2.16)$$

where y is the proportion held in the risk-free asset. The Capital Market Line connecting the risky asset to the risk-free asset is given as

$$\mu_N = r_f + \left(\frac{\mu_R - r_f}{\sigma_R} \right) \sigma_N \quad \dots(2.17)$$

which is a straight line with a slope of $\frac{(\mu_R - r_f)}{\sigma_R}$.

Thus, maximising the utility is equivalent to choosing y in order to maximise:

$$\begin{aligned} \psi &= yr_f + (1-y)\mu_R - \frac{c}{2}(1-y)^2\sigma_R^2 \\ \frac{\partial \psi}{\partial y} &= -(\mu_R - r_f) + c(1-y)\sigma_R^2 = 0 \end{aligned} \quad \dots(2.18)$$

Thus, Tobin found that the optimal proportion of holding the risky asset $x^* = (1-y)$ is

$$x_i^* = \left(\frac{\mu_R - r_f}{c\sigma_R^2} \right) \quad \dots(2.19)$$

2.2 Capital Asset Pricing Model

After Markowitz (1952) laid the majority of the groundwork for modern portfolio theory, it was only 12 years later that any substantial theory was proposed for predicting equilibrium returns. Sharpe (1964), Litner (1965) and Mossin (1966) made major contributions to the theory in the area of asset pricing and the model developed is still referred to as the Capital Asset Pricing Model (CAPM).

The model is summarised in the equation:

$$E(r) = r_f + \beta[E(r_M) - r_f] \quad \dots(2.20)$$

where

$E(r)$	=	the expected return on the asset
$E(r_m)$	=	the expected return on the market
r_f	=	the risk-free rate, and
β	=	the beta of the asset to the market
	=	$\frac{\text{cov}(r_i, r_m)}{\text{var}(r_m)}$

The model is however a simplification of the real-world, and relies on the assumptions that markets are ideal in the sense that:

- They are large and investors are price-takers
- There are no tax or transaction costs
- All risky assets (equity) are publicly traded
- Investors can borrow and lend any amount at a fixed risk-free rate
- Investors have homogeneous expectations

In deriving the CAPM formulation, Sharpe assumes a portfolio of two risky assets: the market portfolio, M , and another risky portfolio i , of proportions x_i and $(1-x_i)$ respectively.

The expected return and standard deviation of the combined portfolio p is:

$$\mu_p = x_i \mu_i + (1 - x_i) \mu_m \quad \dots(2.21)$$

$$\sigma_p^2 = [x_i^2 \sigma_i^2 + (1 - x_i)^2 \sigma_m^2 + 2x_i(1 - x_i) \sigma_{im}] \quad \dots(2.22)$$

In deriving the CAPM, Sharpe then noted that to find the slope of the efficient frontier at the market portfolio, M, we require:

$$\left[\frac{\partial \mu_p}{\partial \sigma_p} \right]_{x_i=0} = \left[\frac{\partial \mu_p}{\partial x_i} \right] \left[\frac{\partial x_i}{\partial \sigma_p} \right] \quad \dots(2.23)$$

where all derivatives are evaluated at $x_i = 0$. From 2.21 and 2.22:

$$\left[\frac{\partial \mu_p}{\partial x_i} \right]_{x_i=0} = \mu_i - \mu_m \quad \dots(2.24)$$

$$\left[\frac{\partial \sigma_p}{\partial x_i} \right]_{x_i=0} = \frac{1}{2} [2x_i \sigma_i^2 - 2(1 - x_i) \sigma_m^2 + 2\sigma_{im} - 4x_i \sigma_{im}] \quad \dots(2.25)$$

At $x_i = 0$ (point M), we know that $\sigma_p = \sigma_m$, and thus,

$$\left[\frac{\partial \sigma_p}{\partial x_i} \right]_{x_i=0} = \frac{(\sigma_{im} - \sigma_m^2)}{\sigma_m} \quad \dots(2.26)$$

and substituting 2.24 and 2.26 in 2.23, we have:

$$\left[\frac{\partial \mu_p}{\partial \sigma_p} \right]_{x_i=0} = \frac{(\mu_i - \mu_m) \sigma_m}{\sigma_{im} - \sigma_m^2} \quad \dots(2.27)$$

However, Sharpe realised that the slope at M was equal to the slope of the Capital Market Line (CML), thus:

$$\frac{(\mu_m - r_f)}{\sigma_m} = \frac{(\mu_i - \mu_m) \sigma_m}{\sigma_{im} - \sigma_m^2} \quad \dots(2.28)$$

and from this we attain the CAPM relationship:

$$\mu_i = r_f + \frac{\sigma_{im}}{\sigma_m^2}(\mu_m - r_f) \quad \dots(2.29)$$

or, in more familiar notation:

$$E(r) = r_f + \beta[E(r_M) - r_f] \quad \dots(2.30)$$

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2.3 Theory of Active Portfolio management

2.3.1 Overview

Bodie, Kane and Marcus (1997) define active portfolio management to be the process of attempting to realise positive active return through adding mispriced shares to the portfolio. The lure of active management lies in the challenge to 'beat the market', yet has become increasingly difficult. Given a market which is generally efficient, Bishop (1990) suggests that active portfolio managers can only consistently beat the average if the fund manager:

1. is lucky,
2. has access to inside information, or
3. concentrates their efforts in an area of the market which is less efficient.

Thus, given the competitive nature of fund managers and the consequent implied market efficiencies, Bishop suggests that the move from active management to more passive management has been enthused. Grossman and Stiglitz (1980) however maintain that there will always be a place for active portfolio management. Their argument follows that in the case that funds under active management dry up, prices will no longer reflect sophisticated forecasts. The consequent opportunity to research share prices will thus lure back active managers who once again expect to take active positions so that their funds benefit from this research [see Grossman and Stiglitz, "On the Impossibility of Informationally efficient Markets", American Economic Review 70 (June 1980)].

Note that a further lure of active management is the potential profits from active management, compared to the low commissions from passive management [as noted by Berkowitz and Logue (2000)].

The central objective of this section is to place this active area of study in the context of modern portfolio theory. The primary focus is on establishing a framework for active portfolio management with specific emphasis on benchmarking and risk monitoring of actively managed portfolios.

2.3.2 CAPM and active portfolio management

One of the valuable results of the CAPM is determining consensus / equilibrium expected returns for assets. Black and Litterman (1991) argue that the value in this result is that consensus expected returns become a neutral point of reference and thus provide a standard of comparison to other generated expected returns. These consensus returns can assist active portfolio managers in isolating non-consensus information and ultimately aid portfolio design.

Black and Litterman (1991) formulated a methodology which incorporates the use of consensus returns and subjective expected returns to tilt the optimal portfolio away from consensus (i.e. benchmark portfolio). This methodology is based on the fact that when consensus expected returns are used as inputs to a mean-variance optimisation, the result would automatically lead to the market (or benchmark) portfolio. The optimal portfolio will thus only differ from the market portfolio if the forecasted returns differ to the CAPM consensus expected returns. The use of Black and Litterman methodology is designed to alleviate the input sensitivity problem suggested by Best and Grauer (1991) to a certain extent, and produces optimal portfolios with a greater degree of diversification.

The rationale behind the using the CAPM for generating consensus expected returns thus provides expected returns against which the manager can contrast his own expectations. The extent to which active managers differ in their expected returns to that of CAPM assists them in making portfolio selection decisions. The relevance of the CAPM to active managers thus lies in creating the focus for managers' attention on how they expect to add value. See Henk and Winkelman (1998), He and Litterman (1999) and Idzorek (2001) for the intuition behind using consensus equilibrium returns and neutral returns.

In the ensuing section, focus will be turned to outlining literature which has founded the basis of active portfolio theory. There is however a lack of extensive literature in this area, as most theory of modern portfolio management is based on the premise that financial markets are in equilibrium and efficient, thus suggesting active portfolio management is futile. Given that a place for active portfolio management does exist, a few academics have proposed foundations on which the theory of active portfolio management is developed. These foundations are discussed in the ensuing section.

2.3.3 Foundations of active management

Richard Grinold and Ronald Kahn were pioneers in the area of active portfolio theory. In their book, *Active Portfolio Management* (1995), they explore the central aspects of active portfolio theory. The primary formulations in the area of active portfolio theory are outlined in the subsequent paragraphs.

To begin, it is noted that in practice, active managers are required to outperform a benchmark portfolio that may not be represented by “the market portfolio”. This requirement encapsulates the aim of active portfolio managers. For that reason, we focus hereafter on the benchmark as a notional portfolio which active managers are required to outperform.

In the context of this section, we begin by defining important considerations of active managers.

For time period t , Grinold and Kahn (1995) define active return as:

$$r_a^t = r_p^t - r_b^t \quad \dots(2.31)$$

where r_a^t = active return in period t
 r_p^t = return on the managed portfolio in period t
 r_b^t = return on the benchmark portfolio in period t

Grinold and Kahn calculate the cumulative return for time period $t = 1, \dots, n$, with the following formula:

$$r_a = \left(\prod_{t=1}^n (1 + r_a^t) \right) - 1 \quad \dots(2.32)$$

If a manager is able to achieve a positive active return, he will thus outperform the benchmark portfolio.

An important measure in active portfolio management is the dispersion of these active returns. This measure is captured by the active risk of the portfolio, more commonly

referred to as tracking error, and is defined by Pope and Yadav (1994), Lee (1998) and Rudolf, Wolter and Zimmerman (1999) as the variance (standard deviation) of relative returns, given as:

$$\begin{aligned} \psi &= \sigma(r_a) \\ &= \sqrt{\frac{\sum_{t=1}^n \left[r_p^t - r_b^t - \left(\overline{r_p^t - r_b^t} \right) \right]^2}{n-1}} \end{aligned} \quad \dots(2.33)$$

By construction, a tracking error of zero would imply a perfect benchmark tracking portfolio.

Rudolf, Wolter and Zimmerman (1999) however argued that the quadratic form of tracking error is difficult to interpret, and propose a linear deviation of mean absolute deviations, defined as:

$$\psi = \frac{1}{n-1} \sum_{t=1}^n |r_p^t - r_b^t| \quad \dots(2.34)$$

Clarke, Krase, and Statman (1994) and Roll (1992) define tracking error as the active return (rather than standard deviation of active return) between the portfolio and the benchmark. As the former method is more widely accepted by practitioners, we employ definition of used in (2.33) in this study.

Similarly to active return, Grinold and Kahn define active beta as:

$$\beta_a = \beta_p - 1, \quad \dots(2.35)$$

where β_p is the beta of the fund relative to the benchmark (as the beta of the benchmark relative to itself is 1).

Residual return, or otherwise know as alpha, is defined by Grinold and Kahn as:

$$\alpha_p = r_p - \beta r_b \quad \dots(2.36)$$

This equation implies that the residual returns in period t for a portfolio with return r_p is the return over-and-above a benchmark portfolio having the same beta. The distinction between residual return and active return is simply that active return is not a beta-adjusted return.

Grinold and Kahn define residual risk thus as the variability of the residual returns:

$$\sigma_e = \sigma(\alpha_p) \quad \dots(2.37)$$

Given the definition of residual risk, we refer back to active risk (or tracking error). We note that

$$\begin{aligned} \psi &= \sigma[r_a] \\ &= \sigma[r_p - r_b] \\ \text{which further simplifies to} \quad &= \sigma[(\beta - 1)r_b + \alpha_p] \\ &= \sqrt{(\beta - 1)^2 \sigma_b^2 + \sigma_e^2} \\ &= \sqrt{(\beta - 1)^2 \sigma_b^2 + (1 - \rho^2) \sigma_p^2} \end{aligned} \quad \dots(2.38)$$

where β = beta of portfolio relative to the benchmark
 σ_b^2 = benchmark volatility
 ρ^2 = correlation squared of the portfolio to the benchmark
 σ_p^2 = portfolio volatility

This decomposition implies that the tracking error is composed of two terms. The first is the relative market risk (also referred to as benchmark timing risk). Inherent in this term is the beta of the portfolio, as a beta of 1 to the benchmark would collapse this term and imply no timing risk. The second term is referred to as residual (or "selection risk"), that is governed by the correlation of the portfolio to the benchmark.

Written in another form:

Tracking error variance (TEV) = Benchmark timing risk + Selection risk

A popular measure of the skill of an active manager is the information ratio, defined by Grinold and Kahn as:

$$IR = \frac{\alpha_p}{\sigma_e} \quad \dots(2.39)$$

Grinold and Kahn have cited an information ratio of 0.5 to be good, and an information ratio of 1 to be excellent. The way to interpret an information ratio of 0.5 would be to say that a manager could expect an expected residual return of 0.5% for every 1% of residual risk taken.

Given the fundamentals of active portfolio management reviewed in the prior section, the focus now shifts to the quantitative application of the theory to practical issues in the area of active portfolio management. As the theme in this thesis is of quantitative applications, the application in the area of risk has far more promise in terms of accuracy – than that of return. Subsequently, instead of concentrating on methods of outperforming a benchmark (return-based strategies), the focus of ensuing chapters thus lies in managing portfolio risk relative to a specified benchmark. In the review and development of quantitative risk frameworks, the objective lies in assisting active managers not only to outperform their benchmark in the long run, but also avoid significant underperformance.

In the following chapter, the focus is on reviewing the standard optimisation framework put forward by Markowitz (1952), and demonstrating its practical application using the Black – Litterman methodology adjustment for return inputs.

CHAPTER 3: PORTFOLIO OPTIMISATION IN ACTIVE PORTFOLIO MANAGEMENT

The optimisation framework introduced by Markowitz in 1952 formed the basis for an inspired quantification of the portfolio design process. The extent of the breakthrough led to a Nobel Prize in Economics in 1990 for Markowitz, and was heralded as the accepted theoretical model to transform the information on assets (in the form of expected returns, covariances and variances) into a portfolio. However, the use of Markowitz's framework has been limited in practice.

3.1 Drawbacks of traditional mean-variance optimization

One would expect that given the straightforward mathematics of the portfolio optimization problem, the accurate estimation procedures for measuring risk and the significant technical advances in computing power – that Markowitz's quantitative framework would be at the forefront of the critical decision of portfolio design. Unfortunately the resulting optimised portfolios have tended to yield extreme and implausible portfolio weights. Additionally these weights also tend to be very sensitive to small changes in expected returns.

To this point, Michaud (1990) argued that mean-variance optimizers are estimation error maximisers. The result is that estimation error of expected returns (of lesser confidence) is not accommodated for. Best and Grauer (1991) also show that the results of the optimisation process is highly sensitive to inputs, and any slight change to the inputs of the optimisation may lead to dramatic changes in the results. Best and Grauer also point out that that the optimal portfolio is often concentrated in a few assets, and is not represented by a well – diversified portfolio. This lack of diversification is revealed in the optimal portfolio dominated by the assets with the highest return. Best and Grauer lastly suggest that return estimates for expected returns in the optimisation are unreliable, being poor predictors of future stock returns.

Henk and Winkelman (1998) later suggested that the resultant optimal portfolio may also not be intuitive given the investment views on each asset. Given practitioners often only

think in terms of portfolio weights, the mapping between expected returns and optimal portfolio weights are difficult to understand.

Another practical problem is that practitioners often do not hold a view in terms of an expected return for every asset, as pointed out by He and Litterman (1999). Asset managers usually have reasonable research information on only a select number of assets – however the Markowitz formulation usually requires expected returns to be specified for every asset in the opportunity set. Thus the useful information on a few assets is typically swamped by the poorer estimates, as the traditional process has no way of distinguishing reliable information from the unreliable information. In practice the effort required by managers to obtain reasonable estimates of returns for every asset is generally thought to be far too onerous – and outweighs the benefits of the quantitative process.

Several attempts have been made to resolve the practical problems of quantitative portfolio design. Most acknowledge that the use of historical returns instead of expected returns are problematic – and that when forecasted returns are used, estimation errors are not taken into account.

3.2 Improvements on standard mean-variance optimisation using Black Litterman

These practical problems led to the pioneering work by Black and Litterman (1992) aimed at reshaping modern portfolio theory in order to make it more amenable to portfolio managers. Black and Litterman initially focused on highlighting the shortcomings of several alternative methods for specifying a starting point for expected returns (for example historical means and equal means). Thereafter they pointed out that a neutral starting reference is needed for expected return inputs - and that market consensus provides the neutral expected returns which yield the benchmark weights under conditions of market equilibrium.

Black and Litterman proceed to develop an approach whereby investors need only specify returns on assets that they have views on. Furthermore their derivation differentiates between views that are more strongly held than others. Additionally their framework allows

views to be stated in a relative fashion. That is, the user need only specify the *relative* amount by which one asset is expected to outperform another, rather than individual absolute returns.

This section demonstrates the intuition behind the Black-Litterman model using a global asset allocation setting – viewed from the perspective of a South African investor. This section follows the demonstration of Black and Litterman(1992) as well as the more recent report by He and Litterman (1999). Our empirical demonstration considers the allocation between the major asset classes from a local investor perspective. Thus our opportunity set of risky assets under consideration are: Local Equity, Local Bonds, Foreign Equity, Foreign Bonds and Foreign Cash. In our later developments we introduce a benchmark comprising the component asset allocation weightings of the Global Alexander-Forbes Top 10 Large Manager Watch. Note our focus is on the optimal choice of risky assets - hence we exclude local cash from our universe of risky assets, as according to conventional theory, the choice of risky assets is made separately - where after the proportionate blend of the risk - free asset follows.

3.3 Empirical examples using traditional mean-variance optimisation

Following Black and Litterman (1992) and He and Litterman (1999) we begin by demonstrating the shortcomings arising from using a variety on non-neutral return specifications.

3.3.1 Equal returns as input

The naïve approach of specifying all expected return inputs as equal was first dealt with; thereafter the results of using historical returns as a frame of reference was considered. All means were set equal to the historical average of the assets under consideration (that is 18% p.a.).

The assumption that the investor has only one relative view is first held: that is that Local Equity is expected to outperform Foreign Equity by 6% in rand terms. To incorporate this

view into the optimisation process, the expected return for Local Equity is shifted up by 3% to 21% (i.e. $18\% + 3\%$) and the return of Foreign Equity is shifted down by 3% to 15% (i.e. $18\% - 3\%$).

Table 3.1 shows the input returns for each asset in the opportunity set.

Table 3.1: Equal returns and returns adjusted for view

Scenario	Local Equity	Local Bonds	Foreign Cash	Foreign Equity	Foreign Bonds
Equal Returns	18%	18%	18%	18%	18%
Returns shifted for view	21%	18%	18%	15%	18%

The optimisation process was run separately using the two return scenarios in Table 3.1. The covariance input requirement was taken as the historical covariance matrix of returns over the prior 5 year period as given in Table 3.2.

Table 3.2: Covariance Matrix of Returns

	Local Equity	Local Bonds	Foreign Cash	Foreign Equity	Foreign Bonds
Local Equity	644.02				
Local Bonds	172.22	148.14			
Foreign Cash	-113.55	-100.28	251.92		
Foreign Equity	200.71	-20.94	180.03	402.49	
Foreign Bonds	-73.63	-69.79	190.42	140.60	170.22

Figure 3.1 shows the resulting optimal portfolio weights arising from the two return input scenarios.

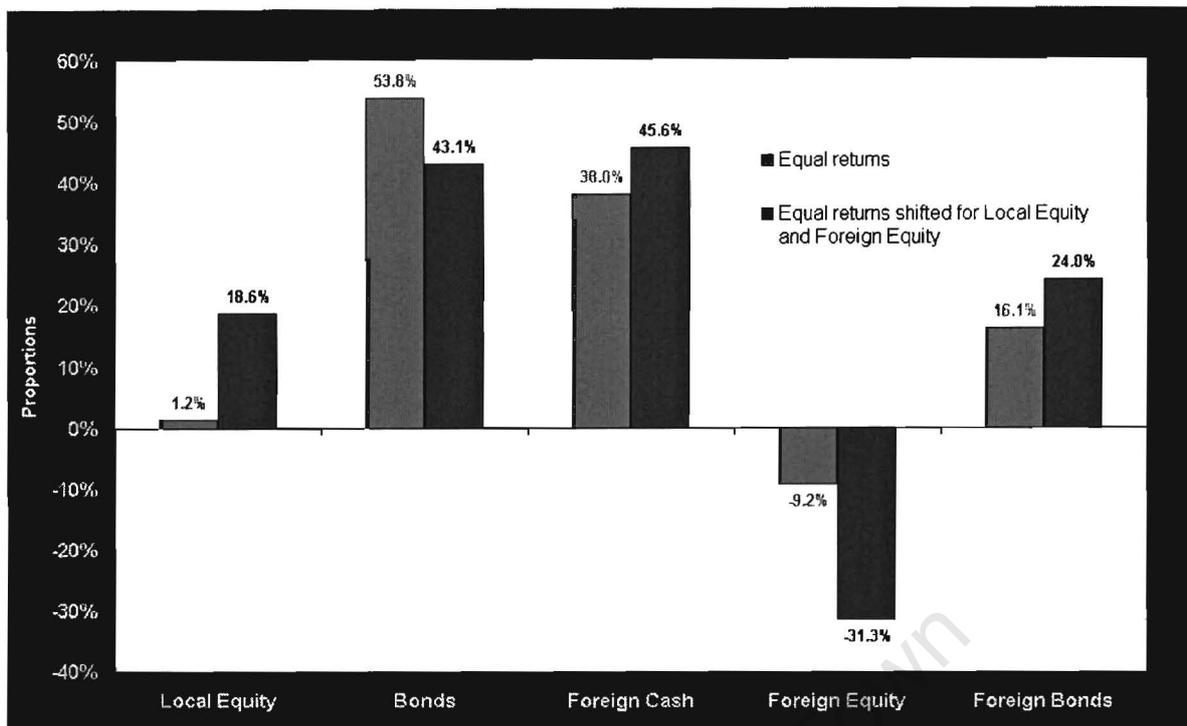


Figure 3.1: Optimal Portfolio using Equal returns and Equal returns shifted for Local Equity and Foreign Equity view

Figure 3.1 shows that using equal means as a starting point yields vastly unintuitive portfolio weights - with only 1.2% in Local Equity and -9.2% in Foreign Equity. Adjusting the return inputs to accommodate the view depicted in the second scenario of Table 3.1 results in the portfolio weights depicted in the maroon bars in Figure 3.1. A small relative shift in the expected return (3% for Local Equity and -3% for Foreign Equity) has translated into huge swings in the weights of these two asset classes. The weight for Local Equity has increased to 18.6% and Foreign Equity has decreased to -31.3%. Additionally the weights for the asset classes for which there were no views also changed significantly. These weights are clearly not practically intuitive, given the return input.

3.3.2 Historical returns and Neutral returns as inputs

The process of using historical averages as forecasts for expected returns are widely known to yield very poor estimates. Nevertheless in our demonstration we use the prior 5 years of return data (from June 1997 to May 2002) for our historical return estimates.

These estimates are depicted in Figure 3.2 together with the *consensus* returns discussed in the ensuing section.

Black and Litterman propose that the only set of “neutral” returns would be the set of expected returns that would clear the market if all investors had identical, i.e. consensus, views. Hence their model starts with equilibrium expected returns generated by the Capital Asset Pricing Model. It is easy to show that these “neutral” returns are the unique set of return inputs that can be effectively derived from reverse optimisation (where the optimal portfolio is the market portfolio). These derivations are generalized, effectively replacing the market portfolio with *any* specified benchmark. Consequently the resulting “neutral” returns are the only ones which lead to the optimal portfolio having the precise benchmark weights. In the demonstration, the specified benchmark specified is the Global top 10 Large Manager Watch peer group.

In Table 3.3 below the set of historical returns together with the set of neutral (or henceforth consensus) returns used as inputs are shown.

Table 3.3: Annualised historical returns and Consensus Returns

	Local Equity	Local Bonds	Foreign Cash	Foreign Equity	Foreign Bonds
Historical Returns	12.8%	17.6%	23.1%	18.8%	21.3%
Consensus Returns	20.2%	14.7%	11.6%	15.6%	12.0%

Figure 3.2 shows the resulting optimal weights using mean-variance optimisation. When historical returns are used, the resulting weights are even more extreme than was the case for equal returns in Figure 3.1.

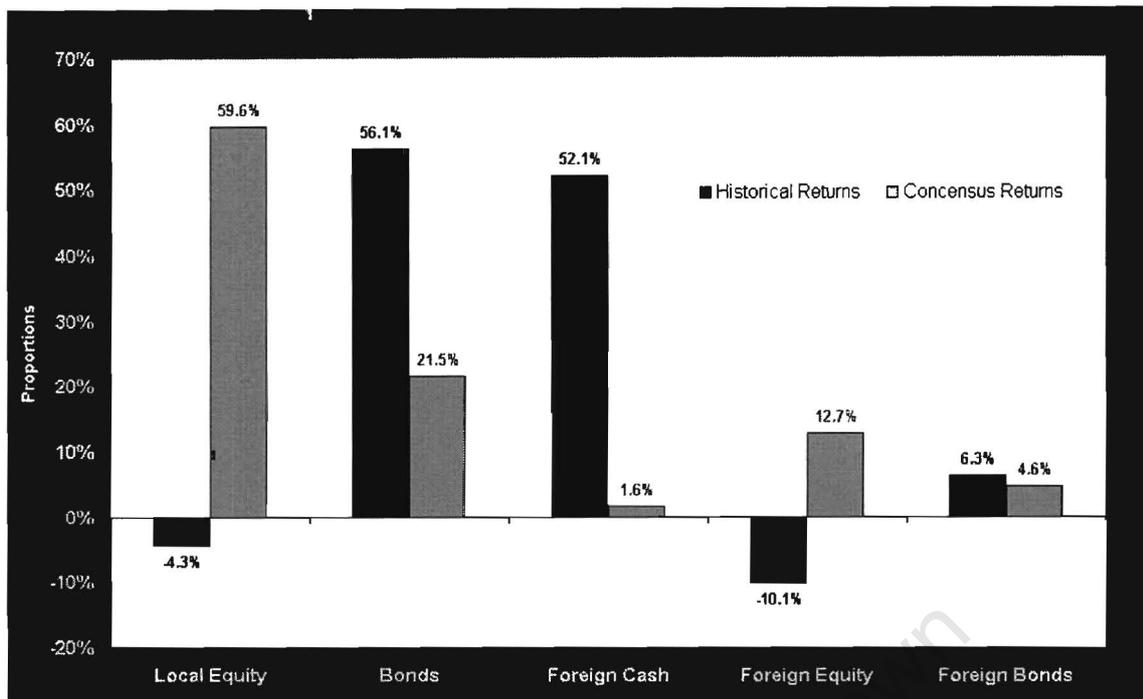


Figure 3.2: Optimal Portfolio using Historical returns and Consensus returns

It is well known that there is substantial forecast error in using historical returns as forecasts for expected returns – and as stated previously estimation error-maximisation is clearly the problem. Clearly a portfolio with -4.3% in local equity and -10.1% in foreign equity is impractical from any portfolio manager perspective.

One would expect that all managers would have positive weighting of all of the asset classes in mind. From Figure 3.2 it is evident that the *consensus* set of input views have resulted in the optimal weights all being positive (and very plausible). Because of the reverse optimisation analogy we are not surprised that these set of weights are precisely the benchmark weights (of the top 10 pension funds) – as they have been formulated with this in mind.

As Black-Litterman state, the neutral (consensus) concept, on its own, is not particularly useful – however its value comes to the fore when one combines consensus views with uncertain views emanating from research.

In the ensuing section the innovative approach devised by Black and Litterman to realistically accommodate input views is demonstrated. As one will see this is one of the more complex features of their approach – but undoubtedly the most innovative.

3.4. Empirical examples using Consensus Returns as neutral returns

There are a variety of ways that one can translate views into expected returns using consensus returns as neutral returns. We will consider each one briefly.

- A direct adjustment to the consensus returns for the assets involved in the view.
- A simple Bayesian-weighted adjustment between the consensus return vector and the direct view – where the weighting scheme reflects the confidence in the view. The objective here is to enable a transparent desensitization of the forecast errors.
- Using the Black and Litterman approach that adjusts the view to be “covariance consistent”, that is, it allows the view to adjust the input returns on other assets according to their covariance between the assets having the view imposed.

3.4.1. Direct adjustment to consensus returns

The same view as previously stated is considered, i.e. that Local Equity is expected to outperform Foreign Equity by 6%. As can be seen from the consensus returns in Table 3.4, under equilibrium conditions, Local Equity is expected to outperform Foreign Equity anyway by 4.6%. Hence to implement our view one needs only shift the expected return on Local Equity up by 0.7% to 20.9% and to reduce Foreign Equity by $-0.7%$ to 14.9% (so that $20.9\% - 14.9\% = 6\%$, the view) as depicted in Table 3.4 below. Otherwise the return inputs are identical to consensus for the remaining assets.

Table 3.4. Consensus Returns and Tilted Consensus Returns for Local Equity and Foreign Equity

	Local Equity	Local Bonds	Foreign Cash	Foreign Equity	Foreign Bonds
Adjusted Consensus	20.9%	14.7%	11.6%	14.9%	12.0%
Consensus Returns	20.2%	14.7%	11.6%	15.6%	12.0%

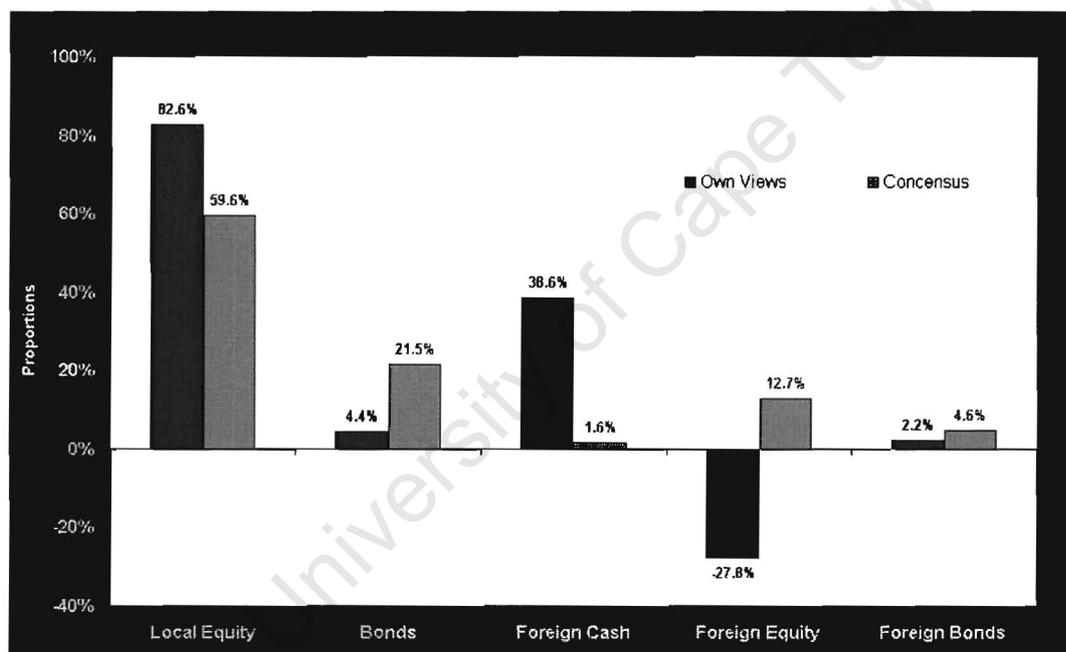


Figure 3.3: Optimal Portfolio using Own Views and Consensus returns

Once more it is evident from Figure 3.3 that the optimal weights of the consensus returns yield the benchmark weights (blue bars). Most puzzling are the weights emanating from the returns adjusted by the view. The difference between the return inputs (between the view and consensus depicted in Table 3.4) was extremely small – yet the difference in the optimal weights is not only extreme, but also result in an unrealistic portfolio (shown in the red bars in Figure 3.3). Most importantly there are significant differences in the weights of

the assets unaffected by the view – this can also be confusing to managers. This latter point is addressed by the Black and Litterman approach and is indeed one of its most innovative features.

