Initial Applications of Statistical Catch-at-Age Assessment Methodology to the Greenland Halibut Resource

Doug S. Butterworth and Rebecca A. Rademeyer

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Summary

In this initial report of the application of SCAA to the assessment of the Greenland halibut resource, a Baseline case with temporally invariant selectivity-at-age vectors is considered together with four variants which allow for serial correlation in survey residuals, temporal variability in commercial selectivity-at-age, and force asymptotically flat commercial selectivity. Only catch- and survey-based data are taken into account, as for the standard NAFO XSA-based assessment. In terms of recent trends, results vary greatly amongst these variants. Two are similar (though slightly above in absolute terms) to the recent negative prognosis indicated by the XSA assessment. Others however (including what reflects the best fit to the data obtained thus far) suggest more optimistic results, in particular indicating increases in abundance over the last decade.

Introduction

This paper presents the results of some initial applications of Statistical Catch-at-Age methodology (SCAA – sometimes known as Age Structured Production Modeling, or ASPM) to data for the Greenland halibut resource in NAFO Subarea 2 and Divisions 3KLMNO. An SCAA-based assessment involves many options for adjusting forms and input parameter choices. The particular reason for circulating this partial analysis at this time to allow other scientists participating in co-operative industry/government project to familiarise themselves with the approach and to request sensitivity runs for factors on which they consider key results might depend..

At this stage, the assessment variants considered are restricted to data inputs from catch and survey information (as in the XSA-based assessment of Healey and Mahe, 2008). No CPUE data are used here in fitting these models.

Data and Methodology

The catch and survey based data (including catch-at-age information) and some biological data are listed in Tables in Appendix A.

The details of the SCAA assessment methodology are provided in Appendix B. Note that as yet not all of the possible variants of the methodology described there have been implemented.

The Baseline SCAA Assessment (B1) assumes a resource at pre-exploitation equilibrium in 1960 (the start of the catch series provided). Annual recruitment is governed (in expectation) by a Beverton-Holt form, with steepness parameter h fixed at 0.9 for the results reported here (this particular value was used because initial model fits treating h as an estimable parameter led to estimates approaching the upper boundary set close to 1).. Selectivity-at-age is taken to be year-independent and estimated in the fit for the commercial catch and for each survey series. These selectivity vectors turn out to be dome-shaped, and a continuing negative exponential trend with age at larger ages has been assumed, in contrast to the asymptotically flat selectivity assumed for the XSA assessment of Healey and Mahe (2008). Note that in contrast to XSA (and similar VPA approaches), the SCAA fits to the age-aggregated survey series rather than to these series disaggregated by age; furthermore while XSA assumes catch-at-age data to be effectively error-free, SCAA allows for the possibility that there is error in such data.

Results for four variants of B1 are also reported. Case 2 takes account of serial correlation in the residuals to the fits to the age-aggregated survey indices of abundance. Cases 3 and 4 admit the possibility of year-dependent variability in selectivity-at-age for the commercial catch, and reflect different degrees to which the selectivity-at-age for any one year may vary from a stationary expectation (a penalty function in the negative log likelihood restricts the extent of variation). Maximal flexibility has been allowed by permitting selectivity to change every two years. The approach is similar to that of Butterworth *et al.* (2003), except that here the expected selectivity-at-age is unchanging in time in contrast to the random walk approach implemented in Butterworth *et al.* (2003).

Results and Discussion

Results for the Baseline SCAA assessment B1 are given in Table 1, and illustrated in Figs 1-5 which show the estimated spawning biomass trend and selectivity-at-age vectors, the stock-recruitment relationship fitted, and the model fits to data for the survey indices of abundance and the various sources of proportions-at-age information. A noteworthy result is the upward trend estimated in spawning biomass over the last decade (Fig.1).

The residuals to the fits to the data shown or evident from Figs 4 and 5 do however show a number of instances of systematic patterns (lack of fit). For example, although the B1 assessment results broadly follow the trends shown by the survey series (Fig. 4), there is clear evidence of serial correlation in the residuals. The estimation of a common serial correlation coefficient (estimation of series-specific coefficients was not AIC-justified) does however remove much of these systematic residual trends (see Fig. 6). The results for this case 2 are shown in Table 1 and do evidence, as might be expected, lower precision (somewhat larger CVs for some estimates) than for the baseline B1 assessment.

Fig. 5 provides evidence of systematic trends in residuals for the fits to proportions-at-age data for both the commercial catch and the surveys, showing that the Baseline assessment assumption of year-invariant selectivities-at-age needs to be relaxed. To date, this has been investigated only for the selectivity-at-age for the commercial catch in assessment variants 3 and 4 in Table 1 (this concern takes precedence, as unlike for the survey selectivities-at-age, changing the selectivity for the commercial catches impacts the resource dynamics). Fig. 7 illustrates the changes residual pattern as the increasing variability in time in the selectivity pattern is admitted. For the larger of the two extents of variability considered (σ_{Ω} =2), the pattern appears reasonably random, and the fit to the data overall is improved in terms of the negative log-likelihood (see Table 1).

The final assessment variant considered (case 5 in Table 1) forces commercial selectivity to be flat above age 10. This mimics the results for XSA, which is implemented under the assumption of flat commercial selectivity for ages 13 and 14+.

The biomass trends for the Baseline assessment B1 and these four variants are compared in Fig. 8, which also includes the corresponding estimates, based on the same data, from the XSA assessment of Healey and Mahe (2008). Fig. 9 compares the commercial selectivity-at-age vectors for these six assessments, while Fig. 10 shows how this selectivity pattern varies with year for variants 3 and 4.

Notable features of Fig. 8 are first that the SCAA assessments all suggest a higher biomass in absolute terms than does the XSA. While SCAA cases 5 and 3 (flat selectivity and a lesser extent of commercial selectivity variation) broadly follow the XSA trends over recent years with their rather negative prognosis for the resource, cases B1, 2, and the (to date) best-fitting case 4 with its greater degree of variability admitted in the commercial selectivity-at-age show a recent increase and more optimistic prognosis.

Importantly, the analysis thus far has indicated that the catch and survey-based data used for past NAFO assessments allow rather varied interpretations, some of which are much more optimistic than that arising from the standard NAFO XSA-based approach.

Further Work Planned

Further aspects of the SCAA assessments still to be investigated include:

a) admitting temporal variation in the survey selectivities-at-age;

b) allowing for differences between reported and actual catches;

c) allowing for natural mortality *M* to vary with age and year; and

d) incorporating CPUE data.

References

Butterworth DS, Ianelli JN and Hilborn R. 2003. A statistical model for stock assessment of southern bluefin tuna with temporal changes in selectivity. *S Afr. J. mar. Sci.* 25:331-361.

Healey BP and Mahé J-C. 2008. An assessment of Greenland halibut (*Reinhardtius hippoglossoides*) in NAFO Subarea 2 and Divisions 3KLMNO. NAFO SRC Doc. 08/48, Ser. No N5550.

Vásquez A and Gonzáles-Troncoso D. 2008. Results from Bottom Trawl Survey on Flemish Cap of June-july 2007. NAFO SRC Doc. 08/34, Ser. No.5535.

Table 1: Results of fits of various SCAA variants (see text for details) to the commercial catch and survey data. The initial rows show the overall penalised negative log likelihood and the contributions thereto. These are followed by parameters of the stock-recruitment function, starting conditions for the first year of the abundance trajectory estimated, and serial correlation in survey residuals (see Appendix B for details); values fixed on input rather than estimated are shown in **bold**. These are followed by estimates of pre-exploitation and current spawning biomass, and their level when MSY is achieved, and an estimate of MSY (biomass units are ‘000t). Next estimates of quantities associated with the data series fitted, particularly standard deviations, are given (see Appendix B for details). Finally σ_R_out is the standard deviation of the stock-recruitment residuals estimated (the input value for which is σ_R = 0.25). Quantities shown in parenthesis are Hessian-based CVs.

