

UNIVERSITY OF CAPE TOWN



M p h i l M a t h e m a t i c a l F i n a n c e
(CM033BUS18)

A Post-Crisis Investigation into the Performance of GARCH-based Historical &
Analytical Value-at-Risk on the FTSE/JSE

Supervisor: Chun-Sung Huang

Course Code: BUS5005W

Alessando de Alessi (DLSALE001)

August 2013

The copyright of this thesis vests in the author. No quotation from it or information derived from it is to be published without full acknowledgement of the source. The thesis is to be used for private study or non-commercial research purposes only.

Published by the University of Cape Town (UCT) in terms of the non-exclusive license granted to UCT by the author.

PLAGIARISM DECLARATION

Student Name

Student Number

1. I know that plagiarism is wrong. Plagiarism is to use another's work and pretend that it is my own.
2. I have used the Harvard convention for citation and referencing. Each contribution to, and quotation in this work from the work(s) of other people has been contributed, and has been cited and referenced.
3. This work is my own.
4. I have not allowed, and will not allow, anyone to copy my work.

Signature:

Date:

Contents Page

| | |
|--|----|
| Abstract | 1 |
| Introduction | 1 |
| Data | 4 |
| Volatility Models | 5 |
| <i>Distributions</i> | 10 |
| Value-at-Risk Methodology | 12 |
| <i>Analytic</i> | 12 |
| <i>Historical</i> | 13 |
| <i>The Practical Nitty-Gritty</i> | 15 |
| <i>A Word on Monte Carlo Simulation</i> | 15 |
| Backtesting Methodology | 16 |
| Results | 18 |
| Conclusion | 25 |
| References | 25 |
| Appendix | 31 |
| <i>Figure 3: VaR Plot</i> | 31 |
| <i>Figure 4: Likelihood Plot</i> | 31 |
| <i>Figure 5: IGARCH Parameter</i> | 31 |
| <i>Figure 6: GED Shape Parameter</i> | 34 |
| <i>Figure 7: Skewed-t Skew Parameter</i> | 35 |
| <i>Figure 8: Log>Returns Density Plot</i> | 36 |
| <i>Distributions</i> | 37 |
| <i>FIGARCH Derivation</i> | 39 |
| <i>Tables 4 –7</i> | 40 |

List of In-Text Tables & Figures

Figure 1: FTSE/JSE Top40 (2000-2013) **4**

Figure 2: FTSE/JSE Top40 Log Returns (2000-2013) **5**

Table 1: GARCH Daily-Analytic VaR Results **20**

Table 2: GARCH Daily-Historical VaR Results **22**

Table 3: Preferred VaR Models Based On Smallest Failure Rates **26**

University of Cape Town

Abstract

This paper is an investigation into the performance of GARCH-based VaR models on the South African FTSE/JSE Top 40 Index. Specifically, this paper investigates whether stability has returned to the VaR measure following its poor performance during the latest global financial crisis (2007). GARCH models are used in both an analytic and historical approach for modeling 1%, 2.5% and 5% daily-VaR for a three year backtest period (2010-2012). Four distributions are used: the normal, generalised error, t-distribution and the skewed t-distribution. A particular question asked by this paper, is whether the data from the latest financial crisis (2007) should be used in estimating VaR in a post-crisis market. To investigate this, all models are re-estimated using data that has the financial crisis and/or high volatility period removed, then the results across the two data sets are compared. The take away point from this research is that the volatility-clustering mechanism inherent in every GARCH model is capable of producing accurate VaR estimates in a post-downturn/lower-volatility market even when the data on which the model was estimated contains financial downturn/volatile data. There is strong evidence suggesting stability has returned to this measure - however caution remains over using over-simplified models.

Introduction

Value-at-Risk has become a ubiquitous risk management tool. The reason driving its ubiquity is simplicity: the model summarises risk and potential losses of a portfolio in a single statistic. This makes it an attractive risk measure for internal use in financial-services as it can be easily passed around from the risk-management department to the trading desk. This same quality also makes it an attractive measure from a regulatory standpoint: as the regulations can be clearly defined and adherence to the regulation can be readily measured through backtesting. For these reasons, the measure has been promoted by Basel II¹ as the preferred tool for banks to use to assess their market risk exposure (Chen, 2013).

¹ the Basel Accords are a set of regulations and recommendations on banking laws issued by the Basel Committee

Given a portfolio, a time horizon and a confidence level, VaR aims to attach a maximum loss to that confidence level. It is a way of saying: with this level of confidence, the portfolio should not incur a loss greater than the VaR amount during the specified time horizon (Degiannakis, Floros & Dent, 2010).

The formal definition of VaR, is that it is the quantile which solves the following equation:²

$$\alpha = \int_{-\infty}^{-VaR} f(t) dt$$

Where α is the significance level and f denotes the probability density function of the returns of the underlying asset/portfolio.

VaR however has come under growing scrutiny, especially after the 2007 financial crisis³, for poor performance during times of market stress (Chen, 2013). A 2012 report by McKinsey may shed light on its poor performance: the report found that most banks⁴ favour historical VaR, attaching a figure of 75% to this finding and, more crucially, it found that 85% of these make use of the equally-weighted historical approach whilst only 15% use some form of return weighting.

This prompts the need for further investigation into the relative merits of GARCH-based weighting of returns over no weighting at all. This paper conducts this research on the South African FTSE/JSE Top 40 Index.⁵ This paper also makes use of GARCH-based analytic VaR, which allows a comparison to be made on the relative advantages and disadvantages of both VaR approaches.

Given VaR's recent poor performance, a key question asked by this paper is whether the problems that dogged VaR during the latest financial crisis, continue to plague VaR in a post-crisis market. What this question is ultimately asking, is whether VaR models should be fed data which contains the recent financial crisis and the prolonged volatility which endured post-downturn in order to predict VaR in what is a post-crisis, post-bullish-recovery market.

McMillan and Thupayagale (2010) did significant research into GARCH analytic VaR for the JSE, however, the data they used ended before the 2007 financial crisis. Thupayagale (2010) corrected this in his

² See Chen (2013) for further details

³ The crisis was precipitated by the US sub-prime mortgage collapse of 2007 (Dolmetsch, 2008)

⁴ The report was based on large banks from Europe and North America

⁵ The FTSE/JSE Top 40 is an index of the largest forty companies by market capitalisation on the FTSE/JSE All Share Index

research which included financial crisis data and again looked at the performance of GARCH analytic VaR for a variety of GARCH models for a host of emerging market indices including the JSE All Share Index.

The research conducted in this paper complements the work done by the above two authors by investigating the post-crisis performance of GARCH VaR models. Together, these three articles are able to cover the performance of GARCH VaR models pre-crisis, crisis and post-crisis for the FTSE/JSE. To add to this, this paper investigates GARCH historical VaR as well and investigates the performance of distributions other than the normal distribution. The other distributions used in this paper have strong theoretical and empirical advantages over the normal distribution.

Ng Cheong Vee, Nunkoo Gunpot and Sookia (2012) used a similar set of distributions and GARCH models as this paper does in their research on analytic VaR performance in emerging markets (not including the JSE). Their result was that there was no one GARCH model and distribution which was superior across all the emerging markets they researched. This paper would nicely complement theirs, by adding the results for the JSE to the list of emerging markets they researched.

It is worth noting that, given the criticisms of the performance of VaR during periods of market stress, Basel III⁶ has moved beyond VaR and now endorses the expected shortfall measure. Expected shortfall is the expected loss conditioned on the VaR limit being exceeded.⁷ Chen (2013), however, raises the concern that this new risk measure lacks the ability to be reliably backtested which was something that made VaR a popular measure.⁸ For this reason, it is not unrealistic to predict that VaR will still be used for internal risk management even if regulation has moved beyond this measure. Continued research into the VaR measure is therefore still relevant.

The layout of this paper is as follows: the data used is discussed in the next section; a section on the volatility models and distributions follows that; then the VaR methodology is discussed; followed by the backtesting procedure; this then leads to the discussion of the results and finally the conclusion.

⁶ Which was agreed upon in 2010-2011 (Chen, 2013)

⁷ See Chen (2013) for a comprehensive look at the expected shortfall measure

⁸ Expected shortfall does however satisfy the criteria of a coherent measure which is something VaR does not (VaR is not sub-additive, see Chen (2013) for details)

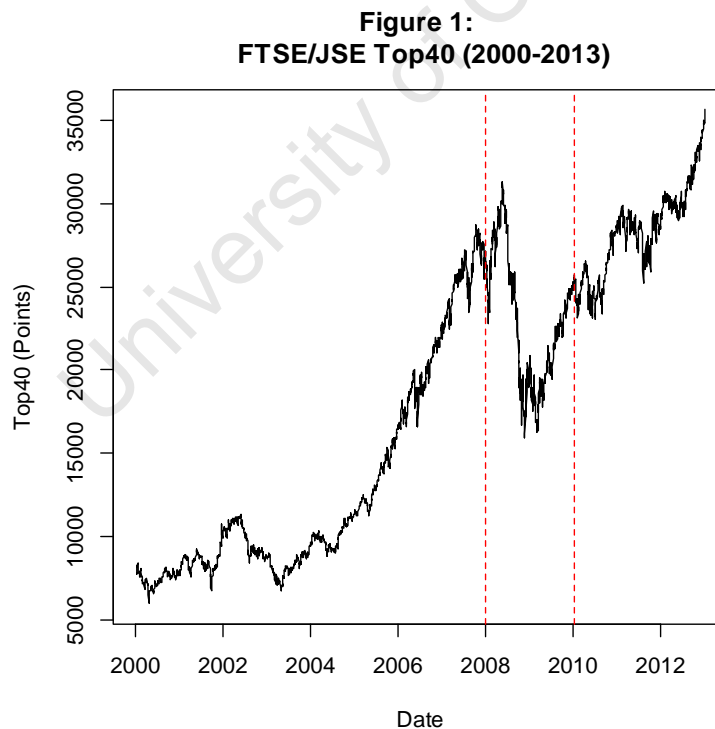
Data

This paper uses the daily closing prices of the FTSE/JSE Top 40 from the start of year 2000 to the end of year 2012. The VaR calculations are based on the log return of the closing prices. The log return is defined as follows:

$$r_t = \ln\left(\frac{p_t}{p_{t-1}}\right) \text{ where } p_t \text{ denotes the closing price at time } t$$

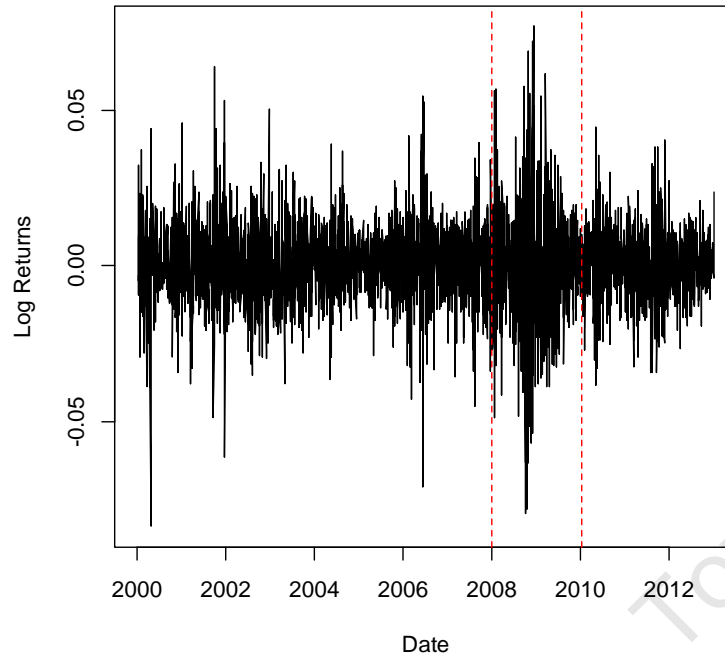
Two approaches are used to calculate VaR: historical and analytic.⁹ Both approaches are used to calculate the daily-VaR at the 1%, 2.5% and 5% significance levels for a three-year period from 2010 until the end of 2012.

Below is a plot of the JSE Top 40 from the start of 2000 to the end of 2012. Below that, is the corresponding plot of the log returns:



⁹ Both approaches are discussed in detail in the section titled Methodology

**Figure 2:
FTSE/JSE Top40 Log Returns (2000-2013)**



Looking at the graphs above, the impact of the latest global financial crisis on the JSE can be prominently seen: there is a significant drawdown from 2008 into the start of 2009. In a four-month period from May 2008, the level of the index almost halved. However, from about the second quarter of 2009, there was a very bullish rebound (in fact, 2009 was a very bullish year for stock exchanges worldwide¹⁰).

Looking at the returns graph (Figure 2) one can see that 2008 and 2009 bracket a time of marked volatility. Given the accepted shortcomings of VaR during this time of market stress, uncertainty and notable increased volatility, there remains the question as to whether to use this data in forecasting future VaR in an environment of relatively decreased volatility and uncertainty. To this end this paper will re-estimate VaR for the three-year period 2010 to 2012 using both the analytic and historical methods with 2008 and 2009 being excluded from the data. Once the VaR estimates have been obtained using both sets of data, a comparison of the results should yield an answer to this question.

This question is ultimately asking whether the patterns emerging from the data of this volatility period - which was an extraordinary and rare time in market history - is of any use in a post-downturn, post-bullish-recovery environment of relatively lower volatility.

¹⁰ Consulting data services like Bloomberg, data shows that many stock indices worldwide bottomed out around early 2009 and began to rebound

Volatility Models

The mean of daily (log) returns is usually very close to zero, and therefore in the case of calculating daily-VaR, the estimate of the volatility of these daily returns is more important (Alexander, 2008). This research considers seven different volatility models - namely: the GARCH; the exponential GARCH (EGARCH); the integrated GARCH (IGARCH); the GJR-GARCH; fractionally integrated GARCH (FIGARCH); fractionally integrated exponential GARCH (FIEGARCH) and the exponentially weighted moving average (EWMA).

Before a (conditional) variance equation can be specified, an equation of the mean is required. The mean equation in this paper is specified as follows:

$$r_t = c + \varepsilon_t \quad \text{where } c \text{ is a constant}$$

This simple mean equation¹¹ is consistent with the one used in the VaR research done by McMillan et al. (2010) on the JSE.

The volatility of the $\{\varepsilon_t\}_t$ process is what is modeled by the aforementioned volatility models. We next describe these models.

A very important feature frequently noticed in financial returns data and described by Mandelbrot (p. 26, 1963) is that "*large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes*" - this phenomenon is now known as volatility clustering.

