Statistics Masters Thesis

Nelson Siegel Parameterisation of the South African Sovereign Yield Curve:

An exploration of its predictors, a link to the main asset classes, and implementation of systematic trading strategies

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1. Introduction

1.1 Problem Identification

While various studies into Emerging Market debt have sought to explain sovereign bond market spreads and their economic, market-based, and fundamental predictors (Baek, Bandopadhyaya & Du, 2005; Min, Lee, Nam, Park, & Nam, 2003; Weigel & Gemmill, 2006), there is a dearth of research into the specifically South African (SA) sovereign yield curve.

A few papers (Sy, 2002; Erb, Harvey, & Viskanta, 1999) include SA bond market yields in their analysis, but such research is limited for two main reasons. Firstly, these papers focus on South African US Dollar-denominated debt issued in international markets and their market spreads relative to US Treasury Bonds. This is definitely more accurate to include in an Emerging Market comparative analysis of "pure sovereign risk" than it would be to look at adjusted local-currency bond spreads, however ultimately such research does not tap into a major portion of South African sovereign debt in issuance. As of end-2013, the South African Government had R1.4bn nominal outstanding in locally-issued Rand bonds, and 87% of all Government marketable debt was attributable to Domestic Rand-Denominated issuance (see Figure 1). That being said, academic research into local ZAR-denominated debt is sorely missing.

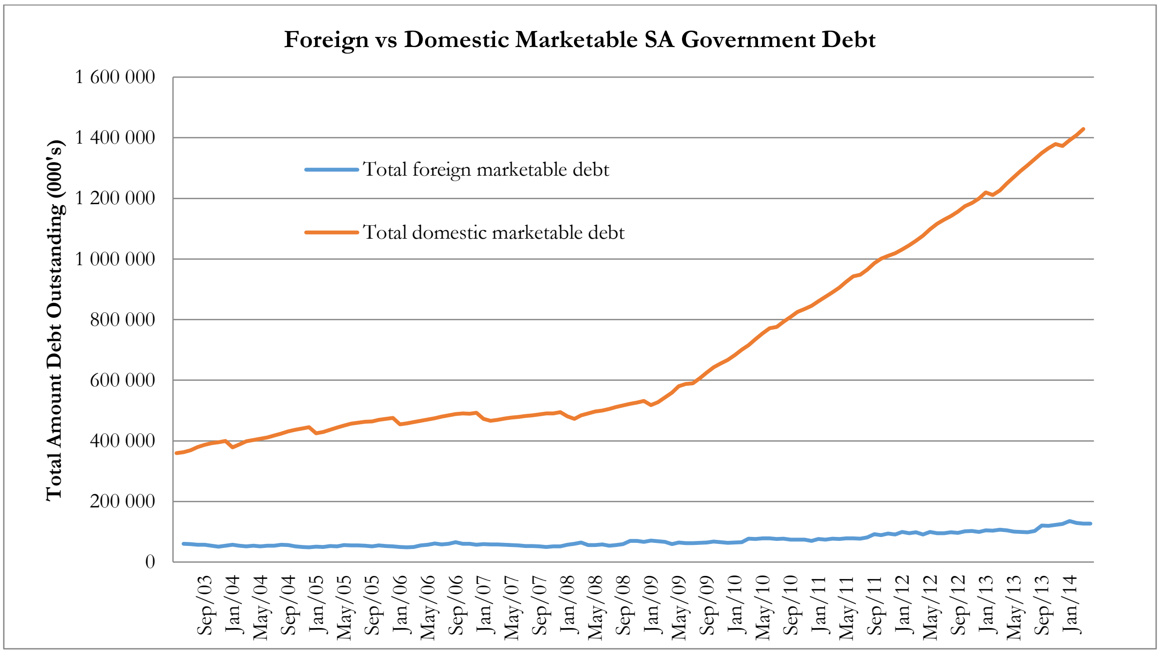


Figure 1: Foreign versus Domestic Marketable SA Government Debt, Nominal Outstanding

Source: South Africa National Treasury database

A second limitation of such research is that it focuses on government bond spreads derived from yield levels only, and largely ignores the other parameters of the yield curve – namely, slope and curvature – and their predictors. This is problematic as such research essentially considers only parallel shifts in the yield curve such that all yields increase / decrease at once, but does not capture yield curve movements owing to changes in the slope (the steepness / flatness of the curve) and the curvature (the relative concavity). Furthermore, even in the few papers where the local South African yield curve slope is looked at as a distinct factor, it is simply estimated as the difference between long-end and short-end rates (Khomo & Aziakpono, 2007; Mehl, 2009), as opposed to the more complex yield curve parameterisation techniques adopted in international literature.

1.2 Aims of the Present Research

The aims of this research are firstly to model the South African Local Government Bond Yield curve according to the Nelson Siegel Parameterisation framework, as implemented in the pivotal work of Diebold and Li (2006) in forecasting the US Treasury curve. In Part 1 of this paper, this yield curve factorization technique will be compared to that of the Svensson (1994) approach, which includes the addition of a 4th parameter also relating to the curvature.

Once the distinct yield curve factors (namely the Level, Slope, and Curvature) have been identified, Part 2 of this paper will investigate the economic, fundamental, and market-based predictors of these parameters via the method of Ordinary Least Squares Regression. This effectively translates such papers as Fabozzi, Martellini, and Priaulet (2005), which focuses on economic and market-based predictors of the US swap curve parameters, for the South African context.

This portion of the analysis also addresses papers such as those of Mehl (2009), Sy (2002), and Erb, Harvey, & Viskanta (1999), which all include SA yields. This research, however, will employ a narrower focus of concentrating specifically on the South African context versus the broader Emerging Market arena, and with the implementation of more complex parameterisation techniques versus simply looking at yield levels only and /or estimating the slope as simplistically as the difference in long-end and short-end rates. One of the aims of this predictive model will also be to ascertain whether global risk sentiment factors or in-country solvency fundamentals are ultimately the better predictor of the shape of the SA government bond curve. This contributes to the discourse of various papers seeking to ascertain whether increasing globalization of the investment community has led to broad-based market risk appetite being a better predictor of sovereign credit curves than the government's own solvency and creditworthiness (Baek, Bandopadhyaya, and Du, 2005; Longstaff, Pan, Pedersen, & Singleton, 2007). Such research has important ramifications for the level of contagion risk amongst the various asset classes in the market and the success of investment managers in using South African Government debt for portfolio diversification purposes.

While the above "two-part" Parameterisation exercise and OLS Regression analysis undoubtedly has some statistical merit, Part 3 of this paper mirrors Diebold, Rudebusch, and Aruoba's (2006) "one-step" Dynamic Latent Factor approach to modeling the US bond yield curve. Using state-space modeling and a nonstructural Vector Autoregressive (VAR) framework, the authors concurrently fit the yield curve parameters and estimate their "lagged-value" and macroeconomic predictors. Using a Kalman Filter, they arrive at optimal maximum-likelihood estimates of these state parameters.

In Part 4 of this research, an analysis linking the South African asset classes (namely Local Equities, Currency, and Bonds) will be conducted. More specifically, the total returns of the various Johannesburg Stock Exchange (JSE) Derivative Indices and Top 40 share returns will be regressed against the All Share Index Total Returns, the USDZAR exchange rate, and the derived local Government Bond Yield Curve factors. Classification of individual JSE shares according to the *Rand Hedge, Rand Leverage,* and *Rand Play* currency categories has been shown in past literature to be a consistent explanatory framework for predicting the share returns of JSE Top 40 companies in relation to exchange rate movements (Barr & Kantor, 2005). However, little research has looked at classification of SA shares according to changes in the shape of the government bond yield curve. In order to get an indication of the robustness and consistency of these fitted models, a 48-month rolling window period will be used.

Finally, in Part 5 of this paper, Nelson Siegel Parameterisation will be adopted for the South African Interest Rate swap curve with a view to develop multiple predictive OLS Regression models of the curve parameters over time. The efficacy of systematic swap trading strategies will then be investigated based on forecasts of the Swap Level, Slope, and Curvature factors extracted from these models. This mirrors the pivotal work of Fabozzi, Martellini, and Priaulet (2005), who perform the same analysis on the US swap rate curve for the period June 1994 to September 2003. Their trading algorithm yields monthly trading returns that ultimately have the greatest relevance for Fixed-Income houses focused on short-term trading strategies, such as Hedge Funds and investment banking proprietary trading desks.

2. Literature Review

2.1 Part 1. Nelson Siegel Parameterisation of the Sovereign Yield Curve

A seminal paper in the field of Government Bond yield curve parameterisation is that of Diebold and Li (2006), in which they adopt the Nelson Siegel structure to model the yield curve dynamically over time. This approach is now widely used by various Central Banks and Fixed Income Portfolio Managers (Annaert, Claes, Ceuster, & Zhang, 2013). Diebold and Li's research was one of the first academic papers to address the issue of forecasting the yield curve over time, as previous work mainly focused on modeling the term structure of interest rates instantaneously at a particular point by applying such principles as the no-arbitrage theorem (Diebold & Li, 2006). By factorizing the yield curve under the Nelson Siegel framework, the authors fitted a constant term and a Laguerre function – which consists of a series of polynomials used to explain second-order linear differential equations and allows one to model the term structure using an exponential time-decay curve. The authors explain that this is clearly important so as to forecast changes in the yield curve over various time buckets.

Diebold and Li (2006) are able to parameterise the yield curve into the three distinct level, slope, and curvature parameters by fitting the following model:

$$y_t(\tau) = \beta_{1t} + \left(\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau}\right)\beta_{2t} + \left(\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau}\right)\beta_{3t}$$

Where $y_t(\tau)$ is the spot rate of the US Treasury bond at time t that matures at "time to maturity" τ

(Diebold & Li, 2006)

(i.e. for $\boldsymbol{t}$ *= 30/06/2005 and* $\boldsymbol{\tau}$ *= 2years,* $y_t(\tau)$ *will be the closing level of the 2 year zero coupon spot rate taken as at 30/06/2005)*

The authors assert that the first constant factor loading for β_{1t} is clearly best described by the overall **Level** of yields across the curve, and represents a long-term factor – as seen in Figure 2.

The second factor loading $\left(\frac{1- e^{-\lambda_t \tau}}{\lambda_t \tau}\right)$ can be interpreted as the **Slope** of the curve (or very loosely as the level of long-term rates minus short-term rates) due to its properties of decreasing monotonically (i.e. in a non-increasing fashion) over time to zero (see Figure 2). Note that this definition assumes an inverted-yield curve shape whose slope is steepest for near-maturity buckets and whose level gradually decreases over time at smaller (or equal) rates of change until such time as the slope is flat at zero at the long-end of the curve. This means that it can be thought of as the level of short-term rates minus long-term rates. The authors describe the slope parameter as representing short-term yield curve changes.

The third factor $\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right)$ is interpreted by the authors as the **Curvature** parameter, which represents an intermediate-term shape factor. This parameter starts at zero, gradually increases, and then returns to zero over time. It thus measures the "humpedness" or concavity of the yield curve, and the point at which the slope is maximized.

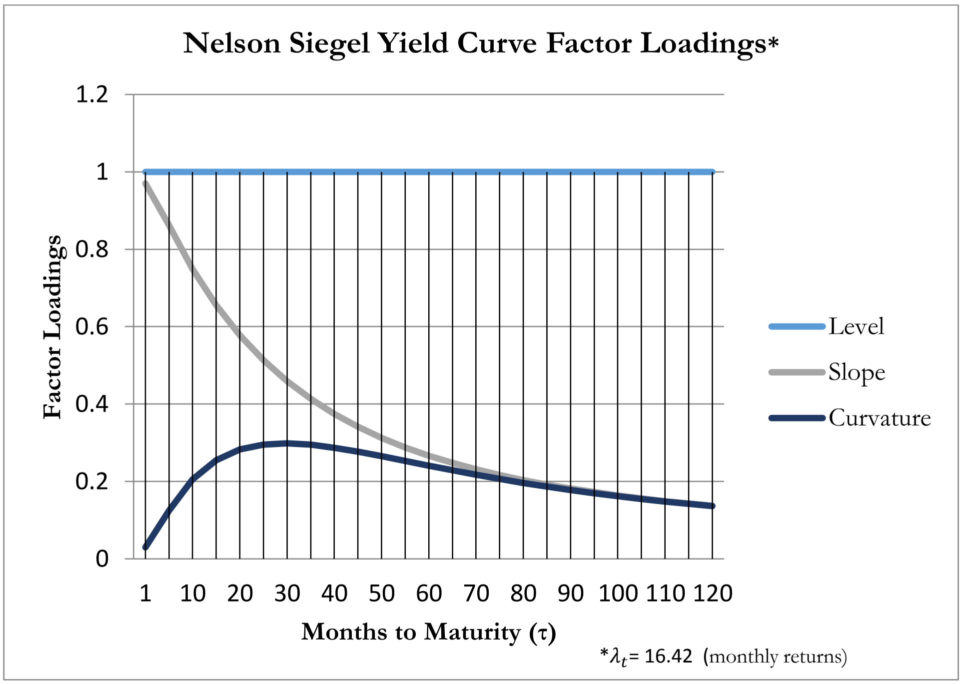


Figure 2: Diebold and Li's (2006) Nelson Siegel Factor Loadings

In order to use an Ordinary Least Squares Regression, the authors must fix the value of λ_t, which is the value at which the yield curve is at maximum concavity for a given time to maturity τ. While nonlinear regression could be used in order to avoid making an assumption as to τ, literature has shown that the parameter estimates will be distorted based on the starting values used, and as such linear estimation is prefered by some researchers as starting values are not needed (Annaert, Claes, Ceuster, & Zhang, 2013).

Diebold and Li (2006) use a value for $\lambda_t$of 0.0609 (or λ_t= 1.37 for annualized data), which they find maximizes the loading on β_3 when time to maturity is taken as 2.5 years. This maturity bucket is cited as being a generally accepted benchmark in the literature on US Treasury rates for the time to maturity at which maximum concavity is reached. Fabozzi, Martellini, and Priaulet (2005) use an annualized estimate of λ_t= 3 (which equates to a hump / trough at the 5 year maturity bucket) when investigating the predictability of the level, slope, and curvature factors for the US interest rate swap curve. While they have mimicked Diebold and Li's earlier (2006) Nelson Siegel Parameterisation approach, their choice of λ_t does not appear to be based on an economic rationale given the shape of the swap rate curve, but rather is chosen in order to minimize the issue of multicollinearity between the Nelson Siegel regressors (Annaert, Claes, Ceuster, & Zhang, 2013).

Annaert, Claes, Ceuster, and Zhang (2013) revisit the assumption of a fixed curvature / shape parameter λ_t given their assertion that a high degree of multicollinearity of the Nelson Siegel regressors is a key (and often ignored) issue when adopting this parameterisation framework. The authors point out that the degree of correlation is contingent upon the various time to maturity buckets selected for the dependent variable (US Treasury bond spot rates). Using various constellations of time to maturity points across the yield curve (see Table 1), they compute the correlation of the regressors assuming different values of λ_t. Specifically, they look at Diebold and Li's (2006) assumption of λ_t = 1.37 (for annualized data) versus Fabozzi et al.'s (2005) choice of λ_t= 3. Given the particular maturity buckets used by each author (seen below), correlation between the slope and curvature factors is a non-issue for the given lambda-value that they have selected.

Table 1: Constellation of Yield Curve Maturity Buckets

Diebold and Li's (2006) Maturity Buckets:

3, 6, 9, 12, 15, 18, 21, 24, 30 months, 3 - 10 years;

λ_t = *1.37*, ***r = -0.051***

Fabozzi et al.'s (2005) Maturity Buckets:

3 and 6 months, 1, 2, 3, 4, 5, 7, 10, 15, 20 and 30 years;

λ_t= *3*, ***r = -0.324***

(Annaert, Claes, Ceuster, & Zhang, 2013).

That said, the authors caution that if one were to use Diebold and Li's (2006) vector of time to maturity buckets with λ_t = 3, the correlation would swell to -0.871. The authors subsequently investigate the ridge regression technique in order to overcome this issue of multicollinearity.

An alternative to the Nelson Siegel framework for forecasting interest rates is presented by Svensson (1994), who extends the Nelson Siegel model by adding a secondary curvature parameter. This involves the addition of 2 parameters β_{4t} and λ_{2t}:

$$y_t(\tau) = \beta_{1t} + \left(\frac{1 - e^{-\lambda_{1t}\tau}}{\lambda_{1t}\tau}\right)\beta_{2t} + \left(\frac{1 - e^{-\lambda_{1t}\tau}}{\lambda_{1t}\tau} - e^{-\lambda_t\tau}\right)\beta_{3t} + \left(\frac{1 - e^{-\lambda_{2t}\tau}}{\lambda_{2t}\tau} - e^{-\lambda_{2t}\tau}\right)\beta_{4t}$$

Where $y_t(\tau)$ is the spot rate of the US Treasury bond at time t that matures at "time to maturity" τ

(Svensson, 1994).

By adding what is interpreted as a 2nd curvature parameter to the model with its own shape parameter λ_{2t}, Svensson (1994) asserts that this second hump-shaped parameter improves model fit for the Swedish term structure of forward interest rates for the period 1992 to 1994. This is attributed to the fact that on certain dates within the sample period the yield curve exhibits a more complex and kinked shape than the "normal" term structure envisioned by Diebold and Li. The 2 shape parameters λ_{1t} and λ_{2t} are not fixed, but rather are estimated at each trade date such that the sum of squared price errors are minimized (Svensson, 1994). The author finds that this method of minimizing price errors unfortunately gives rise to poor goodness-of-fit for short yield maturity buckets where prices are quite insensitive to yield changes given their short duration (*Noting the formula : price elasticity = bond duration*).

Similarly, Laurini and Hotta (2010) find in their investigation of the Brazilian term structure of interest rates for the period 2004 to 2006 that the addition of a second "Svensson" curvature parameter improves the Root Mean Squared Forecast Error of Diebold and Li's (2006) model from 36.61 to 17.05. The authors fix the 2 shape parameters for these particular forecasts and find that the second curvature parameter makes room for multiple yield curve slope movements that may be taking place. This is asserted to be important in order to capture the more fluid and irregular shapes associated with volatile emerging market yield curves.

Arguments against the inclusion of this extra curvature parameter have been made by various economists who emphasize that the 3-Factor Nelson Siegel Model allows for a more unambiguous model interpretation and has greater economic significance and simplicity (Bliss, Sener, Erdogan & Ahi, 2011). The authors also touch on the idea that it is possible to over-fit a model such that it has poor generalizability and explains too many idiosyncratic features of the data over a specific sample period. While they use such an argument to make a case against using non-fixed shape parameters, such ideas surely have relevance to the addition of extra parameters.

2.2 Part 2. Ordinary Least Squares Regression Model of the Predictors of the Yield Curve Parameters

Fabozzi et al.'s (2005) paper on the US swap rate curve, which largely taps into the credit risk of the banking sector, is one of the few to utilize the Nelson Siegel yield curve parameters (as opposed to market spreads) and then to subsequently assess the potential predictors of these factors. The authors fit the Nelson Siegel model on monthly data for the period 1994 – 2003, and differentiate the 3 parameter series extracted due to the presence of unit roots in the level and slope factors. For the level and slope factors, they find no signs of autocorrelation with the "old" series, leading them to believe that simple autoregressive models containing historical data may not perform well or display strong significance. While a simple Autoregressive (1) Model can be fitted to the curvature parameter, the model suffers from lack of out-of-sample predictability in the beta estimates.

The authors then investigate 12 economic and market-based variables that might explain changes in the level, slope, and curvature. They caution that the inclusion of many economic variables in a stepwise Ordinary Least Squares Regression procedure may lead to a high in-sample R-squared but very poor out-of-sample generalizability. They thus provide strong economic rationale for the inclusion of each of the explanatory variables, which fall under the categories of **Interest Rate Risk** (US short-term bond rate, US bond yield spread, and expectations of future interest rates), **Market Risk** (implied market volatility and US high-yield debt spreads), **Cheapness of stock prices** (proxied by dividend yields as a risk premium measure), various **Market sentiment** variables (such as Earnings yield on the S&P500 versus 10 year US Treasury bond yields, and Emerging Market spreads), and **Economic indicators of growth** (US Capacity Utilization rate). They also include default spreads as a measure of **Credit Risk**.