	1) Baseline B1			2) As B1 but with serial correlation in the survey residuals			3) As B1 but with variations in commercial selectivity ($\sigma_\Omega = 0.5$)			4) As 3) but σ_Ω=2 instead of 0.5			5) As B1 but with flat commercial selectivity from age 10		
'-lnL:overall	-52.8			-66.4			-168.1			-226.0			-36.0		
'-lnL:Survey	-27.9			-40.3			-24.2			-30.6			-25.1		
'-lnL:CAA	-56.3			-57.2			-200.1			-234.8			-30.6		
'-lnL:CAAsurv	-4.2			-4.0			-5.5			-2.3			-13.5		
-lnL:SelPen	-			-			28.7			31.8			-		
RecRes penalty	35.6			35.0			33.1			9.9			33.2		
h	**0.90**	-		**0.90**	-		**0.90**	-		**0.90**	-		**0.90**		
θ	**1.0**	-		**1.0**	-		**1.0**	-		**1.0**	-		**1.0**		
ϕ	**0.0**	-		**0.0**	-		**0.0**	-		**0.0**	-		**0.0**		
ρ	**0.0**	-		0.64	-		**0.0**	-		**0.0**	-		**0.0**		
K^{sp}	603	(0.15)		574	(0.15)		352	(0.05)		523	(0.15)		292	(0.04)	
B^{sp}_{2008}	364	(0.34)		320	(0.36)		33	(0.35)		291	(0.38)		20	(0.33)	
B^{sp}_{2008}/K	0.60	(0.20)		0.56	(0.22)		0.09	(0.32)		0.56	(0.24)		0.07	(0.31)	
MSYL^{sp}	0.17	(0.09)		0.17	(0.09)		0.18	(0.13)		0.17	(0.14)		0.20	(0.07)	
B^{sp}_{MSY}	105	(0.23)		100	(0.23)		62	(0.15)		88	(0.27)		57	(0.10)	
MSY	45	(0.15)		43	(0.14)		28	(0.04)		38	(0.14)		25	(0.04)	
σ_{comCAA}	0.14			0.14			0.08			0.07			0.16		
Survey	q's	σ_{surv}	$\sigma_{survCAA}$	q's	σ_{surv}	$\sigma_{survCAA}$	q's	σ_{surv}	$\sigma_{survCAA}$	q's	σ_{surv}	$\sigma_{survCAA}$	q's	σ_{surv}	$\sigma_{survCAA}$
CanFall1	0.0001	0.29	0.11	0.0001	0.28	0.11	0.0002	0.29	0.11	0.0001	0.29	0.11	0.0002	0.30	0.11
CanFall2	0.0002	0.28	0.07	0.0002	0.24	0.07	0.0003	0.22	0.07	0.0002	0.22	0.08	0.0004	0.25	0.07
EU	0.0613	0.48	0.11	0.0647	0.33	0.11	0.1407	0.66	0.11	0.0719	0.49	0.11	0.1559	0.60	0.11
CanSpr	0.0000	0.53	0.12	0.0000	0.44	0.12	0.0000	0.54	0.12	0.0000	0.53	0.12	0.0000	0.51	0.12
σ_R_out	0.24			0.24			0.22			0.23			0.24		

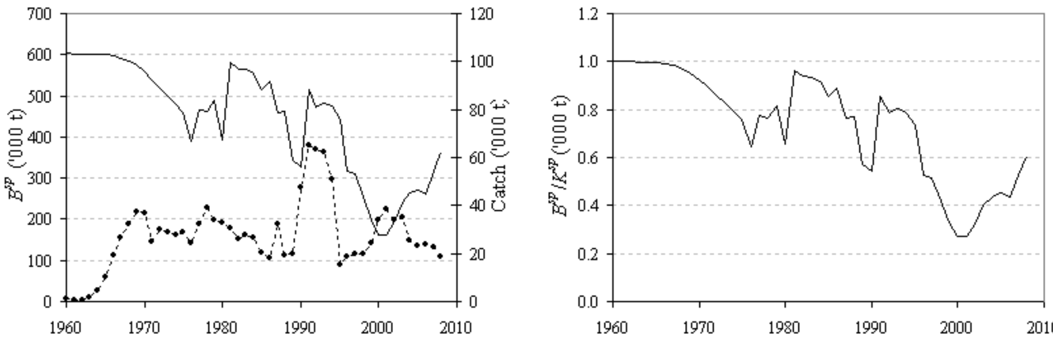

Fig. 1: Spawning biomass trajectories (in absolute terms and relative to pre-exploitation level) for the baseline assessment B1. The total annual catch is also shown.

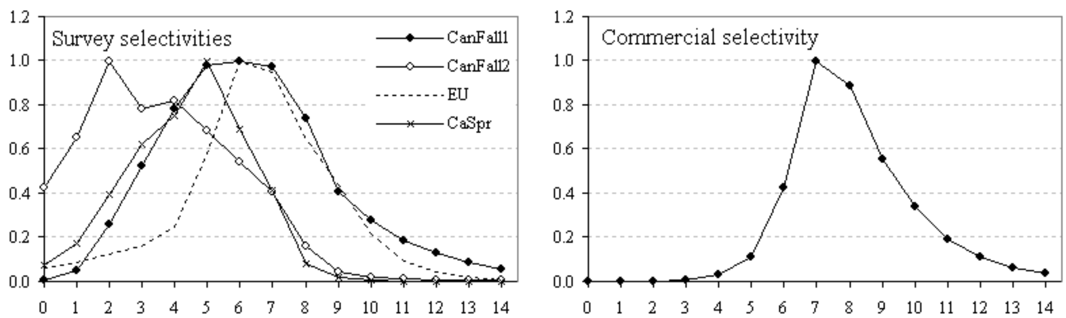

Fig. 2: Survey and commercial fishing selectivities-at-age estimated for the baseline assessment B1. "CanFall1" and "CanFall2" refer to the pre- and post-1995 periods respectively.

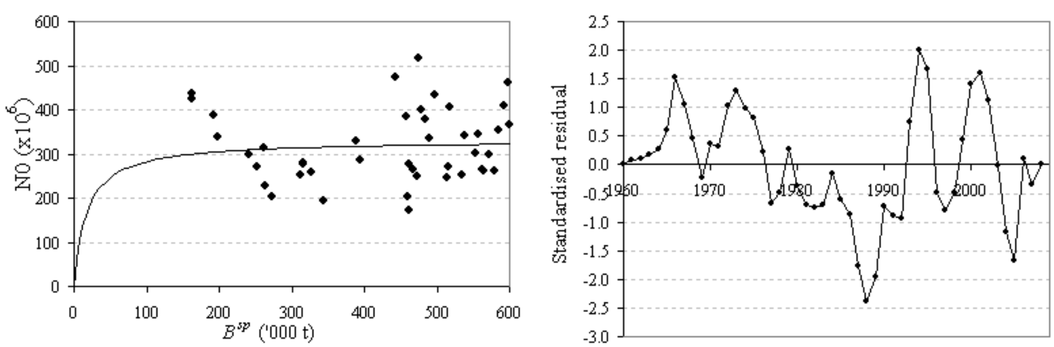

Fig. 3: Stock-recruitment curve and time series of standardised stock-recruitment residuals for the baseline assessment B1.

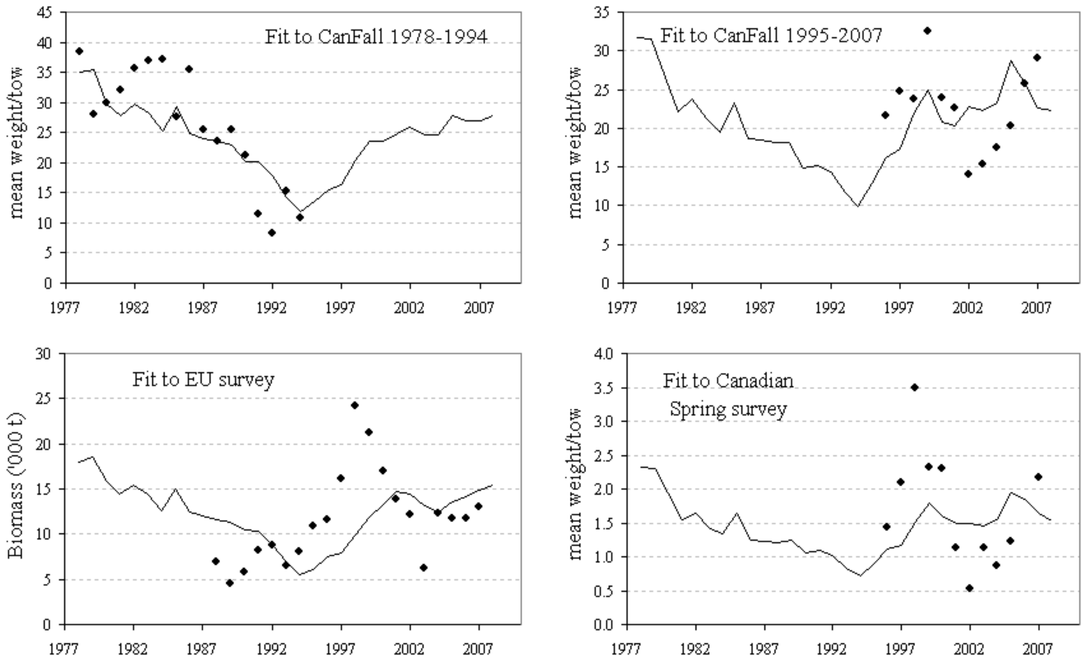

Fig. 4: Fit of the baseline assessment B1 to the survey indices of abundance.

