UNIVERSITY OF CAPE TOWN

DEPARTMENT OF MATHEMATICAL STATISTICS

THE ESTIMATION OF SECURITY BETA COEFFICIENTS ON THE JOHANNESBURG STOCK EXCHANGE

by

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A thesis prepared under the supervision of
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and
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in fulfilment of the requirements for the degree of
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TO CHERYL

my wife, who, although a full-time student herself, has been totally committed to this venture from its inception. Without the countless sacrifices she has made, and her constant belief in its final fruition, this project would never have been completed.

TO THE PRAISE OF HIS GLORY

Ephesians - Chapter 1, Verse 12.

ABSTRACT

Aspects of the use of the market model for the estimation of security beta coefficients are investigated for securities listed on the Johannesburg Stock Exchange. Not surprisingly it is found that the market portfolio is consistently inefficient *ex post*. A Markowitz portfolio selection study using the sector indices of the JSE Actuaries Index shows that some sectors are consistently inefficient *ex post*, yet favoured by the professionally managed Association of Unit Trusts portfolio. The South African exchange control regulations on domestic investors produce conditions which imply that domestic investors should not hold gold shares as portfolio investments. It is shown that holding such shares is more efficient for overseas investors who are able to diversify away virtually the entire risk of investing in these shares in the context of their portfolios.

Homogeneous sector groups are found to exist for the sectors of the JSE Actuaries Index after the overall market effect has been removed. Sectors fall into one of three groups; gold-related, other mining, and industrial and financial. A multi-index model, based on this split of sectors, provides a superior explanation of security returns to the single-index market model.

Security beta coefficients, estimated using a multibeta approach employing a Bayesian adjustment procedure, and recent data on market capitalisations and variance of return, are found to be better predictors of next period's beta coefficients than the ordinary historical betas. The non-stationarity of security betas is noted, and an iterative algorithm, employing an integrated log likelihood function generated from Bayesian statistics and the Chow test for equality of regression parameters, is developed to determine the current valid historical data set from which the current security beta coefficient may be estimated. It is asserted that the method for estimating a security beta coefficient differs according to the intended application of the estimate. Three practical estimation procedures are suggested for estimating respectively, the historical, current and future beta coefficient of a security.

CONTENTS

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Because of the part-time nature of this venture, there were contributions made by my employers while this thesis was being prepared. Here I must especially thank my former colleagues at the Old Mutual, where the bulk of this work was done, for lending their support to me throughout. Access to data, computers and typists was never a problem. My current partners at Allan Gray Investment Counsel, too, have supported the completion of this work.

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CHAPTER ONE

INTRODUCTION AND DISCUSSION OF THE MARKET MODEL

1.1 The Evolution of Stock Markets

It seems that in the thirteenth century a man, Leonardo Fibonacchi, a traveller from Pisa, is credited with introducing the Arabic numerical system to Tuscany in northern Italy. The inventiveness of the human mind soon applied the new tool in the development of bills of exchange and credit notes to aid in the burgeoning merchant trade of the day. As the boom developed it appears that a rather severe energy crisis occurred in Europe because of a wood shortage, which resulted in the Dutch developing a shipping trade to import wood from Finland and Sweden. It did not take long for the ships to be used for other purposes and so it was that financing of trade ventures became a feature of the banking world. This need almost certainly resulted in the development of the Amsterdam Stock Exchange in 1602. Although it was about three hundred and fifty years later before Markowitz (1952) really elucidated the concepts of risk and return in stock markets, it is probably true that these early capitalists understood the risks of project financing extremely well!

Before long the Dutch ships were sailing to America, where the Dutchmen bought an island from the Manhattan Indians and built a wall across the northern boundary of their new acquisition, calling the path alongside it, Wall Street. Here in 1692, after New Amsterdam had been renamed New York, traders met under a buttonwood tree and called the area, the Stock Exchange. Around the same time, and for similar reasons, trading in shares was occurring in London.

In South Africa the project financing need was not shipping, but gold mining, and so it was that in November 1887, one year after the discovery of gold on the Witwatersrand that one Benjamin Woollan founded the Johannesburg Stock Exchange in Simmonds Street.

Stock markets have since become an indelible feature of societies embracing, even partially, the Capitalist ideology. Nevertheless one of history's greatest contributors to economic development, J.M. Keynes (1936) said:

> *"When the capital development of a country becomes a by-product of the activities of a casino, the job is likely to be ill-done. The measure of success attained by Wall Street .... cannot be claimed as one of the outstanding triumphs of laissez-faire capitalism."*

He was, of course, writing after the great Wall Street Crash had left financial scars on virtually all market participants of the time. One of the characteristics of stock markets is that prices move up and down. Because of the obvious advantages of holding shares doing the former, and disadvantages of owning shares doing the latter, researchers and market participants have probably endeavoured to understand markets and individual securities since the first wood shipment arrived in Amsterdam from Finland. It was perhaps inevitable that the coexistence of the "natural" time series of numbers which stock prices represent, and the development of computers would produce a fruitful marriage of endeavour. Thus it has been that for at least the last thirty years academic pursuit into this field of research has been virile. Most work has inevitably been done on the New York Stock Exchange, considerably less in the United Kingdom, and virtually nothing by comparison in South Africa. It seems safe to assume, however, that while stock markets exist, stock market analysts and theorists will abound.

1.2 The Measurement of Risk

It is probably fair to say that the concept of return on a security has achieved a far greater level of understanding and acceptance among investors than the concept of risk associated with that return. Returns are fairly easily and unambiguously measured and perhaps more importantly investors can consume, in an economic sense, the benefits of a positive return. A stock market venture which goes the complete cycle of investing, selling at a profit and collecting dividends along the way results *ex post* in an increment in wealth to the investor (ignoring opportunity cost). The incremental wealth is available for consumption and the risk associated with its production is now a thing of the past. Thus it is that Brealey and Myers (1981, p.129) say:

> "*Risk in investment means that future returns are unpredictable.*"

Jensen (1969) has discussed the notion that investors require additional expected return for the assumption of additional expected risk. These concepts are embodied in the Capital Asset Pricing Model (CAPM) which is described by Jensen (1972). His model is an expectational model and requires an investor utilising it to provide estimates of future expected returns and future expected risks. Because risks and returns can only be measured *ex post*, testing the CAPM is difficult (Roll (1977)). Sharpe (1970) has shown that risk for a security or portfolio can be separated into two components, one specific to the asset and one related to the market. The former component can be potentially eliminated by diversification while the latter is unavoidable whatever the level of diversification. Brealey and Myers (1981, p.126) say:

> "*The risk of a well-diversified portfolio depends on the market risk of the securities included in the portfolio.*"

They call this idea one of the most important in their book. It would seem to behove investors, therefore, to obtain the best possible estimates of market risk for individual securities, in order to be in a position to understand the risk that will be associated with any well-diversified portfolio formed from these securities. The main objective of this thesis is to examine estimation procedures and problems for individual security market risk measurement and this is done primarily by using the market model developed in the next section.

1.3 The Market Model

Markowitz (1959) proposed the use of the market model (MM) for the estimation of security risk parameters. The model may be written as

$$R_{i;t} = \alpha_i + \beta_i R_{m;t} + e_{i;t}$$

where $R_{i;t}$ is the return on security i in time period t

$R_{m;t}$ is the return on the market in time period t

α_i and β_i are parameters unique to security i

$e_{i;t}$ is the disturbance or error term satisfying the following assumptions:

(i) $E(e_{i;t}) = 0$;

(ii) $cov(e_{i;t}, e_{i;s}) = 0$ for all $t \neq s$;

(iii) $var(e_{i;t}) = \sigma^2$ for all t ; and

(iv) $e_{i;t}$ is independent of $R_{m;t}$ for all t.

The parameters α_i and β_i are usually estimated by the technique of regression analysis, provided sufficient past data on the return of the security under consideration and the return on the market, are available. The β_i parameter is argued by Sharpe (1970) to be a measure of the market risk of a security and hence is the only risk parameter required in understanding the impact of that security on the total risk of a well diversified portfolio.

1.4 Using the Market Model

1.4.1 General Comments

Fama, Fisher, Jensen and Roll (1969) have found that the linearity assumption of the MM appears to be reasonably well satisfied. Modigliani and Pogue (1974) established that beta coefficients do provide a good measure of the risk inherent in a security. Beaver, Kettler and Scholes (1970), Rosenberg and McKibben (1973) and Bowman (1979) have noted that the value of beta in any period is related to fundamental characteristics of the firm and its economic environment.

1.4.2 Use of Ordinary Least Squares

The employment of ordinary least squares (OLS) as the regression technique has been used extensively in the application of the MM. It requires the four assumptions for the error term in Section 1.3 to be satisfied. Assumption (i) is achieved by construction while assumption (ii) follows, in principle, from the random walk model which has been shown to apply on the New York Stock Exchange (NYSE) by Fama *et al* (1969) and on the Johannesburg Stock Exchange (JSE) by

Affleck-Graves and Money (1975). Assumptions (iii) and (iv) amount to the regression assumption of homoscedasticity which has been found to be well supported for the NYSE by Fama *et al* (1969) and Martin and Klemkosky (1975). For industrial securities on the JSE, Affleck-Graves (1977) found some 30% exhibited significant heteroscedasticity. Praetz (1969) and Belkaouri (1977) also found significant evidence of heteroscedasticity for the Australian and Canadian stock markets respectively. Dimson (1979) asserted that infrequent trading is a likely explanation for this phenomenon in non-United States markets. Affleck-Graves (1977,p.6.17) concluded that the issue of heteroscedasticity is probably academic if the MM is not a suitable model for a particular security (that is, the "fit" is poor). Chapter Four of this thesis examines the aspect of model fit while Chapter Seven examines alternative regression methods to OLS.

Another of the assumptions required to use OLS is that of normality in the distribution of error terms. Although evidence has been provided by Blattberg and Gonedes (1974) that the assumption of normality in monthly security returns is a reasonable approximation, other researchers (Affleck-Graves (1974) and Fama (1963)) have found support for the stable paretian family of distributions. The impact of this issue, if any, on the use of OLS is also examined in Chapter 7.

One of the characteristics of OLS is the weight given to outliers in the regression methodology. Rejection of outliers

(Anscombe (1960), Anscombe and Tukey (1963)) while advocated by some is cautioned by Draper and Smith (1966, p.95):

> *"Automatic rejection of outliers is not always a very wise procedure. Sometimes the outlier is providing information which other data points cannot due to the fact that it arises from an unusual combination of circumstances which may be of vital interest and requires further investigation rather than rejection."*

These words are felt to be particularly true for stock market data and the MM. Since most regression lines using the MM are positively sloped (that is, beta coefficients are very rarely negative), outliers in the MM can occur when the market portfolio rises (falls) and the security falls (rises), or very exaggerated moves in either variable occur which are not matched by the other. These conditions should be regarded with great interest since they result from price changes brought about by investors presumed to be acting efficiently. As such they contain important information which a market analyst is not necessarily justified in excising from the data set in applying the MM. Therefore this issue is not considered further in this thesis and no trimming of outliers is attempted in this study.

1.4.3 The Market Portfolio

Roll (1977) has pointed out that no portfolio exists which adequately represents all risky assets. Ibbotson and Siegel

(1983), however, have constructed a world market wealth portfolio which probably represents the true universe of risky assets as adequately as possible. Stambaugh (1982) found that even when common stocks represented only 10% of a "market" portfolio, inferences about the CAPM were almost identical to those obtained from a stocks-only portfolio. South African investors are limited to the local JSE for their listed equity investments by the current laws of exchange control. Until September 1978, no adequate overall market index existed for the JSE, but on that date, the JSE Actuaries Index (1978), which has been calculated daily since that time, was launched. Monthly historical data were made available for this Index going back to 1960 in some cases. For this thesis monthly data are used for all chapters except Chapter Seven which employs weekly data for the best "market" index available before the JSE Actuaries Index, namely, the RDM Industrial Index.

It is considered that the JSE All Share Index is an adequate representation of securities listed on the JSE. However, the JSE is a heterogeneous market comprising of securities which can display considerable lack of covariance in their price behaviour. Therefore aspects of the market portfolio as represented by the JSE All Share Index and as applied in the MM are given considerable attention in this thesis, particularly in Chapters Two, Three, Four and Five.

1.4.4 Infrequent Trading

Dimson (1979) has detailed the impact of infrequent trading on risk measurement. Many shares on the JSE probably fall into this category (Strebel (1977) and (1978)) and hence have downward biased beta estimates as outlined by Dimson. Aspects of this problem are discussed in Chapter Four, but in general no attempt has been made to evaluate the use of Dimson's Aggregated Coefficients method for the JSE. It is felt this would be a fruitful line of further enquiry on the JSE.

1.4.5 Time Varying Beta Coefficients

It is well known that beta coefficients of portfolios are more stationary over time than those of individual securities (Blume (1971), (1975)) and yet a considerable body of evidence exists to show that even portfolio betas cannot be regarded as stationary. Chen (1981) noted that the use of OLS will overstate the portfolio residual risk if individual security beta coefficients are changing over time. Fabozzi and Francis (1979B) pointed out that changes in the macro-economic environment can influence the estimation of the parameters in the MM over time. Several studies (Fabozzi and Francis (1978), (1980), Kon and Jen (1978), Miller and Gressis (1980), Thompson (1976) and Kon and Jen (1979)) have regarded beta coefficients as random coefficients following a stochastic process. Ohlson and Rosenberg (1982) found that there were two kinds of stochastic variation in beta in their study. They were:

(i) a stationary first order autoregressive process, which produced wide excursions about the mean value and

(ii) a serially independent random increment.

Other researchers have investigated whether beta coefficients differ in bull and bear market conditions (Chen (1982), Kim and Zumwalt (1979) and Fabozzi and Francis (1979A)). For the JSE, Bradfield, Affleck-Graves and Barr (1982) have found alpha and beta coefficients to be stable over bull and bear markets.

Recognising the time-varying nature of beta coefficients Ball (1972) and Mandelker (1974) advocated use of moving average betas. Other researchers have been concerned to identify the location and frequency of change points in the historical data. This approach is examined in Chapter Six for the JSE and an approach is developed for eliminating non-valid data from the historical data set.

In passing, the issue of the optimal period for the estimation of beta coefficients is mentioned in Chapter Six. It is clear that if beta coefficients are going to be predicted use of an estimation interval of similar length to the length of the prediction interval may be desirable (Eubank and Zumwalt (1977)). In this way an "average" historical beta is used to predict an "average" future beta. The objective of Chapter Six, however, is to establish the best estimate of the *current* beta of a security.

1.5 The Need to Measure Risk

Apart from the purpose of obtaining a security beta estimate to understand the contribution of that security to the total risk of a diversified portfolio, security betas estimated by the market model have been used, and empirically justified, in performance evaluation by Brown and Warner (1980), and Copeland and Mayers (1982). In addition, beta estimates are employed in capital budgeting decisions in finance using the cost of capital concept (Brealey and Myers (1981, Chapter 9)).

1.6 The Organisation of the Thesis

The thesis may be thought of as falling conveniently into two parts. Chapters Two, Three and Four deal with describing and understanding the market portfolio on the JSE, and commenting on related issues of efficiency and composition of this portfolio. At the same time, some peculiarities introduced into the task of investing on the JSE by the laws of exchange control on residents and the heterogeneous nature of the securities listed on the JSE, are discussed. A multi-index model, as an alternative to the market model, is also evaluated in Chapter Four. The inherent attraction of a one-parameter risk model such as the MM provides the justification for the second part of the thesis in Chapters Five to Eight, in which aspects of the practical use of the MM are addressed. A multibeta approach is developed in Chapter Five, while in Chapter Six the issue of time changing betas is treated by developing a method for rejecting data no longer relevant to the current estimate of a security beta. In Chapter Seven aspects of the statistical procedure of beta estimation are examined and finally in Chapter Eight a recommended approach to security beta estimation is suggested which combines the principles outlined in earlier chapters.

CHAPTER TWO

MARKOWITZ PORTFOLIO SELECTION APPLIED TO SECTORS ON THE JOHANNESBURG STOCK EXCHANGE

2.1 Introduction

Thirty years ago Markowitz (1952) published his famous paper on portfolio selection, from which many subsequent articles, theses and papers have followed. The basic notion is that only two factors need be considered in choosing a portfolio, namely, the return an investor can expect to receive from holding the portfolio and the uncertainty associated with this return. For the purposes of this chapter the generally accepted measures of these variables have been used and they are respectively the weighted average return of the securities comprising the portfolio and the standard deviation of the return on the portfolio.

Thus if

E_i = Expected return on the ith security

σ_i = Standard deviation of return on the ith security

E_p = Expected return on the portfolio

σ_p = Standard deviation of return on the portfolio

σ_{ij} = Covariance between security i and security j

ρ_{ij} = Correlation coefficient for the returns on securities i and j

X_i = Proportion of funds invested in security i

N = Total number of securities considered

then

$$E_p = \sum_{i=1}^{N} X_i E_i$$

$$= X'E$$

and

$$\sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} X_i X_j \sigma_{ij}$$

$$= \sum_{i=1}^{N}\sum_{j=1}^{N} X_i X_j \sigma_i \sigma_j \rho_{ij}$$

$$= X'\$X$$

where

$$X' = (X_1, X_2, \ldots, X_N)$$

$$E' = (E_1, E_2, \ldots, E_N)$$

and

$$\$ = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2N} \\ \vdots & & & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_N^2 \end{pmatrix}$$

In addition it is assumed that $\sum_{i=1}^{N} X_i = 1$ and $X_i > 0$ for all i.

This implies that all funds are invested and no short position is held in any security. Under the above conditions any portfolio can be described by a point in the E_p, σ_p plane. In particular Figure 2.1 below shows all the feasible portfolios as filling some region of the E_p, σ_p plane.

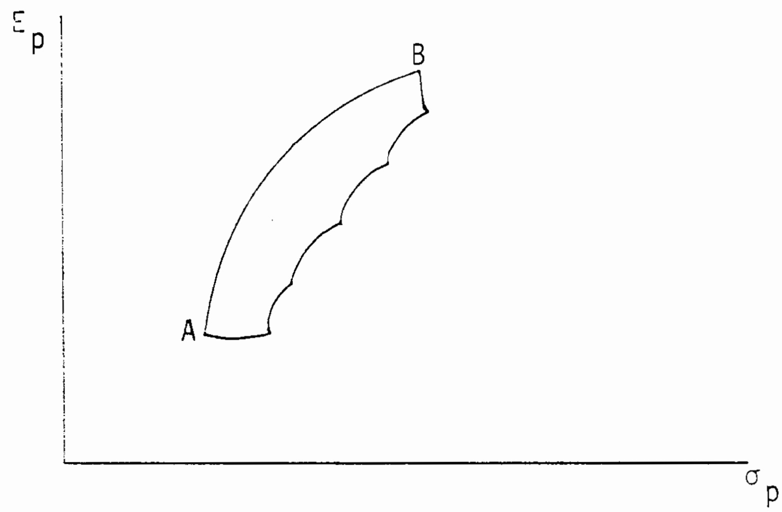


FIGURE 2.1

Portfolios lying along the upper boundary AB dominate all other portfolios and together comprise the efficient set of portfolios because for these portfolios it is not possible to either obtain a greater expected return without incurring greater risk or obtain smaller risk without decreasing expected return. Jensen (1969) says:

> *"In a world dominated by risk averse investors, a risky portfolio must be expected to yield higher returns than a less risky portfolio, or it would not be held."*

Therefore investors will only wish to hold portfolios belonging to the efficient set and each investor is left to choose the single one portfolio (that is, trade off the levels of risk and return) for himself. Hence the problem reduces to the solution of the objective function of the mathematical programming problem minimise $-\lambda E_p + V_p$

for all possible values of λ

subject to $\sum_{i=1}^{N} X_i = 1$ and $X_i \geq 0$ for all i

where $E_p = X'E$ and $V_p = \sigma_p^2 = X'\Sigma X$.

This is the basic Markowitz portfolio selection problem and is a quadratic programming problem. Additional constraints can be added and the basic problem may be expanded to a more general form, termed the 'standard problem' which can be written:

minimise $-\lambda E_p + V_p$

for all possible values of λ

subject to $\sum_{i=1}^{N} X_i = 1$

plus any other linear equality constraints

plus

$$\begin{aligned} L_1 &\leq X_1 \leq U_1 \\ L_2 &\leq X_2 \leq U_2 \\ &\vdots \\ L_N &\leq X_N \leq U_N \end{aligned}$$

where U_i is the upper bound for the proportion of funds invested in security i and L_i is the lower bound.

Sharpe (1970) has proposed an algorithm for the solution of this problem and for this chapter an adaptation of this by Affleck-Graves (1974) was used for all computations.

2.2 The JSE as Described by the JSE - Actuaries Indices

A complete Markowitz portfolio selection exercise on the

Johannesburg Stock Exchange (JSE) would thus involve the selection of efficient portfolios from the universe of 412 quoted stocks. It can be shown that with N stocks the number of different correlation coefficients to be estimated is

$$\frac{N^2-N}{2}$$

and thus in the case of the JSE,84666 estimates would be required. For reasons of computer time and data availability and storage this approach to efficient portfolio selection was not possible. However, the JSE Actuaries Indices (the JSE Actuaries Index (1980)) have been constructed so that the 34 sector indices have a continuous price history from January 1965. Since

(i) the data availability condition was met

(ii) a much lower number of correlation estimates is required (561 in all)

(iii) these indices reflect the behaviour of the total market

it was decided to undertake the selection of *ex post* efficient portfolios regarding the JSE as a 34 'security' market, where each 'security' is in fact a portfolio of like securities aggregated into a sector index.

Appendix A shows the structure of sectoral and composite indices which were used in this study with the percentage contribution that each index made to its immediately superior composite index at the end of January 1980.

Out of the universe of 412 active stocks on the JSE in

January 1980, 153 stocks only comprised the JSE Actuaries Index.

2.3 The Data and Methodology Used

The study was conducted over three equal, non-overlapping time periods, namely

(i) February 1965 to January 1970

(ii) February 1970 to January 1975

(iii) February 1975 to January 1980

These three periods were not chosen to coincide with or represent any particular market cycle but provided three convenient periods for comparison of the results of the study.

Returns for the indices were calculated as follows:

$$R_{i;t} = \frac{P_{i;t} - P_{i;t-1}}{P_{i;t-1}}$$

where $R_{i;t}$ = return for index i in month t

$P_{i;t}$ = price of index i at the end of month t

$P_{i;t-1}$ = price of index i at the end of month t-1.

Thus there were sixty monthly returns spanning five years of data.

Mean monthly returns for each index in each period were calculated ignoring dividends which in any event are excluded from the calculation of the JSE Actuaries Index. The 34×34 covariance matrix was calculated for each period and with this data *ex post* efficient frontiers were established for each of the three periods using the algorithm employed by Affleck-Graves (1974).

It must be emphasised that this study was conducted with *ex post* observed returns while Markowitz portfolio selection uses *ex ante* expected returns. As Sharpe (1970) points out:

> *"The values of capital market theory are ex ante (before-the-fact) estimates. Observed values are ex post (after-the-fact) results. The portfolios that do, in fact, turn out to be efficient will lie along some line, but not necessarily the ex ante capital market line. In fact, the market portfolio invariably proves to be inefficient ex post."*

Therefore the *ex post* nature of this study precludes any conclusions being drawn in regard to the *ex ante* efficiency of either the market portfolio or investors in general. The results presented below are at all times subject to this caveat. Nevertheless *ex post* studies are useful to the extent that they demonstrate the investment opportunities that actually were available in a period.

2.4 Unconstrained Efficient Frontiers

The efficient frontiers for each period of five years were calculated and are graphically displayed in Figure 2.2. These frontiers are unconstrained in the sense that no individual sector was assigned a maximum in terms of the proportion of the total funds which could be invested in that sector.

Unconstrained Efficient Frontiers

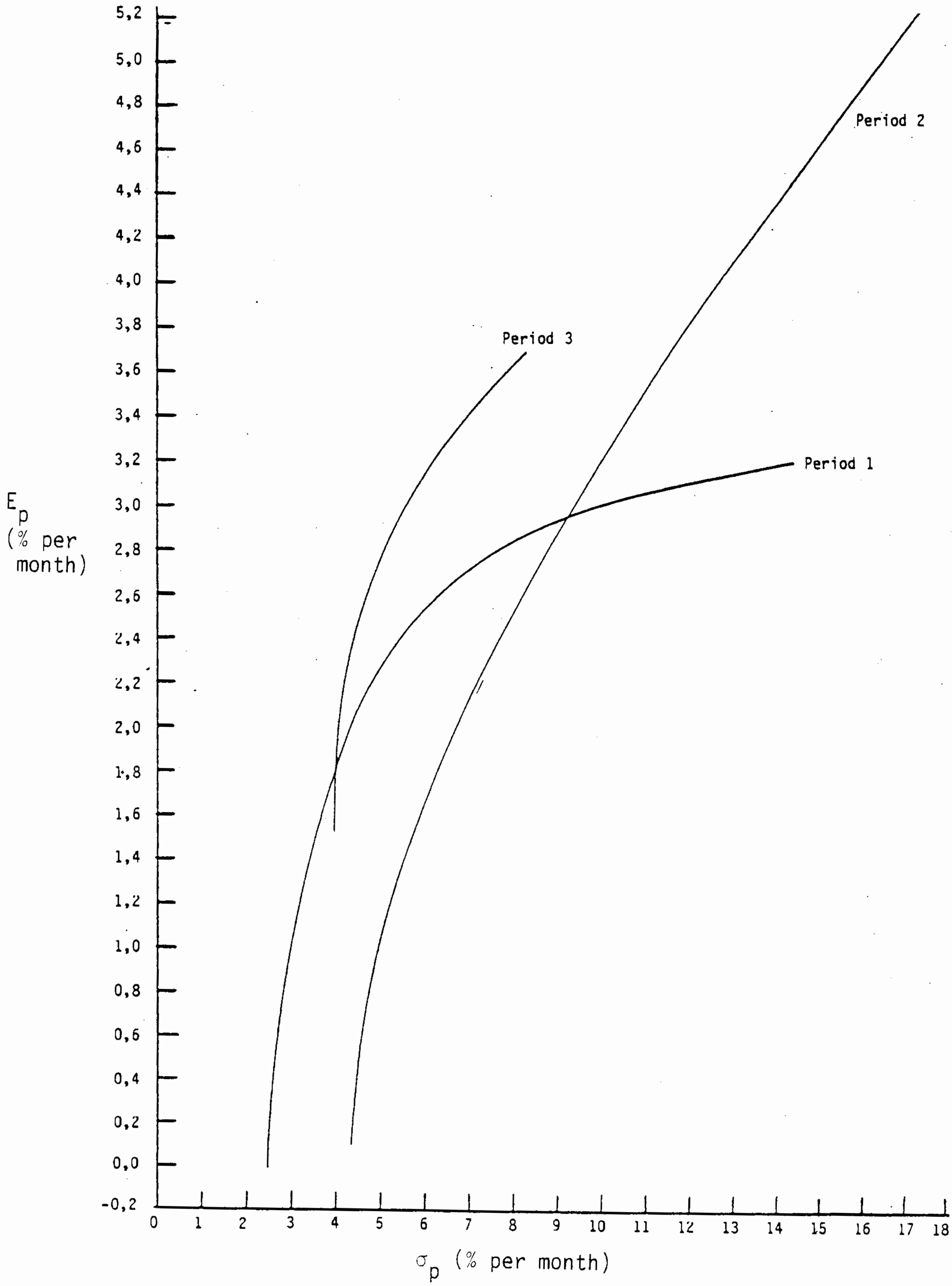


FIGURE 2.2

The minimum proportion of zero was applied throughout which means no short positions were allowed.

An examination of these three frontiers for each period reveals two points:

(i) The range of risk/return combinations obtainable in Period 3 was much more restricted than in Period 1 which in turn was more restricted than in Period 2.

(ii) A given risk level (for example $\sigma_p = 7$) produced dramatically different returns depending on the period under consideration. Similarly the achievement of a specific return (for example $E_p = 2{,}6$) involved bearing quite different risks in each period.

2.5 The Relationship of Market Indices to the Efficient Frontiers

In Figures 2.3, 2.4 and 2.5, the efficient frontiers and risk/return points representing the JSE All-Share Index (M) All Mining Index (A), Mining Financial Index (F) and Industrial and Financial Index (I) for Periods 1, 2 and 3 respectively are plotted. The last three indices represent the three broad sector groupings on the JSE as proposed by the JSE Actuaries Index. The classical capital market theory as developed by Sharpe (1970) requires the market portfolio to plot on the efficient frontier. This conclusion is derived from the basic set of assumptions underlying the theory, in particular the perfect knowledge and agreement of all investors concerning each security's prospects and interrelationship with other securities. From this it is concluded that all investors will agree on the optimal combination of risky securities and each investor will achieve his desired risk level by borrowing or lending at the risk-free rate. From this it is a small step in logic to conclude that (Sharpe (1970)):

> "*In equilibrium, the optimal combination of risky securities must include all securities; moreover the proportion of each security must equal its proportionate value in the market as a whole. The conclusion is inescapable. Under the assumed conditions, the optimal combination of risky securities is that existing in the market.*"

It is apparent from Figures 2.3, 2.4 and 2.5 that *ex post* M proved to be inefficient. However, as noted earlier, this result is not surprising, and is in fact to be expected.

2.6 Composition of Efficient Portfolios

In Figures 2.3, 2.4 and 2.5 portfolios along the efficient frontiers are numbered. It is instructive to examine the composition of efficient portfolios over widely differing risks to observe the level of diversification and magnitude of sector weightings that occur. Tables 2.1, 2.2 and 2.3 refer to Figures 2.3, 2.4 and 2.5 respectively and represent the Periods 1, 2, 3 as defined in Section 2.3. Portfolios

Period 1 - Unconstrained Efficient Frontier
(02/65 - 01/70)

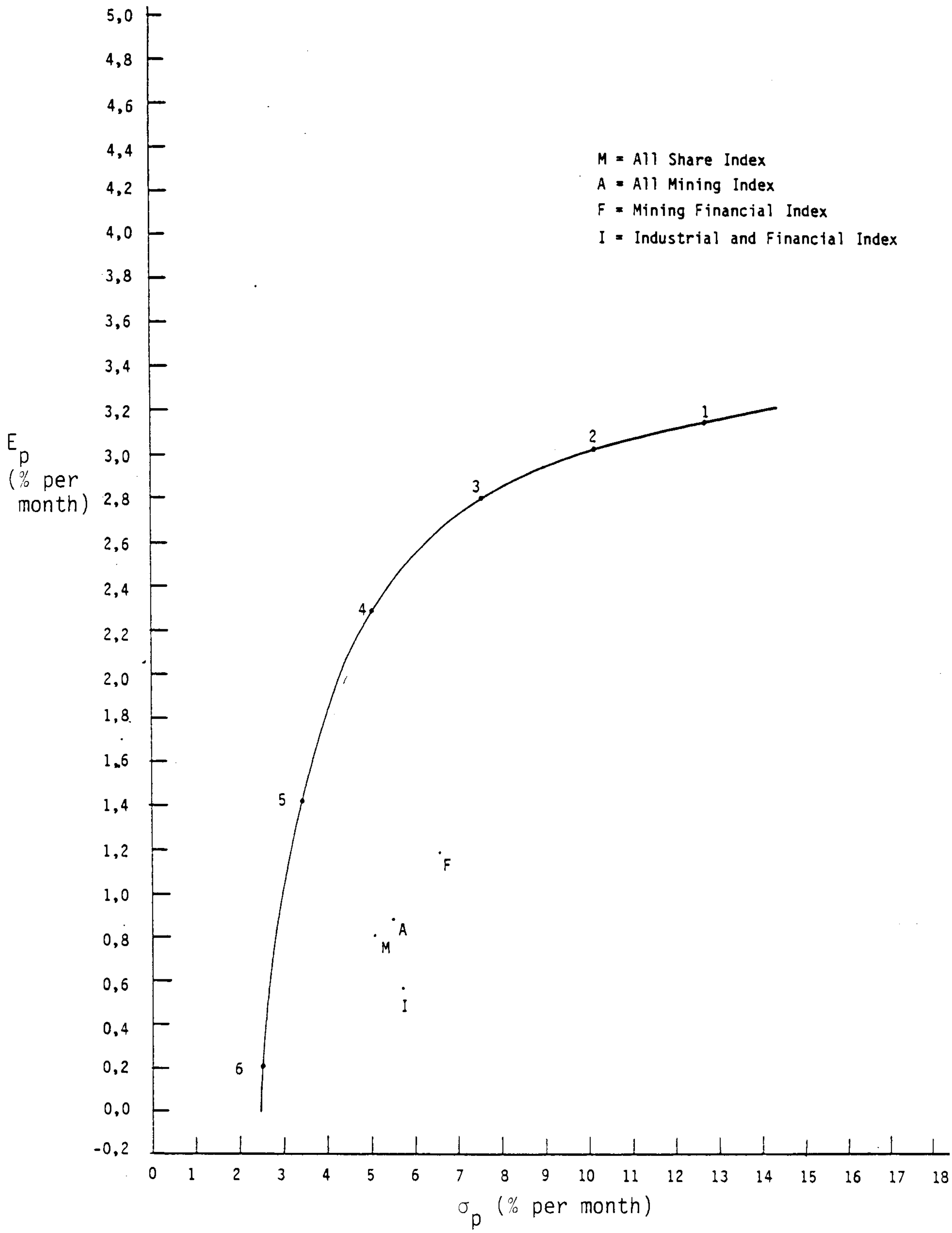


FIGURE 2.3

Period 2 - Unconstrained Efficient Frontier
(02/70 - 01/75)

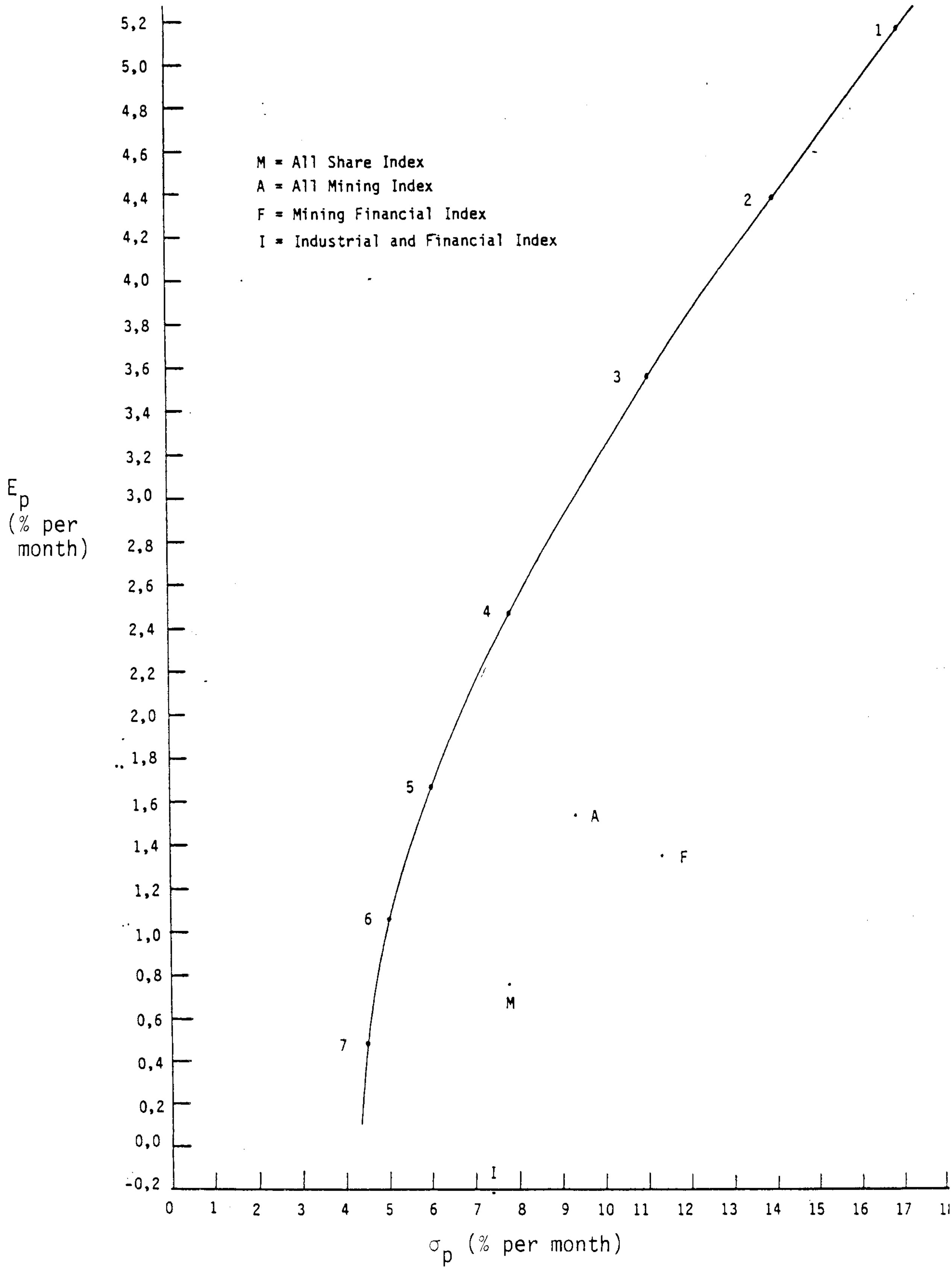


FIGURE 2.4

Period 3 - Unconstrained Efficient Frontier

(02/75 - 01/80)

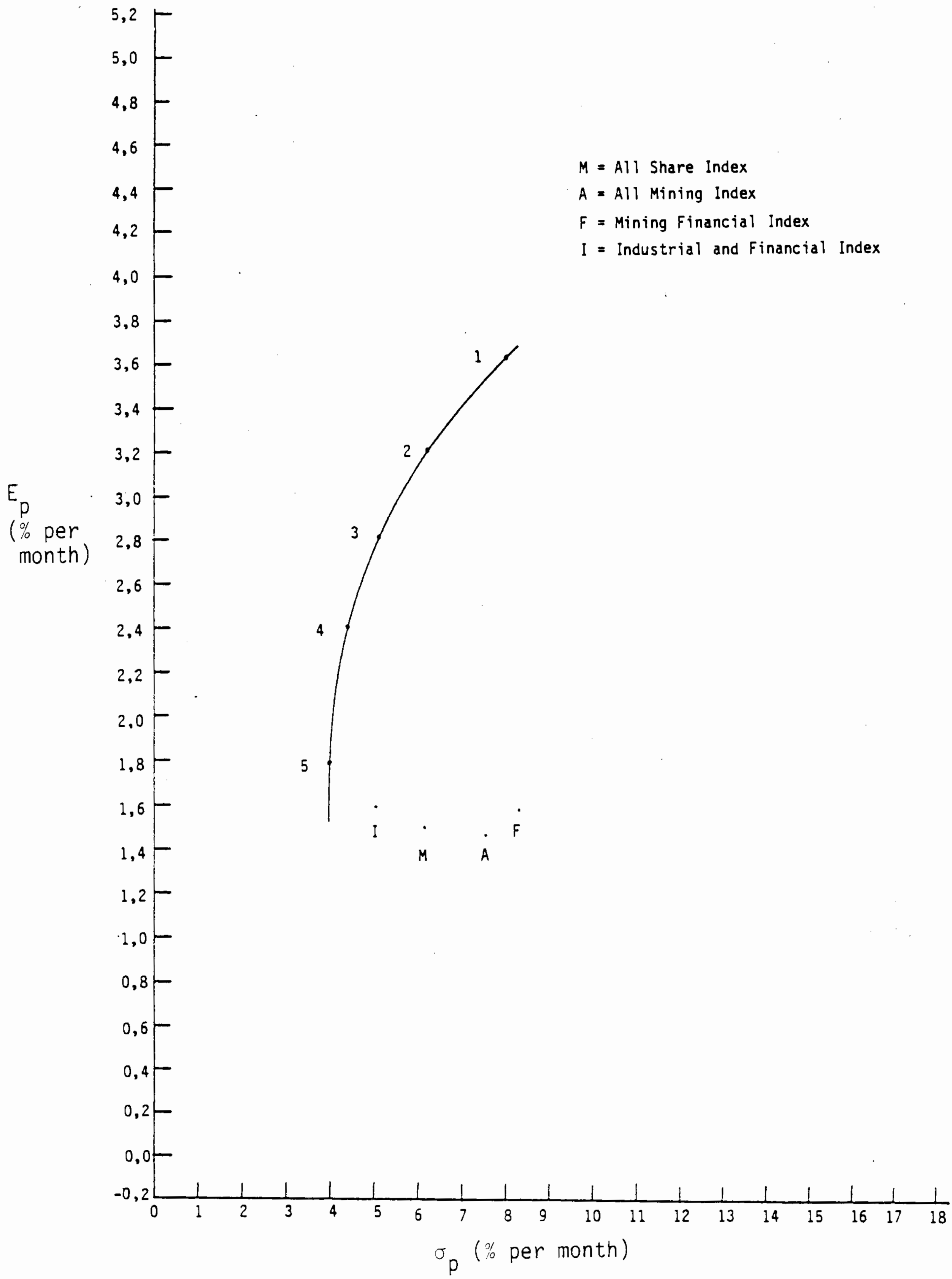


FIGURE 2.5

with an asterisk carry the same risk as the M index. The following points emerge:

(i) At very high risks, limited diversification occurs. This is obviously due to the fact that only a few sectors in each period have a sufficiently high expected return to compensate the investor for the high risk assumed.