Engle (1982) and Bollerslev (1986) introduced the ARCH and GARCH models respectively which both aimed to create a model which could mimic the empirically-evidenced volatility clustering. The impetus behind these models was Mandelbrot's observation and as a result these models have a mechanism whereby the volatility is determined by past observations of the returns process (after being filtered by the mean equation) - and in the case of GARCH - by the past values of the volatility process as well.

¹¹ Empirically, for this data & sample size chosen, it was a struggle to find consistently significant 1st-order autoregressive terms. Thus there was scant evidence to first filter the returns according to an AR(1) process (however this was done for other emerging market indices by Ng Cheong et al. (2012))

All GARCH models describe an equation for the time-evolution of $\{\sigma_t^2\}_t$ which is the conditional daily variance process (daily because the data at hand is daily). For demonstrative purposes, assume that the $\{\varepsilon_t\}_t$ process is normally distributed, then:

$$\varepsilon_t | I_{t-1} \sim N(0, \sigma_t^2)$$

Where I_{t-1} represents the information set at time $t-1$.¹²

The first model is the GARCH(1,1) which takes the following form:

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

Where ω , α and β are non-negative parameters. The magnitude of the parameter α measures how reactive the conditional volatility¹³ process is to market events (Alexander, 2008), whereas the parameter β measures the persistence of conditional volatility. In a sense, if there is a market shock (a large negative return), α measures how immediately responsive the conditional volatility process is to that shock, whereas the magnitude of β gauges for how long afterwards, that shock will have an effect on the conditional volatility process (the higher the value, the longer it will take for the effect of that market shock to die out in the conditional volatility process). For this reason, the β parameter is also known as the decay rate (Hull, 2012).

The second model is the IGARCH(1,1) model. This model is a restricted version of the above GARCH model (where α and β are forced to sum to one) and takes the following form:

$$\sigma_t^2 = \omega + (1-\beta) \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

This model has the drawback that the theoretical unconditional variance is infinite (Alexander, 2008). Despite this drawback, this is still an attractive model:

According to Enders (2010), in financial time series data, conditional volatility is persistent. Persistence is measured by the magnitude of the β parameter as well as the by the sum of $(\alpha + \beta)$ which

¹² The information set is the discrete-time equivalent of the filtration. The information set I_t represents all the information available up to time t (see Alexander (2008))

¹³ Volatility is simply the annualized variance

determines the rate of convergence of the conditional volatility forecasts to its long-term average - the closer the sum of $(\alpha + \beta)$ is to one, the flatter the conditional volatility term structure becomes which implies greater persistence in the conditional volatility process.¹⁴ Enders (2010) also notes that when a model is estimated using a long time series of stock returns, the sum of α and β is generally very close to one. Constraining α and β to equal one can yield a very parsimonious model when dealing with financial data.

The above two GARCH models are symmetric whereas the following two models are asymmetric. Asymmetry in this context refers to a mechanism whereby negative returns have a greater impact on the conditional volatility relative to positive returns. The basis for such models is the so-called leverage effect, which Alexander (2008) explains as the phenomenon whereby equity market volatility increases more markedly following a large negative return than it does following an equally-large positive return.¹⁵

The first of the asymmetric models is the EGARCH(1,1) model, which takes the following form:

$$\ln(\sigma_t^2) = \omega + \alpha \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right) + \lambda \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| + \beta \ln(\sigma_{t-1}^2)$$

This is the model introduced by Nelson (1991). The equation for the conditional volatility is in log-linear form. This has the advantage of guaranteeing that the implied value of σ_t^2 will always be positive - which means the parameters ω , α , λ and β need not be constrained to be non-negative.

Another feature of this model is that instead of using the squared values of the $\{\varepsilon_t\}_t$ process, it uses the standardized values. What this means is that if ε_{t-1} is positive¹⁶ then its standardised effect on $\ln(\sigma_t^2)$ is $(\alpha + \lambda)$. However, if ε_{t-1} is negative, its standardised effect on $\ln(\sigma_t^2)$ is $(-\alpha + \lambda)$. According to Alexander (2008), the EGARCH model usually provides the best in-sample fit of financial data compared to other GARCH models.

¹⁴ See Alexander (2008) for more details

¹⁵ It illustrates the risk aversion of the market as a whole. During periods of market turmoil "risk on" assets sell off as investors flee to "risk off" assets (ie. there is a flight to quality). This triggers further sell offs (for example through stop losses) which results in increased volatility

¹⁶ Given our mean equation, for ε_{t-1} to be positive, r_{t-1} must be bigger than c (opposite applies for a negative value) - remember c is normally very close to zero for daily data

The second asymmetric model is the GJR-GARCH(1,1) - named after Glosten-Jagannathan-Runkle (1993). The model takes the following form:

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \lambda \mathbf{1}_{\{\varepsilon_{t-1} < 0\}} \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

Where the indicator function $\mathbf{1}_{\{\varepsilon_t < 0\}} = 1$ if $\varepsilon_t < 0$ and 0 otherwise. As with the EGARCH model, there is asymmetry: when ε_{t-1} is negative, the effect of ε_{t-1}^2 on σ_t^2 is $(\alpha + \lambda)$; whilst when ε_{t-1} is positive, the effect is just α .

The four GARCH models described above are classified as short memory models. The following two models are long-memory models meaning they have relatively high persistence but still allow volatility to be a mean-reverting process. The unique feature of these two models is that they introduce the fractional integration parameter d which allows the conditional variance process to exhibit a slow hyperbolic rate of decay from shocks (ie. high persistence).¹⁷

The first of these models is the FIGARCH model which introduces fractional integration into the IGARCH model. The model takes the following form:

$$\begin{aligned} \sigma_t^2 &= \omega [1 - \beta(L)]^{-1} + \left\{ 1 - [1 - \beta(L)]^{-1} \phi(L) (1-L)^d \right\} \varepsilon_t^2 \\ &\equiv \omega [1 - \beta(L)]^{-1} + \lambda(L) \varepsilon_t^2 \end{aligned}$$

where L represents the lag operator and any function of L denotes a lag operator polynomial.¹⁸ The term $(1-L)^d$ is the fractional lag operator and when $0 < d < 1$ the FIGARCH model exhibits a slow hyperbolic rate of decay from shocks (Thupayagale, 2010).

The second long-memory model is the FIEGARCH model which is the long-memory version of the EGARCH model. The FIEGARCH model was introduced by Bollerslev and Mikkelsen (1996). In that paper, the general FIEGARCH(p,q) was defined as follows:

$$\ln(\sigma_t^2) = \omega + \phi(L)^{-1} (1-L)^d [1 + \psi(L)] g\left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right)$$

where $g(z) = \theta z + \gamma [|z| - E(|z|)]$

¹⁷ see Baillie, Bollerslev & Mikkelsen (1996) for further details

¹⁸ see Appendix (FIGARCH Derivation) and Baillie et al. for further details and derivation

The FIEGARCH model therefore mixes the long-memory feature of the FIGARCH model with the asymmetrical-modeling abilities of the short-memory EGARCH model.¹⁹

The premise for fractionally integrated models is the finding that the squared returns of various financial data exhibit what is classified as a long memory (McMillan et al.,2010). Being able to more accurately model this persistence is what these models offer over the short-memory models.²⁰

Despite the practical observations and theoretical advantages of long-memory models over short-memory models, it must be mentioned that the extensive research done by Degiannakis et al. (2010) found that allowing for fractional integration in the conditional variance process does not yield greater accuracy for VaR forecasts. Their research did not include the JSE, so its inclusion in this research is still warranted.²¹

The final volatility model that will be used in this paper is the exponentially weighted moving average (EWMA) model made popular by the RiskMetrics methodology developed by JP Morgan in the 1990's.²²

The model takes the following form:

$$\sigma_t^2 = (1-\lambda)\varepsilon_{t-1}^2 + \lambda\sigma_{t-1}^2 \text{ where } \lambda \text{ is a positive constant}$$

As Alexander (2008) explains, the EWMA model is based on an i.i.d.²³ returns model. The fact that returns are assumed independent means there is no conditional variance – there is just a single ‘true’ variance that is constant across time. Such a restrictive model ignores the concept of volatility clustering. However, the redeeming feature of the EWMA model is that the estimate of this constant (unconditional) variance is itself time-varying: the estimate is conditional and can therefore de facto capture volatility clustering. This aspect is a positive but the negative of this positive is that there is no prescribed methodology to obtain a value for λ . The most frequently used is the RiskMetrics proposed value of 0.94 (which is also used in this research).

¹⁹ see Bollerslev et al. (1996) for more context and background for the FIEGARCH model

²⁰ the IGARCH model does incorporate persistence as well, but in this case the volatility persistence is permanent - which is an undesirable property - whereas for fractionally integrated models, the volatility process is still mean-reverting (see McMillan et al. (2010) for further details)

²¹ Their research was conducted over twenty one stock indices from developed markets

²² RiskMetrics is a methodology used to construct the RiskMetrics data which are moving average estimates of volatility and correlation for major risk factors such as equity indices and exchange rates (Alexander, 2008)

²³ i.i.d. stands for independent and identically distributed

One can note that the EWMA model is a special case of the IGARCH model where $\omega = 0$ and where λ is chosen as opposed to estimated through maximum likelihood.

The GARCH models analysed in this paper are much the same as those analysed by McMillan et al. (2010) and Thupayagale (2010) in their respective research. The GARCH models used in this research are also of the same order as those used by NG Cheong Vee et al. (2012) and Alexander (2008).²⁴

Distributions

To obtain estimates for the GARCH parameters, maximum likelihood is used. In order to use maximum likelihood, one must first assume a distributional form of the $\{\varepsilon_t\}_t$ process. Four distributions will be used for each volatility model. The first is the normal distribution:

The normal distribution is the most ubiquitous statistical distribution and there is a wealth of statistical theory surrounding this distribution – such as the Central Limit Theorem²⁵ and constructs such as Brownian motion. The normal distribution has many “nice” properties and characteristics which make it a convenient and a simple distribution to use (Milwidsky and Mare, 2010). One such property is that it is specified by just two parameters (μ and σ^2).

However, this distribution does have its drawbacks in the field of financial return modeling: empirical research into the distribution of financial asset returns reveals the presence of leptokurtosis²⁶ (Aas and Haff, 2006). What this means is that the normal distribution does not have fat enough tails - ie. it assigns too small a probability to extreme events than what is evidenced empirically. What this means for VaR calculations is that using the normal distribution is likely to underestimate the VaR at high confidence levels (Alexander, 2008). This is where the generalised error distribution (GED) and t-distribution come in.

²⁴ That is to say, of the order (1,1)

²⁵ See Gumedze, Stewart and Thiar (2010) for more information

²⁶ Leptokurtosis means a distribution has a higher peak & heavier tails than the normal distribution of the same variance (Alexander, 2008)

Both these distributions can accommodate leptokurtosis. The GED is still symmetric but has higher kurtosis (in fact, the normal distribution is a special case of the GED²⁷). The cost of this benefit is that there is an additional parameter (shape) to estimate. In much the same vein, the t-distribution has higher kurtosis than the normal distribution and should therefore theoretically better estimate VaR at high confidence levels.²⁸

A particular shortfall of both the GED and t-distribution is the inability to accommodate skewness. Skewness is often a feature of asset returns: negative skewness is often seen in equity returns while positive skewness is often seen in commodity returns (Alexander, 2008). To this end, the skewed t-distribution is used.²⁹ It offers both leptokurtosis and skewness and fits heavy-tailed data well (Aas et al., 2006). The skewed t-distribution is in the class of generalised hyperbolic distributions.³⁰

It is also worth noting at this point that Wilhelmsson's (2006) work on the Standard and Poor's 500 into the volatility forecasting abilities of GARCH models yielded the following conclusion: the leptokurtosis in the error distribution leads to significant improvements in volatility forecasts over the normal distribution and that allowing for skewness does not yield improved forecasting abilities. We shall see if a similar sentiment prevails in our VaR results for the JSE.

Aside from the GED, this selection of distributions is the same as those selected by Ng Cheong Vee et al. (2012), however the benefits of the GED over the normal distribution are more or less matched by the t-distribution.

VaR Methodology

Daily-VaR is calculated for a three-year period (2010-2012). VaR is calculated using three different confidence levels (99%, 97.5% and 95%). The methodology of how a VaR estimate is obtained for each approach is described below:

²⁷ See appendix (specifically, under Distributions)

²⁸ A high confidence level is equivalent to a low significance level (for example, a 1% significance level corresponds to a 99% confidence level)

²⁹ See appendix (under Distributions) and Aas et al. (2006) for more details

³⁰ See Dokov, Stoyanov and Rachev (2007) for more details (other representations of the skewed t-distribution are also available, see Aas et al. (2006))

Analytic VaR

Analytic VaR is predicated on assuming a conditional distributional form of the returns process. Once a distribution has been assumed,³¹ the α %-VaR (which translates to a $(1-\alpha)$ % level of confidence) is calculated as follows:

For a normal distribution, the formula is:

$$-\text{VaR}_t = c + \sigma_t \Phi^{-1}(\alpha)$$

Where Φ^{-1} is the inverse of the standard normal cumulative distribution function; c is the parameter from the mean equation (of which we would have a maximum-likelihood estimate); σ_t is the from the GARCH equation (the square-root of it, to be accurate). Note the minus sign in front of the VaR formula - this is because VaR is quoted as a positive number (Alexander, 2008)

The VaR formula for the GED is:

$$-\text{VaR}_t = c + \sigma_t \mathcal{F}^{-1}(\alpha; \theta)$$

Where \mathcal{F}^{-1} denotes the inverse of the generalised error cumulative distribution function and θ is a parameter vector - θ_1 is the scale parameter and θ_2 is the shape parameter. Given the variance of the GED, we have the following equation for the scale parameter θ_1 :

$$\theta_1 = \frac{\sigma_t \Gamma(1/\theta_2)}{\Gamma(3/\theta_2)}$$

Where, again, σ_t is from the GARCH equation.

For the t-distribution, the formula is as follows:³²

$$-\text{VaR}_t = c - \sqrt{\nu^{-1}(\nu-2)} t_{\nu}^{-1}(\alpha) \sigma_t$$

³¹ Which is a requisite step when using a GARCH model if maximum likelihood is used to estimate the parameters of the model

³² See Alexander (2008) for further details

Where ν denotes the degrees of freedom, t_{ν}^{-1} denotes the inverse of the standard t cumulative distribution function and the rest of the parameters are as defined above.

For the skewed t-distribution, there is no neat formula - the VaR estimate comes as a unique root of an equation which is lengthy to derive. For the sake of brevity, this paper omits these calculations and directs the reader to the paper by Dokov, Stoyanov and Rachev (2007) for a comprehensive derivation.