3.4.2 Bayesian-weighted adjustment to consensus returns to capture confidence in views.

The second scenario is based on an attempt to desensitize the forecast errors in expected returns (and consequently the unrealistic portfolio tilts) in order to address the problem of optimisers tilting too much on potential forecast errors.

A straightforward approach of weighting our return view by our confidence (expressed as a proportion) with the consensus return (multiplied by 1 minus our confidence proportion) is used. As consensus returns yield the benchmark weights – this process is intended to ensure that portfolios gravitate towards the benchmark. The weighting scheme is used to generate a new set of returns, formulated as below:

$$E(r_i) = (1 - w)R_{\text{Consensus}} + wR_{\text{OwnView}}$$

where

w	=	confidence in the view of asset i
$R_{\text{Consensus}}$	=	consensus return for asset i , and
R_{OwnView}	=	return of own view on asset i

We note that zero confidence in our own view ensures the final input return is the same as consensus.

Table 3.5 below shows two return input scenarios: the first is the unadjusted consensus returns and the second scenario gives a the Bayesian-weighted approach assuming 50% confidence in our view that Local equity will outperform Foreign Equity by 6%.

Table 3.5: Consensus Returns and Bayesian-weighted Returns

	Local Equity	Local Bonds	Foreign Cash	Foreign Equity	Foreign Bonds
Adjusted Consensus	20.6%	14.7%	11.6%	15.3%	12.0%
Consensus Returns	20.2%	14.7%	11.6%	15.6%	12.0%

From the Table 3.5 it is clear that for assets with no non-consensus views, the returns are the same as consensus.

The resulting optimal portfolio weights for the Bayesian-weighted returns are shown in the Figure 3.4 that follows. They are contrasted to the results without the confidence adjustment discussed in Figure 3.3 (in other words assuming 100% confidence in the view).

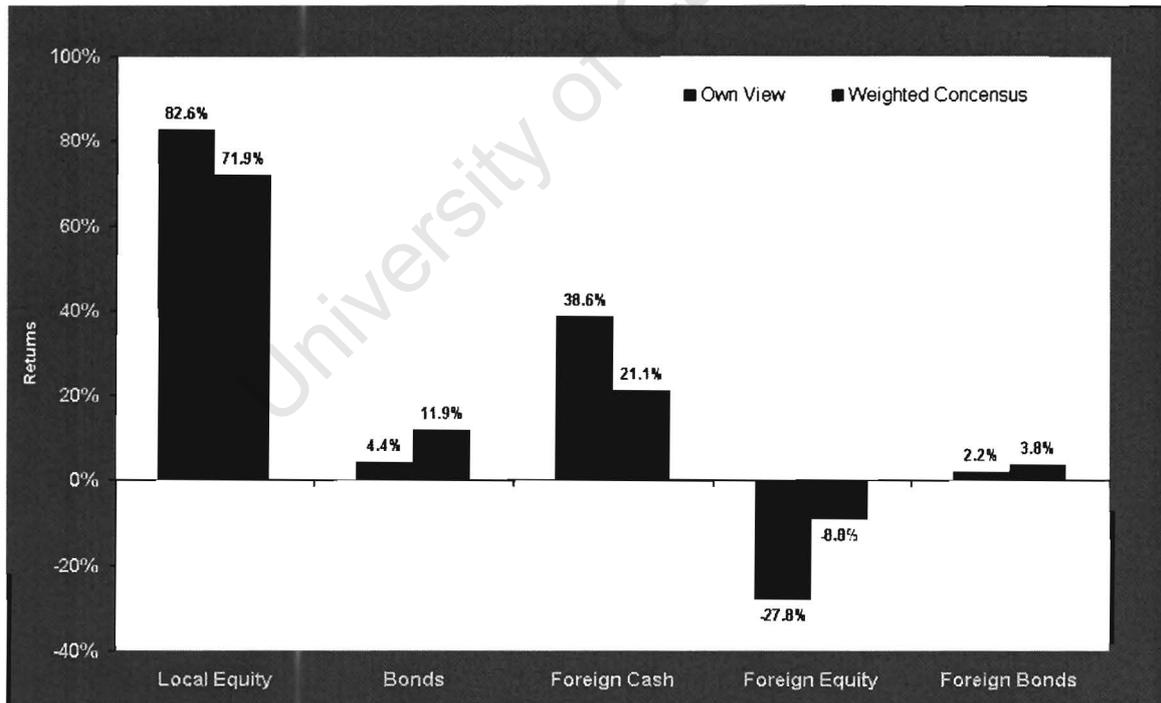


Figure 3.4: Optimal weights for Bayesian-weighted returns at 50% confidence contrasted to 100% confidence

The results of the Bayesian weighted approach show a substantial improvement in the size (and hence realism) of the tilt – yet still has tilts away from the benchmark for the remaining assets having no views.

3.4.3. Black and Litterman adjustment to consensus returns

As stated above, whilst the consensus returns are an important feature in enabling portfolios to gravitate towards the benchmark – when a view *is* imposed, the assets unaffected by the view (i.e. still having consensus return inputs), yield weights which are significantly different from the benchmark.

The multivariate derivation of Black and Litterman result in the expected returns of all assets in the opportunity set being affected by the view imposed. In essence the Black-Litterman model thus adjusts the expected returns away from their consensus values in a manner which is consistent with the underlying covariances and the view being expressed.

Although these adjustments aren't always immediately transparent (because of the complex interaction between the inputs and the optimisation process) – the innovative feature of this approach is observed in the transparency of the optimal weights that intuitively reflect the initial views.

The Black-Litterman conditional expected return formulation has two levels of sophistication:

The first formulation assumes 100% confidence in the views imposed i.e.:

$$\bar{\mu} = \Pi + \tau \Sigma \cdot P' \cdot [P \cdot \tau \Sigma \cdot P']^{-1} \cdot [Q - P \cdot \Pi]$$

The second formulation accommodates the level of confidence in the view

$$\bar{\mu} = [(\tau \Sigma)^{-1} + P' \Omega^{-1} P]^{-1} [(\tau \Sigma)^{-1} \Pi + P' \Omega^{-1} Q]$$

where

$$\Sigma = \text{Covariance matrix of returns for the } n \text{ assets}$$

- τ = scalar which measures the uncertainty of the Covariance matrix
 P = Matrix identifying assets involved in investor's subjective views
 Ω = Diagonal covariance matrix representing the uncertainty of each view
 Π' = vector of consensus / market equilibrium returns

and

$$Q = P \cdot \bar{\mu}'$$

We give some intuition to the rather complex formulation for the case where 100% confidence is assumed. As expected, the formulation begins with the neutral returns, Π' , and proceeds to modify these returns given the covariance structure and views. The term $[Q - P \cdot \Pi']$ is where the investor's view is formulated, and represents the difference between the consensus return and the investor's expected return for those assets for which he has a view. The term $\tau \Sigma \cdot P' \cdot [P \cdot \tau \Sigma \cdot P']^{-1}$ is used to make the investor's return views *covariance consistent*. This is done through ensuring the returns of assets with views are consistent with the returns of assets with no views, as represented in the covariance matrix of returns between all assets.

Again we assume the same view, i.e. that Local Equity outperforms Foreign Equity by 6%. The Black-Litterman is able to implement this view directly without specifying the absolute return differences (required by the previous approaches). The innovative use of the P matrix above captures the relevant assets whilst the Q vector captures the actual view.

We thus define P as:

$$P = (1 \ 0 \ 0 \ -1 \ 0),$$

and Q as

$$Q = 6\%,$$

which satisfies the equation $Q = P \cdot \bar{\mu}'$.

The use of 1 in the position of Local Equity and -1 in the position of Foreign Equity ensures that the difference between the expected return of these two assets is 6%.

Using the Black Litterman formulation to calculate $\bar{\mu}$, the consensus returns is contrasted to the resulting Black-Litterman return inputs obtained in Table 3.6 that follows.

Table 3.6: Consensus Returns and Black Litterman Returns for results view for Local Equity and Foreign Equity tilt

	Local Equity	Local Bonds	Foreign Cash	Foreign Equity	Foreign Bonds
Black Litterman Returns	21.2%	15.1%	11.0%	15.2%	11.6%
Consensus Returns	20.2%	14.7%	11.6%	15.6%	12.0%

From Table 3.6 we see that the Black-Litterman formulation has resulted in the input returns changing for all asset classes – not only those involving the view. While this may at first seem unintuitive, the shifts are in fact a natural consequence of the interrelationships between these asset classes. More important however is the impact of these expected returns on the optimal portfolio. Figure 3.5 shows the resulting optimal weights of the Black-Litterman approach of incorporating the view contrasted to the consensus optimal portfolio. It is evident from Figure 3.5 that the investment weight in Local Equity has increased from 60.3% to 72.4% and the weight in Foreign Equity has declined from 12.1% to -0.1% , entirely consistent with the view expressed.

The most significant feature of the Black-Litterman portfolio however is that optimal weights have only tilted away from the benchmark weights for those asset classes for which a view has been stated. The classes: Local Bonds, Foreign Cash and Foreign Bonds retain their benchmark weights. This feature is consistent across all Black-Litterman applications and emphasizes why views have to impact on all expected return inputs in order to yield intuitive portfolio tilts.

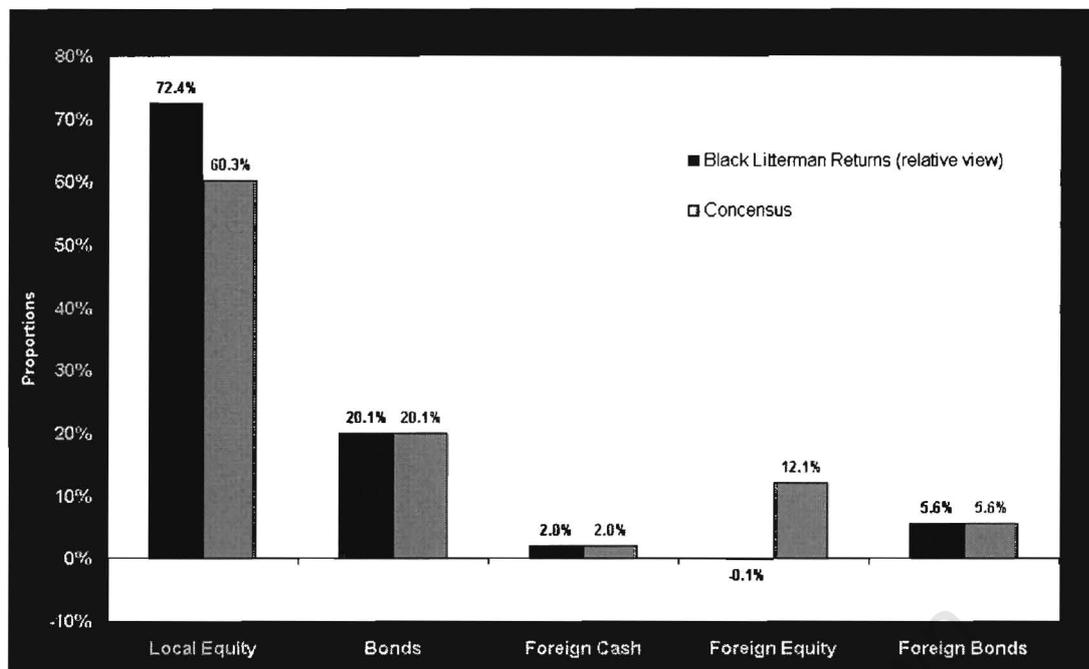


Figure 3.5: Optimal Portfolio using Black Litterman Returns using relative view and Consensus returns

3.5. Summary

The starting point of this section was to demonstrate that consensus returns are an important feature in enabling portfolios to gravitate towards the benchmark weights. The latter focus of the report was on the innovative Black-Litterman approach to accommodating non-consensus views in quantitative portfolio design. We note that the aim of this section was merely to demonstrate the intuitive and realistic results of using Black Litterman returns in a South African optimisation context using an asset allocation setting.

Our empirical demonstration (using a typical global balanced mandate setting) highlighted the advances this approach affords over more traditional approaches. An important feature was the manner in which the expected returns of all assets in the opportunity set are affected by a view imposed on only a select number of assets. In essence the Black-Litterman model adjusts the expected returns away from their consensus values in a manner which is consistent with the underlying covariance's and the view being expressed.

Although these adjustments aren't always immediately transparent (because of the complex interaction between the inputs and the optimisation process) – the innovative feature of this approach is observed in the transparency of the optimal weights that intuitively reflect the initial views.

In the following chapter, the broad area of benchmarking is reviewed, with specific emphasis on the practical application of selecting, constructing and estimating benchmarks in the South African context.

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CHAPTER 4: BENCHMARKING

In this chapter, the important area of benchmarking and its application to active fund management is discussed and documented. Where necessary, an empirical analysis is performed to verify assertions made in this chapter. The importance of benchmarking is discussed in section 4.1 followed by the selection of benchmarks in section 4.2, where after the topic of construction of benchmarks is discussed in section 4.3 and estimation of benchmarks examined in section 4.4.

4.1. Importance of Benchmarking

The fund owner or trustee traditionally has the customary role to provide the fund manager with instructions as to the broad investment performance that the fund manager should produce. Specifying a benchmark portfolio to a fund manager is typically the most efficient manner for a trustee to communicate the investment requirements. The importance of employing a benchmark portfolio is thus evident, as the benchmark portfolio becomes a point of reference and standard of comparison for the fund manager and trustee.

The trustee / investor would additionally require the fund manager to outperform this benchmark, as well as not to take on significant risk against this benchmark, as this may lead to underperformance of the benchmark. The fund manager's performance is thus judged *relative* to the benchmark, and absolute performance is of less importance. Thus, the notion of risk based on relative performance has become ever more important to fund managers and investors alike. This relative risk, referred to as *tracking error*, has emerged as an important measure for managing and monitoring the fund's risks relative to a benchmark. The benchmark allows fund managers to quantify and understand the risks taken against the benchmark, as relative returns cannot be attained without taking any relative risk.

A benchmark is an index most representative of a portfolio's investment objective class. Investors can use the benchmark as a reference point when monitoring fund performance.

The requirements of a benchmark according to Bishop (1999) are as follows:

- i) Investable
- ii) Widely recognised
- iii) Transparent
- iv) Measurable
- v) Pre-specified

The two main types of benchmarks are index benchmarks and peer-group (consensus) benchmarks. Other less widely used benchmarks are those linked to economic indicators e.g. CPI, PPI. There exists a distinct difference between the construction of index benchmarks and peergroup benchmarks as well as the conceptual thinking in using either. An index benchmark replicates the performance of a sector of stocks, constructed on a market capitalisation basis. The constituents of indices are publicly known on a daily basis, making the tracking of the index's performance straightforward to replicate. The use of index benchmarks are primarily for the purpose of exact replication, i.e. index funds, yet are also used when attempting to outperform the index by taking active stock bets against the index compositions.

The construction of a peergroup benchmark involves using the average holdings of competing funds in the investment category, regardless of the size of funds. This benchmark thus aims to replicate the average performance of funds in the investment category. The use of the peergroup benchmarks is primarily linked to the concern of cash in- or outflows based on the performance rankings tables.

This increasing concern of performance ranking among competitors has compelled fund managers to be ever more cognisant of their relative risk against the peer group. As benchmark choice becomes essential, significant departures from a benchmark that differs too radically from peer competitors would expose funds to the risk of significant relative (to competitors) underperformance. From a relative risk perspective this is likely to account for a substantial contribution to tracking error against the peer group. This realization that the peer group concept is important has led many fund managers to use the peer group as their benchmark.

The importance of benchmarks in modern fund management has thus grown due to various factors.

- i) Due to the increasing availability and awareness of investment funds, investors seek to invest in funds that perform well in relation to other competing funds. The value of performance and risk management relative to a benchmark which represents these competing opportunities / funds has thus become important to investors.
- ii) Benchmarks also allow fund managers to keep track of a smaller set of assets (in the benchmark) as opposed to all assets in the market universe. Funds with fewer stocks have lower trading costs and are generally easier to manage. For practical purposes, fund managers often design benchmark proxies, which are a smaller subset of stocks in the actual benchmark. These proxies are constructed and aimed at closely replicating the performance of the actual benchmark, with the least amount of active risk against the benchmark.
- iii) Benchmarks also allow fund managers to focus their efforts on outperforming a specific 'notional' portfolio. The benchmark functions as a point of reference for the fund manager, who focuses on beating his / her benchmark

The benchmark has to be reflective of the investor's risk profile, as well as long –term expectations of the benchmark's performance. Since employing benchmarks in portfolio management is vital, the selection and assessment of the appropriate benchmarks, given the category of investment, is fundamental.

4.2 Selection of Benchmarks

Selection of the appropriate benchmark for the investor is an important decision in the investment process, and should thus be given sufficient consideration. However, the selection of the most appropriated benchmark is difficult since there exists no accepted process for selection of benchmarks. The motive for selecting a benchmark however gives investors some guidance as to appropriated benchmarks, with these motives being either one of risk level desired, of return expectations of the investor, or of both. Thus there exists considerations based on risk as well as considerations based on *return / outperformance*.

When investors select benchmarks based on risk considerations, the success of the benchmarks relies on the assumption that risk characteristics of the benchmark constituents remain fairly stable. Thus, risk estimates / forecasts (volatilities and correlations) using historical data will provide good estimates of future risk characteristics. The validity of this assumption will be tested later in the chapter. Two examples where benchmarks are selected on risk considerations are:

- When attempting to track an investment mandate / process (e.g. indexation benchmark tracking, hedging)
- When accommodating several levels of risk (absolute) to match client profiles

Both of these examples will be discussed in subsequent sections of this chapter. In particular, we highlight some considerations when constructing a benchmark in the case of tracking an investment process.

Selection of benchmarks based on expected return (i.e. outperformance) is in itself difficult. The investor would have to consider the *alpha* of various benchmarks, based either on historical data, or a model forecasting expected returns for the constituents of the benchmark. Thus, an example of a benchmark selected based on return or outperformance would be when an attempt is made to identify a benchmark portfolio that is expected to systematically consistently outperform an index or peer group category. Whether such a benchmark is attainable is questionable, however.

For example some managers in South Africa have created a benchmark by down-weighting the Resource component in the All Share Index. Clearly there is an expectation that Resources will systematically under-perform. Thus, it is anticipated that this benchmark will outperform the All Share Index.

We briefly review this process in a subsequent section of this chapter.

Once a benchmark is chosen the fund manager may need to find a suitable proxy for the benchmark, and construct this proxy to meet the purpose of the benchmark. We now review a framework which will assist us in constructing and assessing the appropriateness of our selected benchmark to our investment category.

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4.3 Construction of appropriate benchmarks

As mentioned, the construction of a benchmark (or benchmark proxy) thus depends on which of the above-mentioned applications are relevant. To begin, the construction of benchmarks when attempting to track an investment process, e.g. indexation, benchmarking or hedging is reviewed in the ensuing section.

4.3.1 Construction of stock proxies for indexation, benchmarking and hedging

There are a variety of applications where a proxy consisting of fewer stocks than some underlying series are typically required. From a practical perspective, series containing a large number of stocks are usually cumbersome to replicate in practice - and stock proxies consisting of a reduced number of stocks are often a practical necessity. Some applications where a basket of fewer stocks are utilised, include:

- index tracking
- benchmark tracking (where the benchmark may for example be some aggregate of peer holdings)
- portfolio hedging

In each of the above-mentioned applications the ultimate success of the stock proxy is related to its ability to generate a *return* commensurate of the return of the underlying series. Traditional approaches to stock proxy construction usually attempt to match the component *holdings* of the underlying series using a smaller set of stocks selected on the basis of their market capitalisation. The success of the stock proxy hinges to a large extent on the return differences between the stocks in the proxy and those *excluded*. From a stock proxy design perspective the *return* of stocks cannot however be predicted in advance - but we are able to predict the *risks* more accurately in advance. Thus the only reliable way one can match the return of a proxy to the return of the series in advance is to ensure that the *risks* of the stock proxy are matched as accurately as possible to the *risks* of the entire underlying series. An important consideration in stock

proxy construction therefore is to monitor whether the underlying *risks* are adequately matched to the underlying series.

As it turns out the larger market capitalisation stocks in the South Africa environment tend to have different systematic risk characteristics to medium and smaller stocks (most often the excluded stocks) creating a relative bias in the risks and consequently the return of stock proxies constructed on the basis of market capitalisations.

Figure 1 for example portrays the betas (versus the ALSI) of the Small Cap, Mid Cap and Large Cap indices. The betas are calculated as the slope of the linear regression between the market cap index and the ALSI. As is evident in figure 1 Large Cap stocks tend to have larger betas (as reflected in the high beta of the Large Cap Index, of 1.06) by contrast to the betas of Mid Cap and Small Cap indices (0.89 and 0.78 respectively). This systematic beta bias inherent in the size of many firms can create problems when proxies are constructed by selecting stocks on the basis of their size first.

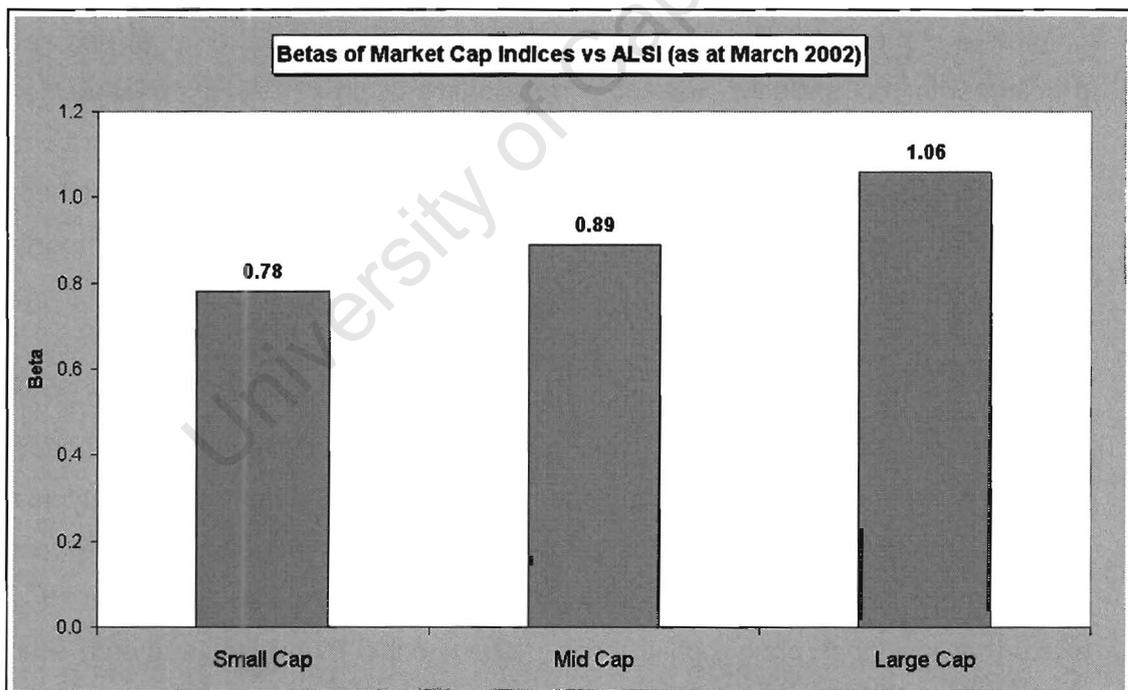


Figure 4.1: Betas (vs ALSI) of the Small, Medium and Large Cap Indices

In this section we highlight how the traditional construction of stock proxies typically lead to an important misalignment of systematic risk and consequently create return

discrepancies. We also give some guidance as to how to counter or rebalance the unwanted risks of the stock proxy.

4.3.1.1 Method and discussion

To assess the success of a stock proxy, one ideally needs to assess the ability the stock proxy has of tracking the *returns* of the underlying series it was designed to proxy. The most appropriate measure for assessing the success of a stock proxy, therefore, is the tracking error, interestingly a measure of *risk*. The extent of the tracking error of the stock proxy relative to the underlying series reflects the disparity of return between the proxy and the underlying series (actually the dispersion of the return differences). Thus the quantitative objective in the stock proxy design should focus on minimising the *risk* measure, tracking error, foremost.

A further rationale for using the tracking error measure is that it encapsulates both of the important aspects of dissimilarity via (1) the beta (relative timing) and (2) the residual risk (stock selection). Often practitioners unwittingly use correlation (related to selection risk only) as a measure of similarity between the stock proxy and the underlying series. The problem with the exclusive use of correlation as a proxy design measure is that the relative timing risk is not considered. The relative timing risk is however of considerable importance, as a stock proxy having a beta different to the series it was designed to track, results in an unwanted timing bet against the underlying series. This is an important point, as any systematic volatility will result in return discrepancies emanating from the proxy.

Typically, the most common method of constructing stock proxies is based on the use of a "market capitalisation" approach, which selects stocks with highest market capitalisation from the component holdings in the series first. This construction approach typically leads to a large market capitalisation bias in the stock proxy. Because large market capitalisation stocks tend to have relatively higher betas in the South African equity market, the potential exists that the stock proxy construction process could inadvertently create a timing bet against the underlying series.

To demonstrate the systematic bias caused by stock proxy construction based on the market capitalisation approach, we use the General Equity peergroup benchmark (based

on an equally weighted aggregate of the Unit Trusts in the General Equity category) as an example of the *underlying series* we will attempt to proxy. Our demonstration considers stock proxies based on market capitalisation for the above series with an ever-decreasing number of stocks.

4.3.1.2 Data

Our data consists of the actual aggregate compositions of the General Equity Unit Trusts as at end of June 2001. We construct market capitalisation-based stock proxies, and compare the impact on the risk characteristics of reducing the number of stocks in the proxy. For simplicity purposes, we consider 150 of the most widely held stocks to comprise the population in the General Equity peergroup benchmark. We depict the relationships graphically in scatter diagrams as well as quantitatively by considering the resulting expected tracking error, beta, correlation, benchmark timing risks, and stock selection risks implicit in the various stock proxies. Our analysis is based on weekly return data from November 1999 to September 2001.

Our scattered diagrams are composed as follows:

On the horizontal (x) axis, we plot the returns of the underlying series. On the vertical (y) axis we plot the matched time series returns of the proposed stock proxy. If the stock proxy was able to track the underlying series, we'd expect the matched pairs of return points to lie exactly on the 45° line. Any departures from the 45° line would reflect the disparity between the proposed stock proxy and the underlying series, as a consequence of different holdings. The extent of the dispersion around the line is represented by *tracking error*, and is visually interpreted as the dispersion around the 45° line.

4.3.1.3 Results

The results of using all 150 shares in the stock proxy are graphically displayed in the scatter plot in Figure 4.2. Clearly, the stock proxy tracks the underlying series (GE peergroup benchmark) perfectly. All matched returns lie on the 45° line and the expected tracking error is zero. However, managing a benchmark based on a 150 stocks may be onerous and many practitioners may prefer a stock proxy based on a fewer number of stocks.

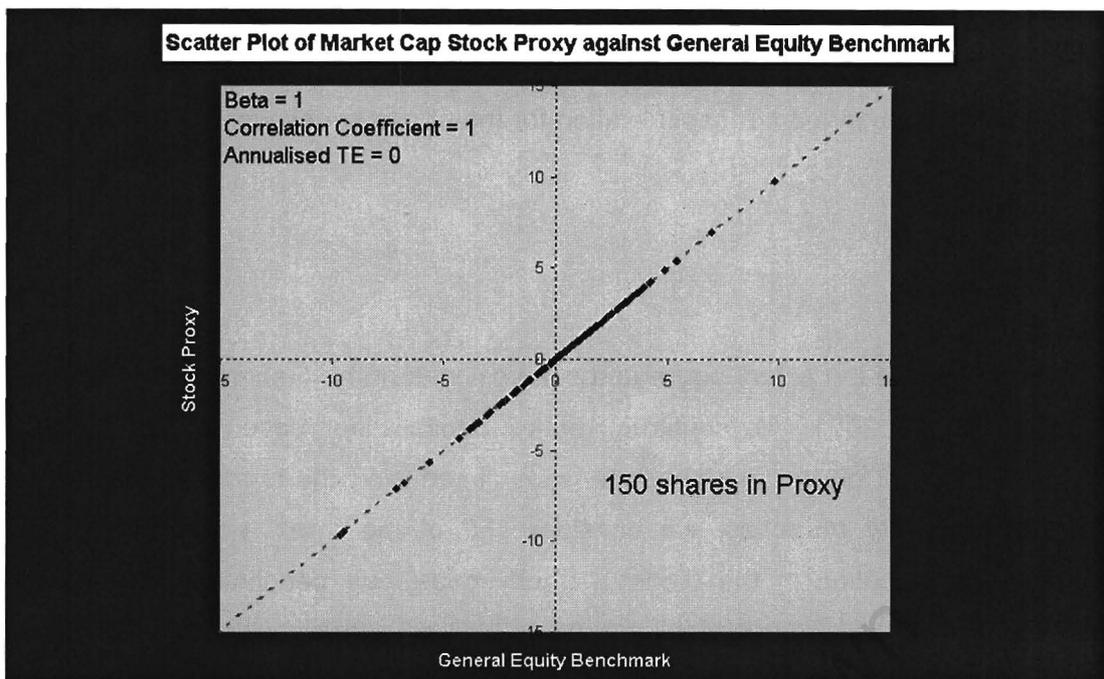


Figure 4.2: Stock proxy comprising 150 stocks vs the GE 150 stock benchmark

100 stock proxy...

The effect of using a 100 share benchmark proxy is graphically displayed in Figure 4.3. Contrasting Figure 4.3 to Figure 4.2 it is evident that the beta against the *underlying series* has increased to 1.02, and the tracking error has increased from 0 to 0.88. Consequently the 100 stock proxy inherently has a small timing bet reflected against the underlying series it was designed to track. We also note that the loss of the 50 smallest market capitalisation stocks has generated an expected tracking error of 0.88, although the correlation is very high at 99.99%. Clearly this stock proxy is a reasonable one as these above-mentioned risks are small, yet the fact that it contains as many as 100 shares may still be practically onerous to maintain.

