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The Centrality Dependence of Thermal
Parameters in Pb-Pb Collisions at
158 GeV/nucleon

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Abstract

A review of the Hadron Gas model and its application to Pb+Pb CERN SPS collision data at a beam energy of 158 GeV/nucleon. The centrality dependence of the freeze-out parameters, characterizing both the hadron multiplicities and the transverse momentum spectra, are determined. This provides valuable information on the effect of the system size on chemical- and thermal freeze-out and contributes towards the systematic understanding of the experimental data.

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Chapter 1

Introduction

Relativistic heavy ion collisions afford us the opportunity of studying strongly interacting matter under conditions of extreme temperature and pressure. It is the ultimate goal of current collision experiments to create a new state of matter (the Quark-Gluon Plasma or QGP) in which the effective degrees of freedom are quarks and gluons, instead of the usual hadrons and hadronic resonances. Such a state of deconfined quarks and gluons is believed to have existed in the early universe up to a few microseconds after the Big Bang, before the temperature dropped sufficiently for hadrons to condense out [1]. In the present universe, this excited state of QGP may exist in the cold interiors of neutron stars, and may cause observable effects in the frequency spectrum of pulsars [2].

In relativistic collisions, the formation of QGP depends not only on the colliding particles and collision energy, but also on the degree to which this matter and energy is transported from the target and projectile nucleons towards the center of mass. This transfer of matter and energy is termed stopping. Observed transverse energy distributions predict a maximum energy density of 3 GeV/fm^3 produced in central fixed target Pb+Pb collisions at 158 A GeV at the CERN SPS [3]. This value is to be compared with recent results from lattice QCD which estimate the phase transition from confined hadronic matter to QGP to occur at a critical temperature between 180 and 200 MeV, corresponding to energy densities of the order of $1\text{--}2 \text{ GeV/fm}^3$ [4]. With the first RHIC results now becoming available, and the prospect of still higher energies being reached in the near future at LHC, it is hoped that new physics, signaling the formation of QGP, will soon be observed.

Although, in principle, the evolution of the system produced in heavy ion collisions is determined by QCD, the hadronization process lies entirely

within its non-perturbative domain. Consequently, one is forced to resort to phenomenological models, such as string-, microscopic transport- or statistical (thermal) models to describe the final state hadrons. It is hoped that signals of QGP formation in the early stages of the collision will survive the subsequent evolution of the system and be reflected in the final state observables.

1.1 The Collision Mechanism

Consider the collision of two relativistic nuclei. Viewed in the center-of-mass frame, the target and projectile approach one another as highly Lorentz-contracted pancakes. The number of nucleons which participate (interact) in the imminent collision is given, to a good approximation, by the geometrical overlap of the target and projectile. The remaining (spectator) nucleons are largely unaffected by the collision, roughly retaining their original momenta.

On colliding, each participating nucleon is decelerated as it interacts with the surrounding nuclear matter. In the process it is excited and fragments into new particles. The degree to which the participating nucleons are stopped is evident in the rapidity distributions of the net baryons (the kinematic variables used throughout this thesis are discussed in Appendix A). In these distributions, it is the fastest baryons that are associated with the original target and projectile nucleons. The use of net baryon distributions for this purpose eliminates the effect of particle-antiparticle pair production.

The two limiting cases for the stopping in symmetric collisions are shown in Figure 1.1. In the case of complete stopping, the target and projectile are effectively opaque to each other, and the baryons are concentrated at mid-rapidity. This scenario might be expected for collisions at low energy or between massive systems. At the other extreme, the target and projectile are essentially transparent. In such a case, two distinct phase-space regions exist; the target- and projectile fragmentation regions, which carry the baryon content of the system, and the central rapidity region, which contains mostly mesons. The fragmentation regions are clearly identified in the net baryon rapidity distributions as peaks centered at rapidities close to those of the original target and projectile. Transparency is expected at ultra-relativistic energies or in elementary collisions.

Figure 1.2 shows the net baryon distributions for central S on S and Pb

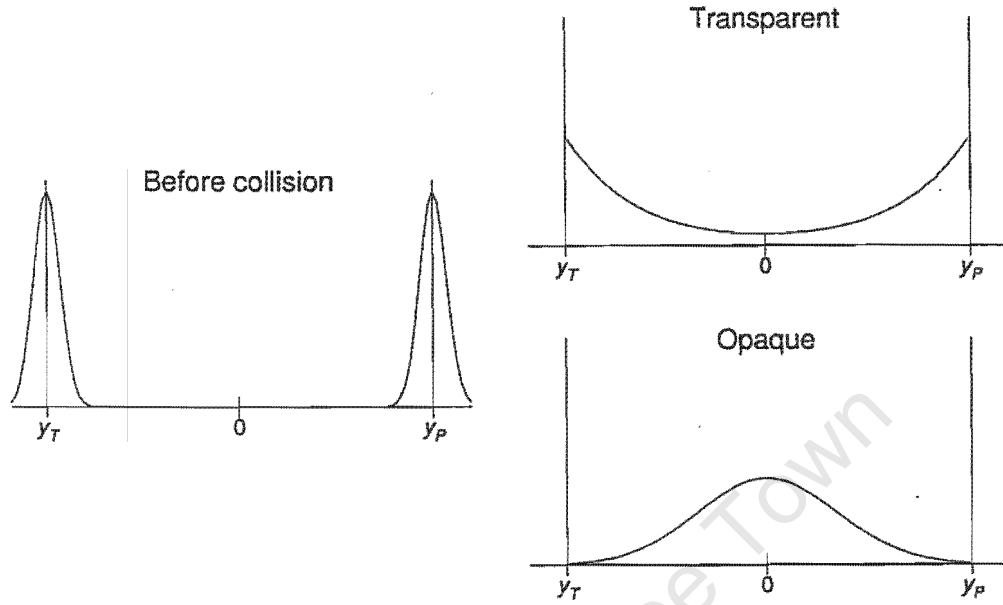


Figure 1.1: The two extreme cases of baryon stopping [6]. The net baryon rapidity distribution before the collision (on the left) compared to the transparent case (above right) and the opaque scenario (below right).

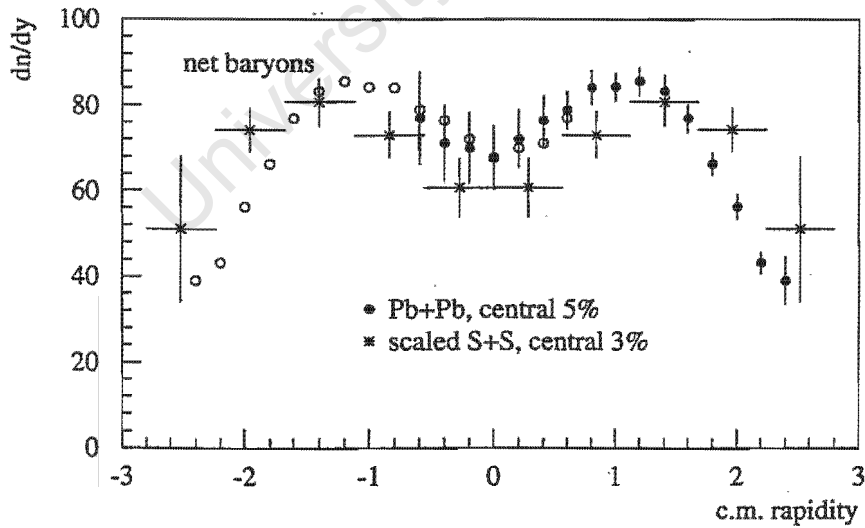


Figure 1.2: Rapidity distributions of the net baryons from central Pb+Pb and S+S collisions (suitably scaled for comparison) at the CERN SPS [7].

on Pb collisions at the CERN SPS. Both show a high degree of transparency (i.e. two peaks with a dip at mid-rapidity), with slightly greater stopping in the larger Pb system [5]. This dependence of the degree of stopping on the system size is consistent with the increased stopping observed in central collisions of Pb on Pb at the CERN SPS, relative to peripheral collisions [6]. At AGS energies, the proton rapidity distribution peaks at mid-rapidity and is roughly Gaussian in shape [8]. This indicates considerable stopping [8] which increases for larger systems [9].

If the energy- and baryon density resulting from the stopping process exceed critical values, a state of QGP will be formed. Such a state is short-lived; as the system expands, it rapidly cools and soon reaches the transition point back to the hadronic phase. If this phase transition is of first order, a mixed phase, in which condensed hadronic matter and QGP co-exist at the transition temperature, will form. Only once all of the QGP condenses into hadrons will the temperature drop further. A second-order phase transition accommodates no such mixed phase; the QGP condenses instantaneously into hadrons. At present, there is still great uncertainty regarding the phase structure of hadronic matter (for a discussion of the current status of the phase structure of hadronic matter see [10]).

If the density of hadrons initially produced by the collision is sufficiently high, these hadrons will collide inelastically with each other and thereby change their identity. When these inelastic collisions cease, chemical freeze-out occurs and all particle yields are fixed. Thereafter, elastic collisions (e.g. $\pi + N \rightarrow \Delta \rightarrow \pi + N$), which have larger cross-sections, continue to occur. These collisions redistribute the momentum of the system until thermal freeze-out is reached and all particle interactions cease. It should be stressed that the observed particle yields and momentum distributions provide a snapshot of the system at chemical- and thermal freeze-out respectively. In particular, the net baryon momentum distributions (from which the degree of stopping is inferred) include the effects of final state rescattering which smear the distributions arising from the initial stopping. This effect is however small in comparison with the large longitudinal momentum scale of the collision, and so rapidity distributions are largely unaffected by such final state interactions [6].

In this thesis, the application of the thermal model to high energy collisions will be considered. These models have the advantage of characterizing hadron production with very few parameters (i.e. the temperature T , fireball volume V , chemical potentials μ_B , μ_S and μ_Q for the baryon number,

strangeness and charge respectively, and an additional parameter γ_s to account for possible undersaturation of the strange sector). The application of statistical mechanics to hadron production in high energy collisions dates back to the work of Fermi [11] and Hagedorn [12]. Naturally, the assumption of thermal behaviour— the backbone of thermal models— requires closer attention.

1.2 Thermalization

Thermal behaviour in relativistic collisions can arise in a number of ways [13]. Firstly, thermalization can result from a statistical filling of hadronic phase-space according to the principle of maximum entropy. In such a case, the temperature and chemical potentials are fixed by the conservation of energy, baryon number, strangeness and charge. Such thermalization requires no scattering; hadrons are created in thermal distributions, with chemical- and thermal freeze-out occurring at the point of particle production. As a result, there is no local equilibrium and no collective flow effects.

Thermalization can also result from an initial non-equilibrium state by rescattering. Such kinetic equilibration is accompanied by pressure and collective flow. Chemical equilibrium (reached at chemical freeze-out) is reflected in the observed particle yields and ratios (from which the temperature and chemical potentials at chemical freeze-out are determined). Thermal equilibrium, reached some time later at thermal freeze-out, is reflected in the momentum spectra (from which the temperature and flow velocity at thermal freeze-out may be determined).

Crucial to the question of thermalization through rescattering, are the time scales involved in high energy collisions. Compiled from a number of sources, Heinz et al. [14] give the following sequence in a QGP:

- The gluons equilibrate thermally after less than 0.5 fm/c and reach chemical equilibrium saturation levels after about 1 fm/c.
- Gluon fusion and the decay of off-shell gluons produces light quark pairs ($q\bar{q}$) which reach chemical equilibrium levels after 2–3 fm/c. Rescattering with gluons results in thermalization of these quarks in less than 1 fm/c.
- The strange quark mass-threshold prolongs the chemical equilibration of strange quarks to about 3–5 fm/c.

and in a hadronic environment:

- In dense systems corresponding to temperatures of the order of 200 MeV, thermal equilibration is extremely fast (typically occurring in less than 0.5 fm/c).
- Owing to their low mass-thresholds, chemical equilibration of non-strange hadrons is fast, occurring on a time scale of a few fm/c.
- Despite the high mass-thresholds of strangeness producing reactions, the strangeness content of the system is rapidly redistributed among the available strange hadron channels. These strangeness exchange processes have very low or vanishing thresholds and lead to relative chemical equilibrium in the strange sector after a few fm/c.
- The small inelastic cross-sections and high mass-thresholds typical of strange hadron pair production mechanisms result in a time of 10–30 fm/c being required for saturation of the strange phase-space for temperatures around 200 MeV.

1.3 Signatures of the QGP

Since direct observation of the QGP is impossible (it is short-lived and its component quarks and gluons are confined), we have to infer its formation from the observed distributions of hadrons, leptons and photons. This is non-trivial as all hadronic observables reflect conditions at freeze-out, by which time final state interactions may have caused all memory of an initial partonic state to be lost. The weakly interacting, directly emitted leptons and photons suffer fewer final state interactions and are a better probe of the early conditions of the system. However, these are not as prolific as the hadrons and need to be extracted from the background of photons and leptons indirectly produced by subsequent hadronic decays.

Once a system has reached thermal- and chemical equilibrium, whether it remains in equilibrium depends on the competition between the collision frequency and the rate of expansion of the system. If the evolution of an initially formed QGP is slow, it is conceivable that (after some initial thermalization time) the system may remain in equilibrium until freeze-out of the produced hadrons. The observed hadrons then reflect the properties of the state at freeze-out (the equilibrium hadron gas) and all information about the initial state is lost. However, if the evolution proceeds too rapidly, it is

possible that the system deviates from equilibrium.

Cleymans et al. [15] compared the predictions for the ratios of produced hadrons in these two scenarios. In the plasma break-up scenario, freeze-out was taken to occur immediately after hadronization and hadronization was considered to proceed at equilibrium. As a result of the equilibrium between the two phases, strangeness conservation in the plasma (requiring the vanishing of the strange quark chemical potential) leads to the relation $\mu_S = \frac{1}{3}\mu_B$ in the hadronic phase in an isospin-symmetric system. This relation is not generally satisfied in the hadron gas picture, as the subsequent evolution of the system allows equilibration away from this value. However, it was found that for $T = 200$ MeV and $\mu_B < 500$ MeV, this relation is approximately satisfied in the hadron gas too (the two pictures are essentially indistinguishable in this region), suggesting that care should be taken in interpreting a value of $\mu_S = \frac{1}{3}\mu_B$ as a signature of QGP formation.

The strange particle data considered in [15] supported a temperature of $T = 200$ MeV and baryon chemical potential $\mu_B = 300$ MeV (i.e. within the range in which the two pictures are indistinguishable), while the non-strange data suggested freeze-out parameters $T = 130$ MeV and $\mu_B = 200$ MeV. This sequential freeze-out arises quite naturally in the hadron gas picture, due to the larger cross-sections of non-strange particles. Furthermore, the entropy per baryon at the non-strange freeze-out was found to be in good agreement with experimental data, and comparable to that found in a plasma at a temperature and baryon density corresponding to the strange freeze-out. This suggested the possibility of an isentropic expansion to an interacting hadron gas, with strange freeze-out occurring soon after the confinement transition.

The hadronization mechanism [15] proposed in the plasma break-up scenario is unsatisfactory, in that the entropy per baryon in the plasma state just before recombination is much greater than that in the hadron phase [16]. This is due to the liberation of gluons which carry almost half of the entropy in the plasma phase [17]. As a result, the plasma break-up picture (of great interest since it preserves relics of the plasma phase) is incomplete. Instead, the system has to re-heat and expand while crossing the mixed phase (assuming a first-order phase transition) [16, 18, 19], before cooling again in the pure hadronic phase. The effect of this re-heating, combined with the $s - \bar{s}$ separation in the mixed phase of a finite baryon density system [20, 21], may lead to characteristic changes in the energy spectra of the K^+ and K^- , signaling the formation of QGP in the early stages of a collision [22, 23].

Amongst the many other proposed signatures of QGP formation are strangeness enhancement [24, 25], J/ψ suppression [26], and those arising from the reduction in the speed of sound during the confinement transition [27]. Strangeness enhancement refers to the enhanced production (per participating nucleon or produced pion) of strange hadrons in heavy ion collisions in which QGP is formed, relative to nucleon-nucleon or nucleon-nucleus collisions with no QGP formation at the same energy. It arises from the shorter time scale for equilibration of strangeness due to the reduced kinematic threshold for strangeness production in the QGP [24]. In a gluon-rich QGP, strangeness enhancement arises mainly due to gluon-gluon interactions which produce $s\bar{s}$ pairs [28]. The equilibration time by this process is comparable to the interaction times in relativistic heavy ion collisions (i.e. a few fm/c), whereas in a purely hadronic environment, the characteristic time for equilibration of strangeness by rescattering is likely to be longer. Enhanced strangeness production was first observed in 1987 in S+S collisions at 200 GeV/c per nucleon (NA35 at CERN SPS) and in Si+Au collisions at 14.6 GeV/c per nucleon (E802 at AGS) [30].

1.4 Thesis Objectives

It is the goal of this thesis to study the centrality dependence of the thermal model parameters for fixed target Pb+Pb collisions at 158 GeV/nucleon at the CERN SPS. Particle multiplicities and momentum distributions will be analysed in order to extract information on both the chemical- and thermal freeze-out. Such an analysis will heighten our understanding of the freeze-out process. In particular, the effect of the size of the excited, strongly interacting system will be determined. Furthermore, the availability of centrality binned proton data will serve as a necessary check on the inclusion of these particles in a thermal model analysis. Due to limited stopping, many of the detected protons in high energy peripheral collisions do not form part of the fireball of secondary particles described by the thermal model.

The remainder of this thesis is structured as follows: Chapter 2 contains a review of the Ideal Hadron Gas model; all results applied in the analysis of the data are discussed in this chapter, while all details and results of the data analysis are presented in Chapter 3.

Chapter 2

Review of the Ideal Hadron Gas Model

The Ideal Hadron Gas model treats the fireball which results from a heavy ion collision as an ideal gas of hadrons and hadronic resonances. At freeze-out, the hadrons are assumed to be described by local thermal distributions, with freeze-out parameters common to all particle species.

The invariant momentum spectrum of particle species i , with spin-isospin degeneracy g_i , emitted directly from the fireball at freeze-out is given by the Cooper-Frye formula [31]

$$E \frac{d^3 N_i}{d^3 p} = \frac{d^3 N_i}{dy p_T dp_T d\phi_P} = \frac{g_i}{(2\pi)^3} \int_{\sigma_f} f_i(x, p) p^\mu d\sigma_\mu \quad (2.1)$$

which involves an integration of the Lorentz-invariant local thermal distribution function $f_i(x, p)$ over the freeze-out surface σ_f with normal $d\sigma_\mu$. Assuming a grand-canonical ensemble, $f_i(x, p)$ at the freeze-out temperature T is given by

$$f_i(x, p) = \frac{1}{e^{(p^\mu u_\mu(x) - \mu_i(x))/T(x)} \pm 1} \quad (2.2)$$

where p^μ and $u_\mu(x)$ are, respectively, the particle 4-momentum and fireball volume-element 4-velocity with respect to the observer frame, and $\mu_i(x)$ is the chemical potential of particle species i . The plus sign refers to fermions and the minus sign to bosons.

Within the grand-canonical ensemble, introduction of a chemical potential for each particle species ensures conservation of the number of each particle type. However, in relativistic collisions, where particle-antiparticle creation and annihilation are possible, it is not the individual particle numbers

that are conserved, but rather the global quantum numbers of the system (i.e. its total charge, baryon number and strangeness— provided strangeness changing weak interactions are absent, a valid assumption given the short time scale of relativistic nuclear collisions). For this reason, at chemical equilibrium (when detailed balance is fulfilled), the chemical potential of particle species i , with baryon number B_i , strangeness S_i , and charge Q_i , is written as

$$\mu_i = B_i\mu_B + S_i\mu_S + Q_i\mu_Q \quad (2.3)$$

which ensures conservation of baryon number, strangeness and charge in an average sense. The values of the chemical potentials μ_B , μ_S and μ_Q cannot be chosen independently; the initial conditions of the collision fix μ_S and μ_Q for a given μ_B .

The freeze-out surface σ_f is a 3-dimensional hypersurface in space-time $\sigma^0(\sigma^1, \sigma^2, \sigma^3)$ defined by some freeze-out criterion (discussed in Section 2.1). When parameterized by three mutually orthogonal coordinates u, v and w , its normal vector is given by [17]

$$d\sigma_\mu = -\epsilon_{\mu\nu\lambda\rho} \frac{\partial\sigma^\nu}{\partial u} \frac{\partial\sigma^\lambda}{\partial v} \frac{\partial\sigma^\rho}{\partial w} du dv dw \quad (2.4)$$

where $\epsilon^{\mu\nu\lambda\rho} = -\epsilon_{\mu\nu\lambda\rho}$ is totally antisymmetric, and $\epsilon^{\mu\nu\lambda\rho} = +1$ if $(\mu\nu\lambda\rho)$ is an even permutation of $(0, 1, 2, 3)$.