Historical VaR

Historical VaR is very simple in design: to calculate $\alpha\%$ daily-VaR, form an empirical distribution of daily returns and calculate the corresponding α -quantile. This approach requires a large amount of historical data as we require the tails of the empirical distribution to be adequately populated if we are to try extract a reasonably-accurate VaR estimate.³³ However the advantage of this approach is that no assumption about the distributional form of the returns is required.³⁴

The technique described above is known as the equally-weighted historical VaR. Its simplicity does have its drawbacks:

Because such a large amount of historical data spanning many years is needed, different market regimes will be represented in the empirical distribution, the problem with this is that the lower tail of the empirical distribution would generally be populated with recession/economic-downturn data - as the most volatile and negative returns are most likely to come from a period when the market was in a recession or economic slowdown. The problem with this is that if one is in a post-recession period, one would derive an estimate of VaR from essentially recession data. This would in all likelihood overestimate VaR.

Put another way, if markets are emerging from a volatile period and entering a tranquil one, VaR would most likely be overestimated. Similarly, VaR would be underestimated if the market was exiting a tranquil period and entering a volatile one. This is a significant drawback of the equally-weighted approach. This could help explain why there was such criticism of VaR after the latest financial crisis,

³³ Although fitting a Johnson distribution to the first four moments may reduce the burden of requiring a very large data set (see Alexander (2008) for details)

³⁴ In contrast to the analytic approach

especially if the majority of VaR models used are based on the equally-weighted approach (as the 2012 McKinsey Report suggests).

What is needed is a way to reflect more volatility in the tranquil returns as the market enters a volatile period and a way to downsize volatile returns as the market enters a tranquil one. What is needed is a volatility-weighting method:

Consider fitting a GARCH model to the set of empirical returns $\{r_t\}_{t=1}^T$ where T is the time at the end of the sample. This would yield a set of corresponding volatility estimates $\{\hat{\sigma}_t\}_{t=1}^T$. One would then base the VaR calculations on the distribution of volatility-adjusted returns which are defined as follows:

$$\tilde{r}_{t,T} = \left(\frac{\hat{\sigma}_T}{\hat{\sigma}_t} \right) r_t$$

Now, there is a mechanism to account for the aforementioned problem: if the market has exited a volatile period and entered a tranquil one,³⁵ the $\hat{\sigma}_t$ values from the volatile period should be relatively large and the $\hat{\sigma}_T$ value should be relatively small given that the market is now in a tranquil period. The adjusted returns would then downsize the volatile returns to more accurately represent a more tranquil market regime. Of course the opposite would happen to tranquil returns if there was currently volatility in the market.

Using GARCH-adjusted historical VaR means that a conditional distributional form of the returns process must be assumed.³⁶ In doing so, the feature that the historical-VaR method is distribution-less is lost, however, the use of a volatility-adjustment has a clear upside.

Despite augmentations like volatility-adjustment, what is always implicit with the historical approach is that only past data is needed to predict the future - this may not always be a correct/accurate assumption.

³⁵ Note this is the situation which prevailed in the FTSE/JSE between 2008-2009 (volatile) leading in to 2010-2012 (less volatile) – see Figure 2 (log returns graph) in the Data section

³⁶ As maximum likelihood is used to get the GARCH parameter estimates

The Practical Nitty-Gritty

Analytic VaR is based on a seven-year moving window,³⁷ but when excluding 2008 and 2009, the length of the window changes to five years. The rationale behind this was to be sure not to introduce new data into the excluded model so that the effect of excluding the two years of crisis data can be isolated. McMillan et al. (2010) made use of a five-year window in their research.

For historical VaR, the window is based on an expanding window using the entire length of the time series. The same applies when the crisis data is removed (ie. when 2008 & 2009 are pulled from the data set, an expanding window is still used).

The models are re-estimated daily. This may be a bit excessive and may be unreasonable to use in the real world – the Basel Accord requires the model be re-estimated only every sixty trading days (Thupagayale, 2010). A rebuttal to this would be to look at this paper as an academic exercise and acknowledge that VaR is being tested under the best case scenario where the parameters estimates themselves are incorporating the latest patterns and information emerging from the latest data (as opposed to having new data pushed into an 'old' model).

A Word on Monte Carlo

Monte Carlo is the third approach available to use in VaR estimation. It is similar in requirements to the analytic approach in that a distributional form must be assumed.³⁸ Monte Carlo is especially useful in a multivariate setting as well as when calculating VaR for option portfolios (non-linear portfolios).³⁹

However, Monte Carlo does not offer much over the analytic approach in this paper for the following reasons:

- Data used is univariate
- VaR is estimated only over a 1-day horizon. If GARCH is being used in an analytic VaR setting over a horizon longer than 1-day, complications do arise - namely that the square-root of time

³⁷ to estimate the GARCH parameters and distribution parameters

³⁸ This then allows simulations to take place and thus the creation of a simulated returns distribution from which the quantiles can be retrieved in order to calculate a VaR estimate at whatever desired significance level

³⁹ See Alexander (2008) for further details

rule no longer applies to scale VaR from a 1-day horizon to whatever desired horizon. This is because a GARCH model introduces dependence⁴⁰ amongst the returns, whereas as the square-root of time rule is predicated on the assumption that the returns are i.i.d.⁴¹ However, such complexities can be avoided by dealing with daily-VaR only

For further information, the paper by Milwidsky et al. (2010) investigates the performance of Monte Carlo VaR on the JSE.

Backtesting Methodology

With VaR, one is simply calculating a quantile of a distribution – whether in the case of analytic VaR, where one works directly with the assumed distribution of tomorrow’s return or in the case of historical VaR, where it is assumed that tomorrow’s return is adequately represented by the empirical distribution of past daily returns. What this means is that for the α %-VaR (corresponding to the α -quantile), there exists a probability α that tomorrow’s return is actually less than the corresponding VaR estimate. When this happens, it is called a VaR failure.⁴² Extrapolating this to a set of n α %-VaR estimates, one expects $n\alpha$ failures. This line of thought leads to the unconditional coverage (UC) test developed by Kupiec (1995).

The UC test has as its null hypothesis that the α %-VaR model is accurate in the sense that it produces a total number of failures which is ‘close enough’ to the expected number (ie. unconditional coverage). The test is a likelihood ratio test and has the following test statistic:

$$LR_{UC} = \frac{\pi_{\text{exp}}^{n_1} (1 - \pi_{\text{exp}})^{n_0}}{\pi_{\text{obs}}^{n_1} (1 - \pi_{\text{obs}})^{n_0}}$$

Where, given a set of n VaR estimates, n_1 is the observed number of failures; $n_0 = n - n_1$ is the observed number of non-failures; π_{exp} and π_{obs} denote the expected and observed proportion of failures respectively (calculated as $n\alpha$ and n_1 / n respectively). The asymptotic distribution of $-2\ln LR_{UC}$ is chi-

⁴⁰ The dependence is second-order

⁴¹ See Alexander (2008) for further details

⁴² It is also known as a violation, exceedance or hit

squared with one degree of freedom. If a tested model leads to a rejection of the null of the UC test, one would conclude that the model is potentially misspecified and inadequate.

A feature important in backtesting, which is not covered by the UC test, is whether VaR failures are independent or whether they come in groups. According to Alexander (2008), clustering of failures is indicative that the VaR model is not adequately responsive to changes in market conditions. For example, if the market leaving one regime and entering another, leads to a string of VaR failures, it suggests there is evidence that the model being used⁴³ is not handling the changing market conditions.

The Christoffersen (1998) test tests the independence of the VaR failures. The test statistic takes the following form:

$$LR_{Ind} = \frac{\pi_{obs}^{n_1} (1 - \pi_{obs})^{n_0}}{\pi_{01}^{n_{01}} (1 - \pi_{01})^{n_{00}} \pi_{11}^{n_{11}} (1 - \pi_{11})^{n_{10}}}$$

Where n_{01} is the number of times a non-failure was followed by a failure; n_{11} is the number of times a failure was followed by a failure (n_{00} and n_{10} are similarly defined). π_{01} is the proportion of failures given that the previous return was a non-failure and π_{11} is the proportion of failures given that the previous return was also a failure - they are both, respectively, defined as follows:

$$\pi_{01} = \frac{n_{01}}{n_{01} + n_{00}} \quad \text{and} \quad \pi_{11} = \frac{n_{11}}{n_{11} + n_{10}}$$

The asymptotic distribution of $-2\ln LR_{Ind}$ is chi-squared with one degree of freedom. The null hypothesis is that the VaR failures occur independently.

The Christofferen independence test does have a drawback in that it only views a dependence relationship as existing if VaR failures occur consecutively - that is to say, a string of uninterrupted failures. This is a problem as failures may still be dependent even if there is a non-failure in between them (Alexander, 2008). Alexander (2008) provides further insight into this problem: the Christofferen test is based on a first-order Markov chain, where what is needed is a higher-order Markov chain if dependencies amongst failures is to be detected even if non-failures occur in between.

⁴³ Perhaps it is an Skewed-t IGARCH Analytic VaR model, for example

However, despite this drawback it is still a widely employed test. Ng Cheong Vee et al. (2012) make use of these two tests as well as the conditional coverage test. The conditional coverage test merges the UC and independence test statistics to form a joint test of unconditional coverage and independence. The reason the conditional coverage test is omitted from this paper is that it is possible that the model passes the joint CC test while still failing either the unconditional coverage or independence tests (Alexander, 2008) - which means one ultimately still needs the results of the independence and UC test in order to get the full picture of what these tests are saying. For this reason, the CC test is omitted from this paper.

The tests described above all benefit from a longer backtest period - this gives the tests more power⁴⁴. This paper uses a three-year backtest period (roughly 750 data points), this is better than the two-year period used by NG Cheong Vee et al. (2012) and the one-year period used by Milwidsky et al. (2010), however it is well short of the seven-year backtest period used by Thupayagle (2010).⁴⁵

Results

This section is about understanding the results of backtesting the various VaR models. Four tables have been produced: two for analytic and two for historical, with the difference between the two tables being one has the 2008 and 2009 data excluded while the other does not. For sakes of brevity, the analytic and historical tables detailing the results when 2008 and 2009 have been excluded, are placed in the appendix⁴⁶ - however the pertinent deductions from comparing the tables will be mentioned here. The table detailing the results for analytic VaR based on the unaltered data set follows:⁴⁷

⁴⁴ Power is the probability that the test will reject a false null hypothesis

⁴⁵ The reason the backtest period is only three years is because of the secondary purpose of this paper to eliminate 2008 and 2009 (volatile data) from the data and re-estimate the VaR models and compare the benefits of eliminating the data versus not eliminating it. This then only left 2010-2012 available for backtesting

⁴⁶ Found in the appendix under Table 5 & Table 7 respectively

⁴⁷ A table reporting the p-values for the tests can be found in the appendix (Table 3)

Table 1:
GARCH Daily-Analytic VaR Results
(Backtest Period: 2010-2012)

| (1- α)-VaR | | 99% | | | 97.5% | | | 95% | | |
|-------------------------------|--------------|--------------|----|-----|--------------|----|-----|--------------|----|-----|
| Volatility Model | Distribution | Failure Rate | UC | Ind | Failure Rate | UC | Ind | Failure Rate | UC | Ind |
| EWMA ($\lambda=0.94$) | Normal | 0.021 | ✗ | ✓ | 0.040 | ✗ | ✓ | 0.067 | ✗ | ✓ |
| | GED | 0.016 | ✓ | ✓ | 0.039 | ✗ | ✓ | 0.067 | ✗ | ✓ |
| | t | 0.016 | ✓ | ✓ | 0.039 | ✗ | ✓ | 0.068 | ✗ | ✓ |
| | Skewed-t | 0.015 | ✓ | ✓ | 0.033 | ✓ | ✓ | 0.065 | ✓ | ✓ |
| GARCH(1,1) | Normal | 0.013 | ✓ | ✓ | 0.028 | ✓ | ✓ | 0.061 | ✓ | ✓ |
| | GED | 0.012 | ✓ | ✓ | 0.027 | ✓ | ✓ | 0.061 | ✓ | ✓ |
| | t | 0.012 | ✓ | ✓ | 0.027 | ✓ | ✓ | 0.065 | ✓ | ✓ |
| | Skewed-t | 0.011 | ✓ | ✓ | 0.025 | ✓ | ✓ | 0.056 | ✓ | ✓ |
| IGARCH(1,1) | Normal | 0.012 | ✓ | ✓ | 0.028 | ✓ | ✓ | 0.064 | ✓ | ✓ |
| | GED | 0.012 | ✓ | ✓ | 0.027 | ✓ | ✓ | 0.064 | ✓ | ✓ |
| | t | 0.012 | ✓ | ✓ | 0.027 | ✓ | ✓ | 0.067 | ✗ | ✓ |
| | Skewed-t | 0.011 | ✓ | ✓ | 0.023 | ✓ | ✓ | 0.055 | ✓ | ✓ |
| EGARCH(1,1) | Normal | 0.012 | ✓ | ✓ | 0.023 | ✓ | ✓ | 0.048 | ✓ | ✓ |
| | GED | 0.011 | ✓ | ✓ | 0.023 | ✓ | ✓ | 0.049 | ✓ | ✓ |
| | t | 0.011 | ✓ | ✓ | 0.023 | ✓ | ✓ | 0.049 | ✓ | ✓ |
| | Skewed-t | 0.007 | ✓ | ✓ | 0.021 | ✓ | ✓ | 0.039 | ✓ | ✓ |
| GJR-GARCH(1,1) | Normal | 0.013 | ✓ | ✓ | 0.025 | ✓ | ✓ | 0.051 | ✓ | ✓ |
| | GED | 0.013 | ✓ | ✓ | 0.025 | ✓ | ✓ | 0.051 | ✓ | ✓ |
| | t | 0.012 | ✓ | ✓ | 0.025 | ✓ | ✓ | 0.051 | ✓ | ✓ |
| | Skewed-t | 0.008 | ✓ | ✓ | 0.021 | ✓ | ✓ | 0.036 | ✓ | ✓ |
| FIGARCH(1,1) | Normal | 0.013 | ✓ | ✓ | 0.031 | ✓ | ✓ | 0.061 | ✓ | ✓ |
| | GED | 0.011 | ✓ | ✓ | 0.029 | ✓ | ✓ | 0.063 | ✓ | ✓ |
| | t | 0.011 | ✓ | ✓ | 0.029 | ✓ | ✓ | 0.061 | ✓ | ✓ |
| | Skewed-t | 0.011 | ✓ | ✓ | 0.023 | ✓ | ✓ | 0.056 | ✓ | ✓ |
| FIEGARCH(1,1) | Normal | 0.012 | ✓ | ✓ | 0.020 | ✓ | ✓ | 0.049 | ✓ | ✓ |
| | GED | 0.013 | ✓ | ✓ | 0.028 | ✓ | ✓ | 0.057 | ✓ | ✓ |
| | t | 0.012 | ✓ | ✓ | 0.029 | ✓ | ✓ | 0.059 | ✓ | ✓ |
| | Skewed-t | 0.011 | ✓ | ✓ | 0.023 | ✓ | ✓ | 0.049 | ✓ | ✓ |
| Expected Failure Rate: | | 0.010 | | | 0.025 | | | 0.050 | | |

Sample Size: 750

*All tests conducted at the 95% confidence level

UC: Unconditional Coverage Test (H0: Correct Number of Failures)

✓ : Fail to Reject H0

Ind: Independence test (H0: VaR Failures are Independent)

✗ : Reject H0

What is clear from this is that the GARCH VaR models perform well across the spectrum of distributions - with all, bar one exception, passing the coverage and independence tests. The EWMA model suffers with achieving an acceptable number of failures (three rejections of the UC test in the 97.5% and 95%-VaR each, with only a single rejection for the normal distribution in the 99%-VaR category).