(ii) As the risk of the efficient portfolios decreases, diversification increases with concomitant smaller sector weightings. This is due to the fact that these portfolios are approaching the area in which M itself lies and since M is by definition fully diversified it is not surprising that these portfolios exhibit more diversification. Lower variance in portfolio returns is also to be expected as diversification increases, and therefore as the risk is decreased it is likely to be accompanied by increased diversification. In fact the least risky efficient portfolios considered in Periods 1, 2 and 3 contained 14, 7 and 10 sectors respectively.

(iii) Certain sectors seem to occur in portfolios grouped in particular risk areas. For example in Period 1 the gold sectors occur in the lower risk portfolios whereas in Period 2 they occur in the higher risk portfolios. This is clearly attributable to the relative volatility and level of the gold price during these two periods. Some sectors like Printing, Electrical and Engineering only appear in lower risk portfolios at all times.

(iv) Some sectors persist in the efficient portfolios over large risk ranges. In Period 2, Gold - West Wits appears in every portfolio considered, while in Period 1, Platinum displays the same behaviour. In Period 3 both the Coal and Clothing sectors display this characteristic. The only way in which a sector can achieve this (for after all each sector is only a point itself in the E_p, σ_p plane) is if it displays not only a high return but also very little covariance with other currently efficient sectors. In this way the algorithm will select it for its high marginal return added for low marginal risk borne.

(v) Some sectors appearing in adjacent portfolios come in at low proportions, rise to a peak and then fall again. Examples are Gold - West Wits and Food in Period 2, Platinum in Period 1 and Clothing in Period 3. Others commence at a high proportion of the portfolio and steadily diminish. Insurance in Period 1, Gold - Rand in Period 2 and Coal in Period 3 are examples of this behaviour. In the former case the sectors plot in mid-range with respect to risk/return while in the latter case the sectors tend to be high risk/high return.

σ_M = 5,020 % per month
R_M = 0,804 % per month

TABLE 2.1: Period 1

Portfolio No.	1.	2.	3.	4.*	5.	6.
σ_p (% per month)	13,000	10,000	7,500	5,020	3,400	2,500
E_p (% per month)	3,193	3,028	2,784	2,312	1,331	0,184
1. Gold - Rand						0,0081
2. Gold - Evander						0,0612
3. Gold - Klerksdorp					0,0427	0,1002
4. Gold - OFS						0,0690
5. Gold - W Wits						
6. Coal					0,0584	0,2010
7. Diamonds				0,1468	0,0811	
8. Platinum	0,0311	0,3560	0,5519	0,3065	0,1503	0,0678
9. Copper, Tin, Others			0,0887	0,2531	0,1615	0,0361
10. Mining Holding						
11. Mining Houses						
12. Inv. Trusts						
13. Insurance	0,9689	0,6440	0,3124	0,0197		
14. Property						
15. Banks						
16. Ind. Holding						
17. Beverages						
18. Building						
19. Chemicals						
20. Clothing						
21. Electrical						
22 Engineering						0,0100
23. Fishing						0,0681
24. Food						
25. Furniture						
26. Motors						
27. Paper, Packaging					0,0331	0,0490
28. Pharmaceutical						
29. Printing					0,3111	0,2491
30. Steel						0,0047
31. Stores						
32. Sugar					0,0395	0,0712
33. Tobacco				0,1576	0,0932	
34. Transport			0,0470	0,1163	0,0291	0,0045
TOTAL	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000

* Portfolio with same risk as the market

σ_M = 7,861 % per month
R_M = 0,749 % per month

TABLE 2.2: Period 2

Portfolio No.	1.	2.	3.	4.*	5.	6.	7.
σ_p (% per month)	17,000	14,000	11,000	7,861	6,000	5,000	4,500
E_p (% per month)	5,190	4,406	3,500	2,392	1,606	1,064	0,6372
1. Gold - Rand	0,8601	0,5268	0,2520	0,0497			
2. Gold - Evander	0,1215	0,1129	0,1005	0,0658	0,0178		
3. Gold - Klerksdorp							
4. Gold OFS							0,0363
5. Gold - W Wits	0,0185	0,3603	0,5320	0,5329	0,4614	0,3398	0,1914
6. Coal				0,0601	0,1305	0,1839	0,2187
7. Diamonds							
8. Platinum							
9. Copper, Tin, Others							
10. Mining Holding							
11. Mining Houses							
12. Inv Trusts							
13. Insurance							
14. Property							
15. Banks							
16. Ind. Holding							
17. Beverages				0,0585	0,0554	0,0536	
18. Building							
19. Chemicals							0,0686
20. Clothing							
21. Electrical				0,0070	0,2091	0,3596	0,4156
22. Engineering							
23. Fishing			0,0520	0,0867	0,0484	0,0157	
24. Food			0,0635	0,1393	0,0702	0,0061	
25. Furniture							
26. Motors							
27. Paper Packaging							
28. Pharmaceutical							
29. Printing							0,0261
30. Steel					0,0072	0,0414	0,0433
31. Stores							
32. Sugar							
33. Tobacco							
34. Transport							
TOTAL	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000

- * Portfolio with same risk as the market

σ_M = 6,292 % per month
R_M = 1,482 % per month

TABLE 2.3: Period 3

Portfolio No.	1.	2.*	3.	4.	5.
σ_p (% per month)	8,000	6,292	5,000	4,400	4,000
E_p (% per month)	3,644	3,235	2,765	2,379	1,802
1. Gold - Rand					
2. Gold - Evander			0,0044		
3. Gold - Klerksdorp					
4. Gold - OFS					
5. Gold - W Wits					
6. Coal	0,9648	0,7228	0,4523	0,2905	0,1442
7. Diamonds					
8. Platinum					
9. Copper, Tin, Others					0,0020
10. Mining Holding					
11. Mining Houses					
12. Inv. Trusts				0,0376	
13. Insurance					
14. Property					
15. Banks					
16. Ind. Holding					
17. Beverages					
18. Building					
19. Chemicals			0,0944	0,1031	0,0406
20. Clothing	0,0346	0,2726	0,3892	0,3283	0,1608
21. Electrical				0,0867	0,1286
22. Engineering					0,2338
23. Fishing					0,0479
24. Food					
25. Furniture					
26. Motors					
27. Paper, Packaging					
28. Pharmaceutical					
29. Printing					0,0558
30. Steel	0,0006	0,0046			
31. Stores					
32. Sugar			0,0134	0,0712	0,0759
33. Tobacco			0,0138		
34. Transport			0,0325	0,0826	0,1104
TOTAL	1,0000	1,0000	1,0000	1,0000	1,0000

* Portfolio with same risk as the market

(vi) The highest proportions for sectors appearing in only one portfolio in any given period are Engineering, 0,2338, in Period 3 and Gold - OFS, 0,0690, in Period 1. Each of these, however, occurred in the lowest risk portfolio considered. The highest proportion for a sector appearing only once in a non-peripheral portfolio was Investment Trusts, 0,0376, in Period 3. In fact the only sectors to appear only once in non-peripheral adjacent portfolios, were during Period 3 and then only on three occasions. This tends to suggest that there is a definite hierarchy of *ex post* efficiency dominance in each period which implies that when a sector is efficient enough to be included in portfolios on the efficient frontier it tends to persist in these portfolios for quite a range in risk. Seldom does a sector, once having achieved efficiency dominance, only appear in a very localised area of the efficient frontier.

(vii) Certain sectors do not appear in any of the efficient portfolios considered in any period. They are Mining Holding, Mining Houses, Property, Banks, Industrial Holding, Building, Furniture, Motors, Pharmaceutical and Stores. It is possible to conclude that these sectors have been dominated in efficiency by the other sectors at all risks for the past fifteen years. Together they represented 34,6% of the All Share Index at end January 1980 and it is perhaps surprising that such a large collective proportion of the market remained relatively inefficient for so long. While it is possible that from an *ex ante* point of view some or all of these sectors may have appeared efficient to investors, the fact remains that *ex post* they were never efficient. This would be of some concern to investors who were heavily invested in these sectors.

(viii) The composition of the lowest risk (that is, most diversified) portfolio in each period is shown in Appendix B.

2.7 Characteristics of the Association of Unit Trusts Portfolio

Following on the amendment of the Unit Trust Control Act, the first South African unit trust was launched in June 1965 with assets totalling R600 000. At 31 December 1980 there were twelve unit trusts in existence with equity assets totalling R566,26m and total assets of R682,81m. The difference between these figures comprises investments in liquid assets (5% minimum of total assets must be in cash at all times), debentures and preference shares and foreign holdings. The twelve unit trusts are controlled by six management companies each of which forms part of one of the country's major financial institutions. The Association of Unit Trusts was established in 1967 to represent the joint interests of its member trusts and their unitholders in dealings with the authorities, to promote the common interests of the industry, and to maintain communication with the media. More information about the movement is contained in its annual review - The Association of Unit Trusts (1980).

For the purposes of this chapter, the Association of Unit Trusts portfolio (T), which is a combined portfolio of the twelve underlying trusts, is significant in that it is the largest professionally managed portfolio, the constituent holdings of which are made public. There are several larger portfolios than this but their structure is a closely guarded competitive secret. In view of the above findings of this chapter with respect to the selection and characteristics of *ex post* efficient portfolios, it was decided to analyse the characteristics of T at 31 December 1980 to see how a large professionally managed portfolio was structured. Table 2.4 contains the Rand million exposure by market value to each sector on the JSE, the proportion that each sector comprises of the portfolio and finally the proportion of each sector in the portfolio relative to that sector's proportion of the market index. The latter parameter indicates the degree to which the unit trust managers are collectively prepared to deviate from the market weighting for each sector. Clearly figures substantially less than one (bounded below by zero) represent a joint vote of no confidence in a sector's prospects and more importantly its possible contribution to overall portfolio efficiency. It is assumed portfolio managers are attempting at all times to choose an *ex ante* efficient portfolio. Figures greater than one indicate the opposite, that is, the expectation of better portfolio efficiency being achieved through higher than average sector exposure.

Table 2.4 shows that two sectors, namely Fishing and Printing, are not present in the portfolio and therefore are the lowest relative exposure at zero. The highest relative exposure is Stores at 4,5408 followed by Furniture, Beverages and Tobacco each with an exposure over three times the market proportion for these sectors. Among the most notable features of the portfolio is its low relative exposure to gold shares. In total 30,7% of the All Share index comprised gold sectors at 31 January 1980 but only 6,4% of the Association of Unit Trusts portfolio comprised gold sectors at 31 December 1980 (a relative ratio of 0,2081) while the corresponding figures for 31 December 1976, the first published amalgamated portfolio, were 4,2% and 0,1364. Thus it can be seen that an exposure to gold sectors far lower than that of the market has always been preferred by unit trust managers. Why this should be so is examined in the next chapter.

Another aspect of T as depicted in Table 2.4 is the high relative and absolute exposure to the consistently *ex post* inefficient sectors of Mining Holding, Mining Houses, Property, Banks, Industrial Holding, Building, Furniture, Motors, Pharmaceutical and Stores noted in Section 2.6 (vii).

TABLE 2.4 Association of Unit Trusts Portfolio at 31 December 1980

	Sector	Rm Exposure	Proportion of Total Portfolio	Relative Sector Proportion in Portfolio to Market Index
1.	Gold-Rand	2,861	0,0047	0,2702
2.	Gold-Evander	2,250	0,0040	0,2658
3.	Gold-Klerksdorp	16,295	0,0288	0,4410
4.	Gold-OFS	8,152	0,0144	0,1928
5.	Gold-W Wits	6,598	0,0117	0,0867
6.	Coal	34,263	0,0605	1,7796
7.	Diamonds	46,301	0,0818	0,8403
8.	Platinum	10,335	0,0183	0,6413
9.	Copper, Tin, Others	3,955	0,0070	0,2287
10,	Mining Holding	22,039	0,0389	0,6918
11.	Mining Houses	87,010	0,1537	1,0120
12.	Inv. Trusts	3,941	0,0070	1,0419
13.	Insurance	4,500	0,0079	1,0638
14.	Property	0,228	0,0004	0,0713
15.	Banks	39,356	0,0695	2,0299
16.	Ind. Holding	82,230	0,1452	2,7514
17.	Beverages	34,473	0,0609	3,7031
18.	Building	8,105	0,0143	0,9288
19.	Chemicals	25,998	0,0459	1,3535
20.	Clothing	1,460	0,0026	0,4086
21.	Electrical	4,706	0,0083	1,1369
22.	Engineering	9,690	0,0171	1,3390
23.	Fishing	-	-	-
24.	Food	10,378	0,0183	1,2989
25.	Furniture	13,742	0,0243	3,8767
26.	Motors	0,740	0,0013	0,2841
27.	Paper, Packaging	10,895	0,0192	1,2750
28.	Pharmaceutical	1,215	0,0021	0,9980
29.	Printing	-	-	-
30.	Steel	4,060	0,0072	0,7802
31.	Stores	48,751	0,0861	4,5408
32.	Sugar	0,355	0,0006	0,0703
33.	Tobacco	20,454	0,0361	3,5448
34.	Transport	1,099	0,0019	0,4757
	TOTAL	R566,255m	1,0000	

In fact 53,6% of T is represented by these sectors whereas they were 34,6% of the All Share Index at end January 1980. Therefore the relative exposure to these sectors is 1,5486.

2.8 Conclusions and Implications

The *ex post* application of the Markowitz Portfolio Selection algorithm on the thirty four sectors comprising the JSE has produced some interesting findings.

(i) The shape and slope of the *ex post* efficient frontiers changed markedly over time as did the composition of the portfolios comprising these frontiers.

(ii) The All Share or market index proved inefficient *ex post* with respect to the *ex post* efficient frontiers. This result is, however, not surprising.

(iii) A definite hierarchy of *ex post* sector efficiency seemed to exist. This manifested itself in that once a sector was selected for an efficient portfolio, it tended to persist in the efficient frontier over quite a range in risk. In addition some sectors representing 34,6% of the market index, were not selected in *ex post* efficient portfolios over the entire fifteen year measurement period. This result is perhaps surprising.

In the next chapter the aspect of low exposure to gold shares in the Association of Unit Trusts portfolio noted in Section 2.7, is examined more closely.

GOLD SHARE INVESTMENT ON THE JOHANNESBURG STOCK EXCHANGE

3.1 Introduction

Under current South African (SA) exchange control regulations, a local investor is not allowed to invest funds overseas without exchange control permission. While companies often obtain this permission to establish manufacturing facilities overseas, holding funds for portfolio investment outside South Africa is not generally permitted. This means that domestic fund managers are limited to the local equity market, The Johannesburg Stock Exchange (JSE) for the listed equity portion of their portfolios.

However, direct investment on the JSE is possible for overseas investors. Until recently (February 1983) such investment had to be effected through the Financial Rand (FR), a currency quoted in United States (US) cents. Investors seeking holdings on the JSE first purchased FR through a stockbroker and then competed with local SA investors on the JSE for stock they sought to buy. Scrip purchased by non-residents is designated as such and any dividends accruing to such shares are subject to a 15% non-resident shareholder's tax. The dividends, less this tax, are paid to the non-resident at the ruling Dollar/Commercial Rand ($/R) rate, which has historically stood at a premium, sometimes quite considerable, to the FR, as is shown in Figure 3.1.

In Chapter 2 it was observed that the level of investment in gold shares in the Association of Unit Trusts (1980) portfolio was very low. In fact at 31 December 1980, it comprised only 6,4% of this portfolio, compared with the total gold share proportion of the JSE All Share Index of 30,7%.

This chapter will show that whereas gold shares have historically provided a similar return to both US and SA investors, the risk associated with these returns has been extremely different for these two groups of investors. This implies that investment in gold shares has a different impact on portfolio efficiency for these investors and accounts for the low level of investment in these shares by SA investors as typified by the Association of Unit Trusts. Investment in gold and gold shares from a non-SA point of view has been addressed by other researchers, notably by Sherman (1983), Renshaw and Renshaw (1983) and McDonald and Solnik (1983).

3.2 Definition of the Return and Risk on Gold Shares

The study was conducted over the fifteen year period May 1967 to April 1982. This period was not chosen to coincide with any particular market trend or cycle but simply because monthly data were readily available for all the variables needed to perform the calculations. For amplification of the results, the fifteen year period was subdivided into three non-overlapping periods of five years each:

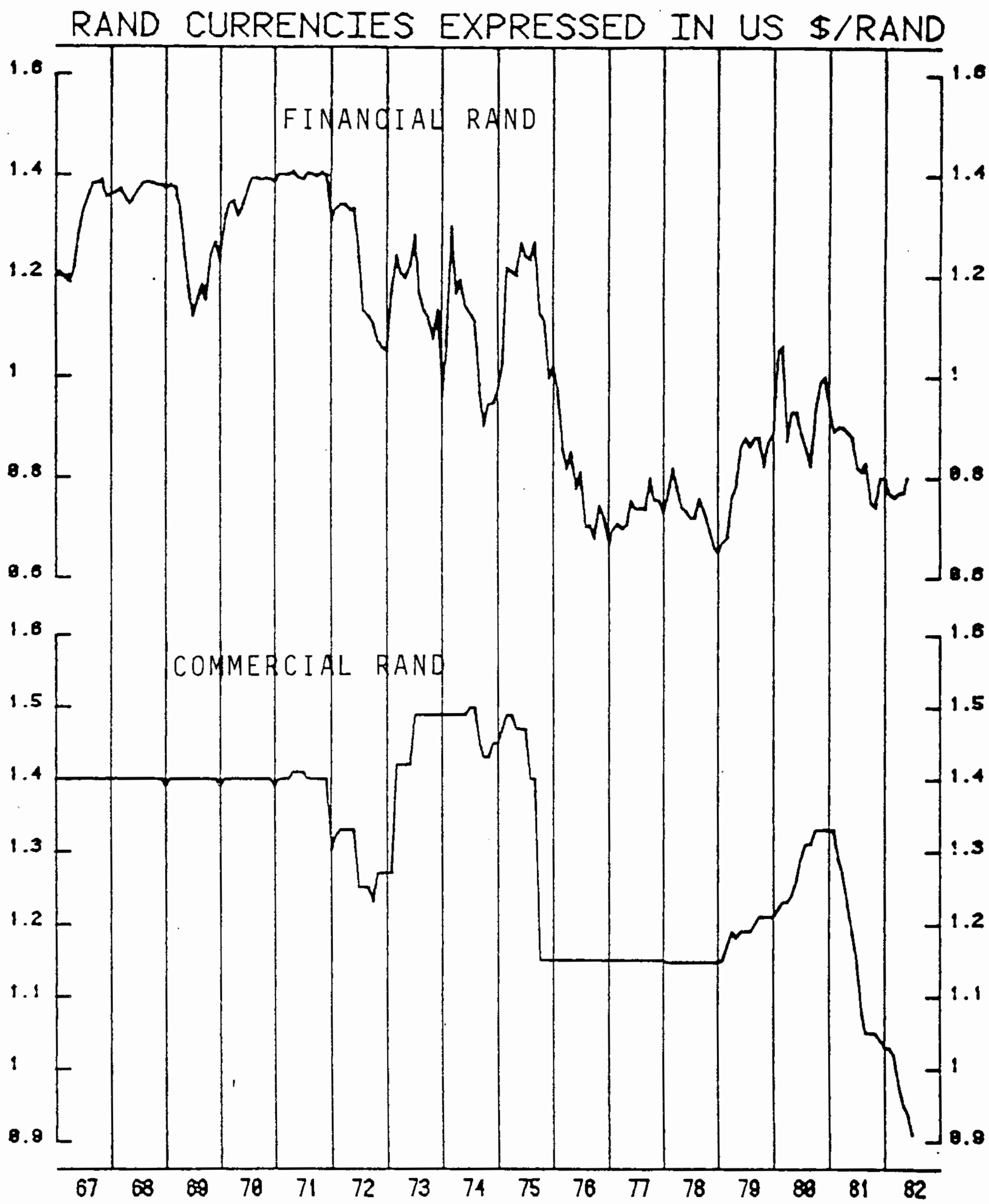

RAND CURRENCIES EXPRESSED IN US $/RAND
FINANCIAL RAND
COMMERCIAL RAND
1.6
1.4
1.2
1
0.8
0.6
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82

FIGURE 3.1

Period 1 May 1967 to April 1972

Period 2 May 1972 to April 1977

Period 3 May 1977 to April 1982

Monthly returns to both the SA and US investor were calculated as follows, using the JSE All Gold index (JSE Actuaries Index (1980)) as a proxy for gold shares on the JSE

$$S_t = \frac{AG_t - AG_{t-1}}{AG_{t-1}}$$

where S_t = return to SA investor in month t

AG_t = JSE All Gold index in month t

AG_{t-1} = JSE All Gold index in month t-1

and $$U_t = \frac{(AG_t \times FR_t) - (AG_{t-1} \times FR_{t-1})}{(AG_{t-1} \times FR_{t-1})}$$

where U_t = return to US investor in month t

AG_t = JSE All Gold index in month t

AG_{t-1} = JSE All Gold index in month t-1

FR_t = financial rand price in month t

FR_{t-1} = financial rand price in month t-1

The standard deviation of these returns is used as a measure of the absolute risk carried, while in a portfolio context risk is measured by the beta coefficients (obtained from the market model of the S_t time series to the JSE All Share index and the U_t time series to the Standard & Poors 500 index (S&P 500).

Since a US investor can invest in virtually any asset market of his choice (including the JSE), it could be argued that some "global" index should be used to more properly represent his choice of constituent options in portfolio formation. Ibbotson and Siegel (1983) have attempted to construct such an index. This problem has also been highlighted by Roll (1977) and discussed by Mayers (1973), Miller and Scholes (1972), Frankfurter and Phillips (1977) and more recently by Rudd and Rosenberg (1980). It is felt that the use of the S&P 500 index for this study does not affect the validity of its conclusions.

3.3 Comparison of the US and SA Investors' Experience

Table 3.1 displays the mean monthly return and standard deviation of return per month (both expressed as percent) of the Standard & Poors 500, JSE All Share index and the JSE All Gold index, the latter from both the US and SA investors' viewpoint.

It can be seen from Table 3.1 that the JSE All Share index outperforms the S&P 500 in each of the sub-periods, while its absolute risk is consistently higher. It is also observed that an investment in the JSE All Gold index produces similar returns for both the US and SA investor over the whole period, revealing that the overall currency effect on returns (mainly FR but also $/R) has not been great. Notably, the US investor experiences a more volatile return than his SA counterpart in each of the sub-periods due to his having to use the currency

TABLE 3.1

Mean monthly returns and risk expressed as percent per month

	Period 1 05/67-04/72 Return	Risk	Period 2 05/72-04/77 Return	Risk	Period 3 05/77-04/82 Return	Risk	Whole period 05/67-04/82 Return	Risk
S&P 500	0,355	3,342	-0,070	4,147	0,316	3,360	0,200	3,641
JSE All Share	0,625	6,760	0,246	7,688	1,944	6,474	0,939	7,031
JSE All Gold to US investor	0,015	6,439	0,303	14,479	1,929	11,996	0,749	11,505
JSE All Gold to SA investor	-0,142	6,333	1,161	13,083	1,479	9,320	0,833	9,993

markets. Of great importance is to note the rather large out-performance of the S&P 500 achieved by the JSE All Gold index held by a US investor.

The risk of an investment in gold shares for a fund manager is not adequately described by the total risk as measured by the standard deviation of return. It is better described by the beta coefficient of those shares to the investors' market surrogate (Sharpe (1970)). For the SA investor the best surrogate available is the JSE All Share index, since he is only able to invest in listed securities which are quoted on the JSE, while for the US investor the S&P 500 is used as a market surrogate.

Table 3.2 shows the regression statistics obtained from regressing the JSE All Gold index returns, from the US and

SA investors' viewpoints, against their respective market surrogates.

TABLE 3.2

Regression Statistics of Gold Share Returns to US and SA investors

Gold Shares	Period 1 05/67-04/72	Period 2 05/72-04/77	Period 3 05/77-04/82	Whole period 05/67-04/82
β to S&P 500	-0,084	0,041	1,200	0,338
Standard error	0,253	0,458	0,442	0,236
F Statistic	0,11	0,01	7,38	2,06
R^2	0,002	0,000	0,113	0,011
β to JSE All Share	0,528	1,484	1,297	1,125
Standard error	0,102	0,109	0,082	0,065
F Statistic	26,97	183,81	249,79	299,06
R^2	0,317	0,760	0,812	0,627

Table 3.2 shows that in the context of portfolios, SA gold shares have dramatically different risk characteristics for US and SA investors. For a SA fund manager, the gold share market has a beta coefficient significantly greater than unity (t-test at 99% level of confidence) in each of Periods 2 and 3, while the Period 1 beta is significantly less than unity but greater than zero. For the whole period it can be said that the beta for the gold share market has a value greater than unity at a confidence level of 90%.

For the US investor, the regressions are not significant for the whole period or Periods 1 and 2. The hypothesis that beta equals zero in these cases (at 99% level of confidence) can not be rejected. In Period 3, the regression is just significant at this confidence level but it can be seen that the beta coefficient has a large standard error and the coefficient of determination of the regression (R^2) is low. In any event it is highly unlikely that any causal information is contained in the tenuous relationship between gold share returns and the S&P 500 returns in this period.

It can also be concluded that even though the US investor in SA gold shares has to accept greater volatility than his SA counterpart in the returns he achieves, virtually all of this risk is diversifiable in the context of a well diversified portfolio. In other words, SA gold shares have added greatly to portfolio returns without adding to the overall portfolio risk. In contrast, a SA investor holding gold shares bears a large element of risk which is not diversifiable (market risk) and which has tended to be greater than the risk borne by an investment in the market index (that is, the beta coefficient has been greater than unity).

The inescapable conclusion is that the SA gold share market has far more attraction to a US investor than a SA investor in a portfolio context and should be preferred by the former investors and not the latter.

3.4 Some Corroborating Evidence

As discussed in Chapter 2 and Section 3.1 above, the level of investment in gold shares in the Association of Unit Trusts portfolio is only 20,8% that of the gold shares in the JSE All Share Index. Therefore the managers of the individual unit trusts consider that investment in gold shares does not contribute greatly to portfolio efficiency. In particular the return obtained for risk assumed is not commensurate with other risk/return opportunities available to these fund managers. Therefore SA fund managers are acting consistently with the assertion that there are other investors for whom it is more efficient to hold gold shares.

A recent study by a SA stockbroker (Davis, Borkum, Hare and Co., Inc. (1982)) reveals that the US holding of SA gold shares has remained virtually unchanged at just over 26% of the latter's market capitalisation during the period 1977 to 1981. Total non-SA ownership of SA gold shares was 39,9% of market capitalisation in December 1981. At the same date a search (McGregor (1982)) indicated that the SA mining houses held some 42,1% of the value of the gold shares quoted on the JSE. These investors have management contracts with the mines, receive fee income and have guaranteed investment outlets because of this 42,1% overall holding. We can regard these investments as strategic and therefore not strictly subject to the assertion that SA investors should not hold high levels of SA gold shares.

The shareholders of SA gold shares can be split into three categories:

Foreign	39,9%
Mining houses	42,1%
Other SA (by deduction)	18,0%
	100,0%

This means that 18% of the SA gold share market capitalisation of R16,6 billion or R3,0 billion, is not owned by overseas investors for whom it appears to be more efficient than for the current shareholders.

3.5 Conclusion

This chapter has presented evidence that SA gold shares have had great attraction as a portfolio diversification to US investors, contributing virtually nothing to portfolio risk while yielding vastly superior returns to the S&P 500. The particular nature of SA exchange control regulations places SA investors at a continual disadvantage to their foreign counterparts in respect of gold share investment and future efficient behaviour by all investors could result in an increasing trend in foreign ownership of SA gold shares.

The results from Chapter 2 and this chapter indicate that the JSE consists of a mixture of heterogeneous sectors, not all of which may have equal utility to SA investors in efficient portfolio formation. Furthermore, the market index, as represented by the JSE All Share Index, has been seen in Chapter 2 to be far from efficient. These facts lead to the need to understand the composition of the JSE more fully and this problem is addressed in Chapter 4.

CHAPTER FOUR

HOMOGENEOUS SECTOR GROUPS AND A COMPARISON OF SINGLE AND MULTI INDEX MODELS

4.1 Introduction

The South African economy must be regarded as essentially resource based. As a country, South Africa exports principally minerals or mineral based commodities and soft commodities (for example, food, sugar) while importing advanced tertiary products (for example, speciality steels, pharmaceuticals), capital goods and high technology products (such as electronic components) (South African Reserve Bank (1982)).

Therefore it is to be expected that as a developing nation with an open, trading economy, the range of industries represented on the Johannesburg Stock Exchange (JSE) through listed securities will reflect both the strong mineral bias and emerging industrial/manufacturing capability of South Africa. Appendix A shows the JSE sectors as defined by the JSE Actuaries Index (JSE Public Relations Department (1980)) and it can be readily seen that many diverse industries are indeed represented. Naturally gold related sectors feature prominently because of that commodity's dominating influence in South Africa's economy. Other mining sectors - Diamonds, Coal, Platinum and the base metals - are also present along with the Mining House and Holding companies which administer the affairs of the mining companies. The range of industrial and financial sectors is also represented.

Portfolios of listed securities in South Africa can only be formed from securities listed on the JSE, because of exchange control regulations, and hence it is assumed for this chapter that the JSE represents the proper universe of securities for equity portfolio selection in South Africa. Estimation of security betas can be made using the familiar market model:

$$R_{i;t} = \alpha_i + \beta_i R_{m;t} + e_{i;t}$$

where $R_{i;t}$ is the return on security i in period t

$R_{m;t}$ is the return on the market surrogate in period t

α_i and β_i are parameters unique to security i

$e_{i;t}$ is a random variable representing the residual error in period t

The parameters α_i and β_i can be estimated by ordinary least squares regression of historical security returns data on historical market surrogate returns data. The random errors, $e_{i;t}$, are usually assumed to satisfy the conditions of error terms in ordinary least squares regression as discussed in Draper and Smith (1966).

A number of issues are raised through the use of the market model. Perhaps the most important has to do with the fact that the market model is a single index model. To be used in practice, the market model, being a single index model, should be demonstrably superior to other models, such as multi-index models, in explaining security returns. In this chapter this issue is assessed for the JSE.

4.2 The Johannesburg Stock Exchange

The producers of the JSE Actuaries Index, The Actuarial Society of South Africa and The Johannesburg Stock Exchange, decided to form some higher or superior combined indices in addition to the basic industry sectoral indices and these are also shown in Appendix A. The final amalgamation of indices is the JSE All Share index (AS), which, for the purpose of the single index market model, is used as the market surrogate in this chapter.

There are three indices immediately inferior to the All Share index and these are the All Mining index (AM), Mining Financial index (MF) and Industrial and Financial index (IF). These three indices apparently represent the three broad categories of securities on the JSE and therefore could be used as the three independent variables in a multi-index model such as:

$$R_{i;t} = \alpha_i' + \beta_{i1} I_{1;t} + \beta_{i2} I_{2;t} + \beta_{i3} I_{3;t} + e_{i;t}' \qquad (1)$$

where $R_{i;t}$ is the return on security i in period t

$I_{1;t}, I_{2;t}, I_{3;t}$ are the returns on the AM, MF and IF indices respectively in period t

α_i', β_{i1}, β_{i2}, β_{i3} are parameters unique to security i

$e_{i;t}'$ is a random variable representing the residual error.

The parameters α_i', β_{i1}, β_{i2}, β_{i3} can be estimated by the multiple regression of historical security returns data on historical index returns data if these are available.

In this chapter the adequacy of the groupings of sectors in the JSE Actuaries Index is investigated using three techniques - multi-dimensional scaling, and two forms of cluster analysis. Then the suitability of the single index market model, using the AS index as market surrogate, to explain security returns is assessed by comparing this model with two multi-index models. Finally the influence of volume traded, value traded and market capitalisation of shares on the suitability of the three models is examined to see whether these models behave differently for different types of shares.

4.3 Other Related Work

An empirical evaluation of single- and multi-index portfolio selection models has been undertaken on the United States stock market (Cohen and Pogue (1967)) and it was found that the single index model was superior in generating efficient sets of portfolios. Other researchers (King (1966) and Farrell (1974) and (1975)) have found that on the US market, factors, additional to the market factor, aided in the explanation of the variation in security returns. Among these was an industry factor as well as a nature of industry factor, such as whether cyclical, stable, growth or oil. In a later study, Arnott (1980) found five clusters of stocks corresponding to major extra-market factors. On average the explanatory power of the resulting clusters for the evaluation of extra-market risk represented better than a 30 per cent improvement over the single-index model.

4.4 Methodology

The grouping of sectors on the JSE into homogeneous groups was conducted over two non-overlapping time periods of sixty months each.

Period 1 - February 1970 to January 1975

Period 2 - February 1975 to January 1980

Two periods were chosen in order to gain some insight into the persistence of the results through time. The actual dates of the periods chosen were not selected to represent any particular state of the economy or stock market. The effect of the market was eliminated for each sector by regressing monthly returns for each sector on the monthly returns of the AS index for each of the two periods using the single index market model described in Section 4.1. The return data for each month was calculated as follows:

$$R_{i;t} = \frac{P_{i;t} - P_{i;t-1}}{P_{i;t-1}}$$

where $R_{i;t}$ is the return on sector index i in month t

$P_{i;t}$ is the price of sector i at the end of month t

$P_{i;t-1}$ is the price of sector i at the beginning of month t

The regression of sector returns on the AS index returns produced thirty four sets of residuals for each of Period 1 and 2. For each period, these sector residuals were correlated with each other to produce a matrix of 561 different correlation coefficients which was used as the raw data for the multi-dimensional scaling and first cluster analysis technique.

4.5 Multi-dimensional Scaling

For each of Periods 1 and 2, a multi-dimensional scaling technique (Chatfield and Collins (1980)) was used to produce a two dimensional scatter diagram containing one point for each of the thirty four JSE sectors. If the single index market model is an adequate model for representing the covariance matrix of sector returns then the scatter diagram produced by the multi-dimensional scaling technique should be totally random with no sectors "grouping" together. If, however, some sectors group together so that the density of sector points changes over the scatter diagram, this would be an indication that factors, additional to the return on the market surrogate, affect the returns on the JSE sector indices. The results for Periods 1 and 2 are displayed in Figures 4.1 and 4.2 respectively with the key to these figures in Table 4.1.

It is apparent from Figure 4.1 that two very strongly separate groups of sectors exist. These comprise the five gold sectors and the Mining House and Holding sectors as one group and the rest (that is, Industrial and Financial sectors and the other mining sectors) as the other group. In Figure 4.2 it is possible to arrive at a similar conclusion and these groups are ringed for convenience in these figures.