2.1 The Freeze-out Criterion

In considering the freeze-out criterion, we assume the fireball, prior to freeze-out, to be in a state of collective expansion (evidence for collective flow will be discussed in Section 2.2.3). Particle freeze-out occurs when local equilibrium can no longer be maintained. In the absence of flow, this occurs for a particular particle species once its mean free path exceeds the linear dimensions of the fireball. With flow, freeze-out may occur before this condition is met if prospective scattering partners recede from the particles of the species under consideration by collective expansion at a rate faster than the thermal motion of this species [32].

Following Heinz et al. [17], the kinetic freeze-out condition for particle species i is best expressed through a comparison of time scales

$$\tau_{\text{scattering}}^i > \min(\tau_{\text{expansion}}, \tau_{\text{escape}}^i). \quad (2.5)$$

The time scale $\tau_{\text{scattering}}^i$, representing the average time between scattering events for particle species i at temperature T , is determined by the expression

$$\tau_{\text{scattering}}^i(T) = \frac{1}{\sum_j \langle v_{ij} \sigma_{ij} \rangle_T \rho_j(T)} \quad (2.6)$$

where the sum is over all potential scattering partners, $\langle v_{ij} \sigma_{ij} \rangle_T$ is the thermally averaged product of the relative velocity between the scattering particles and their total cross-section (measured at the appropriate collision energy) evaluated at temperature T , and $\rho_j(T)$ are the partial densities of the particles at the same temperature.

The expansion time scale $\tau_{\text{expansion}}$ is defined locally by the derivative of the density ρ with respect to the local time $\bar{t}$

$$\frac{1}{\tau_{\text{expansion}}} = -\frac{1}{\rho} \frac{\partial \rho}{\partial \bar{t}}. \quad (2.7)$$

Through the continuity equation, the expansion time scale depends on the form of the collective expansion velocity profile.

Finally, the escape time scale τ_{escape}^i is given by

$$\tau_{\text{escape}}^i = \frac{r_f(t)}{\langle v_i \rangle} \quad (2.8)$$

where $r_f(t)$ is the linear dimension of that portion of the fireball still equilibrated at time t , and $\langle v_i \rangle$ is the average thermal velocity of particle species i .

The kinetic freeze-out condition is particle-specific through the effect of the particle masses on the determination of the average thermal velocities, as well as due to the interaction cross-sections. This implies different freeze-out parameters for different particles. As an example, since the cross-sections involving strange particles are smaller than those involving the non-strange hadrons, strange particles are expected to freeze out first (i.e. at a higher temperature). This should be reflected in steeper m_T -spectra for the strange particles (see Section 2.2). In addition to this, for a general velocity field, the expansion time scale is position-dependent. As a result, the freeze-out criterion may lead to varying freeze-out temperatures for different freeze-out shells. However, in most applications of the thermal model it is assumed that freeze-out is described by thermal parameters common to all particle species and uniform across the emitting surface. The strong temperature dependence of the particle densities indeed prevents a large variation in T across

the freeze-out hypersurface [33], but incomplete baryon stopping may result in significant longitudinal variations in the baryon chemical potential [34]. This sets the limits of the thermal model; without detailed dynamical input on the collision region, the systematic error band of the thermal model allows for discrepancies between model and data at the level of 15–30% [35]. In practice, even hydrodynamical models use a simplified form of the kinetic freeze-out condition. In these models, it is generally assumed that freeze-out occurs at a constant local- or global time, determined by the point at which the freeze-out criterion is met at the surface [36].

2.2 Momentum Spectra and Thermal Freeze-out

Since the particle momenta are fixed at thermal freeze-out, thermal equilibrium at thermal freeze-out will be reflected in the momentum spectra. It is from these spectra that the thermal freeze-out parameters are extracted. As a starting point, we consider the spectral shapes of particles emitted from a stationary, thermalized fireball.

2.2.1 Spectra from a Stationary Thermal Source

In the case of a stationary, thermalized source of temperature T

$$d\sigma_\mu = (d^3x, \vec{0}) \quad (2.9)$$

and the freeze-out criterion for particle species i (equation (2.5)) is met instantaneously throughout the fireball volume V . Averaged over impact parameter, particle emission is isotropic and the ϕ_P integration of equation (2.1) is trivial:

$$\frac{d^2 N_i}{dy p_T dp_T} = \frac{d^2 N_i}{dy m_T dm_T} = \frac{g_i V}{(2\pi)^2} E e^{-(E-\mu_i)/T} \quad (2.10)$$

where we have assumed Boltzmann statistics and a position-independent chemical potential.

Transverse Momentum Spectra

The transverse momentum spectrum of particle species i is obtained by integrating equation (2.10) over the rapidity variable y . Substituting $m_T \cosh y$

for the energy of the particle, and using $K_\nu(z) = \int_0^\infty e^{-z \cosh t} \cosh(\nu t) dt$,

$$\frac{dN_i}{p_T dp_T} = \frac{g_i V}{2\pi^2} m_T K_1\left(\frac{m_T}{T}\right) e^{\mu_i/T} \quad (2.11)$$

where K_1 is a modified Bessel function. Using $\lim_{z \rightarrow \infty} K_1(z) = \sqrt{\frac{\pi}{2z}} e^{-z}$,

$$\lim_{m_T/T \rightarrow \infty} \frac{dN_i}{p_T dp_T} = \frac{g_i V}{2\pi^2} \sqrt{\frac{\pi}{2}} e^{\mu_i/T} T^{1/2} m_T^{1/2} e^{-m_T/T}, \quad (2.12)$$

while using $\lim_{z \rightarrow 0} K_1(z) = z^{-1}$, the transverse momentum spectrum $dN_i/p_T dp_T$ is seen to saturate at low momenta at a level proportional to T .

When plotted logarithmically against m_T , the transverse momentum spectra $dN_i/p_T dp_T$ of particles emitted directly from a stationary fireball resemble straight lines but with a slight convex curvature [36], while logarithmic plots of the transverse mass spectra $dN_i/m_T^{3/2} dm_T$ against $m_T - m_0$ produce straight lines with inverse slope parameter T . If freeze-out occurs at the same temperature for all particles, this will be reflected in the transverse mass spectra of all species sharing the same slope. This universal exponential behaviour in the m_T -spectra of particles emitted directly from a stationary fireball is termed m_T -scaling.

The transverse momentum spectra measured in relativistic heavy ion collisions show deviations from the behaviour expected for particle emission from a stationary, thermal source with fixed temperature. The p_T -spectra ($dN_i/p_T dp_T$) of the pions show a strong concave curvature (“low- p_T enhancement”) [37], as seen in Figure 2.1, while a substantial difference is observed in the slopes of the p_T -spectra of different particles over the energy range of current heavy ion colliders (i.e. from GSI SIS [38] to CERN SPS [5, 39]). This large variation in slope parameters cannot be ascribed to a sequential (particle-specific) freeze-out. In fact, inverse slope parameters are seen to increase linearly with particle mass [40, 38, 41], with the exception of the multi-strange particles (i.e. the Ω and Ξ) which, due to their smaller cross-sections, freeze out before the bulk of the transverse collective flow has developed [43] (refer to Figure 2.2). Furthermore, the extremely high inverse slope parameters often extracted from the p_T -spectra of relativistic heavy ion collisions prevent their interpretation as “real”, physical freeze-out temperatures [5].

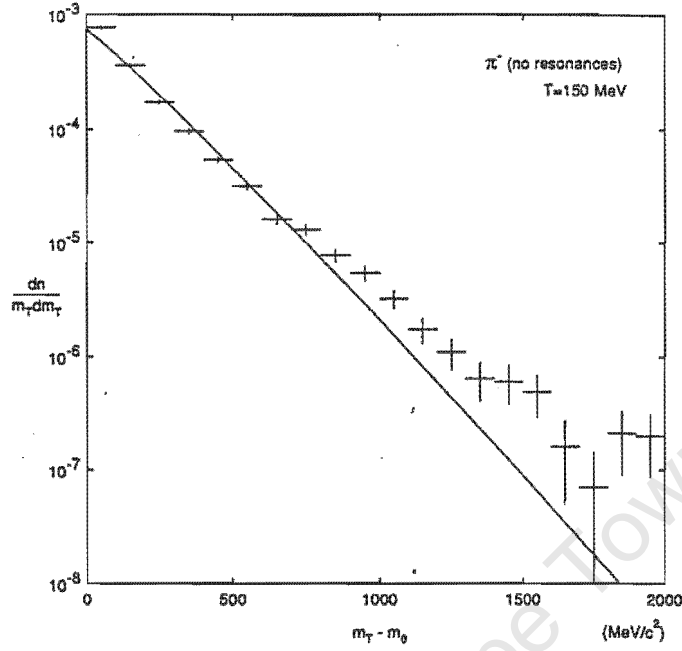


Figure 2.1: A purely thermal fit to the π^- m_T -spectrum from NA35 S+S 200 A GeV [44]. The deviation between the model and the data is significant at medium and high m_T .

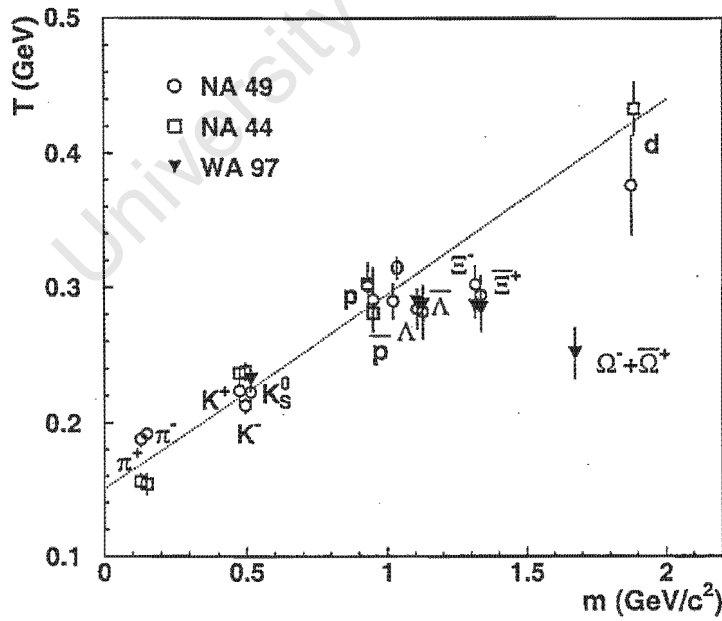


Figure 2.2: The dependence of the inverse slope parameters T of the m_T -spectra on the particle mass m for Pb+Pb collisions at CERN SPS [42].

An enhancement (flattening) of the p_T -spectra of particles produced in relativistic heavy ion collisions is expected at high p_T ($p_T > 1.5$ GeV [44]), on the basis of that observed in pA collisions (Cronin effect [45]). All secondaries produced in pA collisions show a broadening of their transverse momentum distributions, compared to those observed in pp collisions [46]. Even the virtual photons produced by the Drell-Yan process, which do not interact with the nuclear matter, and quarkonia, which escape the nuclear environment before developing any collective effects, are affected. This indicates that the observed broadening must, at least in part, be due to initial state effects [46]. The explanation proposed in [47, 48] relies on multiple hard scattering experienced by nucleons in the initial state. When a projectile nucleon which has already undergone a scattering with a nucleon of the target hits another target nucleon, the collision axis for this collision is generally rotated with respect to the original collision axis. In this way, multiple initial state scattering leads to an enhanced generation of transverse momentum [46]. An enhancement relative to pp collisions has indeed been observed in heavy ion collisions [49], although its identification as an effect of initial state rescattering alone is not possible, owing to the distinct possibility of rescattering also in the final state of these collisions.

Rapidity Spectra

For the rapidity spectrum, we integrate equation (2.10) over p_T , yielding [44]

$$\frac{dN_i}{dy} = \frac{g_i V}{(2\pi)^2} T^3 e^{-m_i \cosh y/T} \left[\frac{m_i^2}{T^2} + \frac{m_i}{T} \frac{2}{\cosh y} + \frac{2}{\cosh^2 y} \right] e^{\mu_i/T}. \quad (2.13)$$

For massless particles, the width of the rapidity distribution is roughly $\Gamma_{\text{FWHM}} \approx 1.76$ [44], while isotropic emission implies a strong narrowing of the distribution for heavier particles [50]. This is in complete disagreement with experimental results at both CERN SPS- [44] and AGS energies [50], where the observed rapidity distributions are considerably wider than those predicted under the assumption of a stationary thermal source. Figure 2.3 compares experimental rapidity distributions for central Si+Al collisions at 14.6 A GeV/c to predictions of the thermal model.

In an attempt to explain the observed deviations of the measured transverse momentum- and rapidity spectra from purely thermal behaviour, a number of extensions to the stationary fireball model have been suggested. The effect of resonance decays is considered in Section 2.2.2, while Sec-

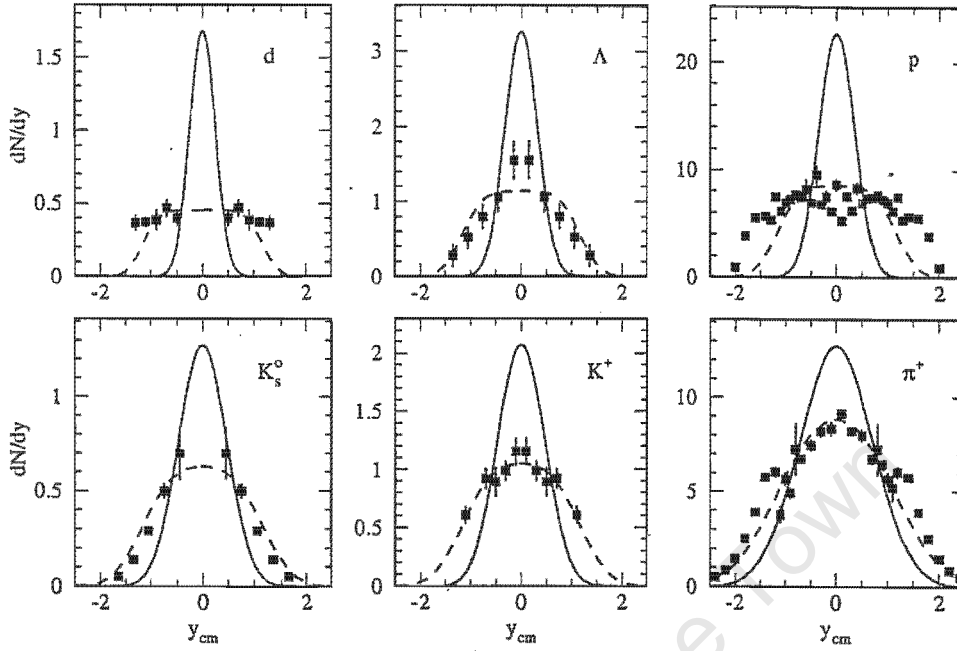


Figure 2.3: Comparison of experimental rapidity distributions for central 14.6 A GeV/c Si+Al collisions with isotropic thermal distributions at $T = 0.12$ GeV (solid lines) [50]. The dashed lines represent the predictions of the thermal model assuming longitudinal expansion (see Section 2.2.4).

tion 2.2.3 describes the inclusion of collective flow of the fireball.

2.2.2 The Effect of Resonance Decays

It was observed in early pp and πp collision experiments that a large fraction of the total pion yield originates from resonance decay (i.e. in π^+p interactions at 16 GeV/c incident momentum, only 10–30% of the pions are of direct origin [51]). This suggests their importance also in relativistic heavy ion collisions. If decay of an hadronic resonance feeding particle species i occurs after thermal freeze-out of this species, or if there is insufficient time between feeding and thermal freeze-out for the particles of species i of direct- and resonance decay origin to reach thermal equilibrium through rescattering, then the momentum spectra of this particular species will deviate from the thermal model predictions. In fact, a considerable fraction of the pions observed in relativistic heavy ion collisions arise from the decay of long-lived resonances such as the η and η' (with approximate lifetimes of 170 000 and 1 000 fm/c respectively), and the moderately long-lived ω meson (with a lifetime of approximately 23 fm/c) [52, 53]. As a result, the measured mo-

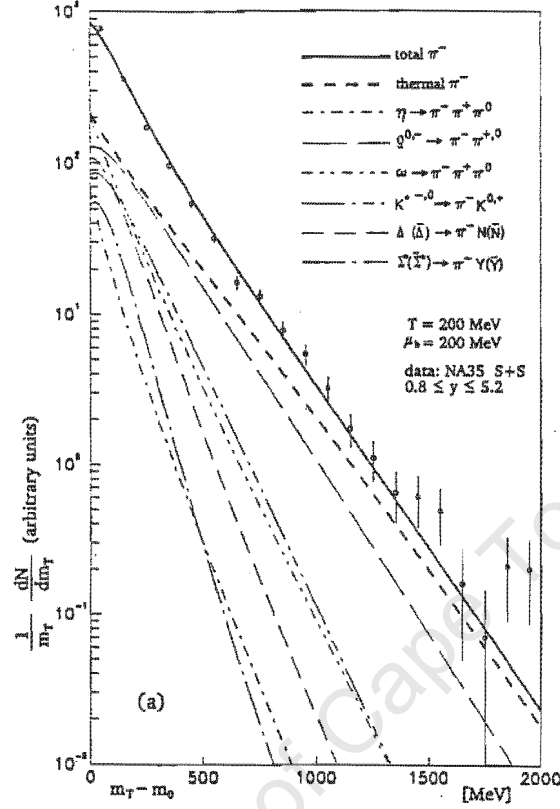


Figure 2.4: The π^- transverse mass spectrum observed in S+S collisions at 200 A GeV by the NA35 experiment [49]. The broken lines indicate contributions to the total theoretical m_T -distribution (solid line) by the decay of resonances in a hadron gas characterized by $T = 200$ MeV and $\mu_B = 200$ MeV.

momentum spectra must be compared with the sum of the primary distributions and the contributions arising from secondary decays of heavier resonances.

In [49, 54], the effect of resonance decays on single-particle transverse momentum spectra was investigated. Assuming thermal resonance distributions, the decay kinematics strongly favour decay to low- p_T daughter particles (especially three-body decays) [49]. With the inclusion of resonance width, the thermal model fits the measured CERN NA35 S+S p_T -spectra extremely well with a temperature $T = 200$ MeV and baryon chemical potential $\mu_B = 200$ MeV [49]. With these parameter values, the decay pions are seen to account for a large proportion of the “low- p_T enhancement” (see Figure 2.4). Although the pion spectrum is significantly affected by resonance feeding, its effect on heavier particles is minor, since the difference in

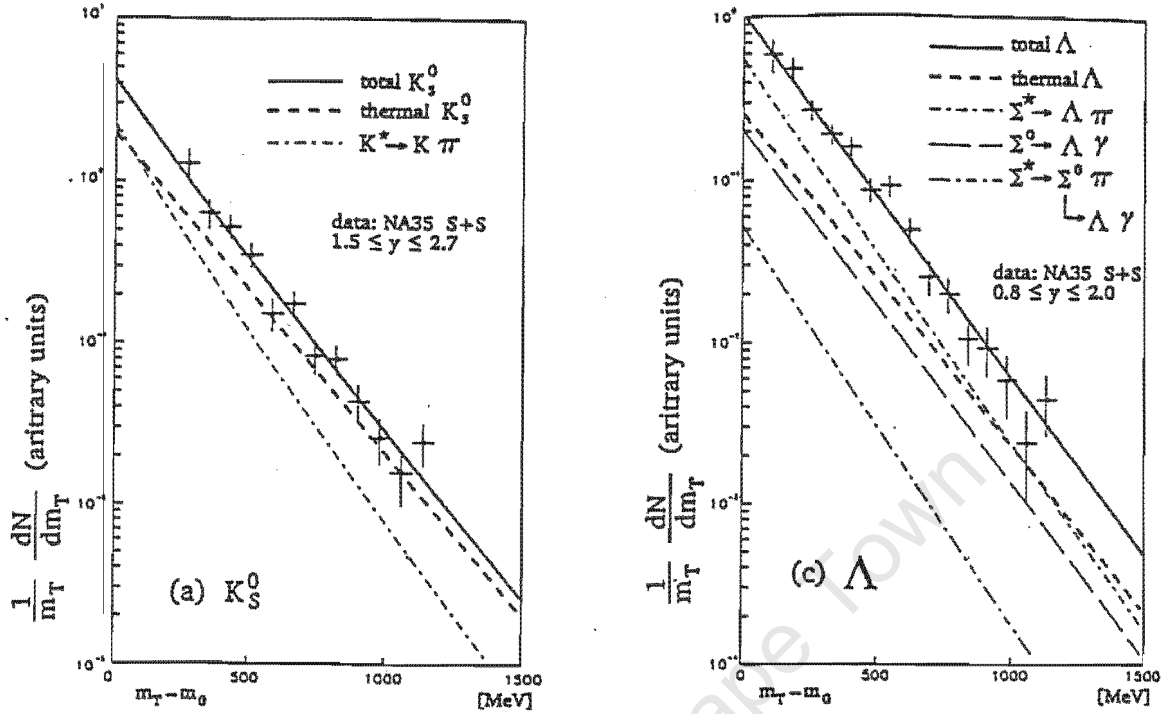


Figure 2.5: Contributions to the m_T -distributions of K_S^0 and Λ arising from various production channels assuming a stationary thermal source at $T = 200$ MeV [49]. The total m_T -distributions are compared with S+S 200 A GeV data measured by the NA35 collaboration.

mass between resonance and stable daughter particle is smaller [44]. This is borne out by the data; the m_T -spectra of the K_S^0 and Λ particles from CERN NA35 S+S resemble more closely the straight exponential drop-off of the pure thermal model (see Figure 2.5).