There is a rejection of the coverage test for the t-IGARCH model for 5%-VaR. However, one would be hesitant to label this model as insufficient as the reality is that it only produced two failures out of a

sample size of 750 more than the normal-IGARCH and GED-IGARCH models did - neither of which failed the coverage test.

It is important to bear in mind that the significance level chosen for the coverage and independence tests, determines which models fail and which do not. The tests in this paper make use of a 5% significance level - however the associated p-values are reported in the appendix if the reader wishes to select a different significance level for the tests. It must be noted that across all the tables, only a handful of models passed the coverage and independence tests with relatively low p-values (there are a few 6% p-values and a couple more with a roughly 9% p-value), the majority of p-values across both tests are relatively large with many having large double-digit percent p-values.

A point to mention whilst scrutinizing the tables is that an EWMA model with any distribution other than the normal distribution is an illogical pairing. The reason being that if any of the other three distributions are used, maximum likelihood is required to get the distributional parameter estimates (shape and skew parameters - where applicable). Therefore in that case, the EWMA model should rather be replaced with the IGARCH model so that maximum likelihood can also be used to get the conditional volatility parameter estimates (as opposed to choosing ad hoc values as with the EWMA model). However the EWMA model with non-normal distributions is still included in the tables in order to see whether it does result in a model which is empirically inferior to the IGARCH.⁴⁸ The results so far indicate that the EWMA model is evidently inferior to the IGARCH model.

The historical VaR table based on the unaltered data set follows:

⁴⁸ See appendix (Figure 5) for a graph of the IGARCH β parameter versus the EWMA value of 0.94

Table 2:
GARCH Daily-Historical VaR Results
(Backtest Period: 2010-2012)

| (1- α)-VaR | | 99% | | | 97.5% | | | 95% | | |
|-------------------------------|--------------|--------------|----|-----|--------------|----|-----|--------------|----|-----|
| Model | Distribution | Failure Rate | UC | Ind | Failure Rate | UC | Ind | Failure Rate | UC | Ind |
| Equally-Weighted | - | 0.000 | ✗ | - | 0.009 | ✗ | ✓ | 0.031 | ✗ | ✓ |
| EWMA ($\lambda=0.94$) | Normal | 0.011 | ✓ | ✓ | 0.028 | ✓ | ✓ | 0.051 | ✓ | ✓ |
| | GED | 0.011 | ✓ | ✓ | 0.028 | ✓ | ✓ | 0.051 | ✓ | ✓ |
| | t | 0.011 | ✓ | ✓ | 0.028 | ✓ | ✓ | 0.051 | ✓ | ✓ |
| | Skewed-t | 0.011 | ✓ | ✓ | 0.028 | ✓ | ✓ | 0.051 | ✓ | ✓ |
| GARCH(1,1) | Normal | 0.008 | ✓ | ✓ | 0.024 | ✓ | ✓ | 0.048 | ✓ | ✓ |
| | GED | 0.008 | ✓ | ✓ | 0.023 | ✓ | ✓ | 0.048 | ✓ | ✓ |
| | t | 0.008 | ✓ | ✓ | 0.021 | ✓ | ✓ | 0.048 | ✓ | ✓ |
| | Skewed-t | 0.008 | ✓ | ✓ | 0.021 | ✓ | ✓ | 0.048 | ✓ | ✓ |
| IGARCH(1,1) | Normal | 0.009 | ✓ | ✓ | 0.023 | ✓ | ✓ | 0.052 | ✓ | ✓ |
| | GED | 0.009 | ✓ | ✓ | 0.021 | ✓ | ✓ | 0.052 | ✓ | ✓ |
| | t | 0.009 | ✓ | ✓ | 0.021 | ✓ | ✓ | 0.051 | ✓ | ✓ |
| | Skewed-t | 0.009 | ✓ | ✓ | 0.024 | ✓ | ✓ | 0.052 | ✓ | ✓ |
| EGARCH(1,1) | Normal | 0.007 | ✓ | ✓ | 0.023 | ✓ | ✓ | 0.053 | ✓ | ✓ |
| | GED | 0.007 | ✓ | ✓ | 0.023 | ✓ | ✓ | 0.053 | ✓ | ✓ |
| | t | 0.007 | ✓ | ✓ | 0.019 | ✓ | ✓ | 0.053 | ✓ | ✓ |
| | Skewed-t | 0.007 | ✓ | ✓ | 0.021 | ✓ | ✓ | 0.053 | ✓ | ✓ |
| GJR-GARCH(1,1) | Normal | 0.007 | ✓ | ✓ | 0.020 | ✓ | ✓ | 0.048 | ✓ | ✓ |
| | GED | 0.007 | ✓ | ✓ | 0.020 | ✓ | ✓ | 0.048 | ✓ | ✓ |
| | t | 0.007 | ✓ | ✓ | 0.019 | ✓ | ✓ | 0.049 | ✓ | ✓ |
| | Skewed-t | 0.007 | ✓ | ✓ | 0.020 | ✓ | ✓ | 0.049 | ✓ | ✓ |
| FIGARCH(1,1) | Normal | 0.009 | ✓ | ✓ | 0.027 | ✓ | ✓ | 0.053 | ✓ | ✓ |
| | GED | 0.009 | ✓ | ✓ | 0.028 | ✓ | ✓ | 0.053 | ✓ | ✓ |
| | t | 0.009 | ✓ | ✓ | 0.027 | ✓ | ✓ | 0.051 | ✓ | ✓ |
| | Skewed-t | 0.009 | ✓ | ✓ | 0.027 | ✓ | ✓ | 0.052 | ✓ | ✓ |
| FIEGARCH(1,1) | Normal | 0.009 | ✓ | ✓ | 0.027 | ✓ | ✓ | 0.056 | ✓ | ✓ |
| | GED | 0.009 | ✓ | ✓ | 0.028 | ✓ | ✓ | 0.057 | ✓ | ✓ |
| | t | 0.009 | ✓ | ✓ | 0.027 | ✓ | ✓ | 0.056 | ✓ | ✓ |
| | Skewed-t | 0.009 | ✓ | ✓ | 0.029 | ✓ | ✓ | 0.063 | ✓ | ✓ |
| Expected Failure Rate: | | 0.010 | | | 0.025 | | | 0.050 | | |

Sample Size: 750

UC: Unconditional Coverage Test (H0: Correct Number of Failures)

Ind: Independence test (H0: VaR Failures are Independent)

*All tests conducted at the 95% confidence level

✓ : Fail to Reject H0

✗ : Reject H0

The standout feature of the historical VaR table is that the equally-weighted VaR model failed the coverage tests across all three significance levels (with zero observed failures for 1%-VaR). The poor results for this model is consistent with the backlash and criticisms leveled at VaR after the latest financial crisis - especially when taking into account the 2012 McKinsey Report which suggests the

equally-weighted historical approach is the most popular. This result suggests stability has yet to return to the equally-weighted historical model.

The GARCH models performed well across all confidence levels and all distributions without a single failure of the coverage or independence tests. The same can be said for the EWMA model, which is surprising given its poor performance in analytic VaR. It seems the volatility weighting the EWMA model provides is sufficient in an historical context, but when using the EWMA volatility estimate with a quantile of a chosen distribution - as in the analytic context - something does go awry (for this sample at least).

The historical EWMA model also has the feature that its failure rates are constant across the distributions for each of the three significance levels used to calculate VaR. It seems, despite what distributional choice is made for the EWMA model, that the results are very similar.

A standout feature which is presented by both sets of VaR approaches, is that despite the documented limitations of the normal distribution, it still performs well in a GARCH setting. Its failure rates across the three confidence levels for both analytic and historical VaR are very similar to the GED - this despite the GED having parameter estimates which indicate leptokurtosis.⁴⁹ What this means is that the normal distribution's tail-modelling abilities, for this data and in the VaR context at least, is perhaps not as inferior as one may have *a priori* expected.

This finding concerning the normal distribution is consistent with the decision made by McMillan et al. (2010) to only use the normal distribution in their analytic-VaR research on the FTSE/JSE. Their supporting reason was the finding by Jorion (1995) that a parametric normal approach may still be preferable even when evidence suggests normality does not hold.

It is also interesting to note that in the historical approach for 1%-VaR, each GARCH model has the same observed failure rates across the four distributions.⁵⁰ Perhaps a point of concern is that these rates are all less than the expected rate of 0.01. The IGARCH, FIGARCH and FIEGARCH are the closest with an observed failure rate of 0.009 whilst the EGARCH and GJR-GARCH have the lowest rate of 0.007 each. The fact that the failure rates are constant across the distributions, may then potentially suggest that the GARCH model specification is the problematic factor in causing the consistently lower-than-expected

⁴⁹ See appendix (Figure 6) for a graph illustrating this

⁵⁰ This is not the case for the other confidence levels, however, despite not being the same, the rates are still in a tight grouping

failure rates. Perhaps the long-memory property of the FIGARCH and FIEGARCH - as well as the greater persistence/longer-memory aspect of the IGARCH model - are performing better than the short-memory asymmetric models in this case. However, such deductions need to be balanced with the fact that there were no failures in terms of the coverage and independence tests - which suggests sufficiency of the GARCH models with these distributions - but again these results are a function of the power of these tests, which would be higher had a larger backtest period been used.

In contrast, the EWMA model was the only model to yield a failure rate greater than expected with a rate of .011 for 1% historical VaR. The fact that that the EWMA model fails more often for this sample than the GJR-GARCH model, for example, is not necessarily a mark against this model as its failure rate is closer to the expected rate meaning it is closer to doing what VaR is meant to do which is to fail 1% of the time (for the 99% confidence level) - for this sample of course.

For 97.5% historical VaR, the FIGARCH and FIEGARCH are the only GARCH models to have failure rates greater the 0.025 expected rate. This is probably the result of the long-memory property of the models but to draw any greater conclusions than this is difficult.

To compare the above two tables with the two tables in the appendix which have the 2008 and 2009 data removed, the following points stand out:

- For the analytic table, there is still a failure of the coverage test for the EWMA normal model for the 1%-VaR. The remaining distributions used in the EWMA model are seemingly sufficient given that they pass the coverage and independence tests. As far as the GARCH models are concerned, the results are much the same across both tables (no failures for either test). This perhaps suggests that the EWMA volatility model is more sensitive to the data it is fed than GARCH volatility models
- For the historical table, the biggest result is that the equally-weighted model passes the coverage and independence tests once 2008 and 2009 have been pulled from the data. This is quite a promising result for the posit that financial crisis and/or high volatility data should not be used in a post-crisis/post-volatility market. However, there is little difference between the historical tables as far as the GARCH models are concerned: the GARCH models across all the distributions still have lower observed failure rates than expected for 1% historical VaR

(however they are still all deemed as sufficient by the coverage and independence tests); the results for 2.5% and 5%-VaR are much the same across the two tables

- Another point to mention is that all-in-all, the historical VaR table with 2008 and 2009 excluded is the only table to have no rejections of the null hypotheses of the coverage and independence tests for any combination of volatility model and distribution (and this also applies to the equally-weighted method)

What can also be said by looking at all four tables is that the fractionally integrated GARCH models do not yield noticeably different or improved results. This can also be extended to all the GARCH models analysed (whether short or long memory) in that the extensions to the basic symmetric GARCH model have not resulted in clearly improved models: in no particular circumstances was the symmetric model outperformed by another model.

Looking across the four tables, the independence test did not fail any model. The fact that the independence test only sees consecutive VaR failures as a sign of dependence probably plays a part in this. This obviously limits how much information this test can really provide.

Looking at all the tables it is hard to try to find a way to rank the various GARCH models and distributions and to try to get an idea whether analytic fares better or worse overall than historical. One can use how close the model's observed failure rates are to the expected failure rates as a way to try rank model performance, however, one must remember that this is just the results for this particular set of data. The more important measure is whether it failed the coverage test or not and - despite the limitations of it - the independence test.

Given that the performance of the various GARCH models are all very similar, this paper borrows the ranking criteria used by McMillan et al. (2010) and Thupayagale (2010) of selecting the preferred model based on minimal number of failures at a given confidence level conditional that the model passed the coverage and independence tests. Using this criteria, the following models are preferred:

Table 3:
Preferred VaR Models Based
On Smallest Failure Rates

| $\alpha\%$ | 99% | 97.5% | 95% |
|------------|---------------------|---------------------------------|-----------------------------|
| Analytic | skw-t EGARCH | skw-t EGARCH skw-t GJR-GARCH | skw-t GJR-GARCH |
| Analytic* | skw-t EGARCH | skw-t EGARCH | skw-t EGARCH |
| Historic | EGARCH GJR-GARCH | t EGARCH t GJR-GARCH | GARCH Norm/GED GJR-GARCH |
| Historic* | Equally-Weighted | Equally-Weighted | Equally-Weighted |

*Denotes when 2008 & 2009 was removed from the data

Using this criteria, the skew-t distribution used in a short-memory asymmetric model, produces the most preferable results for analytic VaR. For historical VaR, it is not as clear cut: when using the unaltered data set, short-memory asymmetric models do dominate the results but no clear distribution emerges; for the altered data set, the equally-weighted method is the preferred model according to this criteria. Something to bear in mind concerning this result is that given the equally-weighted historical method's poor performance when using the unaltered data set, it seems this method is very sensitive as to what data it is fed in order to estimate VaR - as a result, one would be hesitant to label this model as preferred if the data this model is fed is not scrutinised/selectfully-chosen beforehand.

Conclusion

The overall conclusion is that GARCH-based VaR is superior to the simpler EWMA model for analytic VaR, however the same cannot be readily said for historical VaR. As far as GARCH models are concerned, there does not seem to be much in the way of results to separate the six GARCH models used. What this suggests is that the volatility clustering mechanism inherent in every GARCH model yields a sufficient VaR model and the refinements and extensions proposed to Bollerslev's initial symmetric model may not offer more practically despite their theoretical advantages. The same applies to the fractionally integrated models, which also did not yield a clear empirical benefit. This result is consistent with the finding of Degiannakis et al. (2010) that long-memory volatility models do not appear to increase VaR forecasting accuracy in relation to short-memory GARCH models.