It is possible, therefore, that the single index model using the AS index as market surrogate does not provide acceptable explanations of sector returns on the JSE. Indeed the apparent separation into two groups, one gold related and the other not, has some economic appeal and justification. However, the non-gold grouping of twenty-seven sectors requires some further examination. It contains four mining sectors, namely, Coal, Diamonds, Platinum and Copper, Tin, Others. It is known that the companies operating in these sectors each produce a commodity, the price and demand for which are determined by non-South African economic conditions. In this sense they are similar to the gold related sectors but dissimilar to the twenty three industrial sectors with which they group. It is postulated that these four sectors, therefore, should be considered as a group by themselves, thereby splitting the JSE sectors into three separate groups. The scatter diagrams of Figures 4.1 and 4.2 are reproduced in Figures 4.3 and 4.4 respectively with the three groupings ringed. It is interesting to note that in both Figures 4.3 and 4.4 it would be possible to incorporate the Fishing and Sugar sectors with the Coal, Diamond, Platinum and Copper, Tin, Others group. This grouping would have some economic

TABLE 4.1: SECTOR CODES FOR FIGURES 4.1 to 4.12 AND APPENDIX C

Code	Sector
1	Gold - Rand and Other
2	Gold - Evander
3	Gold - Klerksdorp
4	Gold - OFS
5	Gold - West Wits
6	Coal
7	Diamonds
8	Platinum
9	Copper, Tin, Others
10	Mining Holding
11	Mining Houses
12	Investment Trusts
13	Insurance
14	Property
15	Banks
16	Industrial Holding
17	Beverages
18	Building
19	Chemicals
20	Clothing
21	Electrical
22	Engineering
23	Fishing
24	Food
25	Furniture
26	Motor
27	Paper, Packaging
28	Pharmaceutical
29	Printing
30	Steel
31	Stores
32	Sugar
33	Tobacco
34	Transport

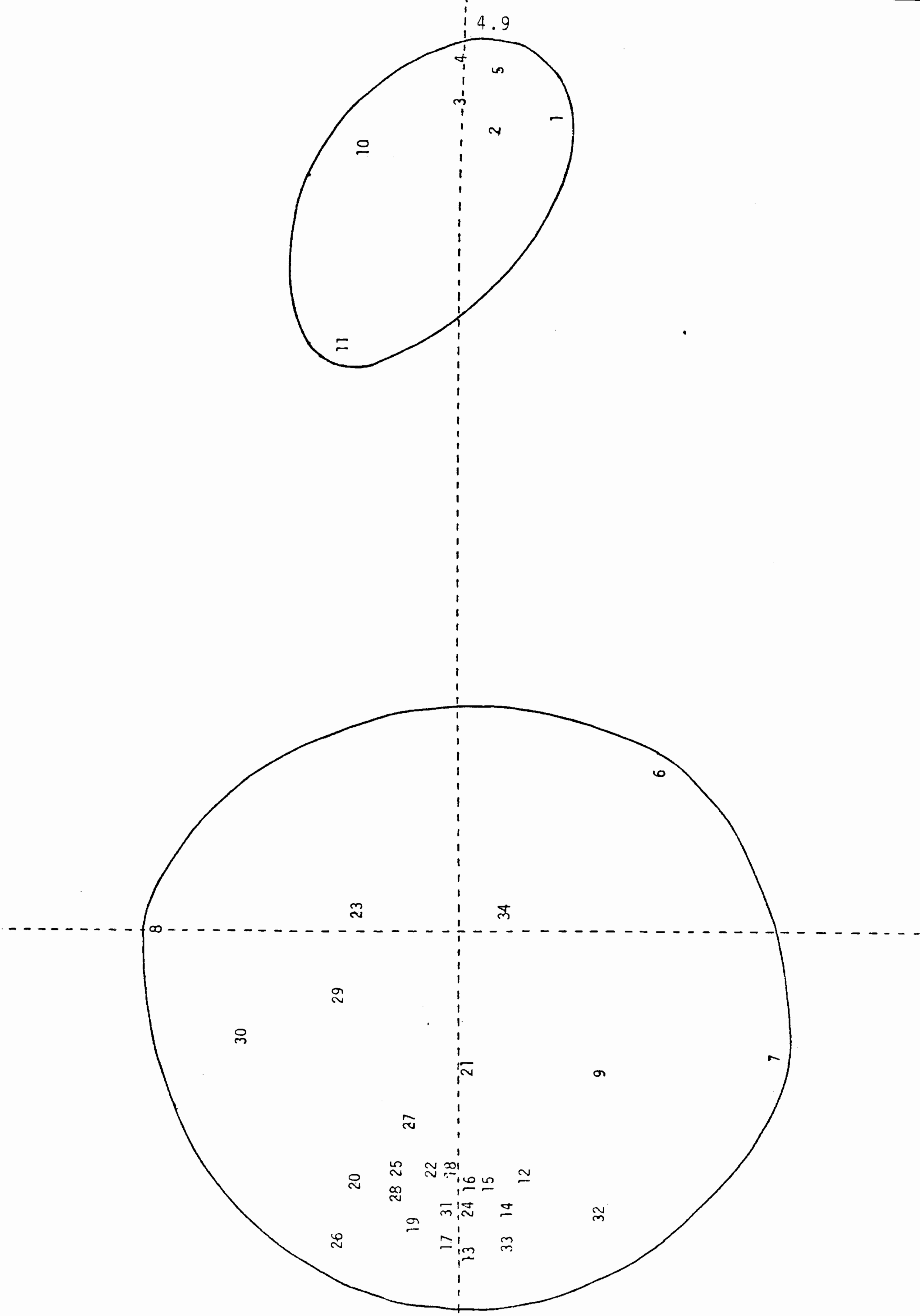


FIGURE 4.1: Multi-dimensional Scaling on JSE Sectors - Period 1

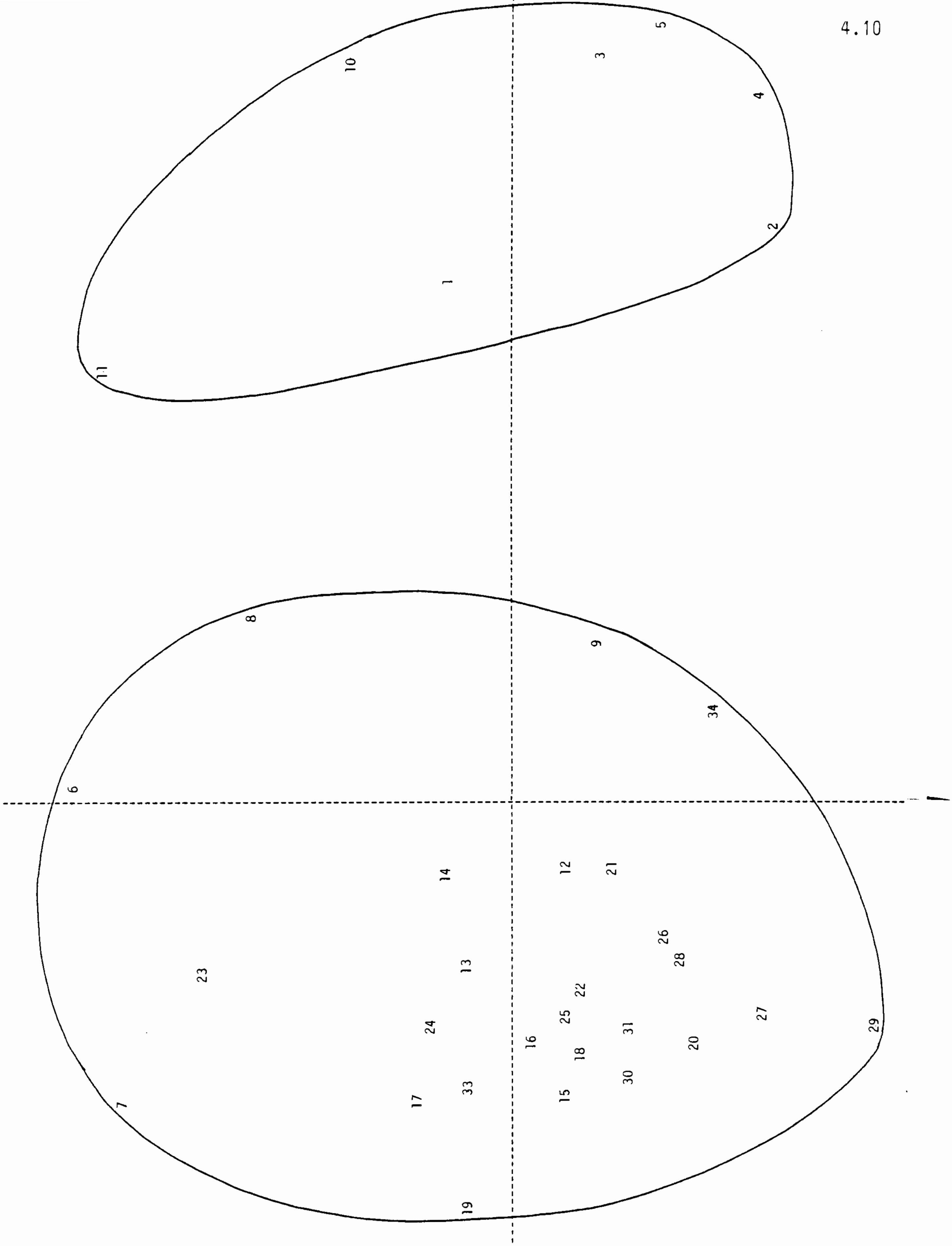


FIGURE 4.2: Multi-dimensional Scaling on JSE Sectors - Period 2

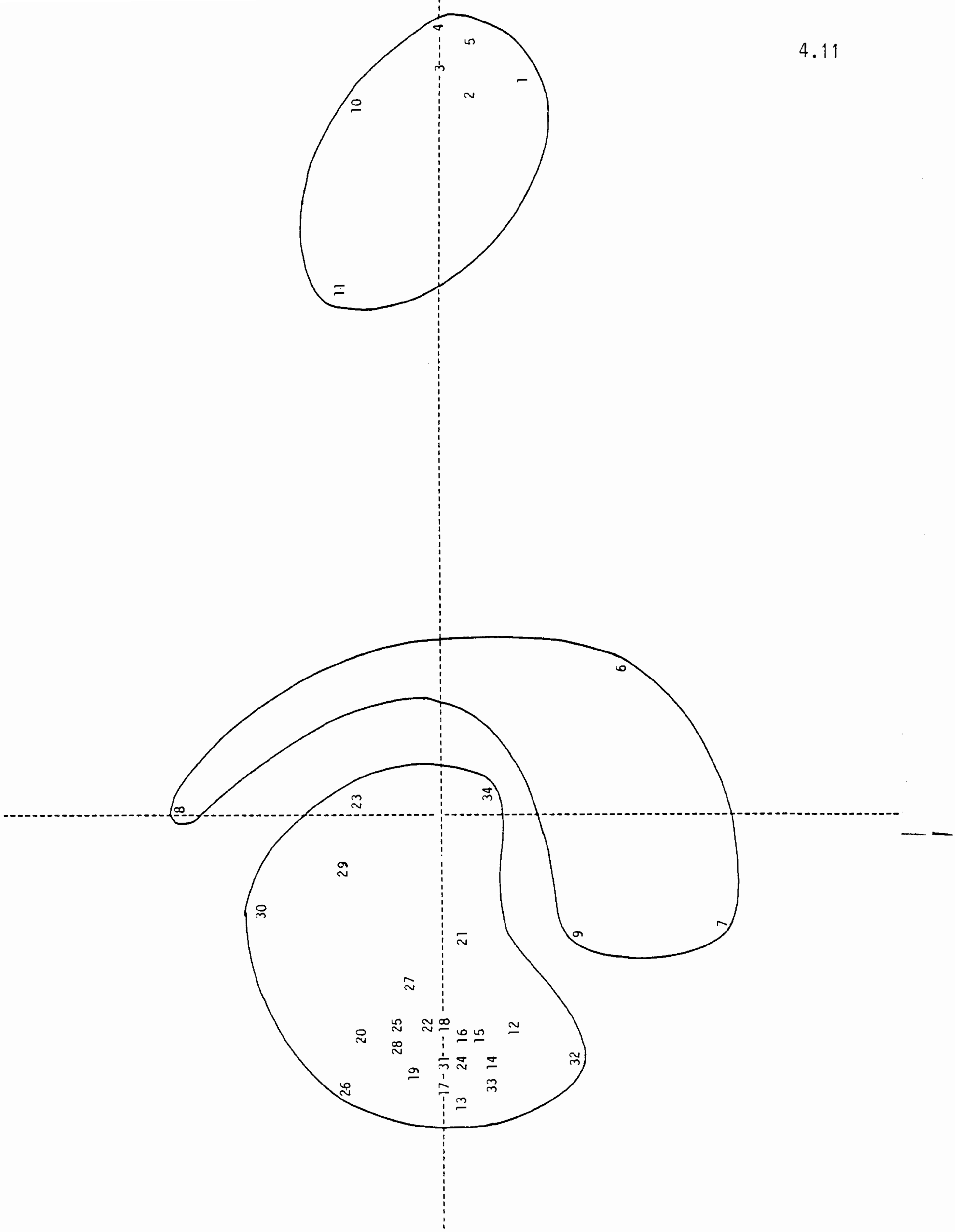


FIGURE 4.3: Multi-dimensional Scaling on JSE Sectors - Period 1

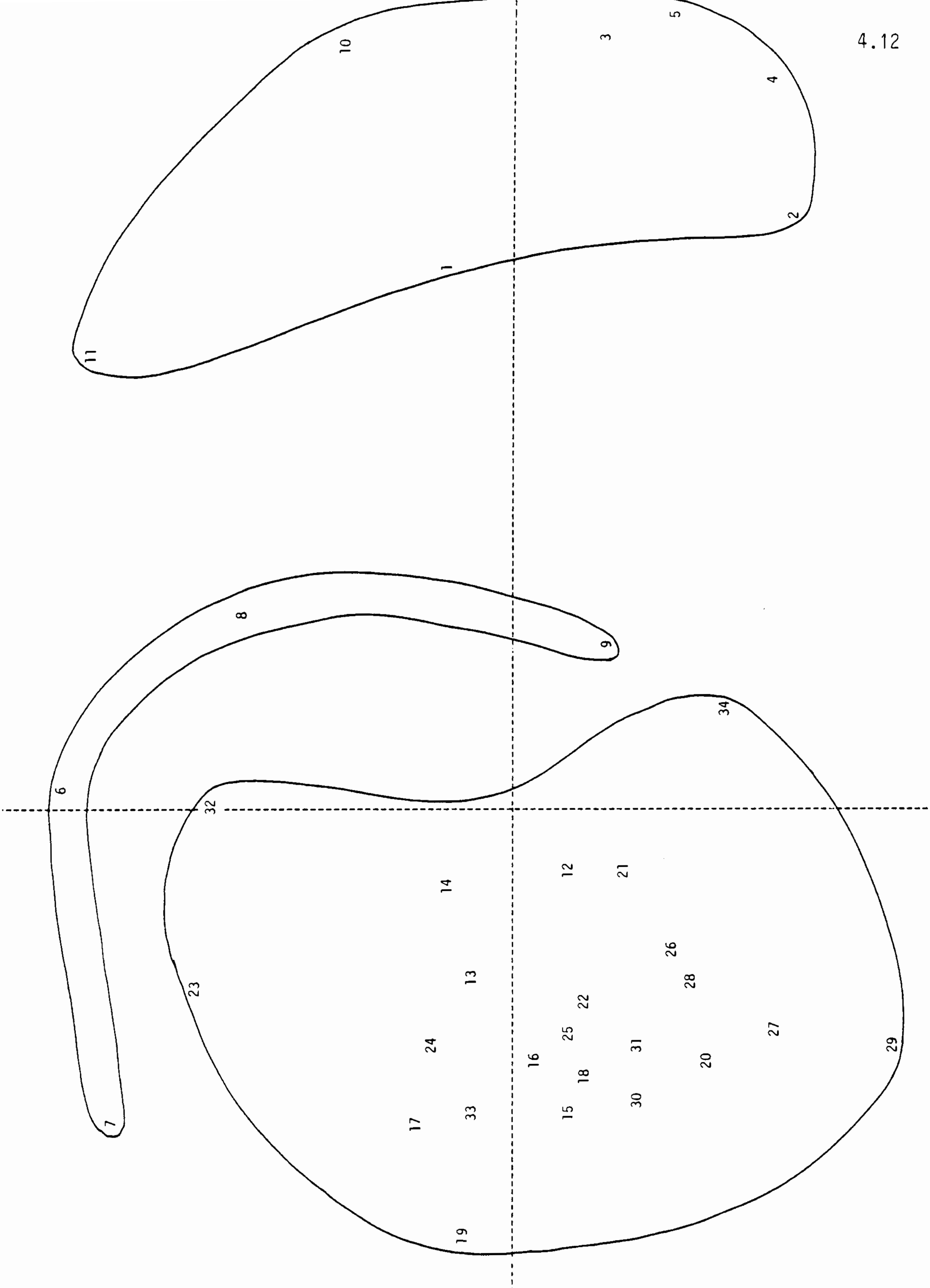


FIGURE 4.4: Multi-dimensional Scaling on JSE Sectors - Period 2

justification because companies in these sectors do produce an internationally priced and traded commodity. However, most of the output of these industries is for South African consumption and hence it was felt they should more properly be grouped with the other industrial sectors. Justification for this is also provided by the results of Section 4.9 below.

The three groupings thus suggested by the multi-dimensional scaling technique are shown below in Table 4.2 and are termed for convenience Gold and Mining Financials (GMF), Other Mining (OMI) and Industrial (IN).

It should be noted that this grouping is different from that shown in Appendix A and adopted in the construction of the JSE Actuaries Index. The JSE Actuaries Index grouping implies that the OMI group of Table 4.2 should more properly be grouped with the direct gold sectors, with the Mining Holding and House sectors as a separate group (Mining Financials). The results of the multi-dimensional scaling technique strongly suggest that the Mining Holding and Mining House sectors should be grouped with the gold sectors in preference to any other sectors, with the OMI group existing as an entity in itself. If the OMI sectors must be grouped with other sectors, Figures 4.1, 4.2, 4.3 and 4.4 suggest that it should be with the Industrial and Financial sectors rather than the gold sectors and Mining House and Holding sectors.

TABLE 4.2: Sector Groupings Suggested by Multi-dimensional Scaling

Gold and Mining Financials (GMF)	Other Mining (OMI)	Industrial (IN)
Gold - Rand	Coal	Investment Trusts
Gold - Evander	Diamonds	Insurance
Gold - Klerksdorp	Platinum	Property
Gold - OFS	Copper, Tin, Others	Banks
Gold - W Wits		Industrial Holding
Mining Holding		Beverages
Mining Houses		Building
		Chemicals
		Clothing
		Electrical
		Engineering
		Fishing
		Food
		Furniture
		Motor
		Paper, Packaging
		Pharmaceutical
		Printing
		Steel
		Stores
		Sugar
		Tobacco
		Transport

4.6 Cluster Analysis with no Recalculation of Correlation Coefficients

A single-linked, step-wise clustering approach similar to that used by Farrell (1974) and Affleck-Graves (1977) and described in Chatfield and Collins (1980) was used as a second method of identifying any residual correlation between sectors on the JSE after the market effect had been removed. The steps of the procedure were:

(i) Search the residual sector correlation matrix for the highest correlation coefficient.

(ii) combine these variables, thereby reducing the dimension of the matrix by one.

(iii) Recompute the correlation matrix to include the correlation between the combined variables and the remaining variables.

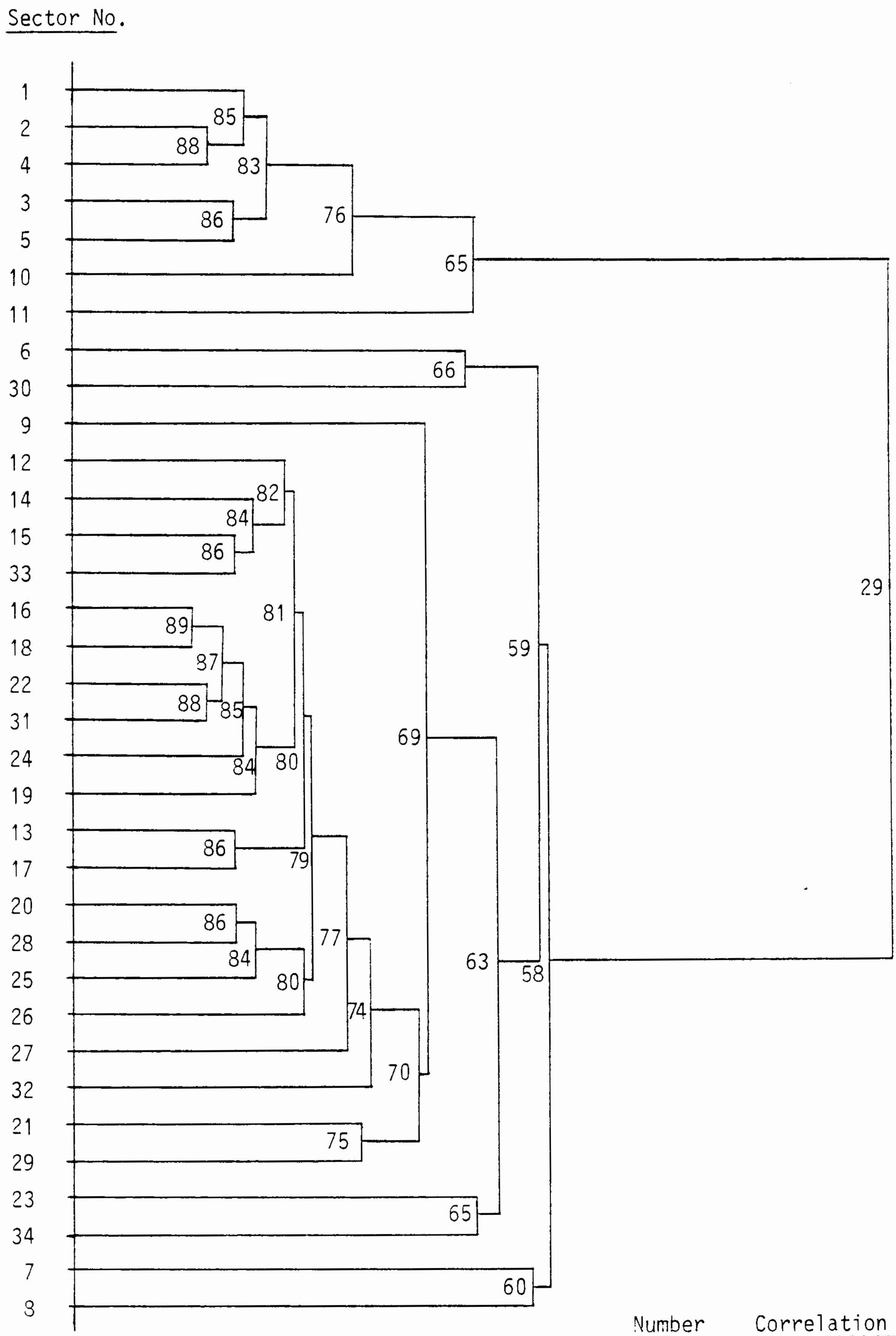
The last step was performed by averaging the correlation coefficients of each set of sector residuals in a cluster with each of the remaining sets of sector residuals or clusters to obtain the correlation of that cluster with the remaining sets of sector residuals or clusters. This technique lacks something in terms of theoretical justification and was improved upon in Section 4.7 below. Nevertheless, the results are in substantial agreement with those of Sections 4.5 and 4.7 and hence are reproduced as confirmatory evidence.

The step-wise nature of this approach means that it is possible to halt the clustering at any determined point. For the purposes of this chapter it was felt that clustering should be stopped at the last group which formed with a positive correlation coefficient. It is accepted that this is essentially arbitrary but it does have intuitive appeal to the extent that homogeneous sectors would not be expected to cluster together with negative correlation.

The results can be displayed easily in the form of a dendrogram. Figures 4.5 and 4.6 are the dendrograms (see Sokal and Sneath (1963)) of the cluster analysis on the JSE sectors for Periods 1 and 2 respectively. The final iteration (the trivial clustering of all sectors into one group) is the only one which occurs at negative correlation for both periods, thereby suggesting that the JSE sectors fall naturally into two groups which are the same in both cases as those of Section 4.5, namely, gold sectors plus Mining Houses and Holding in the first group and the industrial, financial and other mining sectors in the second group. For amplification, the final few iterations at positive correlation coefficients are reproduced below in Table 4.3.

Again it is clear from Table 4.3 that the OMI sectors in Table 4.2 have a greater affinity for themselves and the industrial sectors than the gold sectors, lending further confirmation of the less than optimal grouping of sectors in the JSE Actuaries Index. It should be noted that the correlation coefficients in Period 2 are at a discernibly lower level than those of Period 1. This suggests that the relationships within groups are more tenuous in Period 2, a result which is supported by the more scattered looking Figure 4.2 compared to Figure 4.1 of the multi-dimensional scaling. The tendency for Fishing (in both Periods) and Sugar (in Period 2) to resist grouping with the other industrials is again apparent.

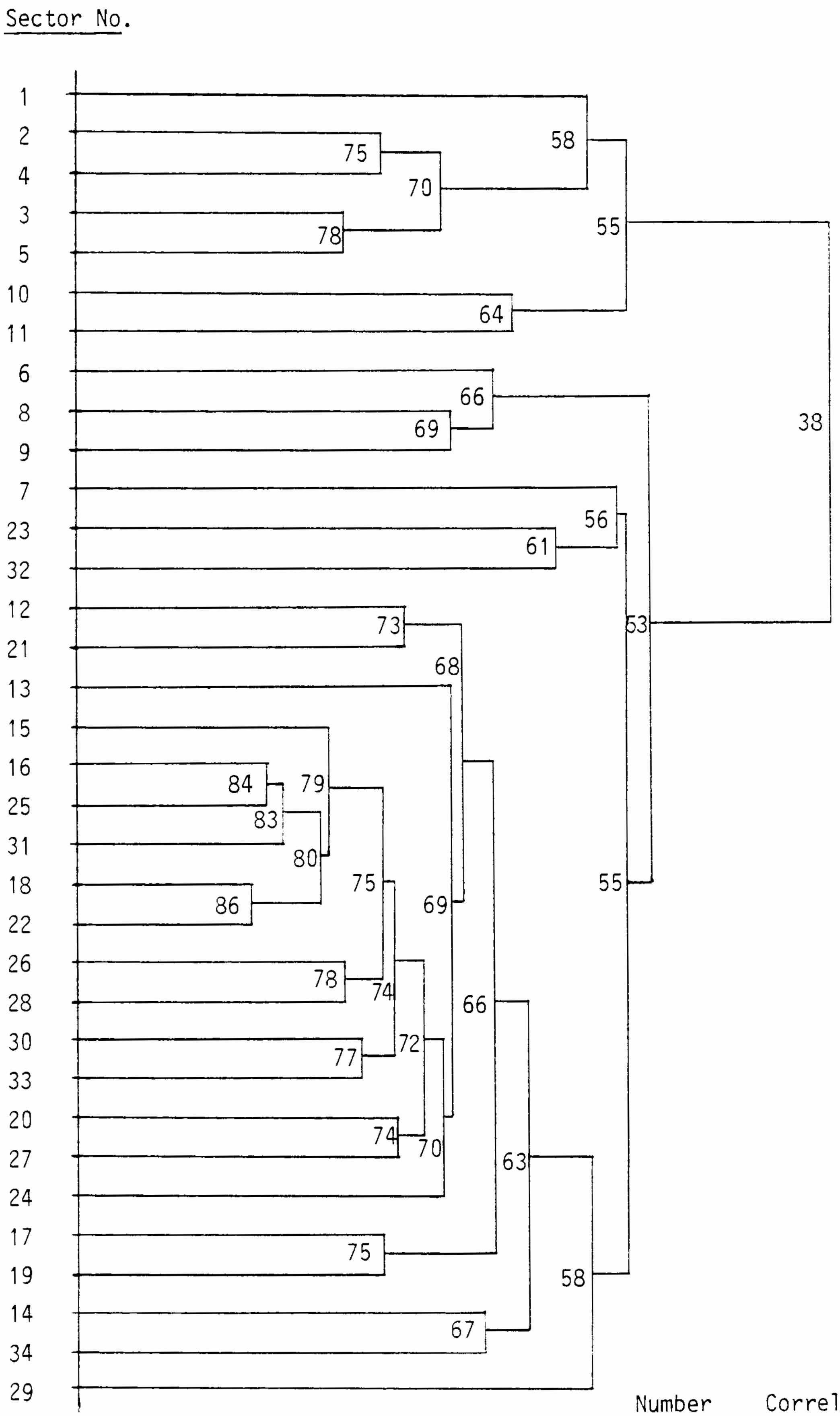
Figure 4.5: CLUSTER ANALYSIS ON JSE SECTORS - PERIOD 1



Note: The numbers shown are from the following linear scale:

Number	Correlation Coefficient
100	1,000
50	0,000
0	-1,000

Figure 4.6: CLUSTER ANALYSIS ON JSE SECTORS - PERIOD 2



Note: The numbers shown are from the following linear scale:

Number	Correlation Coefficient
100	1,000
50	0,000
0	-1,000

TABLE 4.3: Final Iterations of Cluster Analysis

Iteration Number	Correlation Coefficient	First Combining Group	Second Combining Group
Period 1			
28	0,294	Golds, Mining Holding	Mining Houses
29	0,265	Industrials (except Steel), Copper, Tin, Others	Fishing, Transport
30	0,206	Platinum	Diamonds
31	0,184	Industrials group	Coal, Steel
32	0,164	Industrials group, Coal, Steel	Platinum, Diamonds
Period 2			
28	0,163	Industrials (except Sugar, Fishing)	Printing
29	0,116	Diamonds	Sugar, Fishing
30	0,105	Golds	Mining Holding, Mining Houses
31	0,104	Industrials group	Diamonds, Sugar Fishing
32	0,054	Industrials group, Diamonds, Sugar, Fishing	Coal, Platinum, Copper, Tin, Others

4.7 Cluster Analysis with Recalculation of Correlation Coefficients

This procedure is identical to that used in Section 4.6 except that at each combination of sectors, a new set of monthly returns was calculated for the new group by weighting the monthly returns of the constituent sectors equally, removing the market effect by a new regression of the recalculated returns using the market model and re-estimating the correlation coefficients of the new group's residuals with every other set of residuals. In this way any theoretical objection to averaging correlation coefficients is avoided.

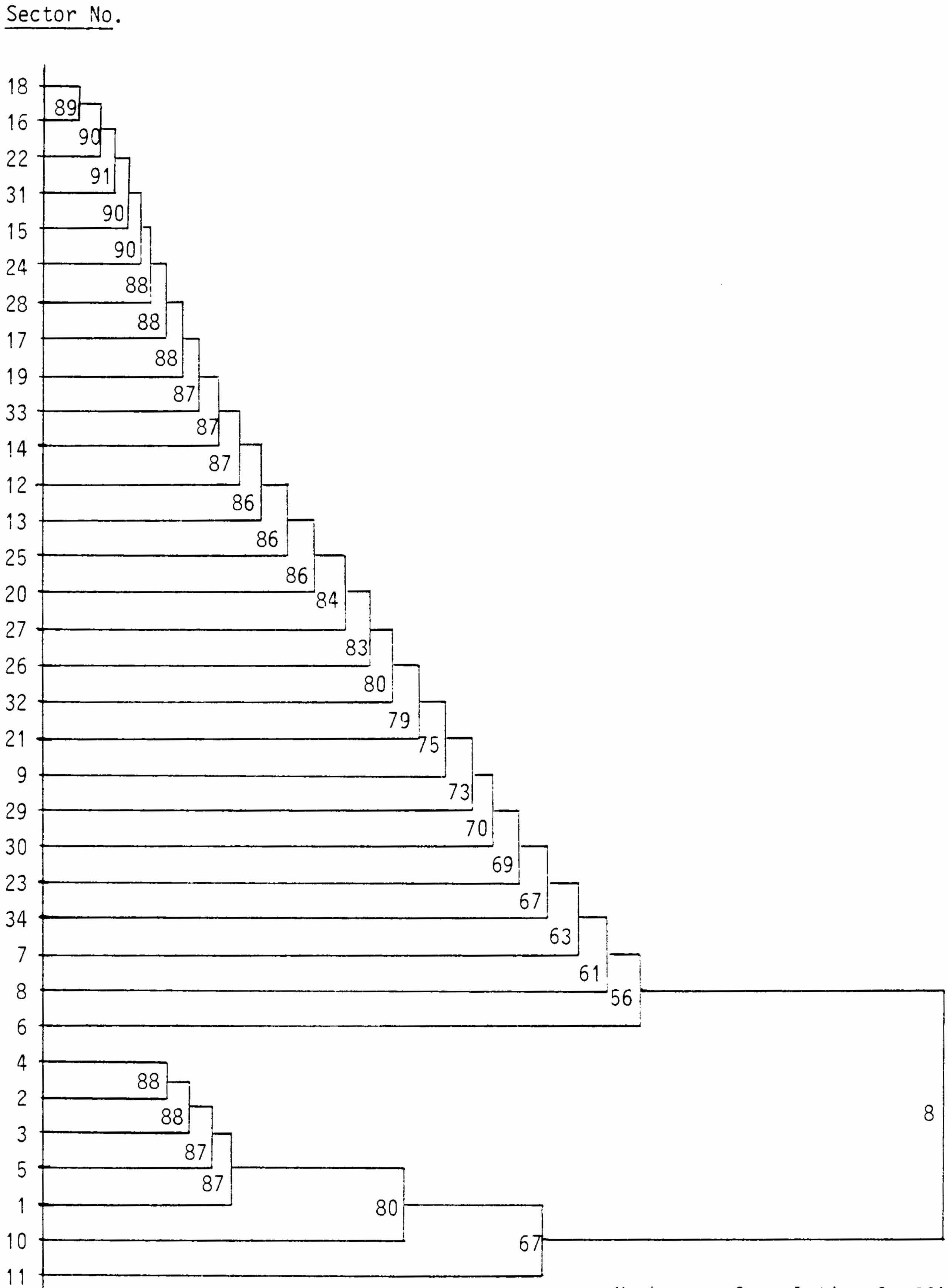
The results are conveniently displayed in Figures 4.7 and 4.8 representing Periods 1 and 2 respectively. As was found in Sections 4.5 and 4.6, two groups emerge at the thirty second iteration at positive correlation for both periods and they are the gold sectors plus Mining Houses and Holding in the first group and the industrial, financial and other mining sectors in the second group. For further amplification the final few iterations at positive correlation coefficients are reproduced below in Table 4.4.

As was the case in Table 4.3 and Figures 4.5 and 4.6 the OMI sectors have a greater affinity for each other and/or the industrial sectors than for the gold sectors. In addition the OMI sectors group together in Period 2 at a correlation of 0,153 thereby supporting the decision taken in Table 4.2 to group these four sectors together as a cluster by themselves. It is noticeable once again that the combining correlation coefficients at which the groups combine are lower in Period 2 than in Period 1.

4.8 Conclusions on Grouping of JSE Sectors

On the basis of the results presented in Figures 4.1 -

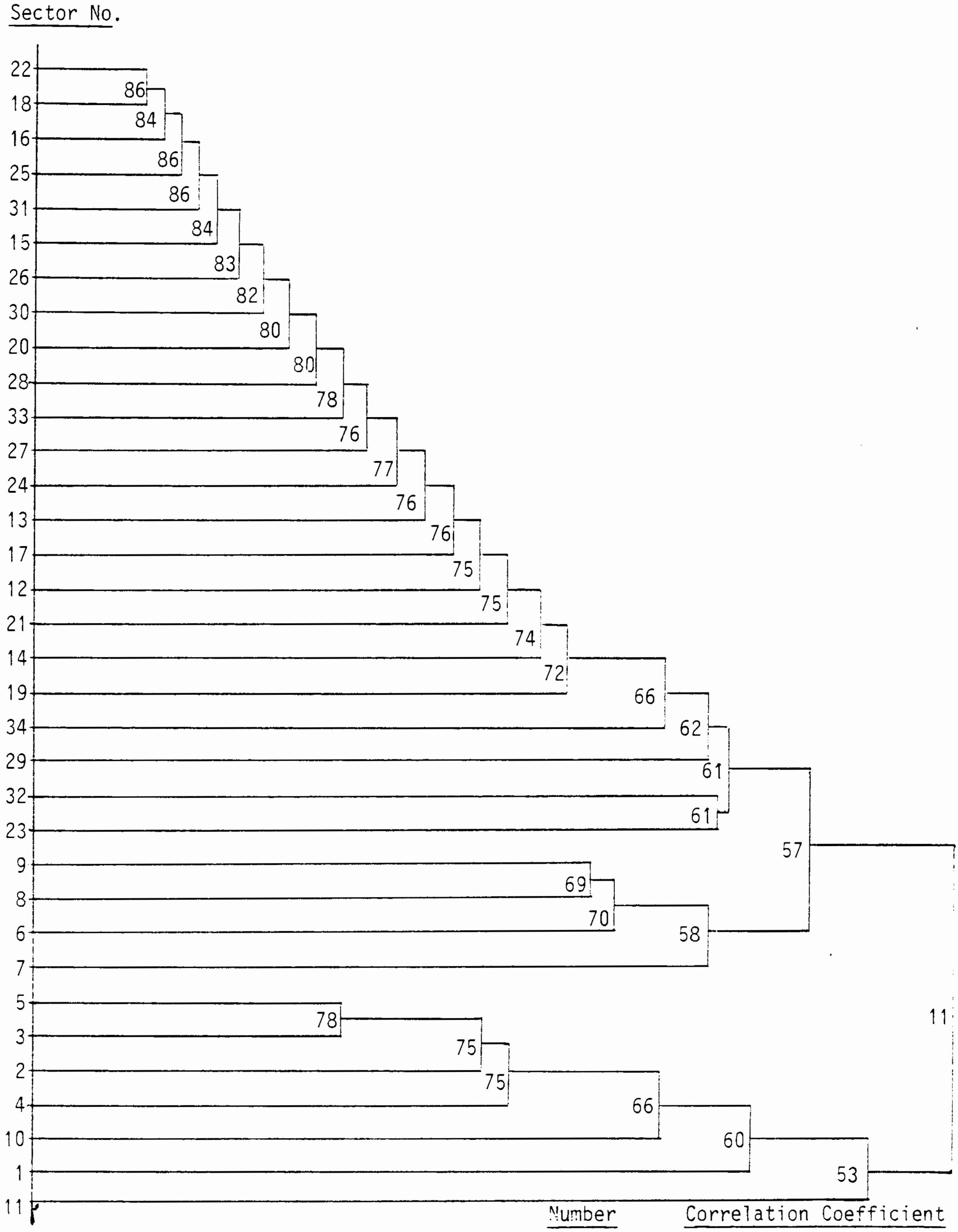
Figure 4.7: CLUSTER ANALYSIS ON JSE SECTORS - PERIOD 1



Number	Correlation Coefficient
100	1,000
50	0,000
0	-1,000

Note: The numbers shown are from the adjacent linear scale. However, the horizontal axis does not represent this linear scale.

Figure 4.8: CLUSTER ANALYSIS ON JSE SECTORS - PERIOD 2



Number	Correlation Coefficient
100	1,000
50	0,000
0	-1,000

Note: The numbers shown are from the adjacent linear scale. However, the horizontal axis does not represent this linear scale.

TABLE 4.4: Final Iterations of Cluster Analysis

Iteration Number	Correlation Coefficients	First Combining Group	Second Combining Group
Period 1			
29	0,332	Golds, Mining Holding	Mining Houses
30	0,251	Industrials, Copper Tin, Others	Diamonds
31	0,219	Industrials group, Diamonds	Platinum
32	0,118	Industrials group, Diamonds, Platinum	Coal
Period 2			
28	0,221	Industrials (except Sugar, Fishing)	Sugar, Fishing
29	0,193	Golds (except Gold - Rand), Mining Holding	Gold - Rand
30	0,153	Copper, Tin, Others, Coal, Platinum	Diamonds
31	0,132	Copper, etc., Coal Platinum, Diamonds	Industrials
32	0,061	Golds, Mining Holding	Mining Houses

4.8 and Tables 4.3 and 4.4, it is concluded that there is a tendency for the JSE sectors to group together in the three groups detailed in Table 4.2 above. The residual correlations of sector residuals after the removal of the market effect at positive correlation coefficients suggests that the single index market model (Sharpe (1963)) is not an effective model in explaining the returns of JSE sectors. This is very likely to be true for individual securities as well. This is because the JSE sectors are portfolios of securities of like nature. However, some portfolio diversification of risk must nevertheless occur in spite of the securities being of like nature. To the extent that the single-index model is inadequate for JSE sectors, therefore, it is likely to be even more inadequate for individual securities where no such portfolio diversification has occurred. This suggests that alternative models may provide a better explanation of the returns of sectors and securities and this is assessed later in Section 4.11.

Furthermore, the aggregation of sectors into the AM, MF and IF indices in the JSE Actuaries Index appears wrongly specified, with the aggregation suggested in Table 4.2 being more appropriate. The extent of the effect of this mis-specification of the JSE Actuaries Index is also examined later in Section 4.11.

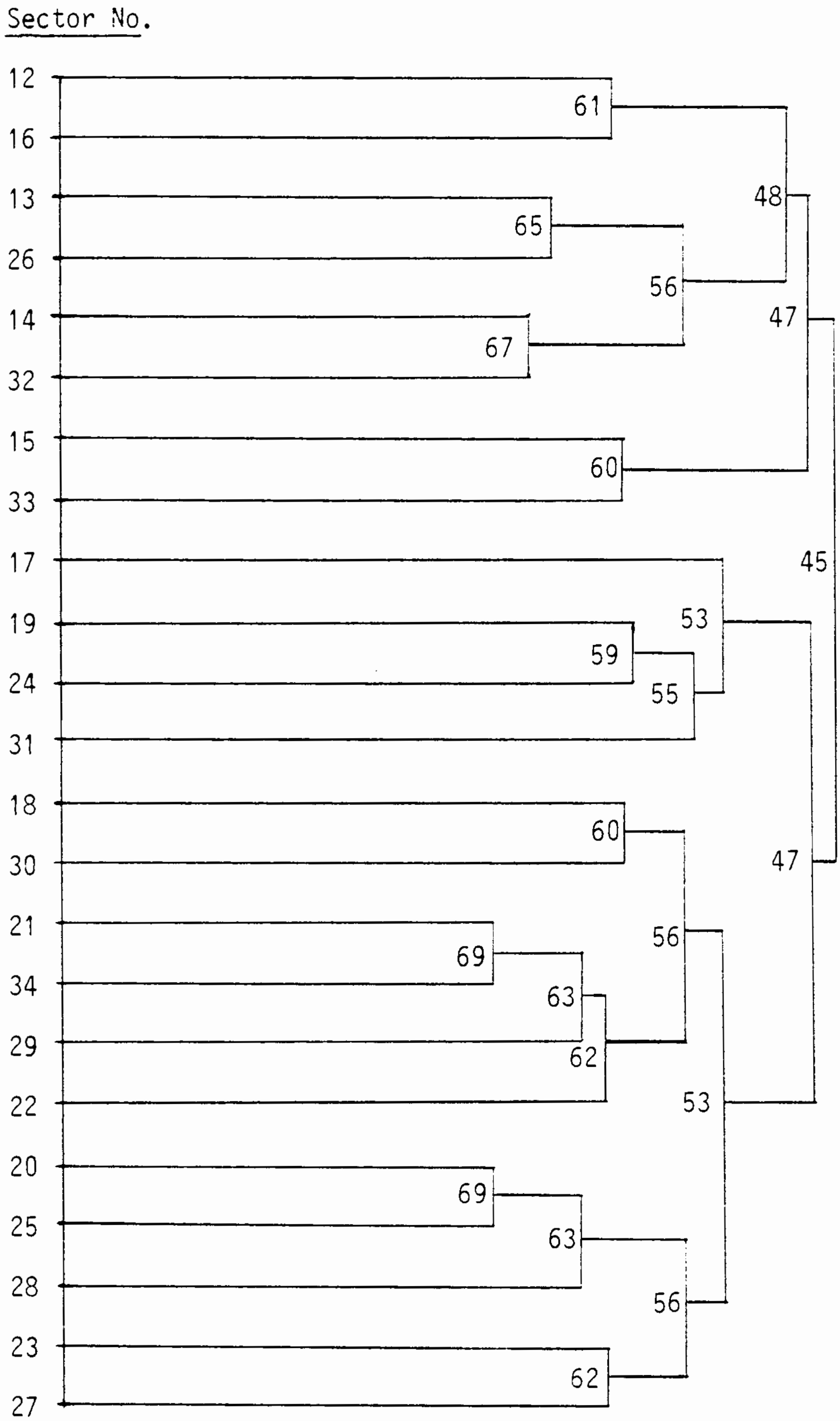
4.9 Clustering of Industrial Sectors

It has been found above that the market and other systematic factors contribute to the explanation of sector returns on the JSE. In addition to these factors, Farrell (1974) found that other systematic factors added to the explanation of security returns on the United States stockmarket and it was decided to see whether the existence of cyclical, growth and stable groups of stocks (as defined in Farrell (1974)) occurred amongst the JSE industrial and financial sectors.

It can be said already that if such groups exist their effect on the covariance matrix of JSE sectors is less than that indicated already in this paper, namely, the grouping of sectors into MI, OMI and IN.