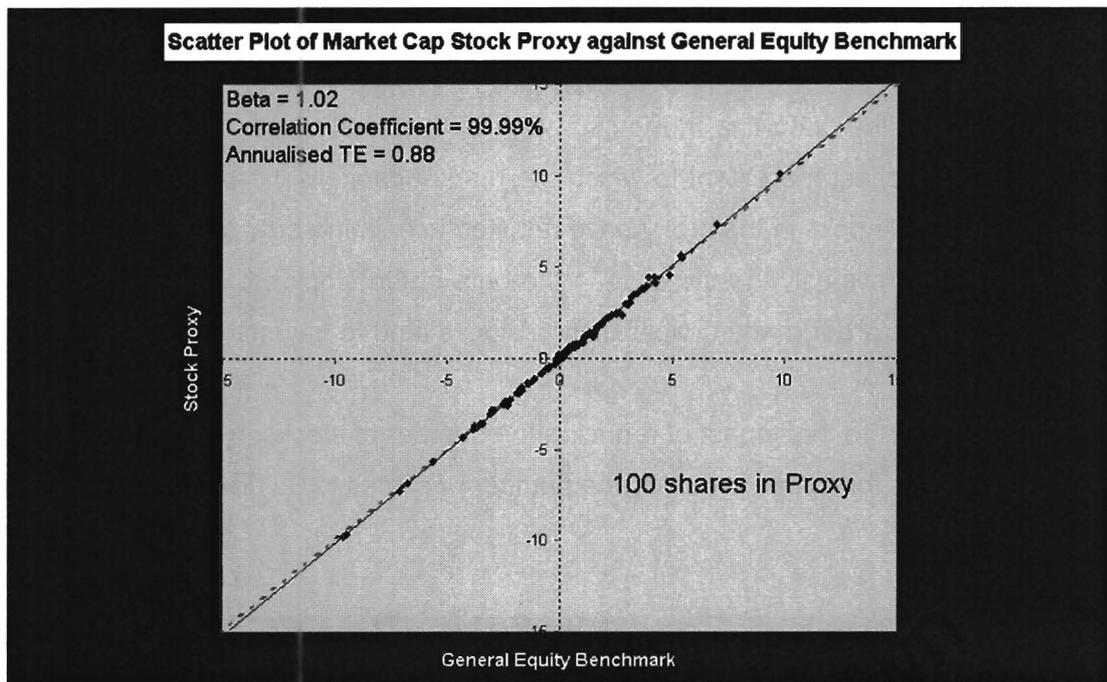


Figure 4.3: Stock proxy comprising 100 stocks vs the GE 150 stock benchmark

80 Stock proxy...

We provide the results of using an 80 stock proxy based on market capitalisations for the underlying GE benchmark series in Figure 4.4.

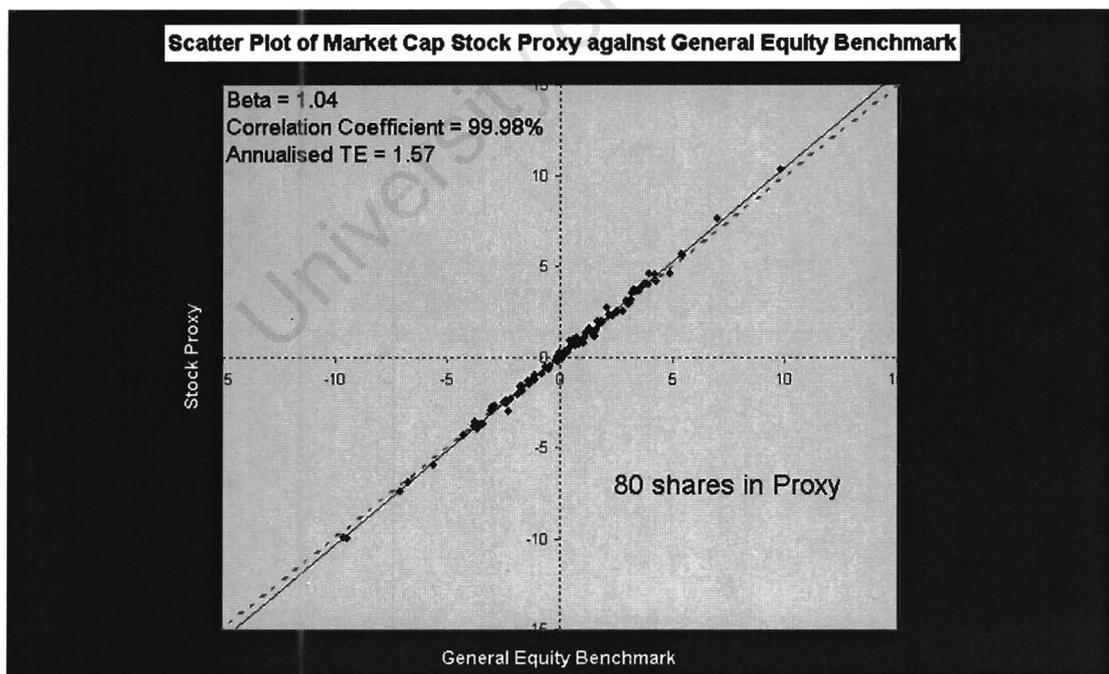


Figure 4.4: Stock proxy comprising 80 stocks vs the GE 150 stock benchmark

From the quantitative data displayed in Figure 4.4 it is evident that the beta of the stock proxy against the *underlying series* increases further to 1.04 as portrayed by the increased slope of the fitted line in Figure 4.4, with the tracking error increasing to 1.57. These measures reflect the extent to which the market capitalisation bias induced by the reduced number of stocks in the proxy have resulted in a timing bet against the underlying series. As stated previously the increase in the beta in the stock proxy is a consequence of the anomaly that larger market capitalisation stocks tend to have larger betas.

To further demonstrate the impact of reducing the number of stocks in a stock proxy we consider the results for further reducing the number of stocks to 60 and 40 stocks as well.

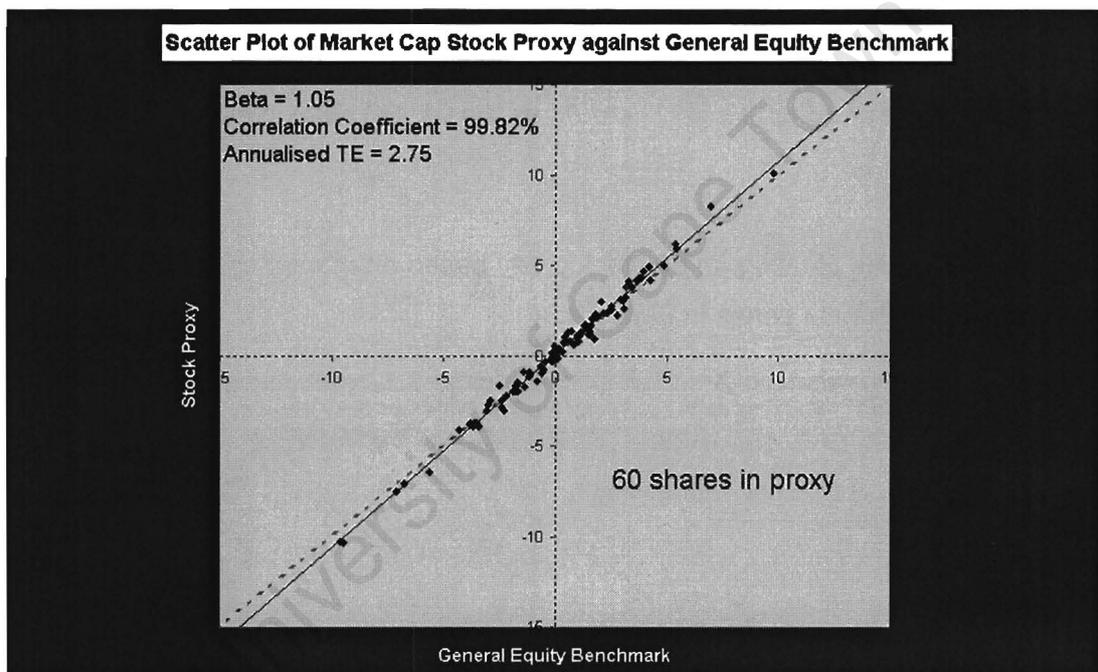


Figure 4.5: Stock proxy comprising 60 stocks vs the GE 150 stock benchmark

60 Stock proxy...

The results for the 60 stock proxy are shown in Figure 4.5. The increasing beta of the stock proxy against the underlying series (GE benchmark) now becomes visually marked. The beta has increased further to 1.05 and consequently the tracking error has increased to a now significant value, i.e. 2.75.

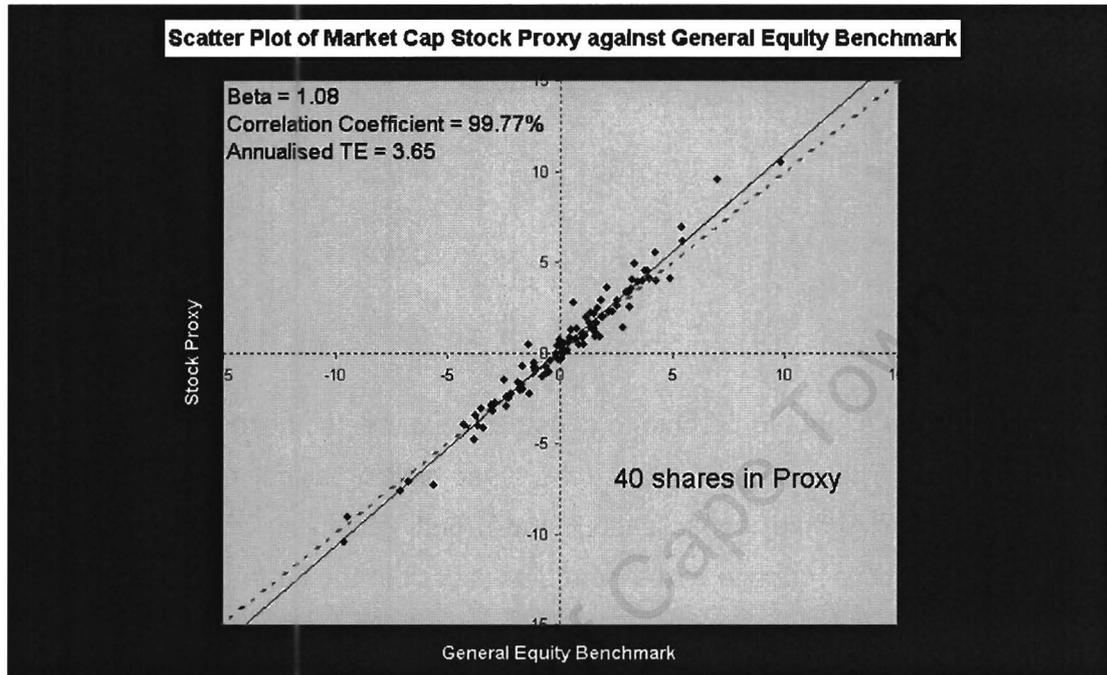


Figure 4.6: Stock proxy comprising 40 stocks vs the GE 150 stock benchmark

40 Stock proxy...

The results of reducing the number of stocks in the proxy to the 40 largest by market capitalisation are shown in Figure 4.6.

As can be seen from Figure 4.6, using a 40 share stock proxy induces an even larger timing risk as seen in the increased beta against the underlying series (to 1.08). This large timing bet also manifests itself in the annualised tracking error of 3.65%. Clearly such a stock proxy would be inadequate for the purposes of tracking the actual benchmark, as too much active risk in the form of timing risk is taken against the underlying series.

Table 1 gives a summary of our quantitative results of reducing the number of stocks in the stock proxies constructed on the basis of selecting stocks with the largest market

capitalisation for our demonstration case, i.e. the General Equity peergroup benchmark as the underlying series.

Table 4.1: Results of reducing the number of stocks in the stock proxy for the General Equity peergroup benchmark

Number of stocks in proxy	Expected Beta	Expected Correlation	Expected Annualised TE
150	1.00	1.0000	0.00
100	1.02	0.9999	0.88
90	1.03	0.9999	1.21
80	1.04	0.9998	1.57
70	1.05	0.9985	2.74
60	1.05	0.9982	2.75
50	1.06	0.9982	3.10
40	1.08	0.9977	3.65
30	1.11	0.9969	4.81

Typically one would expect to be able to construct an acceptable stock proxy with as few as say 60 stocks. The results of table 1 however highlights that using the method of selecting the largest market capitalisation stocks first, results in an unacceptably high tracking error as a consequence of the beta bias implicit in the large capitalisation stocks. Because of this selection bias one ideally needs a stock proxy construction approach that attempts to match the risks of the stock proxy to that of the underlying series more closely. Below we highlight how an optimisation approach can be used to construct a stock proxy that matches the risks more adequately.

4.3.1.4. Suggestions for improving the design of stock proxies

Conceptually the search mechanism for an optimal stock proxy should focus on matching the risks and should therefore give consideration to the correlation structure of the competing stocks.

In order to avoid inadvertently constructing a timing bet against the underlying series we would want the beta of the stock proxy to be as close to 1 as possible. Furthermore we would want the active positions created by the stocks omitted in the stock proxy to be offset (diversified away) by the stocks held in the reduced stock proxy as far as possible.

That is, we would want to reduce our stock selection risk against the underlying series as far as possible.

The objective of the optimisation should be minimise both the timing risk and stock selection risk against the underlying series. Clearly maximising the *correlation* between the stock proxy and the underlying series would be inadequate, as it would not control for the timing risk. Note for example one could have a correlation of 1 (perfect correlation) between two series – but the beta of one series against another could still be significantly different, say 2.

The appropriate quantitative measure that encapsulates both the timing and stock selection risk against the underlying series, is the tracking error. The decomposition of the tracking error into a timing risk and selection risk component follows:

$$\text{Tracking Error, } \psi = \sqrt{(\beta - 1)^2 \sigma_b^2 + (1 - \rho^2) \sigma_p^2}$$

Where β is the beta of the proxy versus the underlying series
 σ_b^2 is the volatility of the underlying series, or benchmark
 σ_p^2 is the volatility of the stock proxy
 ρ^2 is the correlation between the stock proxy and the underlying series.

The optimisation procedure for minimising ψ is equivalent minimising both of the terms capturing the timing and selection risks in the above decomposition. In essence the optimisation procedure results in a search for a stock proxy having a beta as close to 1 as possible, as well as having a correlation with the underlying series as close to 1 as possible. Ultimately the optimisation procedure matches the two central risks as closely as possible to that of the underlying series.

4.3.1.5. Demonstration of the optimisation process

As a demonstration of the optimisation application, we solve for the optimal stock proxy by constraining the optimiser to design a stock proxy based only on two scenarios constraining the number of stocks in the stock proxy. Namely we consider redesigning the stock proxy from (1) the largest 40 stock holdings, and (2) the largest 60 stock holdings of

the General Equity peergroup benchmark. We contrast our results to that of the market capitalisation approach. Here we used EXCEL's Solver, which uses an iterative quadratic optimization to maximize a given function.

The summarised results are shown in Figures 4.7 and 4.8 (for the 40 and 60 stock scenarios). Interestingly, in the first scenario only 36 stocks were selected by the optimisation process, and in the second scenario only 55 stocks were selected by the optimiser.

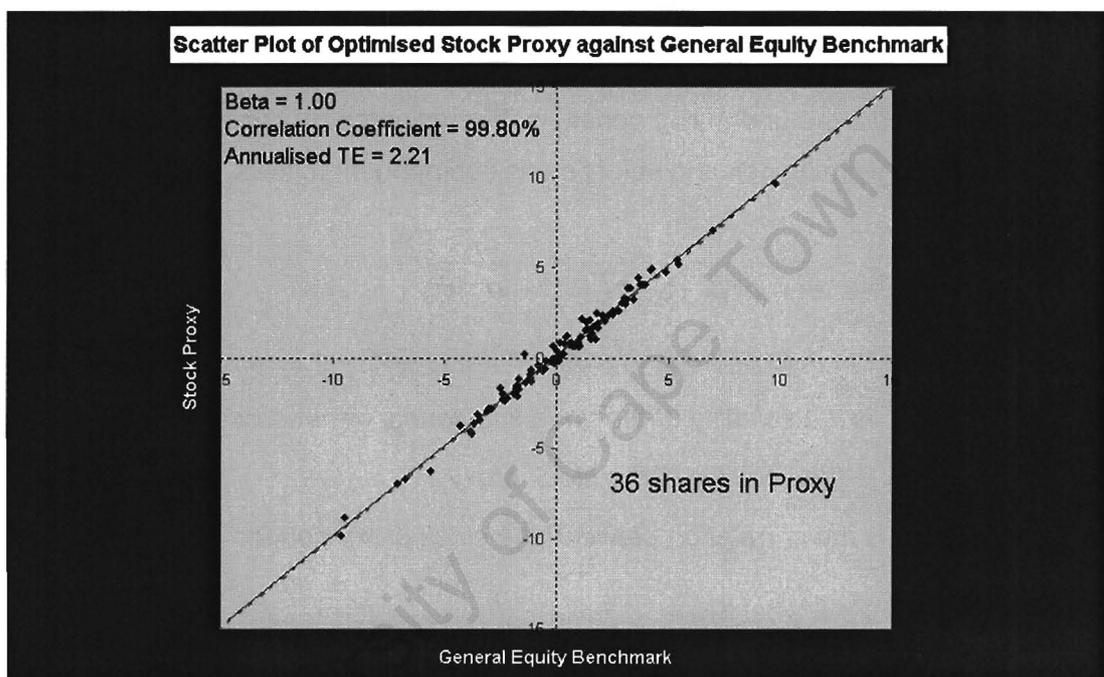


Figure 4.7: Optimisation-based stock proxy with 40 stock constraint

From Figures 4.7 and 4.8 we see that the fit of the optimised benchmark proxy is considerably improved by increasing the number of stocks from 40 to 60. Further when one contrasts these results to that of the 40 and 60 stock market capitalisation-based stock proxy there is a significant decline in the tracking errors. Most importantly, the timing risk, for all practical purposes, is almost totally eliminated, as the beta of the optimised stock proxies are extremely close to 1 against the actual benchmark.

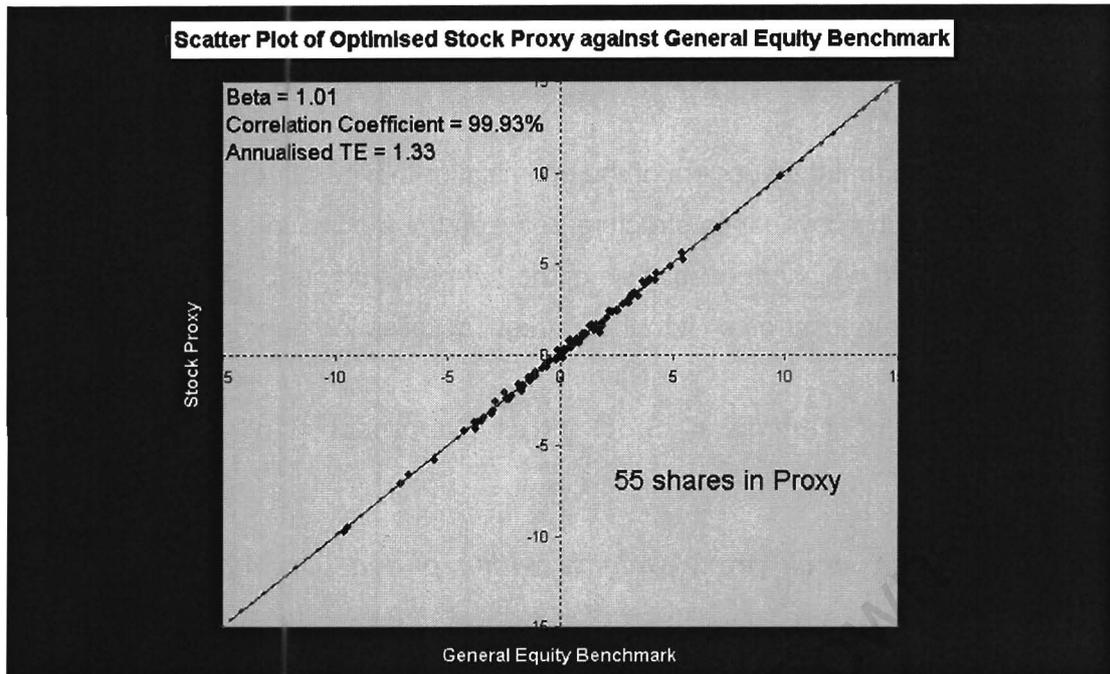


Figure 4.8: Optimisation-based stock proxy with 60 stock constraint

Table 4.2 gives a comparison of the risk decompositions (into timing and stock selection risks) of the various market capitalisation-based stock proxies together with the two optimisation-based stock proxies.

Table 4.2: Tracking error decompositions of market capitalisation-based and optimisation-based stock proxies

Stock Proxy	Selection Risk		Timing Risk		Annualised TEV
	Amount	Proportion	Amount	Proportion	
150	0.00	-	0.00	-	0.00
100	0.14	18.4%	0.63	81.6%	0.77
90	0.26	17.7%	1.21	82.3%	1.46
80	0.47	19.2%	1.99	80.8%	2.46
70	3.97	52.9%	3.54	47.1%	7.51
60	4.60	60.8%	2.96	39.2%	7.56
Optimised 60	1.64	93.3%	0.12	6.7%	1.76
50	4.89	50.9%	4.72	49.1%	9.61
40	6.46	48.5%	6.86	51.5%	13.32
Optimised 30	4.83	99.0%	0.05	1.0%	4.88
30	9.02	39.0%	14.11	61.0%	23.14

From Table 4.2 one obtains a relative perspective of the gains in the risks achieved by using an optimisation-based stock proxy rather than a market capitalisation based stock proxy.

It should be noted that the success of the optimisation-based approach does however rely on the stability of the covariance structure between the stocks under consideration. In the subsequent section we conduct analysis of the *out-of-period* performance of the optimised stock proxy and have contrasted it to the market capitalisation approach.

4.3.1.6 Results of Optimised Stock Proxy approach “in” an “out-of-period” framework

Estimation of the stock proxy weights within period (Oct 1999 – Sept 2000)...

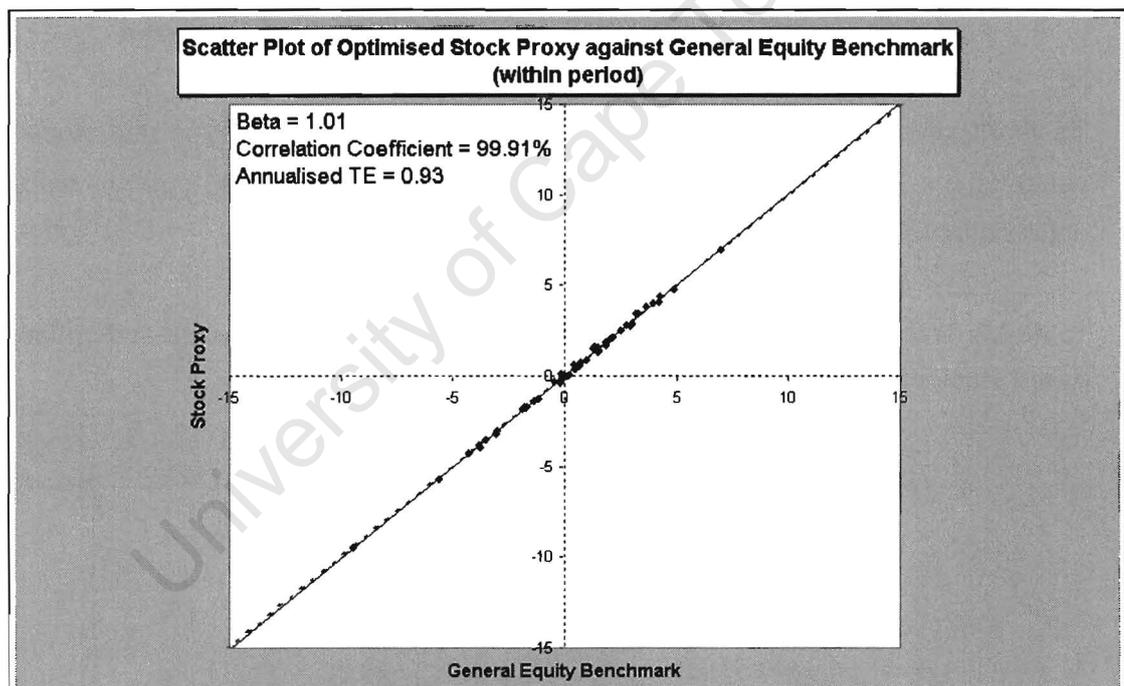


Figure 4.9: Within period results of optimised stock proxy

Using return data for the *first year* of the data period, the stock proxy was optimised using the top 60 market capitalisation stocks to generate portfolio returns which best replicated the benchmark returns during this period (using tracking error as the main criterion). As

can be seen in Figure 4.9, the proxy replicates the benchmark very well, with a low annualised tracking error of only 0.93. This low tracking error is expected as the optimisation is *within* the period assessed.

Testing the proxy out-of-period (October 2000 – September 2001)...

Monitoring the performance of stock proxy in the subsequent one year period (out-of-period) yielded a tracking error of 2.08 with a beta of 0.98 (as seen in Figure 4.10 that follows). This result demonstrates some degeneration of the stock proxy's ability to replicate the benchmark. The test period however consisted of the September market crash - increasing the tracking error of most investments because of the increase in the volatility in the second period (23.6 compared to the first period volatility of 21.6). From this perspective the annualised tracking error of 2.08 thus does not represent excessive risk against the benchmark.

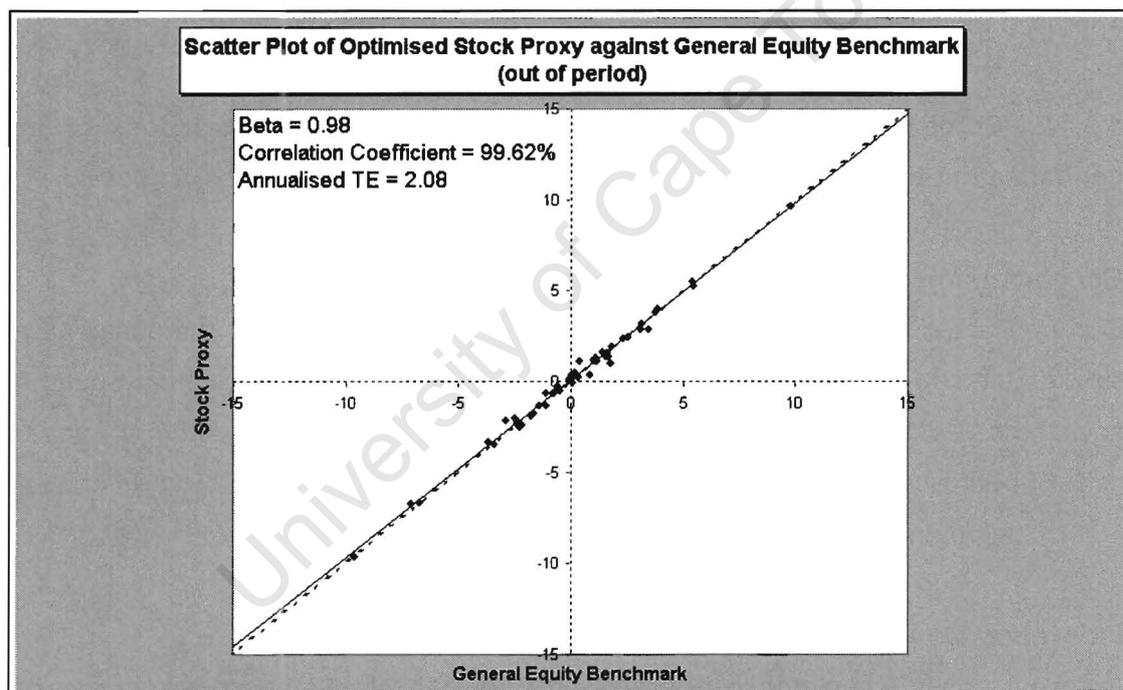


Figure 4.10: Out-of-period results of optimised stock proxy

Contrasting these results to the equivalent analysis of the market capitalisation approach yielded the following tabled summary:

Table 4.3: Comparative Tracking Errors

Stock Proxy Construction method	Tracking Error	
	Within-period	Out-of-period
Optimisation method	0.93	2.08
Market Capitalisation method	2.52	2.80

From Table 4.3 it is evident that the optimisation method of stock proxy design yields a lower out-of-period tracking error – supporting our tentative conclusion that the tradeoff between risk-matching and stability favours the optimisation approach. The cursory results show that the optimised stock proxy remains superior to the market capitalisation-based in an “out-of-period” context. However, one cannot generalize this result, as the results in Table 4.3. demonstrate the optimized proxy’s superiority over the market capitalisation approach for this sample only.

4.3.1.7 Some practical caveats arising from the construction of market capitalisation-based stock proxies

Benchmarking using market capitalisation-based stock proxies...

Our empirical demonstration has highlighted that stock proxies (with a reduced number of stocks) tend to have higher betas than their underlying series in the South African environment. When stock proxies are used to proxy benchmarks in order to manage the risks of portfolios against - the systematic bias in the proxy can lead to systematic errors in matching/managing risks of portfolios.

Indexation using market capitalisation-based stock proxies...

A caveat arises when one constructs a market capitalisation-based stock basket with fewer stocks than the index it is purported to track. As demonstrated in the empirical analysis, the market capitalisation approach can result in the stock proxy having a higher beta relative to the index. Hence the proxy is likely to outperform the index in rising market and to underperform the index in a declining market – creating unwanted tracking error.

Hedging portfolios using market capitalisation-based stock proxies...

A further caveat arises when one constructs a market capitalisation-based stock basket with fewer stocks than the underlying portfolio for the purposes of hedging the portfolio. As highlighted above the market capitalisation approach typically results in the stock proxy having a higher beta relative to the underlying portfolio. Hence the proxy is likely to outperform the underlying portfolio in rising market and vice versa. As the purpose of a hedge is generally to *short* the stock proxy, clearly this would result in a significant shortfall in the hedge position in a rising market and vice versa – highlighting the inadequacy of the market capitalisation-based stock proxy construction approach for hedging purposes.

4.3.1.8. Conclusion

Our empirical demonstration has highlighted that in the South African environment, stock proxies (with a reduced number of stocks) tend to have higher betas than their underlying series. Consequently, the fewer the number of stocks in the proxy - the higher the betas. This problem is particularly relevant where benchmarks are proxied by stock proxies as this systematic bias can lead to systematic errors in matching/managing risks of portfolios. For indexation application, stock proxies can also result in significant under / out performance in volatile markets, thus yielding unacceptably high tracking errors. Additionally, for hedging purposes, these stock proxies can result in significant shortfalls in the hedge return.

In ensuing proposals the attention shifts to the case where benchmarks are required to accommodate several levels (absolute) to match client profiles. We discuss this application with less detail in the subsequent section.

4.3.2 A graphical aid to match client profiles to several levels of risk (absolute)

In this section the contribution has been aimed at the trustees who are typically faced with the task of understanding the risk characteristics of their fund, and most importantly the benchmarks. Most trustees are typically not well versed in the quantitative aspect of finance theory and hence the communication in the context of benchmarks needs to be simple.

Our contribution in this section is very modest – the aim is simply to promote / highlight the use of simple graphical aids to better understand how portfolio risks can be matched to that of differing client risk profiles.

In contrast to attempting to track the investment performance of a benchmark, the client may not be concerned with relative performance of the fund. The objective of the client could be “not to lose money”, in which case the absolute (total) risk of the fund is vital. Clearly, the lower the total risk of the client’s fund, the lower the chance of achieving a negative return. Conversely, clients may prefer a higher absolute risk for the fund, where a larger deviation from the expected return is acceptable. Typically the age profile of investors is an important consideration in matching the absolute risk of the benchmark to client requirements.

We once again refer to the scatter plot of returns which graphically display the performance of risks of the fund against a benchmark in Figure 4.11. Thus, when considering benchmarks with differing levels of absolute risk, we note that absolute risk can be viewed in the scatter plot from the left along the y-axis. Absolute risk is calculated as the standard deviation of the proposed benchmark’s returns. The extent of the dispersion along the y-axis represents the absolute risk of the fund, which is a benchmark-free measure of risk.