Despite the apparent phenomenological success of this extended thermal model, the temperature extracted from the pion p_T -spectra is found to be inconsistent with the freeze-out criterion; the fireball has to cool further before the pions are able to decouple [44]. Furthermore, as shown in Figure 2.6, the CERN S+S data show a high- p_T enhancement relative to pp collisions (at $p_T \geq 500$ MeV). This suggests a higher freeze-out temperature in S+S collisions which does not make sense thermodynamically. A possible explanation for both of these observations is that the high- p_T rise in the S+S collisions is not of thermal origin. The high- p_T slope of the p_T -spectra is then interpreted as an apparent temperature, the true temperature being much lower and consistent with the freeze-out criterion.

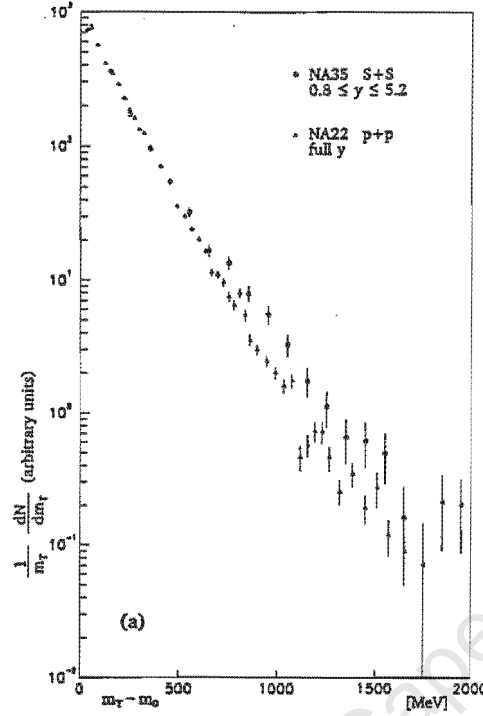


Figure 2.6: Comparison of the rapidity integrated pp NA22 data and the S+S NA35 data, normalized so as to maximize agreement at low p_T [49].

The inability of resonance effects alone to reproduce the momentum spectra observed in relativistic heavy ion collisions is further illustrated by their effect on the rapidity spectra. Inclusion of resonance decays leads to a further narrowing of the predicted rapidity spectra [44] and hence a larger deviation from the experimental distributions. The broadness of the observed rapidity spectra reflects an anisotropy in the system, suggesting the presence of significant longitudinal flow.

2.2.3 Evidence for Collective Flow

Simple models of transverse collective flow in the final state predict a flattening of the transverse mass spectra in the region $p_T < m$ [37, 36]. Since $p_T = m\gamma\beta_T$, a common flow velocity superimposed on the random thermal motion of the particles produced in a collision increases the inverse slope parameter T (which is proportional to $\langle p_T \rangle$) linearly with the mass m of the particles. This assumes that all particles freeze out at the same temperature and with the same velocity field. The random walk model of initial state scattering [46] was long considered an alternative way to explain the observed spectra (i.e. being able to reproduce the linear dependence of the

inverse slope parameter on the particle mass). However, an analysis [3] of Pb+Au data from the CERES collaboration showed a significantly weaker dependence of the inverse slope parameter on collision centrality for protons at SPS energies than that predicted by the random walk model. This, together with the large deviation of the data from the calculated spectra at low m_T , prompted Braun-Munzinger et al. [3] to conclude that initial state scattering alone is incapable of reproducing the observed spectra.

In addition to the evidence for the existence of flow in relativistic heavy ion collisions derived from single-particle momentum spectra (i.e. the broadness of the rapidity distributions, the too high- and particle-dependent inverse slope parameters extracted from transverse momentum distributions, and the enhancement at high p_T relative to both pp results and pure thermal predictions), there is considerable independent experimental evidence for flow effects. At BEVALAC energies, the directed sideways flow of matter provided the first evidence for collective effects [55]. Subsequently, anisotropies in the azimuthal distributions of observables in non-central collisions were observed at SPS and AGS. These anisotropies indicate a back-to-back flow in the transverse plane (elliptic flow) at SPS [56] and directed flow at AGS [57]. See [58] for a review of observed flow effects.

The model assuming spherically symmetric flow of the fireball, developed and applied in [37] and [36], predicts, as a function of m_T , a universal curve of $dN/m_T^{3/2} dm_T$ with significant concave curvature. This curvature, which arises from the different apparent temperatures contributed by the different flow velocities of each fireball shell at freeze-out, leads to a flattening of the m_T -spectra with increasing particle mass. This model fits both the CERN (NA35 and WA80) and AGS (E802) data fairly well, although the flattening of the spectra with increasing m_T is more clearly seen in the AGS data. The strong concave curvature of the CERN pion data is reproduced, except in the low- p_T region ($p_T < 200$ MeV/c) where there is a pronounced enhancement which various choices of freeze-out hypersurface are unable to account for [36]. This enhancement is likely due to the exclusion of resonance decays from their analysis.

Although central Au+Au collisions at 250 MeV/nucleon measured by the FOPI Collaboration are consistent with isotropic radial expansion [38], the same colliding system measured by the E802 Collaboration at an energy of 10.7 GeV/nucleon yields a proton rapidity distribution twice as broad as that predicted by a model of spherically symmetric expansion [59]. Simi-

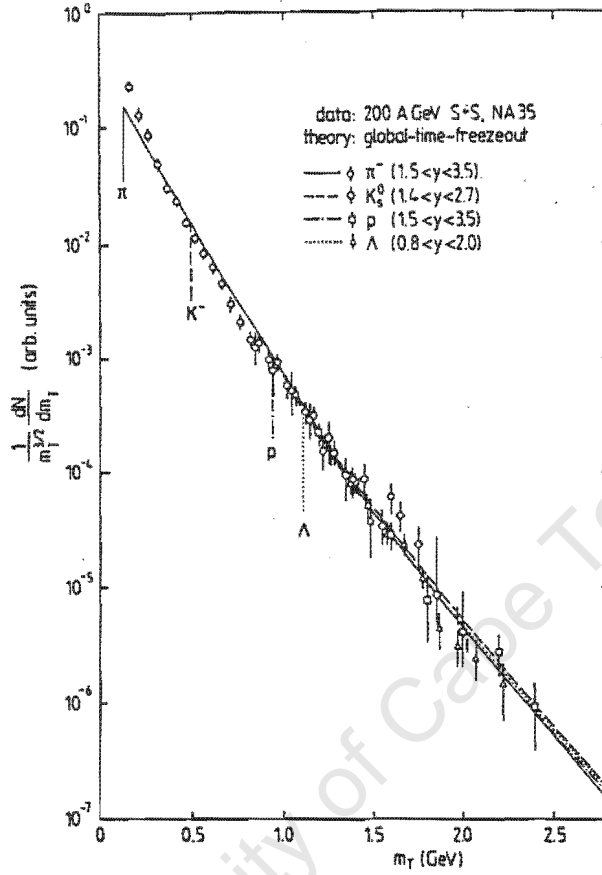


Figure 2.7: The universal curve of $dN/m_T^{3/2} dm_T$ arising from the model of spherically symmetric flow assuming global-time freeze-out applied to suitably normalized data from central 200 A GeV S+S collisions [36].

larly, attempts to describe the CERN SPS rapidity distributions using models of spherical expansion result in spectra that are much narrower than the measured rapidity spectra [36]. This suggests a much stronger longitudinal flow component in these collisions, due either to incomplete stopping, or the strongly anisotropic expansion of a fireball highly compressed in the longitudinal direction [36]. The assumption of cylindrical symmetry is therefore more realistic at highly relativistic energies than that of spherical symmetry, which is best justified in the limit of complete stopping. One of the advantages of using the variables rapidity and transverse mass is that the longitudinal- and transverse flows decouple [44]. The rapidity spectra are largely unaffected by the transverse flow, while the transverse momentum spectra are insensitive to the flow directed along the collision axis.

2.2.4 The Thermal Model Incorporating Cylindrically Symmetric Flow

Rapidity Spectra

As a first attempt at reproducing the measured rapidity spectra, we sum the contributions of isotropic thermal sources within a rapidity interval $[-y'_{\max}, y'_{\max}]$ [44, 60, 50]:

$$\frac{d\tilde{N}_i}{dy} = \int_{-y'_{\max}}^{y'_{\max}} dy' \frac{dN_i(y-y')}{dy} \quad (2.14)$$

where $dN_i(y)/dy$ is given by equation (2.13).

This approach is based on the model of boost-invariant longitudinal expansion of the central rapidity region proposed by Bjorken [61]. The limited beam energy is accounted for in equation (2.14) by the integration limits which restrict boost-invariance to a finite rapidity interval.

Good overall agreement is found with both CERN SPS NA35 and AGS data with $y'_{\max}$ values of 1.7 [44] and 1.15 [50] respectively (see Figure 2.3 for a comparison of this model with experimental AGS data). Only the proton data show a considerable deviation from the model predictions. The width of the proton distribution and the observed dip at central rapidity suggest that the measured spectrum contains a large proportion of protons in the target- and projectile fragmentation regions (due to incomplete stopping) [50, 44]. These protons are not accounted for in this theoretical model which is formulated only for the central rapidity region.

Transverse Momentum Spectra

Although longitudinal flow may dominate in the early stages of a collision, the high pressures also generate transverse flow. Assuming cylindrical symmetry, with the z -axis corresponding to the collision axis of the nuclei, the freeze-out hypersurface is parameterized as

$$\begin{aligned} \sigma^0 &= t_f(r, z) \\ \sigma^1 &= r \cos \phi \\ \sigma^2 &= r \sin \phi \\ \sigma^3 &= z \end{aligned}$$

and from equation (2.4), the normal to the freeze-out hypersurface $d\sigma_\mu$ is given by

$$d\sigma_\mu = \left(1, -\frac{\partial t_f}{\partial r} \cos \phi, -\frac{\partial t_f}{\partial r} \sin \phi, -\frac{\partial t_f}{\partial z} \right) r dr d\phi dz. \quad (2.15)$$

Including both longitudinal- and transverse flow, the 4-velocity of each fireball volume-element, relative to an observer in the center-of-mass frame, is given by (for details see Appendix C)

$$u^\mu = (\cosh \eta \cosh \rho, \sinh \rho \cos \phi, \sinh \rho \sin \phi, \sinh \eta \cosh \rho) \quad (2.16)$$

where ρ and η are the transverse- and longitudinal flow rapidities respectively. In terms of the transverse flow velocity $\beta_T(r)$, the transverse flow rapidity is given by

$$\rho = \tanh^{-1} \beta_T(r). \quad (2.17)$$

A particle with both transverse- and longitudinal momentum has 4-momentum, relative to an observer in the center-of-mass frame, given by

$$p^\mu = (m_T \cosh y, p_T \cos \phi_P, p_T \sin \phi_P, m_T \sinh y) \quad (2.18)$$

where ϕ_P is the azimuthal angle fixing the particle's trajectory in the transverse plane.

Although the general result for the transverse momentum spectrum of hadron species i (including the effects of both longitudinal- and transverse flow) is derived in Appendix C, here we consider the case of instantaneous freeze-out in the r -direction and neglect quantum statistics:

$$\begin{aligned} \frac{dN_i}{p_T dp_T} &= \frac{g_i m_T}{\pi} \int_{\sigma^0(r,z)} r dr dz e^{\mu_i/T} \left(\cosh \eta - \frac{\partial t_f}{\partial z} \sinh \eta \right) \\ &\times K_1 \left(\frac{m_T \cosh \rho}{T} \right) I_0 \left(\frac{p_T \sinh \rho}{T} \right). \end{aligned} \quad (2.19)$$

This factorization of the transverse momentum spectrum is valid only if the temperature and transverse flow are independent of the longitudinal position in a longitudinally comoving frame [44].

In a semi-logarithmic plot, the slope of the transverse momentum spectrum of hadron species i (obtained by fixing r at some value) is

$$\frac{d}{dm_T} \ln \left[\frac{dN_i}{p_T dp_T} \right] = \frac{I_1 \left(\frac{p_T \sinh \rho}{T} \right)}{I_0 \left(\frac{p_T \sinh \rho}{T} \right)} \frac{m_T \sinh \rho}{p_T T} - \frac{K_0 \left(\frac{m_T \cosh \rho}{T} \right)}{K_1 \left(\frac{m_T \cosh \rho}{T} \right)} \frac{\cosh \rho}{T}. \quad (2.20)$$

For large m_T and finite flow, $m_T/p_T \rightarrow 1$, $K_0/K_1 \rightarrow 1$, $I_1/I_0 \rightarrow 1$, and the spectra approach exponential behaviour with a slope given by

$$\begin{aligned} \lim_{m_T \rightarrow \infty} \frac{d}{dm_T} \ln \left(\frac{dN_i}{p_T dp_T} \right) &= -\frac{\cosh \rho - \sinh \rho}{T} \\ &= -\frac{1}{T} \sqrt{\frac{1 - \langle \beta_T \rangle}{1 + \langle \beta_T \rangle}}. \end{aligned} \quad (2.21)$$

At high m_T ($m_T \gg m_0$), the apparent temperature, corresponding to the inverse slope parameter of the transverse momentum spectrum, is larger than the physical freeze-out temperature T_{therm} by a blue-shift factor:

$$T_{\text{apparent}} = T_{\text{therm}} \sqrt{\frac{1 + \langle \beta_T \rangle}{1 - \langle \beta_T \rangle}} \quad (p_T \gg m_0) \quad (2.22)$$

where $\langle \beta_T \rangle$ should be interpreted as the average transverse flow velocity across the freeze-out hypersurface [62]:

$$\langle \beta_T \rangle = \frac{\int_0^\infty dr \, r \tanh \rho(r) C(r)}{\int_0^\infty dr \, r C(r)} \quad (2.23)$$

with $C(r)$ the transverse density distribution. The apparent temperature at high m_T , given by equation (2.22), is the same for all particles and does not allow the unambiguous extraction of the physical freeze-out temperature and the flow velocity.

Since the transverse velocity profile generally has some r -dependence, the m_T -spectrum of a particular species is a superposition of spectra with different blue-shift factors. This leads to a noticeable deviation from exponential behaviour in the intermediate m_T region, where the m_T -spectra show a distinctly concave curvature. This is clearly seen in Figure 2.8 where the effects of both resonance decays and transverse flow on the pion transverse mass spectrum are shown.

In the non-relativistic domain, transverse flow increases the apparent temperature of heavy particles more than that of light particles [36]. For $p_T \ll m_0$, the apparent temperature is approximately [13]

$$T_{\text{apparent}} = T_{\text{therm}} + m_0 \langle \beta_T^2 \rangle \quad (p_T \ll m_0) \quad (2.24)$$

accounting for the experimentally observed linear increase of the inverse slope parameter with particle mass.

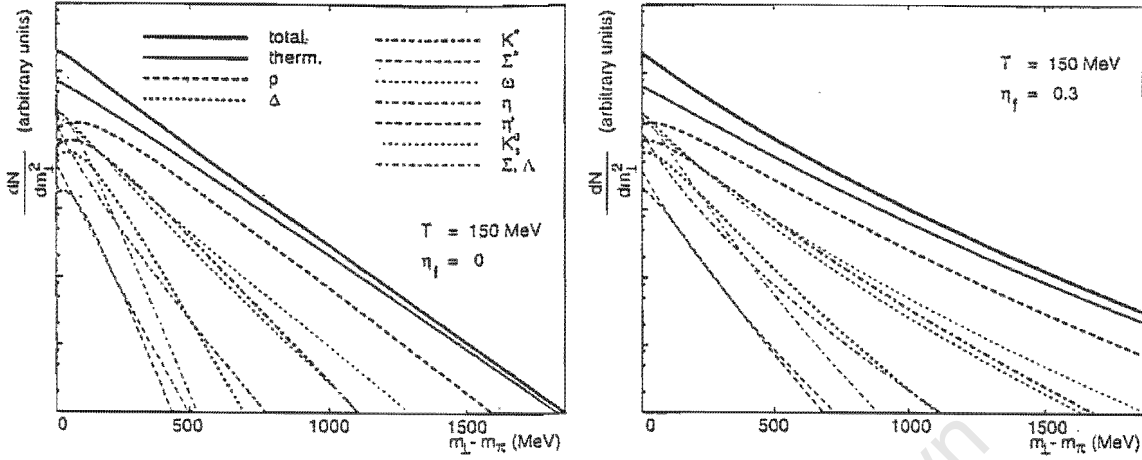


Figure 2.8: The single-pion transverse mass spectrum for $T = 150$ MeV and $\mu_B = \mu_S = 0$ with resonance decay contributions [54]. The left panel is for no transverse flow, whereas the right panel gives the spectrum assuming a linear transverse flow rapidity profile with a surface flow rapidity of 0.3.

It has been shown in several publications (e.g. [50, 63, 64, 44, 62]) that, with an appropriate choice of velocity profile $\beta_T(r)$, the single-particle transverse momentum spectra can indeed be described by such a thermal model incorporating transverse flow. In these models, freeze-out is described by just two parameters; a common thermal freeze-out temperature T_{therm} , and a common flow velocity parameter $\langle\beta_T\rangle$ that characterizes the strength of the flow velocity. However, there is considerable ambiguity in the results of such analyses. In a free fit of equation (2.19) to the observed spectra, the compatible values of T_{therm} and $\langle\beta_T\rangle$ are anti-correlated [62] (high- T_{therm} , low- $\langle\beta_T\rangle$ combinations fit the spectra as well as low- T_{therm} , high- $\langle\beta_T\rangle$ ones do), leading to shallow minima in the goodness-of-fit parameter in the $T_{\text{therm}} - \langle\beta_T\rangle$ parameter space [63]. In order to extract the thermal freeze-out parameters reliably, independent information on the flow velocity is required. Two-particle Bose-Einstein correlations provide one such means of obtaining this information and constraining the results [65]. The ambiguity in the results can also be resolved by requiring consistency of the freeze-out parameters with the dynamical evolution of the system, determined by hydrodynamics, until the point of freeze-out. The kinetic freeze-out criterion provides a direct correlation of the parameters (i.e. large flow leads to earlier freeze-out and, hence, higher T_{therm}) and, together with the free-fit analysis, tightly constrains the parameters [36, 64]. When extracting thermal freeze-out parameters, resonance decays should be taken into account since they have a significant effect on both single-particle momentum distributions [49, 54, 66] and two-particle

Bose-Einstein correlations [67, 53, 54, 68].

2.3 Particle Multiplicities and Chemical Freeze-out

Having investigated the spectral shapes measured in relativistic heavy ion collisions and found them to be consistent with the thermal model incorporating resonance decays and collective flow, we turn now to particle multiplicities to test the assumption of chemical equilibrium at chemical freeze-out. Such an analysis has been performed by many authors, using models which differ slightly in their assumptions and extensions of the basic thermal model. In this section, we discuss some of the most important modifications and results of the thermal model applied in the analysis of particle multiplicities. For a more detailed account see [69], which includes a comparison of the results obtained by the various groups active in this field.

2.3.1 Choice of Ensemble

Within the thermal model, there is a freedom regarding the ensemble with which to treat the quantum numbers B , S and Q which are conserved in strong interactions. As previously stated, the introduction of chemical potentials for each of these quantum numbers (i.e. a grand-canonical description) allows fluctuations about conserved averages. This is a reasonable approximation only when the number of particles carrying the quantum number concerned is large. In applications of the thermal model to high energy elementary collisions, such as pp , $p\bar{p}$ and e^+e^- collisions, a canonical treatment of each of the quantum numbers is required. Within such a canonical description, quantum numbers are conserved exactly. In small systems or at low temperatures (more specifically, low VT^3 values), a canonical treatment leads to a suppression of hadrons carrying non-zero quantum numbers, since these particles have to be created in pairs. In heavy ion collisions, the large number of baryons and charged particles allows baryon number and charge to be treated grand-canonically. However, at the low temperatures of the GSI SIS, the resulting low production of strange particles requires a canonical treatment of strangeness [70]. Particle production within a canonical framework was studied in [71].

2.3.2 Strangeness Suppression

The hadron gas model applied to elementary e^+e^- , pp and $p\bar{p}$ collisions [72, 73] indicates the need for an additional parameter, γ_S [74], to account for the observed deviation from chemical equilibrium in the strange sector. Since a canonical ensemble was considered in these analyses, there is an additional strangeness suppression at work, on top of the canonical suppression. Although strangeness production is expected to be greatly increased in AA collisions, due to the larger interaction region and increased hadron rescattering, we allow for possibly incomplete strangeness equilibration by multiplying the Boltzmann factors in the partition function (or thermal distribution function $f_i(x, p)$) of each particle species by $\gamma_S^{|S_i|}$, where $|S_i|$ is the number of valence strange quarks and anti-quarks in species i . The value $\gamma_S = 1$ corresponds to complete strangeness equilibration.