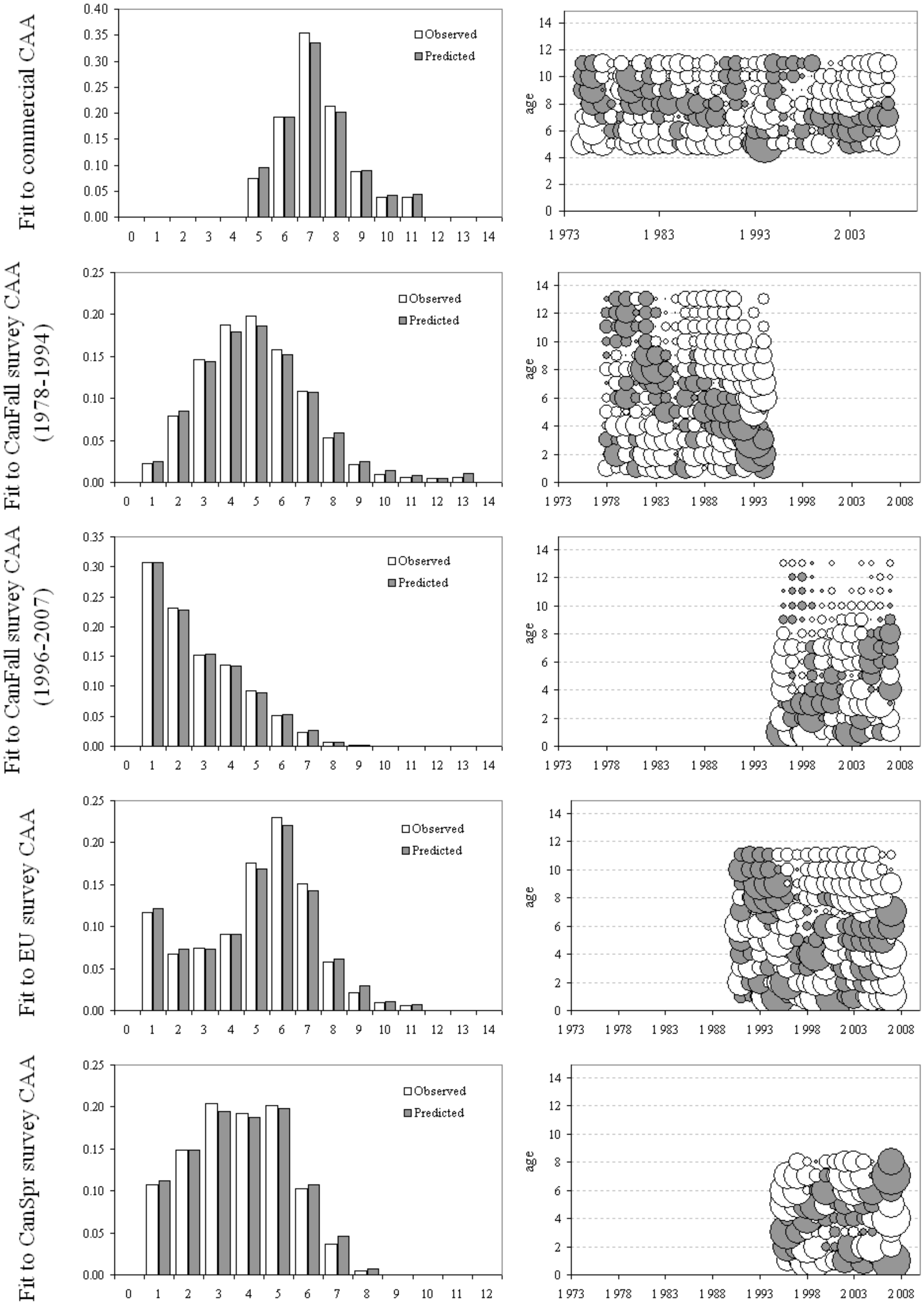

Fig. 5: Fit of the baseline assessment B1 to the commercial and survey catch-at-age data. The first column compares the observed and predicted CAA as averaged over all years for which data are available while the second column plots the standardised residuals, with the size (area) of the bubbles being proportional to the magnitude of the corresponding standardised residuals. For positive residuals, the bubbles are grey, whereas for negative residuals, the bubbles are white.

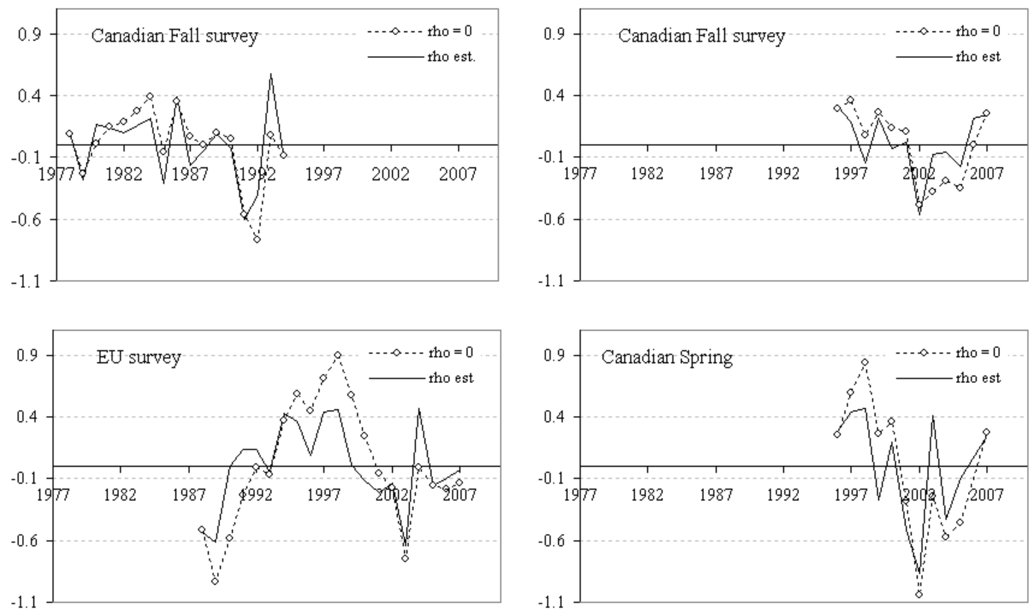

Fig. 6: Survey standardised residuals without (rho=0, baseline assessment B1) and with (rho est) serial correlation.