More specifically, absolute risk is calculated as

$$\sigma_p^2 = \beta^2 \sigma_{benchmark}^2 + \sigma_e^2 \quad \dots(4.1)$$

where $\sigma_{benchmark}^2$ is the benchmark volatility (as reflected by the dispersion along the x-axis), and

σ_e^2 is the residual (unique / diversifiable) risk.

As seen in equation (4.1), in order to adjust the absolute risk of the proposed benchmark to match client profiles, the dominant contribution to the absolute risk of fund is the beta coefficient. The beta (as measured through the slope of the regression line) of the proposed benchmark relative to the actual benchmark is the primary statistic that needs to be changed. This change in beta effectively changes the proposed benchmark's exposure to the actual benchmark.

Clients who have higher aversions to risk (usually older clients) would require more downside protection, and thus lower absolute risk. The easiest means of adjusting the beta and consequently the absolute risk of the fund would be to use tactical asset allocation. This simply implies the changing of the fund's asset allocation to attain a target portfolio beta. Thus lowering of market exposure is easily applied by holding more cash (as cash typically has a beta of zero). Although holding more cash than the benchmark may increase the *relative* risk of the proposed benchmark (tracking error), this may not be of concern to the client, whose primary concern is preservation of capital, i.e. the absolute risk.

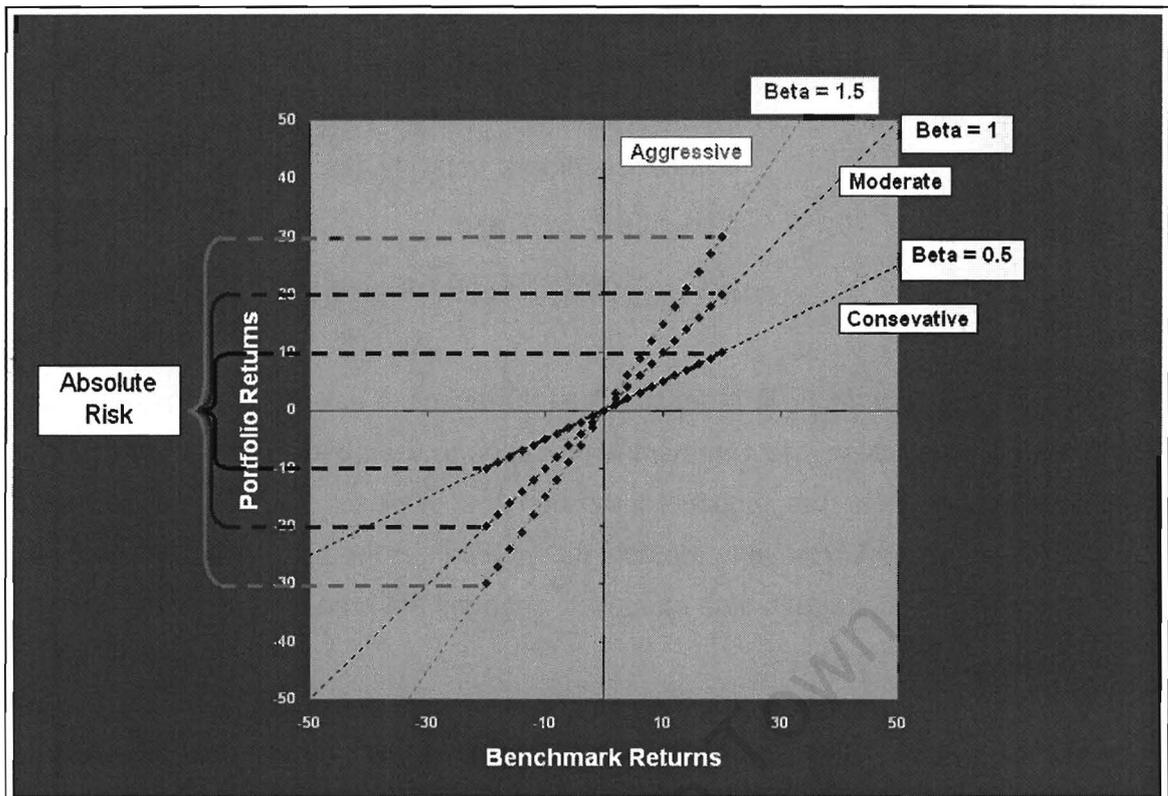


Figure 4.11: Scatter plot of Portfolio returns vs. Benchmark returns for differing levels of portfolio risk

The impact on absolute risk as a consequence of altering the portfolio beta is illustrated in Figure 4.11. As the beta increases from 0.5 to 1 to 1.5, the absolute risk of the fund increases as well according to the formulation in (4.1). Figure 4.11 can be used to effectively communicate how the risk profiles (ages) of clients can be accommodated for by adjusting the overall portfolio beta.

In the subsequent section, we discuss the application of constructing a benchmark to generate an outperformance against a specified benchmark.

4.3.3 A graphical aid for assessing potential outperformance characteristics in an investment category

The aim of any active portfolio manager is to realise positive active return through selecting mispriced shares in the market. The ability to 'beat the market' thus lies in the ability of the manager (or analyst) to select a portfolio of shares which in combination adds value against a benchmark, or has positive *alpha*. Given a manager has this ability to select shares with positive alpha, the important process is transferring this information into meaningful input for portfolio design.

We can transfer these ideas on an "outperformance portfolio" to that of an "outperformance benchmark". It would however seem intuitive that the "outperformance benchmark" would have differing sector weights (rather than stock weights) to some underlying benchmark.

For example, some fund managers in South Africa have constructed envisaged "outperformance benchmarks" by down weighting the Resource sector in the ALSI. Typical down weighted benchmarks have splits of 80% Financials & Industrials (FINDI) and 20% Resources (RESI), whereas the ALSI split would currently average around 60% FINDI and 40% RESI. The implicit expectation in this outperformance benchmark is clearly that the FINDI would outperform the RESI in the foreseen future. In this way, the fund managers hope that such an outperformance benchmark would yield an alpha against the ALSI.

Our objective is once again very modest – we simply aim to highlight how an outperformance benchmark can be assessed in a simple graphical framework. We focus on the outperformance as reflected in the alpha (in much the same way as for alpha for individual stocks).

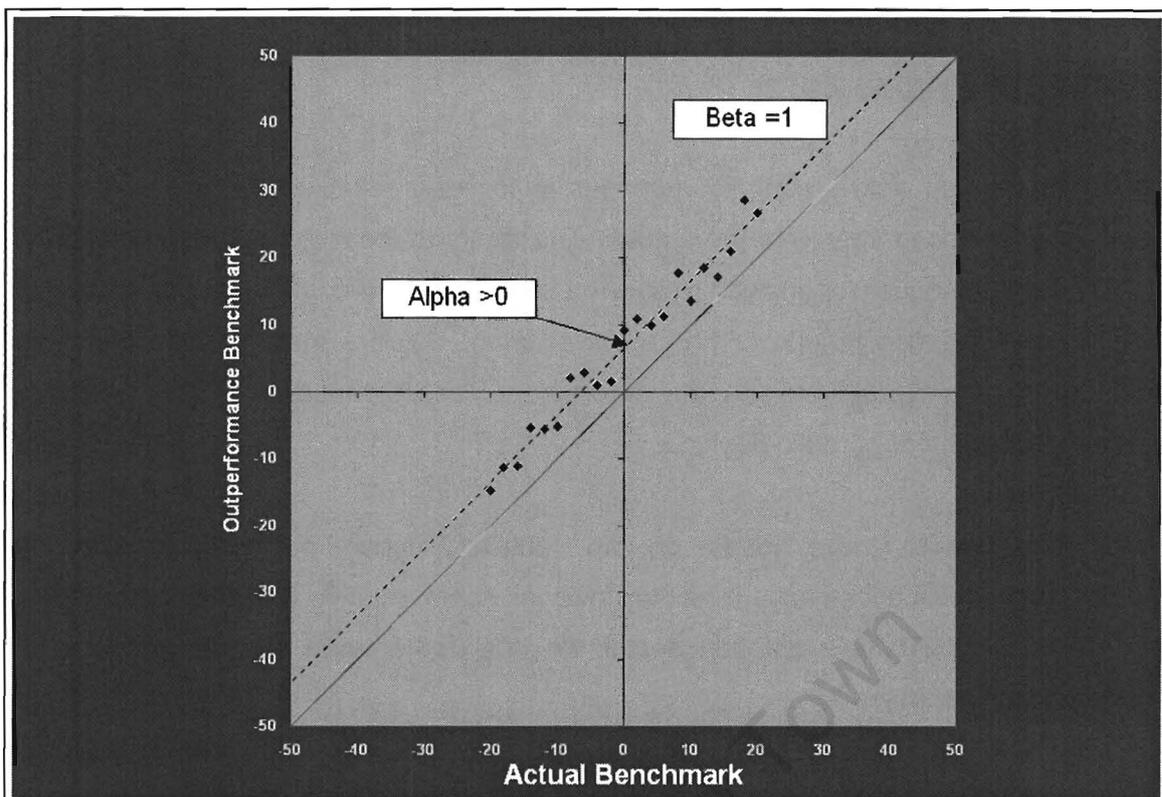


Figure 4.12: Scatter plots of portfolio returns versus benchmark returns with positive alpha and Beta = 1

Graphically, the alpha is the value at the point of intersection of the outperformance benchmark against the actual benchmark. As can be seen in Figure 4.12, the portfolio contains positive alpha and the beta is equal to one.

Besides the fact that this *outperformance benchmark* must contain some alpha, there exist important risk management factors to consider in the portfolio design. When 'chasing alpha', it is vital to ensure the risk taken (tracking error) is taken up largely by selection risk, and not timing risk. Managers thus must ensure that the beta of the outperformance benchmark relative to the actual benchmark is as close to one as possible to avoid unwanted timing risk against the actual benchmark. In a later chapter we will motivate why selection risk is preferred to timing risk, as well as provide recommended beta ranges for which to keep portfolio betas within to avoid significant timing risk.

The proposal in this section is that a fund manager aims to achieve these statistics for the outperformance benchmark.

4.4. Estimating Peer Benchmarks

4.4.1. Introduction and discussion

It is often important for risk management purposes to ensure the portfolio manager considers the *expected* tracking error of the portfolio relative to the peer group benchmark, due to the fact that it encompasses the *current* active risk of the portfolio. Or more loosely speaking, managers often prefer to manage the risk of *underperforming their peers* given the current holdings of the fund – rather than the historical returns. Compositions of peer funds are often not publicly available however, especially in the pension fund industry. The central focus in this section therefore will be on quantitative techniques to estimate the major sector holdings of peers via their returns.

We recall that the computation of expected tracking error requires a covariance matrix of returns of the shares in the portfolio and the benchmark as well as the current relative compositions between the portfolio and benchmark. One practical problem is that current compositions held by peer benchmarks are not always widely known. Whilst compositional data is available only quarterly in the Unit Trust industry, publicly available pension fund data is even scarcer. It is often a necessity therefore to *estimate* the actual compositions through examining the relationship of returns of the benchmark and the observed returns of the assets that the benchmark holds.

Sharpe (1988) pioneered a procedure using constrained regression analysis to form inferences on the influences and compositions of portfolios, based solely on the historical returns of the portfolio. This approach has been used by practitioners attempting to establish the compositions of portfolios or benchmarks for which no composition data is available. This methodology later led to the work by Sharpe (1992) on style analysis.

Sharpe's constrained regression analysis was based on a factor model in the following form:

$$R_p^t = b_o^t + \sum_{k=1}^n b_k^t F_k^t$$

where: R_p^t = return series of portfolio at time period t

F_k^t = return series of factor k

b_k^t = sensitivity of portfolio returns to factor k

b_o^t = time series of portfolio residual returns

In the context of estimating compositions, we use market or sector indices (e.g. Financial Index, Industrial Index or Resource Index) instead of factors. We thus interpret the coefficients of the regression as the sensitivities of the portfolio returns to the factor i (or in our case sector index i).

The regression approach however can yield outputs that can be unintuitive to practitioners, as the coefficients could have negative values (implying unintuitive negative sensitivity or negative weighting) and the independent variables could additionally suffer from multicollinearity problems (if any market indices are highly correlated). Further concerns relating to using regression analysis for estimating index sensitivities are outlined by Christopherson (1995) and Trzcinka (1995).

Sharpe's standard constraints placed on the regression to facilitate intuitive interpretation are

$$0 < b_i^t < 1$$

$$\sum_{i=1}^n b_i^t = 1$$

These constraints assist the problem of multicollinearity, and allow the resultant sensitivities to sector indices as proxies for weights of the portfolio in the specified sector. However, given the constraints of the coefficients, the estimation of the regression no longer uses the traditional estimation method of least squares, but rather a technique of quadratic programming.

Underlying assumptions...

The underlying assumption in the regression requires that the returns of the portfolio attributed to each sector are commensurate of the actual returns of that sector. Stock selection within a sector could cause an element of mismatching of return, however if an aggregated peer group benchmark is used, stock selection would be minimized due to the diversification effect. Another assumption requires that the weights of the portfolio be

constant for the period of analysis. Results can consequently only be interpreted as *average* estimated weights over the period of analysis.

Lobosco and DiBartholomeo (1997) proposed the use of confidence intervals for estimating the weights when using Sharpe's constrained regression. The standard deviation of estimates is thus:

$$\frac{\sigma_a}{\sigma_{B_i} \sqrt{n - k - 1}}$$

where:

σ_a = standard error of regression analysis

σ_{B_i} = unexplained sector volatility

n = number of returns used in the regression analysis, and

k = number of sector indices with non-zero regression coefficient weights.

The derivation of the formula can be found in Appendix 4A at the end of Chapter 4. As expected, the standard deviation of the weight estimated increases with the standard error of the regression analysis. Additionally they decrease with the number of returns used in the regression analysis and also decrease with the unexplained sector volatility.

The aim of multiple linear regression analysis is to minimise the squared errors between the portfolio returns and a portfolio of weighted sector returns, in the form of:

$$\text{Minimise } \sum_{i=1}^n (R_i^p - b_0 - b_1 F_{i1} - b_2 F_{i2} - \dots - b_{n-1} F_{i,n-1})^2$$

Noting that $b_0 + \sum_{k=1}^n b_k F_k$ is equivalent to the return of the estimated sector weighted

portfolio, the regression reduces to

$$\text{Minimise } \sum_{i=1}^n (R_i^p - R_i^*)^2$$

where R_i^* is the portfolio returns of the estimated weighted sector index which best represents the actual portfolio returns. This formulation is thus consistent with the objective of minimising the tracking error between the two series.

4.4.2 Empirical analysis

The objective of the ensuing analysis is to estimate the benchmark composition of the Financial and Industrial Index (FINDI) and Resources (RESI) Index held by the aggregate of fund managers in the Alexander Forbes Top 10 Pension Fund category (henceforth referred to as the *implicit benchmark*). We use the “*equity only*” component of monthly returns in the analysis for the period January 1998 to December 2001 for this purpose. To assess the dynamics of the estimated components of the benchmark through time, we employ a rolling period analysis, whereby a period of 12 months is analysed and rolled forward month by month. We also further decompose the FINDI Index into the Financial Index and Industrial Index in order to establish a more accurate estimate of the FINDI - RESI mix. Peer group benchmarks based on both the *median* and the *mean* Top 10 Manager fund returns in each month are also considered.

4.4.2.1 Results using Peer Median

Because of the small sample size of the AFLMW (10 funds), an “outlying” fund like ALLAN GRAY could potentially influence the *mean* fund return but would have no impact on the *median* fund return. We therefore implemented the iterative methodology described above to estimate the *median* peer group holding using one-year rolling estimation periods. A graphical summary of the results is shown in Figure 4.13.

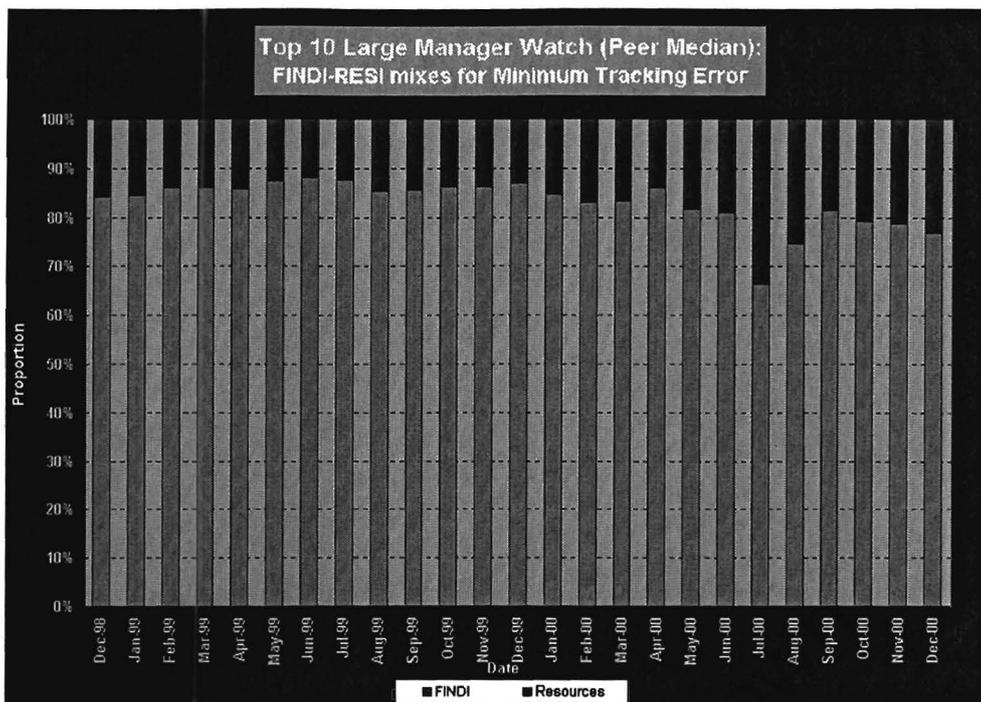


Figure 4.14: FINDI-Resources weights resulting in minimum Tracking Error for Median Top 10 Pension funds.

The results reveal that the *median* fund held between 80% to 88% in the FINDI Index up to the July 2000, thereafter a FINDI weighting fractionally smaller than 80% was apparent. A dip in the FINDI around July 2000 occurred, which is suspected to be an unreliable estimate due to the drastic change.

In an attempt to improve the analysis we include the Financial and Industrial Indices as separate components in the FINDI. The argument is that the additional split into Financials and Industrials, may well result in a lower tracking error generated using 3 assets instead of 2. Whilst we may not be interested in this level of detail, the additional impact on the correlation structure may be important in ascertaining a more accurate overriding FINDI-Resources mix.

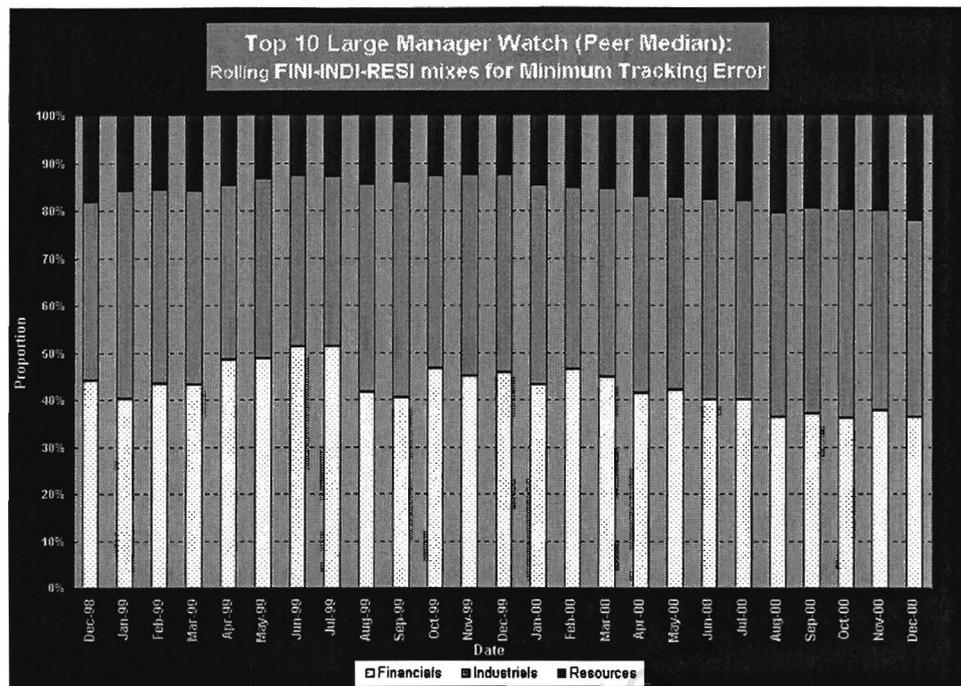


Figure 4.15: Resulting weights considering three indices

It is evident from Figure 4.15 that the overriding FINDI-Resources mix is more stable when the additional decomposition of the FINDI is taken into consideration. This is likely to be a more intuitive result. By comparing the Resource weights for July 2000 in Figures 4.14 and Figure 4.15 it is thus most likely that the more accurate result is found in Figure 4.15.

4.4.2.2 Analysis using Peer Mean

Note that the *mean* fund return in essence uses information from all 10 pension fund returns in each month. Recall however that an "outlying" fund like ALLAN GRAY could potentially influence the *mean* fund return but would have no impact on the *median* fund return.



Figure 4.16: FINDI-Resources weights resulting in minimum Tracking Error for Mean Top 10 Pension funds.

Contrasting Figures 4.16 and 4.17 it is evident that Figure 4.17 depicts slightly less volatile FINDI-Resource mixes than the 2-index analysis in Figure 4.16. This suggests that the 3-index approach does yield a more reliable result. More importantly, contrasting the results of the analysis using the median versus the mean (in Figures 4.15 and 4.17 respectively), it is evident that prior to August 2000 the results of the *median* analysis (Figure 4.15) yielded a marginally smaller weight in Resources than the analysis using the *mean* fund return (Figure 4.17). This influence may be due to an outlying fund such as ALLAN GRAY having a significantly larger relative weight in Resources over this period - impacting on the analysis using the *mean* fund return. Subsequent to August 2000 however there is no significant difference in the weights found using the median and mean fund return.

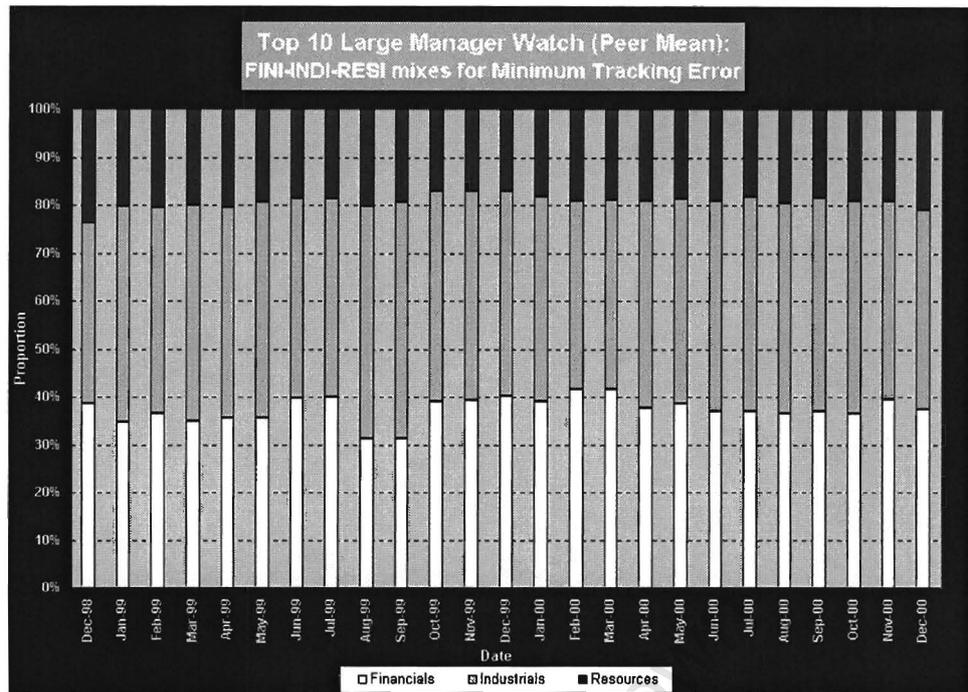


Figure 4.17: Resulting weights considering three indices - analysis using peer mean.

Table 4.4 below gives a quantitative summary of the investment weights in the corresponding sectors of Figures 4.14 through to 4.17. The average minimum tracking error for this procedure was 2.86, compared to the initial approach of using the Peer median with 2-indices with an average minimum tracking error of 3.54. The investment weights are also most stable, with the lowest standard deviation of 1.40. It is clear that the most successful estimation approach for benchmark weights occurs when one uses the Peer Mean with 3 indices.

Table 4.4: Summary of estimated composition of Peer Benchmarks

	Peer Median				Peer Mean			
	2 index		3 index		2 index		3 index	
	FINDI	Resources	FINDI	Resources	FINDI	Resources	FINDI	Resources
Dec-98	84.26	15.74	81.78	18.22	77.26	22.74	76.40	23.60
Jan-99	84.46	15.54	84.09	15.91	78.65	21.35	79.72	20.28
Feb-99	86.04	13.96	84.35	15.65	79.50	20.50	79.43	20.57
Mar-99	85.97	14.03	84.20	15.80	79.37	20.63	80.02	19.98
Apr-99	85.88	14.12	85.34	14.66	79.52	20.48	79.50	20.50
May-99	87.34	12.66	86.74	13.26	80.50	19.50	80.67	19.33
Jun-99	88.04	11.96	87.25	12.75	81.62	18.38	81.39	18.61
Jul-99	87.68	12.32	87.14	12.86	81.52	18.48	81.43	18.57
Aug-99	85.38	14.62	85.59	14.41	79.87	20.13	79.75	20.25
Sep-99	85.64	14.36	85.90	14.10	80.48	19.52	80.72	19.28
Oct-99	86.28	13.72	87.47	12.53	82.48	17.52	82.98	17.02
Nov-99	86.34	13.66	87.65	12.35	82.34	17.66	82.98	17.02
Dec-99	86.92	13.08	87.67	12.33	82.48	17.52	82.90	17.10
Jan-00	84.72	15.28	85.38	14.62	81.48	18.52	81.84	18.16
Feb-00	83.15	16.85	84.80	15.20	80.05	19.95	81.02	18.98
Mar-00	83.35	16.65	84.61	15.39	80.23	19.77	81.21	18.79
Apr-00	86.02	13.98	83.05	16.95	81.72	18.28	80.91	19.09
May-00	81.82	18.18	82.79	17.21	80.61	19.39	81.27	18.73
Jun-00	81.15	18.85	82.24	17.76	80.33	19.67	80.87	19.13
Jul-00	66.36	33.64	81.99	18.01	77.57	22.43	81.90	18.10
Aug-00	74.65	25.35	79.53	20.47	74.65	25.35	80.56	19.44
Sep-00	81.52	18.48	80.41	19.59	81.52	18.48	81.70	18.30
Oct-00	79.12	20.88	80.15	19.85	80.35	19.65	81.01	18.99
Nov-00	78.78	21.22	79.93	20.07	80.01	19.99	80.85	19.15
Dec-00	77.02	22.98	77.90	22.10	78.41	21.59	79.03	20.97
Std Dev	4.92		2.84		1.81		1.40	
Average Min TE	3.54		3.38		2.93		2.86	

4.4.2.3 Reliability of results

In order to give some insights on the reliability of our results, we have documented the minimum tracking error that was obtained, and plotted it alongside the resulting FINDI-Resources weights obtained at these minimum levels of tracking error at each point in time. These tracking errors are displayed in Figure 4.18 for the median analysis (from Figure 4.15) along with the estimate weights.

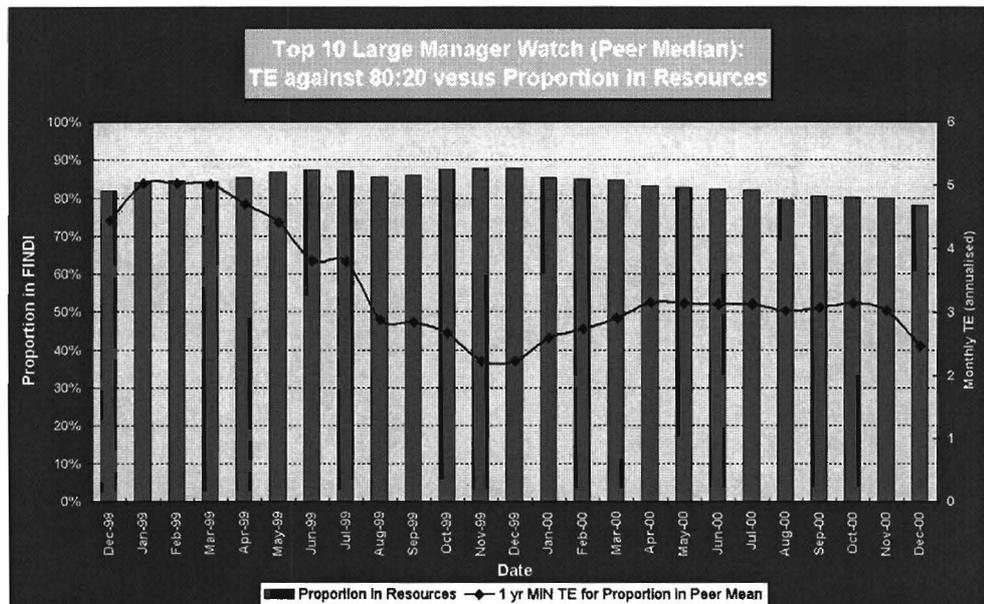


Figure 4.18: Minimum tracking errors and corresponding FINDI-Resources mixes through time using Median fund returns

The minimum tracking error series in Figures 4.18 is seen to be larger up until August 1999 as a consequence of the volatility in fund returns caused by the August 1998 crash. Thereafter the August 1998 returns are rolled out of the estimation period, resulting in significantly lower tracking errors, and consequently more reliability in the estimation process.

4.4.2.4 Conclusion

The objective in this section was to suggest a practical approach for estimating the major sector holdings of an aggregate peer benchmark. Our motivation is that in many circumstances only the returns of the benchmark is publicly available – and compositions are not known.

In this section we have thus proposed using an approach of minimising tracking error, and found it yielded a useful guide for the estimation of the major sector compositions held by the peer benchmark.

4.5. Outperforming benchmarks using Sector Allocation

Outperforming a pre-specified benchmark is the fundamental goal of any active manager. In order to generate outperformance however, active managers need to separate and add value in two distinct processes. The first concerns finding superior information to aid forecast ability, with the second process involving the efficient translation of this information into the construction of portfolio. Practically, beating a benchmark thus requires the aspects of both skill and aggression. Thus, it is evident that it is insufficient for active managers to *only* possess a certain level of skill, as the ability to apply the skill and take active positions against the benchmark is of equal importance.

In this section we consider the potential for an active manager to outperform a pre-specified benchmark through allocation between major sectors on the JSE, namely the Financial & Industrial Index and the Resources Index. We impose various levels of skill and aggression to demonstrate the separate impacts these processes have on both the risk and return of the active manager.