In order to test for these additional factors, the twenty three industrial and financial sectors, indicated in Appendix A as comprising the IF index, were tested via the identical cluster analysis methodologies of Sections 4.6 and 4.7 except that the Industrial and Financial Index was used as the market surrogate in the market model. In other words, it was desired to see whether any systematic effects existed amongst industrial and financial sectors on the JSE apart from the general market effect. The resulting dendrograms for Periods 1 and 2 and the clustering method used in Section 4.6 appear in Figures 4.9 and 4.10 respectively while the results for Periods 1 and 2 and the clustering method of Section 4.7 appear in Figures 4.11 and 4.12 respectively. Analysis of these four figures produces no discernible tendency for sectors to group according to some rational, economically justifiable, basis. Furthermore, the composition of groups does not persist from Period 1 to Period 2 thereby suggesting that no important systematic effects, apart from the market factor, are necessary for a proper appreciation of the covariance matrix of industrial and financial sectors on the JSE. This result has been confirmed by Visser and Affleck-Graves (1982). Furthermore the Sugar and Fishing sectors show no particular tendency to group with each other

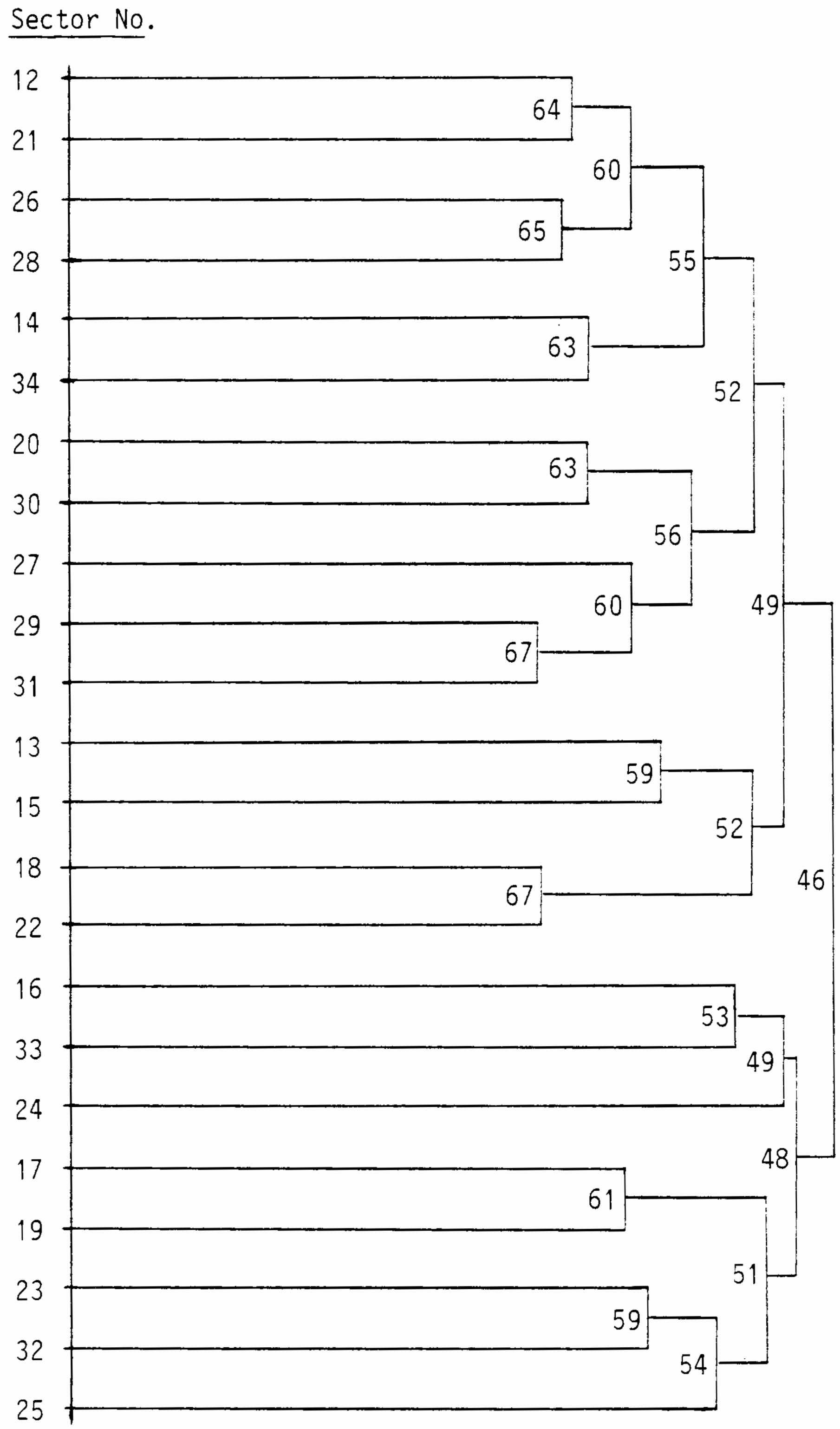
Figure 4.9: CLUSTER ANALYSIS ON INDUSTRIAL SECTORS - PERIOD 1



Note: The numbers shown are from the following linear scale:

Number	Correlation Coefficient
100	1,000
50	0,000
0	-1,000

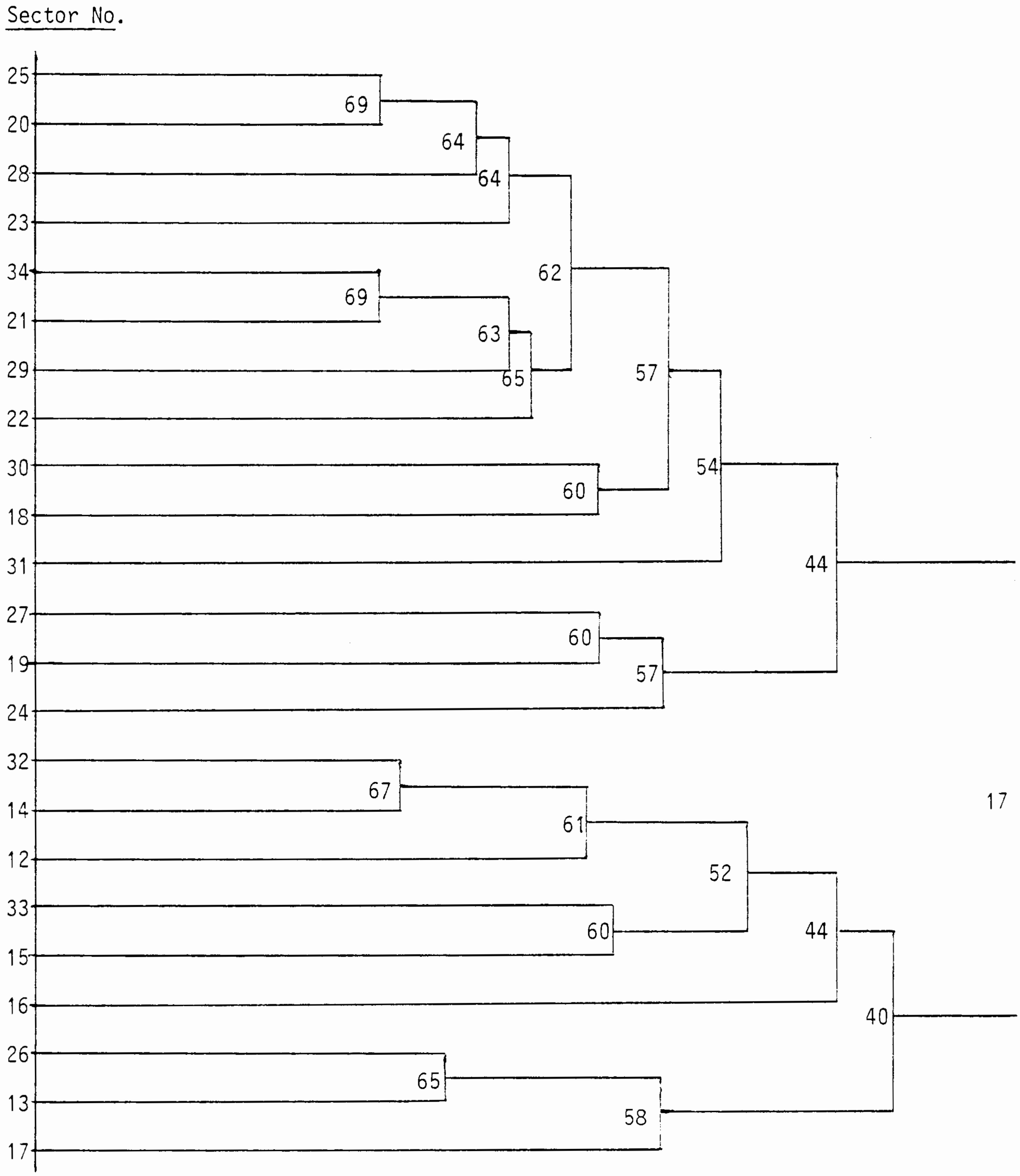
Figure 4.10: CLUSTER ANALYSIS ON INDUSTRIAL SECTORS - PERIOD 2



Note: The numbers shown are from the following linear scale:

Number	Correlation Coefficient
100	1,000
50	0,000
0	-1,000

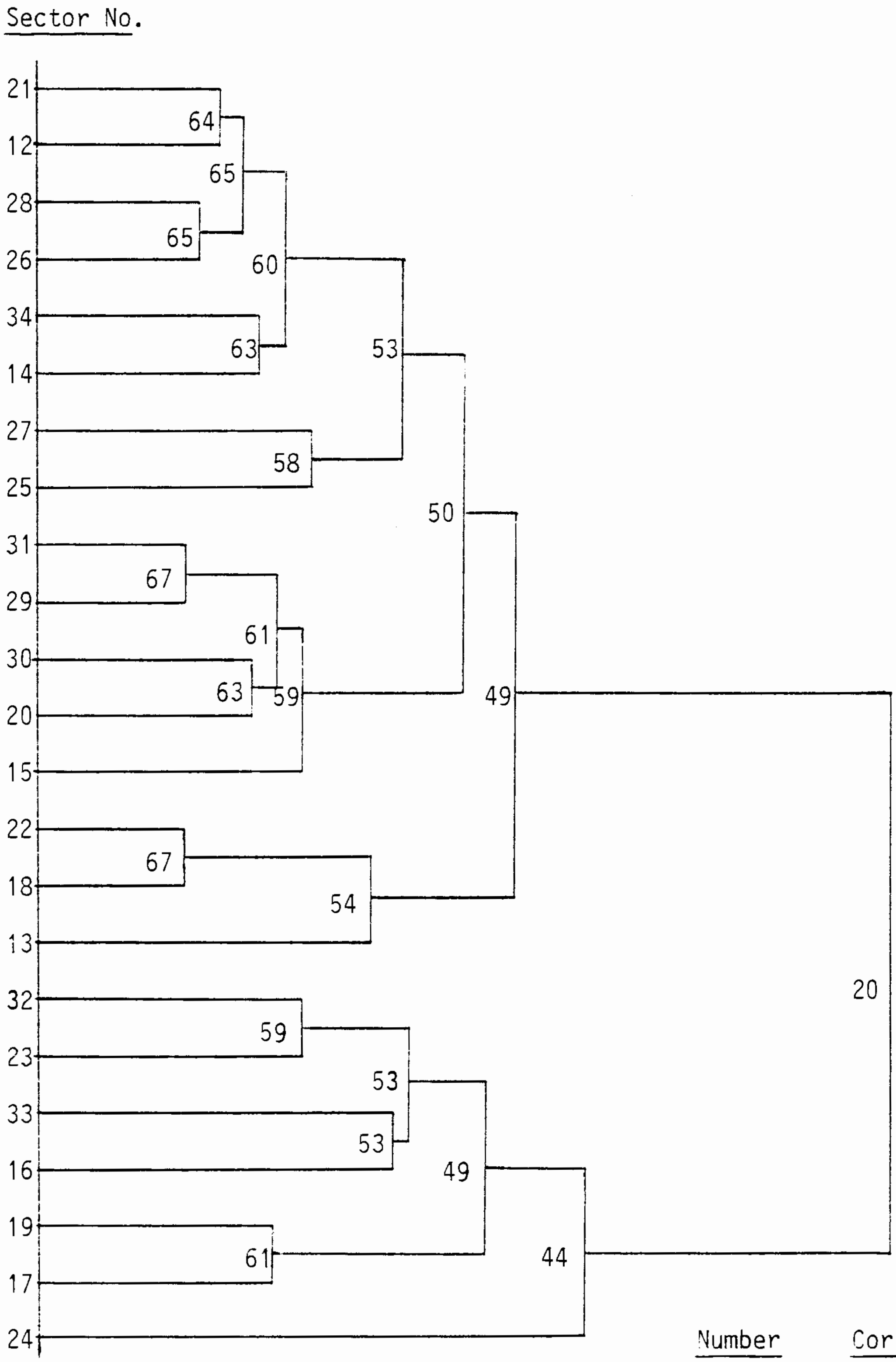
Figure 4.11: CLUSTER ANALYSIS ON INDUSTRIAL SECTORS - PERIOD 1



Number	Correlation Coefficient
100	1,000
50	0,000
0	-1,000

Note: The numbers shown are from the adjacent linear scale. However, the horizontal axis does not represent this linear scale.

Figure 4.12: CLUSTER ANALYSIS ON INDUSTRIAL SECTORS - PERIOD 2



Number	Correlation Coefficient
100	1,000
50	0,000
0	-1,000

Note: The numbers shown are from the adjacent linear scale. However, the horizontal axis does not represent this linear scale.

or separately from the other industrial sectors. This provides additional justification for including them in the IN list of sectors in Section 4.5 above and not with the OMI list of sectors, as may have been indicated by Figures 4.3 and 4.4.

4.10 Formulation of GMF, OMI and IN Indices

The results so far have suggested that the single index market model using the JSE All Share index as the market surrogate may not be the most appropriate way of explaining sector returns on the JSE. In view of the grouping of sectors into the GMF, OMI and IN groups (Table 4.2), it was felt that a multi-index model (equation (1), Section 4.2), using indices formed from the GMF, OMI and IN sectors, could provide a better explanation of security returns on the JSE. Farrell (1974) formulated the indices for his multi-index model by using an arithmetic weighting of constituent returns.

$$I_{kt} = \frac{1}{n} \sum_{j=1}^{n} R_{jt} \qquad k = 1, \ldots, p$$

where I_{kt} is the return of index k in month t

n is the number of constituents in index k

R_{jt} is the return on constituent j in month t

p is the number of indices.

Since the JSE Actuaries indices are market capitalisation weighted indices and since it was desired to compare the GMF, OMI, IN split of sectors with the AM, MF, IF split of sectors, it was decided to construct the GMF, OMI, IN indices on a market capitalisation basis. For this purpose the market capitalisations on the 31 January 1980 of the thirty four JSE sectors were used to determine the weightings applicable. These are shown in Table 4.5. The monthly returns for the GMF, OMI and IN indices were calculated using the following formula:

$$I'_{kt} = \sum_{j=1}^{n} w_j R_{jt} \qquad k = 1,2,3$$

where I'_{kt} is the return on index k in month t

n is the number of constituents in index k

R_{jt} is the return on sector index j in month t

w_j is the weighting factor of sector j

and

$$\sum_{j=1}^{n} w_j = 1 \quad \text{for each } I'_k, \qquad k = 1,2,3$$

Monthly returns for the GMF, OMI and IN indices were calculated for the sixty months of Period 2, that is from February 1975 to January 1980. Monthly returns for the same period for the AM, MF and IF indices were calculated using data provided by the JSE Actuaries Index (1980).

4.11 Comparison of Single-Index Model with Multi-Index Models

4.11.1 Selection of Share Sample

In order to compare the relative effectiveness of the single index market model (using the AS index) with the two multi-index models (using AM, MF and IF indices and GMF, OMI and IN indices) fifty eight shares were chosen as a sample of

TABLE 4.5: Weighting of Sector Returns in the GMF, OMI, IN Indices

GMF		OMI		IN	
Sector	Weighting	Sector	Weighting	Sector	Weighting
Gold - Rand	0,03403	Coal	0,17865	Inv Trusts	0,02268
Gold - Evander	0,02904	Diamonds	0,51134	Insurance	0,02535
Gold - Klerksdorp	0,12674	Platinum	0,14954	Property	0,01916
Gold - OFS	0,14500	Copper, Tin, Other	0,16047	Banks	0,11617
Gold - W Wits	0,26107		1,00000	Ind Holding	0,17907
Mining Holding	0,10925			Beverages	0,05578
Mining Houses	0,29487			Building	0,05227
	1,00000			Chemicals	0,11508
				Clothing	0,02140
				Electrical	0,02482
				Engineering	0,04337
				Fishing	0,00373
				Food	0,04788
				Furniture	0,02125
				Motor	0,01560
				Paper, Packaging	0,05120
				Pharma-ceutical	0,00731
				Printing	0,00372
				Steel	0,03116
				Stores	0,06432
				Sugar	0,03027
				Tobacco	0,03456
				Transport	0,01386
					1,00000

securities whose returns could be explained by the three models. These shares are listed in Appendix C. They were chosen to satisfy a number of criteria.

(i) Each share was listed continually throughout Period 2 so that monthly returns data were available.

(ii) A spread of shares across the entire JSE was desired. As is shown in Table 4.6 this was achieved on the dimensions of number, and market capitalisation, of shares in each broad market category of the JSE.

(iii) A spread of market capitalisations and marketabilities was desired in order to avoid a selection bias in favour of widely held and highly tradeable securities.

TABLE 4.6: Selection of Sample Shares Compared to the JSE

Number of shares	Sample	% of Total	JSE All Share Index	% of Total
Shares in All Mining Index	15	25,9	35	22,9
Shares in Mining Financial Index	3	5,1	6	3,9
Shares in Industrial and Financial Index	40	69,0	112	73,2
TOTAL	58	100,0	153	100,0

Market Capitalisation	% of Sample	% of JSE Index
Shares in All Mining Index	48,2	48,2
Shares in Mining Financial Index	25,9	19,9
Shares in Industrial and Financial Index	25,9	31,9
	100,0	100,0

The shares chosen were all constituents of the JSE Actuaries Index for the entire period under study and together comprised 60,6% of the market capitalisation of the JSE All Share index at 31 January 1980. Since each of the shares was in the JSE Actuaries Index throughout the period under study, the shares chosen do have a selection bias on the criterion of market capitalisation compared with other shares listed on the JSE but not included in the JSE Actuaries Index. However, to the extent that the results presented below are applicable to the shares under study, they will be seen to be even more applicable to shares which are not included in the JSE Actuaries Index.

4.11.2 Adjustment to Coefficient of Determination for Multi-Index Model

For simple ordinary least squares regression the coefficient of determination (R^2) is given by:

$$R^2 = \frac{\text{(sum of squares due to regression)}}{\text{(total sum of squares, corrected for mean)}}$$

In the case of multiple regression, the coefficient of determination obtained must be adjusted for the presence of additional independent variables if a comparison of coefficients of determination is to be made between the single and multi-index models. This adjustment is given by:

$$R^{2''} = 1 - \frac{n-1}{n-k}(1-R^{2'})$$

where $R^{2''}$ is the adjusted coefficient of determination to be compared with R^2 from the single index model

n is the number of data points in the sample

k is the number of regressors

$R^{2'}$ is the coefficient of determination of the multiple regression.

In all succeeding references to coefficients of determination arising from multiple regressions in this chapter, this adjustment has been made. Table 4.7 tabulates, for the share sample, the coefficients of determination from the single index model and each of the multi-index models, as well as some other pertinent statistics which are used later in this chapter.

4.11.3 Coefficients of Determination for Single and Multi-Index Models

Each share's monthly returns were regressed on the corresponding monthly returns for the AS index using the single-index market model. Similarly, multiple regressions were performed for each share on the AM, MF, IF indices and the GMF, OMI, IN indices. This resulted in three coefficients of determination for each share for Period 2. The results are summarised in Table 4.8 and Table 4.9 below.

Some points emerging from Tables 4.8 and 4.9 are:

(i) The coefficients of determination from the single index model averaged 0,277 for the shares under study. This

TABLE 4.7: STATISTICS OF SAMPLE SHARES

Share Code	Beta to AS Index	Mrk Cap 01/80 (Rm)	Value Traded Period 2 (Rm)	Vol Traded Period 2 (million)	R^2 in AS Model	Adj R^2 in AM, MF, IF Model	Adj R^2 in GMF, OMI, IN Model
ABR	0,341	41,1	2,5	0,73	0,079	0,128	0,151
ADK	0,446	30,6	3,7	0,31	0,159	0,305	0,284
ACA	0,290	77,2	1,3	0,22	0,054	0,028	0,042
AFX	0,619	119,3	9,0	4,53	0,236	0,462	0,460
AAL	0,659	76,7	7,2	5,28	0,211	0,369	0,340
AAC	1,184	2981,2	110,2	17,45	0,694	0,754	0,704
AMG	1,499	1833,0	33,7	0,91	0,692	0,869	0,805
AMI	0,530	552,8	31,2	3,09	0,286	0,502	0,515
APE	0,536	49,7	15,7	1,31	0,062	0,085	0,202
BAR	0,715	1005,8	117,4	29,89	0,343	0,406	0,401
CTX	0,383	18,8	0,2	0,16	0,051	0,031	0,037
DBR	0,691	3687,8	206,4	39,61	0,319	0,342	0,738
DLV	0,355	81,6	10,6	1,76	0,075	0,276	0,279
EDS	0,607	102,9	9,5	0,29	0,304	0,396	0,385
EVT	0,663	65,5	3,7	1,96	0,207	0,338	0,344
FUG	0,670	123,0	9,2	11,99	0,344	0,406	0,404
FSG	1,490	569,0	29,1	1,00	0,552	0,623	0,637
GFS	1,222	1142,4	43,4	1,53	0,415	0,552	0,585
HUL	0,482	185,1	21,0	7,82	0,100	0,118	0,103
KLO	1,630	922,3	36,6	3,30	0,647	0,764	0,756
LLA	0,683	210,1	7,6	0,68	0,384	0,487	0,485
MCR	0,761	23,8	3,9	4,79	0,127	0,459	0,439
MBX	0,713	116,5	9,6	3,40	0,265	0,436	0,435
NED	0,731	466,7	56,7	22,11	0,316	0,579	0,583
OKO	0,673	163,7	13,5	1,63	0,233	0,603	0,592
PGS	0,913	75,9	8,5	3,77	0,221	0,378	0,404
PBR	1,560	540,5	27,8	1,39	0,656	0,704	0,719
PPC	0,764	118,8	7,8	2,48	0,355	0,536	0,533
RMT	0,696	352,4	27,9	5,22	0,298	0,556	0,592
RTO	0,301	7,1	0,6	0,16	0,051	0,084	0,037

TABLE 4.7 (cont.):

Share Code	Beta to AS Index	Mrk Cap 01/80 (Rm)	Value Traded Period 2 (Rm)	Vol Traded Period 2 (million)	R^2 in AS Model	Adj R^2 in AM, MF IF Model	Adj R^2 in GMF, OMI, IN Model
ROM	0,400	137,4	6,7	3,11	0,047	0,204	0,222
STH	1,019	320,0	11,0	0,55	0,309	0,318	0,372
SAP	0,950	152,0	18,5	8,11	0,479	0,515	0,502
SCC	0,556	34,7	2,6	1,55	0,077	0,108	0,110
SAN	0,743	10,9	1,1	0,42	0,245	0,222	0,210
SFM	0,467	146,6	7,2	3,09	0,168	0,225	0,220
SLL	0,510	82,0	3,2	1,61	0,133	0,169	0,177
TAV	0,522	98,5	26,2	1,46	0,063	0,092	0,280
ARG	0,437	23,1	2,6	0,23	0,078	0,113	0,121
CLY	0,568	64,6	12,1	5,58	0,186	0,150	0,267
GTA	0,491	13,6	1,7	0,39	0,134	0,225	0,203
ICS	0,744	71,0	9,3	4,64	0,280	0,412	0,416
PML	0,561	191,6	16,5	2,72	0,231	0,375	0,358
RFN	1,207	364,1	101,0	2,31	0,397	0,411	0,417
SAB	0,826	616,1	75,4	56,59	0,395	0,609	0,579
USC	1,072	29,0	4,8	9,56	0,311	0,403	0,363
TIG	0,602	150,8	25,9	2,75	0,387	0,554	0,568
TNC	0,804	335,5	48,6	20,37	0,245	0,225	0,376
TRE	0,211	8,2	0,7	0,20	0,046	0,047	0,051
TWH	0,444	31,5	6,1	0,25	0,103	0,127	0,129
VAR	1,443	1235,0	43,7	1,58	0,683	0,776	0,796
VKS	0,566	118,5	18,2	5,24	0,281	0,505	0,499
WDR	1,137	1098,4	21,1	0,51	0,402	0,461	0,618
WDL	1,476	1025,0	23,6	1,27	0,617	0,693	0,732
WHL	1,512	442,3	14,7	0,40	0,559	0,585	0,577
WHT	0,292	15,1	1,0	1,02	0,042	0,172	0,187
WIC	0,805	302,4	15,6	1,22	0,172	0,180	0,327
WLO	0,569	95,4	10,7	2,65	0,277	0,555	0,548

TABLE 4.8: Average Coefficients of Determination for Share Regressions

Shares in	Number of Shares	From AS Model	From AM, MF, IF Model	From GMF, OMI, IN Model
AM	15	0,391	0,427	0,521
MF	3	0,600	0,725	0,698
IF	40	0,210	0,336	0,333
Weighted Average		0,277	0,379	0,400
GMF	12	0,552	0,626	0,643
OMI	6	0,175	0,179	0,365
IN	40	0,210	0,336	0,333
Weighted Average		0,277	0,379	0,400

TABLE 4.9: Average Percentage Improvement in Coefficient of Determination from Single Index Model

Shares in	From AM, MF, IF Model	From GMF, OMI, IN Model
AM	+ 9,2	+ 33,2
MF	+ 20,8	+ 16,3
IF	+ 60,0	+ 58,6
Weighted Average	+ 36,8	+ 44,4
GMF	+ 13,4	+ 16,5
OMI	+ 2,3	+ 108,6
IN	+ 60,0	+ 58,6
Weighted Average	+ 36,8	+ 44,4

is a poor level of explanation in the return on these shares, but in comparison to the experience in other markets, is not unusual. (Marsh (1980), Farrell (1975), Arnott (1980)).

This result is consistent with other findings on the JSE (Gross (1974)). In addition this low level of explanation supports the assertion that the single index model on the JSE is an inadequate model for explaining security returns.

(ii) The multi-index models enhance the coefficients of determination from the single index model by 36,8% on average for the AM, MF, IF model, and 44,4% on average for the GMF, OMI, IN model. Therefore, on average it is possible that the GMF, OMI, IN model provides a better explanation of security returns than the AM, MF, IF model. The introduction of the sector grouping factor by way of the GMF, OMI and IN indices contributes an additional 12,3 per cent to the explanation of the returns of the shares in the sample. In the case of eight of the fifty eight shares under consideration (13,8%), this factor was more important than the market factor in explaining the share returns.

(iii) Shares in the IF (and IN) indices show a similar improvement in average coefficient of determination for the multi-index models compared to the average coefficient of determination for the single index model, indicating that for the industrial group of shares, use of either multi-index model is equally appropriate.

(iii) It is apparent that shares in OMI, which have on average a low coefficient of determination in the AS model and the AM, MF, IF model, have a substantially higher average coefficient of determination in the GMF, OMI, IN model. In particular, therefore, the single index and AM, MF, IF multi-index models provide a far less adequate explanation of the returns of this group of securities. In Table 4.10 some supporting evidence for this is shown.

TABLE 4.10: Shares with Significant Alpha Values at 95% Level of Confidence

	From AS Model	From AM,MF,IF Model	From GMF, OMI, IN Model
Number of shares	5	3	0
% of sample	8,6	5,2	0

In all but one case (General Tire in the AS Model) these shares fall into the OMI group, thereby suggesting the particular inadequacy of the AS and AM, MF, IF models in explaining the returns on these securities (Farrell (1974)). Thus, the improvement in average coefficient of determination of the GMF, OMI, IN model over the AM, MF, IF Model (Table 4.10) is largely a function of the improvement in the OMI shares, indicating the particular advantage of using the GMF, OMI, IN model for these shares.

(v) There is evidence that shares with a low coefficient of determination from the AS model have a proportionately better improvement in coefficient of determination than other shares when the multi-index models are used to explain the returns on these shares. Table 4.11 shows this phenomenon.

TABLE 4.11: Coefficients of Determination from AS Model in Descending Order in Quartile Averages

Quartile	No. of Shares	From AS Model	From AM, MF, IF Model	% Impro-* vement	From GMF, OMI IN Model	% Impro-* vement
1	14	0,541	0,634	+ 17,2	0,643	+ 18,9
2	15	0,311	0,456	+ 46,6	0,485	+ 55,9
3	15	0,194	0,312	+ 60,8	0,333	+ 71,6
4	14	0,066	0,115	+ 74,2	0,139	+110,6

* on coefficients of determination from AS Model.

It can be seen that shares with lower average coefficients of determination from the AS model have proportionately greater improvements in their coefficients of determination when a multi-index model is used. In particular it should be noted that the performance of the GMF, OMI, IN model relative to the AM, MF, IF model also displays this characteristic. In other words, not only does the GMF, OMI, IN model explain more of the returns than the AM, MF, IF model for each category of shares, but that improvement is greater in percentage terms the lower the coefficient of determination from the AS model.

This is strong evidence that the GMF, OMI, IN model is the superior model and that its relative superiority is greater for shares whose returns are poorly correlated with the AS index. This is likely to be even more true of shares which are not in the JSE Actuaries Index because these shares are of lower market capitalisation and poorer marketability than those in the JSE Actuaries Index. Therefore, the GMF, OMI, IN model is likely to be generally more useful in explaining the returns for this group of shares.

4.11.4 Relationship between the Models, Market Capitalisation, Volume Traded and Value Traded

Dimson (1979) asserted that infrequent trading was a likely explanation for the apparent non-randomness in the price movements of some shares. In particular it was shown that simple regression produces downward biased estimates of the risk of infrequently traded shares. From the simple linear regression model

$$Y = \beta_0 + \beta_1 X$$

it is known that

$$b_1 = R \frac{S_Y}{S_X}$$

where b_1 is the least squares estimate of the slope coefficient β_1

R is the sample correlation coefficient between X and Y

S_Y, S_X are the sample standard deviations of Y and X respectively.

Therefore, for a $\frac{S_Y}{S_X}$ ratio near unity, it can be seen that share regressions using the AS model which have low coefficients of determination will tend to have low beta estimates. Strebel (1977) asserted that unless shares traded at least 250 000 shares per annum on the JSE the market in those shares should be considered inefficient (that is, display non-random price movements) in that shares which are infrequently traded are not amenable to adequate statistical testing to prove efficiency. It might be expected, therefore, that shares on the JSE which trade less than 250 000 shares per annum, would have on average a low coefficient of determination in the AS model. It was decided to investigate the relationships between the coefficients of determination of the share regressions using the three models under investigation and the market capitalisation, value traded and volume traded of these shares. The market capitalisation of each share was taken as that value outstanding in the market in the last month of Period 2, namely, January 1980. The value and volume of shares traded were obtained from the JSE Monthly Bulletins (1975-1980). These figures for each share are shown in Table 4.7, Section 4.11.2. These relationships were examined by computing the correlation coefficients between the coefficients of determination (P) and each of these variables (Q). These results are displayed in Table 4.12.

TABLE 4.12: Correlation between the Coefficients of Determination and Market Capitalisation, Value and Volume Traded

Variable Q	Model from which Coefficients of Determination obtained	Regression Correlation Coefficient
Market Capitalisation	AS	0,576*
Market Capitalisation	AM, MF, IF	0,469*
Market Capitalisation	GMF, OMI, IN	0,617*
Value Traded	AS	0,399*
Value Traded	AM, MF, IF	0,361*
Value Traded	GMF, OMI, IN	0,483*
Volume Traded	AS	0,170
Volume Traded	AM, MF, IF	0,126*
Volume Traded	GMF, OMI, IN	0,276*

* significant at 95% level of confidence.

Table 4.12 shows that the strongest correlation with a share's coefficient of determination is its market capitalisation, irrespective of which model is used. This result is not surprising considering that the models used are market capitalisation weighted indices.

Therefore, shares with larger market capitalisations affect the indices to a greater extent and their returns are expected to show greater covariance with the returns on the market indices than shares with lower market capitalisations, all other things being equal. It is interesting to note from Table 4.12 that there appears to be only a tenuous relationship between volume traded and coefficient of determination for each of the three models. Nineteen out of the 58 shares traded less than 250 000 shares per annum. According to Strebel (1977) these shares are likely to be inefficiently priced, and have a correlation between coefficient of determination from the market model, and volume traded, of 0,405, which is not significant at 95% level of confidence. To the extent that beta coefficients are related to coefficients of determination it would seem that infrequently traded shares are just as likely to have high betas as low betas. However, the correlation between volume traded and beta coefficient for shares which traded less than 250 000 shares per annum is 0,452, which is significant at 95% level of confidence. A similar correlation coefficient calculated for all shares in the sample is 0,042, which is not significant at 95% level of confidence. Therefore the assertions of Dimson (1979) and Strebel (1977) appear to be confirmed for this sample of shares. The result is amplified further in Table 4.13 in which the share sample is split into quartiles based on descending volume traded during Period 2.

From Table 4.13 it is clear that other than in very lowly traded shares (quartile 4) the average coefficients of determination from the regressions using all three models, and average beta coefficients from the AS regressions, are not materially different from quartile to quartile. However, the GMF, OMI, IN model regressions display a descending coefficient of determination ranking indicating that more highly traded

TABLE 4.13: Coefficients of Determination for Shares in Descending Order of Volume Traded in Quartiles

Quartile Number	No. of Shares	Average Annual Volume Traded (million)	Average Beta from AS Model	From AS Model	From AM, MF, IF Model	% Improvement*	From GMF, OMI, IN Model	% Improvement*
1	14	17,49	0,758	0,323	0,424	31,3	0,461	42,7
2	15	3,24	0,743	0,275	0,441	60,4	0,442	60,7
3	15	1,01	0,885	0,303	0,402	32,7	0,439	44,9
4	14	0,34	0,622	0,206	0,245	18,9	0,254	23,3
Weighted Average				0,277	0,379	36,8	0,400	44,4

* on coefficients of determination from AS model.

shares do tend to have higher coefficients of determination in this model. The percentage improvement in coefficients of determination for the multi-index models compared to the single index AS model appears to have no relationship to volume traded except that the improvements to coefficient of determination for quartile 4 shares is discernibly lower than other quartiles. From this it can be concluded that the use of the GMF, OMI, IN model for the explanation of share returns does not have a systematically different impact on shares with different trading volumes, except that the coefficients of determination in the AS regressions of the most infrequently traded shares show proportionately less improvement in coefficient of determination than other shares when this model is used. One reason why trading volume may not be related to efficiency is that it ignores the price of shares. The mining shares in the sample under consideration have a far higher average price than the industrial shares in the sample under study. This is summarised in Table 4.14.

TABLE 4.14: Average Volume, Price and Value per Share in Each Index

Shares In	No. of Shares	Unweighted Average beta to AS Index	Unweighted Average Volume traded in Period 2 (million)	Unweighted Average price in Period 2 (R)	Unweighted Average Value Traded in Period 2 (RM)
AM	15	1,093	5,46	7,73	42,22
MF	3	1,302	6,63	9,42	62,44
IF	40	0,586	5,41	2,65	14,36
AS	58	0,754	5,49	4,38	24,05
GMF	12	1,365	2,68	15,42	41,32
OMI	6	0,654	11,59	4,67	54,12
IN	40	0,586	5,41	2,65	14,36

From this table it can be seen that even though the average volume traded of shares in GMF (consisting of gold shares plus Mining Houses and Holding) was less than half that of the shares in IN (consisting of industrial shares) the average value traded of the GMF shares was nearly three times that of the IN shares because the average price of the GMF shares was nearly six times greater than that of the IN shares. Therefore value of shares traded would seem to be a better parameter to use when addressing the issue of efficiency.

TABLE 4.15: Coefficients of Determination for Shares in Descending Order of Value Traded

Quartile Number	No. of Shares	Average Value Traded (Rm)	Average Beta From AS Model	From AS Model	From AM, MF, IF Model	% Improvement*	From GMF, OMI, IN Model	% Improvement*
1	14	68,67	1,048	0,449	0,569	26,7	0,606	35,0
2	15	18,76	0,865	0,316	0,396	25,3	0,448	41,8
3	15	8,57	0,645	0,239	0,380	59,0	0,375	56,9
4	14	2,05	0,459	0,106	0,174	64,2	0,171	61,3
Weighted average				0,277	0,379	36,8	0,400	44,4

* on coefficients of determination from AS model.

From Table 4.15 it is clear that there is a relationship between value traded and coefficient of determination for each model, as well as with beta to the AS index. As value traded decreases so does the average beta from the AS model and average coefficient of determination for the regression using each model. It is evident that the multi-index models tend to contribute a greater percentage improvement to the coefficients of determination from the AS regressions, the lower the value traded. This again suggests that the single index model is generally inadequate for low beta and low value traded shares which is consistent with the findings of Dimson (1979), since these shares are most likely to have downward biased beta estimates generated from the AS model.

It is interesting to note that all twenty nine shares in quartiles 3 and 4 of Table 4.15 are industrial shares, belonging to the IF and IN indices. It must be concluded therefore that the single index AS model is particularly inappropriate for this group of securities, and conversely the multi-index models much more appropriate in explaining the returns on these shares.

Finally it is worthwhile to consider the relationship between market capitalisation and coefficient of determination for the regressions of the share returns using the three models under study. The results are displayed in Table 4.16.

Table 4.16 shows that there is a relationship between market capitalisation and coefficient of determination for each model, as well as with beta to the AS index. As market capitalisation decreases, so does average beta from the AS model and average coefficient of determination for the regressions of each model. As with value traded, the multi-index models display better improvement in coefficient of determination the lower the market capitalisation.

TABLE 4.16: Coefficients of Determination for Shares in Descending Order of Market Capitalisation

Quartile Number	No. of Shares	Average Market Capital-isation (Rm)	Average Beta from AS Model	From AS Model	From AM,MF, IF Model	% Im-prove-ment*	From GMF, OMI, IN Model	% Im-prove-ment*
1	14	1262,6	1,152	0,501	0,617	23,2	0,655	30,7
2	15	238,5	0,769	0,290	0,384	32,4	0,408	40,7
3	15	91,0	0,604	0,210	0,340	61,9	0,361	71,9
4	14	24,1	0,501	0,112	0,179	59,8	0,180	60,7
Weighted Average				0,277	0,379	36,8	0,400	44,4

* on coefficients of determination from AS Model.

It should be noted that the GMF, OMI, IN model produces a greater average coefficient of determination for shares in each quartile than the AM, MF, IF model, thereby indicating that this model may be superior in explaining share returns across the whole range of market capitalisations. Table 4.16 also invites the conclusion that non-JSE Actuaries Index shares, being generally of lower market capitalisation than shares in the JSE Actuaries Index, should have their returns poorly explained by the AS model. However, use of the GMF, OMI, IN model ought to contribute a great improvement in the explanation of the returns on these shares, although the absolute level of the explanation as measured by the coefficients of determination is not expected to be high.

4.12 Conclusions

The application on the JSE of clustering techniques to identify homogeneous sector groups after allowing for the market effect has produced some interesting findings:

(i) The use of a multi-dimensional scaling technique indicated that the JSE sectors divide naturally into two groups, one consisting of gold related sectors and the other non-gold related sectors.

(ii) This split was confirmed by applying two clustering techniques to the same data. It was argued, and evidence was available, that a third group, consisting of non-gold mining sectors, should be split off from the group of non-gold related sectors. This results in the following three groups of sectors:

1. Gold sectors, Mining Houses, Mining Holding
2. Coal, Diamonds, Platinum, Copper, Tin and Other
3. Industrial and Financial

(iii) The existence of this tendency for JSE sectors to group together after the removal of the market effect casts serious doubt on the validity of the single index market model on the JSE as an adequate model for explaining share returns. Indeed, lowly traded shares, small market capitalisation shares and industrial shares all had their returns poorly explained by the single index market model.

(iv) The homogeneous grouping of sectors is not in accordance with the JSE Actuaries Index, which groups the non-gold mining sectors with the gold sectors into the All Mining index, the Mining Houses with Mining Holding into the Mining Financial index, and the industrial and financial sectors into the Industrial and Financial index.

(v) Sectors within the Industrial and Financial index showed no such tendency to group together in any rational or consistent manner and hence it is concluded that the market factor (Industrial and Financial index) is the only important systematic factor in explaining the returns on shares in these sectors.

(vi) A sample of 58 shares on the JSE had an average adjusted coefficient of determination of 0,277 from the single index model using the JSE All Share index as market surrogate, compared with 0,379 when the Actuaries Index multi-index model was used. When the multi-index model based on homogeneous sector groups was used, this average coefficient of determination increased to 0,400. Even this level of explanation of returns is low, indicating that individual securities always have a large element of unsystematic risk.

(vii) The single index model was particularly inadequate for explaining the returns on industrial and financial shares while the multi-index model based on the Actuaries index was particularly inadequate for explaining the returns on non-gold mining shares.

(viii) The multi-index model based on homogeneous sector groups was seen to be the best model for explaining share returns for the whole range of shares on the JSE. It was particularly superior to the other two models for shares with poor tradeability (measured by value traded) and/or low market capitalisation. This model is therefore expected to be at least as applicable to shares which are not in the JSE Actuaries Index. In particular if such shares are poorly traded they are likely to suffer from downward biased beta estimates in the single index model.

(ix) The single index model using the AS index as market surrogate appears at best, to be an adequate model for explaining the returns of only the most highly value traded shares. These are typically the gold shares and mining financial shares.

(x) While achieving a greater level of explanation of share returns than the single index model, multi-index models do suffer from the lack of intuitive appeal that a one parameter model provides. However, it must be recognised that on the JSE the single index model seems to have such a limited applicability that its intuitive appeal may be insufficient to compensate for this inadequacy.

The remainder of this thesis is therefore devoted to finding methods for making the market model more adequate for, and applicable to, the JSE. In Chapter 5 a multi-beta interpretation of the market model is developed and empirically tested, while in Chapter 6 the issue of temporally changing beta coefficients generated by the market model is addressed. Finally in Chapter 7 a guide to a suitable statistical procedure for estimating beta coefficients in the market model is suggested.