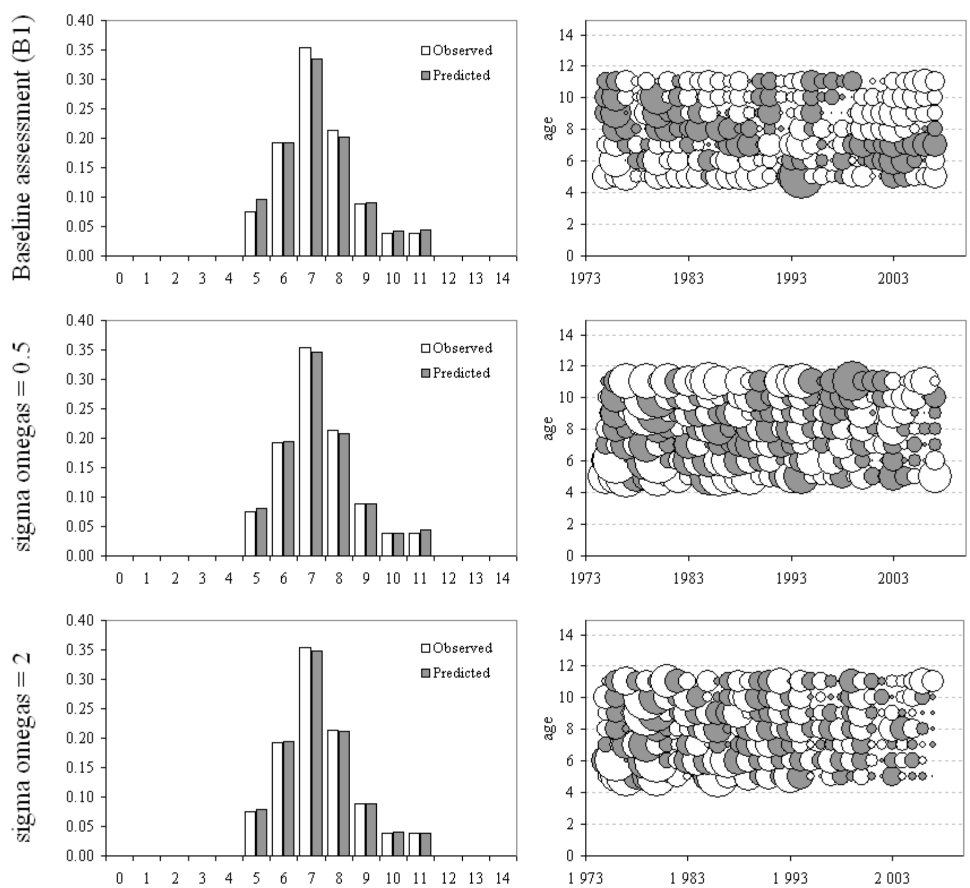

Fig. 6: Fit to commercial CAA for the baseline assessment B1 (top panels) and for two variants (3 and 4) with varying commercial selectivity (in 2 year periods) where the $\Omega_{y,a}$ are estimated.

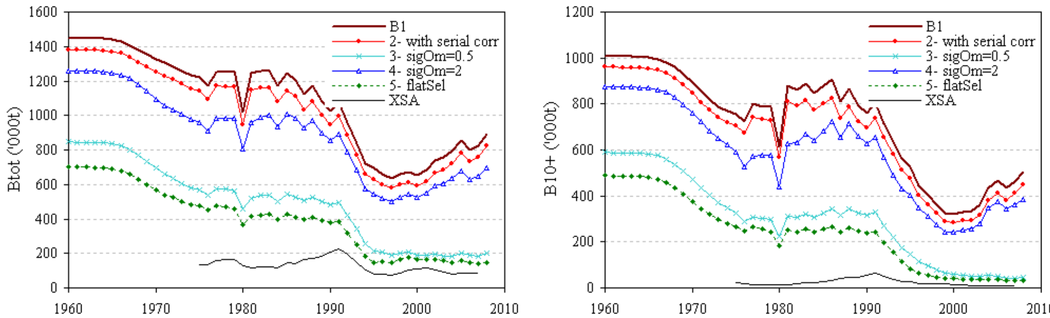

Fig. 8: Comparison of total and 10+ biomass for the five ASPM assessments and the XSA assessment.

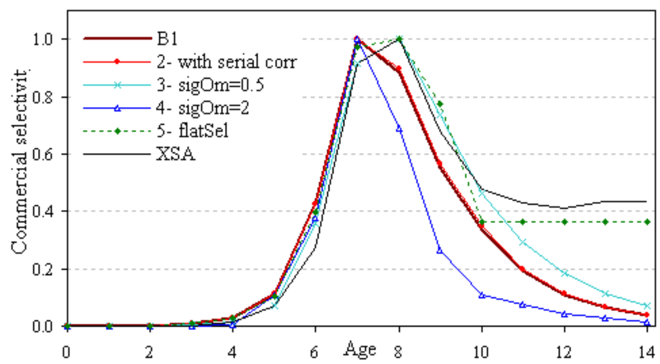

Fig. 9: Comparison of estimated commercial fishing selectivity-at-age (average over 2003-2007) for the five ASPM assessments and the XSA assessment.

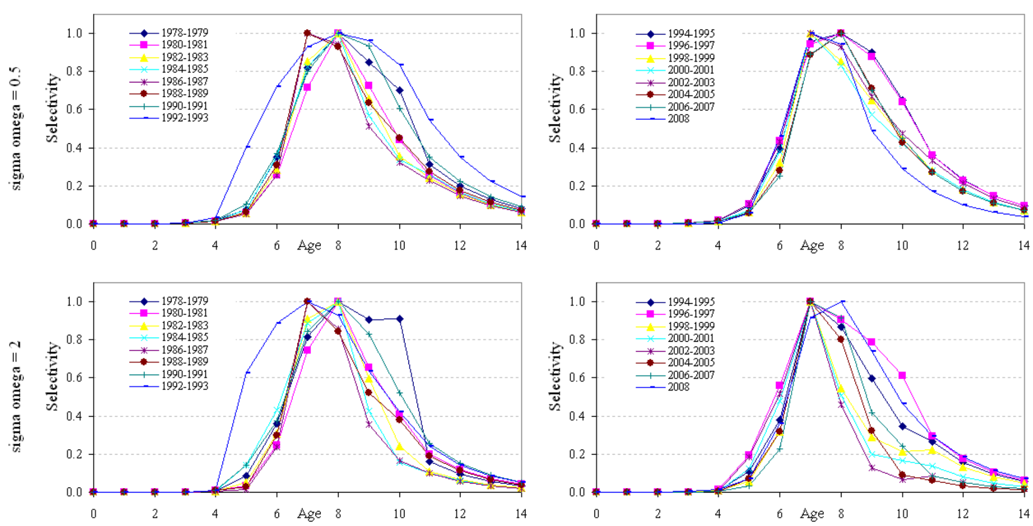

Fig. 10: Estimated commercial selectivities-at-age for the two ASPM assessments (variants 3 and 4) with variations in the selectivity for two-year time periods over time.

APPENDIX A – Data

Table A1: Landings (tons) for Greenland Halibut in Sub-area 2 and Div. 3KLMNO (Healey and Mahé, 2008).

Year	Landings (t)	Year	Landings (t)
1960	938	1984	26711
1961	741	1985	20347
1962	588	1986	17976
1963	1621	1987	32442
1964	4252	1988	19215
1965	10069	1989	20034
1966	19276	1990	47454
1967	26525	1991	65008
1968	32392	1992	63193
1969	37275	1993	62455
1970	36889	1994	51029
1971	24834	1995	15272
1972	30038	1996	18840
1973	29105	1997	19858
1974	27588	1998	19946
1975	28814	1999	24226
1976	24611	2000	34177
1977	32048	2001	38232
1978	39070	2002	34062
1979	34104	2003	35151
1980	32867	2004	25486
1981	30754	2005	23255
1982	26278	2006	23531
1983	27861	2007	22747

Table A2. Catch at age matrix (000s) for Greenland Halibut in Sub-Area 2 and Divisions 3KLMNO (Healey and Mahé, 2008).

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14+
1975	0	0	0	0	334	2819	5750	4956	3961	1688	702	135	279	288
1976	0	0	0	0	17	610	3231	5413	3769	2205	829	260	101	53
1977	0	0	0	0	534	5012	10798	7346	2933	1013	220	130	116	84
1978	0	0	0	0	2982	8415	8970	7576	2865	1438	723	367	222	258
1979	0	0	0	0	2386	8727	12824	6136	1169	481	287	149	143	284
1980	0	0	0	0	209	2086	9150	9679	5398	3828	1013	128	53	27
1981	0	0	0	0	863	4517	9806	11451	4307	890	256	142	43	69
1982	0	0	0	0	269	2299	6319	5763	3542	1684	596	256	163	191
1983	0	0	0	0	701	3557	9800	7514	2295	692	209	76	106	175
1984	0	0	0	0	902	2324	5844	7682	4087	1259	407	143	106	183
1985	0	0	0	0	1983	5309	5913	3500	1380	512	159	99	87	86
1986	0	0	0	0	280	2240	6411	5091	1469	471	244	140	70	117
1987	0	0	0	0	137	1902	11004	8935	2835	853	384	281	225	349
1988	0	0	0	0	296	3186	8136	4380	1288	465	201	105	107	129
1989	0	0	0	0	181	1988	7480	4273	1482	767	438	267	145	71
1990	0	0	0	95	1102	6758	12632	7557	4072	2692	1204	885	434	318
1991	0	0	0	220	2862	7756	13152	10796	7145	3721	1865	1216	558	422
1992	0	0	0	1064	4180	10922	20639	12205	4332	1762	1012	738	395	335
1993	0	0	0	1010	9570	15928	17716	11918	4642	1836	1055	964	401	182
1994	0	0	0	5395	16500	15815	11142	6739	3081	1103	811	422	320	215
1995	0	0	0	323	1352	2342	3201	2130	1183	540	345	273	251	201
1996	0	0	0	190	1659	5197	6387	1914	956	504	436	233	143	89
1997	0	0	0	335	1903	4169	7544	3215	1139	606	420	246	137	89
1998	0	0	0	552	3575	5407	5787	3653	1435	541	377	161	92	51
1999	0	0	0	297	2149	5625	8611	3793	1659	623	343	306	145	151
2000	0	0	0	271	2029	12583	21175	3299	973	528	368	203	129	104
2001	0	0	0	448	2239	12163	22122	5154	1010	495	439	203	156	75
2002	0	0	0	479	1662	7239	17581	6607	1244	659	360	224	126	81
2003	0	0	0	1279	4491	10723	16764	6385	1614	516	290	144	76	85
2004	0	0	0	897	4062	8236	10542	4126	1307	529	289	184	87	75
2005	0	0	0	534	1652	5999	10313	3996	1410	444	244	114	64	46
2006	0	0	0	216	1869	6450	12144	4902	1089	372	136	47	3[illegible]	40
2007	0	0	0	88	570	3732	11912	5414	1230	472	163	80	41	29

Table A3. Catch weights-at-age (kg) matrix for Greenland Halibut in Sub-Area 2 and Divisions 3KLMNO (Healy and Mahé, 2008).