4.5.1 Review

Prior studies on outperforming benchmarks (which have typically been broad market indices) have concentrated on whether active managers have on average outperformed an aggregate benchmark, and whether this outperformance has been due to luck or skill. Early studies have shown that on average, mutual funds have underperformed their benchmarks on a risk-adjusted basis. Ippolito (1993) found that performance of the average fund in the U.S.A was statistically indistinguishable from the S&P 500 index, whereas Malkiel (1995) found that when taking into account survivorship bias, the average U.S.A equity manager significantly underperformed the S&P 500 index for the period 1981 to 1991. Other studies² have found similar results of no evidence for the average active manager producing exceptional returns.

² See Ferson and Schadt (1996) and Daniel, Grinblatt, Titman and Werners (1997).

Fortunately, this says nothing about the potential for successful active portfolio management. Marcus (1990) found that top performing mutual funds exhibited statistically significantly positive relative performance, given the large number of funds in the study.

Fox (1999) ran simulations to assess the ability of U.S. active managers (using data from 1980 to 1995) to outperform a pre-specified benchmark by using tactical asset allocation. According to Fox, the extent of relative performance relied on two factors: forecast abilities and the size of the tilts away from the benchmark mix. He found that a surprisingly little amount of forecasting ability is needed for a U.S. tactical asset allocation manager to provide an attractive return in excess of the benchmark.

4.5.2 Empirical Analysis

Our empirical analysis is thus based on simulations of active equity managers taking sector allocation bets between the Financial & Industrial Index (FINDI) and the Resources Index (RESI) based on historical monthly returns during the period January 1995 to October 2000. To generalise our results, our study design is not reliant on the exact choice of weights in the benchmark. In other words, whether the benchmark choice is 80 : 20 (FINDI : RESI) or 50 : 50 is not material.

We thus run simulations based on active equity managers performing sector allocation between the Financial and Industrial Index (FINDI) and the Resources Index (RESI) based on historical monthly returns during the period January 1995 to October 2001. The manager is compared to a combination of the two sectors, namely a benchmark of 70% FINDI and 30% RESI. Thus, the difference between the active manager's portfolio returns and the benchmark is solely due to the choice of sector weights in the portfolio.

4.5.2.1 Methodology

The skill level of managers and their ability to correctly forecast the best performing sector in a particular month, is reflected in the average proportion of correct choices they make over the time period. For each simulation, a relative return and tracking error is calculated

and recorded. We repeat these simulations five thousand times for each scenario of forecasting ability and tilt size, in order to assess the distribution of relative returns and active risks taken.

4.5.2.2. Forecasting ability

To assess the relationship between the forecasting ability and relative returns, we fix the tilt sizes from the benchmark, and assess relative returns at forecasting abilities of 50%, 60%, 70%, 80% and 90% over the time period 1995 to 2000. In Figure 4.19 we show the median relative return for fixed 5% tilt sizes. We also plot the upper and lower quartile and maximum and minimum relative return in order to assess the distribution of relative returns for various forecasting abilities.

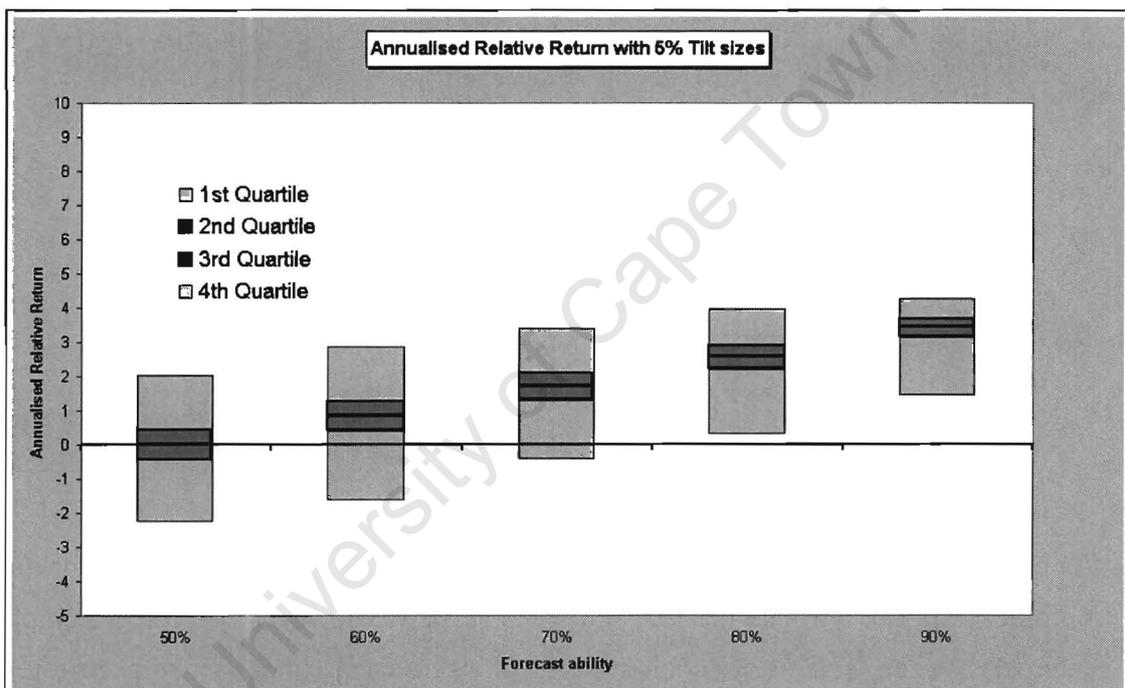


Figure 4.19: Annualised Relative Return with 5% tilt sizes

Figure 4.19 illustrates the relative return quartile distribution for each level of forecasting ability, with the inner two shaded bars representing the 2nd and 3rd quartiles of relative return. The outer lightly shaded bars represent the 1st and 4th quartiles. It is clear that a manager with 50% (random) forecasting ability will have an expected outperformance relative to the benchmark of zero. Clearly, a 50% forecasting ability in essence embodies no skill. It is also clear that as the forecasting ability increases, the median relative return increases almost linearly. However the dispersion of relative returns (as measured by the

inter-quartile range) decreases with increasing forecasting ability. A possible interpretation for these results is that the higher the skill of managers, the more certain or predictable the attainment the relative return. This suggests with higher levels of skill we are more assured of obtaining the outperformance average.

We consider the comparable results in Figure 4.20 where a manager is more aggressive and uses instead larger tilt sizes of 10%.

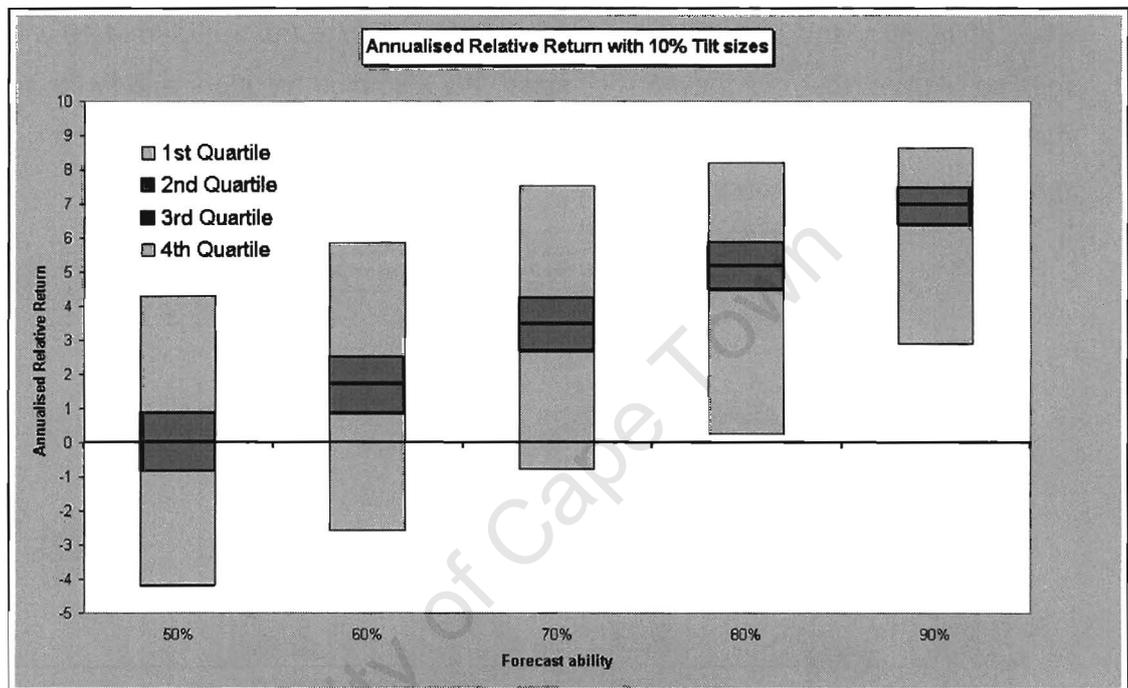


Figure 4.20: Annualised Relative Return with 10% Tilt sizes

As seen in Figure 4.20, the relationship between the median relative return for each level of forecasting ability remains linear with 10% tilt sizes. However, the dispersion of relative returns is wider. The same characteristic of decreasing ranges of relative returns with increasing forecasting ability is seen with 10% tilt sizes, as well as a tendency for increasing positive skewness with forecasting ability.

To confirm the decreasing dispersion of relative returns with forecasting ability, we now plot the tracking error for both 5% and 10% tilt sizes for each level of forecasting ability. This result is shown in Figure 4.21.

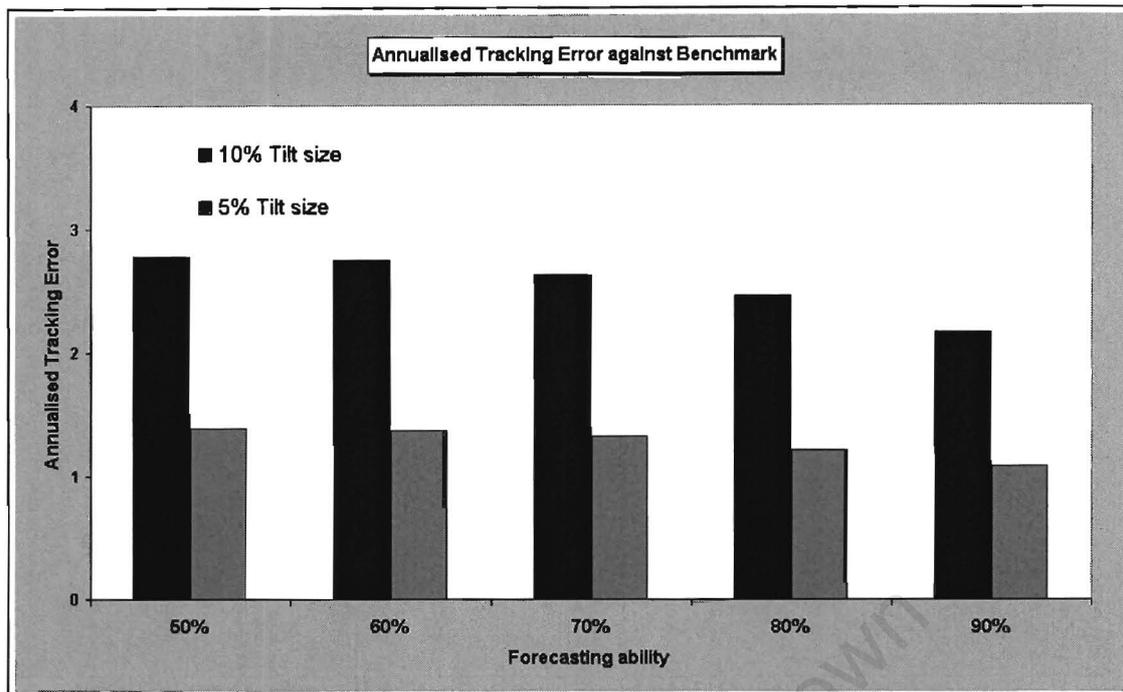


Figure 4.21: Annualised Tracking Error with 10% and 5% Tilt sizes

It is thus clear that as managers exhibit more skill in their ability to forecast the correct asset (above 50%), two positive consequences arise. The first obvious one is that the median outperformance attainable against the benchmark increases. The second consequence is that the certainty of the positive relative return also increases.

4.5.2.3 Aggression

We now consider the dynamics of these results as the degree of tilt sizes change. Since aggression is implicit in the tilt sizes managers take on, this aggression is captured by the tracking error against the benchmark. For the purposes of assessing the impact of changing aggression, we consider forecasting ability of 60% and 70% only. Figure 4.22 shows the results for a forecasting ability of 60%.

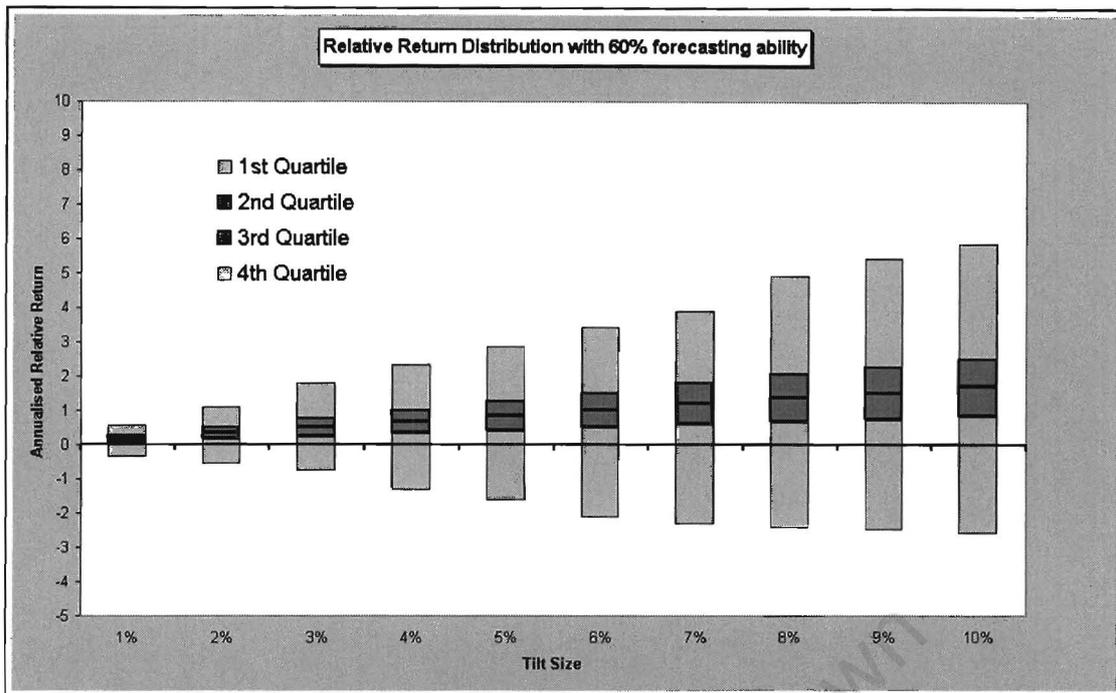


Figure 4.22: Annualised Relative Return with 60% forecasting ability sizes

As can be seen in Figure 4.22, the increase in bet sizes results in the *increase* in the median outperformance for the manager who has 60% forecasting ability. At face value, this result suggests the managers with 60% skill should take on more risk by increasing their tilt sizes.

Unfortunately, and intuitively, the *dispersion* of relative returns also increases with tilt sizes. This also suggests that the risk of obtaining the outperformance also increases. A further implication is that managers with 60% skill should only not take on higher tilt sizes where the risk of underperformance is acceptable. Table 4.5 presents the relevant summary statistics on the relative performance for managers with 60% forecasting ability.

Table 4.5: Performance statistics for managers with 60% forecasting ability

Tilt sizes	Average	Minimum	Lower Quartile	Median	Lower Quartile	Maximum	Prob(underperforming Benchmark)
1%	0.17	-0.32	0.08	0.17	0.25	0.57	8.6%
2%	0.33	-0.52	0.16	0.34	0.51	1.09	8.8%
3%	0.50	-0.74	0.25	0.50	0.76	1.78	8.4%
4%	0.66	-1.30	0.33	0.68	1.00	2.34	9.3%
5%	0.83	-1.59	0.41	0.84	1.25	2.87	9.5%
6%	0.99	-2.11	0.49	1.00	1.52	3.43	9.7%
7%	1.19	-2.31	0.59	1.21	1.81	3.89	9.6%
8%	1.36	-2.39	0.67	1.38	2.07	4.92	9.8%
9%	1.48	-2.48	0.73	1.50	2.28	5.44	9.7%
10%	1.67	-2.55	0.82	1.70	2.50	5.85	9.9%

Similar results for managers with 70% forecasting ability are given in Table 4.6 below.

Table 4.6: Performance statistics for managers with 70% forecasting ability

Tilt sizes	Average	Minimum	Lower Quartile	Median	Lower Quartile	Maximum	Prob(underperforming Benchmark)
1%	0.33	-0.21	0.26	0.34	0.42	0.70	0.4%
2%	0.67	-0.29	0.52	0.68	0.84	1.37	0.2%
3%	1.01	-0.26	0.78	1.02	1.25	2.09	0.3%
4%	1.34	-0.34	1.03	1.36	1.67	2.93	0.3%
5%	1.69	-0.41	1.30	1.71	2.08	3.41	0.2%
6%	2.02	-0.46	1.57	2.04	2.52	4.30	0.4%
7%	2.35	-0.54	1.79	2.39	2.93	5.26	0.3%
8%	2.69	-0.61	2.06	2.71	3.36	5.61	0.3%
9%	3.09	-0.69	2.38	3.11	3.82	6.65	0.3%
10%	3.43	-0.76	2.67	3.46	4.23	7.52	0.2%

Interestingly, the probability of underperforming the benchmark does not increase quickly when increasing the size of tilts. For managers with more skill (70% or more), the probability of underperforming the benchmark becomes negligible. This result is important, as it implies managers with significant skill can enhance the relative return without compromising the probability of underperformance. The result thus points towards managers with more than 70% forecasting ability to take on bigger tilt sizes.

4.5.3 Conclusion

Two important results based on simulating the probabilities of relative performance given various levels of skill and tilt sizes in the context of major sector selection emerge. The first is that the increase in a manager's ability of forecasting leads not only to the increase of expected relative return, but the dispersion of expected relative return would be smaller. The second is that once a manager has significant forecasting ability (70% or more), the payoffs of increasing the tilt sizes are not compromised by the probability of underperforming.

University of Cape Town

Appendix 4A

Approximating the confidence intervals for Sharpe's style weights [Lobsco and DiBartholomeo (1997)]

Following Lobsco and DiBartholomeo (LD, 1997), we define the return series for the *true* style weight combination of sector indices as

$$S = \sum w_i r_i , \quad \dots(A1)$$

where w_i = the true style weight of index i , and

r_i = the time series of returns on index i .

LD also define

$$A = R - S , \quad \dots(A2)$$

where R = the time series of returns to the *subject* portfolio

Series A can be thought of as the residual term of the constrained regression. The extent to which the estimated style weights do not match the true style weights is defined as:

$$\Delta w_i = \omega_i - w_i , \quad \dots(A3)$$

where ω_i = the estimate of the true style weight for index i , and

Δw_i = the amount of error in the estimate of style weight for index i .

The fitting process isolates the portion of the market indexes' returns that are independent of the market indexes used in the style analysis. To isolate the independent portion of each index, LD define

$$T_i = \sum v_m r_m \text{ (for } m \neq i \text{)} , \quad \dots(A4)$$

and

$$\sum v_m = 1 \text{ (for } m \neq i \text{)} , \quad \dots(A5)$$

where T_i = returns on the Sharpe style index for market index i analysed against all market indexes exclusive of i ,

v_m = style weight on index m ,

r_m = returns on market index m .

In defining the extent to which one market index is a linear combination of the others, the intuitiveness of the explanation is no longer necessary. LD thus removed the constraint that the style weights must be in the range of 0 to 1. LD then define

$$B_i = r_i - T_i, \quad \dots(A6)$$

where B_i = portion of the returns on index i not attributable to the other market indexes, subject to the other market indexes, subject to the constraint (A5).

Given that we've got the error terms for both the style weights (Δw_i) and the independent portions of the market index behaviours (B_i). It can be shown that the operative process in style analysis is to try to minimise the variance of $R - S - (\Delta w_i B_i)$ or $A - (\Delta w_i B_i)$. LD thus set an objective function, Z , to this expression:

$$\begin{aligned} Z &= \text{var}(A - \Delta w_i B_i) \\ &= \sigma_A^2 + \Delta w_i^2 \sigma_{B_i}^2 - 2\Delta w_i \sigma_A \sigma_{B_i} \rho_{AB_i} \end{aligned} \quad \dots(A7)$$

To solve for the minimum of the variance, LD set the derivatives of the variance with respect to the style weights equal to zero.

$$\begin{aligned} \frac{\partial Z}{\partial \Delta w_i} &= 2\Delta w_i \sigma_{B_i}^2 - 2\rho_{AB_i} \sigma_A \sigma_{B_i} \\ &= 0 \text{ (iff } \Delta w_i = \rho_{AB_i} \sigma_A / \sigma_{B_i} \text{)} \end{aligned} \quad \dots(A8)$$

Since the standard deviation for ρ is approximately $1/\sqrt{n-2}$, the standard deviation of w_i is approximated by

$$\sigma_{\Delta w_i} \cong \sigma_A / (\sigma_{B_i} \sqrt{n-2}) \quad \dots(A9)$$

where n is the number of data points in the return series.

To solve for σ_A , we have to know σ_a , where

$$a = R - \sum (w_i + \Delta w_i) r_i \quad \dots(A10)$$

Since a has $(n-k)$ degrees of freedom and A has $(n-1)$ degrees of freedom, LD then used the relation

$$\sigma_a^2 = \frac{\sigma_A^2(n-k)}{(n-1)} \quad \dots(A11)$$

where k = number of market indexes with non-zero style weights.

Rearranging (A11) and substituting back into (A9), LD arrived at

$$\sigma_{\Delta w_i} \cong \left(\frac{\sigma_a}{\sigma_{B_i} \sqrt{n-k-2}} \right) \quad \dots(A12)$$

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CHAPTER 5: SOME CONTRIBUTIONS TO RISK MANAGEMENT FOR ACTIVE MANAGERS

5.1 Overview

Active portfolio managers have significant uncertainty to contend with, and thus employ portfolio risk management processes. The popularity of risk management processes is growing, with new paradigms and models emerging relating to the forecasting, estimating and managing of portfolio risk (see "Global Risk Management: Are We Missing the Point?" Richard Bookstaber, *Journal of Portfolio Management*, Institutional Investor Spring 1997)

However, portfolio risk management can be examined within two frameworks, namely absolute and relative. Trustees are concerned about absolute risk (the volatility of absolute returns) and would want to ensure increased certainty of returns through low volatility of absolute returns. Managers however are often not concerned with absolute returns, as their monitoring has become focused on relative returns of funds in their respective investment industry / category. This risk of achieving relative return is referred to as tracking error, as defined in Section 2.3.3. Another term used frequently is *manager risk*. Tracking error captures the risk the manager takes through active bets against the benchmark in an attempt to outperform it. The terms *tracking error*, *active risk* and *manager risk* are thus used interchangeably in this thesis.

The increased cognisance of relative performance of fund managers has led them to employ active risk management processes to avoid severe underperformance, yet at the same time to also achieve a moderate level of outperformance. To this end, significant emphasis of this section will lie on active risk management.

As a result of its popularity, active risk management is winning the favour of a number of influential proponents in the academic, pension fund consulting and asset management worlds. Bob Litterman, co-author with the late Fischer Black of an influential 1992 paper on a quantitative approach to asset allocation, describes himself as a "big fan" of active risk management. Says Litterman, now head of quantitative resources at Goldman Sachs Asset Management: "Risk is a scarce resource that needs to be allocated optimally. That's

where risk management comes into its own. It says that the way to approach the problem is to maximize the return at the margin for each risk allocation within the overall budget."³

Structure of Chapter...

When one considers the importance of a measure such as tracking error in actively managing portfolios, a few aspects regarding tracking error and consequently portfolio risk management have to be considered. This chapter deals with several of these aspects in the context of the South African fund manager. The chapter is thus separated into the following sections:

- 5.2. Managing expected rather than historical active risk
- 5.3. Setting tracking error mandates
- 5.4. Active Risk taking habit of consistent top performers

This chapter will further examine each of these topics on tracking error in detail, providing a background to previous literature on each issue. Additionally, an empirical analysis will be conducted where necessary with the objectives of gaining insights for active managers in managing their portfolio risk against their benchmark. Whilst the above topics by no means exhaust the topic of active risk, they are intended to cover some of the important research areas having current practical relevance. The first consideration, which follows, is that the *expected* tracking error encompasses current conditions of active risk.

³ Chicago Quantitative Alliance Conference, Spring 2000. Website: www.cqa.org/pastconf.html

5.2 Managing Expected rather than Historical Risks

Historical tracking error (as calculated through historical returns) has often been used by practitioners as a measure of manager risk. This measure however encapsulates the average deviation from the benchmark over a historical period of time (often 12 months). The problem with the historical tracking error measure is that it does not convey the current risk against the benchmark, as reflected by the current holdings of the portfolio. This problem was confirmed by Kahn (1997), who found that the use of risk management models in active portfolio management improved the prediction of actual risk, and proposed the use of portfolio-based forecasts (based on current holdings of the portfolio) to that of historical risk estimates (based on historical returns). Dimson and Jackson (2001) also show that when implementing a risk management process, the longer the period of risk measurement, the lower the probability of observing extreme observations.

The measure which best captures the current manager risk is known as 'expected tracking error', which uses the contemporaneous relative weights and the covariance structure of the assets in order to assess the expected level of manager risk currently being taken against the benchmark. This measure of expected tracking error thus reflects the manager risk of the portfolio against the benchmark given that the current relative weights against the benchmark are held over the entire assessment period. We thus calculate expected tracking error as follows (following Hwang and Satchell (2001)):

$$\psi = \sqrt{(w_p^t - w_b^t)' \Sigma (w_p^t - w_b^t)} \quad \dots(3.1)$$

where w_p^t = weights vector for portfolio at time t
 w_b^t = weights vector for benchmark at time t
 Σ = covariance matrix of returns of stocks held in portfolio and benchmark

An important advantage of this measure is its ability to dynamically capture the managers impact on tracking error given conjectured changes in the bet sizes against the benchmark. The expected tracking error then reflects the anticipated tracking error going forward given the covariance structure of the underlying compositions remain constant.

The reliance on this assumption is needed for the use of expected (forecasted) tracking error. In reality, forecast tracking error will not exactly equal actual tracking error. Gardner and Bowie (2000) cite the following five factors as major contributors to the estimation error between forecasted tracking and actual tracking error:

- Significant fluctuations in market volatility
- Unanticipated changes in specific risk
- The presence of autocorrelation
- Extreme market movements
- The emergence of new market trends and themes

It has however been documented that expected tracking error consistently has a downward bias relative to historical tracking error. Hwang and Satchell (2001), as well as Gardner and Bowie (2000) show that ex-ante (expected) and ex-post (historical) tracking errors necessarily differ. In particular, ex-post tracking error is always larger due to the stochastic nature of portfolio and benchmark weights in the period of calculation. They thus conclude that ex-post tracking error (historical) is not suitable to active managers, as it creates the perception that significantly more active risk is being taken.

The next section considers the subject of setting the amount of active risk taken in the South African context.

5.3 Setting Tracking Error mandates

5.3.1. Introduction and background

The focus on active risk through the measure of tracking error has been at the centre of the transition of the investment management industry to a more risk-controlled environment [Gardener and Bowie (2000)]. The use of tracking error has been largely utilised as a control measure of fund managers to restrain relative investment performance, as well as being utilised by trustees as part of the manager selection and evaluation process.

Trustees additionally set tracking error mandates to limit the active risk of fund managers against their specified benchmark. These mandates are typically put in place in order for managers to constrain the fund manager's tracking error against their benchmark, naturally avoiding severe underperformance. However, the cost of curtailing active risk is also the forgoing of potential outperformance. Along with a tracking error mandate, trustees would prescribe a minimum relative performance target against their specified benchmark. This target would typically manifest in the form of a ranking objective i.e. attaining 1st quartile and / or avoiding 4th quartile.

The rationale for restraining a fund's active risk against the benchmark is related to the increasing awareness of relative fund performance in the investment industry. Investor's have become weary about relative performance, and this concern has led fund managers to curtailing tracking errors to avoid severe underperformance.

Few studies however have considered appropriate levels of tracking error for active managers. Most studies have concentrated on maximising expected return for a given level of tracking error [see Larsen and Resnick (1998), and Baierl and Chen (2000)]. Furthermore, the most commonly documented control for active managers has been restraining the bet sizes against the benchmark for each share, sector or asset class. Ammann and Zimmerman (2000) found that imposing active asset allocation bet sizes on fund managers did not adequately restrict active risk, and thus recommended the use of tracking error as a measure of active risk.

Institutions and pension fund consultants have however begun considering limiting tracking error. At ABP (an international risk management consultant), the risk management process has permitted a 2 percent tracking error to the benchmark for active managers and will maintain those allocations within a couple of percentage points and rebalance its mix to keep them relatively constant. Beyond that, ABP has also set a tracking error limit for the sector allocation within each of the asset classes.⁴

PGGM (an international asset management company) are applying risk management to asset classes, and have set a target tracking error for an investment class, such as global fixed income, and measure that against a standard investment benchmark, such as the J.P. Morgan government bond index.⁵

In the ensuing section we perform an empirical analysis so as to review a framework for establishing appropriate active risk parameters (in the form of tracking errors) for investment categories in the South African context. We note that it is imperative that the feasibility of these tracking error mandates be researched empirically to ascertain the likelihood of attaining the specified relative performance objectives of the fund.

In setting tracking error limits for active fund management, a typical first objective would be to avoid ending up in the 4th quartile of fund performance. A conceptual debate for the tracking error limit to avoid 4th quartile performance would be the *median* of the competitor's tracking errors. The argument is that, given a one-to-one relationship between active risk and active return, if a fund takes *moderate* active risk, the fund would be likely to generate *moderate* relative performance, i.e. end up in either the 2nd or 3rd quartile. These moderate active risk-taking funds are most likely to have taken *less* than the median tracking error. The other half of funds that take on more than median tracking error are more likely to fall into the 1st or 4th quartile of fund performance. This is based on the premise that the more active risk a fund takes the higher the fund's chances of out- and under-performing the benchmark would be. However, this premise may not always hold true in practice.

⁴ Website: <http://www.ausbus.com/management.htm#riskman>

⁵ Website: <http://www.globalinvestormagazine.com/issues.asp?i=143>

To illustrate this point, Figure 5.1 plots the annualised tracking errors and relative returns for each individual year from 1995 to 2001. The benchmark used is the peer mean, which is calculated by averaging the returns of the competitors for each month. The fact that distinct years are being used is based on the assumption that performance of a fund in one year is independent from its performance in the following year. It is thus clear that in Figure 5.1, there exist funds which take on relatively low active risk and severely underperform, as well as funds which take on high active risk and attain little active return.