The results of these three chapters are then combined into a suggested method for estimating beta coefficients which recognises some of the difficulties encountered in applying the market model on the JSE. The impetus for finding such a method remains the overwhelming intuitive appeal embodied in a one parameter risk measure such as a beta coefficient.

CHAPTER FIVE

MULTIBETAS ON THE JOHANNESBURG STOCK EXCHANGE

5.1 Introduction

It has been well established (Sharpe (1970)) that in a well diversified portfolio the beta coefficient of an individual security is the only important measure of the risk contributed by that security to overall portfolio risk because any unsystematic risk associated with the security is diversified away. Furthermore it is also well known (Blume (1971), Levy (1971)) that individual securities have unstable beta coefficients over time. This finding has been confirmed for securities listed on the Johannesburg Stock Exchange by Gross (1974), Affleck-Graves (1977) and Silva (1982).

The objective of this and the next two chapters is to investigate ways of obtaining the best current estimate of the beta coefficient of a security using the market model. The approach in this chapter is based on the assumption that the current beta coefficient of a security is the best estimate of its future beta coefficient, in the absence of any special knowledge about future factors which may affect the security's beta. Since the true beta coefficient of a security can never be known the best that can be done is to estimate a set of beta coefficients which are assumed to be the best estimates of the prevailing true beta coefficients of the securities being studied. These sets of betas are called the *prediction* betas. The set of ordinary least squares (OLS) beta estimates of the securities is then obtained for a succeeding period and is called the set of *realised* betas.

This set of *realised* betas is assumed to contain the best estimates of the true prevailing betas of the securities during this period. Then, this set of *realised* betas is regressed on each set of *prediction* betas and the mean square error (MSE) of the regressions calculated. The regression producing the lowest MSE is then deemed to have the best set of *prediction* betas as independent variable and this set is assumed to contain the best estimates of the prevailing true beta coefficients of the securities at the time the predictions were made. In particular the method used to generate this set of betas is of great interest.

5.2 Mean Square Error

The prediction success of each particular method is evaluated by using the mean square error (MSE) criterion (Silva (1982), Wonnacott and Wonnacott (1981)). If each estimation procedure produced unbiased estimates, the one with the minimum variance could be selected as the best predictor. However, biased and unbiased estimators will be compared and in this instance a criterion which takes into account both variance and bias is more appropriate. The MSE criterion is such a measure.

Let Z be the set of computed beta coefficients for the current period which will be used as predictors of beta for the subsequent period. Let Y be the set of estimated betas obtained using the data from the subsequent period. This is the set of beta realisations and is assumed to comprise the set of actual betas for the second period. Then

$$MSE = \frac{1}{K} \sum_{i=1}^{N} (Z_i - Y_i)^2$$

where K is the number of securities for which predictions are available

Z_i is the 'actual' beta coefficient for security i

Y_i is the estimated beta coefficient for security i

and it follows that (Klemkosky and Martin (1975))

$$MSE = (\bar{Z}-\bar{Y})^2 + (1-B)^2 S_Z^2 + (1-r_{YZ}^2) S_Y^2 \qquad (1)$$

where $\bar{Z}$ and $\bar{Y}$ are the means of the 'actual' and estimated sets of betas respectively

B is the estimate of the slope coefficient of the regression of Y on Z

S_Z^2 and S_Y^2 are the sample variances of Z and Y respectively

r_{YZ}^2 is the coefficient of determination of the regression of Y on Z.

In (1) the first term represents bias, the second term inefficiency and the third term random disturbance.

5.3 The Three Methods

5.3.1 Historical Beta

Beta estimates are obtained for a set of securities in the first of two non-overlapping periods using the market model and OLS regression

$$R_i = \alpha_{im} + \beta_{im}R_m + e_{im}$$

where R_i is the return on security i

R_m is the return on the market surrogate, m

α_{im} and β_{im} are parameters unique to security i

e_{im} is a random variable representing the residual error.

The e_{im} satisfy the usual conditions in OLS regression.

The set of betas from the first period (the prediction betas) is used as the set of beta predictions for the second period.

5.3.2 Bayesian Adjusted Historical Beta

Vasicek (1973) suggested a Bayesian approach to the adjustment of beta coefficients. Assume that a set of security beta coefficients is obtained using the market model in the first of two non-overlapping periods. This set of betas has a sample mean and sample variance, and can be regarded as a prior distribution of beta coefficients. Suppose that in the second period, security i has a beta coefficient, β''_{im}. β''_{im} may be estimated by b''_{im}, where

$$b''_{im} = \frac{(b'/s'^2_b) + \left(b_{im}/s^2_{b_{im}}\right)}{(1/s'^2_b) + \left(1/s^2_{b_{im}}\right)} \quad ;$$

where b' = the sample mean of security betas estimated in the first period;

s'^2_b = the sample variance of beta coefficients estimated in the first period;

b_{im} = the estimated beta coefficient (historical beta) for security i in the first period;

$s^2_{b_{im}}$ = an estimate of the variance of b_{im} in the first period.

b''_{im} will be approximately normally distributed for sample sizes larger than twenty.

It should be noted that b''_{im} is *not* an unbiased estimate of β''_{im}. The set of Bayesian adjusted beta estimates from the first period (Bayesian adjusted historical betas) can then be used as the set of beta predictions for the second period.

5.3.3 Adjusted Multibeta of Historical Beta

Assume that each of the securities in the market surrogate, m, is allocated to one and only one of N portfolios. Let X_t be the proportion of total value of m invested in portfolio t and let the return on portfolio t be designated by P_t. Then

$$R_m = \sum_{t=1}^{N} X_t P_t \tag{2}$$

where R_m is the return on the market portfolio, m.

From the market model it is known that

$$\beta_{im} = \frac{\mathrm{cov}(R_i, R_m)}{\mathrm{var}(R_m)} \quad .$$

Substituting for R_m from (2) gives

$$\beta_{im} = \frac{\mathrm{cov}(R_i, \sum_{t=1}^{N} X_t P_t}{\mathrm{var}(R_m)} \quad .$$

Rearranging gives

$$\beta_{im} = \sum_{t=1}^{N} X_t \frac{\mathrm{cov}(R_i, P_t)}{\mathrm{var}(R_m)} \tag{3}$$

But

$$\mathrm{cov}(R_i, P_t) = \beta_{it} \, \mathrm{var}(P_t) \tag{4}$$

where β_{it} is the beta coefficient of the i^{th} security with the t^{th} portfolio. Substituting (4) into (3) gives

$$\beta_{im} = \sum_{t=1}^{N} X_t \frac{\mathrm{var}(P_t)}{\mathrm{var}(R_m)} \beta_{it} \tag{5}$$

which is the *multibeta* form of β_{im}.

This equation implies that the beta coefficient of any security with the market portfolio is the weighted sum of N individual beta coefficients of the security to the N independent portfolios (β_{it}) which together comprise the market portfolio. The weighting factor of the beta of security i to the t^{th} portfolio is equal to the product of the proportion of the market capitalisation of the t^{th} portfolio to the market capitalisation of the market portfolio, (X_t), and the ratio of the variance of returns of the t^{th} portfolio to the variance of returns of the market portfolio, $\left(\text{that is, } \frac{var(P_t)}{var(R_m)}\right)$.

This approach was first suggested by Sharpe (1974), discussed by Schaeffer *et al* (1975) and has been applied by Carter *et al* (1980) in an initial study on the JSE. Regarding (5), it is clear there are three variables of interest in the right hand side of the equation:

(i) X_t - the proportion of the m portfolio represented by the t^{th} portfolio. This may be thought of as the average market capitalisation of the t^{th} portfolio relative to the average market capitalisation of m over the period used to estimate β_{im}.

(ii) $\frac{var(P_t)}{var(R_m)}$ - the variance relative of the returns of the t^{th} portfolio over the period used to estimate β_{im}.

(iii) β_{it} - the beta coefficient of the i^{th} security to the t^{th} portfolio estimated by the market model

$$R_i = \alpha_{it} + \beta_{it}P_t + e_{it} \qquad (6)$$

over the same period used to estimate β_{im}.

5.4 Multibeta Forms

The Johannesburg Stock Exchange (JSE) Actuaries Index, the composition of which is shown in Appendix A, contains three broad indices which together comprise the JSE All Share Index, the market surrogate. These indices are the All Mining Index (AM), the Mining Financial Index (MF) and the Industrial and Financial Index (IF). For the purposes of this chapter these three indices are used for the multibeta calculations of beta in equation (5), (that is, N = 3). The JSE All Share Index is used as the market surrogate, m.

It is possible, using equation (5), to adjust the historical estimate of beta by employing alternative estimates of the market capitalisation relatives (X_t), the variance relatives $\left(\frac{var(P_t)}{var(R_m)}\right)$, and the beta coefficients to the N individual indices (β_{it}). Some of the alternatives are described below:

(i) Instead of using the average market capitalisation of each t^{th} index to the average market capitalisation of the market index, the market capitalisation relatives for the latest period (say, last month, if monthly data are being used) could be used. In fact, market capitalisations follow a random walk, both empirically and by assumption, so theoretically the latest market capitalisation is the best predictor of the future market capitalisation.

(ii) Instead of using variance relatives estimated over the whole period used to estimate β_{im}, a subset of data from the period, ignoring earlier and perhaps outdated data in this period, could be used to estimate the variance relatives. Here a trade-off exists between using up-to-date data and large samples. Clearly a point estimate of the variance at the end of the period would be desirable. Brealey *et al* (1978) employed an autoregressive process to predict variances. This was tried for the variance of monthly returns for annual data of the AM, MF and IF indices. The regressions obtained, however, were not significant. An alternative would be to estimate the prevailing variance by making this the subject of the Black-Scholes option pricing formula (Black and Scholes (1973)). Unfortunately no such option data exists on the JSE. It was concluded, therefore, that an aribtrary decision would have to be taken on the issue of a subset of data to estimate the variance relatives. Since five years of monthly data were employed in the study to estimate betas, it was decided to use variance relatives estimated over the last two years of the period (that is, twenty four data points) as an alternative to the full period estimates using sixty data points.

(iii) Instead of using the historical estimates of beta, β_{it}, for security i to each of the N portfolios (equation (6)), a Bayesian approach could be used to provide new estimates of these beta coefficients for prediction purposes. The choice of a prior is important here and this issue is discussed later.

Clearly combinations of (i), (ii) and (iii) above could be used in equation (5) for prediction purposes. In fact Table 5.1 below shows the eight different multibeta forms which are used in this chapter.

TABLE 5.1: Different Multibeta Forms Used

Multibeta Number	Weight: Average Period	Weight: Last Month	Beta: Unadjusted	Beta: Bayesian Adjusted	Variance: Average Period	Variance: Last 24 months
1*	✓		✓		✓	
2	✓		✓			✓
3	✓			✓	✓	
4	✓			✓		✓
5		✓	✓		✓	
6		✓	✓			✓
7		✓		✓	✓	
8		✓		✓		✓

(*Multibeta 1 is the simple historical beta obtained using the market model.)

5.5 The Data and Methodology Used

Monthly percentage returns data were available for thirty three shares for the period February 1970 to January 1980. These shares are listed in Appendix D. Monthly percentage returns data were also available for the JSE Actuaries sector indices, shown in Appendix A, for the period February 1965 to January 1980. These data were split into three periods of sixty months each as follows:

Period 1 : February 1965 to January 1970

Period 2 : February 1970 to January 1975

Period 3 : February 1975 to January 1980

Thus data for the share sample existed for Periods 2 and 3 while that for the sector indices existed for all three periods. These periods were not chosen to coincide with any particular market conditions which may have prevailed, but merely represented a convenient method of splitting the data available.

The JSE sector indices are industry specific indices which, in effect, are portfolios of like securities ranging in number from one (Diamonds) to eleven (Engineering). Thus it would be expected that as "securities" in the market model, they would demonstrate some, but not all, of the characteristics of diversified portfolios. It was decided to use the sector indices in this study because they represent a complete coverage of the JSE and hence the results of this chapter should not suffer from any industry bias. However, the purpose of the multibeta approach as analysed here is to improve the ability of an investor in predicting the beta coefficients of individual securities, and so the share sample in Appendix D was also used. This sample, however, does not constitute an exhaustive set of shares representing all industries on the JSE. It is felt, however, that an appreciation of the results of the sector indices with those of the share sample, provides a good understanding of the multibeta approach.

For Period 1, Period 2 and Period 3 the following procedure was adopted for the JSE Actuaries sector indices:

(i) Beta estimates of the thirty four sector indices to the m, AM, MF and IF indices using the market model (equations (1) and (6)) were made.

(ii) For each of the four sets of thirty-four beta coefficients obtained from (i), Bayesian adjusted beta estimates (see Section 5.3.2 above) for Periods 1 and 2 were calculated. The prior used for this adjustment process was the arithmetic mean of sector betas to each index in each period, and the sample variance associated with this mean. It is possible that the estimate errors of the beta coefficients to the AM, MF and IF indices are correlated which would require a simultaneous adjustment of a vector of betas to these indices. It is assumed for the purpose of this chapter that these estimate errors are not correlated and hence the approach adopted was to Bayesian adjust the betas to each index separately.

(iii) The ratios of the average monthly market capitalisations of AM, MF and IF to the average monthly market capitalisation of m were calculated.

(iv) The ratios of the last month's market capitalisations of AM, MF and IF to the last month's market capitalisation of m were calculated.

(v) The ratios of the variances of return of AM, MF and IF to the variance of return of m were calculated.

(vi) As for (v) but using only the last twenty-four months of each Period.

(vii) From Period 1 and Period 2 data the eight different multibetas, detailed in Table 5.1 in Section 5.4, were calculated for each of the thirty-four sector indices.

For the share sample step (i) was performed for Periods 2 and 3 while steps (ii) to (vii) were performed for Period 2. As with the sector indices the prior used was the arithmetic mean beta and its associated variance.

5.6 Results Obtained

5.6.1 Comparison of Historical Beta with Bayesian Adjusted Beta

The beta coefficients for each of the thirty-four sectors to the m, AM, MF and IF indices were regressed as follows:

(i) Period 2 on Period 1

unadjusted on unadjusted

unadjusted on Bayesian adjusted

(ii) Period 3 on Period 2

unadjusted on unadjusted

unadjusted on Bayesian adjusted

The results obtained are shown in Tables 5.2A and 5.2B below, for Periods 2 and 3 respectively.

Several observations can be made from Tables 5.2A and 5.2B:

(i) The MSE's from the regressions of realisations on predictions for Period 2 are generally at a far higher level than those of Period 3. It is well known that early in Period 2 the United States authorities abolished the convertibility of the United States dollar with gold. After this event it is now a matter of history that gold bullion appreciated in price from \$35 per ounce to \$850 per ounce by January 1980, a price rise of just more than twenty-four times in the space of ten years. This price rise had a profound and fundamental effect on the price and volatility of mining shares in South Africa during Periods 2 and 3. In particular, therefore, Period 2 and Period 1 were economically fundamentally different for the majority of mining shares and it is no surprise that the betas of these shares changed markedly over this time. Periods 2 and 3, however, were more similar in economic character as far as the gold price is concerned and the beta coefficients of mining shares during this time

TABLE 5.2A: Beta Coefficient Regression of 34 JSE Sectors
(Percent of Mean Square Error in brackets)

Period 2 realisations on
Period 2 predictions

Index	MSE	Bias	Inefficiency	Random
All Mining				
unadjusted on unadjusted	0,18953	0,00284 (1,5)	0,04662 (24,6)	0,14006 (73,9)
adjusted on unadjusted	0,15967	0,00255 (1,6)	0,02092 (13,1)	0,13620 (85,3)
Mining Financial				
unadjusted on unadjusted	0,11369	0,00125 (1,1)	0,03354 (29,5)	0,07890 (69,4)
adjusted on unadjusted	0,09761	0,00107 (1,1)	0,01445 (14,8)	0,08209 (84,1)
Industrial & Financial				
unadjusted on unadjusted	0,10791	0,01068 (9,9)	0,03399 (31,5)	0,06324 (58,6)
adjusted on unadjusted	0,09052	0,01086 (12,0)	0,01819 (20,1)	0,06146 (67,9)
All Share				
unadjusted on unadjusted	0,24590	0,00000 (0,0)	0,13205 (53,7)	0,11385 (46,3)
adjusted on unadjusted	0,20098	0,00000 (0,0)	0,08783 (43,7)	0,11315 (56,3)

TABLE 5.2B: Beta Coefficient Regression of 34 JSE Sectors
(Percent of Mean Square Error in brackets)

Period 3 realisations on
Period 3 predictions

Index	MSE	Bias	Inefficiency	Random
All Mining				
unadjusted on unadjusted	0,02900	0,00194 (6,7)	0,00745 (25,7)	0,01960 (67,6)
adjusted on unadjusted	0,02366	0,00213 (9,0)	0,00270 (11,4)	0,01883 (79,6)
Mining Financial				
unadjusted on unadjusted	0,02342	0,00398 (17,0)	0,00033 (1,4)	0,01911 (81,6)
adjusted on unadjusted	0,02305	0,00433 (18,8)	0,00014 (0,6)	0,01858 (80,6)
Industrial & Financial				
unadjusted on unadjusted	0,13464	0,02073 (15,4)	0,07069 (52,5)	0,04322 (32,1)
adjusted on unadjusted	0,09712	0,01651 (17,0)	0,03836 (39,5)	0,04225 (43,5)
All Share				
unadjusted on unadjusted	0,04637	0,00005 (0,1)	0,00199 (4,3)	0,04433 (95,6)
adjusted on unadjusted	0,04539	0,00036 (0,8)	0,00005 (0,1)	0,04498 (99,1)

proved more stable.

The effect of a change in conditions such as occurred from Period 1 to Period 2 can be illustrated below. Consider again equation (5)

$$\beta_{im} = \sum_{t=1}^{N} X_t \frac{var(P_t)}{var(R_m)} \beta_{it}$$

By setting $\frac{var(P_t)}{var(R_m)} = V_t$ and using $N = 3$ this can be written as

$$\beta_{im} = X_1V_1\beta_{i1} + X_2V_2\beta_{i2} + X_3V_3\beta_{i3} \tag{7}$$

By construction $X_1 + X_2 + X_3 = 1.$ (8)

Now suppose index 1 decreases in price and hence market capitalisation relative to indices 2 and 3, and hence to the overall market as well. Let the market capitalisation relatives change incrementally as follows:

$$X_1 \rightarrow X_1 - \varepsilon < X_1$$

$$X_2 \rightarrow X_2 + \gamma > X_2$$

$$X_3 \rightarrow X_3 + \delta > X_3$$

where ε, γ and δ are positive numbers between 0 and 1.

By construction $(X_1-\varepsilon) + (X_2+\gamma) + (X_3+\delta) = 1.$

Rearranging gives

$$(X_1+X_2+X_3) - (\varepsilon-\gamma-\delta) = 1$$

Therefore (using (8))

$$\varepsilon = \gamma + \delta \tag{9}$$

This means that the incremental decrease in market capitalisation relative of index 1 is exactly balanced by the increase in market capitalisation relatives of indices 2 and 3.

Under these new conditions the beta of security i to the market, β^*_{im} will be given by

$$\begin{aligned}\beta^*_{im} &= (X_1-\varepsilon)V_1\beta_{i_1}+(X_2+\gamma)V_2\beta_{i_2}+(X_3+\delta)V_3\beta_{i_3}\\ &= X_1V_1\beta_{i_1}+X_2V_2\beta_{i_2}+X_3V_3\beta_{i_3}-\varepsilon V_1\beta_{i_1}+\gamma V_2\beta_{i_2}+\delta V_3\beta_{i_3}\\ &= \beta_{im} - \varepsilon V_1\beta_{i_1} + \gamma V_2\beta_{i_2} + \delta V_3\beta_{i_3} \qquad (10)\end{aligned}$$

Let the difference between β^*_{im} and β_{im} be ρ.

Therefore

$$\rho = -\varepsilon V_1\beta_{i_1} + \gamma V_2\beta_{i_2} + \delta V_3\beta_{i_3} \qquad (11)$$

If $V_1\beta_{i_1} = V_2\beta_{i_2} = V_3\beta_{i_3}$ then using equation (9) the value of ρ is zero. However, it must be remembered that security i belongs exclusively to one only of indices 1, 2 or 3, again by construction. Assume it is index 1. If indices 1, 2 and 3 are orthogonal as was found by Farrell (1975) then

$$E(V_2\beta_{i_2}) = E(V_3\beta_{i_3}) = 0$$

because

$$E(\beta_{i_2}) = E(\beta_{i_3}) = 0 \ .$$

In general, if indices x and y are chosen to represent different groups of securities the expected covariance of returns between index x and index y will be low. In general

$$\mathrm{cov}(P_x, P_y) \approx 0 \qquad x \neq y$$
$$x, y = 1, 2, 3 \quad .$$

In particular, therefore, for security i belonging to index 1

$$E(V_1 \beta_{i_1}) > (E(V_2 \beta_{i_2}) + E(V_3 \beta_{i_3})) \tag{12}$$

Therefore from equation (9) and inequality (12) it can be deduced that $E(\rho) < 0$.

Furthermore following from equation (11)

$$\begin{aligned} E(\beta^*_{im}) &= E(\beta_{im} + \rho) \\ &= E(\beta_{im}) + E(\rho) \\ &< E(\beta_{im}) \ . \end{aligned}$$

In other words the beta of security i to the market surrogate, m, is expected to fall if the index to which security i belongs falls in price, relative to the rest of the market, all other things remaining constant.

Suppose further that V_1, the variance relative of index 1 to the market, also falls. In other words the volatility of index 1 decreases relative to the market. This would imply that the volatility of indices 2 and/or 3 increase(s) relative to the market.

Consider β_{i_1}, β_{i_2} and β_{i_3}:

$$\beta_{i1} = \frac{\text{cov}(R_i, P_1)}{\text{var}(P_1)}$$

$$\beta_{i2} = \frac{\text{cov}(R_i, P_2)}{\text{var}(P_2)}$$

$$\beta_{i3} = \frac{\text{cov}(R_i, P_3)}{\text{var}(P_3)}$$

Under the conditions of V_1 falling it is apparent that $\frac{\text{var}(P_2)}{\text{var}(P_1)}$ and/or $\frac{\text{var}(P_3)}{\text{var}(P_1)}$ have to rise.

In other words, if it is assumed that the covariances of the returns of security i to each of the returns of indices 1, 2 and 3 remain constant when V_1 falls, then β_{i2} and/or β_{i3} are expected to fall *relative* to β_{i1}.

Thus in inequality (12), V_2 and V_3 are expected to rise when compared to V_1 while β_{i2} and β_{i3} are expected to fall when compared to β_{i1}. Therefore it is unclear theoretically in what way inequality (12) will behave under these conditions.

The situation of falling market capitalisation relative and falling variance relative is exactly the situation that occurred with the IF index when Period 2 is compared to Period 1. Clearly, if the conditions of the gold price referred to above caused the mining shares to become more volatile relative to the market and increase in relative price and hence relative market capitalisation to the market, the opposite must have occurred to the non-mining shares which together constitute the IF index. This is illustrated in Table 5.3, which shows the arithmetic average beta coefficients of the twenty three industrial and financial sectors listed in Appendix A (the last twenty three sectors) to each of AM, MF, IF and m indices for Periods 1 and 2. The fall in average industrial and financial sector beta to the JSE All Share Index from 0,8447 to 0,6265 reflects the increase in beta of the remaining eleven mining sectors due to the economic conditions described above.

(ii) Tables 5.2A and 5.2B also show that with the exception of the IF regressions of Period 3 realisations on predictions, the MSE of each of the regressions using the AM, MF and IF indices is lower than the corresponding regression using the m index. This means that the sector betas to the AM, MF and IF indices are more stable over time than the betas to the m index. This result increases the validity of using the multi beta approach where sector betas to the AM, MF and IF indices are required.

(iii) The Bayesian adjustment process increases the predictive ability of the historical betas in each of the eight cases under consideration because the MSE's are reduced when the Bayesian adjustments are made. The relative efficiency of two estimators θ' and θ'', whether

TABLE 5.3: Change in Industrial and Financial Sector Betas

		Period 1	Period 2	% change
Average betas (β) to	AM	0,5541	0,3327	-40,0
	MF	0,4748	0,3085	-35,0
	IF	0,8952	0,9442	+ 5,5
	m	0,8447	0,6265	-25,8
Market capitalisation relatives (X)	AM	0,40859	0,44087	
	MF	0,20861	0,18865	
	IF	0,38280	0,37048	
Variance relatives (V)	AM	1,1135	1,3613	
	MF	1,5688	2,0381	
	IF	1,2945	0,8739	
Weighting factors (XV)	AM	0,4550	0,6002	+31,9
	MF	0,3273	0,3845	+17,5
	IF	0,4955	0,3238	-34,7
Multibeta component (βXV)	AM	0,2521	0,1997	-20,8
	MF	0,1554	0,1186	-23,7
	IF	0,4436	0,3057	-31,1
Average multibeta		0,8511	0,6240	-26,7
Percent contributed by component from	AM	29,6	32,0	
	MF	18,3	19,0	
	IF	52,1	49,0	
Total Percent		100,0	100,0	

biased or unbiased, can be defined as:

$$\text{relative efficiency of } \theta' \text{ compared to } \theta'' = \frac{MSE(\theta'')}{MSE(\theta')}$$

Using this definition the effect of the Bayesian adjustment process can be seen in Table 5.4 below:

TABLE 5.4: Relative Efficiency of Adjusted Betas Compared to Unadjusted Betas for JSE Sectors

Regressor Index	Period 2 Realisations on Predictions	Period 3 Realisations on Predictions
All Mining	1,187	1,226
Mining Financial	1,165	1,016
Industrial and Financial	1,192	1,386
All Share	1,224	1,022

In the Period 2 regressions, the adjustment process makes a consistent improvement in each case, whereas in the Period 3 regressions the effect is pronounced for the AM and IF indices, but relatively insignificant for the MF and m indices. From Tables 5.2A, 5.2B and 5.4 it is apparent that the adjustment process produces the greatest gain in efficiency from the highest MSE levels. This perhaps indicates that the Bayesian adjustment process is particularly useful in adjusting a set of betas which for some reason or other are subject to considerable change over time.

(iv) In every case the Bayesian adjustment process either increases or keeps constant the percentage that bias contributes to total MSE, decreases the percentage that inefficiency contributes to total MSE, and increases the percentage that random disturbance contributes to MSE. Therefore it can be concluded that the prediction error of a set of Bayesian adjusted beta coefficients is more genuinely a function of truly changing beta coefficients than the prediction error of a set of unadjusted historical beta coefficients. The inefficiency of the estimation procedure is therefore reduced by use of the Bayesian adjustment process.

5.6.2 Comparison of Multibeta Forms and Adjusted Historical Beta

In Table 5.1 of Section 5.4 above, eight different multibeta forms were defined. Using these multibeta forms for Period 1 and Period 2, beta predictions for Period 2 and Period 3, respectively, were obtained for the JSE sectors, and the MSE of realisations versus predictions calculated as before. The results are tabulated in Table 5.5 below for the JSE Sectors.

The results for the Period 3 realisations on predictions for the sample of thirty three shares are tabulated in Table 5.6 below.

TABLE 5.5: Comparison of MSE of Regressions of Realisations on Predictions for JSE Sectors

Regression	MSE	Rank lowest to highest	MSE Relative to Lowest MSE	Bias (%)	Ineffi-ciency (%)	Random (%)
Period 2 realisations on Period 2 predictions						
Multibeta 1	0,24590	6	1,275	0,0	53,7	46,3
Multibeta 2	0,23261	5	1,206	0,5	50,5	49,0
Multibeta 3	0,20139	2	1,044	0,0	43,5	56,5
Multibeta 4	0,19284	1	1,000	0,6	40,4	59,0
Multibeta 5	0,27420	8	1,422	0,3	57,8	41,9
Multibeta 6	0,25668	7	1,331	0,1	55,2	44,7
Multibeta 7	0,22679	4	1,176	0,3	49,0	50,7
Multibeta 8	0,21512	3	1,116	0,1	46,5	53,4
Period 3 realisations on Period 3 predictions						
Multibeta 1	0,04637	7	1,290	0,1	4,3	95,6
Multibeta 2	0,04164	4	1,159	3,7	14,4	81,9
Multibeta 3	0,04182	5	1,164	0,4	0,0	99,6
Multibeta 4	0,03594	1	1,000	4,2	5,4	90,4
Multibeta 5	0,04204	6	1,170	0,3	11,3	88,4
Multibeta 6	0,04910	8	1,366	0,0	36,2	63,8
Multibeta 7	0,03643	2	1,014	0,3	3,7	96,0
Multibeta 8	0,03913	3	1,089	0,0	23,0	77,0

Table 5.6: Comparison of MSE of Regressions of Period 3 Realisations on Predictions for Share Sample

Multibeta Number	MSE	Rank: lowest to highest	MSE Relative to Lowest MSE	Bias (%)	Inefficiency (%)	Random (%)
Multibeta 1	0,10652	8	1,348	7,0	6,9	86,1
Multibeta 2	0,09762	4	1,235	10,5	14,7	74,8
Multibeta 3	0,09799	6	1,240	7,6	1,8	90,6
Multibeta 4	0,08257	2	1,045	10,8	6,4	82,8
Multibeta 5	0,09785	5	1,238	6,1	12,8	81,1
Multibeta 6	0,10011	7	1,267	1,7	31,9	66,4
Multibeta 7	0,08433	3	1,067	6,7	4,8	88,5
Multibeta 8	0,07902	1	1,000	1,6	19,7	78,8

Table 5.5 shows that Multibeta 4 produces the lowest MSE in both periods. This multibeta uses market capitalisation relatives over the whole period, Bayesian adjusted betas and variance relatives calculated over the final twenty four months of the period. Multibeta 8, which uses end period market capitalisation relatives, Bayesian adjusted betas and variance relatives calculated over the final twenty four months of the period, has the third lowest MSE in both periods. Notably, Multibeta 1, which is in fact the ordinary historical beta is among the poorest predictors (highest MSE) in both periods. The Bayesian adjustment is used in Multibetas 3,4,7 and 8 and these multibetas occupy four of the five lowest MSE positions in the second period, including the lowest three. Therefore it would

seem that the Bayesian adjustment is an important feature in reducing MSE. This will be tested statistically below. In Table 5.6, for the share sample Multibeta 8 has the lowest MSE followed by Multibeta 4 with a 4,5% higher MSE. The fact that Multibeta 8 produced the lowest MSE invites some interesting conclusions:

(i) The theoretically preferable market capitalisation relative, namely, that recorded in the last month of the data, is selected. This provides some empirical support for the assertion that since market capitalisations are random by assumption and empirically, the last month's figure should be preferred to the average over the period.

(ii) The Bayesian adjustment provides superior beta predictions to historical beta estimates.

(iii) A subset of data is better than the whole period for variance relative calculation. This would seem to indicate that the variance of returns probably changes sufficiently over time to render earlier data in the sixty month data set obsolete.

(iv) Points (i) to (iii) taken together suggest that estimating betas of individual securities over a sixty month historical period is probably unwise because the true beta of the security changes over this time. This assertion is tested more formally in Chapter 6 and acted upon in Chapter 7, where a shorter period of weekly data is used.

Since Multibetas 4 and 8 had low MSE rankings in Tables 5.5 and 5.6 it was decided to compare these multibetas directly with the ordinary historical beta and Bayesian adjusted historical beta and this is done in Table 5.7. Two versions of Bayesian adjusted betas are used for the share sample. The first ((mean = 0,8777) in Table 5.7) uses the arithmetic mean beta of the thirty three shares in Period 2 as prior. This mean is 0,8777. It could be argued, however, that a more correct choice of prior would be a beta of unity since in the absence of any knowledge concerning a security beta, an estimate of beta equals unity is the best decision that can be made. This Bayesian adjusted beta is shown in Table 5.7 as mean = 1,0000.

Some observations from Table 5.7 are:

(i) The MSE's of the share sample are higher than those of JSE sectors in the Period 3 realisations on predictions regressions. This indicates that the beta coefficients of the sectors are more stable than those of the individual securities. This is not surprising since the sectors are expected to have some of the characteristics of diversified portfolios (one of which is relatively stable betas) as discussed in Section 5.5.

(ii) In Period 3 both Multibetas 4 and 8 are clearly superior to the alternative betas for both sectors and shares. This however is not the case in Period 2 where the adjusted historical beta is superior to Multibeta 8.

TABLE 5.7: Comparison of MSE of Best Multibetas, Historical and Adjusted Historical Beta

Regression	MSE	MSE compared to lowest MSE	Bias (%)	Inefficiency (%)	Random (%)
Period 2 realisations on Period 2 predictions					
JSE Sectors					
Multibeta 4	0,19284	1,000	0,6	40,4	59,0
Multibeta 8	0,21512	1,116	0,1	46,5	53,4
Historical beta	0,24590	1,275	0,0	53,7	46,3
Adjusted historical beta	0,20098	1,042	0,0	43,7	56,3
Period 3 realisations on Period 3 predictions					
JSE Sectors					
Multibeta 4	0,03594	1,000	4,2	5,4	90,4
Multibeta 8	0,03913	1,089	0,0	23,0	77,0
Historical beta	0,04637	1,290	0,1	4,3	95,6
Adjusted historical beta	0,04539	1,263	0,8	0,1	99,1
Share Sample					
Multibeta 4	0,08257	1,045	10,8	6,4	82,8
Multibeta 8	0,07902	1,000	1,6	19,7	78,8
Historical beta	0,10652	1,348	7,0	6,9	86,1
Adjusted historical beta (mean = 0,8777)	0,10223	1,294	8,3	0,1	96,1
Adjusted historical beta (mean = 1,0000)	0,09369	1,186	5,4	0,1	94,1

(iii) In each case the ordinary historical beta has the highest MSE of all the betas considered, indicating that this beta method is particularly inefficient for beta prediction.

(iv) In the absence of any information about the AM, MF and IF indices, an investor would be advised to Bayesian adjust a set of betas estimated from the single index market model using the JSE All Share as market surrogate, since the Bayesian adjusted historical beta produces a lower MSE in each case considered. In the first period it is 18,3% lower than the historical beta for sectors only, and in the second period it is 2,1% and 12,0% (mean = 1,0000) lower for sectors and shares respectively than the historical beta. This confirms for the JSE the results found on the New York stock exchange by Klemkosky and Martin (1975).

(v) The use of the prior (mean = 1,0000) for the shares in the last period is 8,4% more efficient in terms of MSE than using the prior (mean = 0,8777). This provides support for the theoretical assertion that the best prior beta for a share is one.

5.7 Analysis of Variance of Multibeta Factors

The data in Table 5.5 can be conveniently analysed using a standard four way, completely crossed, analysis of variance, to see which factors have the most effect on the MSE. The four factors, each with two levels, are shown in Table 5.8 below.

TABLE 5.8: Factors and Levels in Analysis of Variance

Factor	Factor Number	Level 1	Level 2
WEIGHT	1	whole period average	last month
BETA	2	unadjusted	Bayesian adjusted
VARIANCE	3	whole period	last twenty-four months
PERIOD	4	Period 2	Period 3

The results are shown in Table 5.9.

TABLE 5.9: Completely Crossed Four Way ANOVA

Interaction (Factor No.)	F Statistic	Tail Probability
1	546,39	0,0272*
2	2123,67	0,0138*
3	144,22	0,0529
4	122671,80	0,0018*
1/2	5,38	0,2591
1/3	9,08	0,2040
2/3	1,38	0,4486
1/4	526,45	0,0277*
2/4	1164,52	0,0187*
3/4	134,93	0,0547
1/2/3	0,24	0,7119
1/2/4	0,02	0,9064
1/3/4	41,17	0,0984
2/3/4	13,89	0,1669

*significant at 5% level of confidence

Table 5.9 shows that the crossed interactions between PERIOD and WEIGHT, and PERIOD and BETA, are significant at 5% level of confidence. No other crossed interactions were significant. The presence of these interactions implies that it is difficult to draw conclusions about the main effects. It appears, however, that three main effects may be significant at the 5% level of confidence and they would be in descending order of significance : PERIOD, BETA and WEIGHT. For reasons described above in Section 5.6.1, it is not surprising that PERIOD appears to be the most significant factor since the beta coefficients changed quite radically from Period 1 to Period 2 but were relatively stable from Period 2 to Period 3.

Because of the presence of the interactions at a significant level and because PERIOD is clearly a significant factor, it was decided to perform a three way, completely crossed, analysis of variance for the remaining three factors separately for Period 2 and Period 3. The results are displayed in Tables 5.10 and 5.11 respectively.

TABLE 5.10: Completely Crossed Three Way ANOVA for Period 2

Interaction (Factor No.)	F Statistic	Tail Probability
1	8124,33	0,0071*
2	24361,28	0,0041*
3	2113,51	0,0138*
1/2	17,85	0,1479
1/3	43,85	0,0954
2/3	91,02	0,0665

*significant at 5% level of confidence

TABLE 5.11: Completely Crossed Three Way ANOVA for Period 3

Interaction (Factor No.)	F Statistic	Tail Probability
1	0,08	0,8205
2	64,75	0,0787
3	0,07	0,8352
1/2	2,76	0,3451
1/3	40,27	0,0995
2/3	2,95	0,3358

It can be seen from Tables 5.10 and 5.11 that no crossed interactions are significant at the 5% level of confidence, implying that the main effects can be tested. In Period 2 (Table 5.10) each of the main effects is significant at the 5% level of confidence, in the following descending order of significance : BETA, WEIGHT and VARIANCE. In Period 3 (Table 5.11), none of main effects is significant at the 5% level of confidence, although the factor BETA is significant at the 10% level of confidence. The factors WEIGHT and VARIANCE are far from significant.

For the share sample it is possible to perform a standard three way, completely crossed, analysis of variance, to see which factors have the most effect on the MSE. The three factors, each with two levels, are the same as those shown in Table 5.8 excluding the factor, PERIOD. The results are shown in Table 5.12.

TABLE 5.12: Completely Crossed Three Way ANOVA for Share Sample for Period 3

Interaction (Factor No.)	F Statistic	Tail Probability
1	496,12	0,0286*
2	3071,26	0,0115*
3	679,47	0,0244*
1/2	110,35	0,0604
1/3	410,35	0,0314*
2/3	180,07	0,0474*

*significant at 5% level of confidence

Table 5.12 shows that the crossed interactions between WEIGHT and VARIANCE, and BETA and VARIANCE, are significant The presence of these interactions makes it difficult to draw conclusions about the main effects. It appears, however, that all three main effects may be significant at the 5% level of confidence and they are in descending order of significance : BETA, VARIANCE and WEIGHT.

5.8 Conclusions

It is apparent from the results presented above that an investor who wishes to obtain a good estimate of beta co-efficients or who wishes to attempt to predict beta coefficients will be helped by adopting a multibeta approach. Some of the conclusions are:

(i) A multibeta calculated for the AM, MF, and IF indices with Bayesian adjusted beta coefficients, average weights and last twenty four months' variance reduced the MSE of beta predictions versus realisations for the JSE sectors by 21,6% in Period 2 and 22,5% in Period 3 from the MSE obtained by using historical betas to the JSE All Share Index to predict next period's betas to the same index.

(ii) For a sample of thirty three shares, beta predictions obtained from Period 2 using a multibeta calculated from the AM, MF and IF indices with Bayesian adjusted beta coefficients, end period weights and last twenty four months' variance reduced the MSE of beta predictions versus realisations (in Period 3) by 25,8% from the MSE obtained by using historical betas as predictors.

(iii) Analysis of variance showed that the Bayesian adjustment in the multibeta seemed to be the vital factor in reducing the MSE. The use of end period weights and last 24 months' variance did not prove consistently desirable for the sectors but was desirable for the shares. This suggests that share betas are so unstable that data covering sixty months of history may be too long and may therefore include out of date information for the estimation of the prevailing beta. The sector results are not surprising in view of the fact that portfolios are known to have more stable beta coefficients. Random portfolios would be expected to display this characteristic even more.

(iv) In the sector analysis the Bayesian adjustment process proved effective whether the MSE was low (as in Period 3) or high (as in Period 2). This suggests that the adjustment process is robust and can be applied equally effectively whether betas change much or little over time.

The selection of end period weights, last twenty four months' variance and Bayesian adjusted betas in the best performing multibeta for the share sample suggests that it is necessary to investigate techniques which can distinguish between return data which is still relevant in the estimation of the current security beta and that which is not. In Chapter Six an approach to this problem is developed.

CHAPTER SIX

THE VALID DATA SET FOR THE ESTIMATION OF THE CURRENT BETA COEFFICIENT

6.1 Introduction

The use of the market model is usually based on the assumption that the regression relationship is constant over time. It has been clearly established by several researchers, however, that estimates of beta coefficients of securities are not stationary through time. Among studies which have addressed this issue are Blume (1975), Brenner (1974), Chen (1981), Chen and Lee (1980), Pettit and Westerfield (1974), Levy (1971), Sunder (1980) and Fabozzi and Francis (1979A). The reasons why beta coefficients change over time is not addressed in this chapter although some important work has been done on this issue by Hamada (1972), Hill and Stone (1980), Fabozzi and Francis (1979B), Rosenberg and Guy (1976), Rosenberg and Marathe (1976), Beaver, Kettler and Scholes (1970), Bowman (1979), Beaver and Dukes (1972), and Beaver and Manegold (1975). Such an investigation for the Johannesburg Stock Exchange (JSE) would be worthwhile.