In addition to the above variables, the authors include changes in the 1-month lagged parameters for the level, slope, and curvature in their Ordinary Least Squares (OLS) regression. In order to assess the robustness of their model, the authors utilize a 4-year rolling estimation window to forecast the beta parameters for the following 1-year period. Their results indicate that only the Slope parameter is significantly explained by the model and exhibits stability of the beta parameters over time, with out-of-sample hit rates of 63 – 67%.

Just as Fabozzi et al. (2005) find the slope of the US swap yield curve to be the most lendable to prediction via OLS regression, so does Mehl (2009) find that the slopes of various Developed and Emerging Market Government curves (in this case defined simply as the difference between long and short-end rates) has itself the ability to predict future inflation rates and industrial production growth. This is because a steeper yield curve points not only to an increasing risk premium at longer-end maturity buckets, but also to the possibility of rising inflation (which is priced in by the market for various forward dates) and greater economic growth. Such studies are not new, with Estrella and Hardouvelis (1991) finding that a 100bp increase in the slope of the US Government bond curve is followed by a real GNP growth increase of 3% one year later. Using data from the period January 1995 – December 2005, Mehl (2009) finds that a 100bp steepening of the South African Government Bond Curve in the 4 months prior is associated with a rise in inflation of 60bps per annum over the next 6 months ($p<0.05$). This verifies that the slope of the yield curve does carry information about market expectations of future inflation, with an increasing slope indicating higher yields (i.e. cheaper future bond prices) in order to compensate investors for the expected erosion in value of fixed-interest debt as a result of price inflation.

Interestingly, while Mehl (2009) also finds significant results for the slope of the yield curve's ability to predict production growth, the sign of the beta coefficient is not stable over the period for many Emerging Markets (including South Africa), whereas in the developed markets a steeper government bond curve and rising inflation is generally a precursor for higher growth. In South Africa, while a steepening of the yield curve is followed by a period of high growth in the 6-month to 1.5-year period ahead, a significant period of decreased production output follows from the 2-year point. Mehl postulates that productivity shocks, fluctuating in-country risk premia, and lack of government bond liquidity lead to a steeper bond slope in the EM space and may be distorting price signals and confounding the analysis.

Also interesting to note is that the slope of the US yield curve does carry strong predictive power for SA industrial production output, with a 100bp steepening in the 2 years prior leading to an increase in SA output of 3.48% per annum in the next 6 months, and 1.14% per annum in the next 2 years (Mehl, 2009). This highlights that growth expectations in the US have implications for various global and emerging markets due to the country's status as an economic superpower.

Returning to the central paper informing this section of the present research, while Fabozzi et al. (2005) are not able to explain the swap curve Level parameter through OLS regression, other studies have been able to explain the Sovereign spreads (above US-Treasury bonds) of Emerging Market Governmnent Bonds via this method. Sy (2002) investigates the ability of a variety of technical and debt / credit fundamentals to explain sovereign bond spreads. He asserts that technical factors include any variables that may enhance liquidity and investor demand for a government's bonds – i.e. rising current account surpluses, corporate deleveraging, and increased liquidity in the interbank market. While such variables may not seem to directly tap into the creditworthiness of the sovereign, they do address the economic "health" of the national financial system and government's national accounting, which ultimately informs both the sovereign's ability to raise taxes and the necessity of the government to fuel economic growth through increased expenditure. Such activities also have rammifications in terms of the government's ability to service the principal and interest payments on its debt. Thus, just as a company's financial health is judged by its balance sheet, so is a Government's creditworthiness judged on a range of economic in-country fundamentals.

Sy (2002) formulates a model for EM Government spreads based on the following variables:

Domestic variables

- Credit Ratings
- Average Duration of debt (with a longer duration equating to increased interest rate exposure for the investor)

External Variables

- Emerging Markets Bond Index (EMBI +) spreads (a measure of risk sentiment towards EM's)
- US High Yield Corporate bond spreads (a global risk appetite measure)
- US Short Term 3 month rates (which is a proxy for global liquidity as low yields on offer in the developed world means that investors may need to seek higher-yielding riskier assets, such as EM debt)
- US Yield curve slope
- Oil Price (As many EM's are commodity exporters they thus should have a vested interest in a rising oil price, which will enhance their economic growth)
- An Economic recession / Crisis dummy variable

Ultimately Sy's (2002) model captures a large proportion of the variability in EM spreads, with adjusted R-squared values of up to 84%.

In a similar vein, Min et al. (2003) investigate liquidity, solvency, macroeconomic, and external shock predictors of sovereign yield spreads in Latin America and Asia:

Liquidity and SolvencyVariables

- Level of Imports and Exports (with increased Exports and a Current Account surplus associated with declining yields/ rising prices)
- GDP Growth (the effect here is uncertain as despite output growth indicating increased health of the economy, this may be accompanied by a rising level of imports / inflation)
- Debt-to-GDP ratio (with a larger ratio decreasing the Government's solvency)
- International reserves-to-GDP (with a falling ratio meaning a greater threat of a liquidity crisis)
- Debt-service-to-Exports ratio (with a larger ratio indicating greater Government debt service payments and reduced solvency)

Macroeconomic Fundamentals

- Inflation Rate
- Terms of Trade (with improvements indicating greater potential Exports, and thus lower yield spreads)
- Real Exchange Rate (with appreciation pointing to a less competitive Exports arena that should be associated with rising Government bond yields)

External Market Shocks

- US 3-month Treasury Bill
- Real Oil Price (Unlike Sy's (2002) rationale, Min et al. (2003) assert that rising oil prices are often a precursor for global recessions, and thus should be associated with a sell-off in government yield spreads)

Min et al. (2003) perform an Ordinary Least Squares regression using White-adjusted standard errors owing to the presence of Heteroskedasticity (non-constant variance of the residuals). Ultimately they find that the liquidity and solvency variables are all significant, with the most important being Debt-to-GDP, International Reserves to GDP, growth of Imports/ Exports, and the Debt-service ratio. All of the macroeconomic fundamentals are also significant to prediction, while only the 3-month US T-bill rate is significant as a market shock/ global liquidity indicator.

What becomes apparent from such analyses is that sovereign yields are not solely determined by in-country debt fundamentals. Longstaff, Pan, Pedersen, and Singleton (2007) explore this issue by performing a regression analysis on the Sovereign Credit Default Swap spreads of various developed and emerging market countries. Their findings indicate that global market sentiment is indeed a far more important predictor of sovereign risk returns than country fundamentals. The authors assert that a potential explanation of this phenomenon might be the increasing globalization of the investment community. They assert that as international capital flows are increasingly being put to work in foreign / emerging markets, foreign holders of EM domestic debt are naturally increasing. Thus, a sudden global risk-off sentiment in the markets is far more likely to cause a sell off in emerging market yields than might previously have been the case due to the fact that a larger base of investors would be selling assets, warranting a larger movement in price. Baek, Bandopadhyaya, and Du (2005) confirm such findings in their analysis of sovereign bond spreads and assert that the market risk appetite index that they develop in their paper explains a more significant proportion of spread variation than such domestic economic fundamentals as real GDP growth or inflation. This has important ramifications for the level of contagion risk amongst the various asset classes in the financial markets, as well as the success of investment managers in using EM debt for portfolio diversification purposes.

2.3 Part 3. Dynamic Latent Factor Approach to the SA Government Yield curve

Diebold, Rudebusch, and Aruoba (2006) expand on the more simplistic "two-step" process of estimating the Nelson Siegel parameters and then applying OLS Regression models to these factors, as seen in Diebold and Li's (2006) research. Using state space modelling, Diebold et al. (2006) are able to **concurrently fit the Level, Slope and Curvature Parameters to the bond yield curve and estimate its fundamental explanatory macro-factors**. The authors assert that such a latent factor modeling approach allows one to better explain the evolution of the yield curve and its associated economic relationships over time. They authors utilize a nonstructural Vector Autoregressive Model (VAR(1)) framework that outlines the linear dynamic relationships between various economic forces and the yield curve shape over time and across multiple time series.

As per before, the authors utilize the Nelson Siegel Model framework with shape parameter λ,

$$y_t(\tau) = L_t + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right) S_t + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) C_t$$

And use L_t, S_t, and C_t to represent the time-varying Level, Slope, and Curvature parameters.

(Diebold et al., 2006).

They assert that if L_t, S_t, and C_t follow a Vector Autoregressive Model of Order 1 (signifying the presence of 1 unit root rendering the data non-stationary) this means that these series automatically exist in a state space system. Harvey (1993) points out that a model need not be of order 1 in order to exist in a state space, and that any Autoregressive Moving-Average (ARMA) model can be put in state space.

Diebold et. al (2006) outline the below **transition equation**, which sets out the dynamics of the state space system.

$$\begin{pmatrix} L_t - \mu_L \\ S_t - \mu_S \\ C_t - \mu_C \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} \begin{pmatrix} L_{t-1} - \mu_L \\ S_{t-1} - \mu_S \\ C_{t-1} - \mu_C \end{pmatrix} + \begin{pmatrix} \eta_t(L) \\ \eta_t(S) \\ \eta_t(C) \end{pmatrix}, \quad (1)$$

Where:

t = 1,…., T represents the month within the sample period.

$\eta_t(L)$, $\eta_t(S)$, and $\eta_t(C)$ are the "white noise transition" (or error) terms, with E(η_t) $= 0$ for all parameters,

And the elements of the vector $\begin{pmatrix} L_t - \mu_L \\ S_t - \mu_S \\ C_t - \mu_C \end{pmatrix}$ are unobservable and have been generated by a first-order Vector Autoregressive process such that they are detemined by their own lagged values and those of other variables.

Next, Diebold et al. (2006) define the **measurement equation**, which outlines the relationship between the yield curve at various maturity buckets and the Level, Slope, and Curvature unobservable factors.

$$\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \cdots \\ y_t(\tau_N) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1- e^{-\lambda\tau_1}}{\lambda\tau_1} & \frac{1- e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_1} \\ 1 & \frac{1- e^{-\lambda\tau_2}}{\lambda\tau_2} & \frac{1- e^{-\lambda\tau_2}}{\lambda\tau_2} - e^{-\lambda\tau_2} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1- e^{-\lambda\tau_N}}{\lambda\tau_N} & \frac{1- e^{-\lambda\tau_N}}{\lambda\tau_N} - e^{-\lambda\tau_N} \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} + \begin{pmatrix} \epsilon_t(\tau_1) \\ \epsilon_t(\tau_2) \\ \epsilon_t(\tau_3) \end{pmatrix}, \quad (2)$$

Where:

t = 1,…., T represents the month within the sample period,

τ represents the chosen time-to-maturity buckets across the yield curve,

The (N x 1) vector $y_t(\tau)$ captures the bond yields at a particular month and for a particular maturity,

And $\epsilon_t(\tau_1)$, $\epsilon_t(\tau_2)$, and $\epsilon_t(\tau_3)$ are the measurement disturbance (or error) terms, with E(ϵ_t) $= 0$ for all t.

Rewriting equations 1 and 2, the authors describe the state-space system as per the below,

$$(f_t - \mu) = A(f_{t-1} - \mu) + \eta_t ,$$

$$y_t = \Delta f_t + \epsilon_t$$

Where:

f_t is the (m x 1) state vector of Nelson Siegel yield curve parameters,

Δ is the (N x m) matrix of Nelson Siegel regressors,

And A is an (m x m) system matrix.

Harvey (1993) asserts that the system matrices A and Δ are assumed to be non-stochastic, meaning that they fluctuate over time in set ways such that the bond yields $y_t$can be expressed as a linear combination of the initial state vector f_0 and the past and current values for η_t and ϵ_t. Furthermore, a key assumption of this procedure is that the error terms η_t and ϵ_t are uncorrelated with each other and with the initial state f_0 (Harvey, 1993). This is stipulated as:

$$\mathrm{E}(\epsilon_t \, {\eta^1}_t) = 0,$$

$$\mathrm{E}(f_0 \, {\eta^1}_t) = 0,$$

And $\mathrm{E}(f_0 \, {\varepsilon^1}_t) = 0$, for all t.

Diebold et al. (2006) assert that while they restrict the matrix of ϵ_t terms to be diagonal such that yield error terms across various maturity buckets are not correlated, they do not place the same restriction on the η_t error terms for the Nelson Siegel parameters. Their rationale behind this thinking is that market shocks that act on the yield curve parameters may naturally have a correlated effect on these factors.

State space models are ideally suited for algorithmic applications, and Diebold et al. (2006) then apply a Kalman filter, which uses linear quadratic estimation to recursively estimate optimal maximum-likelihood estimates of the state vector of unobservable Nelson Siegel yield curve parameters f_t. Kalman smoothing is also utilized as a backwards recursion technique such that estimates throughout the entire sample period are calculated using information obtained after time t (Harvey, 1993).

Letting F_t take on the values of the optimal estimates of the yield curve state vector f_t, Harvey (1993) defines the covariance matrix or "mean squared errors" matrix P_t of the states in F_t:

$$P_t = E\,[\,(f_t - \mathrm{F}_t)(f_t - \mathrm{F}_t)'$$

The Kalman filter utilizies initial values for f_0 and P_0 and then determines the optimal estimate of the state Nelson Siegel Parameters at each time period, given the information available at that time period. Kalman smoothing, on the other hand, starts with the final quantities f_t and P_t and works backwards – thus utilizing information available after the given time period (Harvey, 1993). In their research, Diebold et al. (2006) initialize the Kalman filter using the mean values for the state vector and covariance matrix, and using an initial value for λ as 0.0609 as per Diebold and Li (2006). The authors then use Maximum Likelihood Estimation iteratively to obtain optimal Nelson Siegel parameter values, **while simultaneously estimating the coefficients for the fundamental predictors of these parameters.** The authors assert that this "one-step" approach is superior to the less cohesive two-step procedure that they adopted in earlier papers.

Diebold et al. (2006) implement 2 of the above VAR(1) state-space Models to explain the Government Bond Yield curve using latent and macroeconomic factors. In their first "Yields-Only" Model, $f'_t = (L_t, S_t, C_t)$ in the equation below. This means that the yield curve parameters are modelled based only on their own lagged values.

$$y_t = \Delta f_t + \epsilon_t$$

The authors find that this yields-only model exhibits a good fit for the data, and the standard deviations of the residuals of the measurement equation (ϵ_t) are suitably low. They find that the Level, Slope and Curvature parameters are highly persistent with lagged coefficients of 0.99, 0.94 and 0.84. They also find that the L_t parameter has a significant (albeit small) relationship with the lagged value of the Slope parameter (S_{t-1}) such that a lower yield curve (price rally) is associated with a steeper slope. This potentially points to the fact that the majority of the movement in rates occurs in the more liquid and volatile short-end of the curve.

In their second "Yields-Macro" Model, , $f'_t = (L_t, S_t, C_t,\ CU_t, FFR_t, INFL_t)$, where $CU_t, FFR_t,$ and $INFL_t$ are US Capacity Utilization, the Federal Funds Rate, and the US Inflation rate (Diebold et al., 2006). The Kalman filter in this model provides optimal simultaneous estimates of the yield curve parameter states and the macroeconomic factor states. Again, the mean values of the various state variables are used to intialize this recursion process. The results indicate that while the curvature parameter is not well related to the 3 macroeconomic variables, an increase in capacity utilization, rising inflation, and an increase in the Federal Funds rate are all typically followed by a steeper yield curve slope. Furthermore, the level of yields tends to rise (sells off) in response to climbing inflation, increased capacity utilization, and a decrease in the Federal Funds rate.

Furthermore, Diebold et al. (2006) do not only consider one-way or "unidirectional" macroeconomic relationships (i.e. "macro-to-yield" relations), but instead focus on the potential biodirectional interaction loops that exist between the yield curve and the economy. They thus make the assumption that each in some way informs the other. In this regard, they find that a rise in the slope of the yield curve precipitates an increase in the federal funds rate, which presumably has been utilized as a monetary policy tool to hijack growing inflation. This "yields-to-macro" relationship may be the result of the market's early pricing in of inflationary expectations, or of the Federal reserve reacting to increasing long-end yields in an effort to reduce the cost of long-term borrowing (Diebold et al., 2006). That said, the authors' findings suggest that weaker relationships hold for these yield-to-macro relations than the more traditional macro-to-yield effects.

Diebold et al. (2006) find that the means and standard deviations for the ϵ_t measurement disturbance terms in the yields-only versus yields-macro models are very similar. This indicates that the forecast accuracy of the various yield terms is not vastly improved upon by the addition of the macro-factors – despite being able to describe more complex relationships between the bond market and broader economy.

Krishnan, Ritchken, and Thomson (2007) perform a similar such analysis to Diebold et al. (2006) and investigate forecasts of US credit spreads for various firms in the manufacturing and service-related industries. Using Nelson Siegel parameterisation, the Diebold and Li (2006) 3-factor yield curve framework, and an adapted version of the Vector Autoregressive state-space techniques of Diebold et al. (2006), the authors find that what can be termed their "yields-macro" model does not improve upon the forecasts of credit spreads obtained under their "yields-only" framework. More specifically, the authors find that forecasts based on the spot and forward corporate credit spreads, 3-factor Nelson Siegel credit curve parameters, and data from both the risk-free curve and B-rated corporate spread curve cannot be improved upon by the addition of macroeconomic, stock market and firm-specific variables. The macroeconomic and stock-related variables used for their "yields-macro" model encompass the Real

Activity Index, Inflation rate, share price momentum and volatiliy, book-to-market ratio, and firm leverage.

2.4 Part 4. Linking the SA Asset Classes: An investigation of Equity sectors and their relationship with the Currency and Yield Curve Parameters

In acknowledging fluctuations in the Rand as being a strong determinant of South African market behaviour, Barr and Kantor (2005) and Barr, Kantor, and Holdsworth (2007) classify the companies listed on the Johannesburg Stock Exchange (JSE) according to the classification system of *Rand Hedge, Rand Leverage,* and *Rand Play*. The authors fit a model for predicting the total returns of shares according to movements in the All Share Index (ALSI) and USDZAR fluctuations. The above-mentioned authors identify there as being a scarcity of research into this phenomenon, which is a valuable area of research that assists investors in building a portfolio that safeguards their shares from periods of rand depreciation.

Rand Hedge companies are almost entirely foreign-based companies on the JSE which generate foreign currency income and incur foreign costs, while *Rand Leverage* companies are SA-based and incur their costs in South African rands, while they sell their products in foreign 'hard' currency (Barr & Kantor, 2005). While Rand Hedge shares clearly offer a currency hedge against weakness in the rand, Barr and Kantor (2005) argue that Leverage shares are more likely to offer protection from a weak rand in the short-term following a period of rand depreciation as their incurred costs are initially lowered due to the fact that rand depreciation lowers dollar costs and exerts a leveraged effect on profits. The authors maintain, however, that the principle of Purchasing Power Parity will ultimately restore cost equilibrium, leaving only the profitability of each class of stock (Hedge or Leverage) to determine the strength of relative returns.

Rand Play companies are almost entirely SA-based and almost all costs and revenues are incurred locally in rands. They thus offer the least protection against rand weakness, but instead lose value during times of rand weakness and gain value during periods of rand appreciation (Barr & Kantor, 2005).

Thus, while classification of individual JSE shares according to their reaction to exchange rate movements has been shown in past literature to be a consistent explanatory framework for predicting share returns, little research has looked at classification of SA shares according to changes in the shape of the government bond yield curve. Alam and Uddin (2009) investigate the relationship between share prices and interest rates for developed and emerging market countries over the period 1988 - 2003, and include South Africa in their sample. Unsurprisingly, their results indicate a negative relationship between interest rates and share prices such that a rising interest rate (weaker bond prices) are associated with weaker equity returns due to a higher cost of investor borrowing and a general reining in of economic growth within the economy. This is also consistent with the present value formula, which implies that discounting asset prices at higher rates of interest leads to lower returns (Mangani, 2011). That said, Mangani (2011) uses a GARCH framework to show that the South African discount rate's effect on stock market volatility is not consistent for all definitions of the market portfolio, and is only significant and inverse when using the All Share Index as a whole. Furthermore, results show that the reaction of the stock market to changes in the discount interest rate and Monetary Policy is asymmetric, with a far greater market reaction exhibited during contractionary policy (rising interest rates and lower bond prices) than expansionary monetary policy (Mangani, 2011). What becomes clear from such investigations is that a more in-depth look is warranted, and that the reaction of specific equity sectors and individual shares may shed light on these phenomena.