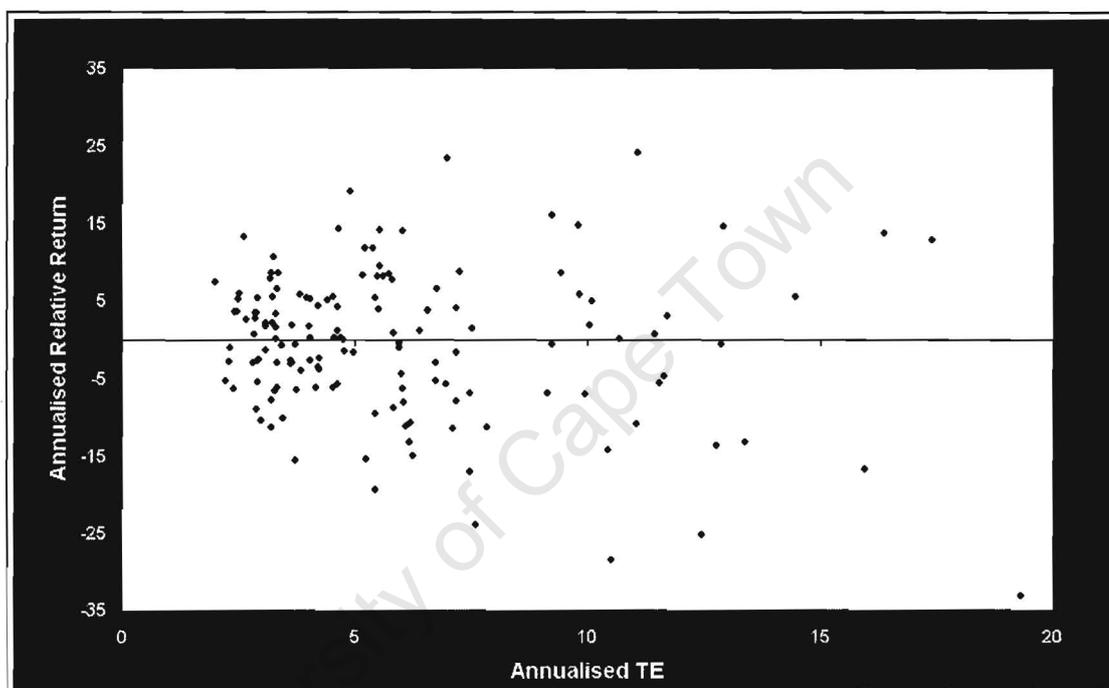


Figure 5.1: Annualised Relative Return and Tracking Error of General Equity funds (1995 – 2000)

Using this conceptual debate, one could argue that should a fund aim to avoid the 4th quartile fund performance, the fund should maintain a tracking error less than the median of tracking errors in the peer group. However, the expense of sacrificing the possibility of achieving 1st quartile performance also has to be considered in conjunction with avoiding underperformance. Thus it is imperative to assess the trade-off between avoiding underperformance and forsaking outperformance at different tracking error levels.

In the next section, an empirical analysis is presented so as to investigate the trade-offs of avoiding underperformance and forsaking outperformance at different tracking error levels.

5.3.2 Methodology

Once again, Figure 5.2 illustrates the relative performances of funds are displayed for various levels of tracking error taken. The median annualised tracking error for the set of funds is calculated to be 4.5%. The clear configuration suggests that funds with highest tracking errors have either achieved the highest or lowest relative return. Conversely, the cross-sectional dispersion of relative returns for those funds that took on low tracking errors is fairly low. The implication is thus that funds taking low tracking errors would thus typically forgo performance in the upper quartile, but at the benefit of avoiding 4th quartile performance.

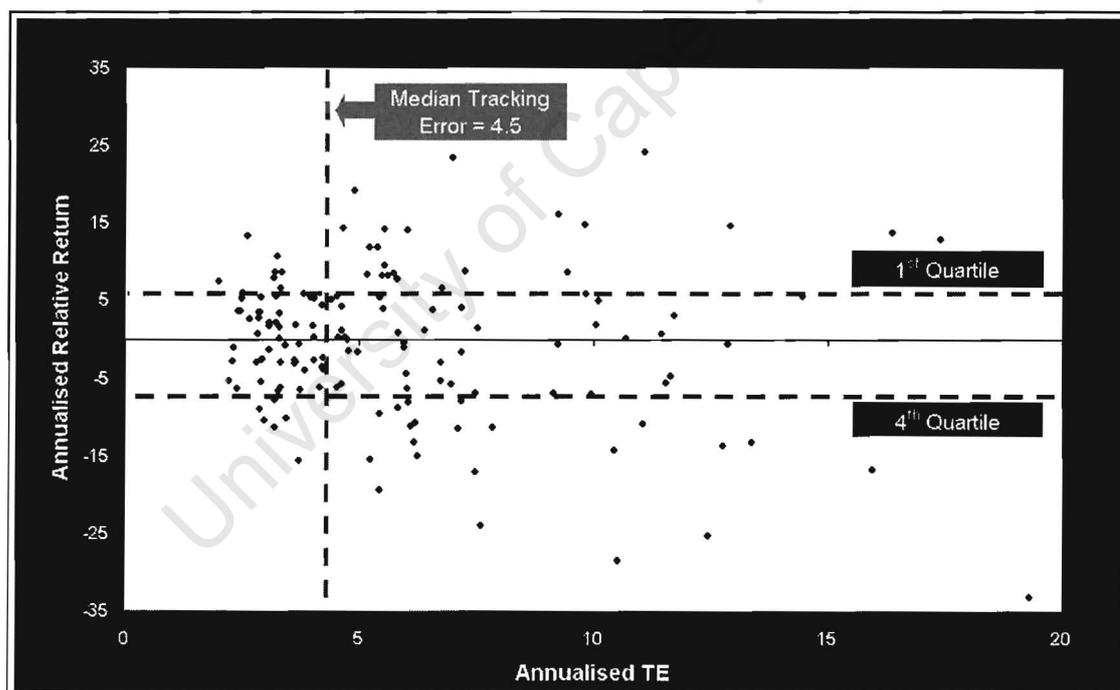


Figure 5.2: Annualised Relative Return and Tracking Error of General Equity funds with Median Tracking Error of 4.5% (1995 – 2000)

Given the upper and lower quartile of returns, it is clear in Figure 5.2 that the majority of funds that have fallen outside these bounds have taken on larger than median tracking errors (approximately 79%). Accordingly, the proportion of funds attaining 4th quartile

performance while remaining under median tracking error is fairly low (at approximately 13% in this example). However, a 13% chance of underperforming while remaining within the median tracking error cannot be guaranteed given the sample size is only 122 funds in this example.

Due to the limited number of Unit Trusts available in each investment category, accurate estimates of reliable probabilities of out- and underperformance are restricted. The unreliability lies in the fact that the number of observations is not enough to make dependable estimates based on this methodology. One needs to make use of a large enough sample size that attempts to represent all possible combinations of relative return and tracking error achievable. A re-sampling method is needed to generate sufficient fund performance data in order to improve the estimates of out and under-performance probabilities in a non-biasing and theoretically rigorous approach.

Bootstrapping is thus used to recreate a fund's return series through randomly re-sampling data points from the actual fund's time series data. These recreated hypothetical funds are then used to re-compute the relative return and tracking errors from the newly created time series against the corresponding benchmark returns series. This re-sampling procedure is repeated one thousand times to ensure reliable approximation of the relationship between relative returns attainable for each level of tracking error. In order to achieve reliability around low tracking error areas where the sample size may be small, different combinations of the benchmark and active funds are re-sampled.

The bootstrapping approach does require the assumption that the performance of a fund is independent from one month to the other, which allows one to sample within the actual fund's return series. This assumption is based on the assertion that the market is efficient and that fund returns are time-variant independent. The disadvantage of this approach is that it does not preserve the sequential integrity of the data, yet a suggestion for future research would be to bootstrap sequences of data. This re-sampling approach is somewhat ad hoc, yet is required in the light of the given data availability. The approach attempts to be as practical as possible, yet maintain a theoretically sound statistical process.

5.3.3. Results

We turn to the General Equity category of Unit Trust funds as our sample in our empirical analysis to assess the relationship between tracking error and relative return. For illustration purposes, we plot the sampled relative return and tracking error for each General Equity fund, bootstrapped using monthly data from January 1995 to April 2001. The benchmark used is the Peer group mean.

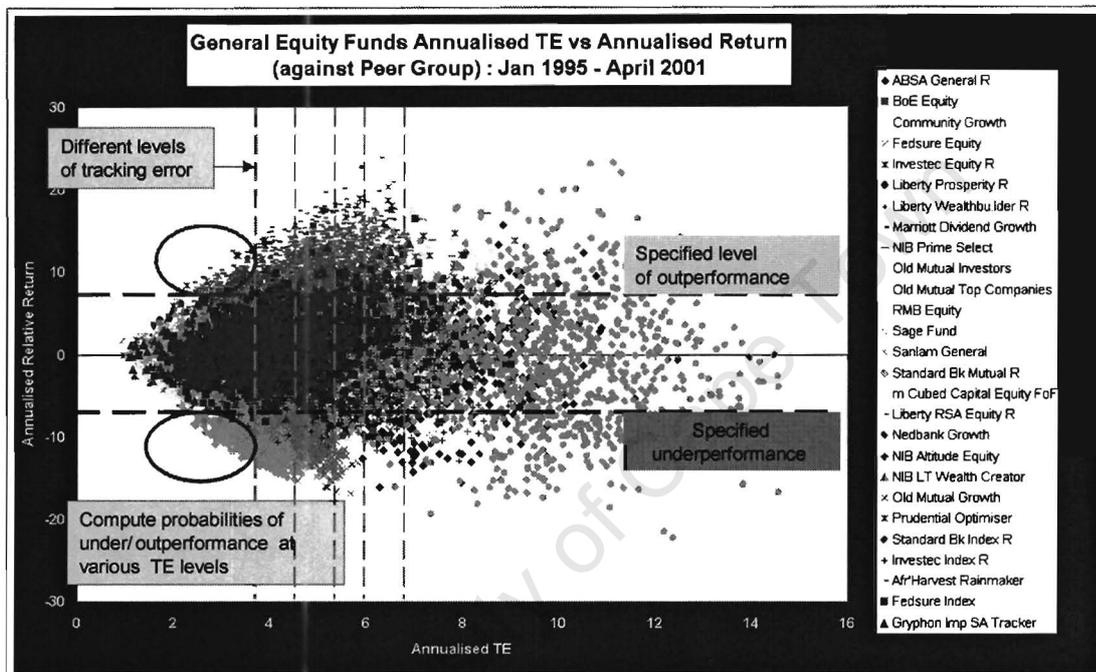


Figure 5.3 Bootstrap of Relative Returns and Tracking errors of General Equity Funds

As can be seen in Figure 5.3, using the bootstrapping approach one is able to generate sufficient data so that the spatial positioning of relative return and tracking error is clearly intuitive. As the tracking error increases, the dispersion of relative return increases as one would expect. This reflects the higher probability of out- and under performing the fund has against the benchmark with increased tracking error. This fan shape is directly related to the assertion that the degree of relative return possible is predominately determined by the tracking error taken. The dispersion of tracking error indicates a large concentration of tracking errors at lower values, with a sparse concentration at higher values. This distribution of tracking error is related to the Chi-squared distribution, which has a

distribution skewed to the right, i.e. has a long positive tail. This skewness is clearly evident through the extreme tracking error points lying to the right.

The approach taken now that the sampling procedure has been completed involves computing the probabilities of out- and underperformance achievable for each level of tracking error. The computation is based on a conditional probability stated as follows: 'given that the tracking error is less than x , what is the likelihood that a relative return of y is exceeded'. In order to compute this probability for each level of tracking error, the number of points falling above / or below the relative return (y) at each tracking error limit, is divided by the total number of points at each tracking error limit. This computation is continuously repeated for each tracking error limit.

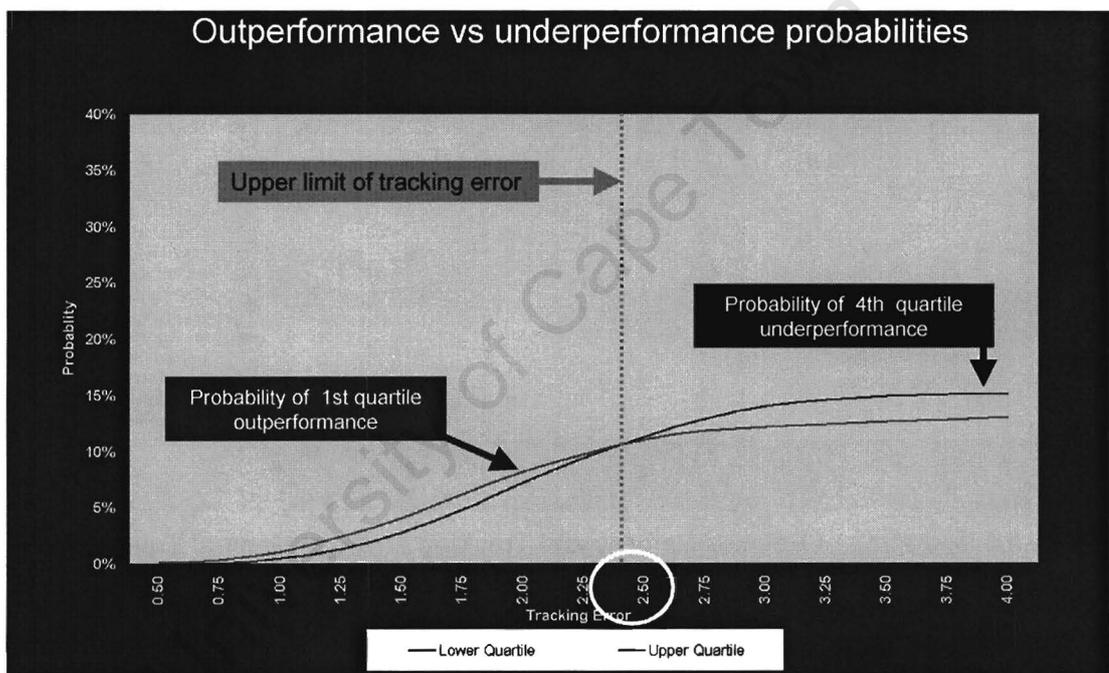


Figure 5.4 Probability of 1st and 4th quartile performance vs Tracking Error

For illustration purposes, we demonstrate the results of the probability plots of the Top 10 South African pension funds. As can be seen in Figure 5.4, the shape of the probability plots are intuitive as the probability of both out- and underperforming increases with tracking error. More specifically, the probability of outperforming the 1st quartile limit as well as underperforming the 4th quartile limit increases monotonically from a tracking error of 0 to the maximum tracking error. These probability increases occur initially at an

increasing rate, then at a decreasing rate at higher tracking error levels. The probability function then asymptotes to a certain probability at the highest tracking error level.

However, as seen in the graph, the probability of outperformance increases at an increasing rate at slightly lower tracking errors than the probability of underperforming. The implication of the difference in the slope of these probabilities is that there exists an optimal tracking error range where the increase in probability of outperforming is higher than the increase in probability of underperforming. Based on this analysis, a recommendation can thus be made that a tracking error of 2.4 should not be exceeded in this Pension fund category, as the probability of underperforming increases above the probability of outperforming. Clearly such a situation must be avoided to ensure severe underperformance is avoided in the long run.

In Table 5.1 we report the tabulated probabilities of attaining specified performance criteria at each tracking error level. This table would be used to set tracking error limits for managers to fall within. For example, in Table 5.1, to have a maximum probability of 10% of underperforming the General Equity benchmark by 4%, the appropriate maximum tracking error would be 4.5%. Conversely, to have a minimum probability of 10% of outperforming the General Equity benchmark by 2%, the appropriate minimum tracking error would be 3%. In Appendix 5A, the corresponding probability tables for the FINDI category is given.

Table 5.1 Probabilities of relative performance given tracking error limits in the General Equity category

Probability of Performance	Tracking Error Limits						
	<3	<3.5	<4	<4.5	<5	<5.5	<6
Prob(< -10%)	0.12%	0.30%	0.50%	0.80%	0.97%	1.34%	1.68%
Prob(< -5%)	2.03%	3.55%	4.79%	6.16%	7.44%	8.78%	9.72%
Prob(< -4%)	3.66%	5.72%	7.46%	9.20%	10.85%	12.31%	13.46%
Prob(< -3%)	6.60%	9.11%	11.24%	13.36%	15.23%	16.89%	18.06%
Prob(< -2.5%)	8.62%	11.29%	13.65%	15.96%	17.83%	19.56%	20.76%
Prob(< -2%)	11.20%	14.08%	16.51%	18.83%	20.73%	22.49%	23.72%
Prob(> 2%)	9.69%	12.70%	15.53%	17.67%	19.36%	20.65%	21.74%
Prob(> 3%)	5.17%	7.69%	10.25%	12.19%	13.83%	15.07%	16.18%
Prob(> 3.5%)	3.99%	6.25%	8.61%	10.37%	11.79%	12.95%	13.98%
Prob(> 4%)	3.15%	5.05%	7.12%	8.70%	10.01%	11.19%	12.22%
Prob(> 5%)	1.81%	3.22%	4.84%	6.12%	7.15%	8.25%	9.20%

Prob(> 10%)	0.19%	0.37%	0.62%	0.77%	1.13%	1.39%	1.74%
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This section was aimed at applying the bootstrapping technique to create more fund performance data, enabling one to reliably make judgments on tracking error mandates. Although the technique is ad hoc, it at least serves to give some indication as to reasonable tracking error ranges.

Now that tracking error mandates can appropriately be set, the subsequent section deals with determining the optimal amount of tracking error to be used on benchmark timing risk and stock selection risk. The study focuses on the split of tracking error of the consistent top-performers.

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5.4. Risk taking habits of consistent top performers

5.4.1. Introduction and discussion

The focus in this section is to assess the risk-taking habits of fund managers in South Africa with a specific focus on the risk-taking habits of the consistent *top performers*. The General Equity category of Unit trusts is used as a laboratory in the ensuing empirical study. The specific emphasis is on the assessment of both their stock selection and benchmark timing risks over each of the prior 6 years. We used the following formulae for the decomposition of tracking error variance:

$$TEV = (\beta - 1)^2 \sigma_{benchmark}^2 + \sigma_e^2, \quad \dots(5.1)$$

$$\text{where benchmark timing proportion} = \frac{(\beta - 1)^2 \sigma_{benchmark}^2}{TEV} \quad \dots(5.2)$$

$$\text{and selection proportion} = \frac{\sigma_e^2}{TEV} = \frac{(1 - \rho^2) \sigma_p^2}{TEV} \quad \dots(5.3)$$

Clearly, the higher the deviation of the fund's beta is from 1, the higher the benchmark timing proportion of TEV. This deviation is often the cause of significant differences in the asset allocation of the fund and benchmark. The selection proportion of TEV is most often affected by significant differences in the stock selection of the fund and the benchmark, as noted by the deviation of the fund correlation from with the benchmark.

In Figure 5.5 we display the 12-month rolling component splits of timing and selection risks as a proportion of the tracking error of the General Equity category of funds. The benchmark used in all our analysis in this report is the aggregate peer group return.

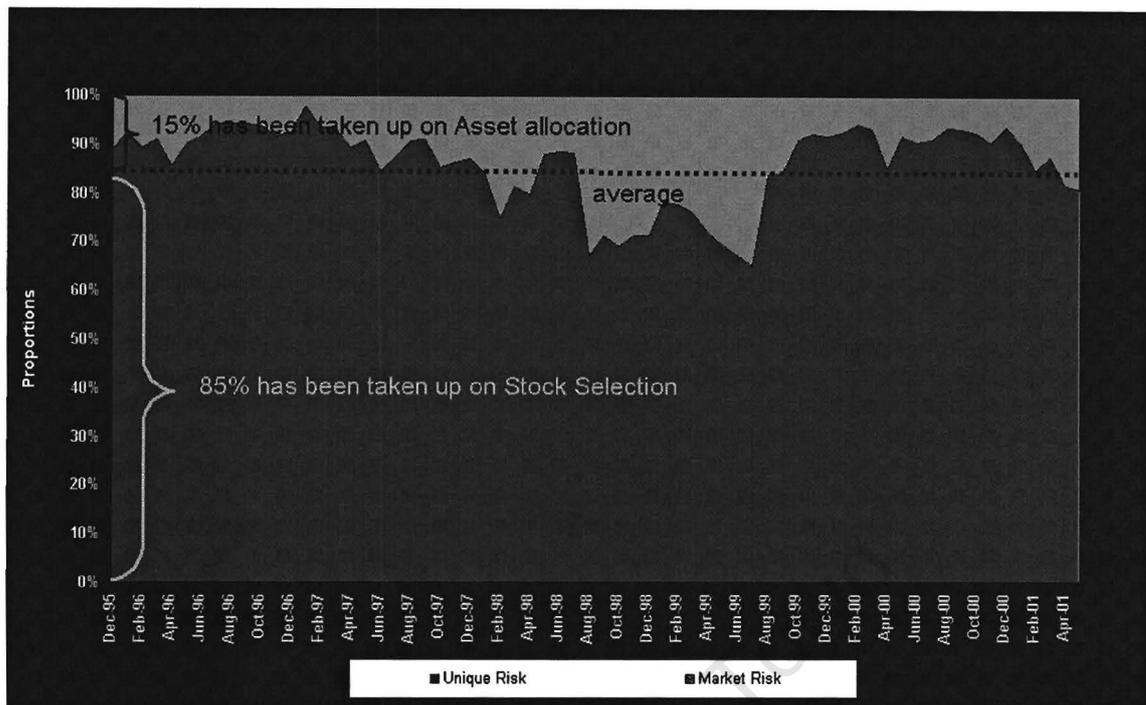


Figure 5.5: Average proportions of Tracking Error taken up by Relative Market Risk and Unique Risk in the General Equity Category

The *rolling* estimation approach assists in gaining a perspective on the dynamics of the component benchmark risks taken through time. From Figure 5.5 it is evident that the majority of the tracking error has consistently been used up on stock selection and a far smaller proportion of risk-taking has been used up on benchmark timing. On average, 85% of the tracking error was used up on stock selection, and only 15% used up on benchmark timing over the prior 6 years (as indicated by the dotted line in Figure 5.5). These results confirm that managers are generally reluctant to use up their risk-taking capacity on benchmark timing.

This preference for avoiding *benchmark timing risk* is perhaps intuitive as taking on asset allocation risks requires research on major asset classes, a far more widely analysed (hence more efficient) component of investments than analysis on local stocks. Hence there is likely to be better payoffs for researching more inefficient series such as local stocks, suggesting taking on *stock selection risks* is indeed more prudent.

5.4.2. Methodology and Data

We continue our focus on the General Equity category to gain insights on the risk-taking habits of *consistent top performers*, as well as poor performers. Our assessment period incorporated the prior seven years of fund performance history.

To identify *consistent* performers, the quartile performance positions of the General Equity funds were averaged (over the 7 year period) and again ranked and partitioned into quartiles. The top quartile of average quartile ranks gives a good indication of the consistent top performing funds and similarly the 4th quartile of average quartile funds represent the consistently poor performers.

5.4.3. Results

Table 5.2 that follows shows the summary results of the partitioning and includes the average percentage of tracking error used up on benchmark timing for each fund.

Table 5.2: the average Quartile ranks for the Funds in the General Equity Category with associated proportion of Tracking Error used up on Timing Risk and Selection Risk.

Funds	Average Quartile	Average Timing Proportion	Average Stock Selection Proportion	Years
FT NIB Prime Select	1.29	9.1%	90.9%	7
Liberty RSA Equity R	1.67	4.7%	95.3%	3
Metropolitan General Equity	1.71	15.3%	84.7%	7
BoE Equity	2.14	12.1%	87.9%	7
Liberty Wealthbuilder R	2.14	9.7%	90.3%	7
Fedsure Index	2.17	8.4%	91.6%	6
Investec Index R	2.17	6.7%	93.3%	6
Investec Equity R	2.29	16.9%	83.1%	7
Liberty Prosperity R	2.29	7.0%	93.0%	7
RMB Equity	2.29	20.6%	79.4%	7
Community Growth	2.43	7.1%	92.9%	7
Sage Fund	2.43	27.2%	72.8%	7
Gryphon Imp SA Tracker	2.50	17.4%	82.6%	4
Standard Bk Index R	2.50	4.7%	95.3%	6
Coronation High Growth	2.60	23.2%	76.8%	5
ABSA Growth FoF	2.67	37.8%	62.2%	3
FNB Growth	2.67	5.5%	94.5%	3
Futuregrowth Core Equity	2.67	19.2%	80.8%	3
Old Mutual Top Companies	2.71	16.8%	83.2%	7
Standard Bk Mutual R	2.71	12.9%	87.1%	7
FT NIB LT Wealth Creator	2.75	17.9%	82.1%	4
Nedbank Growth	2.75	6.4%	93.6%	4
ABSA General R	2.86	12.7%	87.3%	7
Old Mutual Investors	2.86	28.6%	71.4%	7
Fedsure Equity	3.00	19.1%	80.9%	7
m Cubed Capital Equity FoF	3.00	15.4%	84.6%	4
RMB Performance FoF	3.00	30.5%	69.5%	3
Sanlam General	3.00	16.6%	83.4%	7
Sanlam Future Trends	3.33	20.6%	79.4%	3
Brait Accelerated Growth	3.67	3.7%	96.3%	3
Fedsure Pioneer	4.00	9.6%	90.4%	3

In order to quantify the benchmark timing risks of the consistent top performers, we selected all General Equity funds having a history of 5 or more years. Thereafter we partitioned the average quartile positions of these funds once more into quartiles depicting the *consistent* performers, and averaged the benchmark timing risks within each quartile. The resulting average percentage of benchmark timing risk in each performance quartile is shown Table 5.3 which follows.

Table 5.3 The average proportion of Tracking Error used up on Timing Risk by the funds in the different quartiles

Quartile	Average Proportion Timing
1	11.4%
2	12.3%
3	17.6%
4	19.1%

Interestingly there is a clear trend between the performance quartile positions and the proportion of tracking error used up on benchmark timing risk as seen in Table 5.3. The top quartile of consistent performers have clearly used up the smallest percentage of their tracking error on benchmark timing, with an average of only 11.4%. A systematic increase in the percentage of benchmark timing risk is evident as performance quartiles decline to the 4th quartile performers. A plausible reason for these results is that intuition suggests that it's likely that managers require more risk and / or skill to generate outperformance through asset allocation (driven primarily through benchmark timing) than stock selection risk.

These results give some evidence that consistent superior performers have taken on relatively smaller proportions of benchmark timing risk for this sample of General Equity Funds for the prior 7-year period.

Appendix 5A

Table 5A Probabilities of relative performance given tracking error limits for the FINDI category

Probability of Performance	Tracking Error Limits							
	<3	<3.25	<3.5	<3.75	<4	<4.25	<4.5	<4.75
Pr(<-6)	1.61%	2.61%	3.77%	5.24%	6.43%	7.52%	8.23%	8.95%
Pr(<-5)	2.57%	3.85%	5.46%	7.28%	8.82%	10.18%	11.13%	12.02%
Pr(<-4.5)	3.16%	4.65%	6.38%	8.40%	10.17%	11.55%	12.56%	13.51%
Pr(<-4)	4.00%	5.59%	7.52%	9.80%	11.76%	13.21%	14.34%	15.35%
Pr(<-3)	6.13%	8.05%	10.45%	13.08%	15.36%	17.10%	18.52%	19.67%
Pr(<-2)	10.08%	12.36%	15.09%	18.03%	20.54%	22.42%	24.05%	25.27%
Pr(>2)	9.58%	11.38%	12.98%	14.98%	16.83%	18.69%	20.00%	21.25%
Pr(>3)	5.16%	6.74%	8.25%	9.99%	11.69%	13.38%	14.62%	15.81%
Pr(>3.5)	4.03%	5.48%	6.79%	8.35%	9.86%	11.40%	12.49%	13.63%
Pr(>4)	3.13%	4.35%	5.57%	6.97%	8.36%	9.75%	10.76%	11.75%
Pr(>5)	1.94%	2.76%	3.61%	4.74%	5.83%	6.93%	7.79%	8.52%
Pr(>6)	1.14%	1.75%	2.35%	3.19%	4.09%	4.98%	5.62%	6.13%

CHAPTER 6: MULTI - MANAGER TOPICS

Multi-management has become a substantial feature of the South African fund management industry. In this chapter, two studies in the area of multi-management having a quantitative theme are reviewed separately. The first topic investigates the impact of combining managers has on the active risk of a multi-manager portfolio, whereas the following topic reviews the evidence of persistence of fund performance on the South African Unit Trust Industry.

6.1. The Impact of Combining managers on active risk

6.1.1 Introduction and discussion

The popularity of a multi-manager approach to fund management is well entrenched in South Africa, predominantly because this type of fund management has the ability to reduce the manager risk through fund diversification. In the Unit Trust industry, the extent of multi-manager activity is reflected in the large proportion of flows into unit trust funds from multi-manager activity. Its popularity has also been noticed in the pension fund industry, as many institutional pension funds diversify their portfolio by combining funds managed by carefully selected managers.

Multi-managers typically aim to outperform the peer group by optimally selecting a portfolio of funds, which performs better than the average of the peer group (benchmark). In order to avoid the risk of significantly underperforming this benchmark, multi-managers have the difficult task of choosing the skilful managers most often with only short performance track records available. Furthermore, due to the large size of funds, especially in the institutional fund industry, multi-managers most often spread their portfolio across different managers to avoid too much concentration in specific stocks (and also to diversify manager risk away).

Besides the issue of which managers to select, multi-managers face an equally important question of how many to select. The concern is that if too many managers are selected,

the resultant fund will perform like a peer group benchmark tracking fund. Hence not only will the probability of outperforming the peer group aggregate become too low, but the payment of fees for *actively* managed fees becomes unwarranted. Clearly, to outperform the peer group, a multi-managed fund has to take on sufficient active risk (tracking error). Hence the question arises as to how many managers, when combined, yield a resultant *passive* fund. This question is the primary aim of this section.

A further concern for multi-managers is the cost of netting off of transactions between funds within the multi-managed fund. Often some managers may be buying the same stocks another manager is selling, effectively canceling out any active bets taken by each manager, but still incurring transaction costs. The portfolio structure thus remains unchanged, yet costs detract from the portfolio performance.

It is thus imperative to consider the marginal impact of adding additional fund managers to the multi-managed portfolio. Additional to assessing the impact of combining an increasing number of *standard* managers, the report also assesses the impact of adding managers specifically chosen because of various levels of:

- manager aggression,
- fund size, and
- stock selection skill

6.1.2 Data and methodology

The analysis was conducted on the General Equity (GE) category of Unit Trusts (and repeated for Pension fund data using the Top 10 large Manager Watch). Micropal return data was used for the unit trust data, which includes fund returns with dividends reinvested. The study was conducted on data starting in January 1998 through to August 2001. Due to the limited number of funds in many unit trust categories, the study focussed the analysis on the General Equity Category of unit trusts (here forth called GE). Pension fund data is sourced from Alexander Forbes, with the largest 10 Pension funds used in the sample set. We report only on the General Equity fund results here, and summarised results of pension funds can be found in the Appendix 6A.

The method for the analysis uses Monte Carlo Simulation to randomly sample $\binom{m}{n}$ ⁶ possible combinations of funds chosen for n out of m possible managers in a portfolio ($0 < n < m$). For each sample of random funds selected to be combined into one portfolio, an equally weighted average of the funds are taken. The portfolio returns are reconstructed for each sample given the time series of each fund's returns. The multi-managed portfolio returns at time t are thus:

$$R_t = \frac{1}{n} \sum_{i=1}^n r_{it}, \text{ where } r_{it} \text{ are the returns for fund } i \text{ at time } t.$$

This vector of returns is sampled for various combinations of n out of m funds.