The objective of this chapter is to develop an approach which can assist an investor in establishing that set of historical data which is still valid for the estimate of the current beta coefficient of a security. Alexander and Chervany (1980) have pointed out that if β_{1i} and β_{2i} are the beta coefficients of security i in the first and second halves respectively of a period, and $\beta_{1i} \neq \beta_{2i}$, then if $\hat{\beta}_i$ is the estimate of the true beta over the whole period, it follows that:

$$E(\hat{\beta}_i) \neq \beta_{1i} \neq \beta_{2i}$$

In other words $\hat{\beta}_i$ is no longer an unbiased estimate of either β_{1i} or β_{2i}. Therefore while increasing the size of the estimation period will result in less sampling error, it potentially increases the bias of the parameter estimates due to possible structural changes. Chen (1981) has noted that ordinary least squares as a regression method cannot isolate the effect of the variability of the beta coefficient from the estimated residual risk. He asserts that approximately half of what is considered residual risk is in fact due to the variability of the beta coefficient. One implication of this is that naive portfolio diversification will result in a far greater reduction of *pure* residual risk than previously thought.

Various researchers have addressed the question of the optimal estimation interval for security beta coefficients. Gonedes (1973) concluded that seven years of monthly data was optimal while Baesal (1974) found nine years to be appropriate. Eubank and Zumwalt (1977) have suggested a four to six year period of monthly data. While the above studies were all conducted on the New York Stock Exchange, Theobald (1981), working on the United Kingdom market, concluded that a data set of ten to fifteen years was optimal although advantages to be gained by using more than ten years were marginal. He did warn, however, that the application of this average optimal data length indiscriminantly to all securities within a particular sample will lead to considerable problems in cases where stocks have been subject to large structural changes in beta. In some earlier work (Theobald (1980)), he concluded that where betas have changed by 50% or more, it is better, in terms of the mean square error of the resultant beta estimates, to use reduced data sets down to a minimum of fifteen monthly observations rather than to use a data set of sixty mixed observations.

There are three problems posed when the question of beta stationarity over a set of data is considered. They are:

(i) how to test if a change has taken place

(ii) if a change has taken place, how many regimes are actually represented by the data set

(iii) where are the change points, if any.

A number of studies have addressed the issue of the instability of the regression parameters in a simple linear regression model. Quandt (1958) obtained a likelihood ratio estimate of the location of an unknown change point for a simple linear regression model obeying two regimes. Collins and Simonds (1979) used this approach to locate the change point under the assumption that a change in beta had occurred. Quandt (1960) then suggested a test of the null hypothesis that no change has occurred against the alternative that a single change occurred at an unknown location. Chow (1960) developed a method which compares the weighted squared residuals from two regressions (pre- and post- change point) with the squared residuals from the pooled regression to test whether a statistically significant change has occurred at the hypothesized change point. Farley and Hinich (1970) assumed that all possible change points were equally likely over the data set and derived a likelihood ratio test for the null hypothesis that no change took place. McGee and Carleton (1970) recommended hierarchical clustering of adjacent observations successively included in successively recomputed regressions, calling the technique piecewise regression. Some others to have made contributions to the problem are Farley and Hinich (1975), Quandt (1972), Goldfield and Quandt (1972), Garbade (1977), Brown, Durbin and Evans (1975), Ertel and Fowlkes (1976), and Mehta and Beranek (1982).

The emphasis of this chapter is not directed to a comparison of tests available for identifying change points in a regression model although such a study for the Johannesburg Stock Exchange (JSE) would be of value. Rather the emphasis is the development of an iterative procedure which utilises results due to Quandt (1958), Brown, Durbin and Evans (1975), Chow (1960) and Mehta and Beranek (1982) to arrive at that most recent subset of historical data which is valid in the estimate of the current beta coefficient of a security. The outdated data belonging to other regimes is excised from the data set by first determining if beta has changed over a specified historical set of data which is temporally ordered. If it is decided that the beta has changed, the point of change is established and all data before the change point are excluded. The procedure is then repeated until the null hypothesis that there has been no change in the regression model parameters cannot be rejected. The remaining data set is then deemed to comprise the valid set for the estimate of the current beta coefficient of the security. The need for a procedure such as this for the JSE is apparent from the results of Chapters Four and Five of this thesis, in which the non-stationarity of security beta coefficients was noted.

Section 6.2 briefly describes the approaches of Quandt (1958), Brown, Durbin and Evans (1975) (hereafter BDE), Chow (1960) and Mehta and Beranek (1982) (hereafter MB) and describes the methodology used in this study. Section 6.3 describes the data utilised in the study and the results are discussed in Sections 6.4 to 6.6.

6.2 Methodology

6.2.1 The Approach of Quandt

The market model can be specified as

$$\tilde{Y}_t = \alpha_t + \beta_t X_t + \tilde{e}_t \qquad t = 1,2,\ldots,T$$

where $\tilde{Y}_t$ is the return on the security in time period t,

X_t is the return on the market in time period t,

α_t and β_t are parameters unique to the security

$\tilde{e}_t$ is a sequence of independent, random variables obeying the usual conditions for error terms in ordinary least squares regression, namely:

$E(\tilde{e}_t) = 0$

$\text{cov}(\tilde{e}_i, \tilde{e}_j) = 0 \quad i \neq j \quad i,j = 1,2,\ldots,T$

$V(\tilde{e}_t) = \sigma_t^2$ for all t.

If it is believed that the regression relationship has changed once from a relationship specified by β_{1t}, σ_{1t}^2 to another relationship specified by β_{2t}, σ_{2t}^2 at time $t = T_1$, $T_1 = 2,\ldots,T-2$, it is possible to compute the log-likelihood ratio at T_1, $\lambda(T_1)$

$$\lambda(T_1) = \tfrac{1}{2}T_1 \log \hat{\sigma}_{1t}^2 + \tfrac{1}{2}(T-T_1) \log \hat{\sigma}_{2t}^2 - \tfrac{1}{2}T \log \hat{\sigma}_t^2$$

where $\hat{\sigma}_{1t}^2$, $\hat{\sigma}_{2t}^2$ and $\hat{\sigma}_t^2$ are the ratios of the residual sums of squares to the number of observations when the regression is fitted to the first T_1 observations, the remaining $T-T_1$ observations and the whole set of T observations, respectively, and logarithms are computed to the base 10.

The estimate of the point at which the change in the regression relationship has occurred is the value of T_1 when $\lambda(T_1)$ attains its minimum. However, no formal statistical test has been derived for $\min \lambda(T_1)$ since its distribution under the null hypothesis that there has been no change in the regression parameters, is unknown. Therefore the Quandt method can provide evidence of where a single change has taken place but does not assist in deciding if such a change has occurred.

6.2.2 Bayesian Switching Regression of MB

Consider the multiple regression model

$$\tilde{Y}_t = X_t'\beta_t + \tilde{e}_t \quad , \qquad t = 1,2,\ldots,T$$

where $X_t \equiv [1, X_{2t}, \ldots, X_{kt}]'$ is a $k\times 1$ vector of observations on the independent variables at time t

$\beta_t \equiv [\beta_{1t}, \beta_{2t}, \ldots, \beta_{kt}]'$ is a $k\times 1$ vector of regression coefficients at time t

$\tilde{e}_t$ are assumed to be independent normal random variables with $E(\tilde{e}_t) = 0$ and $V(\tilde{e}_t) = \sigma_t^2$ for all t.

In addition the data are assumed to be homoscedastic and display no autocorrelation.

The regression parameters, β_t and σ_t^2, vary with t in the following way. A subset $\{T_1, T_2, \ldots, T_r\}$ of the set of time points $\{1,2,\ldots,T\}$ is designated as the set of change points with the property that

$$0 = T_0 < T_1 < T_2, \ldots, < T_r < T_{r+1} = T$$

and for $\alpha = 1,2,\ldots,r+1$

$$\beta_t = \beta_\alpha,\ \sigma_t^2 = (h_\alpha)^{-1}, \quad \text{if} \quad t \in \{T_{\alpha-1} + 1,\ T_{\alpha-1} + 2, \ldots, T_\alpha\}$$

There are therefore $r+1$ different regression regimes which are separated by the change points $T_1, T_2, \ldots, T_r$.

MB follow a strategy of assigning a prior probability distribution to the change points $\tilde{T}_1, \tilde{T}_2, \ldots, \tilde{T}_r$, and a conditional prior probability distribution to the regression parameters $\{\beta_\alpha, h_\alpha, \alpha = 1,2,\ldots,r+1\}$ given the $\tilde{T}_1, \tilde{T}_2, \ldots, \tilde{T}_r$. The prior probability distributions are then updated using Bayes' Theorem and the observed data. From the joint posterior distribution which is thus obtained, the parameters β_α, h_α, $\alpha = 1,2,\ldots,r+1$ are integrated out, leaving the posterior probability distribution of the r change points, $\tilde{T}_1, \tilde{T}_2, \ldots, \tilde{T}_r$. Having defined the set θ to contain the complete parametric specification for the r change points, MB derive the integrated likelihood function (IL), $\ell(\theta|X,Y)$

$$\ell(\theta|X,Y) = \prod_{\alpha=1}^{r+1} C(n_\alpha)(S_\alpha)^{-(n_\alpha-1)/2} |V_\alpha|^{-\frac{1}{2}}$$

where

$$C(n_\alpha) = 2\pi^{-(n_\alpha-k-2)/2} \Gamma(\tfrac{1}{2}(n_\alpha-k))(n_\alpha-k)^{(k-1)/2}$$

$$S_\alpha = [Y_\alpha - X_\alpha'\hat{\beta}_\alpha]'[Y_\alpha - X_\alpha'\hat{\beta}_\alpha]$$

$$V_\alpha = X_\alpha' X_\alpha$$

and where

$$n_\alpha = T_\alpha - T_{\alpha-1}$$

$$Y_\alpha = [Y_{T_{\alpha-1}} + 1,\ Y_{T_{\alpha-1}} + 2, \ldots, Y_{T_\alpha}]$$

$$X_\alpha = [X_{T_{\alpha-1}} + 1,\ X_{T_{\alpha-1}} + 2, \ldots, X_{T_\alpha}]$$

The IL is then maximised on a data set to give estimates of the location of the change points given their total number, r. In this chapter each IL is shown in its natural logarithmic form as an integrated log likelihood, ILL. It remains to determine the value of r. Unfortunately taking r as the value where the ILL attains its maximum is not valid. MB provide a Bayesian refutation for this approach but the intuitive argument used by them will suffice here (MB, p.252, 253).

> "*Intuitively r represents the dimensionality of the model to which we are fitting the data. As we increase r, the same data are explained by models of greater flexibility; i.e. as r increases, we improve the fit of the data to the model. In the limiting case, if we had a change in the regression regimen after every two observations, we would have a model which fit the data perfectly since each pair of observations would define a unique straight line. Clearly then, even if the data were actually generated by a model of low dimension, a model of higher dimension would fit the same data better. For this reason, the values of the integrated likelihood function at different values of r are not comparable.*"

MB overcome this problem by using a result due to Schwarz (1978), who addressed the issue of the dimensionality of a model. He used an asymptotically optimal rule for adjusting the log likelihood values at different values of r so that they can be compared. This adjustment chooses the value of r which maximises

$$\log(L_r) - \tfrac{1}{2}K_r \log(T)$$

where $$L_r = \prod_{\alpha=1}^{r} (h_\alpha)^{n_\alpha/2} (2\pi)^{-n_\alpha/2} (e)^{-n_\alpha/2} (S_\alpha)^{-n_\alpha/2}$$

and K_r is the additional number of continuous parameters which are introduced because of r regimes.

Although not the prime interest of this chapter, this approach is used to determine the number and location of regression switches in the data of De Beers and the JSE All Share Index over the sixty month period February 1975 to January 1980, in Section 6.6. However, the approach of MB cannot successfully distinguish between $r = 0$ and $r = 1$ which for the iterative procedure of this study is necessary. Appendix E contains a proof of this fact. Therefore the ILL is used in this study to identify the location of the change point if $r = 1$, and is therefore a check on the location of the change point suggested by Quandt's log likelihood ratio described in Section 6.2.1. Other methods have to be found to test the null hypothesis that there has been no change in the regression parameters. These are developed below.

6.2.3 Cusum of Squared Recursive Residuals Test

BDE define a set of recursive residuals as follows, using the market model as specified in Section 6.2.1.

$$w_t = \frac{Y_t - \hat{\alpha}_{t-1} - \hat{\beta}_{t-1}X_t}{\sqrt{\left[1 + \frac{1}{t-1} + \frac{(x_t-\bar{x}_{t-1})^2}{\sum_{i=1}^{t-1}(x_i-\bar{x}_{t-1})^2}\right]}} \qquad t = 3,\ldots,T$$

where $\hat{\alpha}_{t-1}$, $\hat{\beta}_{t-1}$ and $\bar{x}_{t-1}$ are, respectively, the least squares estimate of α, the least squares estimate of β, and the sample mean of market returns, based on the first $t-1$ observations.

If the null hypothesis

$$H_0 : \beta_1 = \beta_2 = \ldots = \beta_T = \beta$$
$$\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_T^2 = \sigma^2$$

is true, the recursive residuals are independent, $N(0,\sigma^2)$. If β_t is constant up to time $t = T_1$, and differs from the constant value thereafter, the recursive residuals, w_t, will have zero means for t up to T_1, but in general will have non-zero means thereafter. BDE suggest two quantities which may be of value in testing the null hypothesis. The first they call the cusum quantity

$$W_t = \frac{1}{\hat{\sigma}} \sum_{j=3}^{t} w_j \qquad t = 3,\ldots,T$$

where $\hat{\sigma}$ is the estimated residual standard deviation for the whole period.

The cusum technique was pioneered by Page (1954) and developed by Woodward and Goldsmith (1964). It has been applied before, but in a different context, to the analysis of stock market data by Carter (1976). The technique is particularly useful for detecting small but systematic changes which over time can accumulate to a large effect.

Although BDE develop a test of H_0 using W_t and results from Brownian motion, Garbade (1977) has shown the power of the test to be quite weak and so it is not considered further in this study. However the second quantity, called by BDE, the cusum of squared recursive residuals, was found to be much more satisfactory by Garbade (1977) as a test of H_0. This quantity, S_t, is defined as

$$S_t = \sum_{j=3}^{t} w_j^2 / \sum_{j=3}^{T} w_j^2$$

and is plotted against t for $t = 3,\ldots,T$.

For ordinary least squares (OLS) regression, BDE suggest drawing a pair of lines

$$m_t = \pm c_0 + (t-2)/(T-2) \qquad t = 3,\ldots,T$$

parallel to the mean value line such that the probability that the sample path, S_t, crosses one or both lines is the required significance level. Durbin (1969) has given values of c_0 which may be used in this procedure.

It is clear that for a data set ordered temporally, the recursive residuals as defined by BDE are not in general likely to be the same for a forward and backward pass through the data. In particular consider the example where the first n of T points defines an OLS line represented schematically in Figure 6.1.

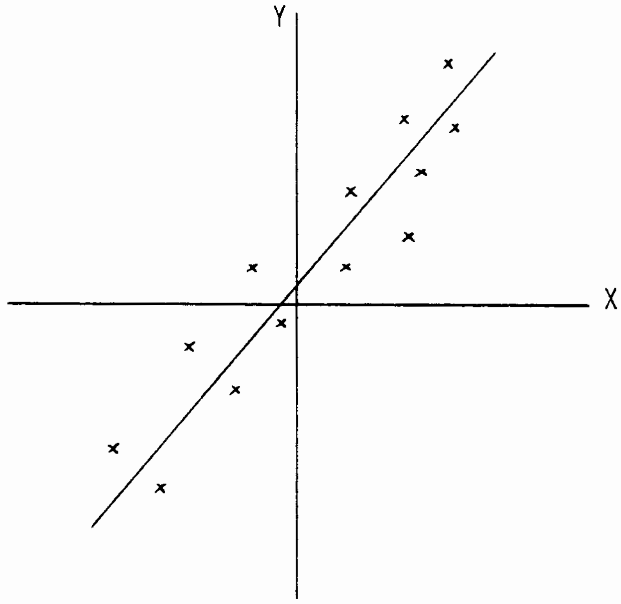


FIGURE 6.1

The crosses in Figure 6.1 represent some of these n points. Now consider the final $T-n$ points, some of which are plotted schematically in Figure 6.2 as dots.

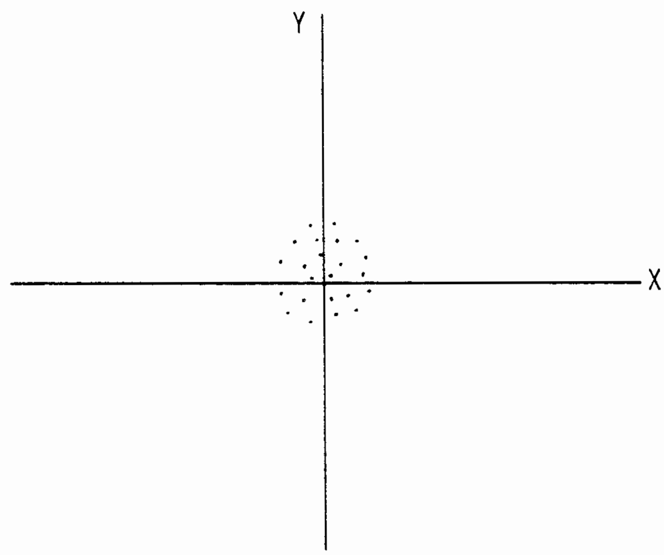


FIGURE 6.2

These $T-n$ points actually define no particular regression line which is significant. If these two sets of points are combined into the T points in the order $1,\ldots,n, n+1,\ldots,T$ and a regression line is estimated, the estimated line is not likely to differ much from that of Figure 6.1 and hence the cusum of squared forward recursive residuals is not likely to reject the null hypothesis. If, however, the data order is reversed, $T, T-1,\ldots,n+1, n,\ldots,1$ and a forward recursion is computed (the equivalent of a backward recursion on the original data), the cusum of squared recursive residuals may well produce a rejection of H_0 as the slope coefficient changes. This particular phenomenon occurred in the data analysed in this study.

It was decided, therefore, to allow a rejection of the null hypothesis from either a forward or backward recursion to be sufficient to require a deletion of data from the set being used to estimate the current beta coefficient of a security.

It remains to determine the confidence level for the test of the null hypothesis. BDE test the null hypothesis at the 99% and 90% level respectively in the two examples contained in their paper. It was felt that the choice of level of confidence should take into account the following factors:

(i) Failure to detect a true change in the regression parameters must be minimised since this will produce a biased estimate of the prevailing beta. Hence a strong test of the null hypothesis must be avoided.

(ii) Incorrect rejection of the null hypothesis is not that serious since the true beta would not have changed in these cases. Therefore the estimate of this beta, while suffering from greater potential sampling error due to a smaller sample size, will not be biased by containing data from a different regression regime.

On balance, therefore the confidence level chosen for testing the null hypothesis was 90%, which is felt to accommodate both of the above factors.

6.2.4 Chow Test

Chow (1960) developed a test of the null hypothesis of no change in the regression parameters which requires the *a priori* hypothesis of the location of the change point. McGee and Carleton (1970) make use of this test as a decision rule for the hierarchical clustering used in their piecewise regression algorithm.

The appropriate test statistic is the F-statistic:

$$F_{(2,T_1+T_2-4)} = \frac{\left[\sum_{t=1}^{T_1+T_2} \hat{e}^2_{Ct} - \sum_{j=1}^{T_1} \hat{e}^2_{Lj} - \sum_{k=1}^{T_2} \hat{e}^2_{Rk}\right]/2}{\left[\sum_{j=1}^{T_1} \hat{e}^2_{Lj} + \sum_{k=1}^{T_2} \hat{e}^2_{Rk}\right]/(T_1+T_2-4)}$$

where L, R, and C refer to the left, right, and combined, set of points, respectively, and the number of points in the left and right sets are T_1 and T_2 respectively.

For the reasons discussed in the previous section, the F test is conducted throughout this chapter at the 90% level of confidence.

6.2.5 Methodology

The objective of this chapter is to determine that most recent set of historical data which is still valid for the estimate of the current beta coefficient of a security. The basic steps in the procedure to be followed are:

(i) Choose an historical set of return data of length T points, synchronised for the security and the market surrogate.

(ii) Reverse the order of the data points so that point 1 is the most recent in time and point T is the most distant (oldest) point in time.

(iii) Regress the set of security returns on the market returns and calculate Quandt's log-likelihood ratio, $\lambda(t)$, and MB's ILL for $r = 1$, for each t, $t = 2,\ldots,T-2$.

(iv) Choose max $ILL(T_1)$ as the most likely change point if $r = 1$. Compare this with the location of the minimum of $\lambda(t)$.

(v) Hypothesise that T_1 is the location of the point of change.

(vi) Test the null hypothesis

H_0 = no change in the regression parameters at T_1

against the alternative

H_1 = a change in the regression parameters at T_1

using the Chow test at 90% level of confidence.

(vii) Calculate the set of forward recursive residuals, w_t, and the cusum of squared recursive residuals, S_t, for $t = 3,\ldots,T$ and test the null hypothesis at the 90% level of confidence. Repeat this for the set of backward recursive residuals, w_t', $t = T-3,\ldots,1$.

(viii) Compare the results of the Chow test with the cusum of squared recursive residuals test.

(ix) If the Chow test rejects the null hypothesis, delete data points T_1+1 to T from the data set and repeat the procedure from point (iii) above with the most recent T_1 points as the new data set.
If the Chow test cannot reject the null hypothesis then assume that the prevailing data set is valid for the estimate of the current beta of the security.

By following this procedure for a sample of securities, the most recent set of historical data which is still valid for the estimation of the current beta coefficient of *each* security can be found. In general, it would not be expected that the most recent set of valid historical data will be of the same length for all securities.

6.3 The Data Used

The data comprised monthly percentage returns for the thirty three shares used in Chapter Five and listed in Appendix D, for the two periods:

Period 1: February 1970 to January 1975

Period 2: February 1975 to January 1980

These periods were not chosen to coincide with any particular market conditions which may have prevailed, but merely represented a convenient method of splitting the data available. Thus two periods of sixty months each were available, and hence for this study, the initial value of T was $T = 60$. Monthly percentage returns were also available for the JSE All Share Index, which was used as the market surrogate, and for the JSE Industrial and Financial Index, which was used as the market surrogate for a separate study on the industrial shares reported in Section 6.5.

6.4 Results for the Complete Share Sample

6.4.1 Period 1

Table 6.1 contains the results of the first iteration of the procedure outlined in Section 6.2.5 for the sixty months of Period 1. Some observations from Table 6.1 are:

(i) All but four of the thirty three securities in the sample indicate a significant change in the regression regime at the hypothesised change point as tested by the

TABLE 6.1 First Iteration of the Share Sample to the JSE All Share Index - Period 1.

SHARE		INDICATED POINT OF CHANGE		SIGNIFICANCE TEST AT 90% LEVEL		ESTIMATED BETA COEFFICIENTS		
CODE	NO.	QUANDT	ILL	CUSUM	CHOW	WHOLE PERIOD	UP TO POINT OF CHANGE	AFTER POINT OF CHANGE
AAC	1	51	51	✓	✓	1,239	1,318	0,992
ADK	2*	49	49	✓	✓	0,415	0,244	0,781
AMG	3	40	40	✓	✓	1,318	1,542	0,720
AMI	4*	40	40	✓	✓	0,502	0,372	0,906
BAR	5*	15	15	✓	✓	0,995	0,426	1,594
DBR	6*	15	15	✓	✓	0,873	0,673	1,083
DLV	7*	36	36	✓	✓	0,490	0,172	1,017
EDS	8*	33	33	x	✓	0,651	0,289	1,185
FSG	9	44	44	✓	✓	1,152	1,411	0,104
FUG	10*	14	14	✓	✓	0,899	0,471	1,277
GFS	11	50	50	x	✓	1,456	1,628	1,218
GTA	12*	14	14	✓	✓	0,479	-0,055	0,838
ICS	13*	14	14	✓	✓	1,056	0,718	1,355
KLO	14	40	40	x	✓	1,028	1,305	0,267
LLA	15*	14	14	✓	✓	0,753	0,050	1,362
MCR	16*	14	14	x	✓	0,779	0,250	1,022
OKO	17*	14	14	x	✓	0,773	0,369	1,076
PBR	18	32	32	✓	✓	0,992	1,458	0,255
PGS	19*	14	14	✓	✓	0,722	0,198	1,146
PML	20	14	14	✓	x	0,671	0,500	0,881
RFN	21	37	37	✓	✓	1,790	2,338	0,584
RMT	22*	29	29	✓	✓	1,043	0,490	1,748
SAB	23*	34	34	✓	✓	0,625	0,310	1,160
SAN	24	37	37	✓	x	0,335	0,308	0,339
SFM	25	49	49	x	x	0,861	0,744	1,239
TIG	26	17	17	✓	x	0,742	0,735	0,750
USC	27*	37	37	x	✓	0,559	0,237	1,237
VAR	28	20	20	✓	✓	1,187	1,517	0,777
VKS	29*	14	14	x	✓	0,593	-0,056	1,086
WDL	30	14	14	✓	✓	0,958	1,541	0,477
WDR	31	14	14	✓	✓	0,755	1,025	0,550
WHL	32	31	31	✓	✓	0,911	1,248	0,400
WLO	33*	34	34	✓	✓	0,881	0,497	1,445

where CODE and NO. are from Appendix D;

QUANDT is the log likelihood ratio as described in Section 6.2.1;

ILL is the integrated log likelihood function of MB as described in Section 6.2.2;

CUSUM is a test on the forward and backward cusum of squared recursive residuals of BDE as described in Section 6.2.3;

CHOW is a test of the F-statistic of Chow as described in Section 6.2.4;

A tick (✓) indicates rejection of the null hypothesis of no change in the regression parameters while a cross (x) indicates the null hypothesis could not be rejected. All tests at 90% level of confidence; and

ESTIMATED BETA COEFFICIENTS up to point of change represent beta estimates on the more recent set of data points, while beta estimates after point of change represent estimates on the older data (the data set of sixty months having been reversed).

Chow test at the 90% level of confidence.

(ii) Change points are located over the range, $t = 14,\ldots,51$, indicating quite disparate change locations. Nevertheless, of the twenty-nine shares displaying a significant change in regression parameters, no less than fourteen show a change between the fourteenth and seventeenth months, indicating that some event occurred around this time which affected many of the shares under consideration. A market analyst might be advised to examine this period carefully to establish the cause(s) of this phenomenon.

(iii) Without exception, the eighteen non-gold securities (asterisked in Table 6.1) which show a significant change in regression parameters show a decrease in the estimated beta coefficient before the point of change (that is, the beta estimated from the more recent set of data) compared to the beta estimated over the whole period. The eleven gold related securities each show the opposite trend, with the estimated beta coefficient before the point of change greater than that for the whole period. As noted in Section 5.6.1 in the previous chapter, Period 2 was characterised by a rising gold price. Therefore a rise in the betas of gold related securities as this period progressed is not surprising. The concomitant fall in non-gold related security beta estimates is an inevitable consequence of this, since the grand mean of security betas is one.

(iv) The cusum of squared recursive residuals test indicates a significant change in regression parameters for twenty five securities. However a significant change is indicated for three of the four securities in which the Chow test cannot reject the null hypothesis of no change. On the other hand, seven of the eight securities for which the cusum of squared recursive residuals test could not reject the null hypothesis, in fact had the null hypothesis rejected by the Chow test. Some of the beta changes indicated in Table 6.1 for these seven securities are quite large and hence this result is disappointing for the cusum of squared recursive residuals test.

(v) The log likelihood ratio of Quandt and the integrated log likelihood of MB agree for every security on the location of the change point, if any. This perfect measure of agreement suggests that probably only one of these methods needs to be employed to indicate the month of change, if any, in regression parameters.

Bearing in mind the conclusion of Theobald (1980), who felt that a data set of less than fifteen months was too small to obtain a meaningful beta estimate, it was decided not to proceed to the second iteration stage for those securities which had a significant change point before and including month fifteen. For these securities the indicated data set in Table 6.1 is therefore taken as the valid data set for the estimation of the current beta coefficient of these securities.

Table 6.2 shows the results obtained for the second and third iterations of the procedure described in Section 6.2.5. The footnotes from Table 6.1 apply to Table 6.2 as well, except that the cusum of squared recursive residuals test was not performed for these iterations. Some observations from Table 6.2 are:

(i) After the third iteration, the valid data set is determined for all securities.

(ii) The log likelihood ratio of Quandt and the integrated log likelihood of MB again agree for every security on the location of the change point, if any.

The summarised results of this section and the next section are presented in Section 6.4.3 below.

6.4.2 Period 2

Table 6.3 contains the results of the first iteration of the procedure outlined in Section 6.2.5 for the sixty months of Period 2. Footnotes from Table 6.1 apply in their entirety to Table 6.3, some comments on which follow:

(i) Twenty one of the thirty three securities indicate a significant change in the regression regime at the hypothesised change point as tested by the Chow test at the 90% level of confidence. This compares with the number of securities of twenty nine showing a significant change in Period 1. Therefore, it is clear that the security betas were more stable in Period 2 compared to Period 1. Again, reasons for this phenomenon were

TABLE 6.2 Second and Third Iteration of the Share Sample to the JSE All Share Index - Period 1.

SHARE		PREVIOUS INDICATED CHANGE POINT	NEW INDICATED CHANGE POINT.		SIGNIFICANCE TEST AT 90%	ESTIMATED BETA COEFFICIENT		
CODE	NO.		QUANDT	ILL	CHOW	WHOLE PERIOD	UP TO POINT OF CHANGE	AFTER POINT OF CHANGE
Second Iteration.								
AAC	1	51	12	12	x	1,318	1,048	1,429
ADK	2	49	39	39	✓	0,244	0,142	1,402
AMG	3	40	11	11	x	1,542	1,521	1,561
AMI	4	40	16	16	x	0,372	0,395	0,356
DLV	7	36	9	9	x	0,172	-0,171	0,366
EDS	8	33	25	25	x	0,289	0,311	-0,932
FSG	9	44	20	20	✓	1,411	1,534	1,297
GFS	11	50	42	42	x	1,628	1,645	-0,588
KLO	14	40	26	26	x	1,305	1,327	1,440
PBR	18	32	14	14	x	1,458	1,627	1,397
RFN	21	37	14	14	x	2,338	2,514	2,891
RMT	22	29	16	16	x	0,490	0,439	0,674
SAB	23	34	12	12	x	0,310	-0,124	0,440
USC	27	37	18	18	✓	0,237	0,126	0,941
VAR	28	20	13	13	x	1,517	1,232	1,373
WHL	32	31	19	19	✓	1,248	1,136	2,542
WLO	33	34	18	18	x	0,497	0,535	-0,106
Third Iteration.								
ADK	2	39	24	24	x	0,142	0,174	0,120
FSG	9	20	6	6	x	1,534	1,079	1,556
USC	27	18	7	7	x	0,126	0,125	0,237
WHL	32	19	12	12	x	1,136	1,206	1,059

TABLE 6.3 First Iteration of the Share Sample to the JSE All Share Index - Period 2.

SHARE		INDICATED POINT OF CHANGE		SIGNIFICANCE TEST AT 90% LEVEL		ESTIMATED BETA COEFFICIENTS		
CODE	NO.	QUANDT	ILL	CUSUM	CHOW	WHOLE PERIOD	UP TO POINT OF CHANGE	AFTER POINT OF CHANGE
AAC	1	12	12	✓	✓	1,184	1,715	1,074
ADK	2	54	54	x	✓	0,446	0,383	1,106
AMG	3	52	52	x	✓	1,489	1,677	0,862
AMI	4	55	55	x	x	0,530	0,509	0,825
BAR	5	51	51	✓	x	0,715	0,718	0,975
DBR	6	10	10	✓	x	0,691	0,661	0,740
DLV	7	46	46	x	x	0,355	0,289	0,455
EDS	8	25	25	x	x	0,607	0,566	0,570
FSG	9	49	49	x	x	1,490	1,571	1,145
FUG	10	26	26	✓	✓	0,670	0,351	0,907
GFS	11	14	14	✓	✓	1,222	0,373	1,351
GTA	12	47	47	✓	✓	0,491	0,518	0,848
ICS	13	49	49	x	✓	0,744	0,608	1,406
KLO	14	37	37	✓	x	1,630	1,811	1,501
LLA	15	40	40	✓	✓	0,683	0,519	0,890
MCR	16	41	41	x	✓	0,761	0,185	1,732
OKO	17	48	48	x	✓	0,673	0,549	1,343
PBR	18	35	35	✓	✓	1,560	1,295	1,746
PGS	19	25	25	✓	✓	0,913	1,082	0,685
PML	20	50	50	x	✓	0,561	0,554	0,877
RFN	21	24	24	✓	✓	1,207	0,872	1,638
RMT	22	23	23	✓	✓	0,696	0,196	0,935
SAB	23	11	11	x	✓	0,826	0,415	0,860
SAN	24	52	52	x	✓	0,743	0,544	1,255
SFM	25	40	40	✓	✓	0,467	0,490	0,687
TIG	26	14	14	x	✓	0,602	0,410	0,694
USC	27	41	41	✓	x	1,072	1,099	1,217
VAR	28	52	52	x	x	1,443	1,496	1,130
VKS	29	29	29	✓	x	0,566	0,354	0,681
WDL	30	9	9	x	✓	1,476	2,046	1,260
WDR	31	38	38	✓	x	1,137	1,202	0,980
WHL	32	46	46	✓	x	1,512	1,626	0,963
WLO	33	12	12	x	✓	0,569	0,232	0,712

discussed in Section 5.6.1 of Chapter Five, and will not be repeated here.

(ii) Change points are located over the range, $t = 9,\ldots,55$, which is somewhat wider than Period 1 ($t = 14,\ldots,51$). Seven of the twenty one shares showing a significant change in regression parameters did so between months 47 and 55, indicating that some event may have occurred at this time which influenced many shares significantly. In any event, tests of regression parameters based on thirteen and less points ought to be treated circumspectly.

(iii) The cusum of squared recursive residuals test indicates a significant change in regression parameters for seventeen securities, but is supported in only ten of these by the Chow test. On the other hand for eleven of the securities which indicate a significant change in regression parameters from the Chow test (that is, over half the total of twenty one indicating a significant change), the cusum of squared recursive residuals test does not reject the null hypothesis. Hence the two tests disagree on eighteen out of thirty three securities (54,5%), which is poor confirmation indeed. In Period 1, the disagreement was ten out of thirty three (30,3%). It must be concluded, therefore, that the cusum of squared recursive residuals is not a suitable test of the null hypothesis as formulated in this study.

(iv) The log likelihood ratio of Quandt and the integrated log likelihood of MB again agree for every security on the location of the change point, if any, as in Period 1. It must be concluded therefore that these two indicators of the point of change can almost certainly be used interchangeably, and need not be duplicated. The ILL, however, while slightly more work computationally, does offer the additional attribute that the probability of a change at a particular location can be established in comparison to another possible change location. This could be particularly useful in certain applications.

Again, it was decided not to proceed to the second interation stage for those securities which had a significant change point before and including month fifteen. For these securities, the indicated data set in Table 6.3 is therefore taken as the valid data set for the estimation of the current beta coefficient of these securities. Securities which indicated a change point in the last fifteen months of the data (that is, after and including month forty five) were taken into the second iteration after the indicated data had been excised. Although it is accepted that some of these securities may have indicated a spurious significant change in regression parameters because of the smallness of the data set, it was felt that not much harm could come from treating all these indicated significant change points as genuine, and deleting the indicated data.

Table 6.4 shows the results obtained for the second and third iterations of the procedure described in Section 6.2.5. The footnotes from Table 6.1 apply, but the cusum of squared recursive residuals was not performed for these iterations. Some observations from Table 6.4 are:

(i) After the third iteration, the valid data set is determined for all securities.

(ii) Securities which had a significant change point up to and including month fifteen in iteration two were not taken to iteration three, and the indicated data set from iteration two is therefore taken as the valid data set for the estimation of the current beta coefficient of these securities.

(iii) Unanimous agreement on the location of the change point, if any, is again displayed by the log likelihood ratio of Quandt, and the integrated log likelihood of MB.

6.4.3 Summary of Period 1 and Period 2

Table 6.5 provides a summary of the latest valid data set for each security in the share sample for each of the two periods. Also shown is the beta estimate for the whole period (that is, sixty months) and the estimates up to and after the indicated change points. Since the data has been reversed in this exercise, the estimates of the prevailing beta coefficient for each security for the period under consideration is shown in the columns headed "Up to Change".

TABLE 6.4 Second and Third Iterations of the Share Sample to the JSE All Share Index - Period 2.

SHARE		PREVIOUS INDICATED CHANGE POINT	NEW INDICATED CHANGE POINT		SIGNIFICANCE TEST AT 90%	ESTIMATED BETA COEFFICIENT		
CODE	NO.		QUANDT	ILL	CHOW	WHOLE PERIOD	UP TO POINT OF CHANGE	AFTER POINT OF CHANGE
Second Iteration.								
ADK	2	54	17	17	x	0,383	0,379	0,475
AMG	3	52	7	7	x	1,677	1,224	1,657
FUG	10	26	18	18	x	0,351	0,349	0,426
GTA	12	47	14	14	x	0,518	0,638	0,607
ICS	13	49	31	31	√	0,608	0,990	0,173
LLA	15	40	15	15	x	0,519	0,611	0,421
MCR	16	41	7	7	√	0,185	-1,295	0,131
OKO	17	48	23	23	x	0,549	0,411	0,436
PBR	18	35	27	27	√	1,295	1,231	1,393
PGS	19	25	17	17	x	1,082	1,129	0,940
PML	20	50	29	29	x	0,554	0,571	0,656
RFN	21	24	10	10	x	0,872	0,445	1,214
RMT	22	23	8	8	√	0,196	0,212	-0,031
SAN	24	52	22	22	x	0,544	0,344	0,454
SFM	25	40	8	8	√	0,490	0,741	0,192
Third Iteration.								
ICS	13	31	22	22	x	0,990	0,885	1,166
PBR	18	27	8	8	x	1,231	0,825	1,450

TABLE 6.5 The Valid Data Set and Estimated Beta Coefficients for the Share Sample to the JSE All Share Index - Periods 1 and 2.

SHARE		PERIOD 1				PERIOD 2			
		POINT OF CHANGE	BETA COEFFICIENTS			POINT OF CHANGE	BETA COEFFICIENTS		
CODE	NO		WHOLE PERIOD	UP TO CHANGE	AFTER CHANGE		WHOLE PERIOD	UP TO CHANGE	AFTER CHANGE
AAC	1	51	1,239	1,318	0,992	12	1,184	1,715	1,074
ADK	2	39	0,415	0,142	1,110	54	0,446	0,383	1,106
AMG	3	40	1,318	1,542	0,720	52	1,489	1,677	0,862
AMI	4	40	0,502	0,372	0,906	-	0,530	0,530	-
BAR	5	15	0,995	0,426	1,594	-	0,715	0,715	-
DBR	6	15	0,873	0,673	1,083	-	0,691	0,691	-
DLV	7	36	0,490	0,172	1,017	-	0,355	0,355	-
EDS	8	33	0,651	0,289	1,185	-	0,607	0,607	-
FSG	9	20	1,411	1,534	0,703	-	1,490	1,490	-
FUG	10	14	0,899	0,471	1,277	26	0,670	0,351	0,907
GFS	11	50	1,456	1,628	1,218	14	1,222	0,373	1,351
GTA	12	14	0,479	-0,055	0,838	47	0,491	0,518	0,848
ICS	13	14	1,056	0,718	1,355	31	0,744	0,990	0,671
KLO	14	40	1,028	1,305	0,267	-	1,630	1,630	-
LLA	15	14	0,753	0,050	1,362	40	0,683	0,519	0,890
MCR	16	14	0,779	0,250	1,022	7	0,761	-1,295	0,770
OKO	17	14	0,773	0,369	1,076	48	0,673	0,549	1,343
PBR	18	32	0,992	1,458	0,255	27	1,560	1,231	1,810
PGS	19	14	0,722	0,198	1,146	25	0,913	1,082	0,685
PML	20	-	0,671	0,671	-	50	0,561	0,554	0,877
RFN	21	37	1,790	2,338	0,584	24	1,207	0,872	1,638
RMT	22	29	1,043	0,490	1,748	8	0,696	0,212	0,758
SAB	23	34	0,625	0,310	1,160	11	0,826	0,415	0,860
SAN	24	-	0,335	0,335	-	52	0,743	0,544	1,255
SFM	25	-	0,861	0,861	-	8	0,467	0,741	0,367
TIG	26	-	0,742	0,742	-	14	0,602	0,410	0,694
USC	27	18	0,559	0,126	1,023	-	1,072	1,072	-
VAR	28	13	1,187	1,232	0,840	-	1,443	1,443	-
VKS	29	14	0,593	-0,056	1,086	-	0,566	0,566	-
WDL	30	14	0,958	1,541	0,477	9	1,476	2,046	1,260
WDR	31	14	0,755	1,025	0,550	-	1,137	1,137	-
WHL	32	19	0,911	1,136	0,544	-	1,512	1,512	-
WLO	33	34	0,881	0,497	1,445	12	0,569	0,232	0,712

It seems that the methodology is able to pick up changes in the beta coefficients reasonably quickly in that six securities in Period 2 had significant change points identified within one year of the end of the period, while five had change points identified within one year of the beginning of the period. There is, however, an admitted difficulty in making beta estimates from small samples because of the potential sampling error in the resultant estimates. In fact such error may account for the rejection of the null hypothesis by the Chow test in some or all of these cases. On the other hand, these change points may in fact be genuine switches in the regression regime. The need to decide which of these possibilities applies in each case, is felt to constitute a strong argument for moving to weekly data for the estimation of beta coefficients of securities on the JSE. In this way the small sample size problem when regimes change near the extremities of the data set may be somewhat alleviated.