A very similar sentiment applies to the normal distribution. Despite its documented shortfalls in tail-modeling; in this paper, there was scant evidence against selecting it to use in a GARCH model. The other three distributions used (GED, t, skewed-t) did not provide more accurate failure rates nor did they fare better in the coverage and independence tests. The normal distribution was the worst faring in the EWMA model in the analytic VaR (most failed null hypotheses), but the shortcomings of the EWMA model in analytic VaR have been noted and also yielded negative results for the other three distributions - although not to the same extent. Based on the evidence from this paper, the EWMA model is inferior to GARCH-based models for analytic VaR.

Ng Cheong Vee et al. (2010) reached a similar, but even stronger, conclusion in their GARCH analytic VaR research:⁵¹ their results indicated the normal distribution actually outperformed the t- and skewed t-distribution.

It is also worth remembering at this point that Wilhelmsson's (2006) conclusion that allowing for leptokurtosis yields improved forecasting abilities for GARCH models whilst allowing for skewness does not. The evidence provided by this paper only partially agrees with Wilhelmsson's finding in that allowing for skewness did not yield clearly superior results.⁵² The finding that can be taken from this paper is that, for this set of data, there is no clear preferred distribution to use in the GARCH models to estimate Value-at-Risk (for both analytic and historic) and the normal distribution is evidently a sufficient choice.

As far as the question of whether the downturn and subsequent volatile recovery data should be used in estimation in a post-downturn, lower-volatility market, there is some evidence supporting this: equally-weighted historical VaR which used data which included the volatile 2008 and 2009 data, was clearly an insufficient model - for the 1%-VaR it had an observed failure rate of zero and the model failed the coverage tests across the three VaR significance levels (1%, 2.5%, 5%). However, once the downturn/volatile data was removed, the results of the model were greatly improved so much so, that it passed all the coverage tests. In the analytic context, the performance of the EWMA model also improved once the 2008 and 2009 was removed.

Concerning the use of downturn/volatile data in a GARCH model, there seems to be no evidence to exclude it: the GARCH-based models fared equally well when the downturn/volatile data was used and

⁵¹ Which was based on emerging indices (but did not include the JSE)

⁵² However, the skewed t-distribution achieves the highest likelihood value for every fitted model - this is the case across all the GARCH models – see Figure 4 in the appendix

when it was not. The volatility-clustering mechanism in every GARCH model is seemingly sufficient to capture different market regimes.

This paper presented little evidence in the way of trying to separate analytic and historical VaR: both approaches are seemingly sufficient in GARCH-based models and yielded similar results. There is a clear difference though, when using the EWMA model: the evidence presented in this paper, suggests its use in analytic VaR is problematic whilst it is sufficient in a historical VaR context.

Some ideas for further research would be to investigate Markov-switch volatility models which can accurately handle different market regimes; one could also answer the question as to whether to include the downturn/volatile data or not, by using a larger backtest period. This would give the coverage and independence tests more power. A larger backtest period may also help separate which models (with which distributions) are better performing - at this stage, there is not a clear way of ranking the various GARCH models as they all performed well in the coverage and independence tests across both VaR approaches.

The McKinsey Report (2012) suggests that the use of equally-weighted historical VaR is widespread and this research has evidenced some of its deficiencies; this may well explain the backlash against VaR in recent times. It may very well be that many financial institutions are relying on an oversimplified version of VaR in the equally-weighted historical method. The adoption of GARCH-based VaR instead may yield more robust risk models as the evidence presented in this paper suggests stability has returned to such models.

Reference List

- Aas, K., Haff, I. (2006). The Generalised Hyperbolic Skew Student's t-distribution. *Journal of Financial Econometrics*, 4:2.
- Alexander, C. (2008). *Market Risk Analysis IV: Value-at Risk Models*. West Sussex: John Wiley & Sons.
- Alexander, C. (2008). *Market Risk Analysis II: Practical Financial Econometrics*. West Sussex: John Wiley & Sons.
- Baillie, R., Bollerslev, T., Mikkelsen, H. (1996). Fractionally Integrated Generalised Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics*, 74, 3-30.
- Bollerslev, T. (1986). Generalised Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics*, 31, 307-327.
- Bollerslev, T., Mikkelsen, H. (1996). Modeling And Pricing Long Memory In Stock market Volatility. *Journal of Econometrics*, 73, 151-184.
- Chen, J. (2013). Measuring Market Risk Under Basel II, 2.5, and III: VaR, Stressed VaR, and Expected Shortfall. Available at SSRN: <http://ssrn.com/abstract=2252463>. (July 2013).
- Degiannakis, S., Floros C., Dent, P. (2010). Forecasting Value-at-Risk (VaR) using Fractionally Integrated Models of Conditional Volatility. *International Review of Financial Analysis*, 27, 21-33.
- Dokov, S., Stoyanov, S., Rachev, S. (2007). Computing VaR and AVaR of Skewed-T Distribution. *Journal of Applied functional Analysis*, 3, 189-209.
- Dolmetsch, C. Subprime Collapse to Global Financial Meltdown. Available at: <http://www.bloomberg.com/apps/news?pid=newsarchive&sid=aleqkSjAAw10>. (August 2013).
- Enders, W. (2010). *Applied Econometric Time Series*. United States of America: John Wiley & Sons.
- Engle, R. (1982). Autoregressive Conditional Heteroskedasticity With Estimates of The Variance of UK Inflation. *Econometrica*, 50, 987-1010.

Gumedze, F., Stewart, T., Thiart, C. (2010). *Probability Theory and Statistical Inference (STA2004F)*. Department of Statistical Sciences of the University of Cape Town course notes.

Hull, J. (2012). *Options, Futures And Other Derivatives*. United Kingdom: Pearson Education Limited.

Jorion, P. (1995). Predicting Volatility In The Foreign Exchange Market. *Journal of Finance*, 50:2, 507-528.

Mandelbrot, B. (1963). The Variation of Certain Speculative Prices. *Chicago Journals*, 36:4, 394-419.

Mare, E., Milwidsky, C. (2010). Value-at-Risk In The South African Equity market: A View From The Tails. *South African Journal of Economic Management Sciences*, 13, 345-361.

McMillan, D. Thupayagale, P. (2010). Evaluating Stock Index Return Value-at-Risk Estimates in South Africa: Comparative Evidence for Symmetric, Asymmetric and Long Memory GARCH Models. *Journal of Emerging Market Finance*, 9, 325-345.

Mehta, A., Neukirchen, M., Pfetsch, S., Poppensieker, T. (2012). McKinsey Working Papers on Risk, Number 32. Available at: <http://www.mckinsey.com/Publications>. (July 2013).

Nelson, D. (1992). Conditional Heteroskedasticity In Asset Returns: A New Approach. *Econometrica*, 59, 347-370.

Ng Cheong Vee, D., Nunkoo Gonpot, P., Sookia, N. (2012). Assessing The Performance of Generalised Autoregressive Conditional Heteroskedasticity-based Value-at-Risk Models: A Case of Frontier Markets. *Journal of Risk Model Validation*, 6, 95-111.

Thupayagale, P. (2010). Evaluation of GARCH-based Models in Value-at-Risk Estimation: Evidence From Emerging Equity Markets. *Investment Analysts Journal*, 72, 13-29.

Wilhelmsson, A. (2006). GARCH Forecasting Performance Under Different Distribution Assumptions. *Journal of Forecasting*, 25, 561-578.

Appendix

Figure 3: VaR Plot

Below is a sample of a VaR plot (it happens to be for the skewed-t IGARCH model). The grey points denote the returns during the backtest period, the black line is the predicted daily-VaR at the 5% significance level and the red points are the VaR failures. This plot conveys the general idea behind VaR. One can also note the changes in the VaR estimates according to the changes in market data.

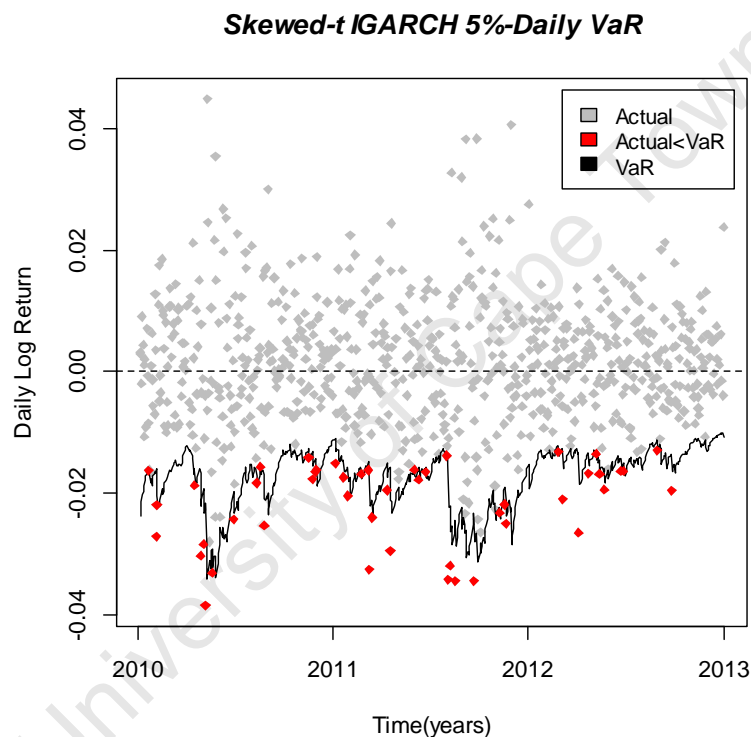
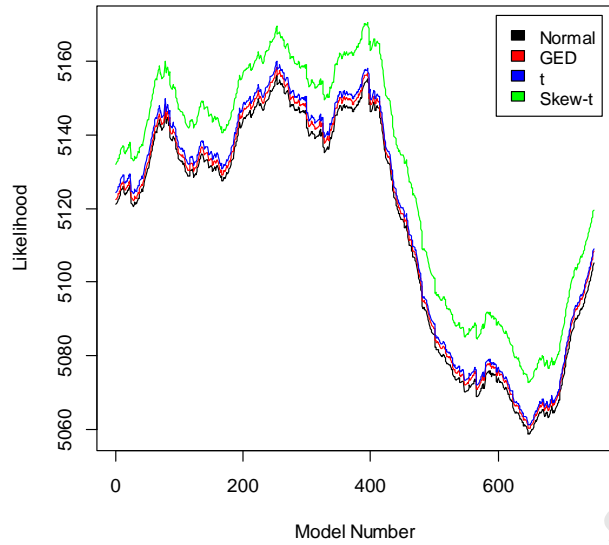


Figure 4: Likelihood Plot

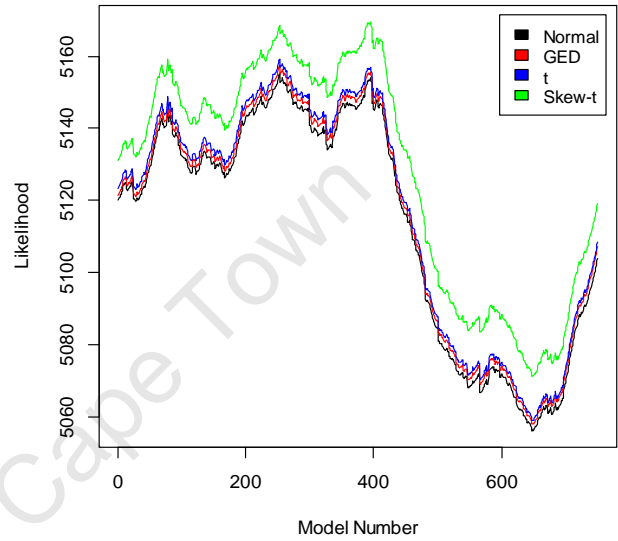
Below are four likelihood plots: one for each GARCH model. They plot the likelihoods of each distribution for each of the 750 estimated models (three-year backtest period with daily refitting). What is clear is that the skewed t-distribution achieves the highest likelihood value for each of the 750 estimated models across the four GARCH models – this suggests that allowing for skewness has merit. Having the highest likelihood means it fits the data the best, but this superiority is not so clear when analysing the VaR results. What can be seen from the graphs below is the drop in the likelihood values

across all four graphs at roughly the 400th estimated model - that translates to roughly the middle of 2011. Possible reasoning behind this is not immediately apparent but there is a spike in the volatility of returns roughly during this period - which can be seen on the returns graph (Figure 2, pg.5) - which can be responsible for the drop in likelihood.

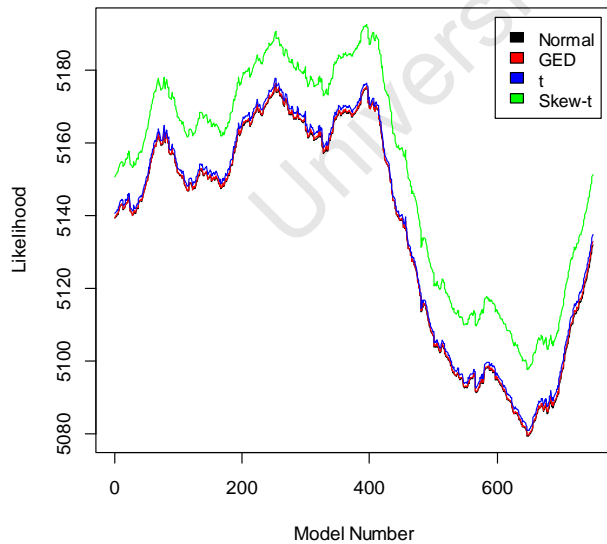
GARCH(1,1) Plot of Likelihoods



iGARCH(1,1) Plot of Likelihoods



eGARCH(1,1) Plot of Likelihoods



GJR-GARCH(1,1) Plot of Likelihoods

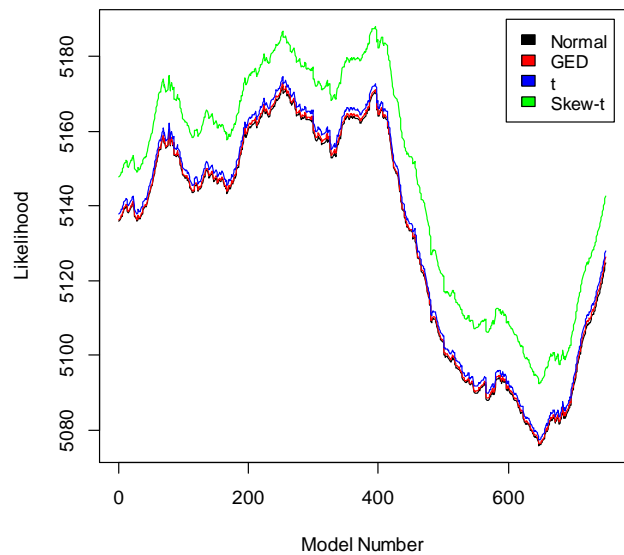


Figure 5: IGARCH Parameter

Below is a graph of the IGARCH β parameter. There is a dotted line at the value of 0.94 corresponding to the RiskMetrics-proposed EWMA value for this parameter. What is clear is that the EWMA value of 0.94 is not optimal in a maximum likelihood sense (based on this sample) but it is quite close. The IGARCH parameter values across the distributions are roughly between 0.88 and 0.91. It is also worth mentioning that the skewed-t consistently has the highest value for the parameter – this translates into having the highest persistence in the conditional volatility process.