This γ_S factor was first introduced by Rafelski as a purely phenomenological parameter [74]. Its inclusion is justified by the short lifetime and rapid dynamics of relativistic collisions, which may prevent complete equilibration. In fact, thermal models treating strangeness canonically and including no additional strangeness suppression are unable to fully account for the observed strange particle yields in heavy ion collisions at CERN SPS [75]. In particular, the below-equilibrium ϕ production, unaffected by canonical suppression, offers strong evidence for the need for significant strangeness suppression at the hadronic level.

2.3.3 Primary Particle Yields

Within the grand-canonical ensemble (a good approximation for the large interaction volumes and high temperatures formed in CERN SPS Pb+Pb collisions), the number of particle species i ($i = \pi^+, K^+, \dots$), with mass m_i , spin-isospin degeneracy g_i , strangeness content $|S_i|$, and chemical potential μ_i , emitted directly from a stationary fireball of volume V and temperature T is given by

$$\begin{aligned}
 N_i^{0(\text{prim})} &= g_i V \int \frac{d^3p}{(2\pi)^3} \frac{1}{\gamma_S^{-|S_i|} e^{(E-\mu_i)/T} \pm 1} \\
 &= V \left(\frac{g_i}{2\pi^2} \sum_{k=1}^{\infty} (\mp 1)^{k+1} \left(\gamma_S^{|S_i|} \right)^k \frac{m_i^2 T}{k} K_2 \left(\frac{km_i}{T} \right) e^{k\mu_i/T} \right) \quad (2.25) \\
 &= V n_i^{0(\text{prim})}
 \end{aligned}$$

where the upper (lower) signs refer to fermions (bosons) (see Appendix B for a derivation of equation (2.25)). In most applications of the thermal model to heavy ion collisions, the Boltzmann approximation (retaining only the $k=1$ term in equation (2.25)) is valid, except for the low-mass pions, for which Bose-Einstein statistics should be used.

Particle multiplicities from flowing sources are derived by dividing the Cooper-Frye formula (equation (2.1)) by E , and integrating over d^3p :

$$N_i^{(\text{prim})} = \frac{g_i}{(2\pi)^3} \int_{\sigma_f} d\sigma_\mu \int d^3p \frac{p^\mu}{E} f_i(x, p). \quad (2.26)$$

Provided the temperature, chemical potentials and strangeness suppression factor γ_S are constant over the freeze-out hypersurface (or else interpreting the freeze-out parameters as global averages) [76],

$$N_i^{(\text{prim})} = \int_{\sigma_f} d\sigma_\mu u^\mu n_i^{0(\text{prim})}. \quad (2.27)$$

This leads to the identification of $\int_{\sigma_f} d\sigma_\mu u^\mu$ as the dynamical (effective) volume of the system (i.e. the sum of the volumes of each fireball element measured in their own rest frames at freeze-out):

$$V_{\text{eff}} = \int_{\sigma_f} d\sigma_\mu u^\mu. \quad (2.28)$$

A consequence of this result is that all flow effects (contained in the dynamical volume) cancel in the ratio of fully-integrated particle multiplicities. It is therefore possible to apply a purely thermal model to fully-integrated data, without any consideration of dynamical effects [76] (i.e. particle ratios are the same as those for a static thermal distribution). One of the reasons for this is that fully-integrated particle yields are Lorentz-invariant. A similar result follows when one considers the superposition of fireballs along the rapidity axis (Section 2.2.4) [76]. As an example, the dependence of the K^+/π^+ ratio on the thermal parameters is shown in Figure 2.9.

2.3.4 Feeding from Unstable Particles

Since the particle yields measured by the detectors include possible feed-down from heavier hadrons and hadronic resonances, the primary hadrons are allowed to decay to particles considered stable by the experiment before

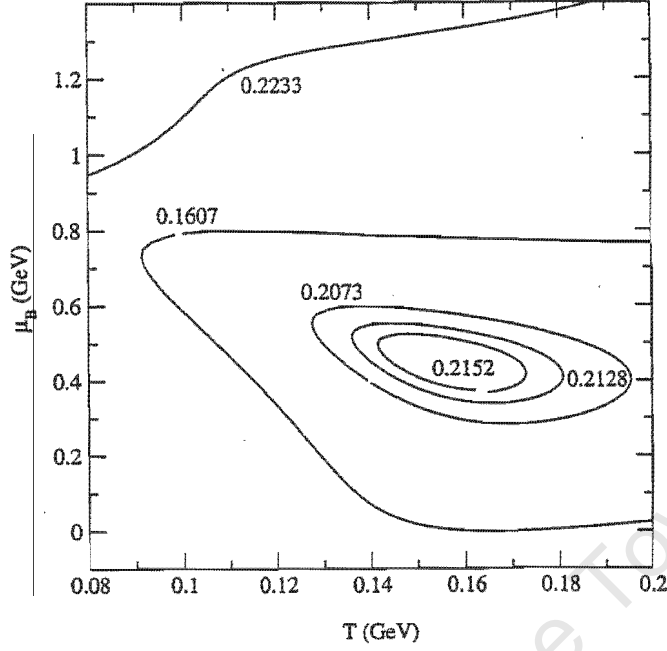


Figure 2.9: Contour plot of the K^+/π^+ ratio in the $T - \mu_B$ plane. The discontinuity corresponds to the onset of Bose-Einstein condensation.

model predictions are compared with experimental data. For example, the total π^+ yield is given by

$$N_{\pi^+} = \sum_i N_i^{(\text{prim})} Br(i \rightarrow \pi^+) \quad (2.29)$$

where $Br(i \rightarrow \pi^+)$ is the branching ratio of the decay of hadron species i into π^+ 's.

2.3.5 Resonances and the Inclusion of Resonance Width

The inclusion of a mass cut-off in the measured resonance mass spectrum is motivated by the realization that the time scale of a relativistic collision does not allow the heavier resonances to reach chemical equilibrium [49]. This assumes that inelastic collisions drive the system to chemical equilibrium. If the hadronization process follows a statistical rule, then all resonances should, in principle, be included [69]. This is problematic, since data on the heavy resonances is sketchy. The situation is saved by the finite energy of the system, resulting in a chemical freeze-out temperature at SPS of approximately 170 MeV which strongly suppresses these heavy resonances and justifies their exclusion from the model. It is, however, important to check the sensitivity

of the extracted thermal parameters to the chosen cut-off.

The finite width of the resonances is especially important at the low temperatures of the SIS. Resonance widths are included in the thermal model by distributing the resonance masses according to Breit-Wigner forms [72, 73, 75, 77, 70, 49]. This amounts to the following modification in the integration of the Boltzmann factor [70]:

$$\begin{aligned} & \int d^3p \exp \left[-\frac{\sqrt{p^2 + m^2}}{T} \right] \\ & \rightarrow \int d^3p \int ds \exp \left[-\frac{\sqrt{p^2 + s}}{T} \right] \frac{1}{\pi} \frac{m\Gamma}{(s - m^2)^2 + m^2\Gamma^2} \end{aligned} \quad (2.30)$$

where Γ is the width of the resonance concerned, with threshold limit $m_{\text{threshold}}$ and mass m , and $\sqrt{s}$ is integrated over the interval $[m - \delta m, m + 2\Gamma]$ where $\delta m = \min[m - m_{\text{threshold}}, 2\Gamma]$.

2.3.6 Repulsive Interaction Between Hadrons

As will be discussed in Section 2.5.1, chemical freeze-out at CERN SPS occurs at a temperature of approximately 170 MeV. As such, the particle density is still relatively large and it is necessary to investigate the effects of density corrections on the particle ratios.

In [78] and [79], the effect of the strong, short-range, repulsive interactions between the hadrons is accounted for by means of an excluded-volume correction, in which each hadron effectively removes a portion of the available spatial volume. The corrected particle density n_i^{phy} of hadron species i is then given by

$$n_i^{\text{phy}} = \alpha^{-1} n_i^0 \quad (2.31)$$

where n_i^0 represents the density calculated for an ideal gas of point-like hadrons. The correction factor α is either determined by the total point-particle energy density ϵ_0 and the bag constant B through $\alpha = 1 + \epsilon_0/4B$ [78], or by $\alpha = 1 + \sum_j V_j^0 n_j^0$ where V_j^0 is the hard-core volume of hadron species j [79]. These corrections leave particle ratios unaffected, since the same factor is applied to each particle species. These models have subsequently been found to be thermodynamically inconsistent and replaced by new models, including that of Rischke et al. [80].

2.4 Hydrodynamics and Flow

Growing evidence for collective flow triggered interest in describing the reaction region formed in relativistic heavy ion collisions within a hydrodynamical framework. The application of hydrodynamics requires local equilibrium. In relativistic heavy ion collisions, this is at best reached after some initial thermalization time, provided sufficient rescattering occurs amongst the reaction products. If the equation of state of the matter is known, the hydrodynamic equations completely describe the space-time evolution of the system from this initial thermalization time until thermal freeze-out, when local equilibrium can no longer be maintained. In a multi-phase- or chemically unequilibrated, multi-component fluid, an additional constraint is required. The simplest choice is to impose entropy conservation throughout the system's evolution.

Defining the local energy density $e(x)$ and pressure $P(x)$, the energy-momentum tensor of an ideal fluid is given by

$$T^{\mu\nu} = (e + P)u^\mu u^\nu - Pg^{\mu\nu} \quad (2.32)$$

where u^μ is the 4-velocity of the fluid rest frame (i.e. the frame in which the energy flux is zero) and $g^{\mu\nu}$ is the metric tensor. Neglecting viscosity and thermal conductivity, the energy-momentum tensor is conserved:

$$\partial_\mu T^{\mu\nu} = 0. \quad (2.33)$$

Now consider a locally conserved scalar quantity N , with associated density n . In the case of a perfect fluid, energy flux is always accompanied by scalar flux. Thus, the flux of any conserved quantity is zero in the rest frame of the fluid, where the energy flux is, by definition, zero. This leads to the local conservation law

$$\partial_\mu (nu^\mu) = 0. \quad (2.34)$$

In relativistic heavy ion collisions, the baryon number, charge and strangeness content, with densities ρ_b , ρ_q and ρ_s respectively, are conserved. This leads to three conservation equations:

$$\partial_\mu (\rho_{b,q,s} u^\mu) = 0. \quad (2.35)$$

Equations (2.33) and (2.35) are Lorentz-invariant and, together with the equation of state, provide a closed system for an initial value problem. In ultra-relativistic heavy ion collisions, two distinct phase-space regions exist; the fragmentation regions (carrying the baryon content of the system), and

the central rapidity region (containing mostly mesons). These regions are vastly different in structure and, consequently, in equation of state. For this reason, hydrodynamical models treat these regions separately.

As an example of a hydrodynamical model, we consider the Bjorken model [61] which treats the space-time evolution of the central rapidity region assuming a one-dimensional (longitudinal) flow along the collision axis. This assumption is valid for times small compared to the radius of the nucleus, whereafter the system is expected to undergo 3-D expansion with a short evolution into the final state hadrons [61].

2.4.1 The Bjorken Hydrodynamical Model

Early results from hadron-hadron collisions at CERN SPS and $p\alpha$ collisions at CERN ISR revealed a “central plateau” structure in the charged particle rapidity spectra, suggesting an invariance under Lorentz transformations of particle production at mid-rapidity [61]. This invariance reflects a symmetry of the system which is assumed to be achieved at the time of thermalization and preserved by the subsequent hydrodynamic flow of the system [61]. As a result, all observables and thermodynamical variables depend only on the Lorentz-invariant proper time τ given by

$$\tau = \sqrt{t^2 - z^2} \quad (2.36)$$

where it is assumed that each fluid element originates from the point of impact of the colliding nuclei. A fluid element a longitudinal distance z from the collision point after a time t then has a longitudinal velocity $v = z/t$. Therefore,

$$\begin{aligned} e &= e(\tau) \\ P &= P(\tau) \\ T &= T(\tau) \end{aligned}$$

and the 4-velocity of the fluid is given by

$$u^\mu = \frac{1}{\tau}(t, 0, 0, z). \quad (2.37)$$

The hydrodynamic equations (2.33) lead to

$$\frac{de}{d\tau} = -\frac{e + P}{\tau} \quad (2.38)$$

subject to the initial condition

$$e(\tau_0) = e_0 \quad (2.39)$$

imposed at the point of thermalization, generically assumed to be reached a proper time of $\tau_0 = 1 \text{ fm/c}$ after the collision.

In order to solve equation (2.38), an equation of state is required relating the pressure and energy density. Irrespective of the equation of state, the first law of thermodynamics with vanishing chemical potentials ($e = Ts - P$) gives

$$\frac{ds}{d\tau} = \frac{-s}{\tau} \quad (2.40)$$

with solution

$$s(\tau)\tau = s(\tau_0)\tau_0. \quad (2.41)$$

The consequence of this result is that the entropy per unit rapidity is conserved, since the volume of a slab is given by $\tau dy d^2x_\perp$ in a frame where the fluid is at rest, as will be shown later.

In order to formulate a complete model for the central rapidity region, transverse flow has to be included. As is well known, the presence of transverse flow greatly alters the appearance of the p_T -spectra and is heavily dependent on the equation of state [81, 82]. In [82], the effect of the equation of state and the inclusion of a QGP phase transition on particle- and photon production at CERN energies was investigated. A boost-invariant Bjorken model was used for the longitudinal flow, supplemented with cylindrically symmetric transverse expansion (referred to as the modified Bjorken model hereafter). When included, the QGP transition was assumed to proceed isentropically, and hadrons were considered to freeze out once the temperature of their environment dropped below a critical value. Two models for the hadronic phase were considered. In the first, the hadronic medium was assumed to consist only of π, ρ, ω and η mesons (the simplified equation of state), while in the second, all hadrons and hadronic resonances were included (the full equation of state). Using the simplified equation of state, the average transverse momentum $\langle p_T \rangle$ of all particles and photons was shown to decrease substantially with the inclusion of a phase transition to QGP [82]. The inclusion of only the lightest mesons is, however, unrealistic at the high temperatures typically reached in heavy ion collisions, where production of heavier particles and resonances cannot be ignored. Comparison of the p_T -spectra of particles and photons assuming the full equation of state, with and without a transition to QGP, showed only a modest difference. From

these results it was concluded that the measured p_T -spectra are incapable of signaling the presence of QGP in the initial state if the hadronic phase can be approximated by the full equation of state [82]. Instead, it is the high initial particle density inferred at the CERN SPS that strongly suggests a QGP initial state.

2.4.2 The Modified Bjorken Model Applied to the Hadron Gas Model

The boost-invariance of the hydrodynamic equations is guaranteed if the longitudinal flow rapidity η corresponds to the space-time rapidity Y , defined by

$$Y \equiv \tanh^{-1} \left(\frac{z}{t} \right). \quad (2.42)$$

This condition is met in the Bjorken model by the particular choice of longitudinal flow velocity $v = z/t$. In terms of the space-time rapidity,

$$z = \tau \sinh Y \quad (2.43)$$

and

$$t = \tau \cosh Y. \quad (2.44)$$

Since freeze-out is Y independent,

$$t_f(r, z) = \sqrt{\tau_f^2(r) + z^2} \quad (2.45)$$

and

$$\frac{\partial t_f}{\partial z} = \frac{z}{\sqrt{\tau_f^2(r) + z^2}} = \tanh \eta \quad (2.46)$$

$$\frac{\partial t_f}{\partial r} = \frac{\tau_f(r)}{\sqrt{\tau_f^2(r) + z^2}} \frac{\partial \tau_f(r)}{\partial r} = \frac{1}{\cosh \eta} \frac{\partial \tau_f(r)}{\partial r}, \quad (2.47)$$

while from equation (2.43), it follows for fixed r that

$$dz = \tau(r) \cosh \eta d\eta. \quad (2.48)$$

Substituting equations (2.46)–(2.48) into equation (C.10) with the γ_S factor

included (since we are here concerned with particle multiplicities fixed at chemical freeze-out), we obtain, after integrating over η ,

$$\begin{aligned} \frac{d^2 N_i^{Bj}}{dy p_T dp_T} &= \frac{g_i}{\pi} \sum_{k=1}^{\infty} (\mp 1)^{k+1} \left(\gamma_S^{|S_i|} \right)^k \int_{\sigma^0(r,z)} dr r \tau_f(r) e^{k\mu_i/T} \\ &\times \left\{ m_T K_1(k\alpha/T) I_0(k\tilde{\alpha}/T) \right. \\ &\quad \left. - p_T \frac{\partial \tau_f(r)}{\partial r} K_0(k\alpha/T) I_1(k\tilde{\alpha}/T) \right\} \end{aligned} \quad (2.49)$$

where $\alpha \equiv m_T \cosh \rho(r)$ and $\tilde{\alpha} \equiv p_T \sinh \rho(r)$, from which it follows that

$$\begin{aligned} \frac{dN_i^{Bj}}{dy} &= \frac{g_i}{\pi} \sum_{k=1}^{\infty} (\mp 1)^{k+1} \left(\gamma_S^{|S_i|} \right)^k \int_{\sigma^0(r,z)} dr r \tau_f(r) e^{k\mu_i/T} \\ &\times \left\{ \int_0^{\infty} dp_T p_T m_T K_1(k\alpha/T) I_0(k\tilde{\alpha}/T) \right. \\ &\quad \left. - \int_0^{\infty} dp_T p_T^2 \frac{\partial \tau_f(r)}{\partial r} K_0(k\alpha/T) I_1(k\tilde{\alpha}/T) \right\}. \end{aligned} \quad (2.50)$$

Using

$$\begin{aligned} \int_0^{\infty} \frac{x^{\mu+1}}{(x^2 - y^2)^{\nu/2}} I_{\mu}(bx) K_{\nu}(c\sqrt{x^2 - y^2}) dx = \\ \frac{\pi b^{\mu}}{2 c^{\nu}} y^{1+\mu-\nu} e^{-i\pi(\nu-\mu-1/2)} \times (c^2 - b^2)^{(\nu-\mu-1)/2} H_{\nu-\mu-1}^{(2)}(y\sqrt{c^2 - b^2}) \end{aligned}$$

together with

$$K_{-\nu}(z) = K_{\nu}(z)$$

and

$$K_{\nu}(z) = -\frac{\pi}{2} i e^{-\nu\pi i/2} H_{\nu}^{(2)}(ze^{-\pi i/2}),$$

the integration over p_T results in

$$\begin{aligned} \frac{dN_i^{Bj}}{dy} &= \int_{\sigma^0(r,z)} dr 2\pi r \tau_f(r) \left(\cosh \rho(r) - \frac{\partial \tau_f(r)}{\partial r} \sinh \rho(r) \right) \\ &\times \left\{ \frac{g_i}{2\pi^2} \sum_{k=1}^{\infty} (\mp 1)^{k+1} \left(\gamma_S^{|S_i|} \right)^k \frac{m_i^2 T}{k} K_2(km_i/T) e^{k\mu_i/T} \right\} \end{aligned} \quad (2.51)$$

with the remarkable consequence [76] that

$$\frac{(dN_i^{Bj}/dy)}{(dN_j^{Bj}/dy)} = \frac{N_i^0}{N_j^0}, \quad (2.52)$$

provided the temperature, chemical potentials and strangeness suppression factor γ_S are the same everywhere on the freeze-out hypersurface.

Comparison of equation (2.51) with equation (2.25), leads to the interpretation of the following expression as the volume in the Bjorken model:

$$V^{\text{Bj}} = 2\pi \int dy \int_{\sigma^0(r,z)} dr r \tau_f(r) \left(\cosh \rho(r) - \frac{\partial \tau_f(r)}{\partial r} \sinh \rho(r) \right) \quad (2.53)$$

which, for instantaneous proper-time freeze-out (i.e. $\frac{\partial \tau_f}{\partial r} = 0$), reduces to

$$V^{\text{Bj}} = \pi R^2 \tau_f \Delta y \quad (2.54)$$

where

$$R^2 \equiv \int_{\sigma^0(r,z)} dr 2r \cosh \rho(r). \quad (2.55)$$

2.5 Results of Thermal Fits

Thermal models have enjoyed considerable success in describing hadron production in both heavy ion- and elementary collisions over a wide range of collision energies (for a review see [69]). In recent years, a universal picture of the freeze-out process has begun to emerge [83, 76, 77].