6.5 Results for the Industrial Shares and the Industrial and Financial Index

Some investigators may wish to allow for the heterogenous nature of the JSE by estimating beta coefficients of industrial shares to an industrial market surrogate, and gold shares to a gold market surrogate. Such betas could not, of course, be used to adjust returns in the context of a random portfolio chosen from the entire JSE, but may have other uses. For example, a fund manager may arrive at a decision to have a certain percentage of his portfolio invested in gold shares, and another percentage in industrial shares. Having taken this decision, he may wish to evaluate the expected risk and return of this strategy both in the context of the total portfolio, and its component parts. Such a fund manager may, therefore, be interested in the betas of individual securities to their respective "homogeneous" market surrogates. With this in mind, and cognisant of the divergent risk movement of gold and industrial shares in Period 1 as discussed in Section 6.2.1, it was decided to test for change points in the regressions of the twenty one industrial shares, which each fall under the Industrial and Financial sector of the JSE, using the JSE Industrial and Financial Index as market surrogate. The methodology described in Section 6.2.5 was used except that only the first iteration step was performed. The results for Period 1 are shown in Table 6.6, while those for Period 2 are shown in Table 6.7. The footnotes of Table 6.1 apply to each of these tables. Some observations from Tables 6.6 and 6.7 are:

(i) In Period 1, of the ten securities showing a significant change, seven show a significant change in the JSE All Share regressions for the same period. For Period 2, the figures are eleven and ten. In contrast of the eleven shares not indicating a significant change in the JSE Industrial and Financial regressions for Period 1, ten show a significant change in the JSE All Share regressions for the same period. For Period 2,

TABLE 6.6 First Iteration of the Industrial Shares to the JSE Industrial and Financial Index - Period 1.

SHARE		INDICATED POINT OF CHANGE		SIGNIFICANCE TEST AT 90% LEVEL		ESTIMATED BETA COEFFICIENTS		
CODE	NO	QUANDT	ILL	CUSUM	CHOW	WHOLE PERIOD	UP TO POINT OF CHANGE	AFTER POINT OF CHANGE
ADK	2	49	49	✓	x	0,881	0,820	0,785
AMI	4	12	12	✓	x	0,718	1,097	0,677
BAR	5	30	30	✓	✓	1,369	1,140	1,531
DLV	7	36	36	x	x	0,794	0,676	0,879
EDS	8	49	49	✓	✓	1,100	1,180	0,852
FUG	10	7	7	x	x	1,145	0,490	1,201
GTA	12	53	53	x	✓	0,771	0,547	1,430
ICS	13	50	50	✓	✓	1,480	1,718	0,978
LLA	15	51	51	✓	✓	1,311	1,108	1,847
MCR	16	14	14	x	x	1,213	0,801	1,172
OKO	17	25	25	x	x	0,991	1,062	0,996
PGS	19	31	31	✓	x	1,150	1,200	1,168
PML	20	13	13	✓	✓	0,983	1,627	0,817
RMT	22	29	29	✓	✓	1,646	1,059	1,894
SAB	23	54	54	x	x	1,109	1,108	1,275
SAN	24	37	37	x	✓	0,480	0,720	0,236
SFM	25	52	52	✓	x	1,018	0,908	1,276
TIG	26	17	17	✓	✓	0,980	1,464	0,758
USC	27	53	53	✓	✓	0,786	0,749	0,795
VKS	29	32	32	✓	x	1,092	0,943	1,165
WLO	33	53	53	✓	x	1,277	1,146	1,407

TABLE 6.7 First Iteration of the Industrial Shares to the JSE Industrial and Financial Index - Period 2.

SHARE		INDICATED POINT OF CHANGE		SIGNIFICANCE TEST AT 90% LEVEL		ESTIMATED BETA COEFFICIENTS		
CODE	NO	QUANDT	ILL	CUSUM	CHOW	WHOLE PERIOD	UP TO POINT OF CHANGE	AFTER POINT OF CHANGE
ADK	2	23	23	✓	x	0,771	0,958	0,861
AMI	4	47	47	x	x	0,884	0,927	0,820
BAR	5	51	51	✓	✓	0,956	1,240	0,566
DLV	7	46	46	x	x	0,826	0,800	0,859
EDS	8	25	25	x	x	0,869	0,860	0,828
FUG	10	26	26	✓	✓	0,908	0,277	1,263
GTA	12	47	47	✓	✓	0,795	0,942	0,868
ICS	13	24	24	x	x	1,137	1,615	0,997
LLA	15	35	35	x	x	0,959	0,737	1,131
MCR	16	41	41	x	✓	1,701	1,224	2,430
OKO	17	46	46	x	x	1,307	1,246	1,514
PGS	19	32	32	✓	✓	1,518	2,191	1,062
PML	20	54	54	✓	✓	0,908	1,004	0,296
RMT	22	22	22	x	✓	1,173	0,768	1,433
SAB	23	34	34	✓	x	1,242	1,320	1,123
SAN	24	22	22	x	✓	0,805	0,296	0,748
SFM	25	29	29	✓	✓	0,697	1,192	0,562
TIG	26	13	13	x	✓	0,902	0,724	1,020
USC	27	32	32	✓	x	1,394	1,575	1,287
VKS	29	27	27	x	x	0,951	1,294	0,780
WLO	33	13	13	✓	✓	0,997	0,767	1,151

the corresponding figures are ten and five. This invites the conclusion that when a security beta estimate does change significantly when estimated using its own homogeneous "market" index, such changes are likely to be significant for the full market regressions as well. However, when a security beta estimate does not change significantly when estimated against its own homogeneous "market" index, it may still indicate a significant change when estimated using the full market surrogate. Such changes are probably due more to a change in *relative* risk of the security to the overall market than to changes in *absolute* risk of the security. This phenomenon has been discussed in Section 5.6.1 of Chapter Five.

(ii) The poor level of confirmation of the cusum of squared recursive residuals test and the Chow test is again noticeable with eight disagreements (38,1%) in Period 1 and seven disagreements (33,3%) in Period 2. This supports the earlier assertion that the cusum of squared recursive residuals is not suitable to test the null hypothesis as formulated in this study.

(iii) The unanimous agreement of the Quandt log likelihood ratio and the integrated log likelihood of MB is again apparent.

In conclusion, it is clear that estimated security beta coefficients change significantly even if the estimates are obtained using a "market" surrogate which is more homogeneous with respect to the securities under consideration, than an overall market surrogate. Findings reported elsewhere in this thesis highlight problems with inter-period stationarity of security beta coefficients. The results of this section and Section 6.4 also point to intra-period instability of security beta coefficients.

6.6 Tracking the Regression Regime Switches of De Beers

In Table 6.3 of Section 6.4.2, the Quandt log likelihood ratio and integrated log likelihood of MB both indicate month ten as a change location for the regression parameters of De Beers to the JSE All Share Index. While the cusum of squared recursive residuals test indicates a rejection of the null hypothesis of no change at the 90% level of confidence, the Chow test is unable to reject the null hypothesis at this same level of confidence. To help resolve this, it was decided to use the Bayesian switching regression methodology as applied by Mehta and Beranek (1982) to the return data of American Telephone and Telegraph (AT&T) to detect changes in that security's beta, to the return data of De Beers for Period 2, namely, February 1975 to January 1980. MB concluded after analysing 100 quarterly returns for AT&T between January 1950 and December 1974, that AT&T changed its regression regime once in the 34th quarter of the period under consideration. However, as shown in Appendix E, their method is unable to test whether $r = 0$ or $r = 1$, that is, whether no change is more likely than one change. MB present evidence of a change by making reference to the rejection of the null hypothesis of no change by the cusum of squared recursive residuals test. However as has been shown in this chapter, this test is not a very reliable test of the null hypothesis and therefore there must be some doubt in MB's paper whether $r = 0$ or $r = 1$ has been adequately tested.

Indeed for the De Beers data under consideration, the cusum of squared recursive residuals test and Chow test disagree, with the former rejecting the null hypothesis and the latter unable to reject this hypothesis. In fact, the Chow test could not reject the null hypothesis at *any* hypothesised change point generated by splitting the sixty months of data into two mutually exclusive sets at the hypothesised change point. It seems reasonable to conclude, therefore, that De Beers beta coefficient did not change significantly during this period.

Table 6.8 presents the integrated log likelihoods for different values of r, the most likely location of the change point(s), and the number of times the location of the change(s) at the indicated point(s) is/are more likely than the other alternatives for each r value. Some observations from Table 6.8 are:

(i) If $r = 1$, it is highly likely that the change point occurred at month 10. The relative log likelihood of month 10 to its nearest rival, month 25, is (-209,8993 - (-211,8049)) = 1,9056 and the relative

TABLE 6.8 Location of Changes in De Beer's Regression to JSE All Share Index Under Different Numbers of Regimes - Period 2.

r VALUE	LOCATION OF CHANGE POINTS	INTEGRATED LOG LIKELIHOOD (ILL)	NUMBER OF TIMES LOCATION AT MAXIMUM ILL IS MORE LIKELY
0	-	-217,4041	-
1	10	-209,8993	-
1	25	-211,8049	6,72
1	19	-211,9314	7,63
1	55	-212,1052	9,07
1	33	-212,5783	14,57
2	10,55	-204,9895	-
2	10,33	-205,9471	2,61
2	10,25	-206,1389	3,16
2	10,19	-206,3424	3,87
3	10,33,55	-201,1787	-
3	10,25,33	-201,3876	1,23
3	10,19,55	-201,4988	1,38
3	10,19,25	-201,6296	1,57
4	10,19,25,55	-196,8783	-
4	10,25,33,55	-196,9151	1,04
4	10,19,33,55	-197,0780	1,22
5	10,19,25,33,55	-192,4058	-
6	10,19,25,33,42,55	-188,5340	-
7	10,19,25,33,42,47,55	-183,6289	-

likelihood of 10 over 25 is then

$$e^{1,9056} = 6,72$$

Therefore, a nonuniform prior distribution favouring the 25th month over the 10th month by odds in excess of 6,42 to 1 would be needed to make month 25 more likely than month 10, as the single change point.

(ii) As the number of change points increases, the ability of the model to discriminate between alternative locations of change points diminishes. When $r = 4$, the likelihood of each of the three alternatives shown is essentially the same. In fact, the alternatives at $r = 3$ could probably be regarded as equally likely in practice.

(iii) Noticeably, month 10 is chosen as a change point in all alternatives at every value of r, adding weight to this location as the change point if $r = 1$. Figures 6.3 and 6.4 show graphically Quandt's log likelihood ratio plotted over the whole period, and the cusum of squared recursive residuals test of BDE. It can be seen from Figure 6.3 that month 10 is indicated clearly as the single change point during the period. Figure 6.4 would be interpreted as a rejection of the null hypothesis of no change during the period with 90% confidence.

(iv) As MB point out, and discussed in Section 6.2.2, it is not possible to determine the most likely value of

DE BEERS REGRESSION ON JSE ALL SHARE INDEX

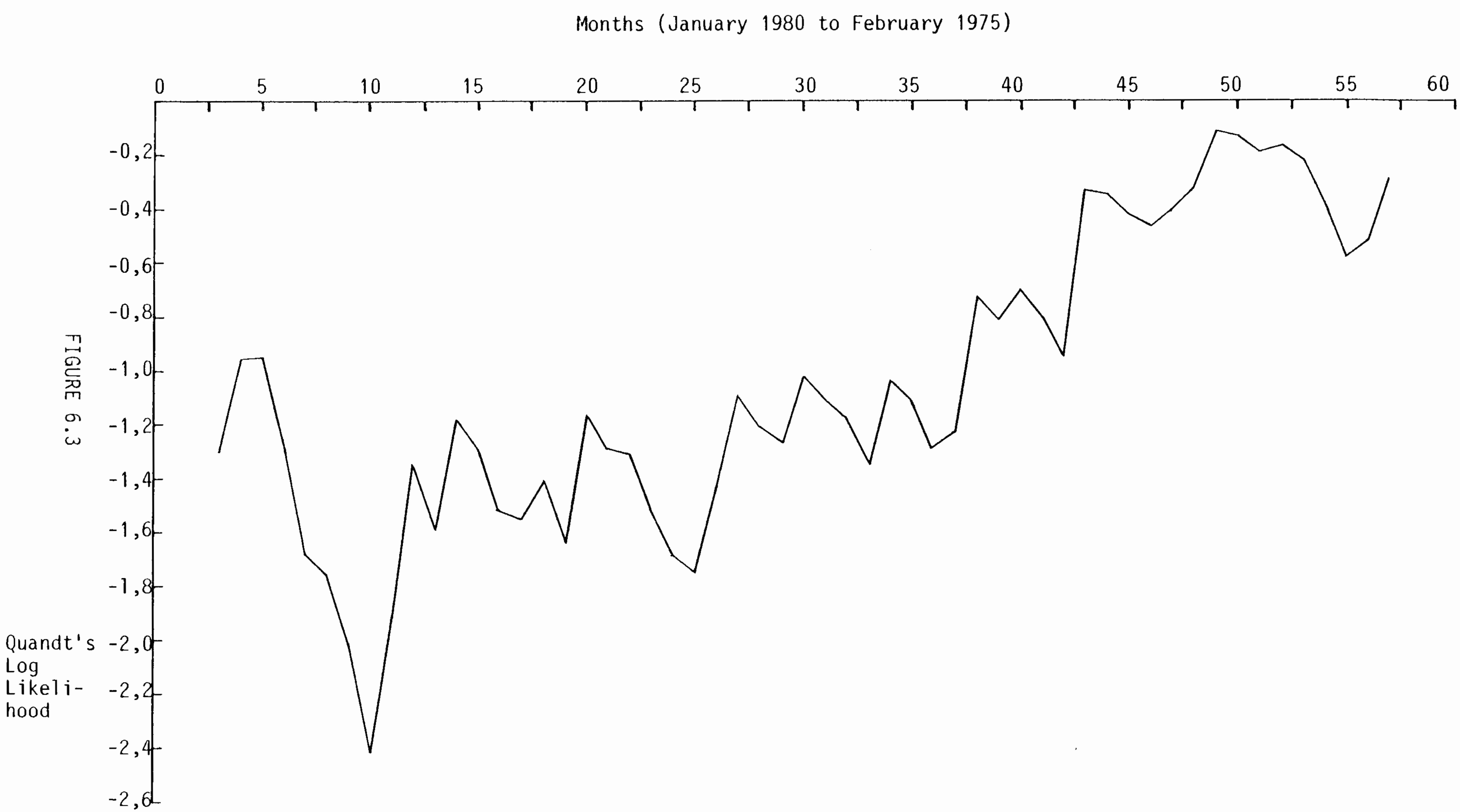


FIGURE 6.3

DE BEERS REGRESSION ON JSE ALL SHARE INDEX

BDE CUSUM OF SQUARED RECURSIVE RESIDUALS TEST

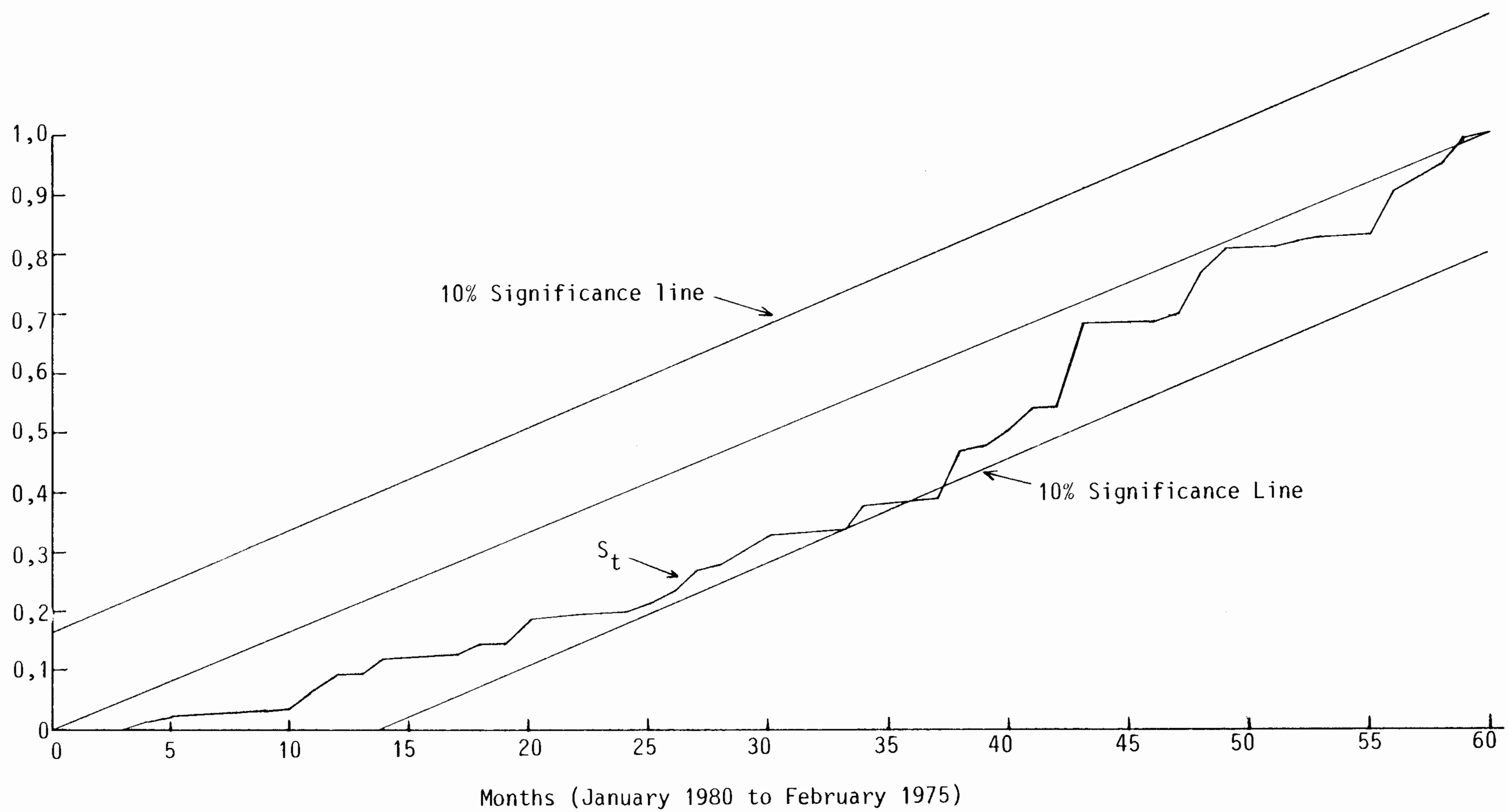


FIGURE 6.4

r by choosing that value of r which maximises the ILL. In fact the ILL in Table 6.8 increases as r increases. This means that the model fits the data better and better as its dimensionality increases, which is not unexpected.

For this reason, MB use a result due to Schwarz (1978) to determine the dimensionality of the model which correctly fits the data. This procedure was described in Section 6.2.2 and therefore here it is only necessary to report the results, which are shown in Table 6.9.

TABLE 6.9: The Dimension of the Switching Regression Model Fitted to De Beers - Period 2

r Value	Location of the Change Points	Value of K_r	$-\frac{1}{2}K_r \log(60)$	Maximum of Log Likelihood (max log (L_r))	Schwarz Adjusted log likelihood
0	-	0	-	-430,4530	-430,4530
1	10	3	- 6,1415	-406,3002	-412,4417
2	10,55	6	-12,2830	-396,9572	-409,2402
3	10,33,55	9	-18,4246	-378,7422	-397,1668
4	10,19,25,55	12	-24,5661	-374,8765	-399,4426
5	10,19,25,33,55	15	-30,7076	-365,1383	-395,8459
6	10,19,25,33,42,55	18	-36,8491	-357,1488	-393,9979
7	10,19,25,33,42,47,55	21	-42,9906	-352,6008	-395,5914

Table 6.9 suggests that the dimension of the switching regression model is $r = 6$, since the Schwarz adjusted log

likelihood achieves its maximum at this value of r. In fact $r = 6$ is 6,35 times more likely than $r = 5$, given by

$$e^{(-393,9979-(-395,8459))} = e^{1,8480} = 6,35$$

and $r = 6$ is 4,92 times more likely than $r = 7$, given by

$$e^{(-393,9979-(-395,5914))} = e^{1,5935} = 4,92$$

If, in fact, De Beer's beta changed six times during the 60 months of Period 2 it would be instructive to examine the beta estimates in each of the regimes. This is done in Table 6.10.

TABLE 6.10: Beta Coefficients for the Regression Regimes - De Beers Period 2

Regime No.	Month Range of Regime	Regime Beta Estimate	Standard Error of Regime Beta	Degrees of Freedom	Range of True Beta at 95% Confidence
1	1-10	0,6610	0,1502	8	0,3146 - 1,0074
2	11-19	0,8544	0,3159	7	0,1074 - 1,6015
3	20-25	1,1128	0,3843	4	0,0459 - 2,1796
4	26-33	1,3939	0,6114	6	-0,1020 - 2,8899
5	34-42	0,4283	0,2983	7	-0,2770 - 1,1336
6	43-55	0,8873	0,3344	11	0,1513 - 1,6233
7	56-60	1,1725	0,6692	3	-0,9573 - 3,3023

Table 6.10 shows that the estimated beta did vary quite dramatically between regimes. Unfortunately the small sample problem occurs again because of the large number of regimes accommodated in only sixty months of data. Therefore it is necessary to look at the significance of the individual

regime betas and this is possible using the standard errors of the betas, degrees of freedom and the t-distribution (Draper and Smith (1966, p.19)). It is apparent that the smallest range for the true beta which may have prevailed in *all* regimes (that is, throughout the whole period of sixty months) is that range for the first regime, namely, 0,3146-1,0074. Now the OLS beta estimate for De Beers for the whole of Period 2 is 0,691 from Table 6.3. Therefore it is possible that the Chow test is correct in not rejecting the null hypothesis of no change since the estimate of 0,691 could have prevailed in each of the regimes generated by the MB model. On the other hand, if the true beta coefficient of De Beers did switch between regimes as shown in Table 6.10, then it is likely that the Chow test, which is looking at the whole period regression, and two mutually exclusive, but exhaustive sub-regressions, would not be able to reject the null hypothesis. One way to test this is to hypothesise a change point at month 10 and allow the "whole" period for the purposes of the Chow test, to be 19 months long. If, in fact, the true beta changed significantly between the regime defined by 1-10 and that defined by 11-19, then the Chow test ought to be able to detect this if a "whole" period of 19 months is used. The value of the F-statistic at month 10 (using the definition in Section 6.2.4) is

$$F_{(2,15)} = 0,4454$$

This F value is far from significant indicating that the regime defined by 1-10 and that defined by 11-19 are not statistically significantly different in terms of the beta parameter. Table 6.10 indicates the most dramatic change in inter-regime beta estimates occurs between regimes four and five. It was decided to perform a Chow test for the period defined by months 26 to 42, hypothesising a change point at month 33. The appropriate F-statistic is

$$F_{(2,13)} = 0{,}6928$$

which is also far from significant.

It is therefore concluded that the strong likelihood is that the beta estimate of De Beers did not in fact change over the duration of Period 2. Since the methodology of MB cannot distinguish between $r = 0$ and $r = 1$, it follows that the results of this section merely say that if beta changed at all, it is more likely to have changed six times, at the identified points, than any other number of changes. Therefore the methodology of MB is only really useful in determining the number and location of change points when it has been established that at least one change point has occurred somewhere in the data set.

6.7 Conclusions

This chapter has examined two methods, namely, the log likelihood ratio of Quandt (1958), and the integrated log likelihood function of Mehta and Beranek (1982), for determining the location of the single change point, if any, in the regression parameters of an OLS regression. The methods were found to be interchangeable since they were in perfect agreement on the locations of possible change points in all cases. It is recommended that the integrated log likelihood method be employed since it provides the additional information of the probability of one location compared to another.

Two methods were examined to test the null hypothesis of no change in the regression parameters at the hypothesised change point. One, the cusum of squared recursive residuals of Brown, Durbin and Evans (1975) was not found to be consistently suitable for this purpose. Therefore the other method, the Chow test, is recommended as the method for testing the null hypothesis.

An iterative procedure was developed to determine the valid historical data set for the estimate of the current beta coefficient of a security. When applied to a sample of thirty three shares over two non-overlapping periods of sixty months, it became apparent that the security betas were very unstable with 29 shares (87,9%) and 21 shares (63,6%), respectively, in the two periods showing a significant change in their betas. The twenty one industrial shares from the sample, however, had more stable betas to the more homogeneous JSE Industrial and Financial Index, with 10 shares (47,6%) and 11 shares (52,4%) respectively, showing significant changes to their betas for the same two periods.

It was argued that the switching regression methodology of Mehta and Beranek (1982) could not distinguish between no, and one, change point. When applied to sixty months of

De Beers data, the method indicated that six change points were most likely, when in fact the Chow test could not reject the null hypothesis of no change anywhere in the data. It was concluded that the method of Mehta and Beranek is likely to be of value only when it has been definitely established that at least one change has taken place in the data set.

It was noted that the iteration procedure developed to determine the prevailing valid data set for current beta estimation, often determined data sets of length less than fifteen months. Since Theobald (1980) found that beta estimates obtained from data sets of less than fifteen monthly points were questionable, it was argued that weekly data may need to be used for the estimation of security betas. Accordingly, the next chapter uses weekly data in a study on the statistical procedure to be employed in obtaining security beta estimates.

CHAPTER SEVEN

ESTIMATION OF BETA COEFFICIENTS IN THE MARKET MODEL

7.1 Introduction

The well known market model can be most simply defined as

$$R_t = \alpha + \beta R_{M;t} + e_t \tag{1}$$

where

R_t is the return on the security in period t,

$R_{M;t}$ is the return on the 'market' in period t,

α and β are parameters unique to the security,

e_t is a random variable representing the residual error at time t.

The following assumptions are usually made with regard to the random error e_t:

(i) $E(e_t) = 0$ for all t

(ii) $Var(e_t) = \sigma_e^2$ for all t

(iii) $E(e_t;R_{M;t}) = 0$ (that is, the e_t's are uncorrelated with the market return)

(iv) $E(e_t;e_s) = 0$ for all $t \neq s$ (that is, the e_t's are not autocorrelated).

Clearly (1) indicates that the beta coefficient represents the relative movement in the security's return to changes in the market return. Numerous authors (for example, Sharpe (1970)) have argued that as such, beta measures the volatility of the security's return relative to the market and hence that beta can be used as a measure of risk. Furthermore, it is easy to show that

$$\mathrm{Var}(R_t) = \beta^2\sigma_M^2 + \sigma_e^2$$

where

σ_M^2 is the variance of the return on the market, and

σ_e^2 is the variance of the residual term.

Sharpe (1970) thus argued that the total risk associated with a security could be broken into a systematic risk part $(\beta^2\sigma_M^2)$ and an unsystematic part (σ_e^2). It is well known (Evans and Archer (1968)) that in a well chosen portfolio the unsystematic risk (that is, that associated with the individual company) can be diversified away, leaving only the systematic risk. Since σ_M^2 is the same for all securities, Sharpe argued that β was therefore a relative measure of the systematic risk of an individual security (and this is the only risk of importance for portfolio purposes).

For these reasons it is clear that a good estimate of beta is most desirable. The aim of this chapter is to examine aspects of the statistical procedure used in estimating these coefficients. In particular, five crucial aspects relating to the estimation of the beta coefficient are examined. They are:

(i) the manner in which the returns (R_t and $R_{M;t}$) should be calculated;

(ii) whether an intercept term (α) should be included in the model or whether the line should be forced through the origin;

(iii) the effect of periods during which the security is not traded;

(iv) the most appropriate statistical procedure for estimating the parameters α and β; and

(v) the effect of adjusting for the risk free interest rate.

Each of these important factors is discussed in greater detail in the next section.

7.2 The Factors

7.2.1 The Return

Traditionally, return on an investment has been defined as

$$\frac{\text{Receipt - Expenditure}}{\text{Expenditure}}$$

In the context of the stock market, this results in the formula

$$R_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}}$$

where

P_t is the price of the security at the end of period t, and

D_t is the amount of dividends accrued in the t^{th} period.

In this study weekly prices are analysed and dividends ignored (Sharpe and Cooper (1972)). As a result, the appropriate traditional formula for return is

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

Unfortunately, this formula might not be suitable for regression purposes as it is clearly non-additive. For example, suppose the price at the end of week 0 is 100 cents and at the end of week 1 is 200 cents. Clearly, R_1 is 2. Now, suppose that at the end of week 2 the price has reverted to 100 cents. Then, R_2 is $-\frac{1}{2}$. However, over the 2 week period the price has remained unchanged at 100 cents and hence the return over the two weeks is zero $\left(\frac{100-100}{100}\right)$. But

$$R_1 + R_2 = 2 - \tfrac{1}{2} = 1\tfrac{1}{2} \neq 0$$

To overcome this difficulty, numerous authors (for example, Fama (1965)) have suggested the use of the continuous rate of interest (or the logarithm of the price relative). This method involves use of the formula

$$R_t = \log_e P_t - \log_e P_{t-1} = \log_e P_t/P_{t-1}$$

It is easy to verify that for the example above R_1 is 0,69 and R_2 is -0,69 while the overall two week return is 0. Clearly,

$$R_1 + R_2 = 0$$

and hence the model is additive.

In the analyses below, both methods of computing the return are used and the results contrasted. In these analyses this factor is referred to as the factor: RETURN (=1 for the traditional return and 2 for the logarithm of the price relative).

7.2.2 The Intercept Term

In the introduction, the market model was defined as

$$R_t = \alpha + \beta R_{M;t} + e_t$$

In practice however, the validity of including the α term in the model can be considered. In fact if none of the individual securities change value (that is, $R_t = 0$ for all securities) then $R_{M;t}$ must also be zero and hence it could be argued that the model should force the line to pass through the origin.

On the other hand Sharpe (1970) among others has argued that the parameter α should be an important consideration in evaluating the worth of an individual security as it indicates the return on the security when there is no overall movement in the market. In addition it is held that the parameter α is an indicator of performance amongst portfolio managers. In fact in the United States particularly, the performance measurement industry has used the α parameter extensively. Some recent work, however, (Roll (1977 and

1980)) has made this subject into an open issue. In this chapter this issue is treated from an empirical point of view only.

In a pilot study, Pienaar (1980) showed that for most of the shares he examined the null hypothesis that $\alpha = 0$ could not be rejected. Thus it could be argued that his study, which shows that in practice the parameter α is usually not statistically significantly different from zero, indicates support for those who advocate the use of an even simpler market model, namely

$$R_t = \beta^* R_{M;t} + e_t^* \tag{2}$$

In the analyses below both models ((1) and (2)) are examined and this factor is henceforth referred to as the factor: INTERCEPT (=1 if α is unrestricted and 2 if $\alpha = 0$).

7.2.3 Untraded Weeks

The issue of non-trading has received some recent attention in the literature. Ball (1977) has shown the effects on estimates of systematic risk of measurement errors generated by the non-trading phenomenon. Roll (1981) has attributed the apparent superior risk-adjusted returns of small firms to infrequent trading inducing measurement errors. Scholes and Williams (1977) and Dimson (1979), amongst others, have proposed methods for dealing with the non-trading problem.

Many shares on the Johannesburg Stock Exchange (JSE) are rather 'thinly' traded and hence the situation arises where shares in a particular company might not be traded during a particular week. If these periods are included in the analysis the results are affected by the presence of these so called "false zeros" (that is, the price assumed to be unchanged so that R_t is zero for that particular period). In fact, it is obvious that in the case of least squares regression, the inclusion of "false zeros" will result in a decrease in the beta coefficient (that is the fitted line will be flatter than if the false zeros are omitted). This results in infrequently traded shares having downward biased beta estimates (see Dimson (1979)).

It is arguable whether this is or is not desirable depending on whether the lack of trading is regarded as information or not. Hence, in the analysis below the effect of these zeros on the beta coefficient is examined. Specifically, all periods during which R_t is equal to zero are initially included in the analyses and later excluded (regardless of whether trading took place or not). In the analysis this factor is referred to as the factor: ZEROS.

It should be noted that all zero weeks are omitted in the exclusion phase for computational simplicity. Clearly, this approach accentuates any differences which might be found if only non-trading zero weeks are omitted.

7.2.4 The Regression Method

Research in the last 10-20 years has indicated that ordinary least squares (OLS), which minimises the sum of the squared errors, is not necessarily the most appropriate estimation procedure for the regression parameters. In particular, Harter (1977) has indicated that alternative methods such as L_1-regression and Chebychev regression might be more suitable in specific cases of the general regression problem.

Barr, Affleck-Graves, Money and Hart (1980) extended this line of thought to the general class of L_p estimators of which OLS is merely a special case ($p = 2$). More precisely, Barr *et al* demonstrated that a more appropriate choice of p could be made if the kurtosis of the error distribution was considered. They showed that if the kurtosis is large then small values of p (1.0 to 1.5) provide more efficient estimates than those obtained with large values of p (2 to infinity) and vice versa.

In the area of stock market research the distribution of the residual term e_t has not received a great deal of attention. However, the distribution of the return on a security (R_t) has received considerable attention although no unanimity exists on the exact distribution which such returns obey. Fama (1965) and Affleck-Graves (1974) among others have found that the distributions do not differ significantly from a class of distributions known as the stable paretian family. Other authors such as Praetz (1969) and Schlosberg (1976) have found support for other distributions such as the t and the compound normal. However a common factor of all these distributions is that the kurtosis is large - certainly greater than that of the Normal Distribution (kurtosis 3). If this is true for R_t then it is likely that the distribution of e_t will also have kurtosis greater than 3. Thus it would appear from the works of Harter and Barr *et al* that OLS is not the most appropriate estimation method.

For very large kurtosis both Harter and Barr *et al* advocate the use of $p = 1$ (that is, minimisation of the sum of the absolute errors). Such studies have been performed on the New York Stock Exchange (Cornell and Dietrich (1978)) but did not indicate superior performance of the L_1 estimates over the $p = 2$ (OLS) estimates. A possible reason for this is that the use of $P = 1$ is too drastic and that a value between 1 and 2 should be used.

Thus, in this chapter, three distinct procedures are examined:

(i) $p = 2$ (OLS)
(ii) $p = 1$ (minimisation of the sum of the absolute errors)
(iii) $p = 1 + \frac{9}{k^2}$ where k^2 is the sample kurtosis of the residuals from an OLS regression (Barr *et al* (1980))

In the analysis below this factor is referred to as the factor: METHOD.

7.2.5 Adjustment by the Risk Free Rate

The market model as defined by (1) is

$$R_t = \alpha + \beta R_{M;t} + e_t$$

This model was first suggested by Markowitz (1959) as a simple method of obtaining the numerous estimates required for his portfolio selection model. This model has been used as a base by many other authors in the field of portfolio selection models with perhaps the most notable being Sharpe (1963). One aspect of this model which has aroused some debate is the question of whether the return on the security and the return on the market should be adjusted by the risk free rate of interest (or at least a surrogate for the risk free rate).

Thus, it is argued that the market model could be written as

$$R_t - r_f = \alpha + \beta(R_M - r_f) + e_t \qquad (3)$$

where r_f is the risk free rate of interest.

However, it is trivial to prove mathematically that if the risk free rate r_f is constant then the two models ((1) and (3)) yield identical estimates of the beta coefficient.

In practice the risk free rate is unlikely to vary markedly relative to the variability in the return on an individual security. In fact, in the short term (say one year) it can be regarded as virtually constant. Hence, as far as annual beta coefficients are concerned the two models are likely to provide almost identical results.

For the purposes of this chapter it is therefore concluded that adjustment of the returns by the risk free rate will not materially affect the estimate of the beta coefficient. For this reason this factor is not considered further.

Before concluding the discussion on the adjustment by the risk free rate it is worth discussing the role of the Capital Asset Pricing Model (CAPM). Sharpe (1964 and 1965) extended the original market model to examine the behaviour of the market under conditions of equilibrium and this led to the well known CAPM. This model is an 'expectations' model and can be defined as

$$E(R_t) = r_f + [E(R_M) - r_f]\beta$$

where $E(R_t)$ is the expected return on the security, and
$E(R_M)$ is the expected return on the market.

Clearly, this model can be written as

$$E(R_t) - r_f = \beta[E(R_M) - r_f]$$

and hence, if r_f is constant, the model is equivalent to the market model with $\underline{\alpha = 0}$. In other words, the beta estimate obtained from the CAPM will be the same as the beta estimate obtained from the market model (using least squares) provided α is excluded from the market model (or preset to zero).

Hence if the inclusion of the intercept term is a significant factor affecting the beta estimate then the CAPM and the market model will yield different beta estimates. If, however, it is not significant then the two models will yield very similar beta estimates.

7.3 The Data

Weekly closing prices were available for 15 shares (see Appendix F) quoted on the JSE for the period 1st January 1969 to 31st December 1975. Thus seven years of weekly data were available for each of the 15 shares.

In the previous section a number of factors which can affect the estimation of the beta coefficient were discussed. Clearly these factors can be combined in any way desired. For example, RETURN can be computed according to the traditional formula: INTERCEPT can be chosen so that the line is forced through the origin; ZEROS can be set so that all zero weeks are excluded; and finally METHOD can be set so that the OLS ($p = 2$) estimates are obtained.

For each combination of these basic factors, annual beta coefficients were computed for each of the 15 shares for each of the 7 years. Thus, 2520 (that is, 2x2x2x3x15x7) different beta coefficient estimates were obtained. These beta coefficients form the basic data to be analysed.

7.4 Analysis and Results

The data described above clearly fall into a standard ANOVA framework. There are six basic factors:

RETURN	(2 levels - traditional and logarithm)
INTERCEPT	(2 levels - $\alpha = 0$ and α unrestricted)
ZEROS	(2 levels - included and excluded)
METHOD	(3 levels - $P = 2$; $P = 1$ and general P)
YEAR	(7 levels - 1969 to 1975)
SHARE	(15 levels - the 15 shares chosen in the sample)

Clearly, this results in a six-way analysis of variance with 1 observation per cell with all factors completely crossed. Unfortunately the size of the problem causes computer storage problems for the standard analysis of variance computer programs. Hence, since it is well accepted that beta coefficients do vary from share to share, the factor SHARE was removed. That is, 15 separate 5-way analysis of variance runs were carried out, one for each particular share.

The results are summarised in Table 7.1 below. Because numerous tests of hypotheses are being performed it was decided to use a 1% level of significance. In addition all third order and higher order interactions are ignored and considered as part of the 'error sum of squares'.

The results presented in Table 7.1 indicate that as far as the interactions are concerned the factors ZEROS, METHOD and YEAR appear to have significant pairwise interactions but not the factors RETURN and INTERCEPT. While it must be admitted that the presence of significant interactions does cause difficulty in interpreting the main effects, it would

TABLE 7.1

Factor	No. of Shares for which the Factor was Significant
Return (R)	0
Intercept (I)	2
Zeros (Z)	15
Method (M)	11
Year (Y)	15
Interactions	
R × I	0
R × Z	0
R × M	0
R × Y	1
I × Z	1
I × M	1
I × Y	8
Z × M	9
Z × Y	15
M × Y	13

appear that the factors RETURN and INTERCEPT are not sigini-ficant. However, the remaining factors do appear to be significant and thus warrant further attention.

Firstly, consider the factor YEAR. Clearly, if the factor YEAR is significant then it can be concluded that the beta coefficient (or at least its estimate) changes significantly from year to year. But, this is not at all surprising. In fact, Blume (1971), Brenner (1974), Levy (1971), Gonedes (1973), Pettit and Westerfield (1974) and others have shown that individual securities have very unstable beta coefficients over time and this fact is now widely accepted. Thus, the fact that results for all 15 shares indicated significance of the factor YEAR only indicates that the beta coefficients change over time - not an unexpected result.

Before leaving the discussion of the factor YEAR it must be mentioned that interactions of the factor YEAR with the other factors are difficult to interpret in a practical sense. In fact, as far as this study is concerned, it can be argued that such interactions are meaningless. In this case all pairwise interactions involving the factor YEAR should be assumed zero and treated as part of the general error term. If this is done then the following table (analogous to Table 7.1) is obtained.