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14+
1975	0.000	0.000	0.126	0.244	0.609	0.760	0.955	1.190	1.580	2.210	2.700	3.370	3.880	5.764
1976	0.000	0.000	0.126	0.244	0.609	0.760	0.955	1.190	1.580	2.210	2.700	3.370	3.880	5.144
1977	0.000	0.000	0.126	0.244	0.609	0.760	0.955	1.190	1.580	2.210	2.700	3.370	3.880	5.992
1978	0.000	0.000	0.126	0.244	0.609	0.760	0.955	1.190	1.580	2.210	2.700	3.370	3.880	5.894
1979	0.000	0.000	0.126	0.244	0.609	0.760	0.955	1.190	1.580	2.210	2.700	3.370	3.880	6.077
1980	0.000	0.000	0.126	0.244	0.514	0.659	0.869	1.050	1.150	1.260	1.570	2.710	3.120	5.053
1981	0.000	0.000	0.126	0.244	0.392	0.598	0.789	0.985	1.240	1.700	2.460	3.510	4.790	7.426
1982	0.000	0.000	0.126	0.244	0.525	0.684	0.891	1.130	1.400	1.790	2.380	3.470	4.510	7.359
1983	0.000	0.000	0.126	0.244	0.412	0.629	0.861	1.180	1.650	2.230	3.010	3.960	5.060	7.061
1984	0.000	0.000	0.126	0.244	0.377	0.583	0.826	1.100	1.460	1.940	2.630	3.490	4.490	7.016
1985	0.000	0.000	0.126	0.244	0.568	0.749	0.941	1.240	1.690	2.240	2.950	3.710	4.850	7.010
1986	0.000	0.000	0.126	0.244	0.350	0.584	0.811	1.100	1.580	2.120	2.890	3.890	4.950	7.345
1987	0.000	0.000	0.126	0.244	0.364	0.589	0.836	1.160	1.590	2.130	2.820	3.600	4.630	6.454
1988	0.000	0.000	0.126	0.244	0.363	0.569	0.805	1.163	1.661	2.216	3.007	3.925	5.091	7.164
1989	0.000	0.000	0.126	0.244	0.400	0.561	0.767	1.082	1.657	2.237	2.997	3.862	4.919	6.370
1990	0.000	0.000	0.090	0.181	0.338	0.546	0.766	1.119	1.608	2.173	2.854	3.731	4.691	6.391
1991	0.000	0.000	0.126	0.244	0.383	0.592	0.831	1.228	1.811	2.461	3.309	4.142	5.333	7.081
1992	0.000	0.000	0.175	0.289	0.430	0.577	0.793	1.234	1.816	2.462	3.122	3.972	5.099	6.648
1993	0.000	0.000	0.134	0.232	0.368	0.547	0.809	1.207	1.728	2.309	2.999	3.965	4.816	6.489
1994	0.000	0.000	0.080	0.196	0.330	0.514	0.788	1.179	1.701	2.268	2.990	3.766	4.882	6.348
1995	0.000	0.000	0.080	0.288	0.363	0.531	0.808	1.202	1.759	2.446	3.122	3.813	4.893	6.790
1996	0.000	0.000	0.161	0.242	0.360	0.541	0.832	1.272	1.801	2.478	3.148	3.856	4.953	6.312
1997	0.000	0.000	0.120	0.206	0.336	0.489	0.771	1.159	1.727	2.355	3.053	3.953	5.108	6.317
1998	0.000	0.000	0.119	0.228	0.373	0.543	0.810	1.203	1.754	2.351	3.095	4.010	5.132	6.124
1999	0.000	0.000	0.176	0.253	0.358	0.533	0.825	1.253	1.675	2.287	2.888	3.509	4.456	5.789
2000	0.000	0.000	0.000	0.254	0.346	0.524	0.787	1.192	1.774	2.279	2.895	3.645	4.486	5.531
2001	0.000	0.000	0.000	0.249	0.376	0.570	0.830	1.168	1.794	2.367	2.950	3.715	4.585	5.458
2002	0.000	0.000	0.217	0.251	0.369	0.557	0.841	1.193	1.760	2.277	2.896	3.579	4.407	5.477
2003	0.000	0.000	0.188	0.247	0.389	0.564	0.822	1.199	1.651	2.166	2.700	3.404	4.377	5.409
2004	0.000	0.000	0.180	0.249	0.376	0.535	0.808	1.196	1.629	2.146	2.732	3.538	4.381	5.698
2005	0.000	0.000	0.252	0.301	0.396	0.564	0.849	1.247	1.691	2.177	2.705	3.464	4.264	5.224
2006	0.000	0.000	0.129	0.267	0.405	0.605	0.815	1.092	1.495	1.874	2.396	3.139	3.747	4.701
2007	0.000	0.000	0.000	0.276	0.389	0.581	0.833	1.137	1.500	1.948	2.607	3.057	3.869	4.954

Table A4: Proportion mature-at-age for Greenland Halibut in Sub-Area 2 and Divisions 3KLMNO (Healy pers. comm.). Note in the assessment, the maturity-at-age in 2008 and pre-1975 is taken as the average over the 1975-2007 period.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14+
1975	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.04	0.04	0.03	0.12	0.21	0.34	0.50	0.77
1976	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.06	0.07	0.06	0.21	0.34	0.50	0.72
1977	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.01	0.04	0.11	0.12	0.14	0.34	0.50	0.79
1978	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.04	0.03	0.08	0.18	0.20	0.29	0.50	0.78
1979	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.06	0.06	0.16	0.28	0.31	0.50	0.80
1980	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.04	0.11	0.12	0.28	0.41	0.45	0.76
1981	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03	0.07	0.18	0.23	0.45	0.55	0.76
1982	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.06	0.13	0.28	0.40	0.63	0.77
1983	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.07	0.12	0.24	0.40	0.59	0.80
1984	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.04	0.11	0.21	0.38	0.54	0.84
1985	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.07	0.19	0.35	0.56	0.78
1986	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.05	0.13	0.30	0.51	0.79
1987	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.10	0.22	0.43	0.77
1988	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.04	0.15	0.17	0.34	0.71
1989	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.02	0.09	0.33	0.29	0.57
1990	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.05	0.16	0.08	0.21	0.58	0.52
1991	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.15	0.97	0.25	0.41	0.74
1992	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.05	0.38	1.00	0.56	0.73
1993	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.10	0.11	0.68	1.00	0.84
1994	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.06	0.26	0.25	0.88	0.99
1995	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03	0.17	0.53	0.47	0.98
1996	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.04	0.08	0.36	0.78	0.80
1997	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.10	0.20	0.61	0.91
1998	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.10	0.21	0.43	0.86
1999	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.09	0.21	0.41	0.80
2000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.04	0.12	0.21	0.41	0.73
2001	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.07	0.18	0.53	0.41	0.69
2002	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.07	0.36	0.53	0.90	0.71
2003	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.17	0.82	0.85	0.93
2004	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.24	0.35	0.97	0.97
2005	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.24	0.56	0.58	1.00
2006	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.24	0.56	0.80	0.86
2007	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.24	0.49	0.79	0.94

Table A5: Survey data (mean numbers per tow) of Greenland Halibut in Sub-Area 2 and Divisions 3KLMNO. Decimalized year reflects the timing of each survey series (e.g. EU Summer survey). (Healey and Mahé, 2008)
Note: 1978-1994 2J3K survey data are direct abundance (in ‘000s) and have not been converted to Campelen equivalents as have the rest of the data.