For each sample of portfolio return, a relative return and tracking error is computed, with the peer group used as a benchmark. This simulation is repeated for n = 1 to m funds. The median tracking error of each set of n funds is computed and compared to the optimal active tracking error ranges suggested for the category to establish whether they fall within the active range.

To represent a realistic sample of core/passive funds in the analysis we use the GE peer group trackers: Futuregrowth Core, NIB Quants and Woolworths (known to be the peer group tracker funds) to set the passive tracking error ranges.

The simulated relative returns and tracking errors are also compared to the peer group passive / core funds' relative returns and tracking errors. These relative returns and tracking errors are generated through the bootstrapping approach, and sampled 1000 times. In order to model the relative space occupied by the passive funds, we use the formulation for a non-centred ellipse⁷, and solve the parameters for the ellipse by using the bootstrapped data.

⁴ where $\binom{m}{n} = \frac{m!}{n!(m-n)!}$

⁷ The formulation of the non-centred ellipse is given as $\frac{(x-a)^2}{k^2} + \frac{(y-b)^2}{l^2} = 1$,

where x = tracking error sample ,y = relative return sample

a = mean tracking error sample, b = mean relative return sample

k = x radius, l = y radius

It is important to note that our research design does not specifically address the case where multi-managers select specialist managers within domestic equity categories.

6.1.3 Results

The results cover the GE category of unit trusts with summarised results for the 10 largest pension funds found in the appendix. We begin with the GE unit trust category.

6.1.3.1 Combining Standard Managers

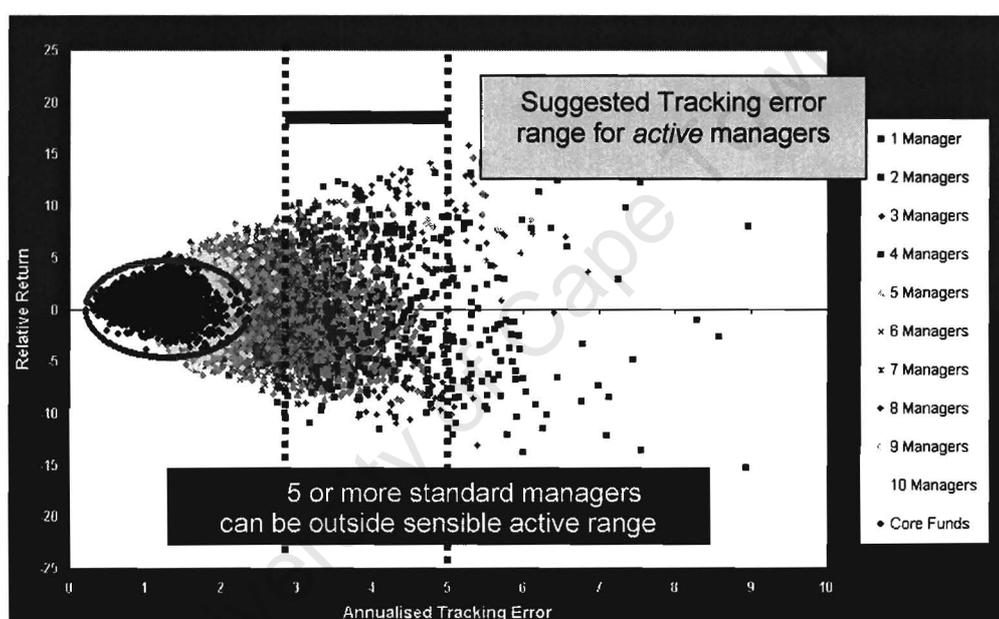


Figure 6.1: Tracking Error versus Relative Return for GE Unit Trusts

Figure 6.1 illustrates the resultant range of relative return and tracking error when combining 1 to 10 out of the 32 GE Funds sampled (each group represented by a different colour). The black cluster of points represents the bootstrapped passive/core funds, with the red ellipse modeling the probable space of the core funds in active risk/return space. The minimum tracking error for these *passive* funds is 0.25%, with the maximum tracking error for passive funds being 2.38%. We superimpose the ideal / optimal range for active funds in the diagram above (this range has been obtained from the prior research on active risks in this category – see section 5.3).

The dispersion of relative returns clearly decreases as the number of combined managers increase, as seen in Figure 6.1. This result is fairly intuitive and can be seen in the fan-shaped dispersion of relative returns and tracking errors.

Figure 6.2 portrays the median tracking error obtained for the various numbers of combined managers. A central insight from Figure 6.2 is that for as few as 4 managers, the combined median tracking error of 4 combined managers falls below 3% (the lower bound for optimal active tracking error range). This implies that when combining standard GE managers with the intention of taking sufficient active risk to outperform the peer group, 4 or more managers would defeat this purpose – clearly having too little active risk. Notice that the median tracking error decreases to below 2.38% (the maximum tracking error for the passive funds) as soon as 6 managers are combined.

Our results suggests that combining 6 or more funds would notionally yield a fund that is essentially a passive fund. There's a marginal decrease in tracking error thereafter, thus the impact of combining additional managers, is small. Combining 8 managers produces a median tracking error below 2%, which is well recognised to be within the passive tracking error range for GE funds.

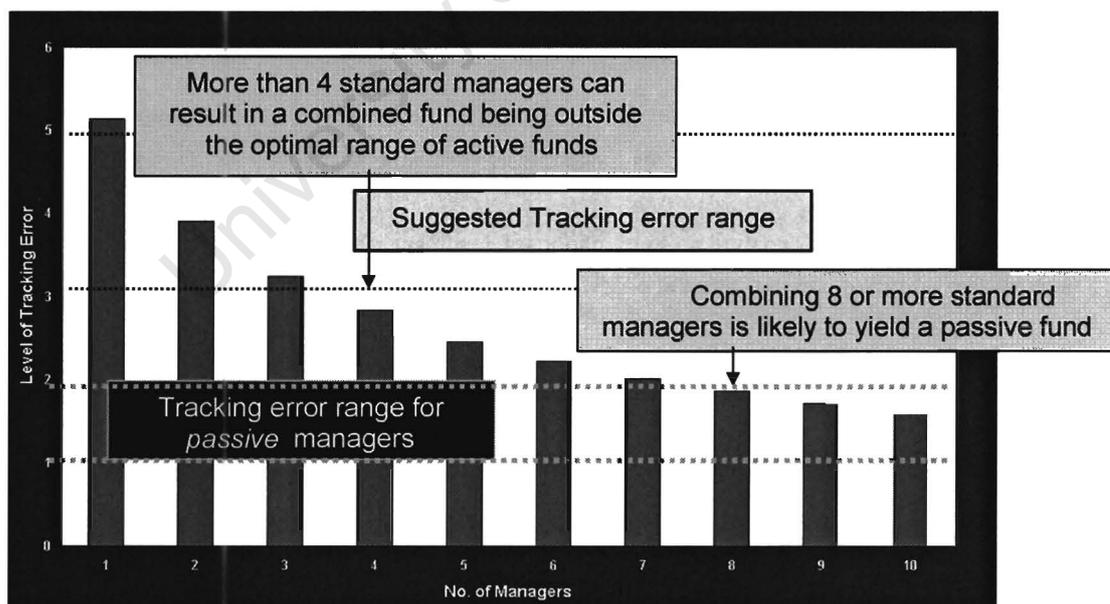


Figure 6.2: Median Tracking Error for combining standard GE Unit Trusts

Next we consider when both the tracking error and active return falls within the passive range⁸. The results are presented as probabilities of resultant “passive” returns and are summarised in Figure 6.3. The figure thus shows the probability of GE funds falling in the passive fund relative risk and return space when combining 1 to 10 managers.

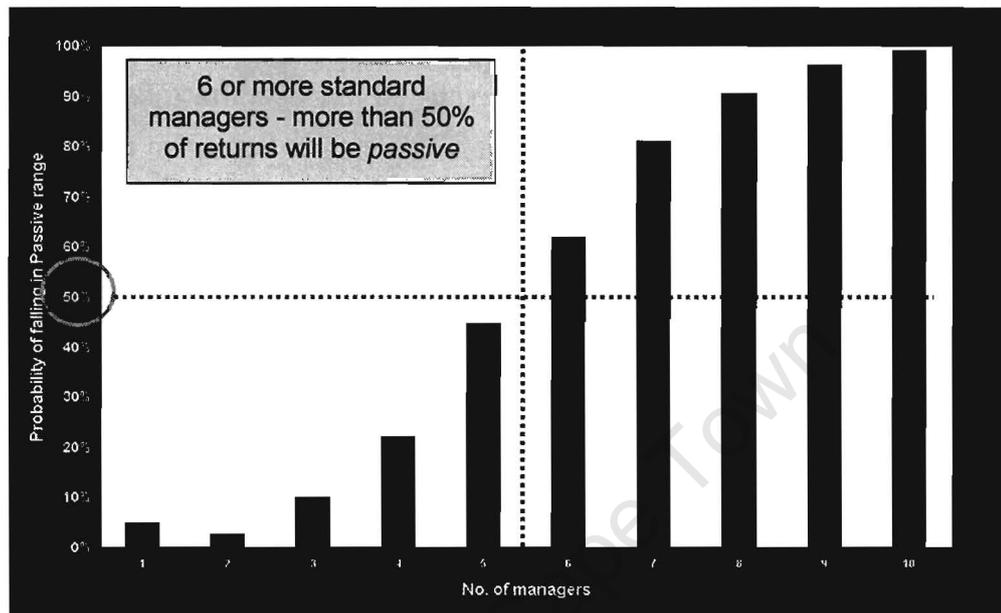


Figure 6.3: Probability of combined GE funds falling in the passive fund relative risk / return space

⁸ We do this by calculating the proportion of the combined funds that fall within the passive funds' perimeter – as a measure of the probability of returns being passive with increasing number of managers.

The computation used for each $n = 1$ to m managers, involves using the combined portfolios' tracking error (x) and relative return (y) and applying the passive non-centred ellipse formulation, i.e.

$$\frac{(x-a)^2}{k^2} + \frac{(y-b)^2}{l^2} \quad (2),$$

thus using the parameters corresponding to the passive funds, we have

$$\frac{(x-1.27)^2}{1.08^2} + \frac{y^2}{4.66^2} \quad (3)$$

The proportion of pairs of points for which (3) is less than unity for $n = 1$ to 32 managers would represent the proportion of relative risk and return points that fall in the passive range

Figure 6.3 clearly illustrates the increasing probabilities of falling into passive relative risk-return space corresponding with the increasing number of combined GE managers. Interestingly, when combining 6 (or more) standard managers, the probability of yielding passive relative risk-return points exceeds 50%. Another interpretation of this result is that when combining 6 or more standard managers, more than half the relative risks and returns generated would be passive.

Clearly there is no incentive to choose 6 (or more) *standard managers* based on this result if a multi-manager wishes to be active against the peer group. This result is entirely consistent with the earlier result where the median tracking error for combining 6 standard funds fell below the maximum tracking error for passive funds. Since the median separates the 6-manager portfolio data set in half, more than 50% of the relative risk / return points would be below the upper bound of passive funds, and less than half would be active.

The corresponding proportion of *passive* relative risks-returns obtained by combining 10 standard managers is 99,2%. Clearly if a multi-managers randomly selected only 10 *standard* GE funds, they would (on average) reproduce tracking errors and relative returns entirely consistent with a passive fund. This strategy would however occur at a higher active management costs.

Table 6.1: Quantitative Results from simulation of combining standard GE Funds

No. of Managers	Tracking Error		Prob. Relative Returns and TE Passive	Percentile of Relative Returns	
	Average	Median		5 th	95 th
1	5.64	5.14	4.9%	-12.24	13.44
2	4.17	3.90	2.7%	-8.04	9.13
3	3.36	3.24	10.0%	-7.33	8.93
4	2.90	2.83	22.0%	-5.94	7.79
5	2.52	2.45	44.7%	-4.83	5.62
6	2.27	2.22	62.0%	-4.86	5.14
7	2.05	2.01	81.1%	-4.12	4.91
8	1.90	1.86	90.6%	-3.97	4.75
9	1.74	1.72	96.3%	-3.83	4.04
10	1.61	1.59	99.2%	-3.28	3.99

The above results suggest that combining *standard* GE funds with the objective of constructing a resultant active fund, is somewhat restrictive in the context of the number of managers that can be combined. Below we now consider the effect of combining aggressive and moderately aggressive GE funds in a portfolio.

6.1.3.2 Combining Managers based on aggression

In this section we investigate the impact of combining only *aggressive* managers on the resultant active risks of the combined fund. The rationale for investigating the selection of more aggressive managers is based on the notion that conservative managers - who take relatively smaller aggregate active bets - are more likely to diversify each other's bets away. Aggressive managers by contrast are less likely to have their larger diversified away as quickly as conservative managers.

In this section we measure aggression as the tracking error, and not only stock selection risk. Thus, aggression may also reflect timing bets taken against the peer group benchmark, and not just stock selection bets. Timing bets (caused largely by asset allocation bets) also have the ability of creating larger tracking errors due to the fact that cash returns are uncorrelated with equity returns.

Recalling the tracking error is decomposed into timing risk and selection risk as follows :

$$TEV = (\beta - 1)^2 \sigma_{benchmark}^2 + \sigma_e^2$$

\Downarrow \Downarrow \Downarrow
tracking error *timing risk* *selection risk,*

we partition the GE funds, into aggressive, moderate and conservative relative risk-takers according to their tracking errors shown in table 2 that follows (over the same period as the period of the analysis).

Table 6.2: GE funds ranked according to aggression based on tracking error

Risk Profile	General Equity Funds	Tracking Error	Timing Risk	Selection Risk
Aggressive	Nedbank Growth	10.92	2.25	10.68
	Allan Gray Equity	10.61	5.44	9.11
	Fedsure Pioneer	9.47	2.61	9.10
	ABSA Growth FoF	7.71	2.86	7.17
	Sanlam Future Trends	7.68	3.41	6.88
	ABSA General R	6.71	1.92	6.43
	Standard Bk Aggressive FoF A	6.38	3.98	4.98
	Sage Fund	6.09	3.08	5.25
	Brait Accelerated Growth	5.87	0.55	5.84
	BoE Equity	5.78	1.42	5.61
Moderate	RMB Performance FoF	5.75	2.71	5.07
	Investec Index R	5.58	0.24	5.58
	FNB Growth	5.56	0.05	5.56
	Fedsure Equity	5.44	1.73	5.16
	Old Mutual Top Companies	5.25	2.11	4.81
	Standard Bk Mutual R	5.23	2.60	4.53
	Metropolitan General Equity	5.18	0.39	5.16
	FT NIB Prime Select	5.10	0.80	5.04
	Fedsure Index	4.79	0.19	4.78
	Liberty Wealthbuilder R	4.64	1.83	4.26
Conservative	Standard Bk Index R	4.61	0.45	4.59
	RMB Equity	4.38	0.62	4.34
	Investec Equity R	4.30	0.29	4.29
	Community Growth	4.21	0.19	4.21
	m Cubed Capital Equity FoF	4.18	0.22	4.17
	Old Mutual Investors	4.10	1.30	3.88
	Coronation High Growth	3.81	0.86	3.71
	Liberty Prosperity R	3.45	0.30	3.43
	Sanlam General	3.41	1.60	3.01
	FT NIB L [™] Wealth Creator	2.86	0.02	2.86

Thereafter we combine the *aggressive* funds randomly in all possible combinations of 1 through to 10 managers using the same methodology as described in the prior section for *standard managers*. The benchmark remains the peer group average of all standard managers, and the relative returns and tracking errors are calculated against this benchmark. We also repeat the analysis for moderate funds and assess the decrease in tracking error with increasing number of funds for both groups.

The summary resulting active returns and risks of the aggressive managers contrasted to the moderate managers are portrayed in figure 6.3.

In Figure 6.3 that follows, it is evident that combining *aggressive* GE funds results in a combined fund having more active risk than would have been attained by combining moderate risk GE funds. Multi-managers can combine up to 5 *aggressive* managers before their active risk falls below the optimal range, by contrast to only 2 for *moderate* managers.

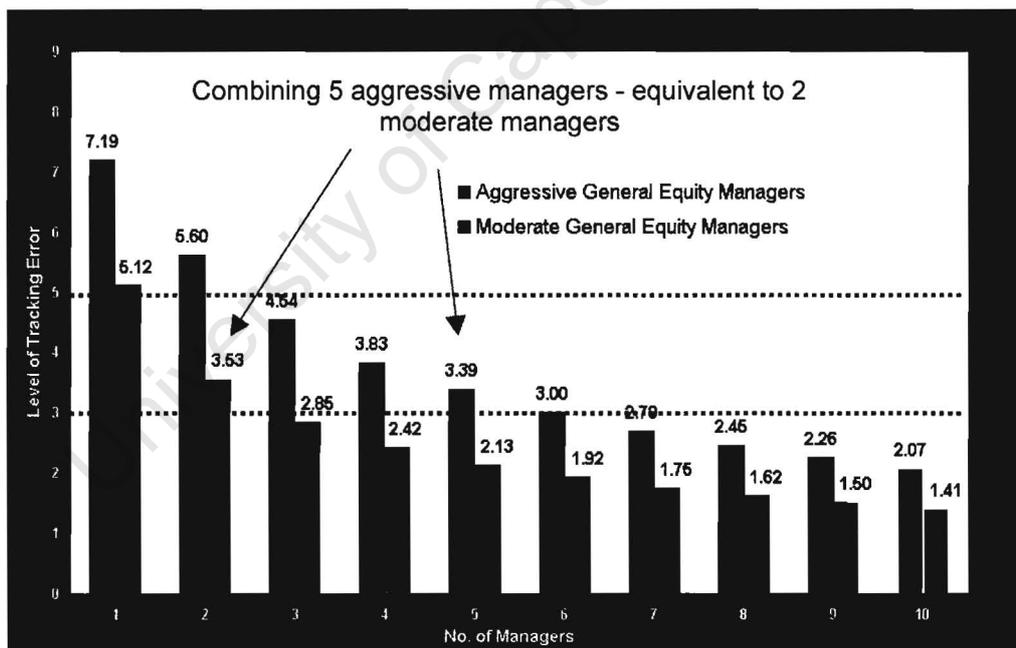


Figure 6.3: Median Tracking Error for combining aggressive and moderate GE Unit Trusts

The above analysis suggests that multi-managers who are able to set higher tracking error mandates for their managers would be able to select several more managers as a consequence.

6.1.3.3 Combining managers based on fund sizes

Another aspect to consider when combining funds is the selection of funds based on size. The intuition is that the size of a fund is associated with the ability of the fund to be aggressive. It has been argued that large funds are unable to obtain enough scrip to construct large active bets against their benchmark.

The converse also applies to small funds. Their ability to actively trade mid- and small-cap shares enables smaller funds to take larger active bets against their benchmark. This ability in turn allows smaller funds to potentially take on higher tracking errors.

Our analysis partitioned the GE unit trusts into 3 groups according to fund size: large, medium and small funds. Based on fund size at August 2001, large funds were selected as having fund sizes larger than R 1 000 000, with medium sized funds between R 250 000 and R 1 000 000, and small funds less than R250 000. The simulation procedure was repeated by selecting from the 3 groups of funds. The results are portrayed graphically in Figure 6.4 that follows.⁹

⁹ Note there are only 7 funds that fell into the large category. These funds were (in descending fund size order - Rm's): Old Mutual Investors (R3480.04), FT NIB Prime Select (R1727.77), Coronation High Growth (R1648.57), Investec Equity (R1442.11), Liberty Wealthbuilder (R1420.21), Sage Fund (R1412.61) and BOE Equity (R1022.83).

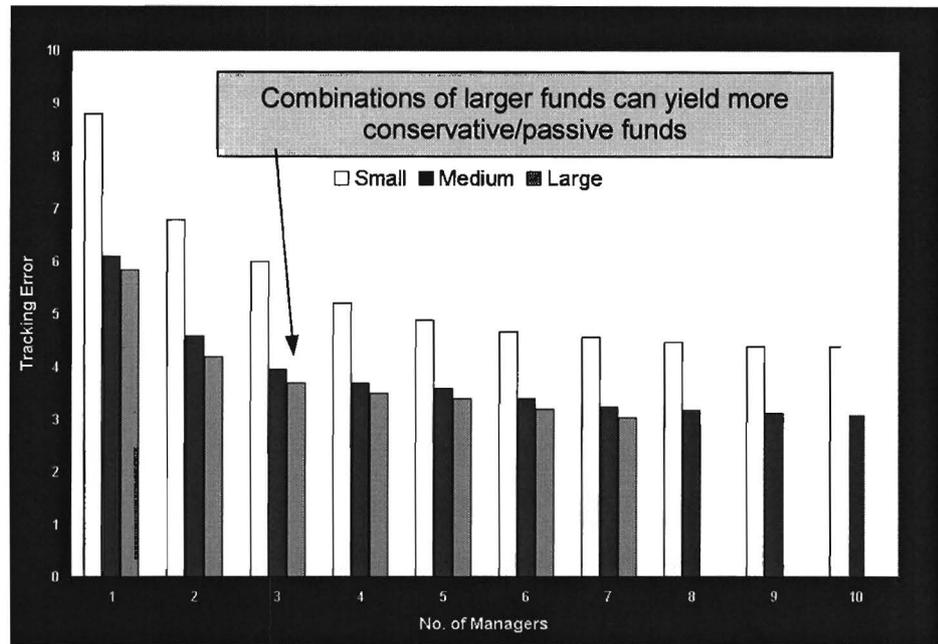


Figure 6.4: Median Tracking Error for combining small, medium and large GE Unit Trusts

The results in figure 6.4 are once again intuitive, as selecting larger funds yielded smaller median tracking errors at each number of managers selected. Furthermore, the tracking errors of small funds are significantly higher than medium and large funds, with a small difference in tracking error between medium and large funds. There is minimal benefit in combining medium sized GE funds as opposed to large GE funds in terms of active risk. However, the result that combinations of large funds yield more conservative funds is confirmed by the above results.

Small sized funds appear to asymptote at a median tracking error of 4%. At this level the unique risk interestingly does not diversify away as reflected by the sluggish decrease in median tracking error of small funds.

6.1.3.4 Combining managers based on skill

The last aspect considered in the multi-manager context is *manager skill*. We use *selection reward* (as measured by the funds alpha) divided by *selection risk* as a measure of manager skill (in essence the information ratio). We measure *persistence* of skill by ranking funds according to their “average rank” of this skill measure over the entire period. The following funds were thus selected based on having “average ranks” less than 2.5 and are portrayed in the table that follows:

Table 6.4: Average rank of General Equity Funds (1995 – 2001)

Funds	Average Rank
Metropolitan General Equity	1.6
BoE Equity	1.8
Investec Equity	2.0
Marriot Dividend Growth	2.0
NIB Prime Select	2.2
Old Mutual Investors	2.2
RMB Equity	2.2
Nedbank Growth	2.3
Sage Fund	2.4
Coronation High Growth	2.5

The results of the simulation based on combining the above funds and computing the combined portfolio's relative returns and risks against the peer group benchmark are portrayed in Figure 6.5 that follows.

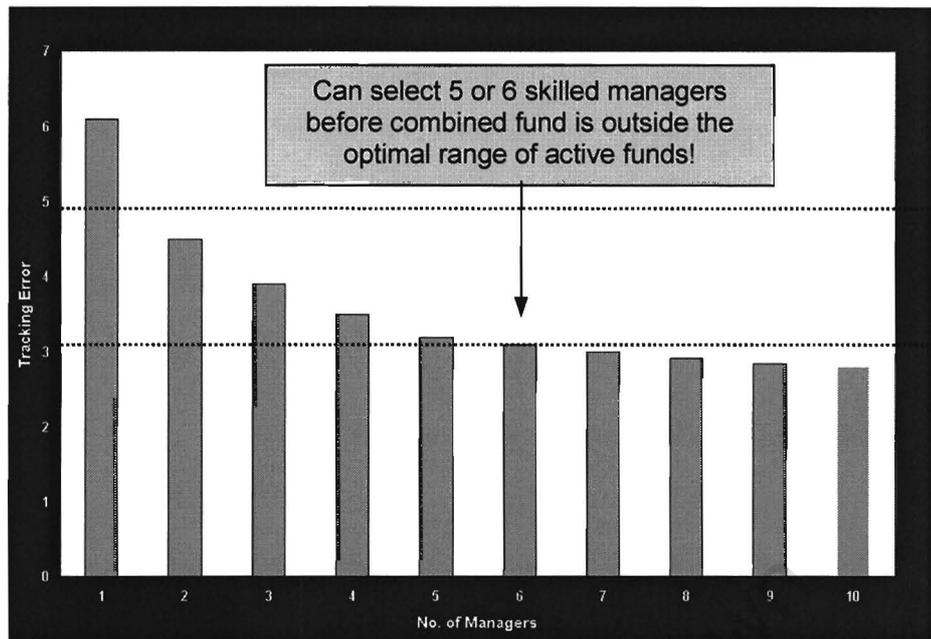


Figure 6.5: Median Tracking Error for combining GE Unit Trusts with persistent skill

The results of Figure 6.5 show that as many as 6 managers with persistent skill can be combined before their median tracking error falls below 3%, thus declining below the optimal range for active GE funds.

6.1.4. Conclusion

Although the focus in this section was somewhat restrictive because we focussed predominantly on the General Equity category of Unit Trusts, the analysis nevertheless yielded some useful practical insights for multi-managers.

The analysis on *combining managers* according to a variety of selection criteria suggest that combining more than 4 *standard* General Equity managers usually results in a combined fund having a passive character, falling outside of the ideal active risk range. The findings also suggested that one can increase the number of managers (to approximately 6) by selecting more aggressive managers, managers with persistent stock selection skill or avoiding larger funds.

6.2. Persistence of Fund performance¹⁰

6.2.1. Introduction and discussion

Marketing strategies of Unit Trusts are commonly based on the notion that investors purchase funds on the basis of their track records. These marketing strategies indirectly imply that funds that have done well in some recent period will continue to do well in the future.

It is commonly accepted amongst academics that stock markets are efficient, suggesting that no persistent superior performance should be expected for individual *stocks* (see Fama (1970)). But managers of investment funds can change their risk-profiles far more quickly than managers of individual stock companies. Does this flexibility in decision-making enable managers of *funds* to achieve what eludes managers of *stock companies* - persistent winning performance?

Clearly evidence on the persistence of fund performance is crucial to investors and multi-managers alike. If persistence is found to be present in South African Unit Trusts, strategies of buying winning funds and / or selling losing funds would prove to be profitable. This section considers the contribution current evidence has on the persistence of the fund performance debate. Not only does this study enable analysis over a longer period, but a significantly larger sample of surviving funds are now available by comparison to other studies.

6.2.2 Review

In a study conducted in the USA, Elton, Gruber and Blake (1996), find tentative evidence that USA funds that have done well in the past tend to do well in the future¹¹. Meyer

¹⁰ This section has been published in Bradfield and Swartz (2001), "Recent evidence on the persistence of fund performance", *SA Journal of Accounting Research*, Vol. 15, No. 2, 99-109.

¹¹ Malkiel (1995), Brown and Goetzmann (1995) and Grinblatt and Titman (1992) all additionally find evidence of persistence of performance of USA funds. See the reference section.

(1997) conducted one of the first rigorous studies¹² on persistence of performance using South African fund performance data over the 1985-1995 period. Meyer also tentatively concluded "some persistence in performance of South African unit trusts, does exist".

In the light of the fact that we have more funds with longer performance history in the unit trust industry, and the recent volatility of the South African market we revisit the issue of fund persistence. In this section we therefore disclose and discuss the results of a study on persistence of fund performance and highlight some implications for fund selection and management.

6.2.3 Data

The data source used is Micropal, which includes fund returns with dividends reinvested. The study was conducted over the prior 7 years, starting January 1995 through to August 2001. To avoid interaction between category effects, the study focussed the analysis on the General Equity Category of Unit Trusts. Because of the structure of our test methodology, all General Equity funds with a history of less than three years were excluded from the analysis.

6.2.4. Results

6.2.4.1 Based on Annual Assessment periods

The measure of performance on which the analysis has been based is the rankings of the absolute annual return of the funds in the General equity category.

Table 6.5 contains a summary of the rankings of the absolute performance measures of the qualifying General Equity funds. The funds in Table 6.1 have been ranked according to their "average rank" over the entire period.

¹² Meyer presenter her evidence on persistence of unit trust performance at the Southern African Finance Association conference in January 1997.

Table 6.5: Ranked and quartile positions of returns of General Equity funds (Jan 1995- August 2001)

	1995		1996		1997		1998		1999		2000		2001		Average	
	Rank	Quartile	Rank	Quartile												
Allan Gray Equity	-	-	-	-	-	-	-	-	1	1	1	1	1	1	1.0	1.0
FT NIB Prime Select	1	1	1	1	4	1	5	1	13	2	4	1	14	2	6.0	1.3
Liberty RSA Equity R	-	-	-	-	-	-	-	-	7	1	10	2	10	2	9.0	1.7
Metropolitan General Equity	2	1	2	1	3	1	4	1	26	4	2	1	23	3	8.9	1.7
BoE Equity	5	2	3	1	2	1	21	4	31	4	13	2	7	1	11.7	2.1
Liberty Wealthbuilder R	9	3	9	2	15	3	22	4	8	1	8	1	5	1	10.9	2.1
Fedsure Index	-	-	11	3	16	4	17	3	6	1	7	1	4	1	10.2	2.2
Investec Index R	-	-	8	2	19	4	9	2	3	1	21	3	3	1	10.5	2.2
Investec Equity R	13	4	5	1	5	1	12	2	12	2	27	4	15	2	12.7	2.3
Liberty Prosperity R	6	2	16	4	10	2	23	4	4	1	11	2	6	1	10.9	2.3
RMB Equity	4	1	6	2	7	2	20	4	16	2	15	2	17	3	12.1	2.3
Community Growth	10	3	18	4	1	1	8	2	27	4	14	2	8	1	12.3	2.4
Sage Fund	12	4	4	1	17	4	13	3	19	3	6	1	2	1	10.4	2.4
Gryphon Imp SA Tracker	-	-	-	-	-	-	16	3	2	1	20	3	22	3	15.0	2.5
Standard Bk Index R	-	-	15	4	18	4	14	3	5	1	5	1	11	2	11.3	2.5
Coronation High Growth	-	-	-	-	13	3	2	1	17	3	23	3	19	3	14.8	2.6
ABSA Growth FoF	-	-	-	-	-	-	-	-	23	3	3	1	28	4	18.0	2.7
FNB Growth	-	-	-	-	-	-	-	-	10	2	25	4	9	2	14.7	2.7
Futuregrowth Core Equity	-	-	-	-	-	-	-	-	22	3	22	3	16	2	20.0	2.7
Old Mutual Top Companies	3	1	7	2	8	2	19	4	20	3	24	3	27	4	15.4	2.7
Standard Bk Mutual R	8	2	13	3	9	2	15	3	24	3	16	2	25	4	15.7	2.7
FT NIB LT Wealth Creator	-	-	-	-	-	-	18	4	9	2	12	2	21	3	15.0	2.8
Nedbank Growth	-	-	-	-	-	-	1	1	15	2	32	4	32	4	20.0	2.8
ABSA General R	11	3	12	3	14	3	6	1	32	4	28	4	12	2	16.4	2.9
Old Mutual Investors	15	4	14	4	12	3	10	2	11	2	19	3	13	2	13.4	2.9
Fedsure Equity	7	2	17	4	6	2	3	1	28	4	30	4	29	4	17.1	3.0
m Cubed Capital Equity FoF	-	-	-	-	-	-	11	2	30	4	17	3	18	3	19.0	3.0
RMB Performance FoF	-	-	-	-	-	-	-	-	25	4	9	2	20	3	18.0	3.0
Sanlam General	14	4	10	3	11	3	7	2	21	3	18	3	24	3	15.0	3.0
Sanlam Future Trends	-	-	-	-	-	-	-	-	14	2	29	4	30	4	24.3	3.3
Brait Accelerated Growth	-	-	-	-	-	-	-	-	18	3	26	4	26	4	23.3	3.7
Fedsure Pioneer	-	-	-	-	-	-	-	-	29	4	31	4	31	4	30.3	4.0

It is evident from the last column in Table 6.5 headed, "Average Quartile", that Allan Gray exhibited persistently superior performance to the extent that they were able to remain in the top ranked position (although based on a restricted 3 year history). NIB Prime Select, has the next lowest (best) average rank, remaining in the 1st quartile for the first 4 years, and slipping to a 2nd quartile performer in 1999 and 2001¹³. On the losing side, Fedsure Pioneer has persistently remained in the 4th quartile (although it too is based on a

¹³ It should be noted that NIB Prime Select changed fund managers in 2001.

restricted 3 year history). The columns headed "Average Rank" and "Average Quartile" are good indicators of the relative aggregate performance of qualifying General Equity funds.