2.5 Part 5. Nelson Siegel Parameterisation of the SA swap rate curve: Implementation of Systematic Trading Strategies

Fabozzi et al. (2005) extend their previously discussed analysis by using their forecasts to implement systematic trading strategies that exploit expected changes in the Slope and Curvature parameters of the interest rate swap curve. Reminiscent of the government yield curve, the swap curve is made up of rates that represent the interest rate at which banks exchange fixed for floating payments with market participants at various maturities. The difference is that this does not represent bank-issued debt, but rather a stream of fixed-for-floating exchange payments in which one party makes a stream of fixed interest rate payments on a particular notional amount (which is never exchanged) to the counterparty, who in turn makes a series of floating interest rate payments. The fixed-rate payer will clearly benefit from a rise in interest rates as they will then be paying below-market rates while receiving at-market rates. This absence of an upfront premium payment or repayment of the notional principal introduces the idea of Leverage, and as such a swap is a Derivative (non-funded) instrument for which the price is derived based on the underlying interest rate.

The authors use subsets of the previously discussed 12 economic variables to build explanatory models for the Level, Slope and Curvature parameters using a 48-month calibration period to estimate the model, a 24-month training period to back-test the model, and a subsequent 36-month trading period (Fabozzi et al., 2005). This is known as recursive modelling and mitigates some of the issues that arise as a result of poor out-of-sample generalizability of the Beta parameters. Fabozzi et al. use this dynamic modelling procedure and acknowledge that the economic factors that are associated with the yield curve parameters evolve over time. They also adopt a Bayesian Model Framework as per Barberis (2000) and Kandel and Stambaugh (1996). Using Bayesian averaging of classical estimates (BACE) (Doppelhofer, Miller, and Sala-i-Martin, 2000), they reject the idea that there is one correct model at any period in time, and instead use a constellation of models and take the average forecast prediction across these models. Fabozzi et al. attach equal probabilities of occurrence to each model under the assumption of non-informative priors, thus meaning that they need not estimate prior distributions for the parameters of all models.

Using at most 4 economic predictor variables in any one model in order to enhance model generalizability, they select models for each period based on the size of the Schwarz Information Criterion and according to the rules that all predictors must be significant ($p<0.05$) and must have exhibited such significance in the previous 12-months (Fabozzi et al., 2005). They also require hit rates of at least 55% for their forecast of the change in slope (steepening / flattening) and curvature (more or less concave), as measured in the back-testing training period.

At a particular point in time, Fabozzi et al. (2005) extract the average probability of a particular up or downward movement in the slope/ curvature across the constellation of models. They also separate their forecasts in terms of the number of standard deviations from a neutral 50% probability of a particular movement in the curve. Their results indicate the best forecast hit rates for the slope parameter (greater than 66% for all standard deviations from the neutral probability) and reasonable hit rates for the curvature parameters (a range of 54 – 71% based on 0 to 2 standard deviations from a neutral view). As earlier discussed, the authors are unable to determine satisfactory models for the Level parameter over their sample period that meet the discussed fit criteria of coefficient significance and hit rates. This leads them to the conclusion that the interest rate level (as proxied by fixed-for-floating exchange swap rates) cannot be effectively explained based on monthly changes.

The authors execute automated trading strategies by putting on 1-month butterfly positions based on forecasts of changes in the slope and curvature parameters. A butterfly made up of swaps is made up of 3 swap positions – a medium (intermediate-term) swap that is called “the body”, and 2 swaps with a maturity longer and shorter than the body that are known as “the wings” (Fabozzi et al., 2005). An example would be a 2/10/30 swap butterfly in which the 10-year swap is the body and the 2 and 30-year swaps are the wings. A payer butterfly position would involve paying the fixed rate on the 10-year swap and receiving the fixed rates (and thus paying the floating rates) on the 2 and 30-year swaps. Putting on a payer butterfly swap expresses the view that the intermediate level of the swap curve (in this case, the 10-year point) will rise, and that the 2 and 30-year maturity rates will decline. By paying the fixed rate of the 10-year point and receiving the 10-year floating rate one would obviously benefit from a rise in 10-year rates. This also equates to a view that the curve's degree of concavity will increase such that it will reflect a more humped/ concave shape. Similarly, an expected rise in the slope of the swap curve could be expressed by paying the fixed rate on a long-term swap and receiving the fixed rate on a short-term swap. Thus, by paying the floating rate on the short-end swap and receiving the floating rate on the long-end swap, one would profit from a steeper curve shape over time.

Another aspect of these swap butterfly positions is the chosen nominal amount on each of the 3 swap trades, which in turn determines the modified duration or "interest rate sensitivity" of the position (Fabozzi et al., 2005). This refers to the change in the price of the swap for a particular movement in interest rates. In order to express a view on the change in the curvature parameter one is essentially looking to profit from changes in the concavity of the swap curve, while eliminating the effect of changes in the overall level of interest rates. Thus, for a payer butterfly, one's position must not yield any profit or loss if the level of swap rates increases in a parallel fashion across the curve such that each point (1-year to 30-year) rises by a certain **equal** number of basis points.

As such, Fabozzi et al. (2005) solve for the nominal amounts on the 3 swap butterfly positions so as to isolate the effect of a particular change in 1 parameter of the yield curve. For example, in order to isolate the effect of a change in the curvature parameter from a change in the Level and Slope parameter, the authors solve for the prinicpal amounts as per the below,

$$q_s \mathrm{D}_s + \alpha \mathrm{D}_m + q_l \mathrm{D}_l = 0$$

$$q_s \mathrm{D}_s = - \alpha \mathrm{D}_m \gamma$$

$$\{ \ \gamma = \frac{S_l - S_m}{S_l - S_s} \ \}$$

Where:

q_s, α and q_l are the Notional amounts on the short, intermediate (body), and long-term swaps;

D_s , D_m and D_l are the modified durations (interest rate risk) of the short, intermediate (body), and long-term swaps; and

S_s, S_m, and S_l are the sensitivity of the short, intermediate (body) and long-term swaps to the Slope Parameter.

What this means is that for a 2/10/30 butterfly payer position if the level of swap rates across the curve were to rise by 2 basis points each (a parallel upward shift in the level of the curve), this would have no impact on the payoff of this position as the **combined** interest rate sensitivity of the position is equal to zero. Thus, only if the "body" was to rise by more than the movement in the wings would a profit be realized (known as a butterfly twist).

Fabozzi et al. (2005) calculate the total return on a 1-month payer butterfly position as per the below,

$$Total\ return\ (basis\ points) = \ \mathrm{D}_m\, \Delta r_m - \frac{q_s \mathrm{D}_s \Delta r_s + q_l \mathrm{D}_l \Delta r_l}{\alpha} + \mathrm{Carry}$$

Where:

Δr_s, Δr_m and Δr_l = (swap rate of the short / medium / long-term swap at month t +1) – (swap rate of the short / medium / long-term swap at month t)

One also sees that the total return is effectively weighted by the nominal amount selected for the body of the butterfly position, which determines the relative leverage of the position. The notion of "Carry" in a swap trade refers to the fact that in a payer swap if one is paying the fixed rate on a long-maturity swap and receiving the fixed rate on a short-maturity swap and the yield curve is "normal" in shape then this means that even if the yield curve does not move one will lose on the "carry" because one is paying a higher rate and receiving a lower (near-maturity) swap rate. The authors assume that carry is negligible for the purposes of their analysis both because they only hold positions for one month, and due to the fact that they put on the same number of payer and receiver butterfly trades over the course of their trading period (Fabozzi et al., 2005).

Fabozzi et al.'s (2005) results indicate that the US swap rate curve over the period June 1994 to September 2003 does lend itself to significant predictability and exploitation via the mechanism of systematic trading strategies – with impressive annual returns of roughly 4.47% to 13.17% for moderately 2x leveraged curve steepener / flattener slope positions and less impressive 2.65% to 2.97% annual returns for a range of butterfly curvature positions (e.g. 2/5/30 and 5/10/30 butterflies). The Sharpe Ratios for these active return strategies, which measures the degree of excess return over the benchmark per unit of risk taken on, are far more moderate at roughly 1.3 – 1.8 for the slope positions, and -0.05 to

-0.4 for the curvature positions. In the case of the curvature butterfly positions, this indicates an inability of this strategy to produce excesss returns above the benchmark (in this case the return on cash or the risk-free rate) per unit of risk taken on.

3. Methodology

3.1 Statistical Procedures

3.1.1 Part 1. Nelson Siegel Parameterisation of the South African Sovereign Yield Curve

For each of the 132 months (denoted "t") in the sample (31/03/2003 to 28/02/2014), the following regression is run:

$$y_t(\tau) = \beta_{1t} + \left(\frac{1-\ e^{-\lambda_t\tau}}{\lambda_t\tau}\right)\beta_{2t} + \left(\frac{1-\ e^{-\lambda_t\tau}}{\lambda_t\tau} -\ e^{-\lambda_t\tau}\right)\beta_{3t}$$

(Diebold and Li, 2006)

Where

$y_t(\tau)$ is the South African zero coupon Government Bond spot rate maturing at time τ,

β_{1t} is the Level Parameter,

β_{2t} is the Slope Parameter,

β_{3t} is the Curvature Parameter, and

λ_t is taken as a constant across each month t, thus allowing the above equation to be estimated using Ordinary Least Squares Regression. The lambda value determines the time point along the yield curve at which maximum concavity is reached, and the value chosen for it is discussed in the Results section.

In addition, the Nelson-Siegel-Svensson Model is fitted to the data as a basis for comparison. This involves the inclusion of 2 extra parameters β_{4t} and λ_{2t}:

$$y_t(\tau) = \beta_{1t} + \left(\frac{1 - e^{-\lambda_{1t}\tau}}{\lambda_{1t}\tau}\right)\beta_{2t} + \left(\frac{1 - e^{-\lambda_{1t}\tau}}{\lambda_{1t}\tau} - e^{-\lambda_t\tau}\right)\beta_{3t} + \left(\frac{1 - e^{-\lambda_{2t}\tau}}{\lambda_{2t}\tau} - e^{-\lambda_{2t}\tau}\right)\beta_{4t}$$

Where $y_t(\tau)$ is the spot rate of the US Treasury bond at time t that matures at "time to maturity" τ

(Svensson, 1994).

In order to get an indication of the robustness and consistency of these fitted models, a 48-month rolling window period is used with a 12-month forecast window. The relative forecast accuracy of the 2 yield curve factorization methods are then compared via back-testing and calculating of the Root Mean Squared Errors of the forecasts.

3.1.2 Part 2. Ordinary Least Squares Regression Model of the Predictors of the Yield Curve Parameters

The following quarterly OLS Regression Models are fitted to the Nelson Siegel Parameters:

$$\Delta level_t = c + (\Delta Economic\ Factor_t)\beta_1 + (Market\ Price_t)\beta_2 + \varepsilon_t$$

$$\Delta slope_t = c + (\Delta Economic\ Factor_t)\beta_1 + (Market\ Price_t)\beta_2 + \varepsilon_t$$

$$\Delta curvature_t = c + (\Delta Economic\ Factor_t)\beta_1 + (Market\ Price_t)\beta_2 + \varepsilon_t$$

Where

$\Delta level_t$, $\Delta slope_t$, $\Delta curvature_t$ is the change in each of the yield curve parameters at Quarter t

(Note that changes are used as these variables are non-stationary as evidenced by the Results of the Dickey-Fuller Unit Roots test)

$\Delta Economic\ Factor_t$ represents the change in each of the economic fundamental variables discussed in the next section (for example, SA Debt to GDP Ratio or SA Credit Rating).

$Market\ Price_t$ is the continuously compounded total return of the particular market price variable for Quarter t (for example, the Barclays SA Inflation-Linked Government Bond Index or the Real Price of Brent Crude Oil).

3.1.3 Part 3. Dynamic Latent Factor Approach to the SA Government Yield curve

As per the work of Diebold, Rudebusch, and Aruoba (2006), this section of the research models the dynamic reciprocal relationships between the yield curve, its latent factors (lagged level, slope and curvature parameters) and the economy (inflation, the deposit rate, and capacity utilization) over time. Using the method of Vector Autoregression (VAR(1)) and the state space modelling framework, a Kalman filter is applied to arrive simultaneously at maximum likelihood estimates of the state parameters – namely the 3 Nelson Siegel factors and core macroeconomic yield curve drivers (Diebold et al., 2006).

The Nelson Siegel 3-parameter model is fitted to the yield curve,

$$y_t(\tau) = L_t + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right) S_t + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) C_t$$

Where L_t, S_t, and C_t represent the time-varying Level, Slope, and Curvature parameters. λ is initialized in the Kalman filter at 0.0609, but then assumes an optimal value over various maximum likelihood iterations.

(Diebold et al., 2006).

As per Diebold et. al (2006), the below VAR(1) **transition equation** is referenced so as to set out the dynamics of the state space system.

$$\begin{pmatrix} L_t - \mu_L \\ S_t - \mu_S \\ C_t - \mu_C \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} \begin{pmatrix} L_{t-1} - \mu_L \\ S_{t-1} - \mu_S \\ C_{t-1} - \mu_C \end{pmatrix} + \begin{pmatrix} \eta_t(L) \\ \eta_t(S) \\ \eta_t(C) \end{pmatrix}, \quad (1)$$

Where:

t = 1,…., T represents the month within the sample period,

$\begin{pmatrix} L_{t-1} - \mu_L \\ S_{t-1} - \mu_S \\ C_{t-1} - \mu_C \end{pmatrix}$ is the vector of lagged latent state (unobservable) variables, And $\eta_t(L)$, $\eta_t(S)$, and $\eta_t(C)$ are the "white noise transition" (or error) terms, with E($\eta_t) = 0$ for all parameters,

The below linear **measurement equation** outlines the relationship between the yield curve at various maturity buckets and the Level, Slope, and Curvature unobservable factors (also known as "state parameters" in the state space framework).

$$\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \cdots \\ y_t(\tau_N) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_1} \\ 1 & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} - e^{-\lambda\tau_2} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda\tau_N}}{\lambda\tau_N} & \frac{1-e^{-\lambda\tau_N}}{\lambda\tau_N} - e^{-\lambda\tau_N} \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} + \begin{pmatrix} \epsilon_t(\tau_1) \\ \epsilon_t(\tau_2) \\ \epsilon_t(\tau_3) \end{pmatrix}, \qquad (2)$$

Where:

t = 1,…., T represents the month within the sample period,

τ represents the chosen time-to-maturity buckets across the yield curve,

The (N x 1) vector $y_t(\tau)$ captures the bond yields at a particular month and for a particular maturity,

And $\epsilon_t(\tau_1)$, $\epsilon_t(\tau_2)$, and $\epsilon_t(\tau_3)$ are the measurement disturbance (or error) terms, with E($\epsilon_t) = 0$ for all t.

Rewriting equations 1 and 2, the state-space framework is described as per the below,

$$(f_t - \mu) = A(f_{t-1} - \mu) + \eta_t ,$$

$$y_t = \Delta f_t + \epsilon_t$$

(Diebold et al., 2006).

Where:

f_t is the (m x 1) state vector of Nelson Siegel yield curve parameters,

Δ is the (N x m) matrix of Nelson Siegel regressors,

And A is an (m x m) system matrix.

Mirroring Diebold et al.'s (2006) paper, 2 models are considered:

- A "Yields-Only" Model in which $f'_t = (L_t, S_t, C_t)$
- A "Yields-Macro" Model in which $f'_t = (L_t, S_t, C_t, CU_t, Depo_t, INFL_t)$

For the yields-only model, optimized least squares estimates of the following 31 parameters are projected:

- The (3x3) system Matrix A found in the transition equation
- The (3x1) vector μ of state parameters
- The λ parameter in the (12 x 3) Δ matrix {corresponding to 12 yield maturity buckets and 3 Nelson Siegel parameters}
- The 12 measurement disturbances ϵ_t
- The 3 white noise error terms η_t and the 3 covariances

(*Note that the log variances are estimated in order to ensure that they are non-negative*).

In order to initialize the Kalman filter, all variances are started out at 1, λ is initialized at 0.0609 (as per Diebold and Li, 2006), and the state vector f_t is initialized at mean values for the data series (Diebold et al., 2006). While the authors initialize the A matrix as per the parameter values they obtained in their 2-step analysis performed in Diebold and Li's (2006) paper, the present research arrives at reasonable estimates by performing a simple OLS regression of the earlier-extracted Nelson Siegel parameters from Part 1 against each of the lagged f_t state parameters.

Once this pattern of parameter values has been entered, the Kalman filter calculates optimal yield forecasts and standard deviations (Diebold et al., 2006). The Gaussian likelihood function of the model is maximized using the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm – which works in a similar way to Newton's iterative optimization method . Contrastingly, Diebold et al. (2006) utilize the Berndt–Hall–Hall–Hausman (BHHH) algorithm. Ultimately, the iterative procedure should converge to optimal estimates of the earlier-mentioned model parameters (i.e. the A transition matrix, λ etc.).

In both of the models, the white noise η_t terms and measurement disturbances ϵ_t must be uncorrelated with each other and with the initial states f_0 (Diebold et al., 2006). The authors assert that this is necessary in order to arrive at least-squares optimal estimates via the Kalman filter, and represent these equations as such:

$$\begin{pmatrix}\eta_t \\ \epsilon_t\end{pmatrix} \sim WN \left[\begin{pmatrix}0 \\ 0\end{pmatrix}, \begin{bmatrix} Q & 0 \\ 0 & H \end{bmatrix} \right],$$

$$E(f_0 \eta'_t) = 0,$$

$$E(f_0 \epsilon'_t) = 0.$$

As per Diebold et al.'s (2006) assumptions, the H matrix of ϵ_t terms is assumed to be diagonal such that the yield maturity-bucket error terms (i.e for the 1y, 2y point etc) are uncorrelated with each other. The Q matrix, on the other hand, is non-diagonal or "unrestricted" – thus allowing for market shocks affecting the yield level, slope, and curvature to be correlated.

3.1.4 Part 4. Linking the SA Asset Classes: An investigation of Equity sectors and their relationship with the Currency and Yield Curve Parameters

Due to the issue of multicollinearity between the Rand currency term and SA Government Bond yield curve parameters, the following regression is performed in order to establish the association between these variables:

$$USDZAR_t = c + (\Delta level_t)\beta_1 + (\Delta slope_t)\beta_2 + (\Delta curvature_t)\beta_3 + \varepsilon_t$$

Where

$USDZAR_t$ is the continuously compounded change in the USD/ZAR Exchange rate for month t

$\Delta level_t$, $\Delta slope_t$, $\Delta curvature_t$ is the change in each of the yield curve parameters for month t

The residual of the above regression analysis is extracted in order to represent fluctuations in USDZAR that are not explained by the SA Government Bond yield curve.

Next, the following OLS Regression Models are fitted to the Total Returns of the JSE Derivative Indices and the individual Top 40 shares:

$$y_{it} = c + (ALSI_t)\beta_1 + (USDZAR_t)\beta_2 + (\Delta level_t)\beta_3 + (\Delta slope_t)\beta_4 + (\Delta curvature_t)\beta_5 + \varepsilon_t$$

Where

y_{it} is the continuously compounded total return in the JSE Index or Top40 share for month t

$ALSI_t$ is the continuously compounded total return of the ALSI for month t

$USDZAR_t$ is now the residual of the earlier regression at time t; representing the change in the currency not explained by the Government Bond Yield Curve factors

$\Delta level_t$, $\Delta slope_t$, $\Delta curvature_t$ is the change in each of the yield curve parameters for month t

In order to get an indication of the robustness and consistency of these fitted models, a 48-month rolling window estimation period is used.

3.1.5 Part 5. Nelson Siegel Parameterisation of the SA swap rate curve: Implementation of Systematic Trading Strategies

After performing the Nelson Siegel Parameterisation seen in Part 1 on the South African interest rate swap curve, OLS Regression Models are fitted to the Nelson Siegel Parameters. A 36-month rolling window estimation period is used with a lag of 1 month between each window. Forecast estimates of the level, slope and curvature are then obtained for each proceeding month.

The models take on the following form:

$$level_t = c + (\Delta Economic\ Factor\ 1_{\ t})\beta_1 + (\Delta Economic\ Factor\ 2_{\ t})\beta_2 + \ldots\ldots + \varepsilon_t$$

$$\Delta slope_t = c + (\Delta Economic\ Factor\ 1_{\ t})\beta_1 + (\Delta Economic\ Factor\ 2_{\ t})\beta_2 + \ldots\ldots + \varepsilon_t$$

$$\Delta curvature_t = c + (\Delta Economic\ Factor\ 1_{\ t})\beta_1 + (\Delta Economic\ Factor\ 2_{\ t})\beta_2 + \ldots\ldots + \varepsilon_t$$

Where:

$level_t$ is the level parameter for the swap curve at Month t

$\Delta slope_t$, $\Delta curvature_t$ is the change in each of the swap parameters at Month t

(Note that changes are used as these variables are non-stationary as evidenced by the Results of the Dickey-Fuller Unit Roots test)

$\Delta Economic\ Factor_t$ represents the lagged change in each of the economic fundamental variables discussed in the next section (for example, SA Capacity Utilization Rate or SA Banking Sector Dividend Yield).