2J3K Canadian Fall, 1978-1994

	1	2	3	4	5	6	7	8	9	10	11	12	13	14+
1978.9	2538	25686	54708	55914	57650	45141	28923	13379	6983	5112	4237	2541	1611	1184
1979.9	2805	22523	28846	25799	35886	38805	18843	7378	3316	3179	2102	1843	1520	1834
1980.9	2994	8911	15315	22680	35995	42154	27942	9511	4207	3229	3601	2393	1551	1419
1981.9	7563	22486	30875	21226	34277	38654	26647	11458	5281	2824	2255	1030	579	450
1982.9	2137	5991	23971	31204	31061	29062	32070	32617	13535	5375	2801	1790	1276	2517
1983.9	1004	5905	19036	31465	40182	34742	38908	31538	11559	3040	2049	1497	1089	1100
1984.9	1452	7148	21435	36094	72180	38931	30683	21712	10222	4132	1869	1216	964	1665
1985.9	7460	18147	20024	36224	44886	37715	22359	12761	6293	3498	1592	1218	517	1337
1986.9	13005	22185	32997	55685	45213	57886	45327	12676	3306	1430	960	961	441	686
1987.9	1491	8685	47694	35752	35854	33486	33956	20722	7621	2156	1065	642	504	461
1988.9	4025	12436	28404	50345	58938	39603	29733	9257	2525	809	542	309	267	480
1989.9	3407	10414	35816	69334	77935	56524	32108	9627	2884	675	558	161	56	173
1990.9	547	5347	14506	68019	65410	48199	28837	6828	1839	718	488	267	160	191
1991.9	5814	6726	11369	37832	38273	27416	9020	2155	475	231	104	61	14	7
1992.9	1684	14858	26664	34313	23316	17109	8406	962	95	48	13	0	0	0
1993.9	7510	62818	97955	46098	18385	6912	2520	739	63	0	0	13	0	0
1994.9	14541	30412	42221	43669	31165	7237	3136	947	114	38	7	0	4	0

2J3K Canadian Fall, 1995-2007

	1	2	3	4	5	6	7	8	9	10	11	12	13+
1996.9	98.68	47.82	32.01	9.54	6.28	2.47	0.84	0.19	0.18	0.04	0.02	0.01	0.02
1997.9	28.05	58.62	43.61	21.13	10.37	5.01	2.00	0.64	0.20	0.06	0.03	0.02	0.01
1998.9	23.35	25.07	31.19	21.87	10.86	4.45	2.07	0.57	0.13	0.06	0.03	0.02	0.01
1999.9	15.99	34.42	24.07	28.28	20.04	10.53	3.81	0.70	0.14	0.07	0.02	0.01	0.03
2000.9	38.57	21.94	16.43	13.20	13.76	7.21	2.16	0.50	0.06	0.03	0.02	0.00	0.00
2001.9	43.90	22.72	17.00	14.07	9.77	7.59	3.40	0.69	0.11	0.02	0.01	0.00	0.01
2002.9	40.67	24.08	12.50	9.68	6.03	1.97	0.72	0.19	0.04	0.01	0.00	0.00	0.00
2003.9	45.70	26.67	11.69	9.49	6.39	2.27	0.89	0.27	0.04	0.02	0.01	0.01	0.00
2004.9	32.49	32.93	13.89	12.31	9.21	2.68	1.20	0.36	0.08	0.03	0.01	0.00	0.01
2005.9	16.06	16.15	8.56	13.84	10.98	6.85	3.96	0.66	0.12	0.03	0.03	0.01	0.01
2006.9	32.34	17.98	8.50	17.60	13.03	9.11	4.18	1.15	0.18	0.03	0.02	0.01	0.00
2007.9	32.61	14.51	12.81	18.77	9.57	10.35	6.17	2.14	0.34	0.08	0.04	0.02	0.01

EU Summer, 1995-2007

	1	2	3	4	5	6	7	8	9	10	11	12+
1995.6	12.41	2.54	2.23	1.91	2.66	5.10	3.77	2.12	1.31	0.26	0.07	0.02
1996.6	5.84	7.97	2.42	3.04	4.20	5.82	2.49	1.62	0.42	0.09	0.03	0.04
1997.6	3.33	3.78	6.00	6.50	7.11	8.46	4.99	2.15	0.66	0.22	0.03	0.02
1998.6	2.74	2.13	7.69	11.00	12.33	11.30	7.84	2.62	0.75	0.20	0.03	0.01
1999.6	1.06	0.70	3.01	10.47	13.41	12.58	5.55	1.82	0.35	0.10	0.01	0.00
2000.6	3.75	0.29	0.60	2.17	7.09	14.10	5.40	2.32	0.45	0.11	0.05	0.00
2001.6	8.03	1.43	1.81	0.99	2.79	7.79	6.63	3.21	0.18	0.05	0.01	0.00
2002.6	4.08	2.94	2.80	1.67	3.79	5.59	5.73	1.28	0.13	0.06	0.02	0.01
2003.6	2.20	1.00	0.61	1.51	2.48	2.94	1.93	0.47	0.13	0.10	0.02	0.01
2004.6	2.19	3.29	4.37	1.97	6.97	7.80	2.54	0.64	0.29	0.13	0.08	0.05
2005.6	0.54	0.81	3.18	2.50	6.89	7.59	2.92	0.61	0.11	0.12	0.06	0.02
2006.6	0.68	0.40	0.65	1.17	5.98	7.46	3.31	0.77	0.22	0.18	0.13	0.06
2007.6	0.42	0.09	0.57	0.34	3.44	7.37	5.76	1.51	0.31	0.21	0.08	0.05

3LNO Canadian Spring, 1996-2007

	1	2	3	4	5	6	7	8+
1996.4	1.62	4.24	4.60	2.18	0.83	0.28	0.06	0.00
1997.4	1.16	3.92	5.16	3.23	1.46	0.51	0.10	0.01
1998.4	0.22	0.81	3.85	6.19	4.96	1.24	0.33	0.07
1999.4	0.29	0.55	1.15	1.98	3.39	1.09	0.24	0.05
2000.4	0.79	1.07	1.07	1.51	1.95	2.04	0.56	0.03
2001.4	0.57	0.71	0.74	0.68	0.80	0.72	0.28	0.02
2002.4	0.64	0.57	0.60	0.58	0.61	0.21	0.05	0.01
2003.4	0.93	2.14	1.66	1.57	1.06	0.21	0.05	0.01
2004.4	0.66	0.57	1.18	1.18	1.16	0.26	0.04	0.02
2005.4	0.35	0.31	1.09	0.95	1.37	0.82	0.21	0.03
2006.4			Survey not completed					
2007.4	1.60	0.52	0.80	0.40	1.41	1.49	1.12	0.18

Table A6: Survey data in terms of weight for ages combined: 2J3K Fall and 3LNO Spr (Healey, 2008), EU survey (Vázquez and González-Troncoso, 2008).

Year	2J3K Fall Mean weight (kg)/tow	EU survey Biomass (tons)	3LNO - Spr Mean weight (kg)/tow
1978	38.4		
1979	28.1		
1980	30.0		
1981	32.1		
1982	35.6		
1983	36.9		
1984	37.2		
1985	27.5		
1986	35.4		
1987	25.5		
1988	23.6	6926	
1989	25.4	4472	
1990	21.2	5799	
1991	11.5	8169	
1992	8.2	8728	
1993	15.3	6529	
1994	10.8	8037	
1995	14.1	10875	
1996	21.6	11594	1.43
1997	24.8	16098	2.10
1998	23.8	24229	3.50
1999	32.5	21207	2.33
2000	23.9	16959	2.30
2001	22.7	13872	1.13
2002	14.1	12100	0.53
2003	15.3	6214	1.13
2004	17.5	12292	0.87
2005	20.3	11698	1.23
2006	25.7	11706	
2007	29.1	13040	2.17

Appendix B - The Age-Structured Production Model

The model used for these assessments is an Age-Structured Production Model (ASPM) (e.g. Hilborn, 1990). Models of this type fall within the more general class of Statistical Catch-at-Age Analyses. The approach used in an ASPM assessment involves constructing an age-structured model of the population dynamics and fitting it to the available abundance indices by maximising the likelihood function. The model equations and the general specifications of the model are described below, followed by details of the contributions to the (penalised) log-likelihood function from the different sources of data available and assumptions concerning the stock-recruitment relationship. Quasi-Newton minimization is used to minimize the total negative log-likelihood function (the package AD Model Builder™, Otter Research, Ltd is used for this purpose).