2.5.1 Chemical Freeze-out Parameters

Going from SIS energies of 1 A GeV to SPS energies of 200 A GeV, the thermal parameters describing the collision systems at chemical freeze-out vary considerably; the temperature increases from 50 MeV to 160 MeV, whereas the baryon chemical potential decreases from 820 MeV to 240 MeV [77]. Despite this wide variation, the thermal parameters are consistent with an average energy per hadron at freeze-out of 1 GeV, independent of both the collision energy and the colliding system [83, 76]. This universality of the chemical freeze-out parameters is displayed in Figure 2.10. A study of the excitation functions of particles produced at AGS energies confirms that increased beam momentum leads primarily to increased particle production, rather than the generation of additional transverse momentum [84]. The observed decrease of the baryon chemical potential with increasing beam energy signals increasing transparency. In heavy ion collisions, the strangeness suppression factor γ_S is consistent with a value of 0.7–0.8 [77], although the large errors in the extracted values allow the possibility of a fully equilibrated

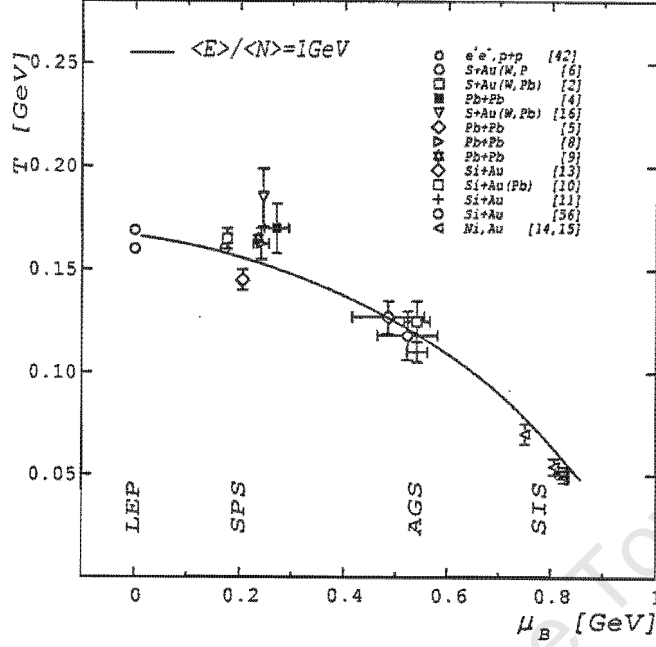


Figure 2.10: The universality of the chemical freeze-out parameters T_{chem} and μ_B [76].

hadron gas to be excluded only at CERN SPS energies (see Figure 2.11) [77].

Remarkably, hadron production in elementary e^+e^- , pp and $p\bar{p}$ collisions can also be described by the thermal model within the canonical formalism [72, 73]. Owing to the small particle densities produced in such elementary collisions, this thermal behaviour can only be understood if it is assumed that the hadronization process active in such systems leads directly to a statistical filling of hadronic phase-space [13, 73, 35]. The extracted chemical freeze-out temperature of approximately 170 MeV is independent of both the collision energy and the collision system [73]. This suggests that hadronization in such elementary collisions occurs at critical parameters of the prehadronic matter [73, 75]. The temperature of 170 MeV is consistent with lattice QCD predictions of the critical temperature of hadronic matter, as well as with the limiting (“Hagedorn”) temperature of an equilibrated hadron gas [73].

A thermal analysis of heavy ion results at CERN SPS reveals a similar temperature to that derived from elementary collisions [75, 77]. Together with the independence of the chemical freeze-out temperature on the collision system (S+S or Pb+Pb) [75], this suggests that the same mechanism of hadronization at equilibrium levels is present in heavy ion collisions at this

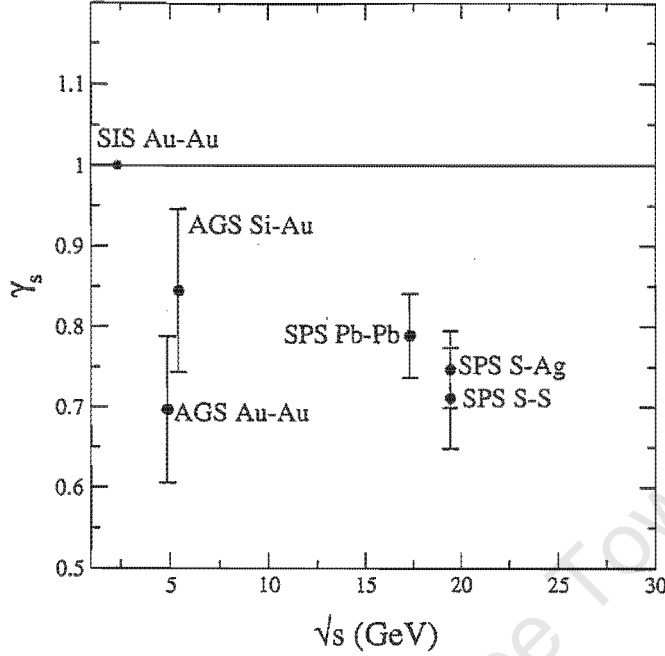


Figure 2.11: The strangeness suppression factor γ_s as a function of nucleon-nucleon center-of-mass energy [77].

energy. The chemical composition of the system formed at CERN SPS energies is largely unaffected by hadronic final state rescattering [85, 86, 87]. If rescattering effects were responsible for driving the system towards chemical equilibrium, the freeze-out temperature in the larger Pb+Pb system would be lower, since it would take this system longer to satisfy the kinetic freeze-out criterion [13]. The near chemical equilibrium levels of the multi-strange hyperons and anti-hyperons (i.e. the Ξ and Ω) at CERN SPS, which cannot be due to rescattering on the short time scale of these collisions, lends further support to the notion of statistical filling of phase-space. It would appear that particle multiplicities at AGS are also fixed very soon after hadronization (since $T_{\text{chem}} = 130\text{--}140$ MeV [35]). At SIS however, chemical freeze-out occurs at a much lower temperature than hadronization. This suggests that the longer lifetime of the system produced at this lower energy allows for kinetic equilibration.

The similarity between heavy ion- and elementary collisions ends when one considers strangeness production. Relative to elementary collisions, heavy ion collisions exhibit considerable strangeness enhancement (in addition to that brought about by the relaxation of canonical suppression) [75]. Strangeness production is best studied by calculation of the Wróblewski factor λ_s [88] which measures the ratio of newly created $s\bar{s}$ pairs to newly

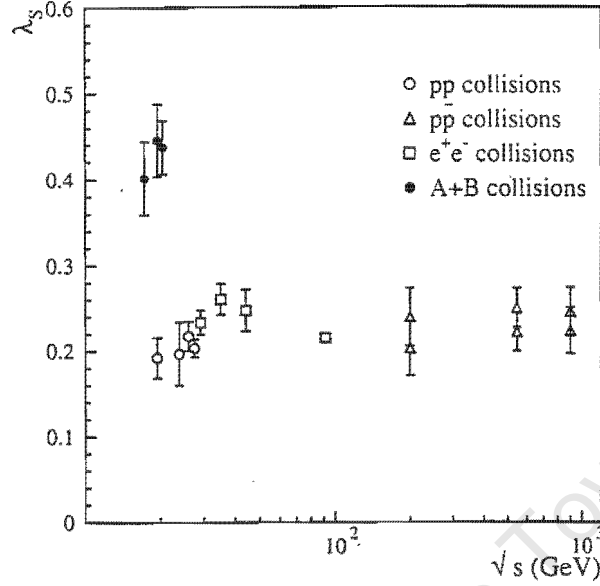


Figure 2.12: The strangeness suppression factor λ_S in high energy collisions as a function of center-of-mass energy [75].

created non-strange valence quark pairs at the primary hadron level:

$$\lambda_S = \frac{2\langle s\bar{s} \rangle}{\langle u\bar{u} \rangle + \langle d\bar{d} \rangle}. \quad (2.56)$$

The quark strangeness suppression factor λ_S in heavy ion collisions is approximately a factor of two greater than the value of 0.2 observed in elementary collisions [75]. Figure 2.12 displays the factor λ_S as a function of center-of-mass energy. In terms of the parameter γ_S , this translates into a value of 0.5 in elementary collisions [73], compared with the 0.7–0.8 observed in heavy ion collisions [77]. There is convincing evidence that strangeness enhancement in Pb+Pb collisions at CERN SPS increases with the number of strange quarks a hadron contains (refer to Figure 2.13) [41, 29]. Such an observation rules out the kinetic equilibration of strangeness by final state interactions, in which multi-strange particles are suppressed owing to higher thresholds [35]. The universality of γ_S from S+S to Pb+Pb collisions at CERN SPS [75] further illustrates the minor importance of secondary hadron rescattering on strangeness production at this energy [75]. Instead, the observed strangeness enhancement must be due to a fast strangeness production mechanism in the prehadronic stage of these heavy ion collisions (such as gluon-gluon interactions in a QGP) which is not present in elementary collisions [35].

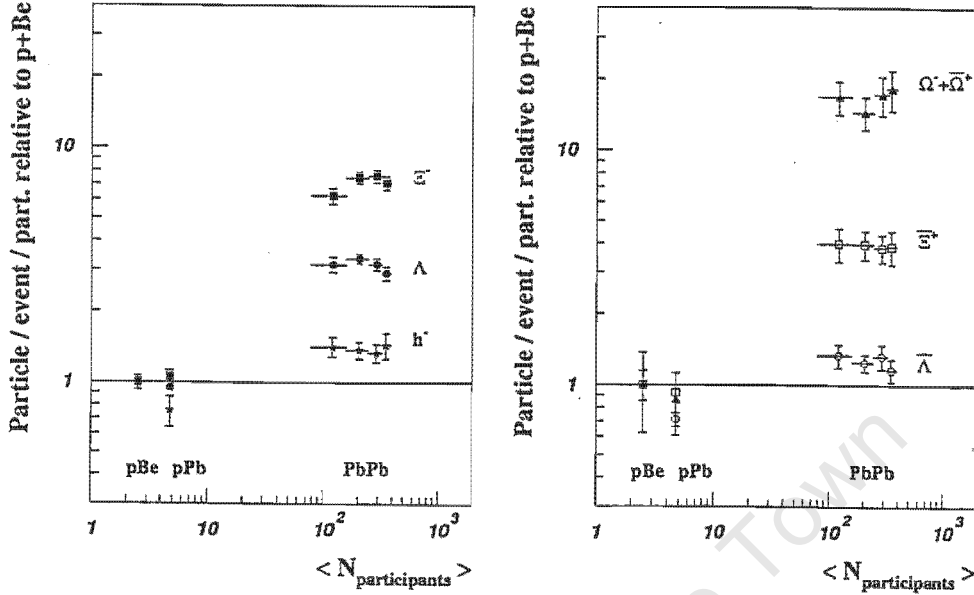


Figure 2.13: Yields per unit rapidity per participant relative to p+Be collisions [41].

2.5.2 Thermal Freeze-out Parameters

Thermal freeze-out parameters are extracted from single-particle momentum spectra and two-particle momentum correlations. Either one performs a simultaneous fit to single-particle m_T -spectra of different species [63], in which one has to assume that these hadrons all freeze out at the same point, or one combines the spectra and correlations of a single particle species [62]. Since the system has by this stage already departed from chemical equilibrium, the baryon chemical potential μ_B is not well defined at this point. In order to plot thermal freeze-out points in the nuclear phase diagram, it is assumed that the fireball evolution to thermal freeze-out is isentropic. In this way, Cleymans et al. [76] combined chemical- and thermal freeze-out parameters in a single diagram (Figure 2.14).

Kinetic models agree that most of the entropy is produced early in a collision [89]. At SPS and AGS [89], these models predict an initial non-equilibrium stage lasting 8–10 fm/c. This is followed by a stage of approximate local thermal equilibrium (lasting for approximately 10 fm/c), in which there is a strong build-up of collective flow at roughly constant entropy. It is intense elastic rescattering that drives the system to this thermalization. The ratios of entropy to baryon number extracted from kinetic analyses [89] agree

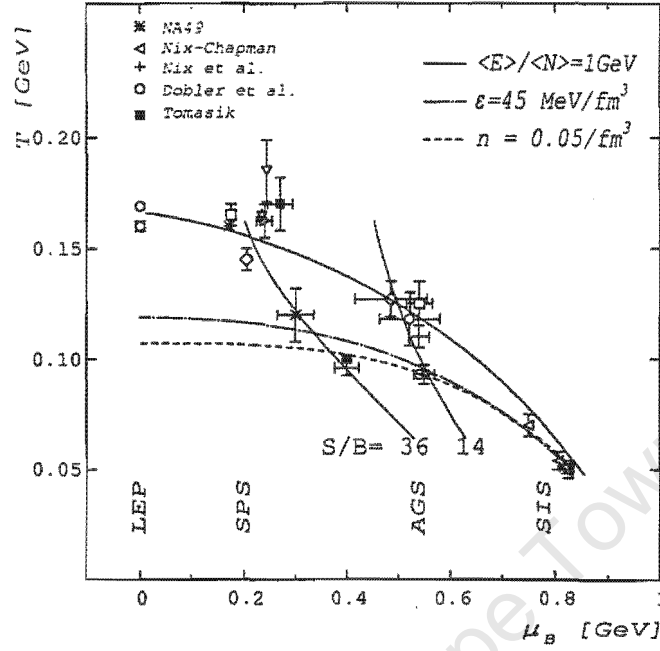


Figure 2.14: Thermal freeze-out and chemical freeze-out curves [76].

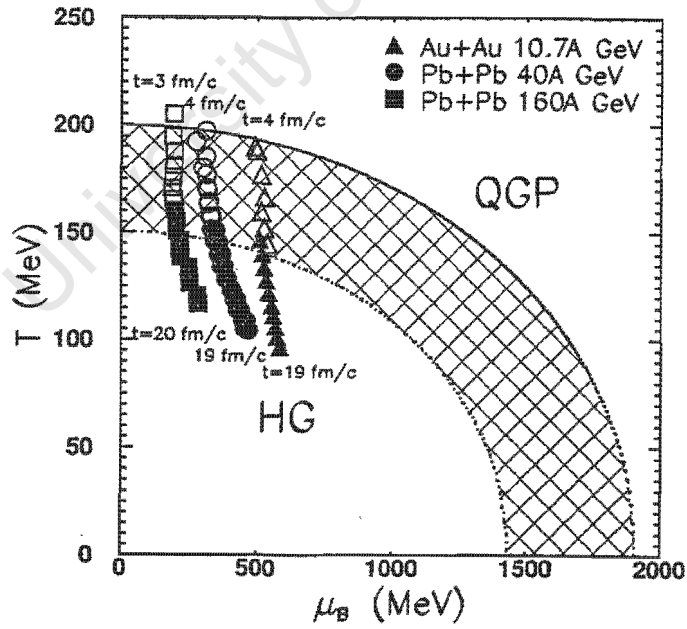


Figure 2.15: Expansion trajectories from URQMD simulations [89].

with those predicted by the thermal model.

In [66], transverse momentum spectra of identified hadrons produced in πp , pp , Kp and e^+e^- collisions were analysed within a statistical model incorporating resonance decays. In pp collisions at $\sqrt{s} = 27.4$ GeV, the extracted thermal freeze-out temperature is approximately 162 MeV, while at $\sqrt{s} = 21.7$ GeV, the temperature in π^+p and K^+p collisions increases to around 175 MeV [66]. The good agreement between the temperature extracted by fitting the transverse momentum spectra and that obtained from a multiplicity analysis suggests that thermal- and chemical freeze-out occur in quick succession in such elementary collisions between 20 and 30 GeV [66].

It is clear from interferometry [62] that the systems formed in ultra-relativistic heavy ion collisions expand greatly prior to freeze-out. In thermal model analyses, this is evident in the large differences between the thermal- and chemical freeze-out temperatures at SPS- and AGS energies, while thermal- and chemical freeze-out occur in quick succession at SIS. At CERN SPS, thermal freeze-out in the large Pb+Pb system is reached later than in the smaller S+S system [13]. This suggests that larger systems evolve collectively for longer, and so freeze out at lower temperatures [13].

Figure 2.16 shows the thermal freeze-out temperature and average transverse expansion velocity $\langle\beta_T\rangle$ extracted from data over a wide range of collision systems and energies [90]. As the collision energy and system size increase, the average transverse flow at freeze-out is seen to increase sharply at first and then flatten off.

Table 2.1 lists the results obtained from various analyses of CERN SPS Pb+Pb data. The wide range of extracted parameters should be noted, as well as the apparent anti-correlation of the temperature and flow velocity (i.e. lower freeze-out temperatures are compensated for by higher transverse flow velocities).

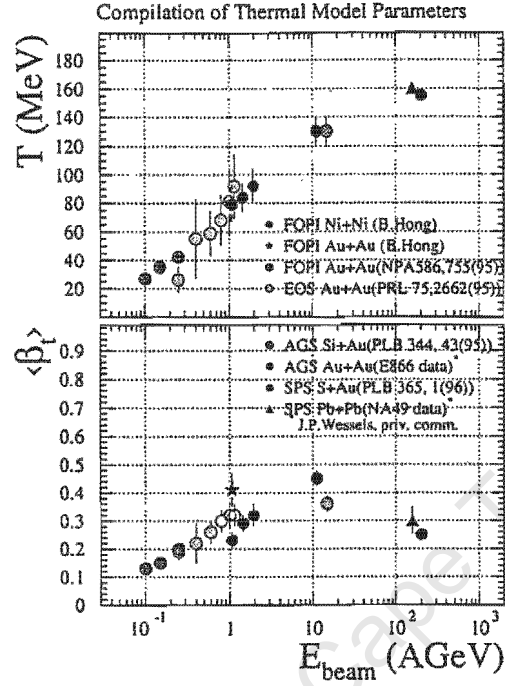


Figure 2.16: Beam energy dependence of the thermal freeze-out parameters [90].

T_{therm} (MeV)	$\langle\beta_T\rangle$	Reference
130	0.4	[91]
95 ± 15	0.55 ± 0.07	[62]
120	0.55	[65]
110–120		[92]
120	0.43	[93]

Table 2.1: Thermal freeze-out parameters for Pb+Pb collisions at CERN SPS compiled from a number of sources.

University of Cape Town

Chapter 3

Application of the Thermal Model

In this chapter, recent results from centrality selected fixed target Pb+Pb collisions at a beam energy of 158 GeV/nucleon are analysed within the framework of the thermal model incorporating hydrodynamic flow. The data [6] comprises centrality binned p , $\bar{p}$, π^+ , π^- , K^+ and K^- momentum spectra measured in the NA49 heavy ion experiment at the CERN Super Proton Synchrotron (SPS). Six centrality bins were defined by the energy E_0 deposited in the zero degree calorimeter (i.e. the greater E_0 , the more peripheral the collision). In Table 3.1, we reproduce the E_0 ranges defining each of the bins, as well as estimates of the collision impact parameter b and the number of participating nucleons N_{part} for each bin. See [6] for a detailed account of the determination of b and N_{part} .

Bin	E_0/E_0^{beam} Range	Fraction of Cross-Section	b Range (fm)	N_{part}
I	0–0.251	0.05	0–3.4	366 ± 8
II	0.251–0.399	0.075	3.4–5.3	309 ± 10
III	0.399–0.576	0.11	5.3–7.4	242 ± 10
IV	0.576–0.709	0.10	7.4–9.1	178 ± 10
V	0.709–0.797	0.10	9.1–10.2	132 ± 10
VI	0.797–	0.57	10.2–	85 ± 6

Table 3.1: The centrality bins as defined in [6].

Three separate analyses were performed on the data to study the centrality dependence of both the chemical- and thermal freeze-out parameters. In Analysis 1, the use of fully-integrated particle yields allowed the chemical freeze-out parameters to be extracted without any assumptions on the hydrodynamic flow of the fireball. In order to extract thermal freeze-out parameters, as well as chemical freeze-out parameters from particle multiplicities in a restricted phase-space region, a model for the hydrodynamic flow is required. Our chosen hydrodynamical model is one assuming boost-invariant longitudinal expansion with cylindrically symmetric transverse flow (the modified Bjorken model). Since the rapidity distributions of the NA49 data do not exactly resemble the flat distributions predicted by the Bjorken model, the applicability of this model is restricted to a narrow interval about mid-rapidity. In Analyses 2 and 3, the modified Bjorken model was applied to the rapidity- and transverse momentum spectra at mid-rapidity respectively.

3.1 Chemical Freeze-out Analysis

The statistical-thermal model applied in the extraction of the chemical freeze-out parameters contained the following standard features:

- An ideal gas of all hadrons and hadronic resonances listed by the particle data group [94] was considered.
- The grand-canonical ensemble was used throughout, with the additional parameter γ_s included to account for possibly incomplete equilibration in the strange sector.
- The correct quantum statistics were used for each particle.
- Resonances were distributed according to a Breit-Wigner form over an interval $[m - \delta m, m + 2\Gamma]$ where $\delta m = \min[m - m_{\text{threshold}}, 2\Gamma]$ [70]. Resonance widths Γ were taken from the Particle Data Booklet [94].
- All resonances were allowed to decay with branching ratios taken from [94]. Weak decays were excluded, since the data in [6] were corrected for such decays.
- No excluded-volume corrections were incorporated.