The 12 economic variables considered mirror those used in the pivotal Fabozzi et al. (2005) paper, except with a variation for the South African context. They are discussed in the following section. As per Fabozzi et al. (2005), the models also consider the lagged values of the changes in the Nelson Siegel swap curve parameters { $d\beta_1(-1), d\beta_2(-1), and\ d\beta_3(-1)$ from Part 1 } as potential explanatory variables.

Model Building Rules

For the initial 36-month window period (e.g., August 2003 to August 2006), a model is fitted with a maximum of 4 explanatory variables such that model robustness is addressed (Fabozzi et al., 2005).

The following rules must be adhered to:

1. The explanatory variables must all be significant ($p<0.05$) and must have displayed such significance over the previous 12-months
2. The R-squared value cannot be below 20% and the Adjusted R-squared value cannot fall below 10%
3. The selected model must offer the best fit for the data of all possible explanatory variables, as determined via a Combinatorial Stepwise Regression procedure

As per Fabozzi et al. (2005), a constellation of models is sought for each period such that once the initial model has been selected, the explanatory variables chosen are removed from the possible sample of predictors, and a secondary model is chosen that satisfies the above criterion 1 and 2. This theoretically continues for as many models as can be chosen.

Forecasts of the following month (e.g. September 2006, continuing the above example) are then obtained, and the models chosen for the first 36-month period are then fitted to the second 36-month rolling period (September 2003 to September 2006). For the sake of robustness and model continuity, the same models as those initially selected are used, however criterion 1 and 2 must still be satisfied. In the event that they are no longer satisfied (i.e. R-squared and Adjusted R-squared values fall below the selected minimum restriction) then the model is discarded and a new model is selected that satisfies criterion 1, 2, and 3.

Systematic Trading Strategies

Using the forecast value for the particular parameter (level, slope, or curvature), a trading position is entered into for a period of one month.

For the level parameter, a simple long / short position is entered into such that a predicted rise in the level of the swap curve for month t +1 predicates that a swap be entered into at month t as the fixed-rate payer.

For the slope positions, curve steepeners / flatteners are considered for the 1-/ 10-year, 1-/ 20-year, and 10-/30-year areas of the swap curve. For example, if the forecasted change in the slope is a decrease in the slope of (for example) 26.8 basis points for month t + 1, a 1/10, 1/20, and 10/30 curve flattener is put on at month t. For the first of these positions, this would involve entering into a 1-year swap as the fixed rate payer and a 10-year swap as the fixed-rate receiver. This position would profit if the back-end of the curve should fall relative to the front-end as one would be paying the floating rate on the 10-year swap and receiving the floating rate on the 1-year swap.

For the curvature parameter, 3 butterfly twist strategies are considered – 1/5/10, 1/10/30, and 10/20/30 positions. If the forecasted change in the curvature is an increase in concavity (more humped shape) for month t + 1, a 1/5/10, 1/10/30, and 10/20/30 butterfly payer position is put on at month t. For the first of these positions, this would involve entering into a 5-year swap as the fixed rate payer, and a 1-year and 10-year swap as the fixed-rate receiver. This position would profit if the 5-year swap rate rose relative to the rise in the 1-year and 10-year points.

The nominal amounts chosen for the curve steepeners / flatteners and curve butterfly positions are done so that the overall trade is delta-neutral and is thus not receptive to parallel increases or decreases in the level of the yield curve across all maturity buckets. This is reminiscent of Fabozzi et al.'s (2005) nominal selection procedure, however the process is simplified such that for a particular forecasted change in the slope parameter, the nominal is solved for such that this movement is isolated from the effect of a change in the Level parameter only. Thus, for a given month t, the nominal amounts on the curve steepener / flattener position are solved as per the below,

$$q_s \mathrm{D}_s + q_l \mathrm{D}_l = 0$$

Where:

q_s and q_l are the Notional amounts on the short and long-term swaps; and

D_s and D_l are the modified durations (interest rate risk) of the short and long-term swaps.

Similarly, for a particular forecasted change in the curvature parameter, the nominal amounts are solved for such that this movement is isolated from the effect of a change in the Level parameter only.

$$q_s \mathrm{D}_s + \alpha \mathrm{D}_m + q_l \mathrm{D}_l = 0$$

Where:

q_s, α and q_l are the Notional amounts on the short, intermediate (body), and long-term swaps; and

D_s , D_m and D_l are the modified durations (interest rate risk) of the short, intermediate (body), and long-term swaps.

The rationale for this simplification involves the issue of multicollinearity between the slope and curvature regressors used in the parameterisation technique. The Nelson Siegel analysis conducted involves a specific matrix of swap maturities of 3-months out to 30-years that yields a correlation of 0.3 between the slope and curvature regressors. As earlier discussed, Annaert et al. (2013) assert that a high degree of multicollinearity of the Nelson Siegel regressors is a key (and often ignored) issue when adopting this parameterisation framework. While a 0.3 correlation is conferred to be reasonable for Fabozzi et al.'s (2005) analysis, given this relatively sizeable correlation it is presumed that a forecasted increase in the curvature parameter should thus have an impact in terms of a coincident rise in the slope. To isolate the effect of curvature versus slope would thus potentially negate some of the profit yielded from a payer butterfly position. This argument is strengthened by the fact that the explanatory models of the yield curve parameters need only meet a 10% adjusted R-squared value in order to be used under the trading algorithm.

An example of selected nominal amounts ensues. The interest rate delta for a 1-year, 5-year, and 10-year swap as at 30th June 2014 is 96, 419, and 697 "Rand per point" respectively (Data Source: Bloomberg). Furthermore, the fixed swap rates for the 1-year, 5-year, and 10-year swap as at 30th June 2014 are 6.37, 7.54, and 8.2 respectively (Data Source: Thomson Reuters). Note that interest rate delta is a function of the fixed swap rate and the relative maturity bucket, and deltas for the given swap maturities thus change over time as the swap rates move.

In order to set up a delta-neutral 1-/10-year curve steepener position, one enters into a 10-year fixed rate payer swap position in which one pays fixed at 8.2% with a nominal of R1,446,198 (interest rate delta = R1,000 rand per point), and receives the fixed rate of 6.37% on the 1-year swap with a nominal of R10.5million (interest rate delta = -R1,000 per basis point move in the 1-year rate). For a 1-basis point increase in the 1-year swap rate and a 1-basis point increase in the 10-year swap rate, this position will yield zero profit as the overall interest rate delta is equal to zero.

Similarly, in order to set up a delta-neutral 1/5/10 butterfly payer position, one enters into a 5-year fixed rate payer swap position in which one pays fixed at 7.54% with a nominal of R4,811,456 (delta = R2,000 per point). One enters into fixed-rate receiver positions on the 1-year and 10-year swaps such that one receives the fixed rate of 6.37% on the 1-year swap with a nominal of R10.5million (interest rate delta = -R1,000) and receives the fixed rate of 8.2% on the 10-year swap with a nominal of R1,446,198 (interest rate delta = -R1,000). Again, if the entire swap curve were to increase by 1-basis point this position would yield zero return. Note that all positions taken on in the present research are "2x" leveraged.

As per Fabozzi et al.'s (2005) work, the carry on all trade positions is assumed to be zero. When calculating the return on a butterfly payer position, the following formula is referenced,

$$Total\ return\ (basis\ points) = \mathrm{D}_m\,\Delta r_m - \frac{q_s \mathrm{D}_s \Delta r_s + q_l \mathrm{D}_l \Delta r_l}{\alpha}$$

(Fabozzi et al., 2005)

Where:

q_s, α and q_l are the Notional amounts on the short, intermediate (body), and long-term swaps;

D_s , D_m and D_l are the modified durations (interest rate risk) of the short, intermediate (body), and long-term swaps; and

Δr_s, Δr_m and Δr_l = (swap rate of the short / medium / long-term swap at month t +1) – (swap rate of the short / medium / long-term swap at month t).

As a result of the fact that all butterfly positions entered into in the present analysis have equal interest rate exposure on each of the wings (i.e. Nominal amount x Modified Duration for the short- and long-maturity swaps are equal), the formula used in this analysis is:

$$Total\,\%\,return = \;\; 2(\Delta r_m) - (\,\Delta r_s + \Delta r_l)$$

Where:

Δr_s, Δr_m and Δr_l = [(swap rate at month t +1) – (swap rate at month t)] / (swap rate at month t) for the relevant maturity swap.

Similarly, for the steepener / flattener positions that reference changes in the slope, the formula used is:

$$Total\,\%\,return = \;\; \Delta r_l - \Delta r_s$$

Δr_s and Δr_l = [(swap rate at month t +1) – (swap rate at month t)] / (swap rate at month t) for the relavant maturity swap.

3.2 Overview of Data

The data used in this research was sourced from Inet and Reuters Datastream, and consists of the variables discussed below.

3.2.1 Part 1. Nelson Siegel Parameterisation of the South African Sovereign Yield Curve

Sample Period: Monthly Data June 2003 – Feb 2014

- South African Local Currency Government Bond Yield Curve: 3M, 6M, 1Y - 30Y zero coupon bond spot rates

3.2.2 Part 2. Ordinary Least Squares Regression Model of the Predictors of the Yield Curve Parameters

Sample Period: Quarterly Data Q3 2003 – Q4 2013

The following 19 explanatory variables were considered when predicting changes in the Level, Slope, and Curvature of the SA Yield Curve. The majority of these variables relate to economic fundamentals, and thus are released quarterly. Provided below is the economic rationale for their consideration in the OLS Regression Model.

South African Economic and Debt Fundamentals

- Government Debt to GDP Ratio

  This ratio looks at the amount of debt issued by the SA Government relative to the Gross Domestic Product. One line of thinking is that as the nominal amount of debt issued by government increases, the cost of their debt should also increase as potential investors view worsening debt fundamentals as a sign of weakening ability to meet interest / principal repayment obligations (Min et al., 2003). This is scaled by the amount of output in the SA economy as one would expect that if debt levels are rising but GDP is falling (or not rising by as much) that this generates an even poorer outlook for SA's ability to meet credit obligations as they fall due This is compounded by the fact that weak GDP growth indicates that the sovereign may need to generate even more debt in the future so as to aid dwindling economic activity via the mechanism of fiscal policy expenditure on such variables as infrastructure. Another line of thinking, however, is that as the amount of domestic traded debt in the market increases, so might more investors with various yield-curve maturity preferences be enticed by SA's well developed government bond curve. An increased number of market players may improve liquidity conditions and thus reduce bond yields.

- Real GDP Growth rate

The effect of a rising GDP Growth rate and better economic prospects should lower the level parameter of the yield curve (which is representative of long-term bond yields) (Min et al., 2003), as well as potentially flatten the curve via the slope factor given a lower risk premium needed in the long-end of the curve versus the short-end. This is consistent with Liquidity Preference Theory, which assigns a greater risk premium to long-end versus short-end bonds (Cox, Ingersoll, & Ross, 1985).

- Foreign Direct Investment to GDP Ratio

A rising level of foreign investment in local business and production points to strengthening economic fundamentals and thus an elevated creditworthiness of the SA sovereign (Min et al., 2003). Such a variable may of course be a coincident indicator with yield curve parameters, as a more positive outlook for SA should fuel investment activity and at the same time improve bond yields and the ability of government to raise cheap debt financing. Note that foreign non-direct (or Financial) investment is not considered owing to the fact that the debt investment portion of this variable is already accounted for under the "Government Debt to GDP Ratio" variable.

- Current Account Deficit / Surplus to GDP Ratio

The current account measures the difference between trade exports and imports, with a deficit indicating that a country is importing more than it is exporting (Min et al., 2003). Typically, this should lead to a depreciation of the local currency rate (in this case the ZAR) as foreign exporters must sell ZAR in order to recognize profits in their home currency. Apart from indicating a weaker export arena (in SA's case potentially the mining and resources sector), according to national accounting principles a current account deficit needs to be balanced with a financial account surplus. Thus, government's need for financial investment and debt issuance would be greater. This expected increase in borrowing should appropriately raise government bond yields.

- Government Budget Balance to GDP Ratio

The Budget Balance looks at the difference between government spending and taxes raised, with a Budget Deficit indicating that spending needs have overtaken government "income" in the form of taxes (IMF Staff Report, 2013).

The interpretation of this ratio of course depends on the economic climate at hand. If government has increased spending so as to stimulate economic activity and such policy action is perceived as being both credible and likely to succeed then it is possible that the market will interpret this as being indicative of improving future economic growth (thus lowering the level of yields and potentially flattening the slope of the yield curve).

If, however, the market perceives the deterioration of this ratio as being indicative of the inability of government to collect sufficient taxes or infers that government spending is not effective and is not targeting the appropriate areas of the economy, this may lead to a sell-off across the yield curve. Such a situation has arisen in the past when the "cannibalization" of the SA government budget by paying for above-inflation wage increases in the mining sector caused the budget balance to deteriorate, while key infrastructure spend on aspects of the economy was perceived to be lacking (IMF Staff Report, 2013).

- Inflation rate

A rise in the inflation rate should lead to an erosion of the value of fixed income debt, such as government bonds, which earn a fixed interest rate (Mehl, 2009). Apart from raising the level of bond yields in order to cheapen them, the slope of the yield curve might also be expected to rise if long-term inflation is expected.

- Average SA Sovereign Credit Rating (as measured by Oxford Economics)

  Sourced from Reuters Datastream, this data series comprises of numerical average credit rating scores compiled by Oxford Economics. The highest rating reflects a numerical score of “20”, which equates to an AAA-rated ‘risk-free’ issuing entity. Composite averages are determined using rating scores assigned by large rating agencies.

- South Africa Foreign Investment Climate Rating (as measured by World Economic Services)

  Sourced from Reuters Datastream, this data series comprises of numerical average Foreign Investor Climate Rating scores compiled by World Economic Services. This captures the degree of political stability within SA and the extent to which perceived instability affects foreign investor sentiment and the ease of terms of trade. A panel of economic experts within South Africa are surveyed and asked to assign scores from 1 to 9 (with “9” equating to the highest level of political stability and foreign positive sentiment towards investment).

- Average Maturity of Domestic Marketable Bonds

  Sy (2002) uses this variables as an indicator of the magnitude of interest rate exposure or risk that a sovereign’s debt exposes its investors to, with longer maturity debt profiles increasing the interest rate risk for debtholders.

- Proportion of Foreign ownership of Short Term Debt

  While domestic investors tend to fall into the “buy-and-hold” category owing to specific fund obligations to hold a specific minimum proportion of local debt and equities, foreign investors are far more likely to trade in and out of the market (Peiris, 2010). As a result of South Africa’s rising proportion of foreign ownership of domestic debt in recent years, this has made national sovereign bond levels more sensitized to large-scale sell-offs that occur when “risk-off” sentiment pervades the global markets (IMF Report, 2013).

South African Market-traded variables

- 3-Month JIBAR

Sourced from Reuters Datastream, 3-Month JIBAR (Johannesburg Interbank Agreed Rate) is the consensus 3-month benchmark rate (released daily) at which the major South African banks lend money to each other in the Interbank market. This level of short term rates may play a role in both the longer-term level and slope factors of the yield curve.

- Barclays South Africa Inflation-Linked Bond Index

Given that the SA Inflation rate has already been included as an economic variable, the addition of this variable is deemed necessary as it does not measure the SA inflation rate per se, but rather the total return on the SA Inflation Linked bonds in issue (As sourced from Reuters Datastream). The key differences are firstly that this index measures inflation expectations at multiple yield buckets from short term (1 – 3 years) out to the 30-year point. Additionally, the returns on the inflation-linked bonds should rise based on rising inflation **expectations**. This taps into a more complex notion than simply the inflation rate, but rather the market's expectations about future inflation and the credibility of the South African Reserve Bank (SARB) in terms of its ability to keep inflation within South Africa's 3 % - 6 % target band in the future (Woglom, 2003).

If future inflation expectations are expected to rise owing to the inability of the SARB to rein in inflation, one might expect the slope factor of the yield curve to steepen. Alternatively, a rise in expected short-term inflation could cause the curve to flatten.
That said, whether rising inflation is the result of "demand-led" core factors pertaining to the domestic economy (such as increased retail spending, growing economic activity, and rising consumer wealth) or global supply-side pressures (which most often impact the Food and Fuel components of the Consumer Price Index basket) will also play a role. The SARB's credibility should only presumably be judged based on its ability to mitigate demand-led domestic factors that are within its control.

United States Economic and Market-traded Variables

- US Real GDP Growth Rate

While SA's Real GDP Growth rate has already been included as a potential variable, the US's real GDP Growth rate is also deemed necessary based on the US's status as a superpower whose economic outlook greatly affects the global financial markets (Mehl, 2009). US economic data releases, such as US GDP rate, Federal Open Market Committee meeting decisions, and US unemployment figures, are closely watched by market players and give direction as to the risk sentiment in the market.

Weaker US GDP growth could point to a broad-based investor risk-off sentiment and a reduced investor interest in riskier Emerging Market assets, such as SA Government bonds. At the same time, if weaker US growth was perceived to be accompanied by an expansionary Monetary Policy regime, this could be supportive of Fixed Income on a global level as it might send the market the message that interest rate hikes are unlikely for the foreseeable future. This could possibly cause the level parameter of the yield curve to decline (i.e. a bond rally).

- US 3- Month Deposit Rate

The interest rate for 3-month US-Dollar deposits is considered due to its strong ability both to influence the global markets (Min et. al, 2003) and because of its relation relative to the long-end of the yield curve and subsequent implications for the level and slope factors. A fall in yields (rising prices) of US 3-month deposits might indicate that Fixed-Income investments seem attractive and be supportive of the SA yield curve, or may signify that an investor "flight to quality" is taking place such that secure high-investment grade assets are preferred over riskier Emerging market assets.

- JP Morgan US Government Bond Index

The spread between various global sovereign bonds and the US government bonds are watched by market speculators and investors as a signal of so-called pricing inefficiencies or an indicator of the relative value of global sovereigns in comparison to the US as a "risk-free" benchmark (Sy, 2002). South African bond yields might thus be expected to move somewhat in tandem with their US counterpart bonds.

- US BAA-Rated Corporate Bond Yield Index (Lower Medium Grade Corporates)

A rise in price (lower yields) in lower-grade US corporate bonds might signify an increase in risk appetite in the global financial markets, which could be supportive of riskier emerging market debt (Sy, 2002).

International Traded Variables

- Real Brent Crude Oil price

  The real (inflation-adjusted) oil price fluctuates with demand and supply shocks in the global oil markets. Rising prices are often seen as an indicator of future economic downturns or even a recession, as was seen in the late 1970's (Min et al., 2003). This variable is included as a measure of external market shocks and their effects on SA government yields.

- JP Morgan Emerging Markets Bond Index (EMBI)

  The JP Morgan EMBI is used so as to account for the emerging market (EM) risk factor and account for global sentiment towards these developing countries (Sy, 2002). An increase in the EMBI spread (higher yield) should indicate a widespread "risk-off" market sentiment towards EM's, and such contagion risk is the explanation for increased Emerging Market asset correlations during financial downturns (Baek, Bandopadhyaya & Du, 2005).

- CBOE Volatility Index

  While Baek, Bandopadhyaya, and Du (2005) develop their own market risk appetite index as an explanatory variable of government debt spreads, this model uses the Chicago Board Options Exchange (CBOE) Volatility Index. The authors explain that higher volatility in the markets is seen during economic crises and often indicates reduced desire for riskier assets.

3.2.3 Part 3. Dynamic Latent Factor Approach to the SA Government Yield curve

Sample Period: Monthly Data June 2003 – Feb 2014

This section of the research mirrors the analysis and data in Diebold et al.'s (2006) paper, with the following monthly variables utilized:

- South African Local Currency Government Bond Yield Curve: 3-month, 6-month, 1Y - 10Y zero coupon bond spot rates

- South African Deposit Rate

- SA Inflation Rate

- SA Capacity Utilization

  This variable measures the extent to which the full productive capacity in the economy is being utilized, and is often viewed as a coincident indicator of the business cycle. Increases in aggregate demand signify that manufacturers and producers are utilizing more of their productive capacity in order to meet consumer appetite (Diebold et al., 2006).