B.1. Population dynamics

B.1.1 Numbers-at-age

The resource dynamics are modelled by the following set of population dynamics equations:

$$N_{y+1,1} = R_{y+1} \tag{B1}$$

$$N_{y+1,a+1} = \left(N_{y,a}\, e^{-M_a/2} - C_{y,a}\right) e^{-M_a/2} \qquad \text{for } 1 \le a \le m-2 \tag{B2}$$

$$N_{y+1,m} = \left(N_{y,m-1}\, e^{-M_{m-1}/2} - C_{y,m-1}\right) e^{-M_{m-1}/2} + \left(N_{y,m}\, e^{-M_m/2} - C_{y,m}\right) e^{-M_m/2} \tag{B3}$$

where

$N_{y,a}$ is the number of fish of age *a* at the start of year *y* (which refers to a calendar year),

R_y is the recruitment (number of 0-year-old fish) at the start of year *y*,

M_a denotes the natural mortality rate for fish of age *a*,

$C_{y,a}$ is the predicted number of fish of age *a* caught in year *y*, and

m is the maximum age considered (taken to be a plus-group).

These equations reflect Pope's form of the catch equation (Pope, 1972) (the catches are assumed to be taken as a pulse in the middle of the year) rather than the more customary Baranov form (Baranov, 1918) (for which catches are incorporated under the assumption of steady continuous fishing mortality). Pope's form has been used in order to simplify computations. As long as mortality rates are not too high, the differences between the Baranov and Pope formulations will be minimal.

B.1.2. Recruitment

The number of recruits at the start of year *y* is assumed to be related to the spawning stock size (i.e. the biomass of mature fish) by a Beverton-Holt stock-recruitment relationship (Beverton and Holt, 1957), parameterised in terms of the "steepness" of the stock-recruitment relationship, *h*, and the pre-exploitation equilibrium spawning biomass, K^{sp}, and recruitment, R_0 and allowing for annual fluctuation about the deterministic relationship:

$$R_y = \frac{4hR_0 B_y^{sp}}{K^{sp}(1-h) + (5h-1)B_y^{sp}}\, e^{(\varsigma_y - \sigma_R^2/2)} \tag{B4}$$

where

ς_y reflects fluctuation about the expected recruitment for year y, which is assumed to be normally distributed with standard deviation σ_R (which is input in the applications considered here); these residuals are treated as estimable parameters in the model fitting process.

B_y^{sp} is the spawning biomass at the start of year y, computed as:

$$B_y^{sp} = \sum_{a=1}^{m} f_{y,a} w_{y,a}^{strt} N_{y,a} \tag{B5}$$

where

$w_{y,a}^{strt}$ is the mass of fish of age a during spawning, and

$f_{y,a}$ is the proportion of fish of age a that are mature.

In the fitting procedure, K^{sp} is estimated while h has thus far been fixed at 0.9 for reasons elaborated in the main text.

B.1.3. Total catch and catches-at-age

The catch by mass in year y is given by:

$$C_y = \sum_{a=1}^{m} w_{y,a}^{mid} \, C_{y,a} = \sum_{a=1}^{m} w_{y,a}^{mid} \, N_{y,a} \, e^{-M_a/2} \, S_{y,a} \, F_y \tag{B6}$$

where

$w_{y,a}^{mid}$ denotes the mass of fish of age a landed in year y,

$C_{y,a}$ is the catch-at-age, i.e. the number of fish of age a, caught in year y,

$S_{y,a}$ is the commercial selectivity (i.e. combination of availability and vulnerability to fishing gear) at age a for year y; when $S_{y,a} = 1$, the age-class a is said to be fully selected, and

F_y is the proportion of a fully selected age class that is fished.

The model estimate of the mid-year exploitable ("available") component of biomass is calculated by converting the numbers-at-age into mid-year mass-at-age (using the individual weights of the landed fish) and applying natural and fishing mortality for half the year:

$$B_y^{ex} = \sum_{a=1}^{m} w_{y,a}^{mid} S_{y,a} N_{y,a} e^{-M_a/2} (1 - S_{y,a} F_y \;/ 2) \tag{B7}$$

whereas for survey estimates of biomass in spring:

$$B_y^{surv,spring} = \sum_{a=1}^{m} w_{y,a}^{mid} S_a^{surv} N_{y,a} e^{-M_a/4} \left(1 - S_{y,a} \, F_y \;/ 4\right) \tag{B8}$$

Summer:

$$B_y^{surv,summer} = \sum_{a=1}^{m} w_{y,a}^{mid} S_a^{surv} N_{y,a} e^{-M_a/2} \left(1 - S_{y,a}\, F_y \,/2\right) \quad \text{(B9)}$$

and fall:

$$B_y^{surv,fall} = \sum_{a=1}^{m} w_{y,a}^{mid} S_a^{surv} N_{y,a} e^{-M_a 3/4} \left(1 - S_{y,a}\, F_y \;3/4\right) \quad \text{(B10)}$$

where

S_a^{surv} is the survey selectivity for age a (which is sometimes generalised to be year-dependent).

B.1.4. Initial conditions

For the first year (y_0) considered in the model therefore, the stock is assumed to be at a fraction (θ) of its pre-exploitation biomass, i.e.:

$$B_{y_0}^{sp} = \theta \cdot K^{sp} \quad \text{(B11)}$$

with the starting age structure:

$$N_{y_0,a} = R_{start} N_{start,a} \qquad \text{for } \; 1 \le a \le m \quad \text{(B12)}$$

where

$$N_{start,1} = 1 \quad \text{(B13)}$$

$$N_{start,a} = N_{start,a-1} e^{-M_{a-1}} (1 - \phi S_{a-1}) \qquad \text{for } \; 2 \le a \le m-1 \quad \text{(B14)}$$

$$N_{start,m} = N_{start,m-1} e^{-M_{m-1}} (1 - \phi S_{m-1}) \Big/ (1 - e^{-M_m} (1 - \phi S_m)) \quad \text{(B15)}$$

where ϕ characterises the average fishing proportion over the years immediately preceding y_0.

Unless indicated otherwise though, the stock is assumed to be at pristine equilibrium in 1960, i.e. θ=1 and ϕ=0 for the results reported here.

B.2. The (penalised) likelihood function

The model can be fit to (a subset of) CPUE and survey abundance indices, and commercial and survey catch-at-age data to estimate model parameters (which may include residuals about the stock-recruitment function, the fishing selectivities, the annual catches or natural mortality, facilitated through the incorporation of penalty functions described below). Contributions by each of these to the negative of the (penalised) log-likelihood (- $\ell\mathrm{n}L$) are as follows.

B.2.1 CPUE relative abundance data

The likelihood is calculated assuming that an observed CPUE index for a particular fishing fleet is log-normally distributed about its expected value:

$$I_y^i = \hat{I}_y^i \exp\left(\varepsilon_y^i\right) \quad \text{or} \quad \varepsilon_y^i = \ell\mathrm{n}\left(I_y^i\right) - \ell\mathrm{n}\left(\hat{I}_y^i\right) \quad \text{(B16)}$$

where

I_y^i is the CPUE index for year y and series i,

$\hat{I}_y^i = \hat{q}^i \hat{B}_y^{ex}$ is the corresponding model estimate, where $\widehat{B}_y^{ex}$ is the model estimate of exploitable resource biomass, given by equation (B7)[1],

$\hat{q}^i$ is the constant of proportionality (catchability) for CPUE series i, and

ε_y^i from $N\left(0,\left(\sigma_y^i\right)^2\right)$.

The contribution of the CPUE data to the negative of the log-likelihood function (after removal of constants) is then given by:

$$-\ell n L^{CPUE} = \sum_i \sum_y \left[\ell n\left(\sigma_y^i\right) + \left(\varepsilon_y^i\right)^2 / 2\left(\sigma_y^i\right)^2 \right] \tag{B17}$$

where

σ_y^i is the standard deviation of the residuals for the logarithm of index i in year y.

Homoscedasticity of residuals is assumed, so that $\sigma_y^i = \sigma^i$ is estimated in the fitting procedure by its maximum likelihood value:

$$\hat{\sigma}^i = \sqrt{1/n_i \sum_y \left(\ell n(I_y^i) - \ell n(q^i \widehat{B}_y^{ex})\right)^2} \tag{B18}$$

where

n_i is the number of data points for CPUE index i.