At chemical freeze-out, the system is fully specified by its temperature T_{chem} , effective volume V_{eff} , strangeness suppression factor γ_S , and chemical potentials μ_B , μ_S and μ_Q . The initial conditions of the collision fix μ_S and μ_Q for a given μ_B ; μ_S is fixed by the requirement of strangeness conservation (since strangeness changing weak decays were corrected for in the data), while μ_Q is determined by the constraint that the ratio of the total baryon number to total charge of the system is conserved. For the Pb+Pb system, these constraints are given by

$$\sum_i S_i = 0 \quad (3.1)$$

and

$$\sum_i B_i / \sum_i 2Q_i = \frac{208}{164} \quad (3.2)$$

where each of the sums is taken over all of the particles in the hadronic gas, and S_i , B_i and Q_i are the strangeness, baryon number and charge of particle species i respectively.

3.1.1 Analysis 1 - Analysis of Fully-integrated (4π) Particle Yields

The chi-square minimization routine MINUIT was used to fit the chemical freeze-out parameters to the fully-integrated experimental yields using

$$N_i = \sum_j Br(j \rightarrow i) n_j^{0(\text{prim})} V_{\text{eff}} \quad (3.3)$$

where $Br(j \rightarrow i)$ is the branching ratio of the decay of hadron species j into species i , $n_j^{0(\text{prim})}$ is the density of particle species j emitted directly from a stationary, thermalized fireball, and N_i is the fully-integrated yield of particle species i . In this way, the chemical freeze-out temperature T_{chem} , effective fireball volume V_{eff} , strangeness suppression factor γ_S , and baryon chemical potential μ_B were extracted.

The thermal model parameters obtained from the fit to the experimental multiplicities of all the particle species considered in [6] (i.e. the p , $\bar{p}$, K^+ , K^- , π^+ and π^- yields) are listed in Table 3.2. The fireball radius R was calculated from the effective volume V_{eff} under the assumption of a spherical fireball. Although this is certainly never the case, it allows for easy comparison with the radius of a static Pb nucleus. The high χ^2 -values are a result of

	BIN I	BIN II	BIN III
χ^2	602.32	363.41	174.43
T_{chem}	$148.98^{+0.85}_{-0.87}$ MeV	$151.12^{+0.72}_{-0.73}$ MeV	$158.11^{+0.80}_{-0.80}$ MeV
μ_B	$279.39^{+2.06}_{-2.03}$ MeV	$294.26^{+1.97}_{-1.90}$ MeV	$315.51^{+2.02}_{-1.90}$ MeV
γ_s	$0.764^{+0.006}_{-0.006}$	$0.736^{+0.006}_{-0.005}$	$0.713^{+0.005}_{-0.004}$
R	$10.408^{+0.124}_{-0.120}$ fm	$9.421^{+0.096}_{-0.094}$ fm	$7.690^{+0.081}_{-0.085}$ fm
	BIN IV	BIN V	BIN VI
χ^2	183.36	86.57	57.90
T_{chem}	$160.24^{+1.02}_{-1.01}$ MeV	$161.35^{+1.52}_{-1.50}$ MeV	$160.17^{+2.01}_{-1.98}$ MeV
μ_B	$325.73^{+2.91}_{-2.90}$ MeV	$331.59^{+4.24}_{-4.22}$ MeV	$333.15^{+5.56}_{-5.52}$ MeV
γ_s	$0.674^{+0.005}_{-0.005}$	$0.620^{+0.007}_{-0.007}$	$0.547^{+0.008}_{-0.008}$
R	$6.602^{+0.096}_{-0.095}$ fm	$5.780^{+0.125}_{-0.124}$ fm	$5.102^{+0.146}_{-0.144}$ fm

Table 3.2: Chemical freeze-out parameters extracted from a fit to the 4π -data of all species considered in [6]. Errors are statistical only.

the small errors in the experimental yields quoted in [6]. As is evident from Table 3.3, there is generally good agreement with the data. Only the calculated proton multiplicities show substantial deviations from the experimental data, particularly in the most peripheral collisions. However, the experimental proton multiplicities show an erratic behaviour, increasing from Bin V to Bin VI—most likely due to the uncertain extrapolation of the measured proton multiplicities to full phase-space. This prompted a repeat analysis with the protons excluded. The resulting thermal parameters are shown in Table 3.4, while the calculated multiplicities are listed in Table 3.3.

The centrality dependence of the chemical freeze-out temperature T_{chem}

Particle Species	BIN I			BIN II		
	Exp. Data	Model		Exp. Data	Model	
		all	no p		all	no p
π^+	563.7 ± 1.8	561.55	553.52	466.1 ± 1.3	459.63	454.78
π^-	606.6 ± 1.4	604.03	612.19	495.8 ± 0.9	497.55	500.85
K^+	102.1 ± 0.7	91.44	100.87	81.1 ± 0.6	74.06	79.10
K^-	45.68 ± 0.35	51.01	46.33	36.46 ± 0.27	39.42	37.31
p	137.1 ± 1.7	155.93	224.93	126.2 ± 1.8	141.68	176.42
$\bar{p}$	4.90 ± 0.18	4.29	4.86	3.62 ± 0.11	3.43	3.59
Particle Species	BIN III			BIN IV		
	Exp. Data	Model		Exp. Data	Model	
		all	no p		all	no p
π^+	345.6 ± 0.9	339.76	338.26	243.2 ± 0.7	235.56	235.87
π^-	370.0 ± 0.6	371.91	372.90	256.5 ± 0.4	258.89	258.66
K^+	58.4 ± 0.4	55.67	57.16	38.25 ± 0.31	37.20	36.92
K^-	26.33 ± 0.17	27.41	26.79	17.30 ± 0.15	17.78	17.92
p	112.3 ± 1.9	123.39	134.33	98.5 ± 2.8	91.36	88.93
$\bar{p}$	2.88 ± 0.08	2.80	2.86	1.95 ± 0.06	1.94	1.93
Particle Species	BIN V			BIN VI		
	Exp. Data	Model		Exp. Data	Model	
		all	no p		all	no p
π^+	168.6 ± 0.6	164.76	165.25	109.1 ± 0.5	106.69	106.78
π^-	179.8 ± 0.4	181.40	181.20	116.67 ± 0.27	117.34	117.30
K^+	24.71 ± 0.28	24.31	23.99	14.38 ± 0.21	13.99	13.93
K^-	11.31 ± 0.13	11.46	11.63	6.48 ± 0.09	6.63	6.65
p	87 ± 4	66.52	63.38	105 ± 13	43.55	42.94
$\bar{p}$	1.35 ± 0.06	1.35	1.34	0.84 ± 0.05	0.831	0.83

Table 3.3: Comparison of the experimental 4π -yields with the results of the thermal model arising from the fit to all particle species considered in [6], as well as the fit in which the protons were excluded (using the best-fit parameters listed in Tables 3.2 and 3.4 respectively).

	BIN I	BIN II	BIN III
χ^2	54.54	128.43	106.97
T_{chem}	$164.26^{+1.29}_{-1.31}$ MeV	$159.61^{+1.02}_{-1.03}$ MeV	$162.00^{+0.98}_{-0.99}$ MeV
μ_B	$334.94^{+3.18}_{-3.14}$ MeV	$328.72^{+2.98}_{-2.97}$ MeV	$330.27^{+2.76}_{-2.75}$ MeV
γ_s	$0.776^{+0.006}_{-0.006}$	$0.751^{+0.005}_{-0.005}$	$0.720^{+0.005}_{-0.005}$
R	$8.226^{+0.150}_{-0.149}$ fm	$8.238^{+0.123}_{-0.122}$ fm	$7.237^{+0.103}_{-0.102}$ fm
	BIN IV	BIN V	BIN VI
χ^2	174.62	56.30	35.33
T_{chem}	$159.05^{+1.07}_{-1.06}$ MeV	$159.13^{+1.51}_{-1.43}$ MeV	$159.51^{+1.99}_{-1.96}$ MeV
μ_B	$321.09^{+3.27}_{-3.26}$ MeV	$323.07^{+4.42}_{-4.24}$ MeV	$330.70^{+5.55}_{-5.51}$ MeV
γ_s	$0.673^{+0.005}_{-0.005}$	$0.618^{+0.007}_{-0.006}$	$0.546^{+0.008}_{-0.008}$
R	$6.725^{+0.104}_{-0.104}$ fm	$5.984^{+0.124}_{-0.128}$ fm	$5.155^{+0.146}_{-0.144}$ fm

Table 3.4: Chemical freeze-out parameters extracted from the 4π -data [6] with protons excluded from the fit. Errors are statistical only.

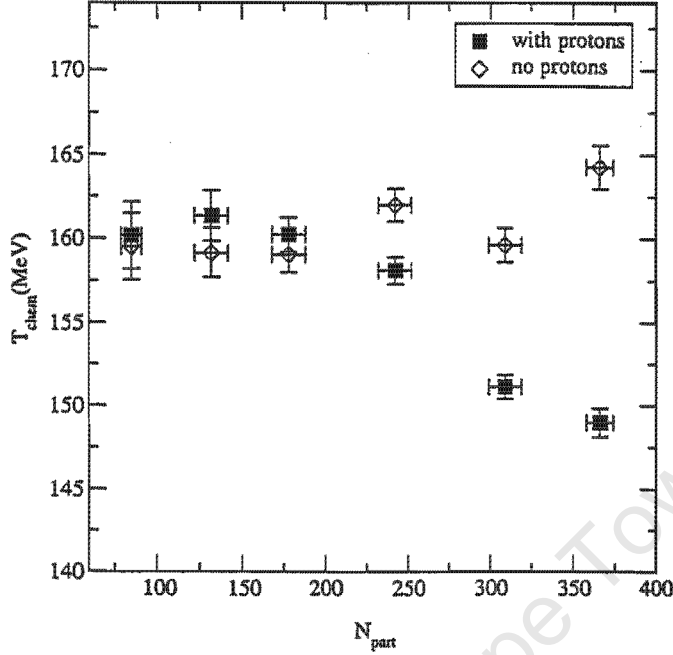


Figure 3.1: Centrality dependence of the chemical freeze-out temperature T_{chem} . Errors are statistical only.

is displayed in Figure 3.1. With the protons excluded, the freeze-out temperature is approximately centrality independent, and is in good agreement with that extracted from elementary collision data [72, 73]. This suggests freeze-out soon after hadronization, with the chemical composition of the fireball largely unaffected by inelastic collisions. Furthermore, this temperature agrees with that extracted in the analysis of central Pb+Pb collisions at CERN SPS, performed by Becattini et al. [77]. Including the protons in the fit, the temperature is seen to decrease with increasing centrality. This may be explained by the view [13] that despite reaching higher initial temperatures, larger systems evolve collectively for longer and so freeze out at lower temperatures than smaller, more peripheral collision systems do.

In Figure 3.2, the centrality dependence of the baryon chemical potential at chemical freeze-out is shown. With the protons excluded from the fit, μ_B is roughly constant in the interval 328 ± 7 MeV. Since only one baryon is then included in the analysis, there is considerable uncertainty in the values of μ_B . In comparison, Becattini et al. [77] obtained a value for the baryon chemical potential of 238 ± 13 MeV in an analysis of central collision data. This discrepancy arises from the $\bar{p}$ experimental yield used in [77] being a factor of two greater than the Bin I value listed in [6], as well as the inclusion

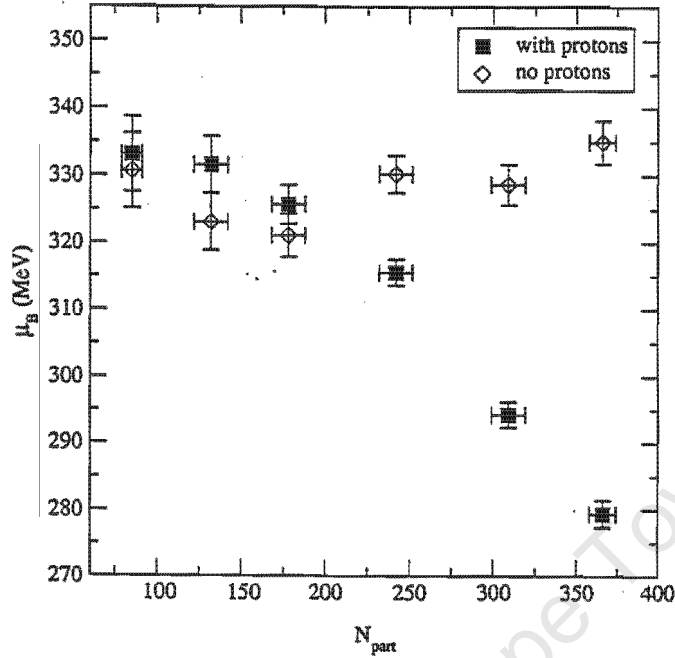


Figure 3.2: Centrality dependence of the baryon chemical potential μ_B at chemical freeze-out. Errors are statistical only.

of additional baryons in the analysis of Becattini et al. [77]. Including the protons, μ_B is seen to decrease with increasing centrality. This is consistent with the picture that central collision systems are hot, long-lived and freeze out late. As a result, many new particles (predominantly mesons) are created, and the overall baryon density decreases.

Figure 3.3 shows the radius R as a function of the number of participating nucleons N_{part} . As the collisions become more and more central, the radius increases smoothly. With the exclusion of the protons, the radius reaches a maximum value of 8.2 fm; only slightly greater than the radius of a static Pb nucleus. Thus, the system does not expand very much prior to chemical freeze-out. The observed decrease in the chemical freeze-out temperature with increasing centrality, when protons are included, is compensated for by a more rapid, roughly linear increase in the radius of the system with the number of participants.

The centrality dependence of the strangeness suppression factor γ_s is displayed in Figure 3.4. It is seen to increase smoothly from an approximate value of 0.55 in the most peripheral collisions, to that of 0.78 in the most central bin. This is consistent with the notion that as the collisions become

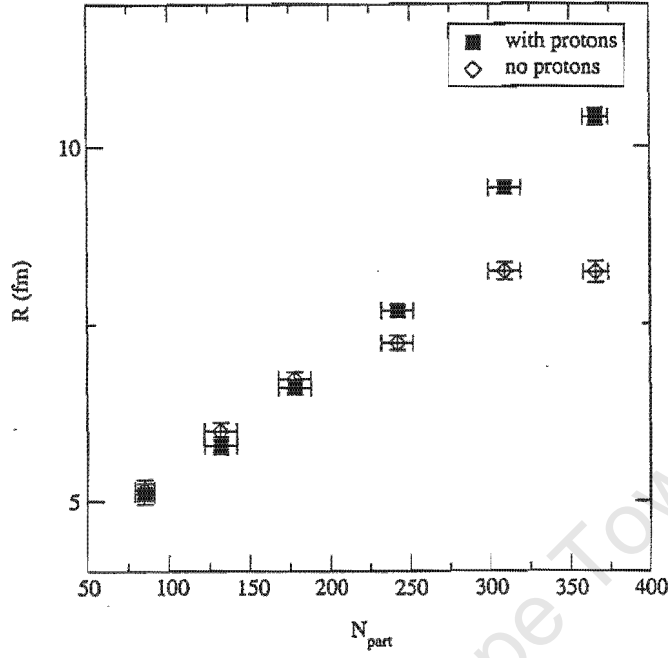


Figure 3.3: Centrality dependence of the fireball radius R at chemical freeze-out. Errors are statistical only.

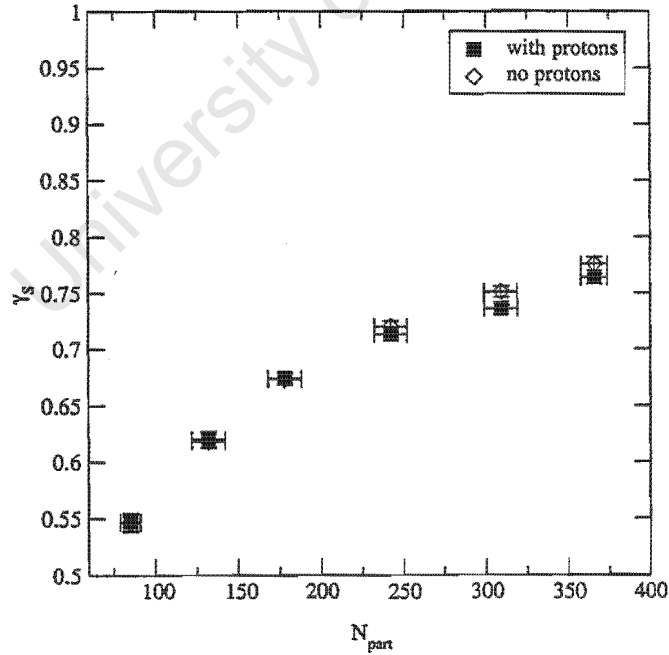


Figure 3.4: Centrality dependence of the degree of strangeness equilibration γ_s at chemical freeze-out. Errors are statistical only.

more central, more strange particles are produced. There is still, however, considerable deviation from complete strangeness equilibration, even in the most violent collisions. The value of γ_S for the most central collisions is in good agreement with that calculated by Becattini et al. [77].

3.1.2 Analysis 2 - Application of the Modified Bjorken Model to $(dN/dy)_{y=0}$ Data

As a reminder, for the modified Bjorken model:

$$\frac{(dN_i/dy)_{y=0}}{(dN_j/dy)_{y=0}} = \frac{n_i^0}{n_j^0}. \quad (3.4)$$

Although equation (3.2) is certainly true in the case of fully-integrated particle multiplicities, its enforcement in the analysis of mid-rapidity data from CERN SPS, where there is partial transparency, is questionable. For this reason, this constraint was replaced in this analysis by the requirement that the π^+/π^- ratio calculated by the model correspond to its experimental value. In fact, it was found that equation (3.2) is approximately valid; the results are not greatly affected by constraining the baryon to charge ratio at mid-rapidity to its fully-integrated value (see Figure 3.5).

For the three most central bins and the most peripheral bin, bands were plotted in the $T - \mu_B$ plane, consistent with the constraints and the measured particle ratios at mid-rapidity (similar to what was done in [70]). Errors in the ratios (reflected in the width of each band) were determined by adding the individual experimental errors in quadrature. The value $\gamma_S = 0.859$ was taken from the 4π -analysis of central Pb+Pb SPS data performed by Becattini et al. [77].

It is evident from Figure 3.5 that the mid-rapidity CERN SPS data [6] do not support the modified Bjorken model; no common overlap region exists for any of the bins considered. Clearly, the energies at CERN SPS are too low for the Bjorken model (formulated for infinite energies) to hold at chemical freeze-out. It is, however, believed that the required plateau structure of the particle distributions in the central rapidity region will be seen at the significantly larger RHIC- and LHC energies. This should allow a chemical analysis based entirely on mid-rapidity data.

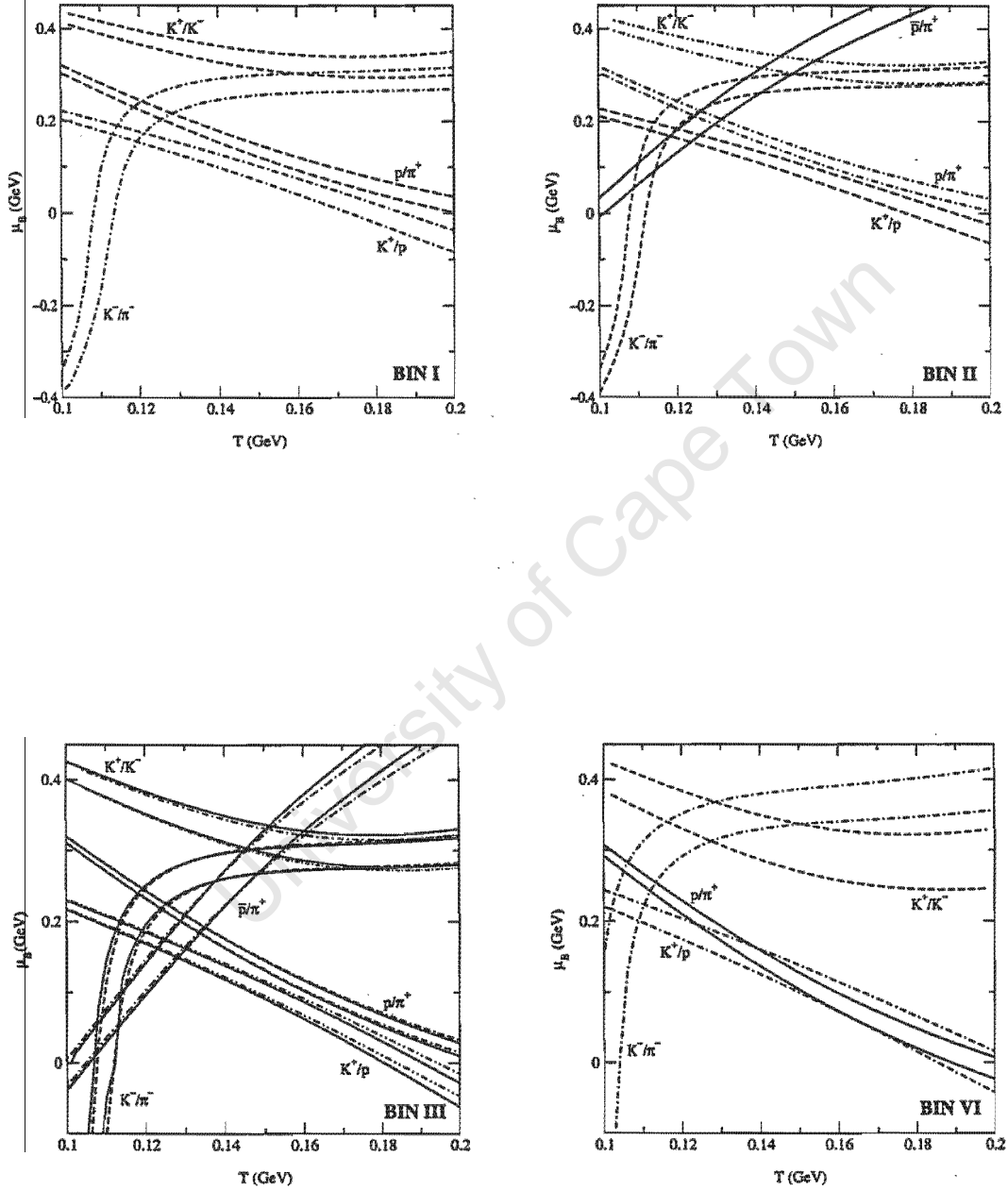


Figure 3.5: Bands in the $T - \mu_B$ plane consistent with the experimental particle ratios at mid-rapidity [6] assuming the modified Bjorken model. The solid lines in the Bin III plot correspond to the results obtained with the ratio of baryon number to charge fixed. In each plot, γ_S was fixed at 0.859.