3.2.4 Part 4. Linking the SA Asset Classes: An investigation of Equity sectors and their relationship with the Currency and Yield Curve Parameters

Sample Period: Monthly Data June 2003 – Feb 2014

- South African Local Currency Government Bond Yield Curve: 1Y - 30Y zero coupon bond spot rates (with annual benchmark levels)

- USD/ZAR spot closing levels

- Total Returns for the Johannesburg Stock Exchange (JSE) Derivative indices:

    - RESI10 (Resources Index)
    - INDI25 (Industrials Index)
    - FINI15 (Financials Index)
    - FINDI30 (Financials and Industrials Index)

- JSE ALSI and Top40 closing share prices and dividend yields

3.2.5 Part 5. Nelson Siegel Parameterisation of the SA swap rate curve: Implementation of Systematic Trading Strategies

As per the key work of Fabozzi et al. (2005) on the US Swap rate curve and their investigation of 12 potential explanatory variables, South African data is collected to mirror their selection of variables.

Sample Period: Monthly Data August 2003 – June 2014

- South African Swap Curve: 3-month, 6-month, 1-, 2-, 3-, 4-, 5-, 7-, 10-, 15-, 20-, and 30-year swap rates

Interest-Rate Variables

- Lagged Values of the Swap Level, Slope, and Curvature Parameters $\{d\beta_1(-1), d\beta_2(-1), and\ d\beta_3(-1)$ as extracted from the Nelson Siegel Parameterisation }
- 1- / 30-year Government Bond Yield Curve Slope
- 3-month JIBAR
- One-year USDZAR forward rates

  Fabozzi et al. (2005) look at the one-year forward rates on bonds ranging from 1 to 5-years in maturity as a proxy for future interest rate expectations. Due to lack of liquidity for these financial instruments in the South African context, reliable and complete data is not available for the sample period. As such, the present research utilizes the one-year forward USDZAR rate, which should contain expectations of the future currency value and thus the level of inflation / interest rates. The link is obviously somewhat more tenuous, however.

- Barclays South Africa Inflation-Linked Bond Index

Market-Risk Variables

- S&P 500 Volatility Index (VIX)

While many of the predictors (inflation / forward rates) used mirror that of Fabozzi et al.'s (2005) data but reference the South African economy, the Volatility Index for the All Share Index / Johannesburg Stock Exchange Top 40 does not contain enough data history to be included in the analysis. That said, given the status of the US as an economic superpower and the high correlations between its asset classes and those of the EM world, the S&P 500 VIX is also used in the present research.

- US High-yield Corporate Bond spreads

Given the lack of liquidity and stale market pricing seen in the SA corporate bond market for much of the sample period, the US level of high-yield corporate bond spreads is utilized as a proxy for market sentiment towards riskier asset classes.

- Brent Crude Oil price

"Stock-Price cheapness" Variables

- JSE Banks Sector Average Dividend Yield

Fabozzi et al. (2005) assert that a rising dividend yield points to the fact that dividends are falling more slowly than share price, which is in turn indicative of stock weakness and a greater risk premium being attached to shares. The dividend yields for the banking sector in particular were selected as the swap rate curve incorporates the default risk of the major SA banks (Liu, Longstaff & Mandell, 2002, 2006).

Economic Growth Variables

- SA Capacity Utilization

4. Data Exploration

4.1 South African Government Bond Yield Curve Exploration

The South African ZAR Government Bond Yield curve has undergone a dramatic transformation from 2003 to 2014 – with the yield curve shape shifting from being totally inverted to taking on a "normal" term structure with long-end rates trading at much higher yields than those in the short-end (see Figures 3 and 4). Apart from this change in the slope of the curve, yield levels have fallen quite dramatically over time. This fall in yields in the 2000's is common of all Emerging Market (EM) Sovereign debt, and is a trend attributed to increased foreign participation in local-currency EM bond markets over the period (Peiris, 2010).

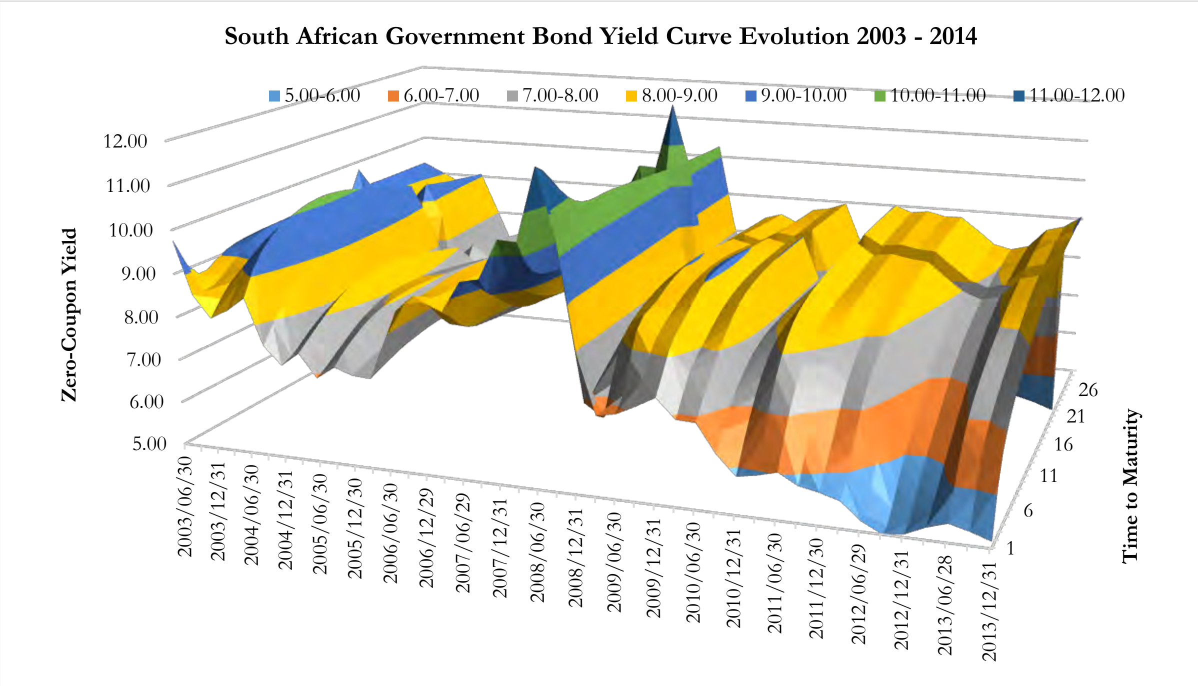


Figure 3: Evolution of the South African Government Bond Yield Curve, March 2003 to Feb 2014

Source: Reuters Datastream

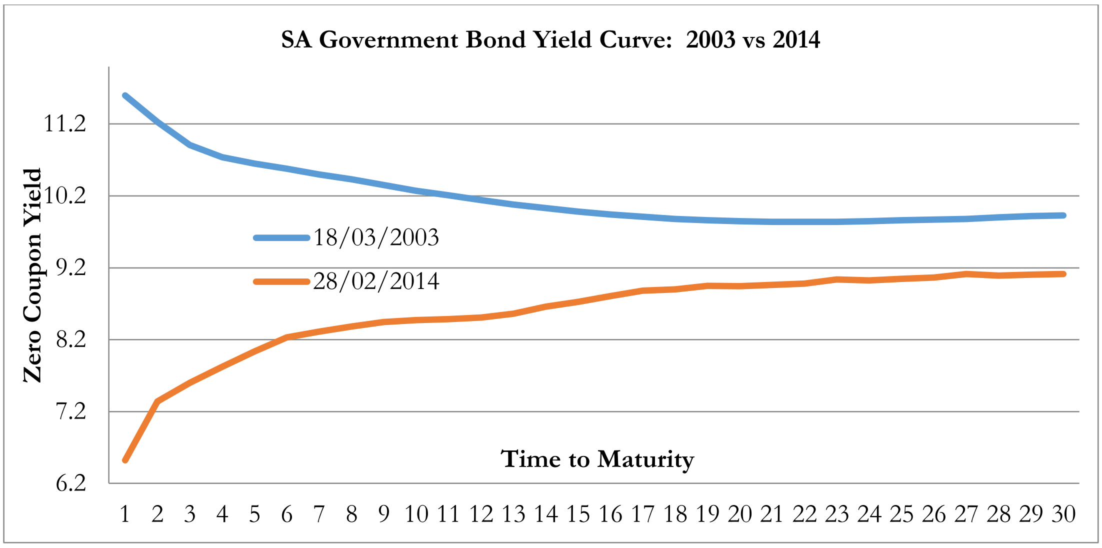


Figure 4: SA Government Bond Yield Curve - Snapshots as at 18/03/2003 vs 28/02/2014

Source: Reuters Datastream

4.1.1 Theories of the Term Structure of Interest Rates

Theories of the term structure of interest rates postulate under the "Liquidity Preference Theory" that long term rates under a normal yield curve structure should be greater than short-term rates in order to compensate investors for the greater risk associated with investing for a longer term (Cox, Ingersoll, & Ross, 1985). Contrastingly, the authors explain that the Culbertson's "Market Segmentation Hypothesis" of interest rates asserts that supply and demand dynamics in various parts of the curve will determine the shape of the term structure and the relative level of yields, with longer-term maturity buckets not necessarily exhibiting higher yields. The authors also explain that the "Preferred Habitat Theory" explanation of the term structure emphasises that different investors and market players have a preference to invest in certain areas of the curve, and that they will thus require yield compensation to move into another maturity bucket. An example of this is that Life Insurance companies generally have a vested interest in trading within the long-end of the curve so as to hedge their future long-term insurance policy liabilities. Thus, an influx of Life Insurance Institutions within a given bond market may serve to flatten the yield curve over time as they demand less of a premium to invest in the long-term, and may alter demand and supply dynamics for that area of the curve.

Armed with these theories, one might hypothesise that the reasons for this dramatic movement in the SA yield curve, and specifically in the steepening of its slope, could be multi-fold. As per the Liquidity Preference theory, investors may have demanded more of a risk premium for holding long-term South

African debt owing to worsening Government debt fundamentals and deteriorating creditworthiness, which may have been driven by anything from political instability to weaker GDP Growth.

Under the Market Segmentation theory, various demand and supply dynamics might have contributed to these slope changes – for example increased issuance and supply of Government debt, and a greater emphasis being placed on long-end versus short-end issuance of debt. More difficult to assess would be whether, as per the "Preferred Habitat Theory", the type of investor within the local ZAR Government bond market has changed such that there has been a possible increase in shorter-term market players, such as Hedge Funds and market speculators, that has driven short and medium term yields lower in comparison to the long-end.

Additionally, Cox, Ingersoll, and Ross (1985) propose the "Rational Expectations Hypothesis", which suggests that the term structure is representative of investors' expectations concerning the path of future interest rates. Such expectations would undoubtedly be influenced by inflationary concerns, with a steeper curve indicating expectations of higher future inflation and interest rates that lead to an erosion in the value of fixed-interest debt. A rise in yields (i.e. lower prices) would then take place in order to compensate investors for this erosion in value.

In the following data exploration, the ability of these term structure theories to explain changes in the shape of the yield curve will be evaluated by looking at evidence of South Africa's deteriorating creditworthiness, adjustments in bond supply and demand dynamics, changes in the proportional investor ownership of Government debt, and inflation expectations.

4.1.2 Inflation and the Yield Curve

The most simple of the discussed theories to assess is arguably the effect of inflation and the expected path of interest rates on the term structure. The inverted term structure seen in 2003 was proceeded by a period of dramatic decline in the South African Inflation rate from 6.7% year-on-year at the beginning of the sample period to just 0.2% in May 2004 (see Figure 5). Thus, the shape of the curve reflected falling interest rates and increasing bond prices. By the end of the sample period, inflation was relatively stable at 6.1% year-on-year (the upper-end of the South African Reserve Bank's target of 3 – 6%).

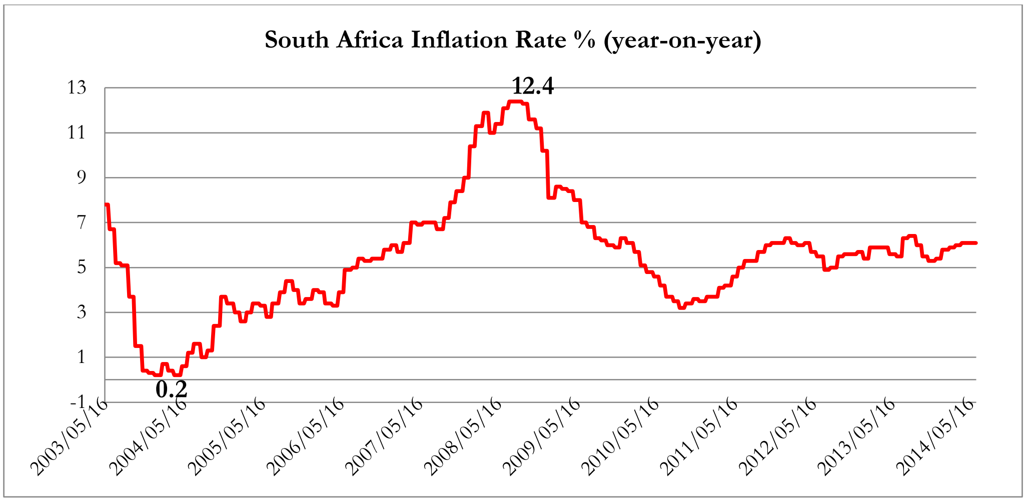


Figure 5: SA Quarterly Inflation Rate figures (y/y %)

Source: Reuters Datastream

Examining the level of 10-year SA Government bonds, the 1-30-year SA Government Bond slope, the SA inflation rate, and USDZAR exchange rate in Figure 6 reveals some interesting trends amongst these series. Rand strength and weakness appears to track bond level rallies and sell-offs such that risk-off sentiment in bonds is accompanied by rand weakness. From an implied inflationary perspective such a relationship makes sense as a weakening currency should be associated with higher inflationary pressures in the short-term and a subsequent erosion in the value of fixed income debt (Woglom, 2003).

The more volatile swings in inflation appear to be more closely captured in the Government bond slope. While in more recent periods (2011 to present day) the slope seemed to rise and fall concurrently with inflation, for much of the 2003 to 2010 sample period it would appear that the slope flattened when inflation rose. This could represent the market's pricing in of a rise in short-term (1-year) rates as the result of inflation, with longer-term rates seen to not react as strongly to perceived inflationary fears. Such trends could be indicative of the market's inherent confidence in the South Africa Reserve Bank and its perceived credibility in terms of being able to use monetary policy as an effective tool to combat inflation in the longer-term (Woglom, 2003).

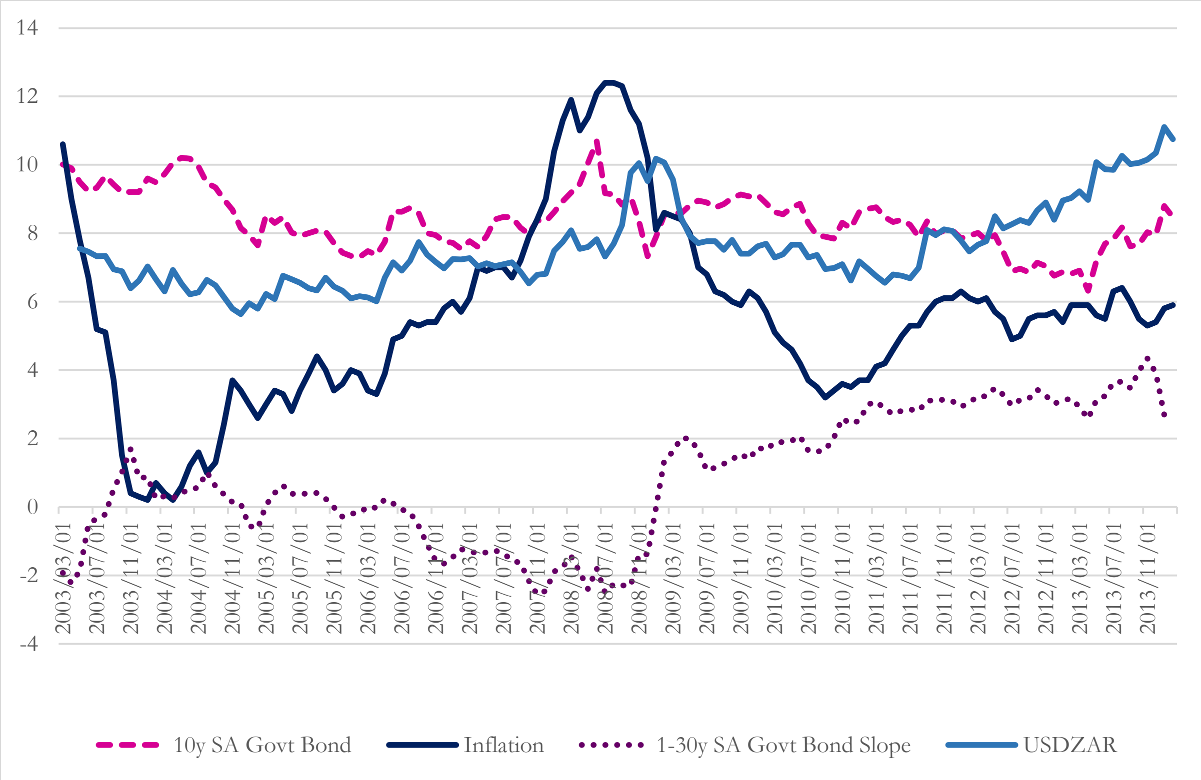


Figure 6: Potential Predictors of the 10-year SA Government Bond Level

Source: Reuters Datastream

4.1.3 Changing Issuance, Maturity Profiles and Ownership of Government Debt

The proportion of SA Gross Government Debt as a percentage of GDP has risen from 37% to 45% over the research sample period of 2003 to early 2014 (see Figure 7). During the pre-crisis boom years of the 2000's, the South African Government was able to reduce its debt levels from 51% of GDP to a multi-year low of 23%. This allowed the sovereign some leg room to increase fiscal policy spending and increase its debt in the after-math of the crisis, but since then the situation has spiralled (IMF Report, 2013). The IMF warns that it is critical that SA maintain a below-40% Debt-to-GDP ratio, which is the limit that they recommend for all emerging markets in order so that they can have spare capacity to raise debt in the event of an economic crisis.

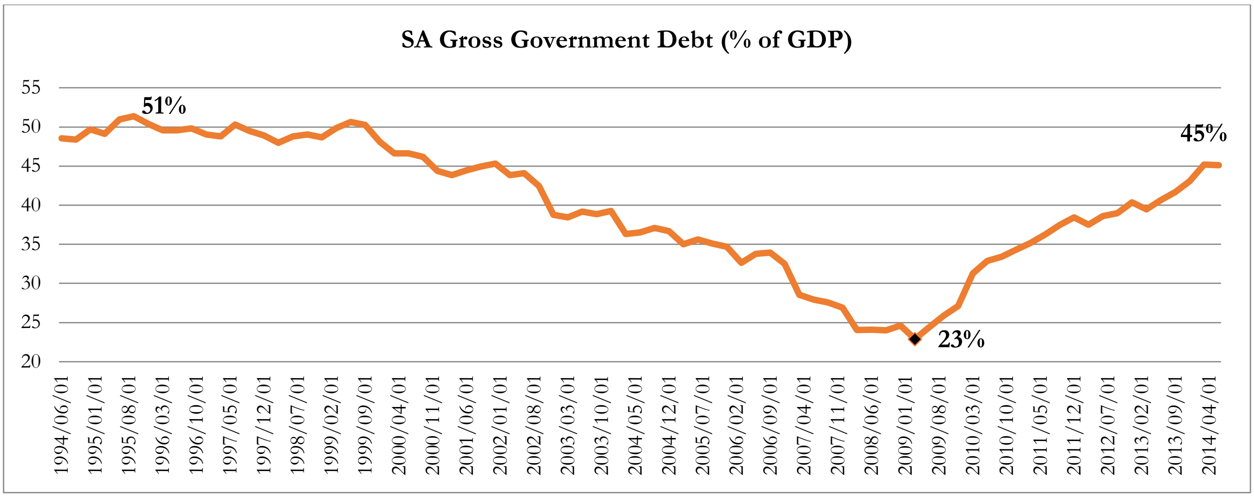


Figure 7: SA Gross Government Debt as a % of GDP, 1994 to 2014

Source: Reuters Datastream

Thus, while situating South Africa's Net percentage of Government debt to GDP amongst its global sovereign counterparts in Figure 8 might **appear** to shed more of a positive light on the situation, the IMF's upper limit for developed economies is a far greater 60% ratio.