To gauge to what extent funds have exhibited persistence in performance over the 7-year period, Table 6.6 below shows a more cursory summary of Table 6.5. Table 6.6 gives a summary of the percentage of funds whose rankings moved both up and down by 1, 2, and 3 quartiles over each consecutive two-year period.

Table 6.6: Percentage of funds changing quartiles in consecutive years

	1995-1996	1996-1997	1997-1998	1998-1999	1999-2000	2000-2001	Average
Moved up 3 quartiles	0.00%	5.56%	5.26%	13.04%	0.00%	3.13%	4.50%
Moved up 2 quartiles	13.33%	5.56%	15.79%	13.04%	18.75%	6.25%	12.12%
Moved up 1 quartile	26.67%	11.11%	21.05%	13.04%	12.50%	18.75%	17.19%
Unchanged	26.67%	50.00%	10.53%	21.74%	40.63%	43.75%	32.22%
Moved down 1 quartile	20.00%	11.11%	31.58%	8.70%	9.38%	15.63%	16.06%
Moved up 2 quartiles	0.00%	11.11%	15.79%	21.74%	15.63%	12.50%	12.79%
Moved up 3 quartiles	13.33%	5.56%	0.00%	8.70%	3.13%	0.00%	5.12%

In Table 6.6 above it is evident that in the 1996-1997 period, half of the funds remained in the same quartile positions, and in the 1999-2000 period over a third of the funds remained in the same quartile positions. The periods 1997-1998 and 1998-1999 represented periods having the most interquartile movements with only 11% and 22% (respectively) of funds remaining in the same quartile positions. Interestingly 1998 was a period characterised by high market volatility. On average, 32% of funds have remained in the same quartile positions, with the average proportion of interquartile movements decreasing the higher the magnitude of the quartile position movement.

We also subjected our results to more rigorous statistical scrutiny. To more formally test persistence, we implemented a test to assess whether the consecutive annual rankings of fund performance was significantly correlated. Spearman's rank correlation measure was appropriate for this type of test, since the emphasis is on rankings of funds, and not performance of funds. A summary of the results is shown in Table 6.7.

Table 6.7: Spearmans rank correlations of consecutive annual performance ranks

	1995 – 1996	1996 – 1997	1997 - 1998	1998 - 1999	1999 - 2000	2000 - 2001
No. of Funds	15	18	19	23	32	32
Spearman's Rank Correlation	0.429	0.205	0.128	-0.342	0.248	0.498
p-value	0.054	0.199	0.293	0.054	0.084	0.003

Note: The p-value conveys the degree of statistical significance. The smaller the p-value the more significant the result.

The Spearman's Rank correlation is positive in 5 of the 6 consecutive periods analysed - suggesting a positive relation between the ranked performance position in 5 of the periods. The exception was the 1998-1999 period yielding a negative ranked correlation (suggesting instead evidence of winners becoming losers and vice versa).

More formal statistical inferences can be made by assessing the magnitude of the p-values (levels of statistical significance) reflected in the last row of Table 6.7. Under the hypothesis that correlations of zero imply evidence of "lack of persistence of fund performance", our attention shifts to the degree of departures of the correlations from zero captured in the p-values. The *smaller* the p-value, the more significant the departure of the correlation from zero. From Table 6.7 it is evident that in 4 of the 6 periods the p-values were less than 0.1, suggesting some statistical evidence of non-zero correlations of fund performance rankings. In 3 of these 4 periods, including the last two periods the underlying correlations were positive (with the last period yielding a highly statistically significant p-value of 0.003) suggesting *recent* evidence of strong *persistence* of fund performance.

Interestingly the 1998-1999 was the only period yielding a negative ranked correlation (with a p-value of 0.054), confirming that this period represented a reversal of persistence, i.e. winners becoming losers and vice versa.

Even though conducting a rank correlation test is a rigorous measure of testing persistence (or lack thereof), it fails to identify the persistence of *each* quartile position. We next consider the persistence within each individual quartile, by assessing the proportions of various quartile movements in subsequent periods. Figure 6.6 shows the results of each quartile's proportions of quartile positions in the subsequent period.

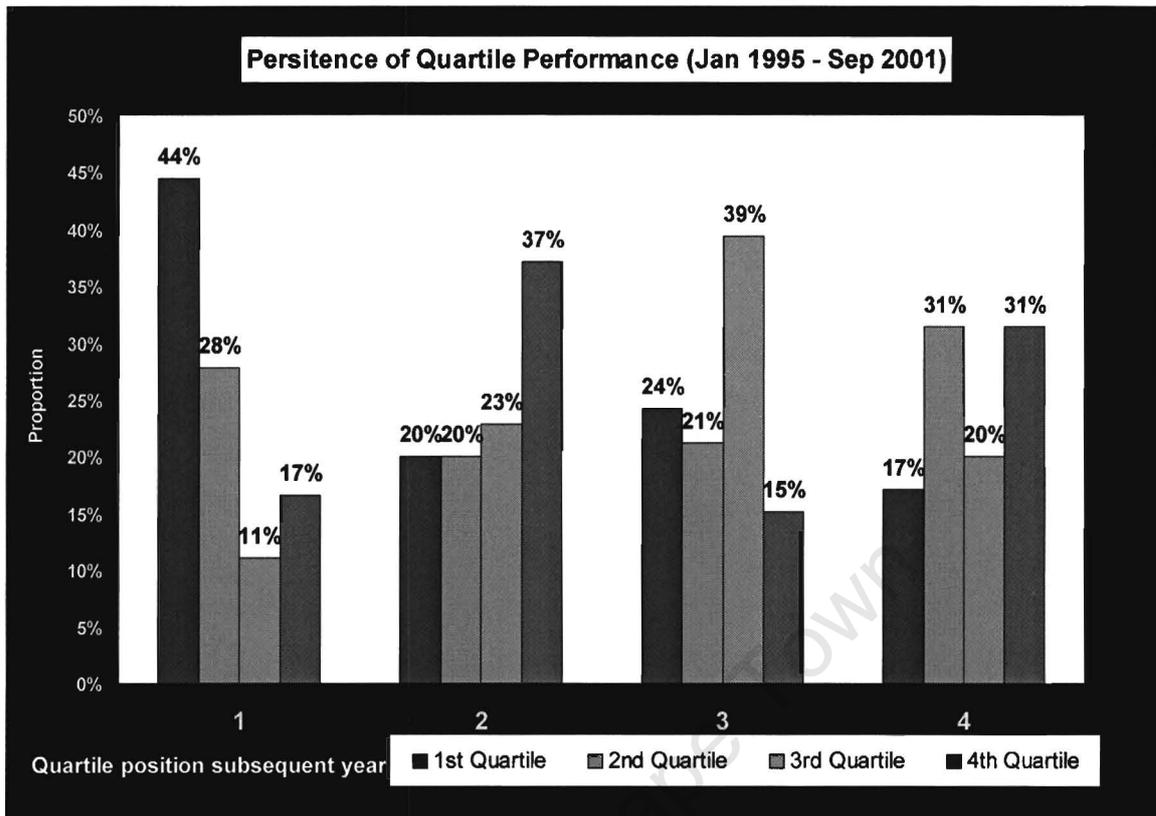


Figure 6.6: Distribution of quartile position of funds in their subsequent year

From Figure 6.6 it is evident that the strongest persistence is found in 1st quartile funds as is evidenced here by the 44% occurrence again in the 1st quartile in the subsequent periods. The next best level of persistence was found in the 3rd quartile, where 39% of the time, 3rd quartile funds have remained 3rd quartile in the subsequent period. A more interesting result has been the lack of persistence found in the 2nd quartile, where a high proportion of interquartile movement (37%) has been from the 2nd to the 4th quartile.

Theses results give confirmation that the persistence found is predominantly a consequence of the superior, 1st quartile performers. Thus there does seem to be evidence of skill, rather than luck.

Simulating the investment strategies based on historical quartile performance...

To assess persistence from a practical investment perspective, we considered the impact of constructing portfolios based on historical top quartile performers, second quartile performers etc.. Portfolios were rebalanced annually by equally weighting the funds in each performance quartile (based on the quartile performance of the prior year). Returns were accumulated on the subsequent year of "unseen" data.

The portfolios were first formed at the end of December 1995 (based on 1995 performance) and rebalanced annually and monitored through to August 2001.

Figure 6.7 shows the results of investing in these rebalanced portfolios based on the historical performance quartiles of their component funds.

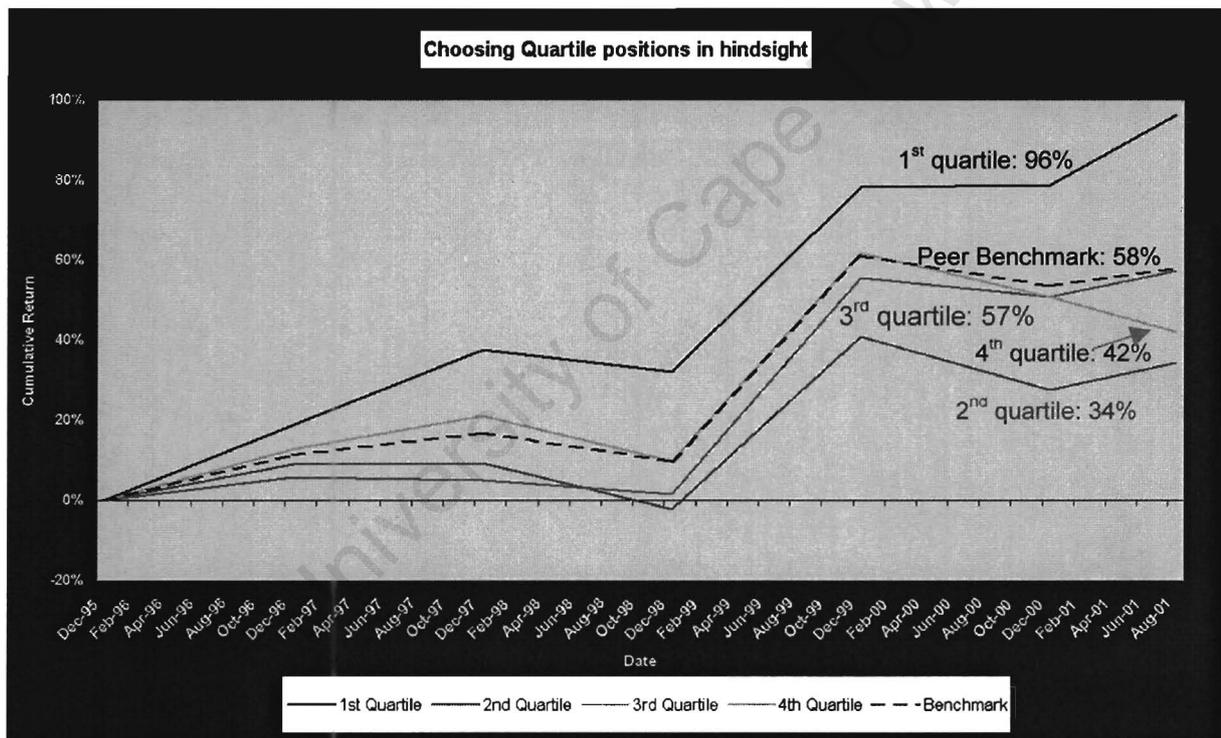


Figure 6.7: Cumulative returns from annual rebalancing of portfolios in the same performance quartile

The most startling result portrayed in Figure 6.7, is the unexpectedly high return obtained by investing in historical 1st quartile performers. This strategy yielded a cumulative return of 96%. Contrary to popular local belief there does seem to be evidence that superior skill

exists and is being translated into superior performance, consistently! Interestingly the second best strategy was to invest in the category benchmark (yielding 58%)- and the worst was to invest in second quartile performers yielding only 34%.

We also assessed a variety of alternate strategies based on historical quartile performance. For example we considered a portfolio constructed from funds that improved by 1 and 2 quartiles and contrasted these to a portfolio of funds that moved down by 1 and 2 quartiles. The results of the cumulative returns of portfolios constructed on this basis are shown in Figure 6.8.

It is evident from Figure 6.8 that none of the alternate strategies of investing in funds that improved/declined in their quartile performance was superior to the strategy of investing in the peergroup benchmark.

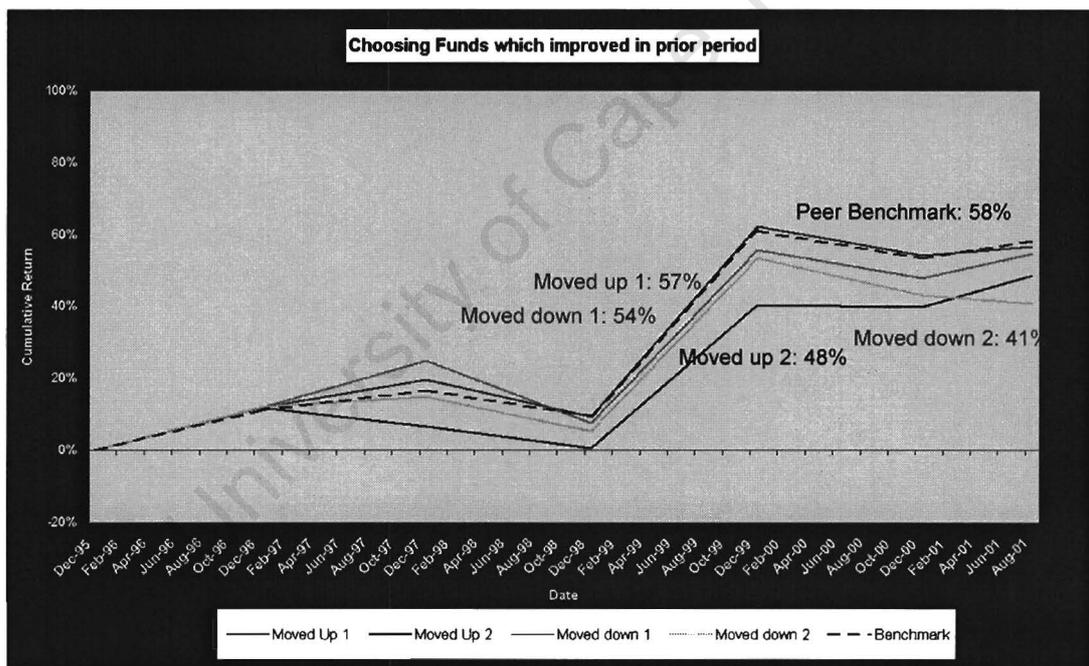


Figure 6.8: Cumulative returns from annual rebalancing of portfolios based on various scenarios of moving up and down the quartiles

Lastly we also considered a strategy based on investing only in funds that never obtained a 4th quartile position in the seven year history analysed. We compared this to the strategy of investing in the peergroup benchmark in Figure 6.9.

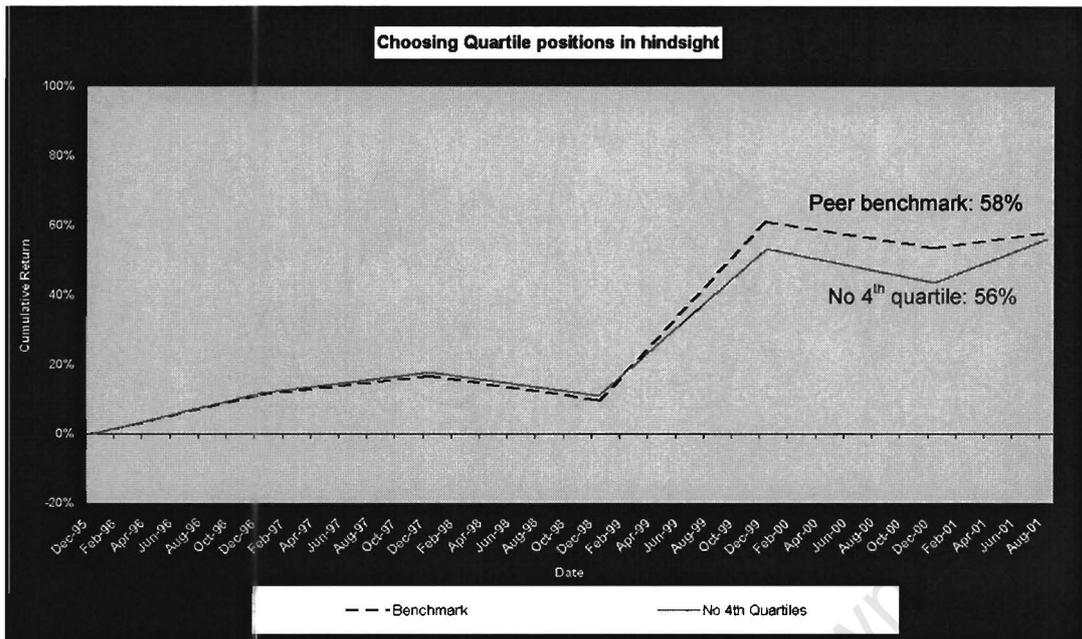


Figure 6.9: Cumulative returns from annual rebalancing of a portfolio based on all funds that have never had 4th quartile performance.

It is evident from Figure 6.9 that holding the peer group benchmark was also superior to the strategy of combining all funds that have never had annual performance resulting in a 4th quartile placing. In summary it seems that amongst all the alternatives considered above, only the strategy of investing in historical 1st quartile performers was superior to investing in the peer group benchmark.

6.2.4.2 Based on quarterly Assessment periods

The analysis was repeated by considering the shorter quarterly assessment periods. Even though superior performers show evidence of persistence on an annual assessment basis, it is of interest to assess to what extent persistence manifests itself in shorter periods.

Based on the quartile ranking of the funds, we once again compute Spearman's rank correlation for each *quarterly* period, along with the associated p-value (reflecting the significance of the result). These results are found in Figure 6.10.

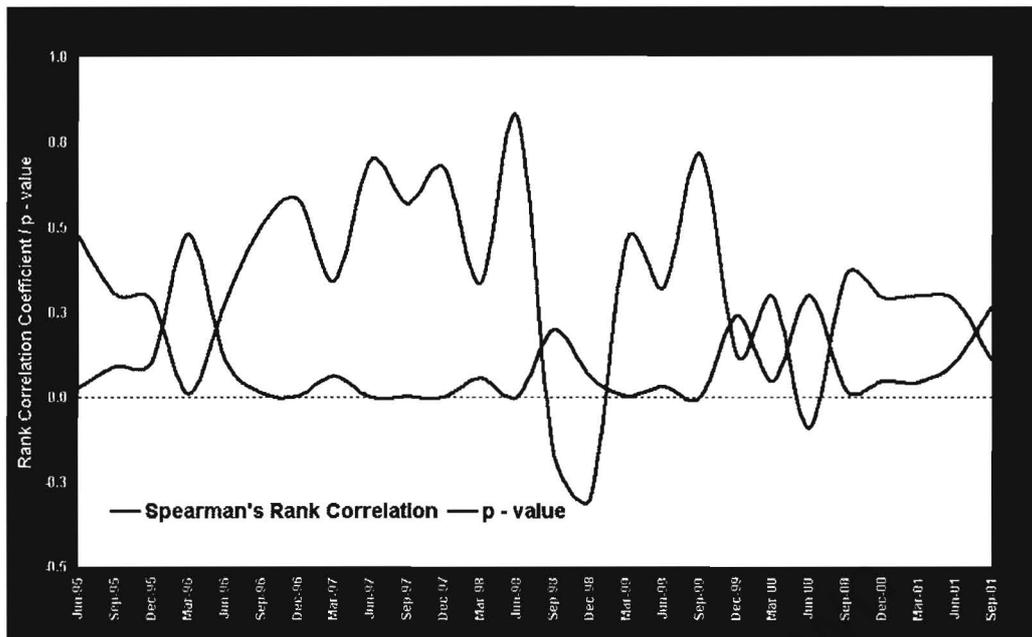


Figure 6.10: Spearman's Rank Correlation and associated p-value for testing persistence of General Equity fund performance based on a quarterly basis.

It is evident from Figure 6.10 that the quarterly time series of Spearman's Rank correlation displays significant volatility. The rank correlation reached a high of 0.82 during the 2nd quarter 1998, and a low of -0.30 during the 4th quarter of the same year. It should also be noted that General Equity funds generated a negative return of -24.1% during this quarter. It was thus during this period that winners became losers and visa versa. The persistence analysis based on annual assessment periods also yielded negative persistence during 1998.

The Spearman's correlation coefficient also displayed high levels of positive significance at certain quarters. This significance was especially evident from September 1996 to the high of 0.82 in June 1998. Throughout this period, Spearman's Rank correlation yielded an average of 0.57. Similar periods of high rank correlation and low p-value occur during the periods December 1998 to September 1999 and September 2000 and June 2001.

To identify the persistence present in *each* quartile position, we assess the proportions of various quartile movements in subsequent periods. Figure 6.11 shows the results of each quartile's proportions of quartile positions in the subsequent period.

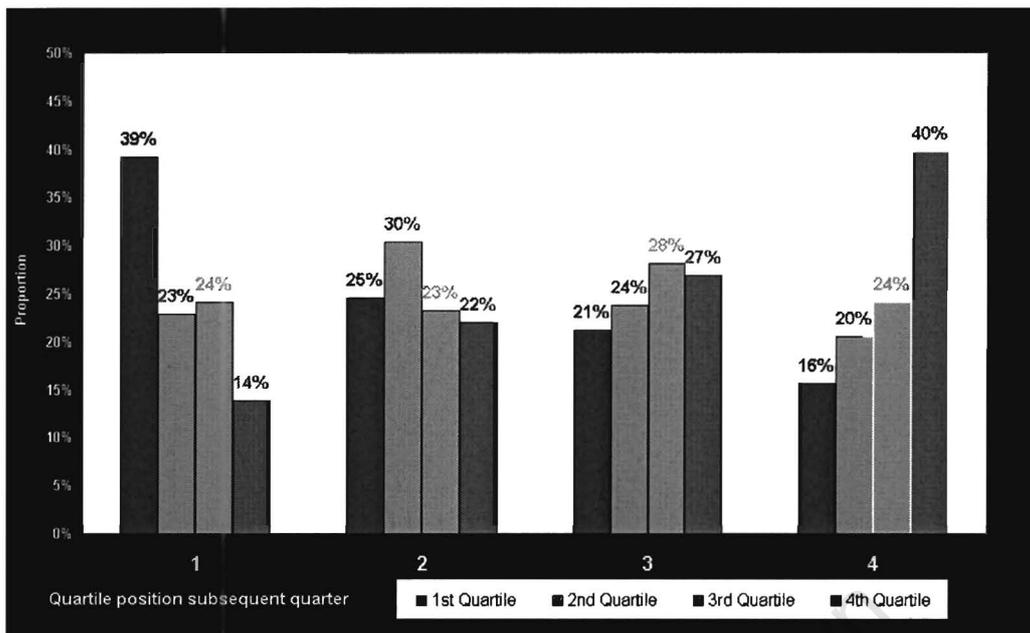


Figure 6.11: Distribution of quartile position of funds in their subsequent quarter

Assuming that no persistence existed, i.e. the results showed a zero Spearman's Rank correlation, each proportion is expected to be 25%. From Figure 6.11 however it is apparent that there are significant departures from this. This is evident by the maximum proportion of quartile movement being to the same quartile in the subsequent periods – for each quartile. The most persistence is found in the 4th quartile where 39% of the time, 4th quartile funds remained 4th quartile in the subsequent period. Marginally less persistence was found in the 1st quartile, where 39% of the time, 1st quartile funds remained 1st quartile in the subsequent period. Even though persistence was evident in the 2nd and 3rd quartiles, these were not as significant as 1st and 4th quartiles.

Simulating investment performance based on quarterly assessment periods...

To consider persistence from a practical investment perspective, we constructed portfolios based on historical top quartile performers, second quartile performers etc... Portfolios were now rebalanced quarterly instead of annually, by equally weighting the funds in each performance quartile (based on the quartile performance of the prior year). Returns were accumulated on the subsequent quarter of "unseen" data.

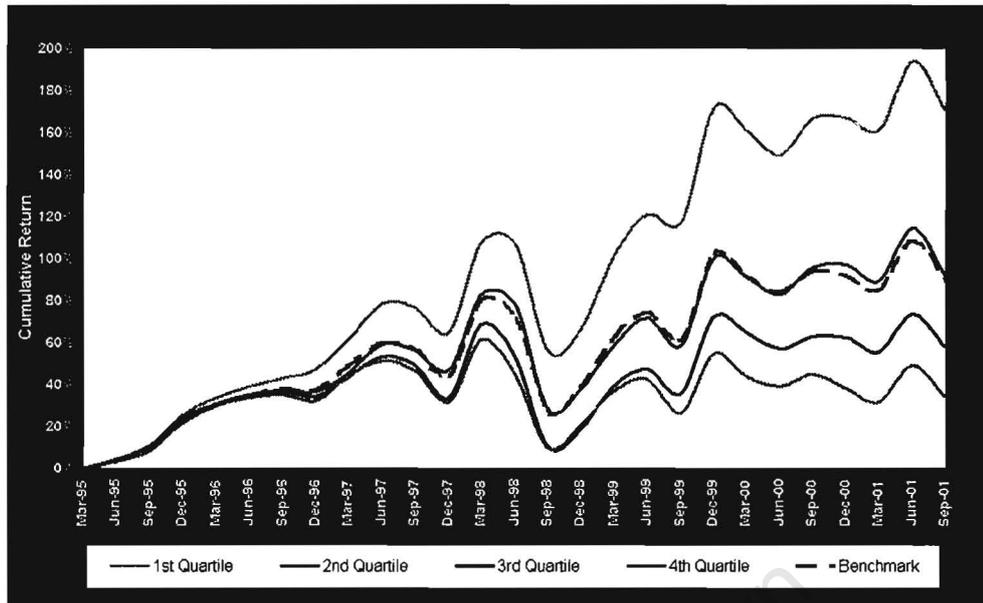


Figure 6.12: Cumulative returns from annual rebalancing of portfolios in the same performance quartile

Figure 6.12 shows the results of investing in these quarterly rebalanced portfolios based on the historical performance quartiles of their component funds. The results are reasonably consistent with the annual rebalancing of Section 6.2.4.1. Clearly, the first quartile funds performed best, with second quartile funds performing marginally better than the benchmark. Third quartile funds according were placed third in terms of performance over the period, with 4th quartile funds performing worst. This result once again confirms persistence of fund performance, with an even stronger corroboration for 1st quartile performers using quarterly assessment periods.

6.2.5. Conclusion

Contrary to popular local belief the picture that has now emerged in this study points to persistence of fund performance, rather than lack of it. In 5 out of the 6 consecutive years analysed we found positive correlation between the ranked performance of the funds (and in 3 of them the results were statistically significant at the 10% level). This latest evidence seems to be primarily driven by the consistently superior performance of some top performers (rather than persistent poor performers). It seems that there are managers

who have significant skill, to the extent that they have systematically been able to outperform their peers.

As an exercise to assess the practical advantages of persistence of fund performance, we considered the impact of investing in historical top quartile performers, second quartile performers etc., rebalancing annually, and accumulating returns during the subsequent year of "unseen" performance data. The results are indeed surprising and compelling. Contrary to popular local belief, selecting historical top quartile performers yielded vastly superior performance results to a wide range of alternate combinations. The relevance of the persistence result to multi-managers is that there exists evidence that selecting top performing funds is a superior strategy, and interestingly, the second best strategy was that of selecting a passive peergroup of funds.

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CHAPTER 7: CONCLUSIONS AND RECOMMENDATIONS FOR PRACTITIONERS

The aim of this thesis was to provide the practitioner with some applications of quantitative techniques in the area of asset management.

The first significant contribution in the thesis was in the area of portfolio optimisation – specifically on the demonstration of the Black and Litterman (1992) technique of quantitative portfolio design based on managers' views in the local South African context. This practical application of the traditional Markowitz optimisation was shown to be useful to practitioners, as the resultant optimal portfolios was not accompanied by the characteristic problems of input sensitivity and portfolio concentration. The most encouraging result found however was that manager views are incorporated into the optimisation process so that investment weights tilt away from a benchmark in an intuitive and practical way. This procedure of applying the Black and Litterman adjustment to the return inputs is thus recommended for practitioners when endeavouring to optimise active portfolios.

The topical areas of benchmarking were also reviewed, with specific emphasis placed on the selection and construction of benchmarks for various client requirements. Here the aim was to demonstrate the use of graphical aids to better understand the basic concepts of matching the portfolio risks to that of the benchmark, as well as constructing an outperformance benchmark. These contributions serve primarily as useful aids for trustees. Two other topics of benchmarking were considered, with primary contributions being to demonstrate optimisation techniques in estimating competitor benchmarks, and assessing the ability of outperforming benchmarks given various levels of skill and aggression.

Issues in portfolio risk management for active managers were considered. Given the practical difficulty in determining appropriate tracking errors, a bootstrapping technique helps with the objective of setting risk mandates to avoid specified underperformance of active managers. It was also shown that funds that consistently perform well use a greater

proportion of their active risk on stock selection, and thus this strategy is recommended for practitioners.

Lastly, two studies of multi-manager topics were reviewed. It was shown that when combining more than 4 *standard* General Equity managers, this usually results in a combined fund having a passive character, falling outside of the ideal active risk range. It was also found that there exists evidence of persistence in the Unit Trust industry, and it seems to be primarily driven by the consistently superior performance of some top performers.

Directions for future research...

A direction for future research is possibly applying the Black and Litterman adjustment to the estimated covariance matrix. Since this thesis focused on using conditional distribution theory to adjust only the expected mean vector, the demonstration of adjusting the covariance matrix in an optimisation context is a useful and relevant direction for future research.

Estimating the covariance matrix has also become a relevant area of research for practitioners. Given the volatility of financial markets in today's day and age, it would be useful for practitioners to model covariance structures for both turbulent and calm periods of financial markets. Practitioners could then make asset allocation decisions based on the extent to which they are averse to turbulent periods.

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