3.2 Thermal Freeze-out Analysis

In the extraction of the thermal freeze-out parameters, all contributions of resonance decays were ignored. This was to avoid the need for additional, poorly constrained parameters such as the momentum-space distributions of the parent hadrons. As will be shown, the use of quantum statistics, and in particular the non-zero pion chemical potential, already accounts sufficiently for the low- p_T enhancement in the pion spectra due to resonance decays. Owing to limited experimental $\bar{p}$ data at mid-rapidity in Bin I and Bin VI, anti-protons were excluded entirely from the thermal freeze-out analysis.

3.2.1 Analysis 3 - Application of the Modified Bjorken Model to the Transverse Momentum Spectra

A free-fit analysis of the transverse momentum distributions at mid-rapidity $(d^2N_i/dy m_T dm_T)_{y=0}$ was performed using the modified Bjorken model with instantaneous freeze-out at constant proper time τ_f .

In terms of the variable $\xi = r/R_f$, where R_f is the transverse radius of the fireball at thermal freeze-out, the momentum distributions of the particles considered in [6] (from equation (2.49) with $\frac{\partial \tau_f(r)}{\partial r} = 0$ and the γ_S factors omitted) are given by

$$\begin{aligned} \frac{d^2 N_{\pi^\pm}^{Bj}}{dy m_T dm_T} &= m_T \left[\frac{g_{\pi^\pm}}{\pi} \tau_f R_f^2 \right] \sum_{k=1}^{\infty} e^{k\mu_{\pi^\pm}/T} \\ &\times \int_0^1 d\xi \xi K_1 \left(\frac{km_T \cosh \rho(\xi)}{T} \right) I_0 \left(\frac{kp_T \sinh \rho(\xi)}{T} \right), \quad (3.5) \end{aligned}$$

$$\begin{aligned} \frac{d^2 N_{K^\pm}^{Bj}}{dy m_T dm_T} &= m_T \left[\frac{g_{K^\pm}}{\pi} \tau_f R_f^2 e^{\mu_{K^\pm}/T} \right] \\ &\times \int_0^1 d\xi \xi K_1 \left(\frac{m_T \cosh \rho(\xi)}{T} \right) I_0 \left(\frac{p_T \sinh \rho(\xi)}{T} \right), \quad (3.6) \end{aligned}$$

$$\begin{aligned} \frac{d^2 N_p^{Bj}}{dy m_T dm_T} &= m_T \left[\frac{g_p}{\pi} \tau_f R_f^2 e^{\mu_p/T} \right] \\ &\times \int_0^1 d\xi \xi K_1 \left(\frac{m_T \cosh \rho(\xi)}{T} \right) I_0 \left(\frac{p_T \sinh \rho(\xi)}{T} \right). \quad (3.7) \end{aligned}$$

Although the use of Bose-Einstein statistics (contained in the sum over k in equation (3.5)) is essential for the pions, Boltzmann statistics were found to be an excellent approximation for the kaons and protons. The normalization factors contained in the square brackets of equations (3.5)–(3.7) were treated as single, independent parameters in the analysis. For the transverse flow, a linear velocity profile $\beta_T(r) = \beta_S (r/R_f)$ was assumed, with β_S the transverse flow velocity on the fireball surface. The average transverse flow velocity of the fireball $\langle\beta_T\rangle$ is then related to its surface velocity through

$$\langle\beta_T\rangle = \frac{2}{3}\beta_S. \quad (3.8)$$

With these assumptions, the transverse flow rapidity is given by

$$\rho(\xi) = \tanh^{-1}(\beta_S \xi). \quad (3.9)$$

As a first step, each momentum distribution was analysed separately. For the temperature range $T = 60\text{--}190$ MeV, the remaining free parameters in the parameterization of the spectrum of the species under consideration (i.e. the transverse flow velocity at the fireball surface β_S , the normalization factor, and the chemical potential in the case of the pions) were fitted. For each centrality bin, the results for each particle species were plotted as lines in the $T - \beta_S$ plane (see Figure 3.6). If our assumption of simultaneous freeze-out is valid, all lines should intersect at a point corresponding to the common temperature and surface transverse flow velocity at thermal freeze-out for the particular impact parameter considered. It is evident from Figure 3.6 that no such idealized situation is realized by the data.

Following this, a chi-square fit was performed to the p , K^+ , K^- , π^+ and π^- transverse momentum spectra simultaneously, bin by bin. The results of this chi-square minimization are listed in Table 3.5. In view of the deviation from thermal behaviour exhibited by the proton data in the chemical analysis, as well as discrepancies [95] between the proton spectra listed in [6] and those published by earlier Pb+Pb collision experiments at CERN SPS, we repeated this minimization procedure with the proton distributions excluded. These results are listed in Table 3.6. In Figures 3.7 and 3.8, the Bin I experimental transverse momentum distributions at mid-rapidity are compared with the fitted spectra, using the thermal parameters listed in Table 3.5. There exists fairly good agreement between the data and the model. Even the strong concave curvature of the pion spectra is reproduced.

Our extracted thermal freeze-out parameters lie within the range of those obtained in other analyses (compare with Table 2.1). Comparing the results

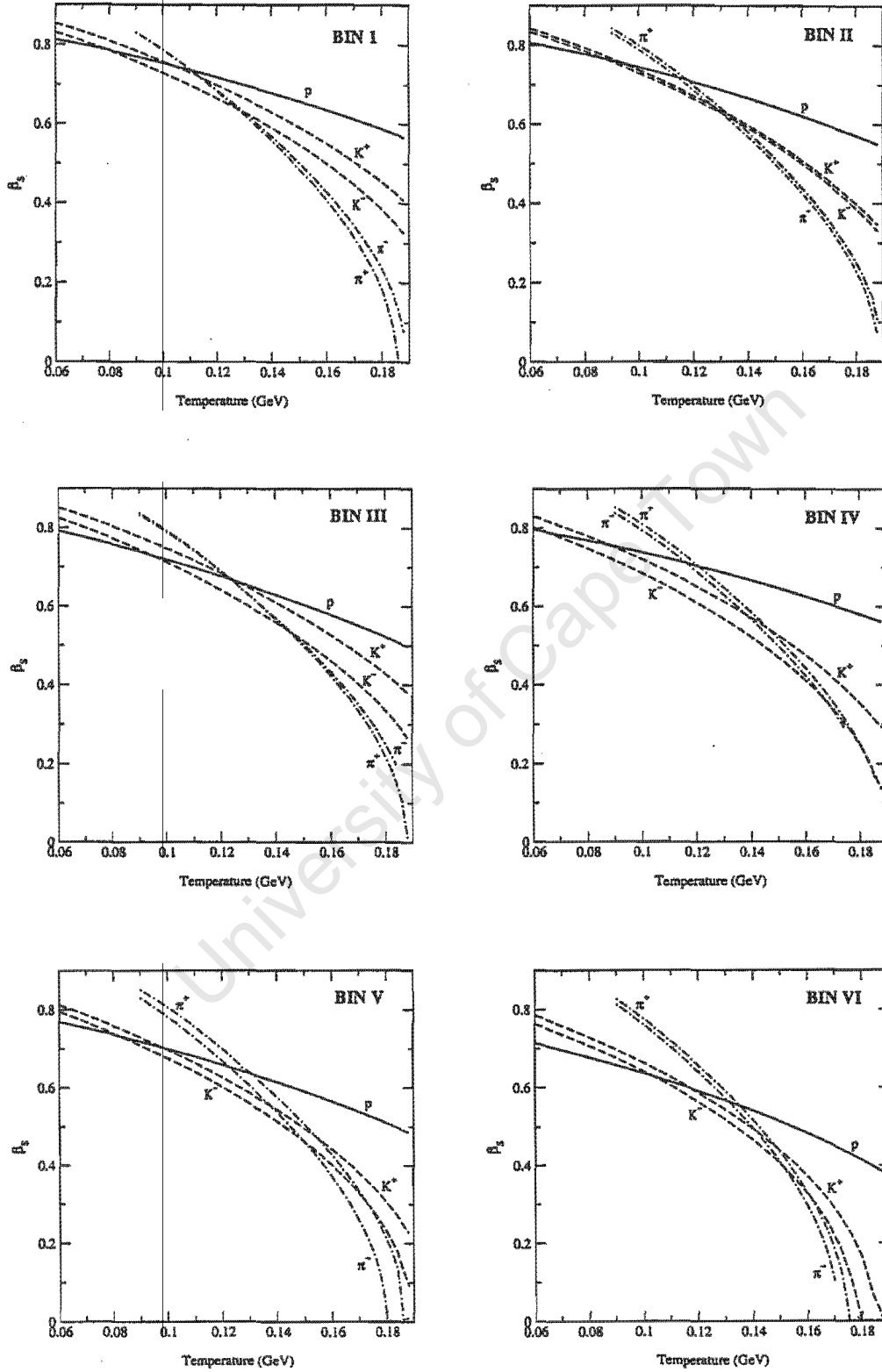


Figure 3.6: Lines of minimum χ^2 in the $T-\beta_S$ plane for each of the centrality bins.

Bin	χ^2	T_{therm} (MeV)	$\langle\beta_T\rangle$	μ_{π^+} (MeV)	μ_{π^-} (MeV)
I	13.27	114.9	0.4729	97.7	97.0
II	24.61	109.6	0.4855	103.1	92.2
III	61.46	103.2	0.4927	75.8	48.8
IV	97.28	101.9	0.4893	86.7	28.7
V	64.20	102.0	0.4865	95.1	60.0
VI	23.25	113.3	0.4428	86.1	94.9

Table 3.5: The thermal freeze-out parameters extracted from the p , K^+ , K^- , π^+ and π^- transverse momentum distributions at mid-rapidity assuming the modified Bjorken model.

Bin	χ^2	T_{therm} (MeV)	$\langle\beta_T\rangle$	μ_{π^+} (MeV)	μ_{π^-} (MeV)
I	13.16	117.1	0.4661	96.2	99.9
II	19.29	116.1	0.4659	94.3	101.9
III	34.08	113.4	0.4687	80.3	93.7
IV	36.22	122.4	0.4407	102.1	111.3
V	21.17	129.0	0.4100	109.7	115.2
VI	23.98	124.2	0.3891	81.8	96.3

Table 3.6: The thermal freeze-out parameters extracted from the K^+ , K^- , π^+ and π^- transverse momentum distributions at mid-rapidity assuming the modified Bjorken model.

listed in Tables 3.5 and 3.6, the extracted parameters are considerably altered by the exclusion of the protons, in all but the most central bin. With the protons excluded, the pion chemical potential (particularly that of the π^-) no longer varies erratically.

The centrality dependence of the thermal freeze-out temperature is displayed in Figure 3.9. Excluding the proton distributions, it would appear as if the thermal freeze-out temperature drops with increasing centrality. This supports the view [13] that larger systems evolve collectively for longer and so freeze out at lower temperatures. Inclusion of the proton data seems to suggest the opposite.

In Figure 3.10, the average transverse flow velocity is shown as a function of the number of participants in the collision. Exclusion of the protons suggests a centrality independent, limiting velocity of approximately 0.47 reached in the three most central bins. For the three most peripheral bins, the average transverse flow velocity is seen to drop sharply to a value of 0.39 in Bin VI. This is consistent with the behaviour of the temperature; central collision systems freeze out later than peripheral ones do, by which time more of the thermal energy has been transferred to the hydrodynamic flow of the fireball. With the proton data included, no obvious trend is observed.

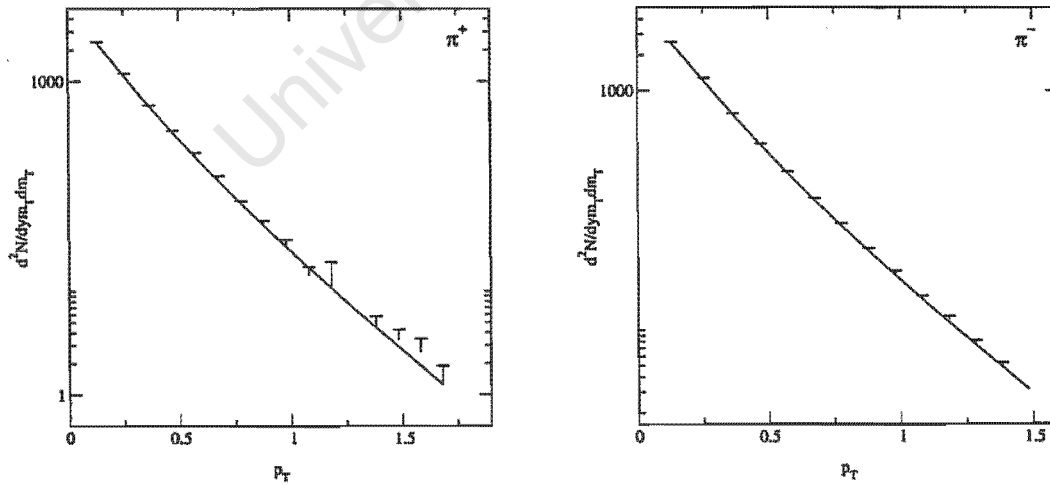


Figure 3.7: Fits to the pion transverse momentum spectra of Bin I assuming the modified Bjorken model.

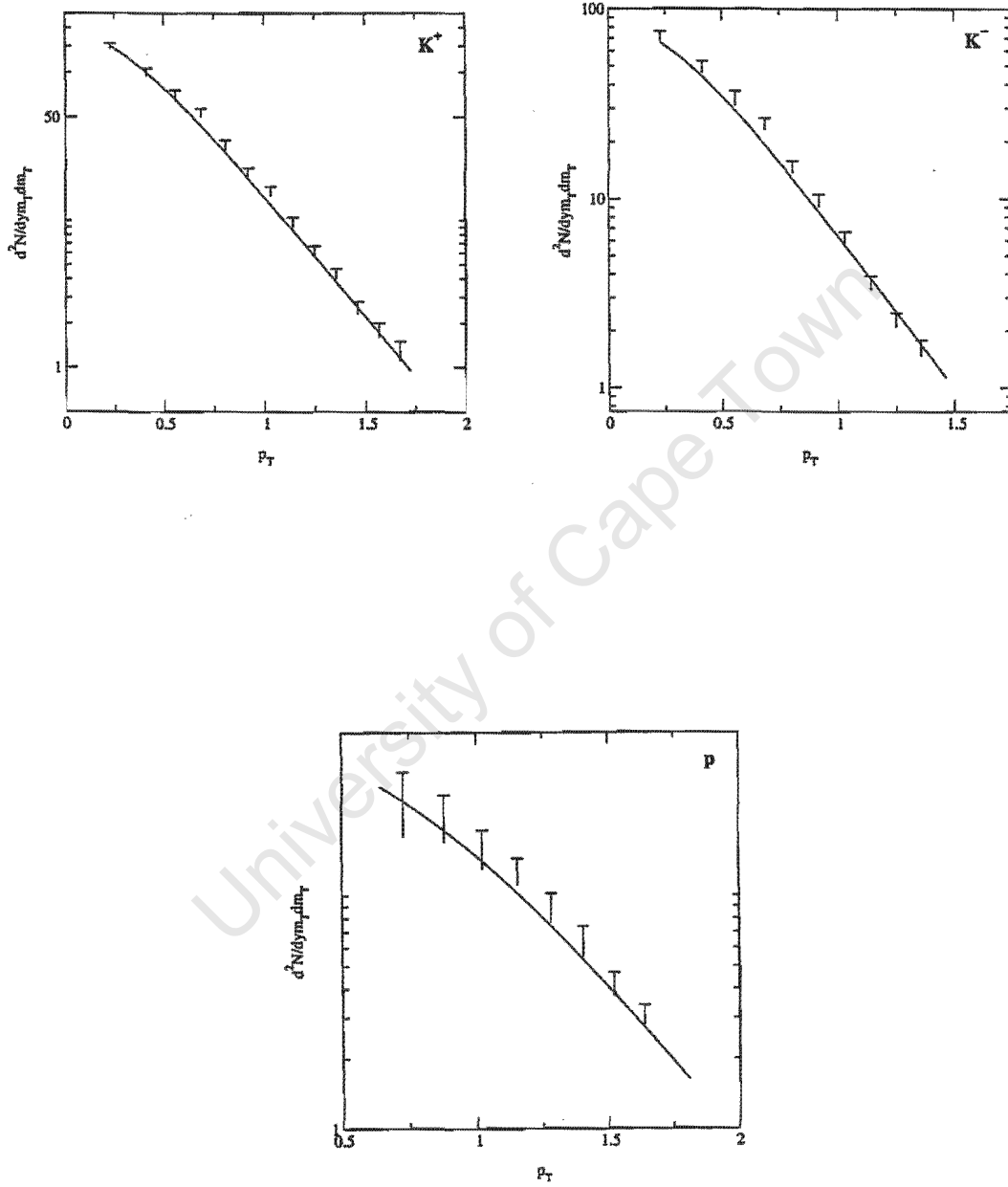


Figure 3.8: Fits to the kaon- and proton transverse momentum spectra of Bin I assuming the modified Bjorken model.

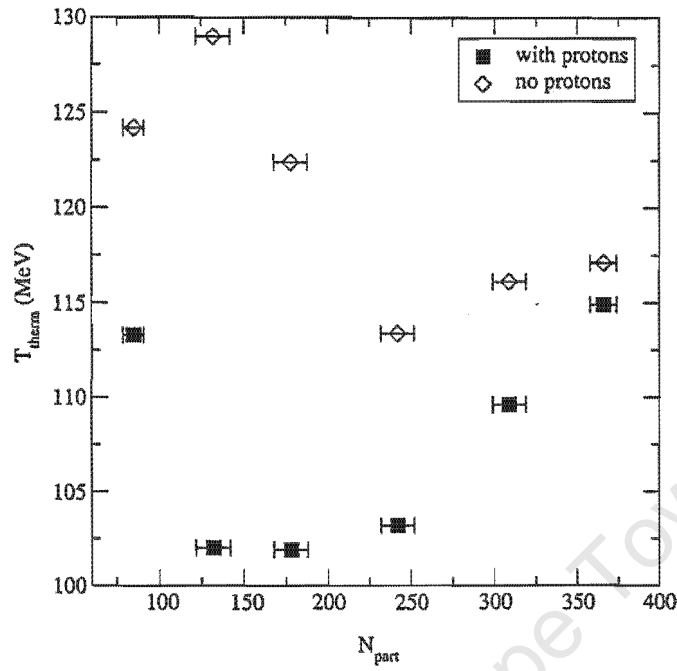


Figure 3.9: Centrality dependence of the thermal freeze-out temperature T_{therm} .