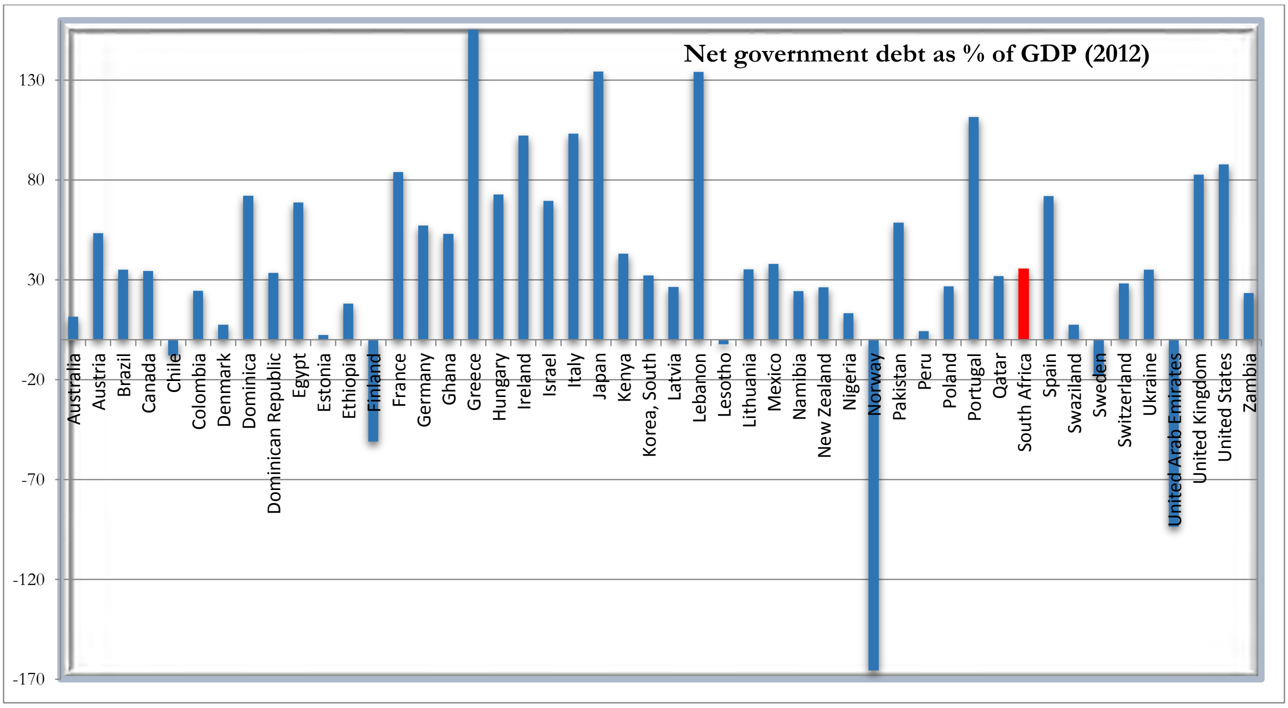


Figure 8: Net Government Debt for Global Sovereigns, 2012

Source: World Bank Database

In fact, while Emerging Market sovereigns have been able to reduce their Debt-to-GDP ratios following the post-crisis fiscal policy expenditure period, as per Figure 9 one can see that South Africa's ratio has been trending even higher, and is now well above the EM average (IMF Report, 2013).

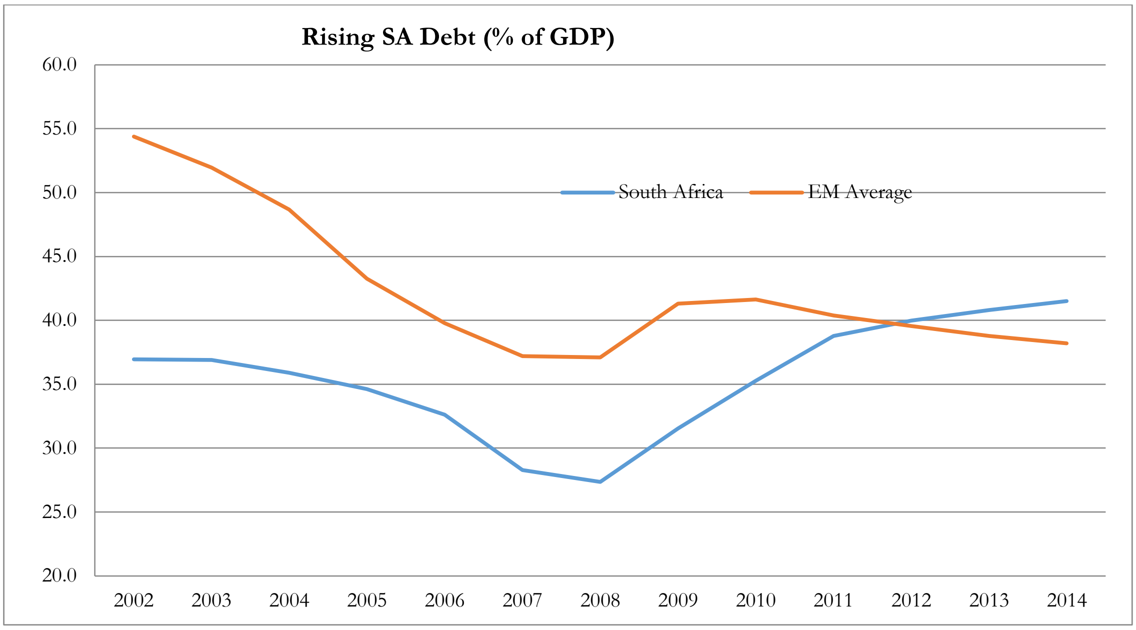


Figure 9: South African vs Emerging Market Debt, % of GDP: 2002 to 2014

Source: Reuters Datastream

An evaluation of the composition of Government Marketable debt in Figure 10 reveals that it is 85% comprised of Local-Currency ZAR-denominated bonds issued in the domestic market. That said, the IMF cautions that in the 2012 – 2013 period, foreign ownership of all SA Government debt climbed to 60% (with 36% foreign ownership in local currency SA bonds up from 13% in 2008). Figure 11 illustrates the climb in foreign ownership of short-term ZAR Government debt.

This puts South African bonds in an extremely vulnerable situation due to the enlarged price effect of any risk-off sentiment in the global markets. Peiris (2010) asserts that while domestic investors tend to fall into the "buy-and-hold" category owing to specific fund obligations to hold a specific minimum proportion of local debt and equities, foreign investors are far more likely to trade in and out of the market. During May 2013 when foreigners sharply pulled out of Emerging Markets the effect on South Africa debt was thus amplified, sparking a colossal sell-off in yields and dramatically increasing the level at which Government could fund itself (IMF Report, 2013). The IMF asserts that this sell-off in the SA currency and local bond market was amongst the largest of any Emerging Market.

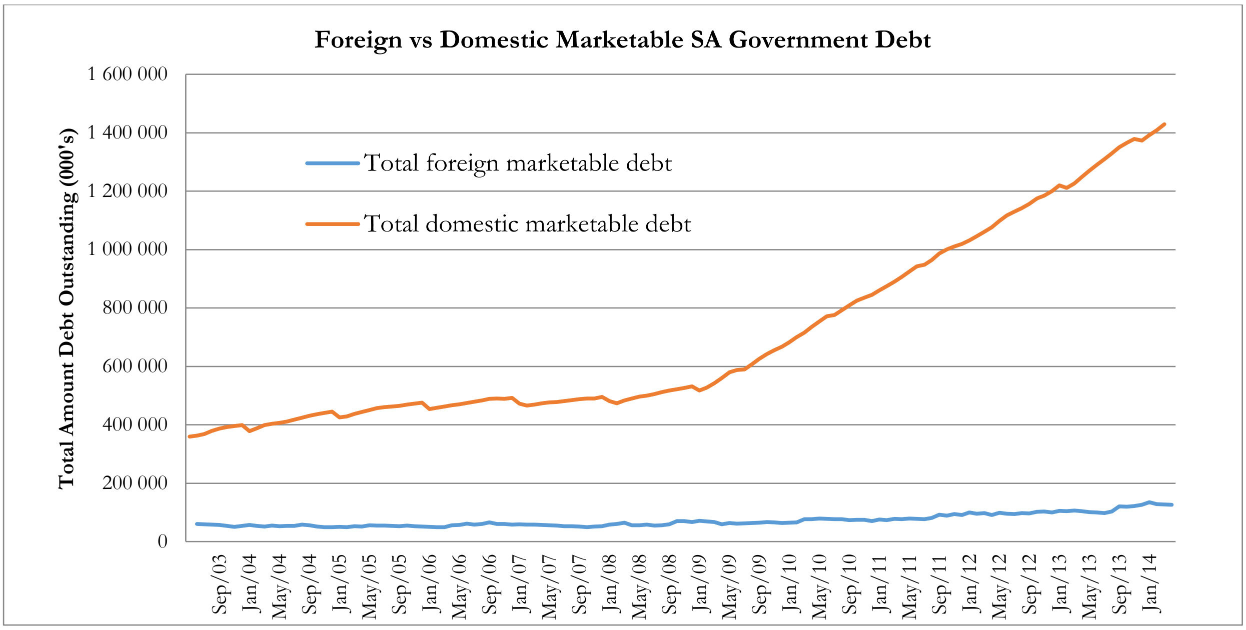


Figure 10: Foreign versus Domestic SA Marketable Debt, Nominal Outstanding

Source: South Africa National Treasury database

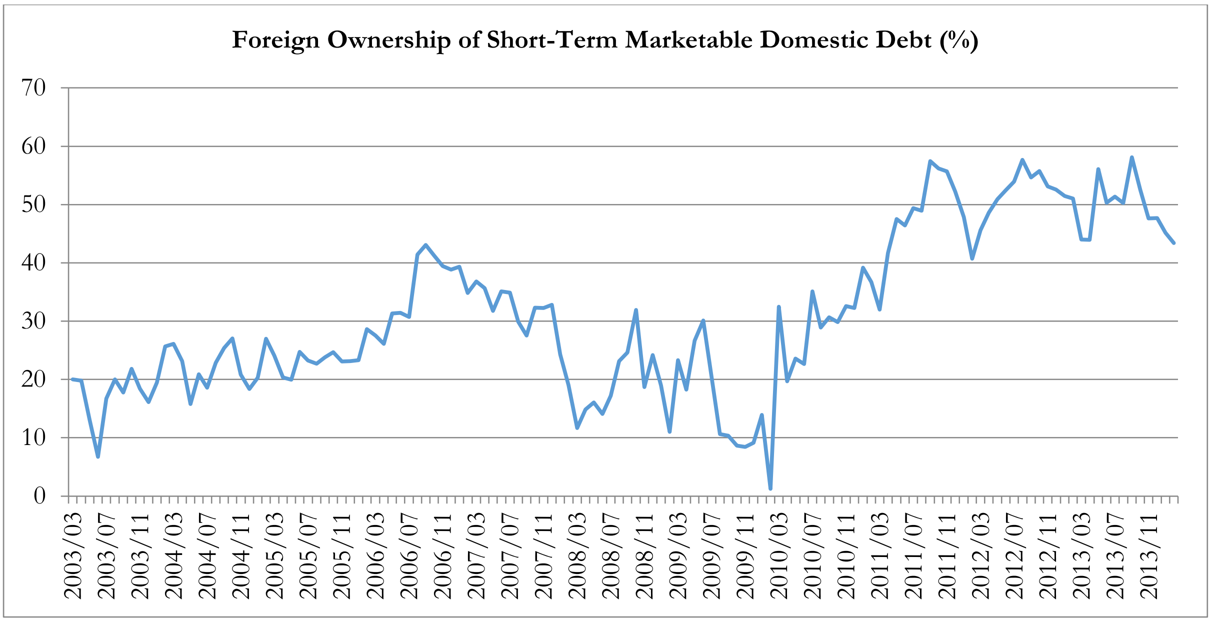


Figure 11: Foreign Ownership of Short Term SA Govt Marketable Debt, %

Source: South Africa National Treasury database

Another problematic issue that the IMF highlights is that of the long-average tenor of South Africa bonds (IMF Report, 2013), which increases the interest rate risk of Government debt. Clearly, the amount of debt outstanding is not as important to the slope of the yield curve as the relative duration of the bonds in issuance. As seen in Table 2, 50% of the current debt outstanding is situated in the long-end of the curve (10 – 30 years).

Table 2: Nominal Outstanding on SA Rand Government Bonds (by maturity)

	Years to Maturity	Amount Government ZAR Traded Bonds (Nominal Outstanding)	% of Total Debt
Short Term	1 – 5yrs	R 248 316 000	26%
Intermediate Term	5 – 10yrs	R 227 885 000	24%
Long Term	**10 – 30yrs**	**R 472 030 000**	**50%**

Source: Bloomberg

A bond's duration or "interest rate risk" is a key measure in the Fixed Income market that taps into the sensitivity of a bond's price to a change in the level of the yield curve (Litterman & Scheinkman, 1991). It is a function of both the bond's time to maturity and its coupon rate, with a longer time to maturity and a lower coupon rate leading to a higher duration as the bond will take longer to be repaid to the investor – leaving them with a greater exposure to changes in the level of interest rates (Litterman & Scheinkman, 1991). A comparison of the average duration of the South African Government bond index with that of the average for the Emerging Markets reveals that South Africa has a relative oversupply of higher-duration bonds (see Figure 12). While the gap between these series seemed to be closing when entering the 2008 crisis period, an increase in the average duration of issued South African bonds in the aftermath of the crisis caused the interest rate risk to trend higher. This may have contributed to the vivid steepening of its local ZAR Bond yield curve over the sample period.

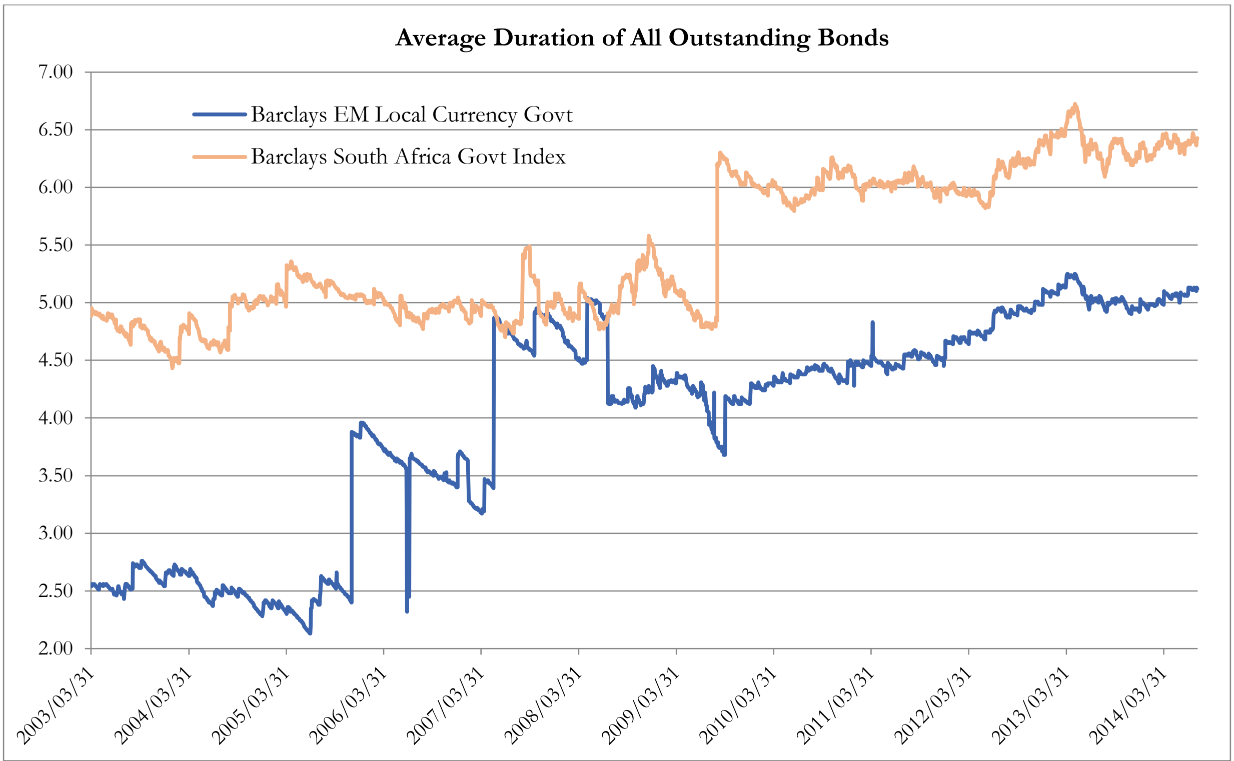


Figure 12: Average Duration of SA Government vs EM Local Government Bonds

Source: Reuters Datastream

Indeed, as per Figure 13, the relative growth in SA 10-year+ maturity bonds has outstripped that of all other tenors. Furthermore, the Nominal Amount outstanding in the 10-year + category has overtaken that of 3 – 10-year intermediate-term bonds. This change in the concentration of Government debt was in part due to increased "new issuance" of longer duration debt in the weekly SA Government bond debt auctions occurring every Tuesday, but also due to the introduction of a new auction by the National Treasury – the "Switch Auctions" (South African Reserve Bank, 2014). Taking place every Thursday, these auctions are a form of "debt restructuring" that allows the National Treasury to buy back short-dated debt from investors in the market, and to switch it out with them in exchange for an issuance of longer-dated debt – thus accomplishing the goal of extending the maturity profile of Government debt and avoiding the impending Principal Repayment due on the shorter-maturity bond. As per Figure 14, this has increased the average maturity of Marketable SA Government debt.

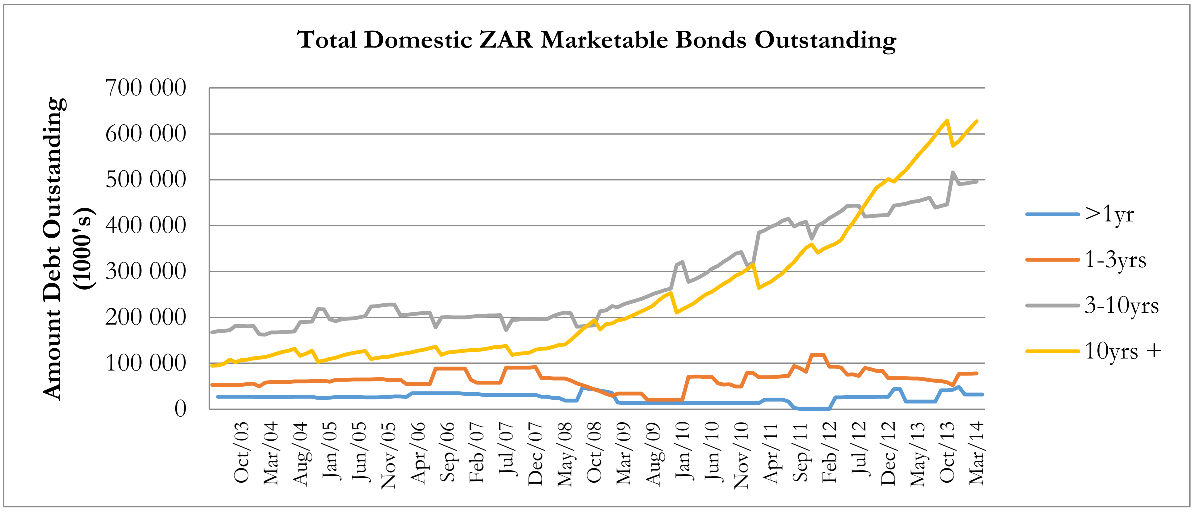


Figure 13: Total Government ZAR Bonds (Nominal Outstanding by maturity)

Source: South Africa National Treasury database

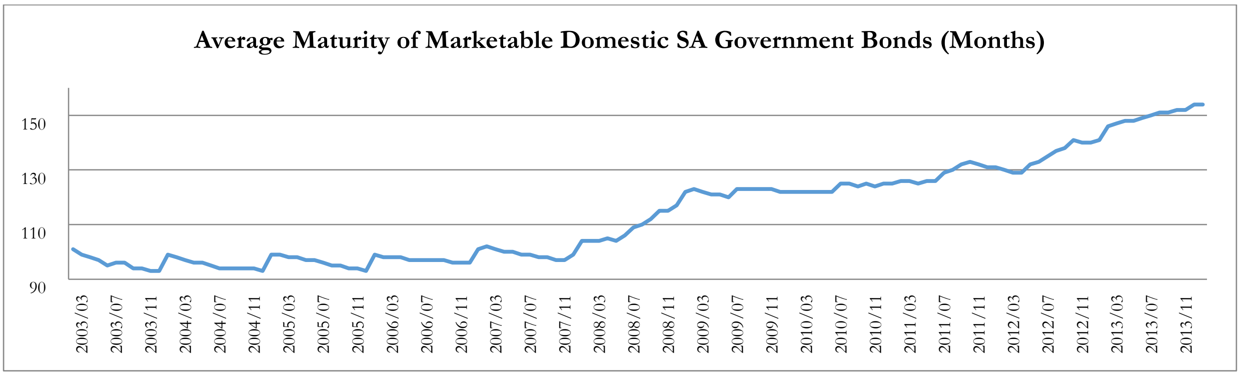


Figure 14: Average Maturity in Months, SA Govt Bonds

Source: South Africa National Treasury database

4.1.4 Government Creditworthiness, Risk Premiums, and the Yield Curve

The reason for these increases in Government debt and subsequent deterioration of the sovereign's creditworthiness are in part due to a need to stimulate the economy with extra spending in the aftermath of the 2008 crisis. Indeed, the IMF reports that South Africa's average of roughly 3% Real GDP growth in the post-crisis period (see Figure 15) was below the EM average of roughly 5% -- indicating a need to enhance economic growth and catch up to the level of its peers. That said, the composition of such expenditure is important. National expenditure has to a great extent been cannibalized by the wage bill and Government's grants of above-inflation wage increases to striking workers, which accounted for 35% of all spending in 2012 – far above the Emerging Market norm (IMF Report, 2013). According to the report, a deterioration in public economic investment into areas of infrastructure (roads, buildings, machinery, and schools) has unfortunately been the result of such over-allocations in spending.