The catchability coefficient q^i for CPUE index i is estimated by its maximum likelihood value:

$$\ell n\, \hat{q}^i = 1/n_i \sum_y \left(\ln I_y^i - \ln \hat{B}_y^{ex}\right) \tag{B19}$$

B.2.2. Survey abundance data

In general, data from the surveys are treated as relative abundance indices in exactly the same manner to the CPUE series above, with survey selectivity function S_a^{surv} replacing the commercial selectivity $S_{y,a}$. Account is also taken of the time of year when the survey is held. For these analyses, selectivities are estimated as detailed in section B.4.2 below.

To allow for serial correlation between the survey residuals, the ρ input to equation B17 is given by::

$$\varepsilon_y^i = \lambda_y^i - \rho \lambda_{y-1}^i \tag{B20}$$

where

$$\lambda_y^i = \ell n\left(I_y^i\right) - \ell n\left(\hat{I}_y^i\right)$$

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[1] Ideally $\widehat{B}_y^{ex}$ should be fleet specific, corresponding to the selectivity for the fleet linked to CPUE index i. However, this requires the total annual catch and catch-at-age data to be provided on a fleet-disaggregated basis, and these data are not immediately available in this form.

ρ is the serial correlation coefficient, which is estimated (or set to zero in the case of the Baseline assessment B1). Note that ρ could be series dependent, but analyses for the data set available indicated that estimation of series-specific values was not justified in AIC terms. The standard deviation of the ε^i_y is termed σ_{surv} in Table 1.

B.2.3. Commercial catches-at-age

The contribution of the catch-at-age data to the negative of the log-likelihood function under the assumption of an "adjusted" lognormal error distribution is given by:

$$-\ell n L^{CAA} = \sum_y \sum_a \left[\ell n\left(\sigma_{com} / \sqrt{p_{y,a}}\right) + p_{y,a}\left(\ell n\, p_{y,a} - \ell n\, \hat{p}_{y,a}\right)^2 / 2\left(\sigma_{com}\right)^2 \right] \quad \text{(B21)}$$

where

$p_{y,a} = C_{y,a} / \sum_{a'} C_{y,a'}$ is the observed proportion of fish caught in year y that are of age a,

$\hat{p}_{y,a} = \hat{C}_{y,a} / \sum_{a'} \hat{C}_{y,a'}$ is the model-predicted proportion of fish caught in year y that are of age a,

where

$$\hat{C}_{y,a} = N_{y,a}\; e^{-M_a/2}\, S_{y,a}\, F_y \quad \text{(B22)}$$

and

σ_{com} is the standard deviation associated with the catch-at-age data (termed "σ_{comCAA}" in Table 1), which is estimated in the fitting procedure by:

$$\hat{\sigma}_{com} = \sqrt{\sum_y \sum_a p_{y,a}\left(\ell n\, p_{y,a} - \ell n\, \hat{p}_{y,a}\right)^2 / \sum_y \sum_a 1} \quad \text{(B23)}$$

The log-normal error distribution underlying equation (B21) is chosen on the grounds that (assuming no ageing error) variability is likely dominated by a combination of interannual variation in the distribution of fishing effort, and fluctuations (partly as a consequence of such variations) in selectivity-at-age, which suggests that the assumption of a constant coefficient of variation is appropriate. However, for ages poorly represented in the sample, sampling variability considerations must at some stage start to dominate the variance. To take this into account in a simple manner, motivated by binomial distribution properties, the observed proportions are used for weighting so that undue importance is not attached to data based upon a few samples only.

Commercial catches-at-age are incorporated in the likelihood function using equation (B21), for which the summation over age a is taken from age a_{minus} (considered as a minus group) to a_{plus} (a plus group).

B.2.4. Survey catches-at-age

The survey catches-at-age are incorporated into the negative of the log-likelihood in an analogous manner to the commercial catches-at-age, assuming an adjusted log-normal error distribution (equation (B21)) where:

$p_{y,a} = C^{surv}_{y,a} / \sum_{a'} C^{surv}_{y,a'}$ is the observed proportion of fish of age a in year y,

$\hat{p}_{y,a}$ is the expected proportion of fish of age a in year y in the survey, given by:

$$\hat{p}_{y,a} = S_a^{surv} N_{y,a} \Big/ \sum_{a'=0}^{m} S_a^{surv} N_{y,a} \qquad \text{for begin-year surveys.} \tag{B24}$$

The residual standard deviation (analogous to σ_{com}, alternatively termed "σ_{comCAA}" in the previous section, is termed σ_{surv} in Table 1.

B.2.5. Stock-recruitment function residuals

The stock-recruitment residuals are assumed to be log-normally distributed. Thus, the contribution of the recruitment residuals to the negative of the (now penalised) log-likelihood function is given by:

$$-\ell n L^{SRpen} = \sum_{y=y1}^{y2} \left[\varepsilon_y^2 / 2\sigma_R^2 \right] \tag{B25}$$

where

ε_y from $N\left(0,(\sigma_R)^2\right)$, which is estimated for year $y1$ to $y2$ (see equation (B4)), and

σ_R is the standard deviation of the log-residuals, which is input.

B.2.6. Selectivity residuals

In some instances, variations around the fishing selectivity functions are estimated in two-year periods.

$$S_a \to S_{y,a} = S_a e^{\Omega_{z,a}} \tag{B26}$$

The contribution of the selectivity residuals to the negative of the penalised log-likelihood is given by:

$$-\ell n L^{Selpen} = \sum_{y=y1}^{y2} \sum_{a=a1}^{a2} \left[\Omega_{y,a}^2 / 2\sigma_\Omega^2 \right] \tag{B27}$$

where

$\Omega_{y,a}$ from $N\left(0,(\sigma_\Omega)^2\right)$, which is estimated for year $y1$ to $y2$ and age $a1$ to $a2$, and

σ_Ω is the standard deviation of the residuals, which is input.

B.2.7. Annual catch residuals

In some instances, differences between the reported and the actual annual catches are estimated.

$$C_y \to C_y^{reported} = C_y^{actual} e^{\omega_y} \tag{B28}$$

The contribution of the catch residuals to the negative of the penalised log-likelihood is given by:

$$-\ell n L^{Cpen} = \sum_{y=y1}^{y2} \left[\omega_y^2 / 2\sigma_C^2 \right] \tag{B29}$$

where

ϖ_y from $N\left(0,(\sigma_C)^2\right)$, which is estimated for year $y1$ to $y2$, and

σ_ω is the standard deviation of the residuals, which is input.

B.2.7. Mortality residuals

In some instances, variations about the default age- and year-independent natural mortality are estimated.

$$M_a \rightarrow M_a e^{\varsigma_{y,a}} \qquad \text{(B30)}$$

The contribution of the catch residuals to the negative of the penalised log-likelihood is given by:

$$-\ell n L^{Mpen} = \sum_{y=y1}^{y2} \sum_{a=a1}^{a2} \left[\varsigma_{y,a}^2 / 2\sigma_M^2 \right] \qquad \text{(B31)}$$

where

$\xi_{y,a}$ from $N\left(0, (\sigma_M)^2\right)$, which is estimated for year $y1$ to $y2$, and age $a1$ to $a2$, and

σ_M is the standard deviation of the residuals, which is input.

B.3. Estimation of precision

Where quoted, CVs are Hessian-based.

B.4. Model parameters

B.4.1. Fishing selectivity-at-age:

The commercial fishing selectivity, S_a, is estimated separately for ages 5-12. The estimated decreases from ages 6 to 5 and ages 11 to 12 are assumed to continue exponentially to ages 0 and 14+ respectively. Similarly, the selectivities for the surveys are estimate separately for ages 1-11 for the Canadian Fall and EU surveys and for ages 1-8 for the Canadian spring surveys. The estimated decreases from ages 2 to 1 and from ages 10 to 11 (7 to 8 for the Canadian spring survey) are assumed to continue exponentially to ages 0 and 14+ respectively.

B.4.2. Other parameters

Plus group:	
m	14
Commercial CAA:	
a_{minus}	5
a_{plus}	12
Survey CAA:	
a_{minus}	1
a_{plus}	11/8
Stock-recruitment residuals:	
σ_R	0.25
y_1	1960
y_2	2008
Natural mortality:	
M	age independent, fixed at M=0.2 (unless otherwise specified)
Age-at-maturity:	
$f_{y,a}$	input, see Table A4
Weight-at-age:	
$w_{y,a}$	input, same for begin-and mid-year, see Table A3
Initial conditions:	
θ	1 (unless otherwise specified)
ϕ	0 (unless otherwise specified)
Survey serial correlation:	
ρ	0 (unless otherwise specified)

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