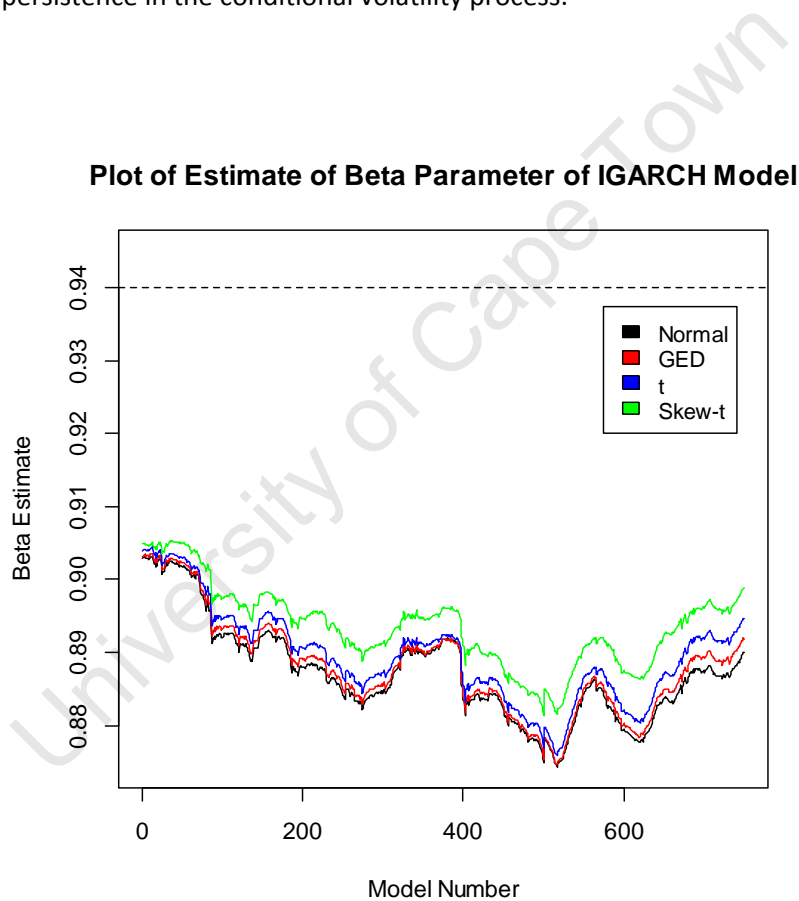


Figure 6: GED Shape Parameter

Below is a graph of the GED shape parameter for each GARCH model across the 750 estimated models. What is clear across the GARCH models, is that the shape parameter is always less than two. A value less than two indicates leptokurtosis – ie. greater kurtosis than the normal distribution. It is also something to note that the symmetric models (GARCH, IGARCH) have values closer to two than the asymmetric models (EGARCH, GJR-GARCH). But all values are within the 1.7 to 1.82 region. However the differences for VaR between using the normal distribution or the GED are not very apparent.

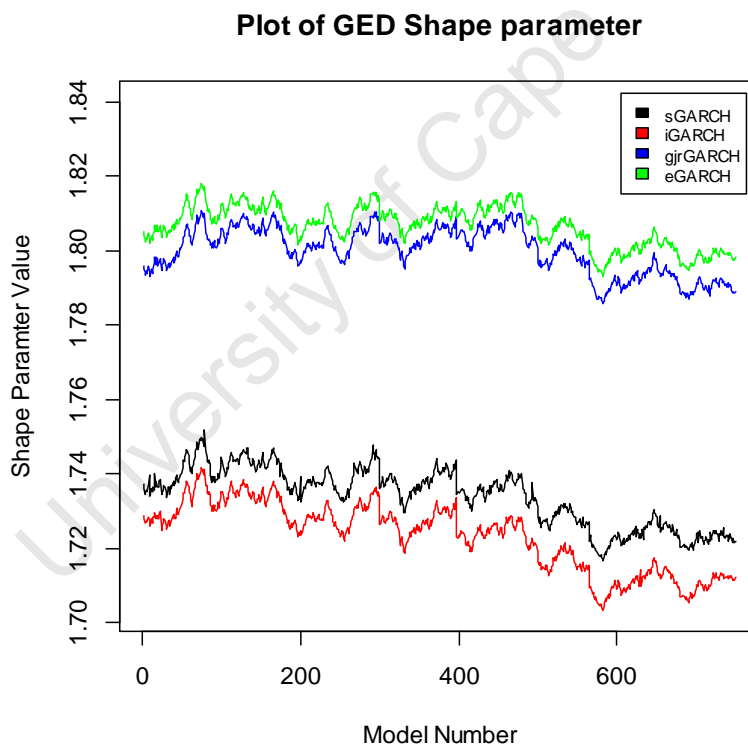
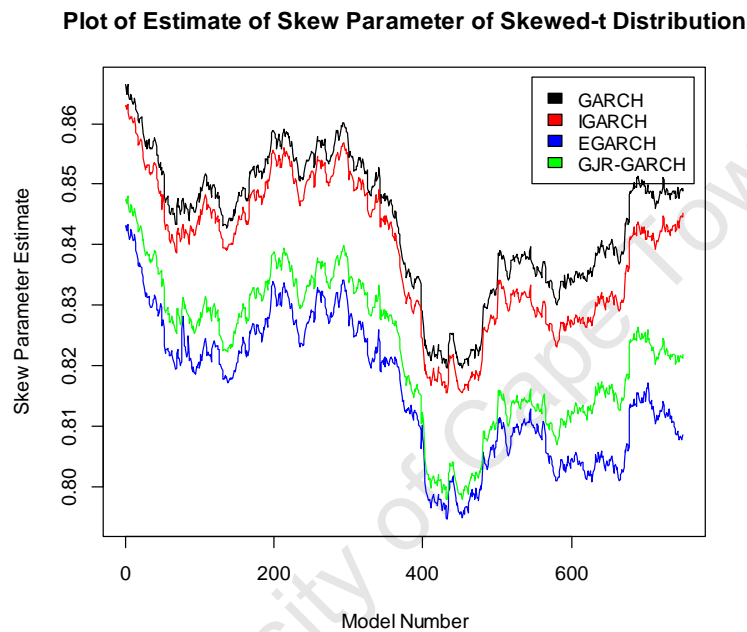


Figure 7: Skewed t-distribution Skew Parameter

Below is a graph of the skew parameter of the skewed t-distribution as it evolved across the 750 estimated models. Skew parameter values less than one correspond to negative skew (longer left tail).



All the estimated values are consistent with a negative skew. This is consistent with the statement by Alexander (2008) that equity returns generally have a left tail (especially for daily data). The values for all the models are in a narrow band of one another. However, the symmetric models achieve higher values (implying less skewness) than the asymmetric ones. The paths of the symmetric models are very close to another. Similarly, the paths of the asymmetric models are also close to another. However, there is a noticeable difference between symmetric and asymmetric models.

On the next page, there is a graph of the pdf for the skewed-t distribution for skew parameter values of 1 (which reduces to the standard, symmetric, t-distribution), 0.86 and 0.8 (which are roughly the values from the graph above). A slight left tail is apparent.

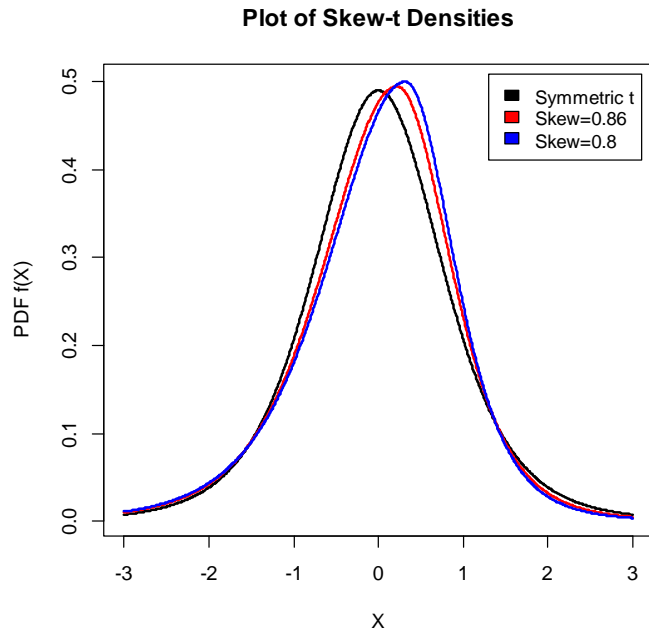
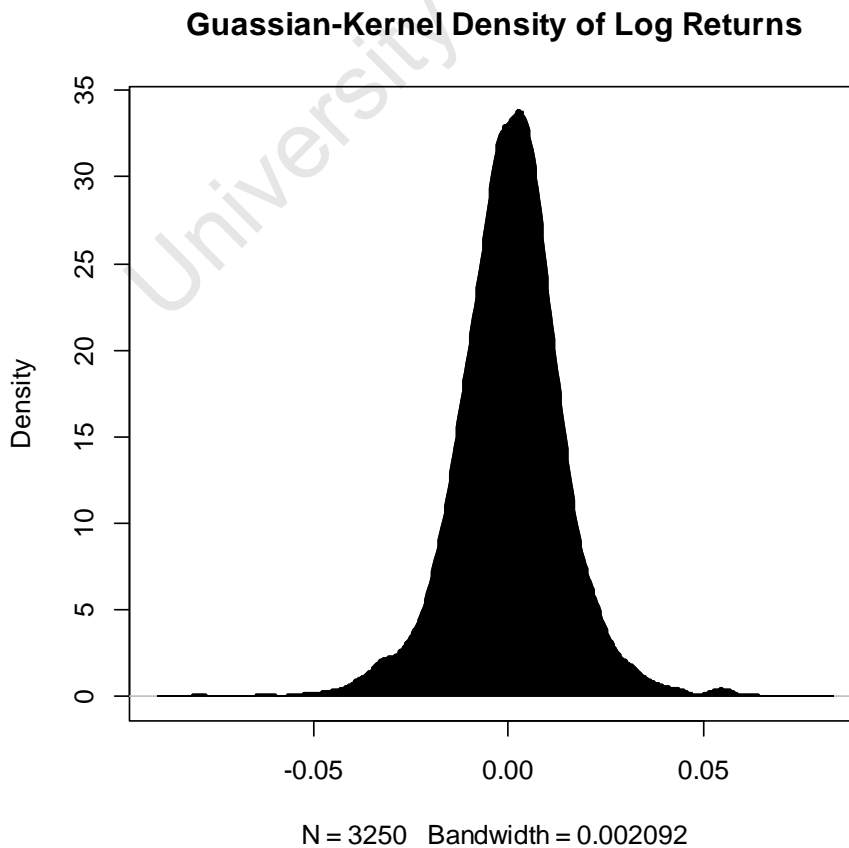


Figure 8: Log-Return Density Plot

Below is a density plot of the entire series of log returns from 2000 to 2012. The density is based on the Gaussian kernel (see Alexander (2008) for details). There definitely seems to be more density in the left tail than in the right - although the right tail does have a little clump of density at the end of it.



Distributions:

If a random variable X is normally distributed with mean μ and variance σ^2 then X has the following pdf:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$

If a random variable X has a generalized error distribution (GED) with μ, θ, δ as location, scale and shape parameters respectively, then X has the following pdf:

$$f(x) = \frac{\delta}{2\theta\Gamma(1/\delta)} \exp\left\{-\left(|x-\mu|/\theta\right)^\delta\right\}$$

If a random variable X has a t-distribution with ν degrees of freedom, then X has the following pdf:

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$

If a random variable X has a standardised generalised hyperbolic skewed t-distribution with ν degrees of freedom and skewness γ , then X has the following pdf (see Dokov et al., 2007 for mode details):

$$f(x) = \frac{\nu^\lambda \gamma^{2\lambda}}{\Gamma\left(\frac{\nu}{2}\right) \sqrt{\pi\nu} 2^{2\lambda}} \int_0^\infty t^{-\lambda-1} \exp\left\{-t - \frac{\nu\gamma^2}{4t} + \gamma x - \frac{(\gamma x)^2}{4t}\right\} dt$$

Where $\lambda = \frac{\nu+1}{2}$

To notice that when $\delta = 2$ the GED reduces to a normal distribution with mean μ and variance $\frac{\theta^2}{2}$,

one must have the result that $\Gamma(1/2) = \sqrt{\pi}$ which is proven below:

$$\Gamma(1/2) := \int_0^{\infty} t^{-0.5} e^{-t} dt$$

$$\text{let } z = t^{0.5}$$

$$\therefore \Gamma(1/2) = 2 \int_0^{\infty} e^{-z^2} dz$$

$$= \int_{-\infty}^{\infty} e^{-z^2} dz \text{ because its an even function}$$

$$\therefore (\Gamma(1/2))^2 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(y^2+z^2)} dy dz$$

Now change to polar coordinates:

$$\therefore (\Gamma(1/2))^2 = \int_0^{2\pi} \int_0^{\infty} e^{-r^2} r dr d\theta$$

$$= 2\pi * \left[-\frac{1}{2} e^{-r^2} \right]_{r=0}^{\infty}$$

$$= \pi(0 - (-1))$$

$$= \pi$$

$$\therefore \Gamma(1/2) = \sqrt{\pi} \quad \because \Gamma(1/2) > 0$$

University of Cape Town

FIGARCH Derivation:

To derive FIGARCH conditional volatility equation as shown in this paper, one must start off with the GARCH(p,q) model:

$$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2$$

where $\alpha(L)$ and $\beta(L)$ are lag operator polynomials of order q and p respectively. The equation is then modified as follows:

$$\begin{aligned}\sigma_t^2 &= \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2 \\ \Rightarrow [1 - \beta(L)]\sigma_t^2 &= \omega + \alpha(L)\varepsilon_t^2 \\ \Rightarrow [1 - \alpha(L) - \beta(L)]\varepsilon_t^2 &= \omega + [1 - \beta(L)](\varepsilon_t^2 - \sigma_t^2)\end{aligned}$$

Now for the case of the IGARCH, $[1 - \alpha(L) - \beta(L)]$ would have a unit root, which would yield the following representation:

$$\phi(L)(1-L)\varepsilon_t^2 = \omega + [1 - \beta(L)](\varepsilon_t^2 - \sigma_t^2)$$

Now the FIGARCH models extends the IGARCH model by replacing the term $(1-L)$ with the fractional differencing operator $(1-L)^d$ yielding the following:

$$\phi(L)(1-L)^d \varepsilon_t^2 = \omega + [1 - \beta(L)](\varepsilon_t^2 - \sigma_t^2)$$

This equation can then be manipulated to yield the equation seen in this paper:

$$\begin{aligned}\sigma_t^2 &= \omega[1 - \beta(L)]^{-1} + \{1 - [1 - \beta(L)]^{-1}\phi(L)(1-L)^d\}\varepsilon_t^2 \\ &\equiv \omega[1 - \beta(L)]^{-1} + \lambda(L)\varepsilon_t^2\end{aligned}$$

See Baillie et al. (1996) for further details. Similar steps are used to arrive at the FIEGARCH model from the EGARCH model (see Bollerslev et al. (1996) for details).