Figure 15: SA Real GDP Growth Rate, % y/y

Source: Reuters Datastream

Looking for evidence of the South African Government's worsening creditworthiness involves investigation of a wide range of variables. As evidenced by World Economic Service's "Foreign Investor Climate Rating" (Figure 16), which encompasses the attractiveness of a country to foreign investors in terms of political stability and terms of trade, South Africa's rating has clearly deteriorated over the sample period. Similarly, South Africa's average numerical credit rating, as measured by Oxford

Economics, has dipped lower in recent years – albeit with something of a lag in comparison to its deteriorating political stability (Figure 17).

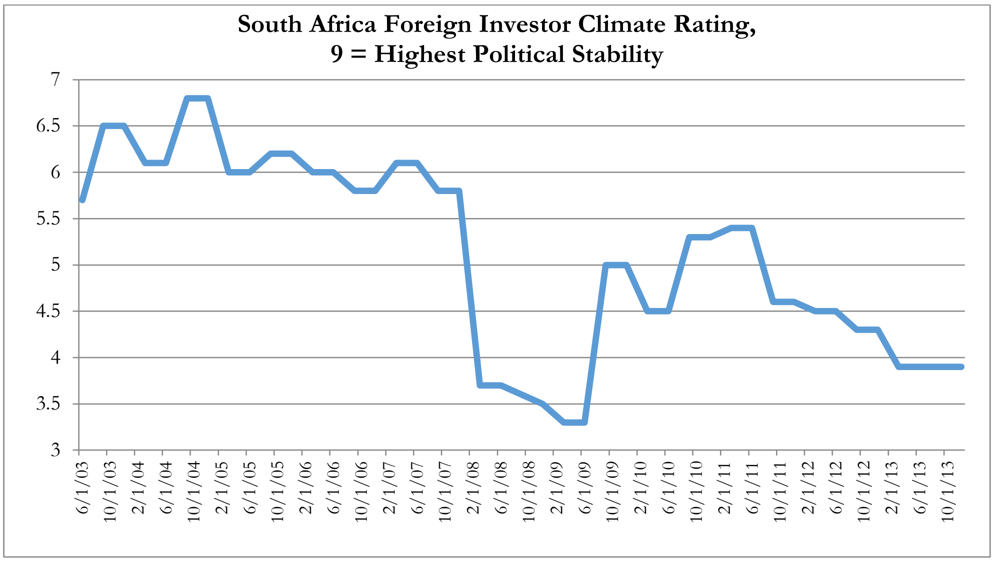


Figure 16: SA Foreign Investor Climate Rating

Source: World Economic Services via Reuters Datastream

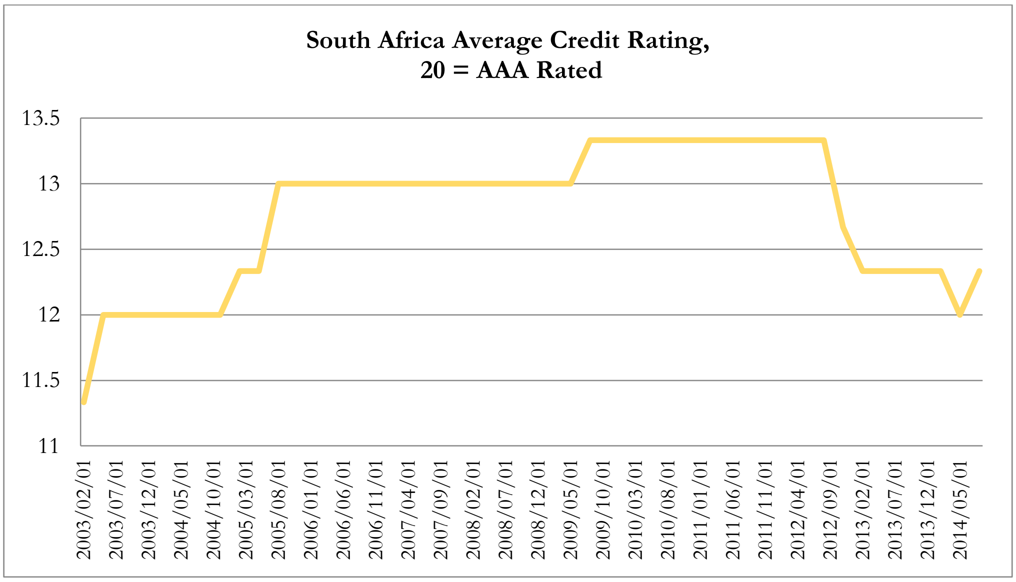


Figure 17: SA Average Credit Rating

Source: Oxford Economics via Reuters Datastream

The dramatic 2008-crisis decline in SA's Foreign Investor Climate Rating has recovered to some extent, but has not been able to get back to pre-crisis levels. Asiedu (2006) finds that Foreign Direct Investment flows into Sub-Saharan Africa are greatly impacted by perceptions of political stability, corruption, and riots. Undoubtedly encapsulated in the more-recent deterioration in SA's Foreign Investor Climate Rating are foreign investor reactions to such attention-grabbing News headlines around SA government corruption and illegal tenders, crippling strikes in the mining sector, above-inflation wage increase demands, a shortage of national electricity production, threats of banking sector and land nationalization, and social unrest and riots as evidenced by the killings at Marikana (IMF Staff Report, 2013).

Thus, equipped with the knowledge that the distribution of Government spending may not be optimal and with evidence of a Fiscal Deficit ranked 3rd largest in the Emerging Market sphere in 2012 (see Figure 18), and improving only slightly in 2013, South Africa's borrowing credibility is called into question (IMF Report, 2013).

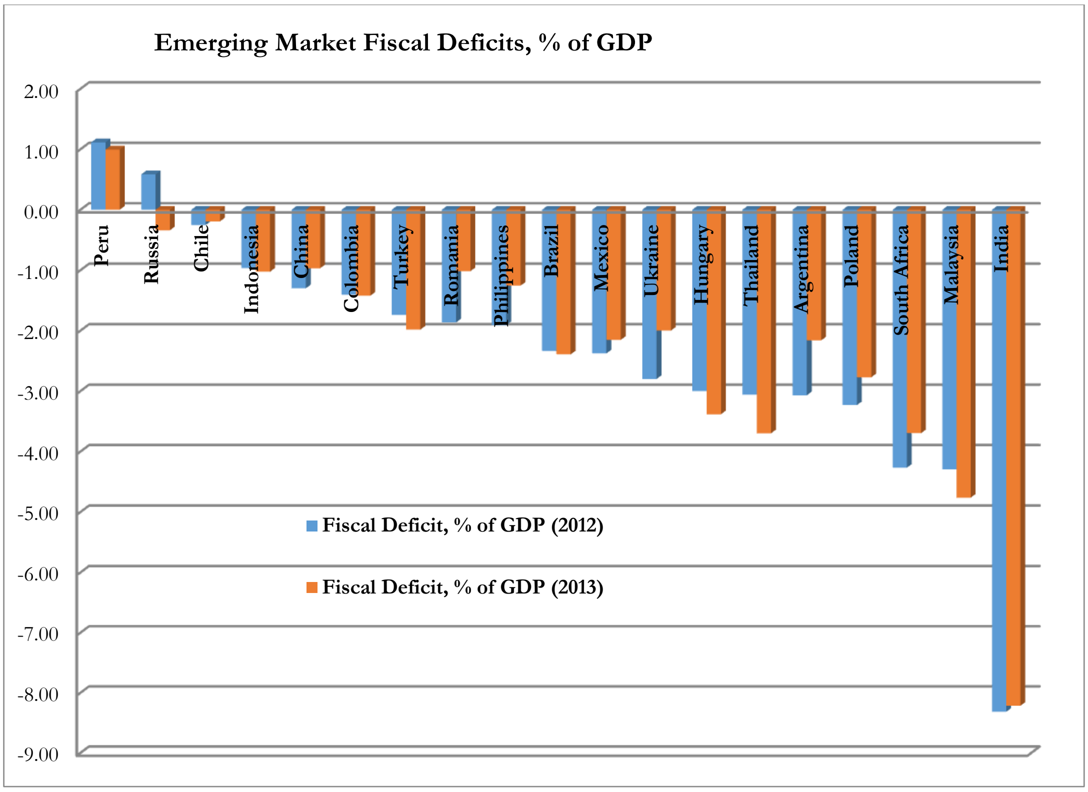


Figure 18: EM Fiscal Deficits (% GDP) for 2012, 2013

Source: Reuters datastream

The IMF also discusses the issue of the "twin peaks" South African Deficits in that a burgeoning Budget Deficit is paired with a wide Current Account Balance deficit (see Figure 19). This issue of lack of global export competitiveness and a declining level of exports relative to imports is similarly off-putting to investors as it means that Government will need to finance the current account deficit with even greater portfolio liabilities, such as marketable debt.

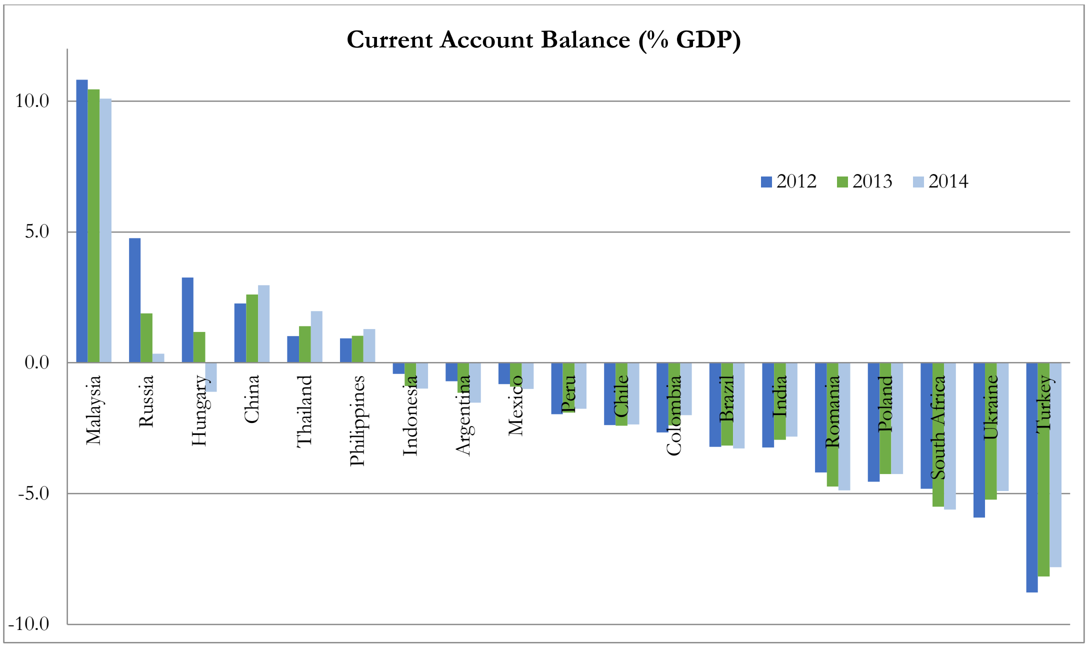


Figure 19: EM Current Account Balances (% GDP) for 2012, 2013, 2014

Source: Reuters datastream

While the Annual Current Account balance is now well below that of its EM peers (Figure 20), a look at the monthly data for the sample period in question reveals that in late 2010 this figure almost reached a surplus following a period of sustained Rand decline, which was supportive for global export competitiveness (Figure 21).

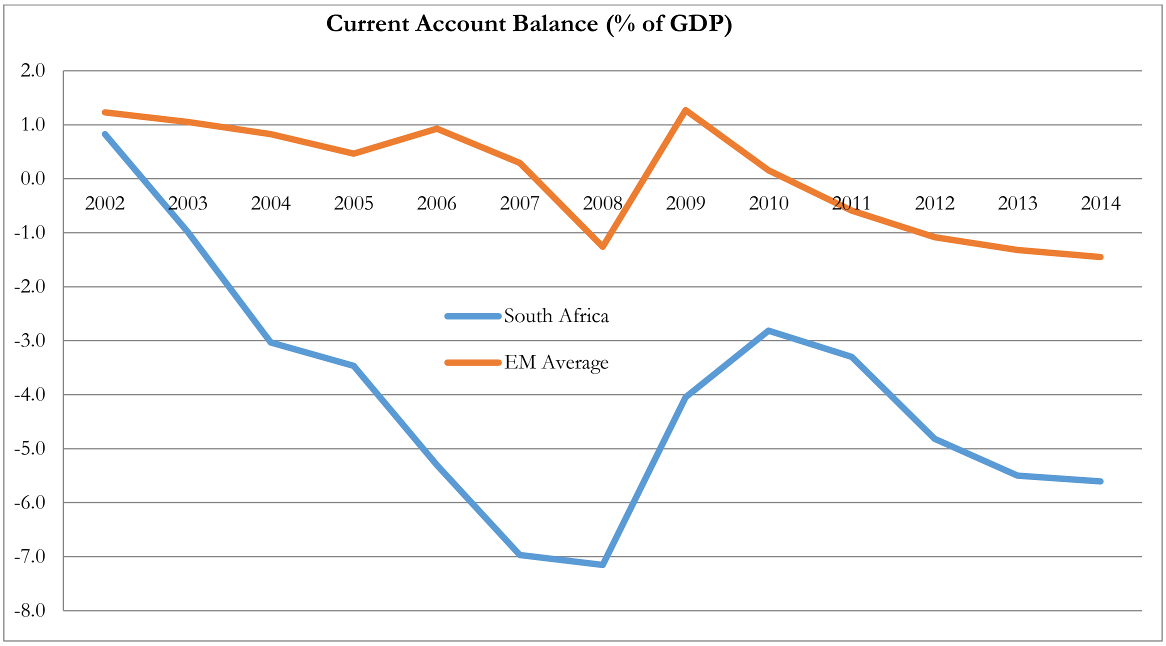


Figure 20: SA Annual Current Account Balance (% GDP) versus EM Average

Source: Reuters datastream

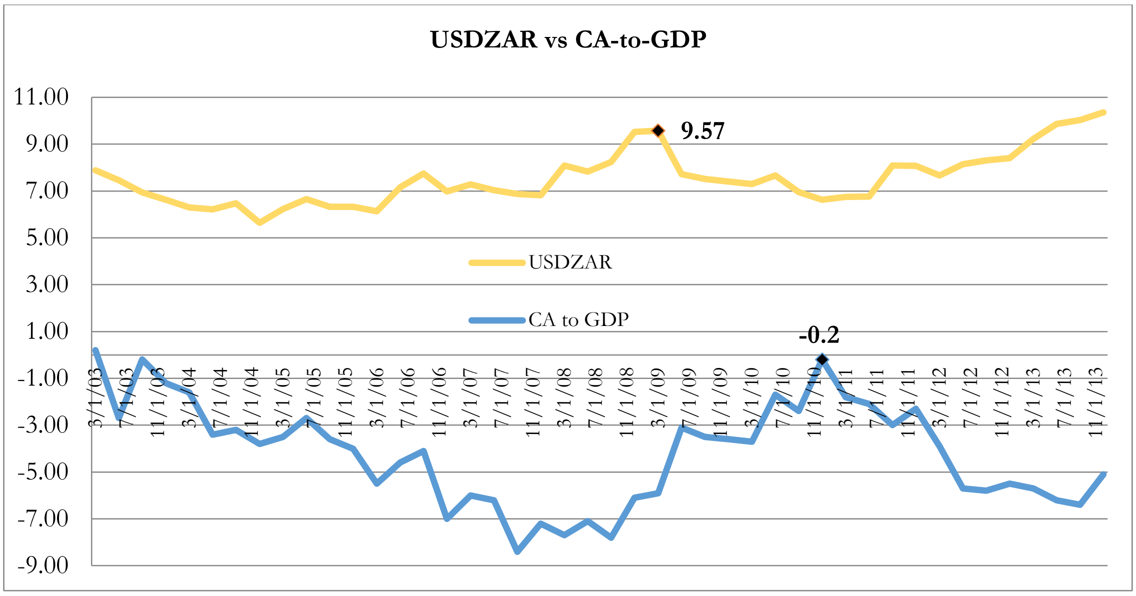


Figure 21: SA Monthly Current Account Balance (% GDP) versus USDZAR fluctuations

Source: Reuters datastream

That said, given the highest unemployment rate of its EM peers at a staggering 25% in 2013 and almost the lowest rate of National Savings at 15% of GDP for the same year, the South African sovereign's creditworthiness and future ability to raise necessary debt in the financial markets is undoubtedly shaky (Figure 22).

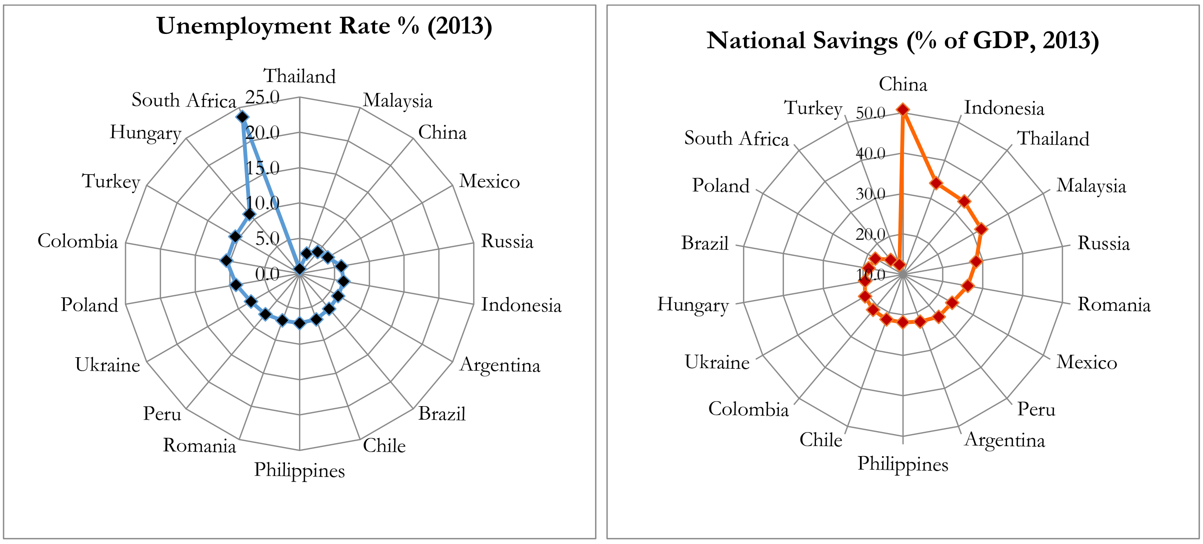


Figure 22: EM Unemployment Rate (%) and National Savings (% of GDP), 2013

Source: Reuters datastream

4.2 South African Swap Curve Exploration

4.2.1 Interest Rate Swap Theory

Interest rate swaps emerged as a new class of derivative instrument in the 1980s that aided financial institutions in managing their interest rate risk in the face of increasing interest rate volatility (Bicksler & Chen, 1986). By allowing such institutions to swap out their floating LIBOR (London Interbank Offered Rate) commitments that they paid on their liabilities for a fixed rate at the given market swap rate, interest rate swaps allowed parties to hedge their interest rate exposure (Bicksler & Chen, 1986).