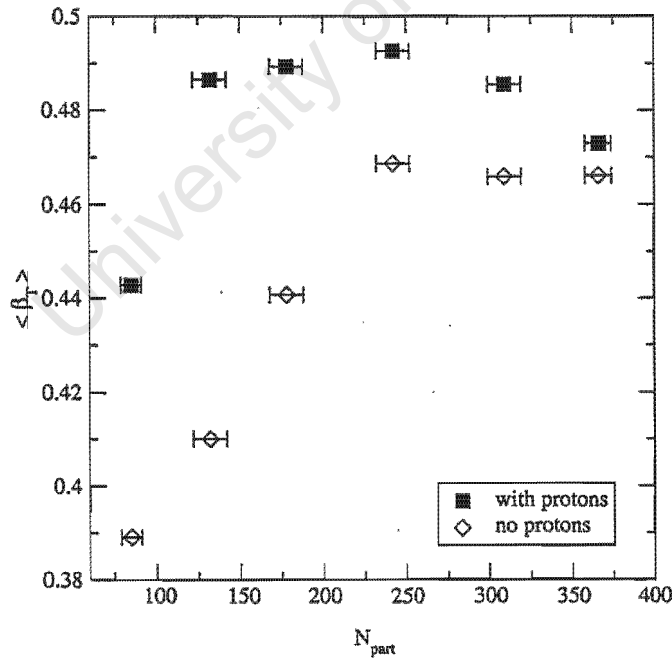


Figure 3.10: Centrality dependence of the average transverse flow velocity $\langle\beta_T\rangle$ at thermal freeze-out.

3.3 Conclusions

The thermal model has been found to be incapable of describing the proton multiplicities published in [6]. The nature of this discrepancy, however, is not clear. The erratic behaviour of the proton multiplicities (i.e. the increase from Bin V to Bin VI), together with the conflict of the proton data [6] with that published by earlier Pb+Pb collision experiments at CERN SPS, raises doubts as to the reliability of the proton data in [6]. This prevents the clean interpretation of the inability of the thermal model to reproduce the proton data as evidence for the exclusion of the protons from a thermal model analysis. It is, however, believed that protons should be excluded from thermal model analyses of high energy peripheral collisions. In such collisions, due to the limited stopping, the protons do not form part of the fireball of secondary particles to which the thermal model applies. For these reasons, we include here the results of the thermal analysis with the proton data excluded.

Analysis of the centrality dependence of the thermal parameters at chemical freeze-out reveals that, with increasing centrality:

- the fireball volume increases smoothly,
- the baryon chemical potential is roughly constant in the interval 328 ± 7 MeV, although more baryon data is required to fix this value more exactly,
- the strangeness suppression factor γ_s increases (i.e. the strange particle multiplicities edge towards full chemical equilibrium),
- the temperature remains roughly constant in the interval 162 ± 3 MeV, consistent with the independence of the chemical freeze-out temperature on system size at CERN SPS, established by comparison of S+S and Pb+Pb collision data [75];

while at thermal freeze-out, with increasing centrality:

- the average transverse flow velocity appears to increase to a limiting, centrality independent value of 0.47,
- the temperature drops to approximately 115 MeV in the most central collisions, suggesting later freeze-out in large systems,
- the pion chemical potential (of the order of 100 MeV) displays no clear centrality dependence.

Therefore, the goal of this thesis has been realized, having determined the centrality dependence of both the chemical- and thermal freeze-out parameters for Pb+Pb collisions at 158 GeV/nucleon. In general, these parameters agree well with those extracted in other analyses, although the centrality binned momentum distributions of additional hadrons are required to verify the results. Furthermore, since our kinetic freeze-out analysis was based on a very simple model, further investigation is required before the trends in the thermal freeze-out parameters proposed in this thesis can be confirmed. In particular, the effects of resonance decays should be included in subsequent studies.

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Appendix A

Kinematic Variables

Throughout this thesis, a system of units is used in which

$$\hbar = c = k_B = 1. \quad (\text{A.1})$$

In order to fully exploit the cylindrical symmetry of ultra-relativistic heavy ion collisions, it is convenient to introduce the variables longitudinal rapidity y and transverse mass m_T . With the spatial z -axis chosen to correspond to the beam direction, the longitudinal rapidity of a particle with energy E is defined as

$$y \equiv \tanh^{-1} \left(\frac{p_z}{E} \right) \quad (\text{A.2})$$

and its transverse mass as

$$m_T \equiv \sqrt{m_0^2 + p_x^2 + p_y^2} = \sqrt{m_0^2 + p_T^2} \quad (\text{A.3})$$

where m_0 is the rest mass of the particle, p_x , p_y and p_z are its momentum components, and p_T is the particle's transverse momentum.

In terms of these variables, a particle with both transverse- and longitudinal momentum has 4-momentum

$$p^\mu = (E, \vec{p}) = (m_T \cosh y, p_T \cos \phi_P, p_T \sin \phi_P, m_T \sinh y) \quad (\text{A.4})$$

where ϕ_P fixes the direction of the particle's motion in the transverse plane.

Next, we consider the differential momentum element $d^3p = dp_x dp_y dp_z$. In cylindrical coordinates,

$$d^3p = p_T dp_T d\phi_P dp_z. \quad (\text{A.5})$$

Using $E = m_T \cosh y$ and $p_z = m_T \sinh y$,

$$dp_z = m_T \cosh y \, dy = E \, dy, \quad (\text{A.6})$$

while from the definition of m_T , it follows that

$$m_T \, dm_T = p_T \, dp_T \quad (\text{A.7})$$

and so

$$d^3p = E \, dy \, p_T \, dp_T \, d\phi_P = E \, dy \, m_T \, dm_T \, d\phi_P. \quad (\text{A.8})$$

The Lorentz transformation matrix Λ_ν^μ , representing a boost of velocity v in the z -direction, is given by

$$\Lambda_\nu^\mu = \begin{pmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{pmatrix} \quad (\text{A.9})$$

where

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} \quad (\text{A.10})$$

and

$$\beta = v. \quad (\text{A.11})$$

Now consider a particle of rest mass m_0 . In its rest frame, it has 4-momentum

$$p_{RF}^\mu = (m_0, 0, 0, 0), \quad (\text{A.12})$$

while viewed in a frame moving along the z -axis relative to the particle rest frame, its 4-momentum is given by

$$p^\mu = (E, 0, 0, p_z). \quad (\text{A.13})$$

The 4-momenta in these two frames are related by

$$p^\mu = \Lambda_\nu^\mu p_{RF}^\nu \quad (\text{A.14})$$

where Λ_ν^μ is the Lorentz transformation matrix introduced above. It follows that

$$y \equiv \tanh^{-1} \left(\frac{p_z}{E} \right) = \tanh^{-1} \left(\frac{\gamma\beta m_0}{\gamma m_0} \right) \quad (\text{A.15})$$

and so

$$\beta = \tanh y. \quad (\text{A.16})$$

From the definition of γ (equation (A.10)), it follows that

$$\gamma = \cosh y \quad (\text{A.17})$$

and so

$$\gamma\beta = \sinh y. \quad (\text{A.18})$$

Finally, in terms of the longitudinal rapidity, the Lorentz transformation matrix is given by

$$\Lambda_{\nu}^{\mu} = \begin{pmatrix} \cosh y & 0 & 0 & \sinh y \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sinh y & 0 & 0 & \cosh y \end{pmatrix}. \quad (\text{A.19})$$

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Appendix B

Derivation of Equation (2.25)

The number of particle species i emitted directly from a stationary, thermalized fireball of temperature T and volume V is given by

$$N_i^{0(\text{prim})} = g_i V \int \frac{d^3 p}{(2\pi)^3} f_i(x, p) \quad (\text{B.1})$$

where g_i is the spin-isospin degeneracy of particle species i , and $f_i(x, p)$ is the thermal distribution function for this particle.

Within the grand-canonical ensemble, with strangeness suppression

$$\begin{aligned} f_i(x, p) &= \frac{1}{\gamma_S^{-|S_i|} e^{(E-\mu_i)/T} \pm 1} \\ &= \gamma_S^{|S_i|} e^{-(E-\mu_i)/T} \times \frac{1}{1 \pm \gamma_S^{|S_i|} e^{-(E-\mu_i)/T}} \end{aligned}$$

where the upper (lower) signs refer to fermions (bosons).

Using $(1 - x)^{-1} = \sum_{k=0}^{\infty} x^k$,

$$\begin{aligned} f_i(x, p) &= \gamma_S^{|S_i|} e^{-(E-\mu_i)/T} \times \sum_{k=0}^{\infty} (\mp 1)^k \left(\gamma_S^{|S_i|} e^{-(E-\mu_i)/T} \right)^k \\ &= \sum_{k=0}^{\infty} (\mp 1)^k \left(\gamma_S^{|S_i|} e^{-(E-\mu_i)/T} \right)^{k+1} \\ &= \sum_{k=1}^{\infty} (\mp 1)^{k+1} \left(\gamma_S^{|S_i|} e^{-(E-\mu_i)/T} \right)^k \end{aligned} \quad (\text{B.2})$$

where the $k=1$ term corresponds to the Boltzmann approximation. Substi-

tuting equation (B.2) into equation (B.1), and exploiting spherical symmetry,

$$\begin{aligned} N_i^{0(\text{prim})} &= g_i V \sum_{k=1}^{\infty} (\mp)^{k+1} (\gamma_S^{|S_i|})^k \int \frac{d^3 p}{(2\pi)^3} e^{-(E-\mu_i)k/T} \\ &= g_i V \sum_{k=1}^{\infty} (\mp)^{k+1} (\gamma_S^{|S_i|})^k e^{k\mu_i/T} \frac{1}{2\pi^2} \times \int_0^{\infty} dp p^2 e^{-kE/T}. \end{aligned}$$

Substituting $E = m_i \cosh y$ and $p = m_i \sinh y$,

$$\int_0^{\infty} dp p^2 e^{-kE/T} = \int_0^{\infty} dy m_i^3 \sinh^2 y \cosh y e^{-km_i \cosh y/T},$$

and integrating by parts,

$$\int_0^{\infty} dp p^2 e^{-kE/T} = \frac{m_i^3 \sinh^3 y}{3} e^{-km_i \cosh y/T} \Big|_0^{\infty} + \frac{km_i^4}{3T} \int_0^{\infty} dy \sinh^4 y e^{-km_i \cosh y/T}$$

where the boundary term vanishes and the integral over y resembles the integral expansion of a modified Bessel function:

$$K_{\nu}(z) = \frac{\pi^{1/2} \left(\frac{1}{2}z\right)^{\nu}}{\Gamma\left(\nu + \frac{1}{2}\right)} \int_0^{\infty} e^{-z \cosh t} \sinh^{2\nu} t dt$$

with

$$\Gamma\left(\nu + \frac{1}{2}\right) = \frac{1 \cdot 3 \cdot 5 \cdot 7 \dots (2\nu - 1)}{2^{\nu}} \Gamma\left(\frac{1}{2}\right).$$

Therefore,

$$\int_0^{\infty} dp p^2 e^{-kE/T} = \frac{m_i^2 T}{k} K_2\left(\frac{km_i}{T}\right)$$

and, finally

$$N_i^{0(\text{prim})} = \frac{g_i V}{2\pi^2} \sum_{k=1}^{\infty} (\mp)^{k+1} (\gamma_S^{|S_i|})^k \frac{m_i^2 T}{k} K_2\left(\frac{km_i}{T}\right) e^{k\mu_i/T}. \quad (\text{B.3})$$

Appendix C

Cylindrical Symmetry

Under the assumption of cylindrical symmetry (with the z -axis corresponding to the collision axis of the nuclei), the freeze-out hypersurface is parameterized as

$$\begin{aligned}\sigma^0 &= t_f(r, z) \\ \sigma^1 &= r \cos \phi \\ \sigma^2 &= r \sin \phi \\ \sigma^3 &= z\end{aligned}$$

and from equation (2.4), the normal to the freeze-out hypersurface $d\sigma_\mu$ is given by

$$d\sigma_\mu = \left(1, -\frac{\partial t_f(r, z)}{\partial r} \cos \phi, -\frac{\partial t_f(r, z)}{\partial r} \sin \phi, -\frac{\partial t_f(r, z)}{\partial z} \right) r dr d\phi dz. \quad (\text{C.1})$$

Therefore, using equation (A.4),

$$\begin{aligned}p^\mu d\sigma_\mu &= \left\{ m_T \cosh y - \frac{\partial t_f(r, z)}{\partial z} m_T \sinh y \right. \\ &\quad \left. - \frac{\partial t_f(r, z)}{\partial r} p_T \cos(\phi - \phi_P) \right\} r dr d\phi dz. \quad (\text{C.2})\end{aligned}$$

To derive the 4-velocity of a fluid-element with transverse- (radial) as well as

longitudinal flow, we begin with its 4-velocity when at rest:

$$u^\mu = (1, 0, 0, 0), \quad (\text{C.3})$$

then boost it in the x -direction with rapidity ρ :

$$u^\mu \rightarrow \begin{pmatrix} \cosh \rho & \sinh \rho & 0 & 0 \\ \sinh \rho & \cosh \rho & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \cosh \rho \\ \sinh \rho \\ 0 \\ 0 \end{pmatrix}, \quad (\text{C.4})$$

rotate it through an angle ϕ in the $\hat{r}$ -direction:

$$u^\mu \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi & 0 \\ 0 & \sin \phi & \cos \phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cosh \rho \\ \sinh \rho \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \cosh \rho \\ \sinh \rho \cos \phi \\ \sinh \rho \sin \phi \\ 0 \end{pmatrix}, \quad (\text{C.5})$$

and, finally, boost it in the z -direction with longitudinal flow rapidity η :

$$u^\mu \rightarrow \begin{pmatrix} \cosh \eta & 0 & 0 & \sinh \eta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sinh \eta & 0 & 0 & \cosh \eta \end{pmatrix} \begin{pmatrix} \cosh \rho \\ \sinh \rho \cos \phi \\ \sinh \rho \sin \phi \\ 0 \end{pmatrix} = \begin{pmatrix} \cosh \eta \cosh \rho \\ \sinh \rho \cos \phi \\ \sinh \rho \sin \phi \\ \sinh \eta \cosh \rho \end{pmatrix}. \quad (\text{C.6})$$

Starting from the Cooper-Frye formula,

$$E \frac{d^3 N_i}{d^3 p} = \frac{g_i}{(2\pi)^3} \int_{\sigma_f} f_i(x, p) p^\mu d\sigma_\mu, \quad (\text{C.7})$$

we substitute equation (C.2) for $p^\mu d\sigma_\mu$, as well as the series expansion for $f_i(x, p)$ from equation (B.2), with E replaced by

$$p^\mu u_\mu = m_T \cosh \rho \cosh (y - \eta) - p_T \sinh \rho \cos (\phi - \phi_P). \quad (\text{C.8})$$

Provided we are considering thermal freeze-out, by which time the system

has departed from chemical equilibrium, we may omit the strangeness suppression factor γ_S , yielding

$$\begin{aligned}
E \frac{d^3 N_i}{d^3 p} &= \frac{g_i}{(2\pi)^3} \sum_{k=1}^{\infty} (\mp 1)^{k+1} \int_{\sigma^0(r,z)} r dr d\phi dz e^{k\mu_i/T} e^{-k\alpha \cosh(y-\eta)/T} \\
&\quad \times \left\{ m_T \cosh y - \frac{\partial t_f}{\partial z} m_T \sinh y \right. \\
&\quad \left. - \frac{\partial t_f}{\partial r} p_T \cos(\phi - \phi_P) \right\} e^{k\tilde{\alpha} \cos(\phi - \phi_P)/T} \\
&= \frac{g_i}{(2\pi)^3} \sum_{k=1}^{\infty} (\mp 1)^{k+1} \int_{\sigma^0(r,z)} r dr dz e^{k\mu_i/T} e^{-k\alpha \cosh(y-\eta)/T} \\
&\quad \times \left\{ \int_0^{2\pi} d\phi' e^{k\tilde{\alpha} \cos \phi'/T} m_T \left(\cosh y - \frac{\partial t_f}{\partial z} \sinh y \right) \right. \\
&\quad \left. - \int_0^{2\pi} d\phi' e^{k\tilde{\alpha} \cos \phi'/T} \frac{\partial t_f}{\partial r} p_T \cos \phi' \right\} \quad (C.9)
\end{aligned}$$

where $\alpha \equiv m_T \cosh \rho$, $\tilde{\alpha} \equiv p_T \sinh \rho$ and $\phi' \equiv \phi - \phi_P$.

Using $I_0(z) = \frac{1}{\pi} \int_0^\pi d\phi' e^{\pm z \cos \phi'}$, the first term in the curly brackets of equation (C.9) becomes

$$2\pi I_0\left(\frac{k\tilde{\alpha}}{T}\right) m_T \left(\cosh y - \frac{\partial t_f}{\partial z} \sinh y \right)$$

and the second term can be written as

$$-p_T \frac{\partial t_f}{\partial r} \frac{d}{d(k\tilde{\alpha}/T)} \left[2\pi I_0\left(\frac{k\tilde{\alpha}}{T}\right) \right]$$

which, on using $\frac{dI_0(z)}{dz} = I_1(z)$, becomes

$$-2\pi p_T \frac{\partial t_f}{\partial r} I_1\left(\frac{k\tilde{\alpha}}{T}\right).$$

Combining these results,

$$\begin{aligned}
E \frac{d^3 N_i}{d^3 p} &= \frac{g_i m_T}{(2\pi)^2} \sum_{k=1}^{\infty} (\mp 1)^{k+1} \int_{\sigma^0(r,z)} r dr dz e^{k\mu_i/T} e^{-k\alpha \cosh(y-\eta)/T} \\
&\quad \times \left\{ \left(\cosh y - \frac{\partial t_f}{\partial z} \sinh y \right) I_0\left(\frac{k\tilde{\alpha}}{T}\right) - \frac{p_T}{m_T} \frac{\partial t_f}{\partial r} I_1\left(\frac{k\tilde{\alpha}}{T}\right) \right\}. \quad (C.10)
\end{aligned}$$

Transforming d^3p to cylindrical coordinates and integrating over ϕ_P , which, due to the cylindrical symmetry, simply introduces a factor of 2π ,

$$\begin{aligned} \frac{dN_i}{p_T dp_T} &= \frac{g_i m_T}{2\pi} \sum_{k=1}^{\infty} (\mp 1)^{k+1} \int_{\sigma^0(r,z)} r dr dz e^{k\mu_i/T} \\ &\times \left\{ I_0 \left(\frac{k\tilde{\alpha}}{T} \right) \int_{-\infty}^{+\infty} dy \left(\cosh y - \frac{\partial t_f}{\partial z} \sinh y \right) e^{-k\alpha \cosh(y-\eta)/T} \right. \\ &\quad \left. - I_1 \left(\frac{k\tilde{\alpha}}{T} \right) \int_{-\infty}^{+\infty} dy \frac{p_T}{m_T} \frac{\partial t_f}{\partial r} e^{-k\alpha \cosh(y-\eta)/T} \right\}. \quad (\text{C.11}) \end{aligned}$$

Introducing $t = y - \eta$, using the evenness of $\cosh t$, the oddness of $\sinh t$, and the definition of the Bessel function,

$$\begin{aligned} \int_{-\infty}^{+\infty} dy \cosh y e^{-k\alpha \cosh(y-\eta)/T} &= \int_{-\infty}^{+\infty} dt \cosh(t + \eta) e^{-k\alpha \cosh t/T} \\ &= \int_{-\infty}^{+\infty} dt (\cosh t \cosh \eta + \sinh t \sinh \eta) e^{-k\alpha \cosh t/T} \\ &= \int_{-\infty}^{+\infty} dt \cosh \eta \cosh t e^{-k\alpha \cosh t/T} \\ &= 2 \cosh \eta K_1 \left(\frac{k\alpha}{T} \right), \\ \int_{-\infty}^{+\infty} dy e^{-k\alpha \cosh(y-\eta)/T} &= \int_{-\infty}^{+\infty} dt e^{-k\alpha \cosh t/T} \\ &= 2 K_0 \left(\frac{k\alpha}{T} \right), \\ \int_{-\infty}^{+\infty} dy \sinh y e^{-k\alpha \cosh(y-\eta)/T} &= \int_{-\infty}^{+\infty} dt \sinh(t + \eta) e^{-k\alpha \cosh t/T} \\ &= \int_{-\infty}^{+\infty} dt (\sinh t \cosh \eta + \sinh \eta \cosh t) e^{-k\alpha \cosh t/T} \\ &= \int_{-\infty}^{+\infty} dt \sinh \eta \cosh t e^{-k\alpha \cosh t/T} \\ &= 2 \sinh \eta K_1 \left(\frac{k\alpha}{T} \right), \end{aligned}$$

and, finally

$$\begin{aligned}
\frac{dN_i}{p_T dp_T} &= \frac{g_i m_T}{\pi} \sum_{k=1}^{\infty} (\mp 1)^{k+1} \int_{\sigma^0(r,z)} r dr dz e^{k\mu_i/T} \\
&\times \left\{ \left(\cosh \eta - \frac{\partial t_f}{\partial z} \sinh \eta \right) K_1 \left(\frac{k m_T \cosh \rho}{T} \right) I_0 \left(\frac{k p_T \sinh \rho}{T} \right) \right. \\
&\quad \left. - \frac{p_T}{m_T} \frac{\partial t_f}{\partial r} K_0 \left(\frac{k m_T \cosh \rho}{T} \right) I_1 \left(\frac{k p_T \sinh \rho}{T} \right) \right\}. \quad (\text{C.12})
\end{aligned}$$

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