Table 4:
Extended GARCH Daily-Analytic VaR Results
(Backtest Period: 2010-2012)

| $\alpha\%$ -VaR | | 99% | | | | | 97.5% | | | | | 95% | | | | |
|-------------------------------|--------------|--------------|----|-------|-----|-------|--------------|----|-------|-----|-------|--------------|----|-------|-----|-------|
| Volatility Model | Distribution | Failure Rate | UC | p-val | Ind | p-val | Failure Rate | UC | p-val | Ind | p-val | Failure Rate | UC | p-val | Ind | p-val |
| EWMA ($\lambda=0.94$) | Normal | 0.021 | ✗ | 0.007 | ✓ | 0.382 | 0.040 | ✗ | 0.015 | ✓ | 0.515 | 0.067 | ✗ | 0.046 | ✓ | 0.702 |
| | GED | 0.016 | ✓ | 0.129 | ✓ | 0.505 | 0.039 | ✗ | 0.026 | ✓ | 0.436 | 0.067 | ✗ | 0.046 | ✓ | 0.702 |
| | t | 0.016 | ✓ | 0.129 | ✓ | 0.528 | 0.039 | ✗ | 0.026 | ✓ | 0.436 | 0.068 | ✗ | 0.032 | ✓ | 0.794 |
| | Skewed-t | 0.015 | ✓ | 0.230 | ✓ | 0.566 | 0.033 | ✓ | 0.164 | ✓ | 0.566 | 0.065 | ✓ | 0.065 | ✓ | 0.566 |
| GARCH(1,1) | Normal | 0.013 | ✓ | 0.383 | ✓ | 0.602 | 0.028 | ✓ | 0.606 | ✓ | 0.270 | 0.061 | ✓ | 0.168 | ✓ | 0.927 |
| | GED | 0.012 | ✓ | 0.594 | ✓ | 0.638 | 0.027 | ✓ | 0.772 | ✓ | 0.294 | 0.061 | ✓ | 0.168 | ✓ | 0.927 |
| | t | 0.012 | ✓ | 0.594 | ✓ | 0.638 | 0.027 | ✓ | 0.772 | ✓ | 0.294 | 0.065 | ✓ | 0.065 | ✓ | 0.646 |
| | Skewed-t | 0.011 | ✓ | 0.560 | ✓ | 0.677 | 0.025 | ✓ | 0.953 | ✓ | 0.320 | 0.056 | ✓ | 0.459 | ✓ | 0.667 |
| IGARCH(1,1) | Normal | 0.012 | ✓ | 0.594 | ✓ | 0.638 | 0.028 | ✓ | 0.606 | ✓ | 0.270 | 0.064 | ✓ | 0.091 | ✓ | 0.990 |
| | GED | 0.012 | ✓ | 0.594 | ✓ | 0.638 | 0.027 | ✓ | 0.772 | ✓ | 0.294 | 0.064 | ✓ | 0.091 | ✓ | 0.990 |
| | t | 0.012 | ✓ | 0.594 | ✓ | 0.638 | 0.027 | ✓ | 0.772 | ✓ | 0.294 | 0.067 | ✗ | 0.046 | ✓ | 0.817 |
| | Skewed-t | 0.011 | ✓ | 0.856 | ✓ | 0.677 | 0.023 | ✓ | 0.678 | ✓ | 0.374 | 0.055 | ✓ | 0.563 | ✓ | 0.610 |
| EGARCH(1,1) | Normal | 0.012 | ✓ | 0.594 | ✓ | 0.638 | 0.023 | ✓ | 0.678 | ✓ | 0.374 | 0.048 | ✓ | 0.800 | ✓ | 0.807 |
| | GED | 0.011 | ✓ | 0.856 | ✓ | 0.677 | 0.023 | ✓ | 0.678 | ✓ | 0.374 | 0.049 | ✓ | 0.933 | ✓ | 0.896 |
| | t | 0.011 | ✓ | 0.856 | ✓ | 0.677 | 0.023 | ✓ | 0.678 | ✓ | 0.374 | 0.049 | ✓ | 0.933 | ✓ | 0.896 |
| | Skewed-t | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.021 | ✓ | 0.510 | ✓ | 0.402 | 0.039 | ✓ | 0.139 | ✓ | 0.126 |
| GJR-GARCH(1,1) | Normal | 0.013 | ✓ | 0.383 | ✓ | 0.602 | 0.025 | ✓ | 0.953 | ✓ | 0.320 | 0.051 | ✓ | 0.933 | ✓ | 0.960 |
| | GED | 0.013 | ✓ | 0.383 | ✓ | 0.602 | 0.025 | ✓ | 0.953 | ✓ | 0.320 | 0.051 | ✓ | 0.933 | ✓ | 0.956 |
| | t | 0.012 | ✓ | 0.594 | ✓ | 0.638 | 0.025 | ✓ | 0.953 | ✓ | 0.320 | 0.051 | ✓ | 0.933 | ✓ | 0.956 |
| | Skewed-t | 0.008 | ✓ | 0.568 | ✓ | 0.757 | 0.021 | ✓ | 0.510 | ✓ | 0.402 | 0.036 | ✓ | 0.065 | ✓ | 0.154 |
| FIGARCH(1,1) | Normal | 0.013 | ✓ | 0.383 | ✓ | 0.602 | 0.031 | ✓ | 0.337 | ✓ | 0.187 | 0.061 | ✓ | 0.168 | ✓ | 0.482 |
| | GED | 0.011 | ✓ | 0.856 | ✓ | 0.677 | 0.029 | ✓ | 0.459 | ✓ | 0.158 | 0.063 | ✓ | 0.125 | ✓ | 0.533 |
| | t | 0.011 | ✓ | 0.856 | ✓ | 0.677 | 0.029 | ✓ | 0.459 | ✓ | 0.158 | 0.061 | ✓ | 0.168 | ✓ | 0.482 |
| | Skewed-t | 0.011 | ✓ | 0.856 | ✓ | 0.677 | 0.023 | ✓ | 0.678 | ✓ | 0.395 | 0.056 | ✓ | 0.459 | ✓ | 0.296 |
| FIEGARCH(1,1) | Normal | 0.012 | ✓ | 0.594 | ✓ | 0.638 | 0.020 | ✓ | 0.364 | ✓ | 0.299 | 0.049 | ✓ | 0.933 | ✓ | 0.896 |
| | GED | 0.013 | ✓ | 0.383 | ✓ | 0.602 | 0.028 | ✓ | 0.606 | ✓ | 0.131 | 0.057 | ✓ | 0.367 | ✓ | 0.127 |
| | t | 0.012 | ✓ | 0.594 | ✓ | 0.638 | 0.029 | ✓ | 0.459 | ✓ | 0.158 | 0.059 | ✓ | 0.289 | ✓ | 0.150 |
| | Skewed-t | 0.011 | ✓ | 0.856 | ✓ | 0.677 | 0.023 | ✓ | 0.678 | ✓ | 0.395 | 0.049 | ✓ | 0.933 | ✓ | 0.137 |
| Expected Failure Rate: | | 0.010 | | | | | 0.025 | | | | | 0.050 | | | | |

Sample Size: 750

UC: Unconditional Coverage Test (H0: Correct Number of Failures)

Ind: Independence test (H0: VaR Failures are Independent)

*All tests conducted at the 95% confidence level

✓ : Fail to Reject H0

✗ : Reject H0

Table 5:
Extended GARCH Daily-Analytic VaR Results (2008-2009 excluded)
(Backtest Period: 2010-2012)

| $\alpha\%$ -VaR | | 99% | | | | | 97.5% | | | | | 95% | | | | |
|-------------------------------|--------------|--------------|----|-------|-----|-------|--------------|----|-------|-----|-------|--------------|----|-------|-----|-------|
| Volatility Model | Distribution | Failure Rate | UC | p-val | Ind | p-val | Failure Rate | UC | p-val | Ind | p-val | Failure Rate | UC | p-val | Ind | p-val |
| EWMA | Normal | 0.021 | ✘ | 0.007 | ✓ | 0.382 | 0.035 | ✓ | 0.109 | ✓ | 0.172 | 0.063 | ✓ | 0.125 | ✓ | 0.966 |
| | GED | 0.015 | ✓ | 0.230 | ✓ | 0.566 | 0.035 | ✓ | 0.109 | ✓ | 0.172 | 0.063 | ✓ | 0.125 | ✓ | 0.966 |
| | t | 0.015 | ✓ | 0.230 | ✓ | 0.566 | 0.035 | ✓ | 0.109 | ✓ | 0.172 | 0.064 | ✓ | 0.091 | ✓ | 0.990 |
| | Skewed-t | 0.013 | ✓ | 0.383 | ✓ | 0.602 | 0.029 | ✓ | 0.459 | ✓ | 0.248 | 0.061 | ✓ | 0.168 | ✓ | 0.584 |
| GARCH(1,1) | Normal | 0.015 | ✓ | 0.230 | ✓ | 0.566 | 0.027 | ✓ | 0.772 | ✓ | 0.294 | 0.061 | ✓ | 0.168 | ✓ | 0.584 |
| | GED | 0.012 | ✓ | 0.594 | ✓ | 0.638 | 0.027 | ✓ | 0.772 | ✓ | 0.294 | 0.063 | ✓ | 0.125 | ✓ | 0.533 |
| | t | 0.012 | ✓ | 0.594 | ✓ | 0.638 | 0.027 | ✓ | 0.772 | ✓ | 0.294 | 0.064 | ✓ | 0.091 | ✓ | 0.990 |
| | Skewed-t | 0.009 | ✓ | 0.853 | ✓ | 0.715 | 0.024 | ✓ | 0.860 | ✓ | 0.346 | 0.055 | ✓ | 0.563 | ✓ | 0.860 |
| IGARCH(1,1) | Normal | 0.012 | ✓ | 0.594 | ✓ | 0.638 | 0.028 | ✓ | 0.606 | ✓ | 0.270 | 0.060 | ✓ | 0.223 | ✓ | 0.634 |
| | GED | 0.012 | ✓ | 0.594 | ✓ | 0.638 | 0.025 | ✓ | 0.953 | ✓ | 0.320 | 0.061 | ✓ | 0.168 | ✓ | 0.584 |
| | t | 0.012 | ✓ | 0.594 | ✓ | 0.638 | 0.025 | ✓ | 0.953 | ✓ | 0.320 | 0.064 | ✓ | 0.091 | ✓ | 0.490 |
| | Skewed-t | 0.011 | ✓ | 0.856 | ✓ | 0.677 | 0.021 | ✓ | 0.510 | ✓ | 0.402 | 0.045 | ✓ | 0.552 | ✓ | 0.625 |
| EGARCH(1,1) | Normal | 0.011 | ✓ | 0.856 | ✓ | 0.677 | 0.024 | ✓ | 0.860 | ✓ | 0.346 | 0.051 | ✓ | 0.933 | ✓ | 0.448 |
| | GED | 0.009 | ✓ | 0.853 | ✓ | 0.715 | 0.025 | ✓ | 0.953 | ✓ | 0.320 | 0.052 | ✓ | 0.803 | ✓ | 0.499 |
| | t | 0.009 | ✓ | 0.853 | ✓ | 0.715 | 0.025 | ✓ | 0.953 | ✓ | 0.320 | 0.053 | ✓ | 0.678 | ✓ | 0.553 |
| | Skewed-t | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.016 | ✓ | 0.091 | ✓ | 0.536 | 0.040 | ✓ | 0.194 | ✓ | 0.835 |
| GJR-GARCH(1,1) | Normal | 0.011 | ✓ | 0.856 | ✓ | 0.677 | 0.024 | ✓ | 0.953 | ✓ | 0.320 | 0.051 | ✓ | 0.803 | ✓ | 0.990 |
| | GED | 0.011 | ✓ | 0.856 | ✓ | 0.677 | 0.025 | ✓ | 0.953 | ✓ | 0.320 | 0.055 | ✓ | 0.563 | ✓ | 0.610 |
| | t | 0.011 | ✓ | 0.856 | ✓ | 0.677 | 0.027 | ✓ | 0.772 | ✓ | 0.294 | 0.055 | ✓ | 0.563 | ✓ | 0.610 |
| | Skewed-t | 0.008 | ✓ | 0.568 | ✓ | 0.757 | 0.019 | ✓ | 0.245 | ✓ | 0.465 | 0.043 | ✓ | 0.345 | ✓ | 0.732 |
| FIGARCH(1,1) | Normal | 0.012 | ✓ | 0.594 | ✓ | 0.638 | 0.028 | ✓ | 0.606 | ✓ | 0.614 | 0.059 | ✓ | 0.289 | ✓ | 0.382 |
| | GED | 0.011 | ✓ | 0.856 | ✓ | 0.677 | 0.028 | ✓ | 0.606 | ✓ | 0.614 | 0.061 | ✓ | 0.168 | ✓ | 0.482 |
| | t | 0.011 | ✓ | 0.856 | ✓ | 0.677 | 0.028 | ✓ | 0.606 | ✓ | 0.614 | 0.057 | ✓ | 0.367 | ✓ | 0.176 |
| | Skewed-t | 0.008 | ✓ | 0.568 | ✓ | 0.757 | 0.023 | ✓ | 0.678 | ✓ | 0.395 | 0.052 | ✓ | 0.803 | ✓ | 0.499 |
| FIEGARCH(1,1) | Normal | 0.013 | ✓ | 0.488 | ✓ | 0.749 | 0.027 | ✓ | 0.566 | ✓ | 0.441 | 0.061 | ✓ | 0.385 | ✓ | 0.382 |
| | GED | 0.009 | ✓ | 0.815 | ✓ | 0.677 | 0.029 | ✓ | 0.677 | ✓ | 0.614 | 0.061 | ✓ | 0.385 | ✓ | 0.223 |
| | t | 0.012 | ✓ | 0.606 | ✓ | 0.244 | 0.025 | ✓ | 0.903 | ✓ | 0.745 | 0.052 | ✓ | 0.855 | ✓ | 0.176 |
| | Skewed-t | 0.008 | ✓ | 0.568 | ✓ | 0.880 | 0.021 | ✓ | 0.592 | ✓ | 0.403 | 0.053 | ✓ | 0.803 | ✓ | 0.412 |
| Expected Failure Rate: | | 0.010 | | | | | 0.025 | | | | | 0.050 | | | | |