Examination of Table 7.2 reveals that virtually all pairwise interactions (ignoring the factor YEAR) are not significant. The main effect factors RETURN and INTERCEPT are also not significant. The factors ZEROS and YEAR are definitely significant factors being significant for all 15 shares examined. However, the factor METHOD is now significant in only approximately half of the shares examined.

TABLE 7.2

Factor	No. of Shares for which the Factor was Significant
Return (R)	0
Intercept (I)	1
Zeros (Z)	15
Method (M)	7
Year (Y)	15
Interactions	
R × I	0
R × Z	0
R × M	0
I × Z	0
I × M	0
Z × M	4

Secondly, consider the factors ZEROS and METHOD. Significance of the factor ZEROS would indicate that the estimate of the beta coefficient changed significantly depending on whether weeks with zero returns were included or excluded from the analysis. Examination of the actual estimates obtained suggested that this significance might be due to one of the METHOD alternatives, namely $p = 1$. The reason for this is that $p = 1$ regression is equivalent to a median-type estimator and results in a regression line which passes through at least two of the data points. Now, if a large number of data points are such that the dependent variable (R_t) is zero then a large number of the data points lie on the horizontal $(R_{M;t})$ axis. If the fit of the line is poor then, as the number of zero R_t's increases so does the possibility that the median line will pass through two of the points on the horizontal axis (the median line is in fact the median of all possible lines drawn through 2 of the data points). In such a case the estimates of the parameters α and β are both zero (that is, $\hat{\alpha} = 0$; $\hat{\beta} = 0$). This phenomenon can be illustrated by the simple sketch below.

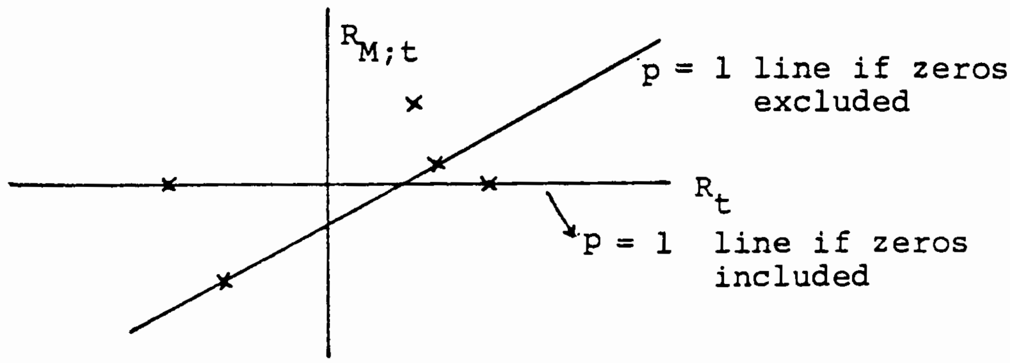


FIGURE 7.1

Such a situation arises surprisingly frequently in the data set under examination (approximately 5% of the time). Hence, it is felt that significance of the factor METHOD (in approximately 50% of the shares examined) might be due to the periodic odd behaviour of the p = 1 estimates when zeros are included. To examine if this is indeed the case the analysis was repeated with the factor ZEROS removed. All zero weeks are removed (to remove the odd p = 1 effect) and the following results are obtained. (Again all third order and higher interactions are ignored as are the second order interactions involving the factor YEAR.)

The results presented in Table 7.3 clearly illustrate that

TABLE 7.3

Factor	No. of Shares for which the Factor was Significant
Return (R)	0
Intercept (I)	1
Method (M)	2
Year (Y)	15
Interactions	
R × I	0
R × M	0
I × M	0

the factor METHOD is no longer a significant factor. Specifically, it can be concluded that if all zero weeks are excluded (avoiding the p = 1 problem) the beta coefficients are relatively unaffected by the use of p = 1 or the use of a general p in preference to OLS (p = 2) estimates.

It must be noted, however, that in the above analyses all weeks with zero return are omitted. That is, any week in which the opening and closing price are the same are omitted regardless of whether the share is traded or not. Since the p = 1 estimates are sometimes affected by zero weeks (whether or not the share is traded) all such weeks must be omitted if this problem is to be avoided. But, there is no economic, financial or statistical justification for omitting weeks during which the share has traded merely because the opening and closing prices are the same. As a result, it is argued that p = 1 regression should not be considered as an estimation procedure for beta coefficients.

But, the general p method does not suffer from such a disadvantage and hence it remains an alternative. In order to examine the behaviour of the general p estimate *vis-a-vis* the ordinary least squares (p = 2) estimates, the analysis was repeated with the p = 1 option removed. That is, the following factors are included:

RETURNS	(2 levels - traditional and logarithmic)
INTERCEPT	(2 levels - $\alpha = 0$ and α unrestricted)
ZEROS	(2 levels - included and excluded)
METHOD	(2 levels - p = 2 and general p)
YEAR	(7 levels - 1969 to 1975)

Once again 15 5-way analyses of variance were performed and all third order and higher interactions are ignored as are the second order interactions involving the factor YEAR. The results are summarised in Table 7.4.

As can be seen from Table 7.4 the factor METHOD is again not significant for most of the shares examined. This indicates that there is little or no difference between the beta estimate obtained using ordinary least squares and that obtained using the general p method. Therefore, in the light of the results presented in Tables 7.3 and 7.4, it seems reasonable to assume that the significance of the factor METHOD in Tables 7.1 and 7.2 is due to the odd behaviour of the p = 1 estimates when zero weeks are included in the data set. Once p = 1 is discarded or zero weeks are omitted the METHOD factor is no longer significant.

TABLE 7.4

Factor	No. of Shares for which the Factor was Significant
Return (R)	0
Intercept (I)	3
Zeros (Z)	15
Method (M)	3
Year (Y)	15
Interactions	
R × I	0
R × Z	0
R × M	0
I × Z	0
I × M	0
Z × M	0

7.5 Conclusions

Before presenting and discussing the main conclusions of this chapter it must be stressed that this study is not intended as an exhaustive study of all possible factors affecting the estimation of the beta coefficient of an individual security. Rather the study is intended as a guide as to which factors under consideration are important as regards the estimation procedure and which are relatively unimportant.

The main conclusions of the chapter are discussed below in terms of the factors analysed in the study.

(i) Return. The method of computing the return is not a major factor in estimating the beta coefficient. Although use of the logarithmic approach might be preferable on theoretical grounds (that is, the model is additive), the weekly returns are of such a low order that the traditional return is almost equal to the logarithm of the price relative. This substantiates the argument of Granger and Morgenstern (1970). Thus it is felt that in practice the logarithm of the price relative should be used (because it is theoretically preferable). However, previous results in the literature which have used the traditional return nevertheless remains valid. Because the absolute value of the return is likely to be larger as the period between observations increases it is possible that the non-additivity of the traditional return may become a problem in such cases.

(ii) The Intercept Term. In general it appears that the restriction of α to be zero does not significantly affect the estimate of the beta coefficient. This means that in general the null hypothesis that $\alpha = 0$ cannot be rejected and thus it could be argued that exclusion of the α term results in a better specification of the model (that is, needless terms are not included). Thus it is recommended that in practice the original model (1) be fitted and the hypothesis that $\alpha = 0$ be tested. If this hypothesis cannot be rejected the beta may be reestimated using the abbreviated model (2).

In addition, since the intercept is generally not a significant factor the market model and the CAPM yield very similar beta estimates and hence any conclusions regarding beta supported by one model will be generally valid for the other model.

(iii) Adjustment of the Returns by the Risk Free Rate.

Provided the risk free rate of interest does not vary markedly relative to the return on an individual security (R_t) the adjustment will make no difference to the estimate of the beta coefficient. Therefore it would appear that obtaining the additional information required for the risk free rate is not worthwhile. If the risk free rate is easily available it can be included as it might give the model greater theoretical acceptability. As far as previous results in the literature are concerned any conclusions reported using non-adjusted returns are probably as valid as those made using risk adjusted returns. Moreover the above argument indicates that the actual surrogate used for the risk free rate is not important provided it does not vary markedly within the period over which the beta coefficient is being estimated. This is of considerable importance when the theoretical difficulties inherent in defining exactly what the risk free rate is and how it can be measured, are considered.

(iv) The Treatment of Zero Return Weeks. The exclusion of zero weeks (whether or not the share is traded) is unlikely to result in a statistically significant difference in the beta coefficient estimate unless a regression that minimises the sum of the absolute errors ($p = 1$) is used. The question of whether no trade in a week constitutes information to the investor or not is an open issue and hence it is not advocated that the so-called 'false zeros' be omitted. Techniques suggested by Dimson (1979) could be used in these cases.

It should be noted that if observations are taken over longer intervals in time (for example, monthly instead of weekly) the problem of zero returns will not be as serious as fewer periods will result in observations of zero return.

(v) The Regression Method. The statistical procedure used to estimate the beta coefficients does not appear to have as marked an effect on the results as might have been expected. Certainly, regression minimising the sum of absolute errors, because of its peculiarities when a number of R_t's are zero, should be avoided unless all zero weeks are excluded. Of the remaining methods examined it is concluded that there is not a great deal of difference between the OLS estimates and the general p-estimates of Barr *et al* (1980). As the use of OLS is much less expensive in computer time and mathematically simpler to understand, it is recommended that ordinary least squares be used.

(vi) The Year. The year was the most significant of the factors examined. This provides additional support for the view that beta is not stable over time. More specifically, the year was a highly significant factor for each of the 15 shares examined. Practically, this means that grave dangers exist if past betas are naively used for future predictions. This issue has, however, been examined in Chapters 5 and 6.

Finally, in conclusion the following procedure is proposed for the estimation of the beta coefficient of a particular security, using the market model:

(i) Compute R_t and $R_{M;t}$ using the logarithm of the price relative ($\log_e P_t/P_{t-1}$).

(ii) Use ordinary least squares to estimate the parameters α and β of the model

$$R_t = \alpha + \beta R_{M;t} + e_t$$

(iii) Test the hypothesis: $H_0 : \alpha = 0$

$H_1 : \alpha \neq 0$

If this hypothesis can be rejected then β is the required estimate.

(iv) If the hypothesis cannot be rejected the beta coefficient can be estimated using ordinary least squares and the model

$$R_t = \beta^* R_{M;t} + e_t^* .$$

It should be noted that in the event that the hypothesis that $\alpha = 0$ can not be rejected, use of β as the required estimate can not be regarded as wrong. However, use of β^* as the estimate of the beta coefficient of the security in this case is also admissible on the evidence of the results of this chapter, and arguably provides a better specification of the return generating model.

CHAPTER EIGHT

SUGGESTED PROCEDURES FOR SECURITY BETA ESTIMATION

8.1 Introduction

This thesis has made extensive use of the market model (MM) due to Markowitz (1959):

$$R_{i;t} = \alpha_i + \beta_i R_{m;t} + e_{i;t}$$

where $R_{i;t}$ is the return on security i in time period t

$R_{m;t}$ is the return on the market in time period t

α_i and β_i are parameters unique to security i

$e_{i;t}$ is the disturbance or error term satisfying the following assumptions:

(i) $E(e_{i;t}) = 0$;

(ii) $cov(e_{i;t}, e_{i;s}) = 0$ for all $t \neq s$;

(iii) $var(e_{i;t}) = \sigma^2$ for all t ; and

(iv) $e_{i;t}$ is independent of $R_{m;t}$ for all t.

Several aspects which affect the variables, parameters and use of this model have been addressed in the earlier chapters of this thesis. The objective of this chapter is to cull some principles from these findings which may be of use to an investigator who needs to work with security beta estimates. Therefore, this chapter should be interpreted on a practical level.

It seems apparent that the way a researcher uses the MM depends on the purpose for which he requires the risk estimates. In other words, the findings of this thesis imply that there is no single beta estimate for a security which is applicable for all purposes. This is so, primarily because security beta estimates are unstable over time. Therefore the *time horizon* of the reseracher in his intended application of the beta estimate is critical in determining the method by which he obtains the beta estimate. Furthermore, beta is defined as

$$\beta_i = \frac{\mathrm{cov}(R_{i;t}, R_{m;t})}{\mathrm{var}(R_{m;t})}$$

Therefore, the choice of market surrogate is also critical to a researcher and may also depend on the intended application of the beta estimate. With these aspects in mind the following applications of beta estimates will be discussed in this chapter:

(i) Obtaining past beta estimates;
(ii) Obtaining current beta estimates; and
(iii) Obtaining future beta estimates.

8.2 Past Beta Estimates

This thesis has confirmed results found for other markets, namely, that security beta estimates are not stationary. Accordingly the major application for past beta estimates is likely to be performance evaluation. This situation arises when a client wishes to judge the performance of a fund manager who he retains to manage his portfolio. Although there are problems in the theory of performance measurement which have been addressed by Roll (1980) and others, one principle seems to clearly follow from the time varying nature of betas. It would seem sensible to estimate the market risk, which will be used to adjust the returns obtained by the fund manager, over the same period as the performance evaluation is being conducted. In other words, risk adjustment of returns in a given holding period should be done with risk estimates obtained from the same period. If risk estimates are obtained from a non-synchronous period to that of the performance measurement it is likely that bias, due to structural changes in risk, will be introduced.

Accordingly the proposed method for historical beta estimation is as follows:

(i) Choose the length of historical period from which the beta estimate is to be made equal to the length of the period for which it is required.

(ii) Using an overall market surrogate such as the JSE All Share Index, calculate monthly continuously compounded returns (log price relatives) for the security and the market. If the historical period is less than fifteen months, weekly data should be used to provide sufficient sampling points. Dividends, which in any event are excluded from the calculation of the JSE Actuaries Indices, may be ignored (Sharpe and Cooper (1972)).

(iii) Using ordinary least squares (OLS), obtain estimates of the alpha and beta parameters using the MM and the returns calculated above. Zero return weeks should be retained, whether traded or not.

(iv) The beta estimate obtained can be used as the estimate of the true beta coefficient of the security over the historical period in question.

The results of Chapters Two and Three provide an interesting variation to this procedure. In these chapters it was seen that it does not appear efficient for South African investors to have, in their portfolios, an exposure to gold shares in the same proportion as gold shares in the JSE All Share Index. In this case the JSE All Share Index may not be the ideal market surrogate for performance evaluation because it does not really represent a proper balance of the efficient alternatives open to a domestic fund manager. A fruitful line of further enquiry, therefore, would be the estimation of beta coefficients using a market surrogate which more correctly reflects the efficient alternatives available to a local fund manager. Such a market surrogate would almost certainly have a much lower than current exposure to gold shares.

The procedure outlined above permits the inclusion of the alpha parameter in the model specification. This is felt to be justified because of the *ex post* inefficiency of the market index. Thus the alpha coefficient is likely to be a legitimate parameter in the MM and its retention is therefore advocated for all securities.

This procedure for estimating historical security beta coefficients will produce an estimate of the *average* market risk over the period in question. This is in order, however, because the adjustment to returns in performance evaluation should be done by the average risk borne over the period being assessed.

8.3 Current Beta Estimates

To obtain the current beta estimate of a security, the possibility of structural changes in the risk parameters in the historical data becomes the most important consideration to the researcher. Therefore the method of beta estimation must eliminate data from past risk regimes which do not apply at the time of estimation. The procedure outlined in Chapter Six is recommended for this form of beta estimation:

(i) Choose an historical period for which continuously compounded monthly returns are available for the security and market surrogate. This period can be as long as desired but should probably be at least sixty months long.

(ii) Using OLS and the MM, calculate the integrated log likelihood value (ILL) of Mehta and Beranek (1982) at each month of the historical data. This is done for $r = 1$, that is, assuming only one change took place in the regression regime.

(iii) Choose that point at which the ILL reaches a maximum as the most likely location of a single change point in the data set, and set up the hypothesis:

H_0 : no change in the regression parameters at the suggested point

H_1 : a change in the regression parameters at the suggested point.

(iv) Test this hypothesis using a Chow test (1960) at the 90% level of confidence. If the null hypothesis cannot be rejected, accept the original full data period as being valid for the estimate of the current beta co-efficient of the security and use the procedure of Section 8.2 to obtain this estimate. If the hypothesis can be rejected return to step (ii) above having excised all data older in time than the hypothesised location of the change point.

(v) Continue until the null hypothesis cannot be rejected in the test of step (iv).

(vi) If, at any time, the length of the current valid data set is less than fifteen months, the researcher would be advised to switch to weekly data and continue from step (ii).

This procedure should provide the researcher with a beta estimate which is the best estimate of the true beta prevailing in the current valid regression regime. Clearly future changes in the regression regime are unlikely to be identified immediately the change becomes a part of the historical data set. Therefore it is accepted that for a while, the researcher will be estimating the beta coefficient in this instance from data obtained from two regimes. Unless methods of detecting instantaneous changes in regression parameters are developed, however, this problem will always exist. The results of Chapter Six seem to indicate that a researcher following the suggested procedure stands a reasonable chance of detecting regime changes quite quickly.

8.4 Future Beta Estimates

Estimates of beta coefficients are required by all researchers using expectational models in capital market theory and who are required to form portfolios *ex ante*. However, the method adopted for prediction of security betas is again conditional on the period for which prediction is required. It is reasonably safe to conclude from the results of Chapter Six that in a five year projection of a security beta coefficient, the true beta coefficient is likely to undergo one or more structural changes. Therefore the researcher predicting five year security betas must be aware that he is predicting an *average* market risk of the security over this time. On the other hand, one period expectational models, like the Capital Asset Pricing Model (CAPM), may simply require a prediction which can be best based on the current beta estimate of the security. Therefore two methods are suggested for the prediction of beta coefficients:

(i) One period beta prediction; and

(ii) Multiperiod beta prediction.

8.4.1 One Period Beta Prediction

Investigators using the CAPM, for example, are interested in predicting betas for one period. This period is assumed for the purposes of this section to be a short time, perhaps a month or a quarter. In this case, unless the investigator has knowledge to the contrary, the predicted beta should be largely a function of the current beta estimated by the procedure of Section 8.3. It is felt that the investigator may choose, if desired, to generate the predicted beta by a Bayesian adjustment of the current beta estimate of the security. The use of a prior of one would seem justifiable on the basis that the set of current beta estimates for all securities is the set of all best linear unbiased estimates of the true security beta coefficients which it is known has a grand mean of one. The variance prior could be the sample variance of all the current beta estimates of securities available to the researcher in his sample.

8.4.2 Multiperiod Beta Prediction

In this instance the researcher may be concerned to estimate beta coefficients over a forecast period which is quite long in time, and thus comprises several quarters or years. In a sense this may be thought of as a one period prediction in which the period length is long. An investor may have this need because portfolio revision can only be performed infrequently, perhaps for tax reasons. Therefore the investor may assume that the securities in his portfolio could obey several risk regimes in the prediction period under consideration. In this case the current beta estimate of Section 8.3 may not be the most suitable as the beta prediction. The following procedure, which assumes (Eubank and Zumwalt (1977)) a prediction interval of equal length to the estimation interval, is therefore suggested:

(i) Choose an historical set of data for the security and market indices of equal length to that required for the beta prediction.

(ii) Using continuously compounded returns of monthly data (or weekly, if less than fifteen months has been chosen as the interval), OLS and the MM, estimate the beta coefficient of the security to each of N mutually exclusive and exhaustive sub-market indices which together comprise the overall market surrogate. For example use of the JSE All Mining, Mining Financial and Industrial and Financial Indices could be made.

(iii) According to the method of Chapter Five, Bayesian adjust each of these beta estimates.

(iv) Using the multibeta approach of Sharpe (1974), generate a prediction beta using the Bayesian adjusted beta estimates from step (iii), the most recent point estimates of the market capitalisation relatives of the sub-market indices to the overall market, and the variance of return relatives of these sub-market indices to the market obtained from a recent data history, say the last twenty four months.

This prediction beta, according to the results of Chapter Five, should give the investor a good estimate of the true beta coefficient of the security as it will be estimated *ex post* when the prediction period becomes actual history. Its value *ex ante* should therefore be obvious to a portfolio manager whose performance is going to be judged *ex post* by a method such as that suggested in Section 8.2.

The results of Chapter Four suggest that an alternative specification of the three sub-indices employed in the above procedure may have particular value for securities in the "Other Mining" sector of the Johannesburg Stock Exchange. These are the Coal, Diamonds, Platinum and Copper, Tin and Others sectors. The benefit for securities in other sectors is probably not worth the additional computational effort of regrouping the sectors in the JSE Actuaries Index as suggested in Chapter Four.

8.5 Final Thoughts

It is again stressed that this chapter has tried to use the results of the earlier chapters to aid in the practical estimation of security beta coefficients.

It is felt there is nothing, in principle, to prevent a researcher applying the procedures outlined in this chapter to portfolios as well as securities. However, the results reported in this thesis are for individual securities, and it would be preferable to test the suggested procedures for random portfolios to confirm the above assertion.

APPENDIX A

JSE ACTUARIES INDICES - JUNE 1980*

SECTOR No	Name	% of Immediate Superior Index	Group	%	Group	%	Group
1.	Gold - Rand	7,5%					
2.	Gold - Evander	4,7%					
3.	Gold - Klerksdorp	20,1%	All Gold	67,4%			
4.	Gold - OFS	24,5%					
5.	Gold - W Wits	43,2%					
6.	Coal			6,5%	All Mining	48,4%	
7.	Diamonds			16,5%			
8.	Platinum	51,5%	Metals and Minerals	9,6%			
9.	Copper, Tin, Others	48,5%					
10.	Mining Holding			21,1%	Mining Financials	20,0%	
11.	Mining Houses			72,9%			
12.	Investment Trusts	13,1%					
13.	Insurance	13,0%	Financial	16,0%			All Shares 100%
14.	Property	11,3%					
15.	Banks	62,6%					
16.	Industrial Holding	19,5%					
17.	Beverages	7,4%					
18.	Building	5,3%					
19.	Chemicals	22,1%					
20.	Clothing	2,1%			Industrial & Financial	31,6%	
21.	Electrical	2,8%					
22.	Engineering	6,0%					
23.	Fishing	0,5%					
24.	Food	5,3%					
25.	Furniture	2,8%	Industrial	84,0%			
26.	Motors	1,6%					
27.	Paper, Packaging	5,7%					
28.	Pharmaceutical	0,7%					
29.	Printing	0,4%					
30.	Steel	3,0%					
31.	Stores	6,2%					
32.	Sugar	4,0%					
33.	Tobacco	3,0%					
34.	Transport	1,6%					

* The indices have been expressed excluding Property Trusts which only entered in 1976.

APPENDIX B

SHARES IN LOWEST RISK EFFICIENT PORTFOLIOS

Period 1 Portfolio 6		Period 2 Portfolio 7		Period 3 Portfolio 5	
Share	Proportion	Share	Proportion	Share	Proportio
Ergo	0,0029	Pres. Brand	0,0070	Amcoal	0,0645
Randfontein	0,0027	Free State Geduld	0,0067	Trans Natal	0,0415
ERPM	0,0010	Harmony	0,0065	Wits Colls	0,0287
Grootvlei	0,0009	Pres. Steyn	0,0061	Tavistock	0,0096
Durban Deep	0,0006	Western Holdings	0,0058	Palamin	0,0007
Winkelhaak	0,0370	St. Helena	0,0042	Samanco	0,0008
Kinross	0,0242	East Drie	0,0463	Ass. Mang	0,0003
Vaal Reeks	0,0457	Western Deep	0,0404	Rooiberg	0,0001
Southvaal	0,0277	West Drie	0,0354	Cons Murch	0,0001
Harties	0,0268	Kloof	0,0303	AECI	0,0173
Pres. Brand	0,0132	Elands	0,0214	Sasol	0,0135
Pres. Steyn	0,0115	Deelkraal	0,0176	Sentrachem	0,0099
Free State Geduld	0,0128	Amcoal	0,0976	Romatex	0,0939
Harmony	0,0123	Trans Natal	0,0630	I L Back	0,0258
Western Holdings	0,0111	Wits Colls	0,0435	Consol Tex	0,0148
St. Helena	0,0081	Tavistock	0,0145	Rex Trueform	0,0096
Amcoal	0,0898	AECI	0,0292	Seardel	0,0071
Trans Natal	0,0579	Sasol	0,0228	Dubin	0,0062
Wits Colls	0,0400	Sentrachem	0,0167	S A Wool	0,0035
Tavistock	0,0133	Altech	0,1573	Altech	0,0487
Rustenburg	0,0422	African Cables	0,0848	African Cables	0,0262
Impala	0,0256	Asea	0,0612	Asea	0,0189
Palamin	0,0129	Scottish Cables	0,0573	Scottish Cables	0,0177
Samancor	0,0148	Aberdare	0,0549	Aberdare	0,0170
Ass Mang	0,0049	Argus	0,0145	Haggie	0,0498
Rooiberg	0,0015	Saan	0,0064	Afrox	0,0449
Cons Murch	0,0012	Afrikaanse Pers	0,0053	Dorbyl	0,0310
Gefco	0,0007	Hiveld	0,0387	Stewarts & Lloyds	0,0282
Haggie	0,0021	Cullinan	0,0046	Abercom	0,0196
Afrox	0,0019			Hulamin	0,0137
Dorbyl	0,0013		1,0000	NEI Africa	0,0085
Stewarts & Lloyds	0,0012			Afgate	0,0071
Abercom	0,0008			Reunert & Lenz	0,0071
Hulamin	0,0006			Genrec	0,0065
NEI Africa	0,0004			Standard Brass	0,0063
Afgate	0,0003			Cemenco	0,0060
Reunert & Lenz	0,0003			E L Bateman	0,0051
Genrec	0,0003			Lamberts Bay	0,0208
Standard Brass	0,0003			Oil	0,0152
Cemenco	0,0003			Kaap Kunene	0,0119
E L Bateman	0,0002			Argus	0,0309
Lamberts Bay	0,0296			Saan	0,0136
Oil	0,0217			Afrikaanse Pers	0,0113
Kaap Kunene	0,0169			Huletts	0,0305
Nampak	0,0151			Tongaat	0,0229
Sappi	0,0150			C G Sugar	0,0225
Metal Box	0,0087			Safmarine	0,1052
Kohler	0,0065			Trencor	0,0050
Cons Glass	0,0038				
Argus	0,1380				1,0000
Saan	0,0608				
Afrikaanse Pers	0,0502				
Hiveld	0,0042				
Cullinan	0,0005				
Huletts	0,0286				
Tongaat	0,0215				
C G Sugar	0,0211				
Safmarine	0,0043				
Trencor	0,0002				
	1,0000				

APPENDIX C

SHARES IN THE SAMPLE USED IN CHAPTER FOUR

The Sector Number code is found in Table 4.1 on page 4.8

Share Code	Sector Number	Share Name
ABR	21	Aberdare Cables Africa Ltd
ADK	28	Adcock Ingram Ltd
ACA	21	African Cables Ltd
AFX	22	African Oxygen Ltd
AAL	18	Anglo-Alpha Ltd
AAC	11	Anglo American Corporation of South Africa Ltd
AMG	10	Anglo American Gold Investment Company Ltd
AMI	16	Anglo American Industrial Corporation Ltd
APE	6	Apex Mines Ltd
BAR	16	Barlow Rand Ltd
CTX	20	Consolidated Textile Mills Investment Corporation Ltd
DBR	7	De Beers Consolidated Mines Ltd
DLV	22	Dorbyl Ltd
EDS	31	Edgars Stores Ltd
EVT	18	Everite Ltd
FUG	12	First Union General Investment Trust Ltd
FSG	4	Free State Geduld Mines Ltd
GFS	11	Gold Fields of South Africa Ltd
HUL	32	Huletts Corporation Ltd
KLO	5	Kloof Gold Mining Company Ltd
LLA	13	Liberty Life Association of Africa Ltd
MCR	26	McCarthy Group Ltd
MBX	27	Metal Box South Africa Ltd
NED	15	Nedbank Group Ltd

APPENDIX C (continued)

Share Code	Sector Number	Share Name
OKO	31	OK Bazaars (1929) Ltd
PGS	16	Plate Glass and Shatterprufe Industries Ltd
PBR	4	President Brand Gold Mining Company Ltd
PPC	18	Pretoria Portland Cement Company Ltd
RMT	33	Rembrandt Group Ltd
RTO	20	Rex Trueform Clothing Company Ltd
ROM	20	Romatex Ltd
STH	4	St Helena Gold Mines Ltd
SAP	27	Sappi Ltd
SCC	21	Scottish Cables Ltd
SAN	29	South African Associated Newspapers Ltd
SFM	34	South African Marine Corporation Ltd
SLL	22	Stewarts and Lloyds of South Africa Ltd
TAV	6	Tavistock Collieries Ltd
ARG	29	The Argus Printing and Publishing Company Ltd
CLY	6	The Clydesdale (Transvaal) Collieries Ltd
GTA	26	The General Tire and Rubber Company (South Africa) Ltd
ICS	24	The Imperial Cold Storage and Supply Company Ltd
PML	24	The Premier Group Ltd
RFN	1	The Randfontein Estates Gold Mining Company Witwatersrand Ltd
SAB	17	The South African Breweries Ltd
USC	30	The Union Steel Corporation of South Africa Ltd
TIG	24	Tiger Oats and National Milling Company Ltd
TNC	6	Trans-Natal Coal Corporation Ltd

APPENDIX C (continued)

Share Code	Sector Number	Share Name
TRE	34	Trencor Ltd
TWH	31	Truworths
VAR	3	Vaal Reefs Exploration and Mining Company Ltd
VKS	15	Volkskas Group Ltd
WDR	5	West Driefontein Gold Mining Company Ltd
WDL	5	Western Deep Levels Ltd
WHL	4	Western Holdings Ltd
WHT	26	Williams Hunt South Africa Ltd
WIC	6	Witbank Colliery Ltd
WLO	31	Woolworths Holdings Ltd

APPENDIX D

Shares in the Sample Used in Chapters Five and Six

Share Code		Share Name
1	AAC	Anglo American Corporation of South Africa Limited
2	ADK	Adcock Ingram Limited
3	AMG	Anglo American Gold Investment Company Limited
4	AMI	Anglo American Industrial Corporation Limited
5	BAR	Barlow Rand Limited
6	DBR	De Beers Consolidated Mines Limited
7	DLV	Dorbyl Limited
8	EDS	Edgars Stores Limited
9	FSG	Free State Geduld Mines Limited
10	FUG	First Union General Investment Trust Limited
11	GFS	Gold Fields of South Africa Limited
12	GTA	The General Tire and Rubber Company (South Africa) Ltd.
13	ICS	The Imperial Cold Storage and Supply Company Limited
14	KLO	Kloof Gold Mining Company
15	LLA	Liberty Life Association of Africa Limited
16	MCR	McCarthy Group Limited
17	OKO	OK Bazaars (1929) Limited
18	PBR	President Brand Gold Mining Company Limited
19	PGS	Plate Glass and Shatterprufe Industries Limited
20	PML	The Premier Group Limited
21	RFN	The Randfontein Estates Gold Mining Company Witwatersrand Limited
22	RMT	Rembrandt Group Limited
23	SAB	The South African Breweries Limited
24	SAN	South African Associated Newspapers Limited
25	SFM	South African Marine Corporation Limited
26	TIG	Tiger Oats and National Milling Company Limited
27	USC	The Union Steel Corporation of South Africa Limited
28	VAR	Vaal Reefs Exploration and Mining Company Limited
29	VKS	Volkskas Group Limited
30	WDL	Western Deep Levels Limited
31	WDR	West Driefontein Gold Mining Company Limited
32	WHL	Western Holdings Limited
33	WLO	Woolworths Holdings Limited

APPENDIX E

Proof that $r = 1$ is always more likely than $r = 0$ at the midpoint of any data set, n, where $n > 16$, using the Schwarz (1978) adjustment as outlined in Mehta and Beranek (1982). All logarithms are to the base e.

Assume that the data set contains n points. Let the unbiased residual variance be $\sigma^2\hat{e} = \frac{\Sigma\hat{e}^2}{n-2}$ (1)

From Mehta and Beranek (1982, p.253), the likelihood function, L_r, is:

$$L_r = \Pi_{\alpha=1}^{r}(h_\alpha)^{n_\alpha/2} \; (2\pi)^{-n_\alpha/2} \; e^{-n_\alpha/2} \; (S_\alpha)^{-n_\alpha/2}$$

where r is the number of change points

n_α is the location of a change point

h_α is the reciprocal of the residual variance for the regime ending at n_α $(= (\sigma^2\hat{e}_\alpha)^{-1})$

S_α is the sum of squared errors for the regime ending at n_α $(= \Sigma\hat{e}^2_\alpha)$.

Therefore L_0 (L_r when $r = 0$) as calculated by Mehta and Beranek and shown in Table 3, p.259 is given as

$$L_0 = ((\sigma^2\hat{e})^{-1})^{n/2}(2\pi)^{-n/2}e^{-n/2}(\Sigma\hat{e}^2)^{-n/2}$$

From (1)

$$L_0 = (\sigma^2\hat{e})^{-n/2}(2\pi)^{-n/2}e^{-n/2}((n-2)(\sigma^2\hat{e}))^{-n/2}$$

Therefore

$$\log L_0 = -n/2 \log(\sigma^2\hat{e}) - n/2 \log(2\pi) - n/2 - n/2 \log((n-2)(\sigma^2\hat{e})) \qquad (2)$$

Now assume that two lines of best fit are fitted to the data, one to the points 1 to $n/2$, the other to the points $n/2 + 1$ to n. That is, a change point is assumed at the midpoint of the data set. Further assume that the residual variance for the first line is $x\sigma^2\hat{e}$, some proportion of the total residual variance, while that for the second line is $y\sigma^2\hat{e}$. x and y are clearly both positive numbers. Then the log likelihood function for $r = 1$, is:

$$\log L_1 = [-n/4 \log(x\sigma^2\hat{e}) - n/4 \log(2\pi) - n/4 - n/4 \log((n/2-2)(x\sigma^2\hat{e}))] +$$

$$+ [-n/4 \log(y\sigma^2\hat{e}) - n/4 \log(2\pi) - n/4 - n/4 \log((n/2-2)(y\sigma^2\hat{e}))]$$

Gathering like terms

$$= [-n/2 \log(2\pi) - n/2] + [-n/4 \log x - n/4 \log(\sigma^2\hat{e}) - n/4 \log y - n/4 \log(\sigma^2\hat{e}) +$$

$$+ [-n/4 \log((n/2-2)(x\sigma^2\hat{e})) - n/4 \log((n/2-2)(y\sigma^2\hat{e}))]$$

$$= [-n/2 \log(2\pi) - n/2] + [-n/2 \log(\sigma^2\hat{e})] + [-n/4 \log x - n/4 \log y] +$$

$$+ [-n/4 \log((n/2-2)(\tfrac{n-2}{n-2})(x\sigma^2\hat{e})) - n/4 \log((n/2-2)(\tfrac{n-2}{n-2})(y\sigma^2\hat{e}))]$$

$$= [-n/2 \log(2\pi) - n/2 - n/2 \log(\sigma^2\hat{e})] + [-n/4 \log x - n/4 \log y] +$$

$$+ [-n/4 \log((\tfrac{n/2-2}{n-2})(n-2)(x\sigma^2\hat{e})) - n/4 \log((\tfrac{n/2-2}{n-2})(n-2)(y\sigma^2\hat{e}))]$$

$$= [-n/2 \log(2\pi) - n/2 - n/2 \log(\sigma^2\hat{e})] + [-n/4 \log x - n/4 \log y] +$$

$$+ [-n/4 \log x - n/4 \log((\tfrac{n/2-2}{n-2}) - n/4 \log((n-2)(\sigma^2\hat{e})) -$$

$$- n/4 \log y - n/4 \log(\tfrac{n/2-2}{n-2}) - n/4 \log((n-2)(\sigma^2\hat{e}))]$$

$$= [-n/2 \log(2\pi) - n/2 - n/2 \log(\sigma^2\hat{e})] + [-n/4 \log x - n/4 \log y] +$$

$$+ [-n/4 \log x - n/4 \log y] + [-n/2 \log(\frac{n/2-2}{n-2})] + [-n/2 \log((n-2)(\sigma^2\hat{e}))]$$

$$= [-n/2 \log(2\pi) - n/2 - n/2 \log(\sigma^2\hat{e}) - n/2 \log((n-2)(\sigma^2\hat{e}))] +$$

$$+ [-n/2 \log x - n/2 \log y] + [-n/2 \log(\frac{n/2-2}{n-2})]$$

Substituting from (2)

$$\log L_1 = \log L_0 + [-n/2 \log x - n/2 \log y] + [-n/2 \log(\frac{n/2-2}{n-2})] \quad (3)$$

Let $\log L_1'$ be the adjusted log likelihood function using the adjustment due to Schwarz (1978). That is

$$\log L_1' = \log L_1 - \tfrac{1}{2}K_r \log n.$$

When $r = 1$, $K_r = 3$ (Mehta and Beranek (1982)).

Therefore

$$\log L_1' = \log L_1 - \frac{3}{2} \log n \ .$$

Rearranging gives

$$\log L_1 = \log L_1' + \frac{3}{2} \log n \quad (4)$$

Substituting (4) into (3) gives

$$\log L_1' + \frac{3}{2} \log n = \log L_0 + [-n/2 \log x - n/2 \log y] +$$

$$+ [-n/2 \log(\frac{n/2-2}{n-2})]$$

Rearranging gives

$$\log L_1' - \log L_0 = -\frac{3}{2} \log n + [-n/2 \log x - n/2 \log y] +$$

$$+ [-n/2 \log(\frac{n/2-2}{n-2})]$$

Now $r = 1$ is more likely than $r = 0$ whenever $\log L_1' - \log L_0 > 0$.

Therefore $r = 1$ is more likely than $r = 0$ when

$$-\frac{3}{2}\log n - n/2[\log x+\log y] + [-n/2\ \log(\frac{n/2-2}{n-2})] > 0$$

Therefore

$$\frac{3}{2}\log n + n/2[\log x+\log y] + n/2[\log(\frac{n/2-2}{n-2})] < 0$$

$$\therefore \quad n/2[\log x+\log y] < -\frac{3}{2}\log n - n/2\ \log(\frac{n/2-2}{n-2})$$

$$\therefore \quad [\log x+\log y] < -\frac{3}{n}\log n - \log(\frac{n/2-2}{n-2})$$

$$\therefore \quad \log\ (xy) < \log(n)^{-3/n} - \log(\frac{n/2-2}{n-2})$$

$$\log\ (xy) < \log\left(\frac{(n)^{-3/n}}{(\frac{n/2-2}{n-2})}\right)$$

This occurs when

$$xy < \left(\frac{(n)^{-3/n}}{(\frac{n/2-2}{n-2})}\right) \tag{5}$$

That is, whenever (5) is true, $r = 1$ is more likely than $r = 0$.

Now consider the situation where the regression line has *not changed* over the set of n data points, that is, $r = 0$.

Let $\Sigma\hat{e}_1^2 = (n/2-2)(x\sigma^2\hat{e})$ and $\Sigma\hat{e}_2^2 = (n/2-2)(y\sigma^2\hat{e})$.

Now $\Sigma\hat{e}^2 = \Sigma\hat{e}_1^2 + \Sigma\hat{e}_2^2$

because the line has not changed.

Therefore using (1)

$$(n-2)(\sigma^2\hat{e}) = (n/2-2)(x\sigma^2\hat{e}) + (n/2-2)(y\sigma^2\hat{e})$$

$$= (n/2-2)(x+y)(\sigma^2\hat{e})$$

Therefore when $r = 0$

$$(n-2) = (n/2-2)(x+y)$$

Rearranging gives

$$x + y = \frac{(n-2)}{(n/2-2)} \tag{6}$$

It can be shown that if

$$x + y = k$$

then $x \times y$ reaches its maximum when

$$x = y = k/2 \ . \tag{7}$$

Therefore $\max(xy)$ from (6) and (7) is

$$\max(xy) = \left[\frac{(n-2)}{2(n/2-2)}\right]^2 \tag{8}$$

Using (8) and (5) it can be shown

$$\left[\frac{(n-2)}{2(n/2-2)}\right]^2 < \left[\frac{(n)^{-3/n}}{(\frac{n/2-2}{n-2})}\right]$$

for all $n > 16$.

Therefore using the Schwarz adjustment of Mehta and Beranek (1982) $r = 1$ will *always* be more likely than $r = 0$ at point $n/2$ for all $n > 16$. Therefore this method cannot be used to test whether $r = 0$ or $r = 1$.

APPENDIX F

Shares Used for Chapter Seven

1. The South African Breweries Limited
2. Murry and Roberts Holdings Limited
3. AECI Limited
4. Sentrachem Limited
5. The Premier Milling Company Limited
6. Tiger Oats and National Milling Company Limited
7. Sappi Limited
8. OK Bazaars (1929) Limited
9. Hulett's Corporation Limited
10. Tongaat Group Limited
11. Rembrandt Group Limited
12. Barlow Rand Limited
13. Federale Volksbeleggings Beperk
14. The Trust Bank of Africa Limited
15. Volkskas Beperk

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