Sample Size: 750

UC: Unconditional Coverage Test (H0:Correct Number of Failures)

Ind: Independence test (H0: VaR Failures are Independent)

*All tests conducted at the 95% confidence level

✓ : Fail to Reject H0

✘ : Reject H0

Table 6:
Extended GARCH Daily-Historical VaR Results
(Backtest Period: 2010-2012)

| $\alpha\%$ -VaR | | 99% | | | | | 97.5% | | | | | 95% | | | | |
|-------------------------------|--------------|--------------|----|-------|-----|-------|--------------|----|-------|-----|-------|--------------|----|-------|-----|-------|
| Model | Distribution | Failure Rate | UC | p-val | Ind | p-val | Failure Rate | UC | p-val | Ind | p-val | Failure Rate | UC | p-val | Ind | p-val |
| Equally-Weighted | - | 0.000 | ✗ | 0.000 | ✗ | - | 0.009 | ✗ | 0.002 | ✓ | 0.540 | 0.031 | ✗ | 0.009 | ✓ | 0.228 |
| EWMA ($\lambda=0.94$) | Normal | 0.011 | ✓ | 0.856 | ✓ | 0.677 | 0.028 | ✓ | 0.606 | ✓ | 0.131 | 0.051 | ✓ | 0.933 | ✓ | 0.448 |
| | GED | 0.011 | ✓ | 0.856 | ✓ | 0.677 | 0.028 | ✓ | 0.606 | ✓ | 0.131 | 0.051 | ✓ | 0.933 | ✓ | 0.448 |
| | t | 0.011 | ✓ | 0.856 | ✓ | 0.677 | 0.028 | ✓ | 0.606 | ✓ | 0.131 | 0.051 | ✓ | 0.933 | ✓ | 0.448 |
| | Skewed-t | 0.011 | ✓ | 0.856 | ✓ | 0.677 | 0.028 | ✓ | 0.606 | ✓ | 0.131 | 0.051 | ✓ | 0.933 | ✓ | 0.448 |
| GARCH(1,1) | Normal | 0.008 | ✓ | 0.568 | ✓ | 0.757 | 0.024 | ✓ | 0.860 | ✓ | 0.446 | 0.048 | ✓ | 0.800 | ✓ | 0.115 |
| | GED | 0.008 | ✓ | 0.568 | ✓ | 0.757 | 0.023 | ✓ | 0.678 | ✓ | 0.395 | 0.048 | ✓ | 0.800 | ✓ | 0.115 |
| | t | 0.008 | ✓ | 0.568 | ✓ | 0.757 | 0.021 | ✓ | 0.510 | ✓ | 0.345 | 0.048 | ✓ | 0.800 | ✓ | 0.115 |
| | Skewed-t | 0.008 | ✓ | 0.568 | ✓ | 0.757 | 0.021 | ✓ | 0.510 | ✓ | 0.345 | 0.048 | ✓ | 0.800 | ✓ | 0.115 |
| IGARCH(1,1) | Normal | 0.009 | ✓ | 0.853 | ✓ | 0.715 | 0.023 | ✓ | 0.678 | ✓ | 0.368 | 0.052 | ✓ | 0.803 | ✓ | 0.191 |
| | GED | 0.009 | ✓ | 0.853 | ✓ | 0.715 | 0.021 | ✓ | 0.510 | ✓ | 0.345 | 0.052 | ✓ | 0.803 | ✓ | 0.191 |
| | t | 0.009 | ✓ | 0.853 | ✓ | 0.715 | 0.021 | ✓ | 0.510 | ✓ | 0.345 | 0.051 | ✓ | 0.933 | ✓ | 0.163 |
| | Skewed-t | 0.009 | ✓ | 0.853 | ✓ | 0.715 | 0.024 | ✓ | 0.860 | ✓ | 0.070 | 0.052 | ✓ | 0.803 | ✓ | 0.191 |
| EGARCH(1,1) | Normal | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.023 | ✓ | 0.678 | ✓ | 0.395 | 0.053 | ✓ | 0.678 | ✓ | 0.221 |
| | GED | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.023 | ✓ | 0.678 | ✓ | 0.395 | 0.053 | ✓ | 0.678 | ✓ | 0.223 |
| | t | 0.007 | ✓ | 0.320 | ✓ | 0.793 | 0.019 | ✓ | 0.245 | ✓ | 0.465 | 0.053 | ✓ | 0.678 | ✓ | 0.223 |
| | Skewed-t | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.021 | ✓ | 0.510 | ✓ | 0.402 | 0.053 | ✓ | 0.678 | ✓ | 0.223 |
| GJR-GARCH(1,1) | Normal | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.020 | ✓ | 0.364 | ✓ | 0.299 | 0.048 | ✓ | 0.800 | ✓ | 0.353 |
| | GED | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.020 | ✓ | 0.364 | ✓ | 0.299 | 0.048 | ✓ | 0.800 | ✓ | 0.353 |
| | t | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.019 | ✓ | 0.245 | ✓ | 0.257 | 0.049 | ✓ | 0.933 | ✓ | 0.399 |
| | Skewed-t | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.020 | ✓ | 0.364 | ✓ | 0.299 | 0.049 | ✓ | 0.933 | ✓ | 0.399 |
| FIGARCH(1,1) | Normal | 0.009 | ✓ | 0.853 | ✓ | 0.715 | 0.027 | ✓ | 0.772 | ✓ | 0.108 | 0.053 | ✓ | 0.678 | ✓ | 0.223 |
| | GED | 0.009 | ✓ | 0.853 | ✓ | 0.715 | 0.028 | ✓ | 0.606 | ✓ | 0.131 | 0.053 | ✓ | 0.678 | ✓ | 0.223 |
| | t | 0.009 | ✓ | 0.853 | ✓ | 0.715 | 0.027 | ✓ | 0.772 | ✓ | 0.108 | 0.051 | ✓ | 0.933 | ✓ | 0.163 |
| | Skewed-t | 0.009 | ✓ | 0.853 | ✓ | 0.715 | 0.027 | ✓ | 0.772 | ✓ | 0.108 | 0.052 | ✓ | 0.803 | ✓ | 0.191 |
| FIEGARCH(1,1) | Normal | 0.009 | ✓ | 0.853 | ✓ | 0.715 | 0.027 | ✓ | 0.772 | ✓ | 0.108 | 0.056 | ✓ | 0.459 | ✓ | 0.296 |
| | GED | 0.009 | ✓ | 0.853 | ✓ | 0.677 | 0.028 | ✓ | 0.606 | ✓ | 0.131 | 0.057 | ✓ | 0.367 | ✓ | 0.338 |
| | t | 0.009 | ✓ | 0.853 | ✓ | 0.715 | 0.027 | ✓ | 0.772 | ✓ | 0.557 | 0.056 | ✓ | 0.459 | ✓ | 0.296 |
| | Skewed-t | 0.009 | ✓ | 0.853 | ✓ | 0.506 | 0.029 | ✓ | 0.459 | ✓ | 0.158 | 0.063 | ✓ | 0.125 | ✓ | 0.553 |
| Expected Failure Rate: | | 0.010 | | | | | 0.025 | | | | | 0.050 | | | | |

Sample Size: 750

UC: Unconditional Coverage Test (H0: Correct Number of Failures)

Ind: Independence test (H0: VaR Failures are Independent)

*All tests conducted at the 95% confidence level

✓ : Fail to Reject H0

✗ : Reject H0

Table 7:
Extended GARCH Daily-Historical VaR Results (2008-2009 excluded)
(Backtest Period: 2010-2012)

| α%-VaR | | 99% | | | | | 97.5% | | | | | 95% | | | | |
|-------------------------------|--------------|--------------|----|-------|-----|-------|--------------|----|-------|-----|-------|--------------|----|-------|-----|-------|
| Model | Distribution | Failure Rate | UC | p-val | Ind | p-val | Failure Rate | UC | p-val | Ind | p-val | Failure Rate | UC | p-val | Ind | p-val |
| Equally-Weighted | - | 0.005 | ✓ | 0.159 | ✓ | 0.825 | 0.019 | ✓ | 0.245 | ✓ | 0.465 | 0.044 | ✓ | 0.442 | ✓ | 0.234 |
| EWMA (λ=0.94) | Normal | 0.008 | ✓ | 0.568 | ✓ | 0.757 | 0.021 | ✓ | 0.510 | ✓ | 0.402 | 0.047 | ✓ | 0.672 | ✓ | 0.772 |
| | GED | 0.008 | ✓ | 0.568 | ✓ | 0.757 | 0.021 | ✓ | 0.510 | ✓ | 0.402 | 0.047 | ✓ | 0.672 | ✓ | 0.772 |
| | t | 0.008 | ✓ | 0.568 | ✓ | 0.757 | 0.021 | ✓ | 0.510 | ✓ | 0.402 | 0.047 | ✓ | 0.672 | ✓ | 0.772 |
| | Skewed-t | 0.008 | ✓ | 0.568 | ✓ | 0.757 | 0.021 | ✓ | 0.510 | ✓ | 0.402 | 0.047 | ✓ | 0.672 | ✓ | 0.772 |
| GARCH(1,1) | Normal | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.021 | ✓ | 0.510 | ✓ | 0.345 | 0.049 | ✓ | 0.933 | ✓ | 0.399 |
| | GED | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.021 | ✓ | 0.510 | ✓ | 0.345 | 0.048 | ✓ | 0.800 | ✓ | 0.353 |
| | t | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.021 | ✓ | 0.510 | ✓ | 0.345 | 0.048 | ✓ | 0.800 | ✓ | 0.353 |
| | Skewed-t | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.021 | ✓ | 0.510 | ✓ | 0.345 | 0.048 | ✓ | 0.800 | ✓ | 0.353 |
| IGARCH(1,1) | Normal | 0.008 | ✓ | 0.568 | ✓ | 0.757 | 0.021 | ✓ | 0.510 | ✓ | 0.345 | 0.052 | ✓ | 0.803 | ✓ | 0.501 |
| | GED | 0.008 | ✓ | 0.568 | ✓ | 0.757 | 0.021 | ✓ | 0.510 | ✓ | 0.345 | 0.052 | ✓ | 0.803 | ✓ | 0.499 |
| | t | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.021 | ✓ | 0.510 | ✓ | 0.345 | 0.053 | ✓ | 0.678 | ✓ | 0.553 |
| | Skewed-t | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.021 | ✓ | 0.510 | ✓ | 0.345 | 0.052 | ✓ | 0.803 | ✓ | 0.499 |
| EGARCH(1,1) | Normal | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.021 | ✓ | 0.510 | ✓ | 0.402 | 0.053 | ✓ | 0.678 | ✓ | 0.223 |
| | GED | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.021 | ✓ | 0.510 | ✓ | 0.402 | 0.055 | ✓ | 0.563 | ✓ | 0.087 |
| | t | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.021 | ✓ | 0.510 | ✓ | 0.402 | 0.053 | ✓ | 0.678 | ✓ | 0.072 |
| | Skewed-t | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.021 | ✓ | 0.510 | ✓ | 0.402 | 0.055 | ✓ | 0.563 | ✓ | 0.087 |
| GJR-GARCH(1,1) | Normal | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.025 | ✓ | 0.953 | ✓ | 0.500 | 0.052 | ✓ | 0.803 | ✓ | 0.499 |
| | GED | 0.007 | ✓ | 0.320 | ✓ | 0.793 | 0.023 | ✓ | 0.678 | ✓ | 0.374 | 0.052 | ✓ | 0.803 | ✓ | 0.499 |
| | t | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.021 | ✓ | 0.510 | ✓ | 0.402 | 0.051 | ✓ | 0.933 | ✓ | 0.448 |
| | Skewed-t | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.021 | ✓ | 0.510 | ✓ | 0.402 | 0.051 | ✓ | 0.933 | ✓ | 0.448 |
| FIGARCH(1,1) | Normal | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.025 | ✓ | 0.953 | ✓ | 0.500 | 0.051 | ✓ | 0.933 | ✓ | 0.448 |
| | GED | 0.007 | ✓ | 0.320 | ✓ | 0.793 | 0.023 | ✓ | 0.678 | ✓ | 0.395 | 0.051 | ✓ | 0.933 | ✓ | 0.448 |
| | t | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.023 | ✓ | 0.678 | ✓ | 0.395 | 0.051 | ✓ | 0.933 | ✓ | 0.448 |
| | Skewed-t | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.023 | ✓ | 0.678 | ✓ | 0.395 | 0.051 | ✓ | 0.933 | ✓ | 0.448 |
| FIEGARCH(1,1) | Normal | 0.008 | ✓ | 0.568 | ✓ | 0.677 | 0.025 | ✓ | 0.953 | ✓ | 0.402 | 0.055 | ✓ | 0.563 | ✓ | 0.383 |
| | GED | 0.008 | ✓ | 0.568 | ✓ | 0.715 | 0.023 | ✓ | 0.678 | ✓ | 0.131 | 0.056 | ✓ | 0.502 | ✓ | 0.338 |
| | t | 0.007 | ✓ | 0.329 | ✓ | 0.793 | 0.025 | ✓ | 0.953 | ✓ | 0.557 | 0.053 | ✓ | 0.459 | ✓ | 0.296 |
| | Skewed-t | 0.008 | ✓ | 0.568 | ✓ | 0.757 | 0.028 | ✓ | 0.459 | ✓ | 0.158 | 0.055 | ✓ | 0.563 | ✓ | 0.741 |
| Expected Failure Rate: | | 0.010 | | | | | 0.025 | | | | | 0.050 | | | | |

Sample Size: 750

UC: Unconditional Coverage Test (H0: Correct Number of Failures)

Ind: Independence test (H0: VaR Failures are Independent)

*All tests conducted at the 95% confidence level

✓ : Fail to Reject H0

✗ : Reject H0