As per the earlier exploration of the South African Government Bond yield curve, one sees that the SA Swap rate curve has undergone a similar such evolution over the sample period of 2003 to 2014 (see Figures 23 and 24). Again, the entire curve has traded at lower rates over time and has steepened. That said, this steepening trend is not as dramatic as that seen in the Government bond curve – in which a near-perfect inversion of the term structure took place.

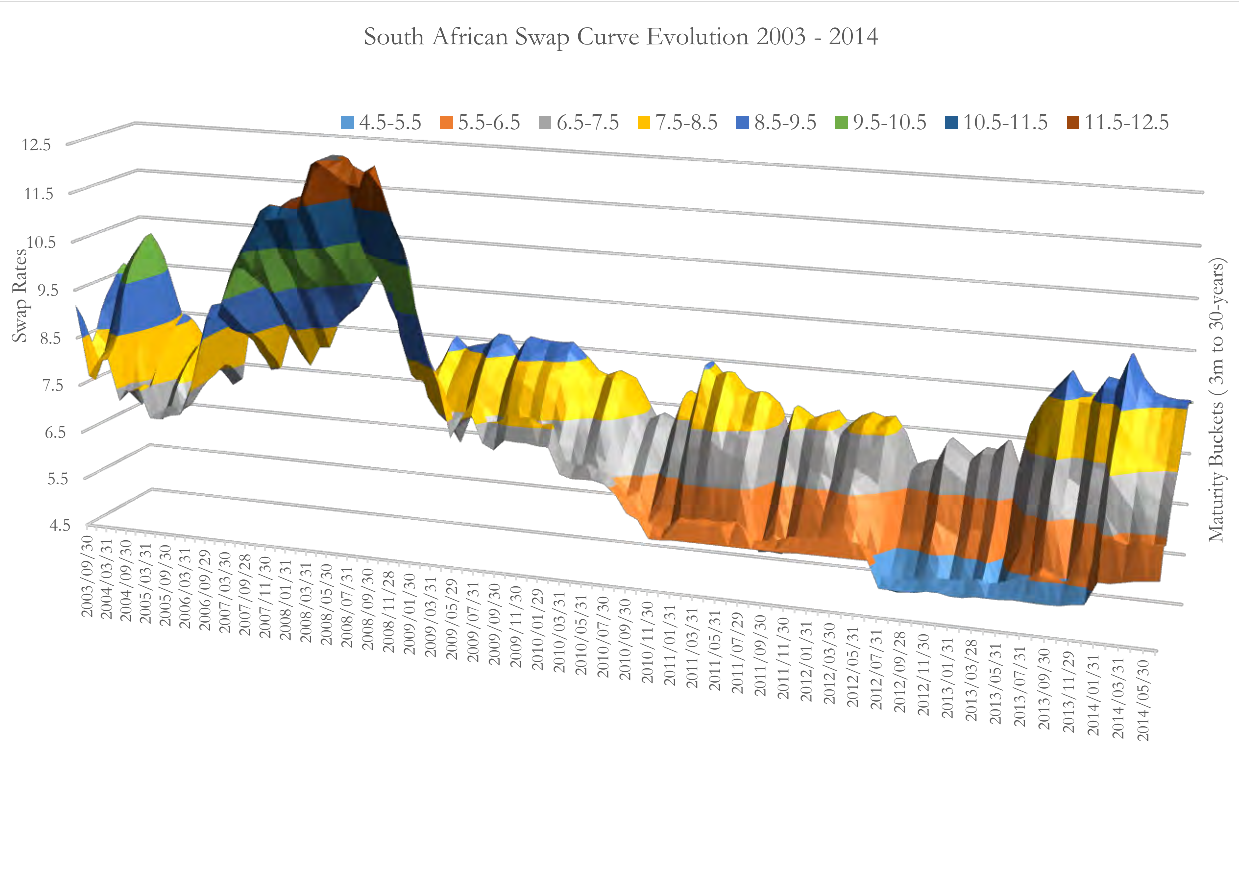


Figure 23: Evolution of the South African Swap Curve, September 2003 to June 2014

Source: Reuters datastream

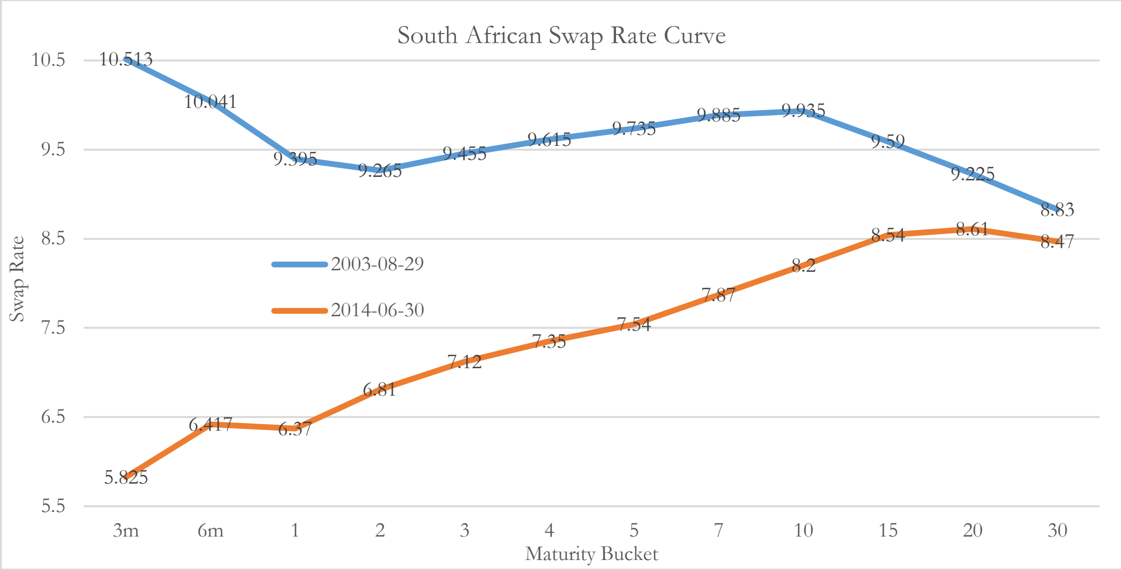


Figure 24: SA Interest Rate Swap Curve - Snapshots as at 29/08/2003 vs 30/06/2014

Source: Reuters datastream

Several studies assert that the difference between the US Government Treasury Curve and the US Swap curve is in part attributable to the fact that the US Government curve is considered riskless and free of any probability of default, whereas the US swap curve incorporates the default risk of the banking sector (Liu, Longstaff & Mandell, 2002, 2006). This sector effectively prices all interest rate swaps in the market by quoting USD-LIBOR rates over the course of the trading day, which in turn are the reference point for the floating rate in an interest rate swap (Liu, Longstaff & Mandell, 2002, 2006). Thus, the authors explain that the LIBOR rate, which is the level at which banks are presumed to fund themselves in the Interbank market, incorporates the average credit risk of the subset of banks that submit LIBOR quotes.

Furthermore, interest rate swaps incorporate liquidity risk given assumed poorer liquidity than sovereign bonds (Liu, Longstaff & Mandell, 2002, 2006). Given the presumed fewer market players in the derivatives market due to fund restrictions in this more exotic and leveraged product space one might expect the Government bond market to be far larger than that of the swap arena. However, Liu, Longstaff and Mandel (2006) asserts that it was as early as 2003 that the Nominal amount outstanding in the US swap market stood at 15x the amount in the US Treasury arena. Important to note here is that the Nominal outstanding on a swap is never actually exchanged being that it is a leveraged derivative instrument. Thus, it is just a reference amount for the stream of interest rate payments.

4.2.2 Bond-Swap Basis

Liu, Longstaff and Mandell (2002) stress that while one would expect the swap curve to trade at a higher level than the sovereign bond curve given that it captures the credit risk of a far weaker credit entity (banks), they are puzzled by the fact that the implied premia in the US swap curve were actually negative throughout the 1990s. The authors hypothesize that this might represent weakened liquidity of bonds over that period.

An investigation of the level of the SA bond versus swap term structure over the present research's sample period reveals that while the swap curve traded above the Government Bond yield curve on 30/09/2003, this situation had reversed by the post-crisis period such that on 30/06/2009, the Government Bond curve out to 10-years was trading above the swap curve (see Figure 25). Looking closer to the present day, both curves are seen to trade at similar levels on 28/02/2014.

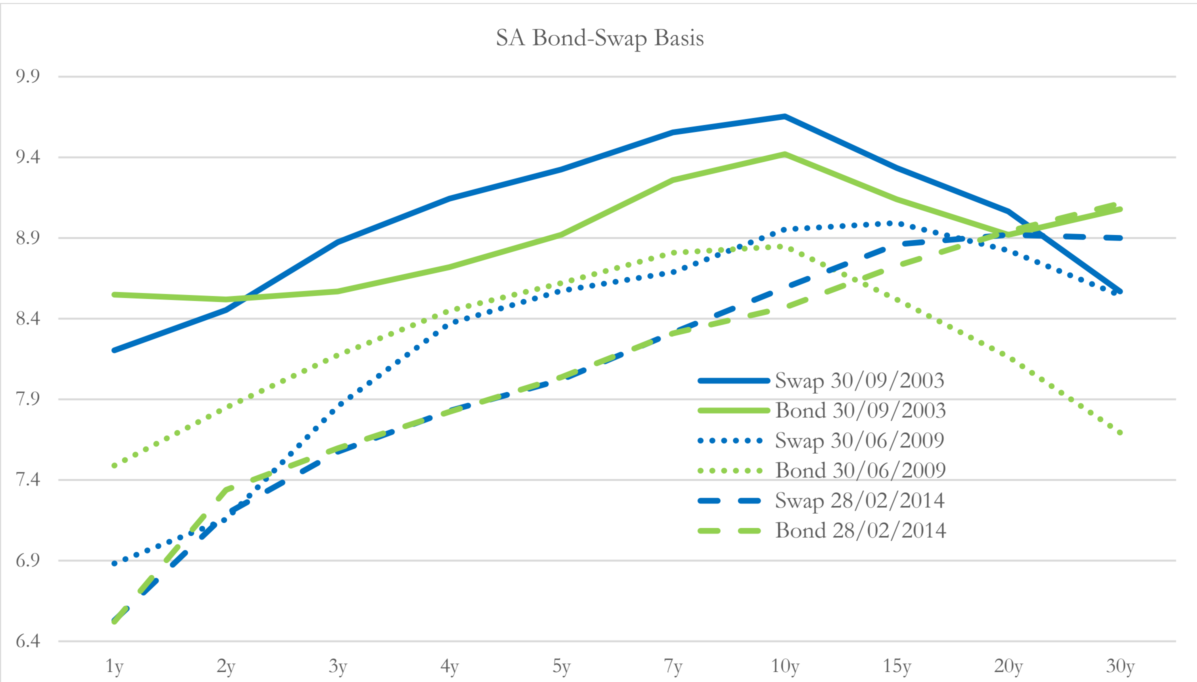


Figure 25: SA Bond and Swap Curve Pairs - Snapshots as at 30/09/2003, 30/06/2009, 28/02/2014

Source: Reuters datastream

Caceres, Guzzo, and Segoviano (2010) assert that an inversion of the bond-swap basis took place in the crisis period such that bond yields began to trade at higher levels than swap rates in the majority of EU countries. The authors assert that this was the result of weakening market confidence in the creditworthiness of many sovereigns. Looking at the spread between South Africa's 10-year swap rate and 10-year Government bond in Figure 26 reveals that in the aftermath of the 2008 crisis, the 10-year

Government Bond began to trade at a higher level than that of the equivalent swap rate, with this relationship only normalizing very briefly in 2010 and for a more sustained period over 2013.

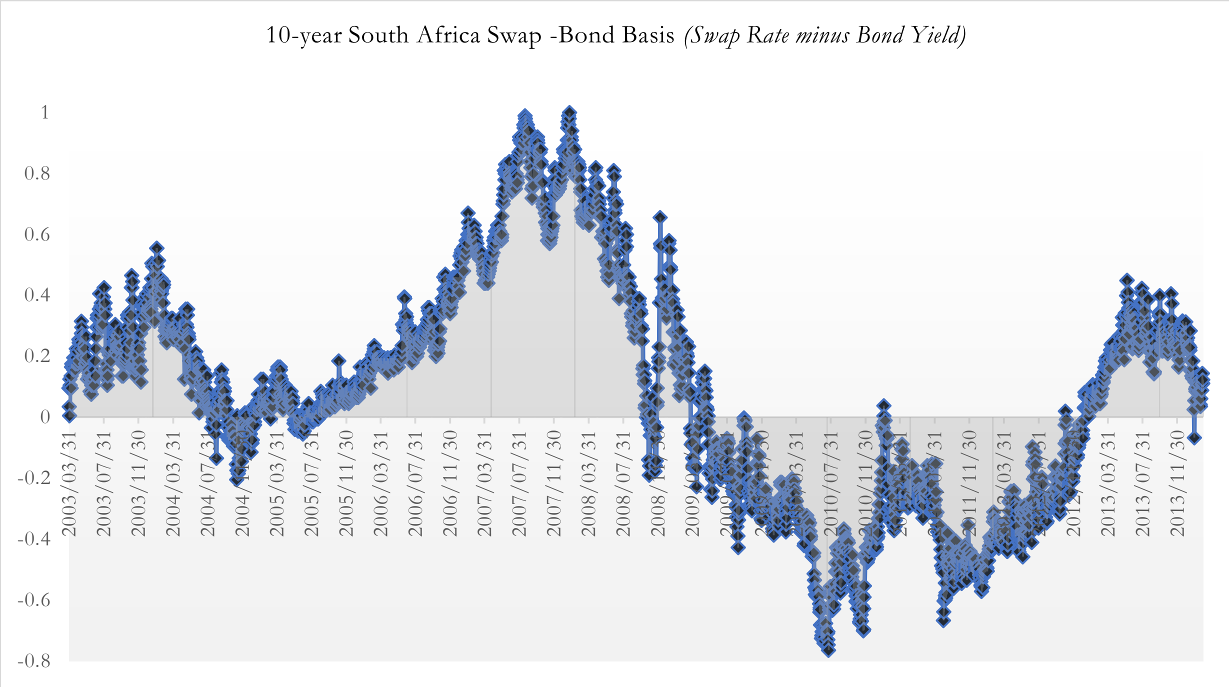


Figure 26: 10-year SA Swap Rate – 10-year SA Bond Yield

Source: Reuters datastream

This phenomenon might then be interpreted as indicating that the market is placing less confidence in the default probability of the national government than that of the banking sector. Such a market perception might have been intensified by the fact that many sovereigns, such as South Africa, were faced with a growing urgency for debt issuance after the crisis so as to stimulate the economy with Fiscal Policy infrastructure and investment spending (IMF Staff Report, 2013). Aside from the ensuing worsening creditworthiness of the sovereign, demand and supply dynamics might have thus played a role here too. An alternative explanation, however, is that due to the leveraged nature of a swap and the absence of an initial cash outlay, market players may have been attracted to this financial instrument in the aftershock of the credit crisis period when funding was scarce and liquidity had all but dried up (Krishnamurthy, 2009).

4.2.3 Swap Curve Predictors

As discussed, Fabozzi et al. (2005) investigate 12 potential predictors of the shape of the US swap curve. Included in their analysis is the dividend yield (i.e. dividends per share / price per share) of the S&P 500, whereas this research focuses in on the dividend yields for the SA banking sector in particular as the swap rate curve is theorized to incorporate the banking arena's default risk (Liu, Longstaff & Mandell, 2002, 2006). Furthermore, drawing from the potential list of swap predictors used by Fabozzi et al. (2005), the USDZAR forward rate and the short-term level of interest rates (in this case, 3-month JIBAR) are investigated.

As seen in Figure 27, it would appear that these series all loosely move in line with the 10-year SA swap rate over time. 3-month JIBAR appears to be far more reactive in terms of the size of its movements, which is to be expected given that the short-end of the curve is more liquid and will more strongly price in a rise / fall in the current policy rate as the SARB puts into effect its monetary policy decisions. The USDZAR forward rate does not always move in tandem with 10-year swap rates, and the forward expectations for the currency remained quite low throughout 2007 and some of 2008 despite swap rates selling off quite dramatically. Similarly in the 2012 to 2014 time periods one sees expectations for the currency dramatically rising while the movement in swap rates is more tempered. The average dividend yield for the SA Banking stocks that trade on the All Share Index is a much smoother and less volatile series than the 10-year swap rate. Furthermore, it would appear that despite the sudden sell off in fixed rates seen around the 2008 crisis period, there was something of a lag before the dividend yields rose (reflecting declining price per share relative to dividends and a cheapening of the banking sector stocks). The blow to the interest rate arena is seen to have occurred earlier than the equities arena as liquidity dried up and borrowing / lending rates skyrocketed.

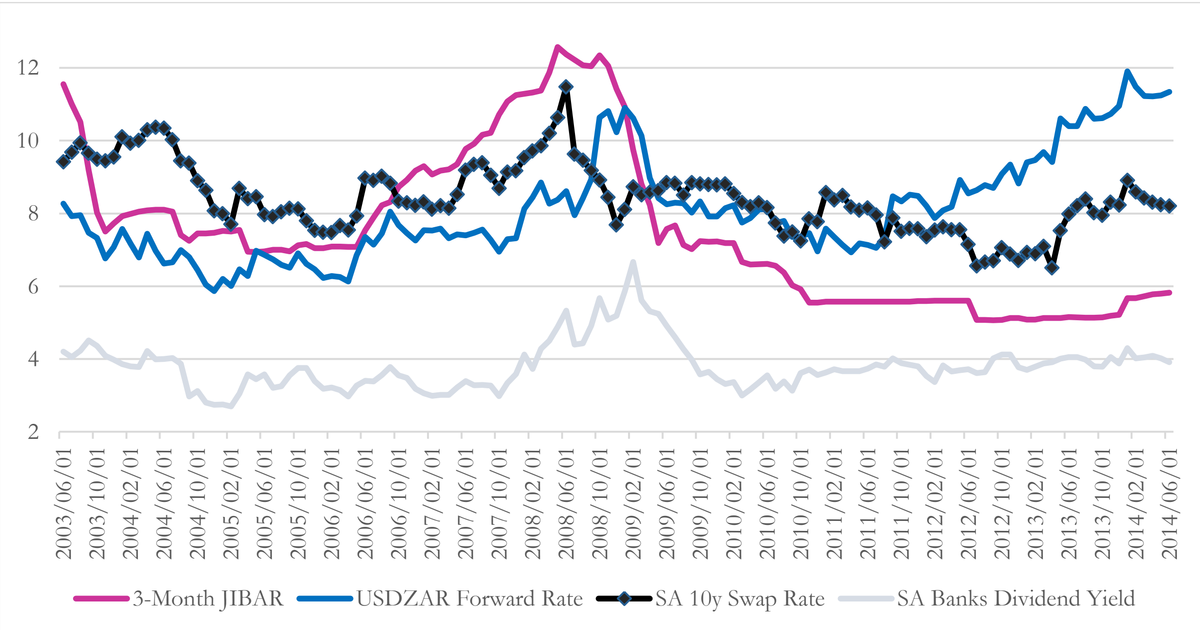


Figure 27: Potential Predictors of the 10-year SA Swap Curve Level

Source: Reuters datastream

Fabozzi et al. (2005) also investigate the slope of the government bond curve in explaining the shape of the swap rate curve. The also use high yield corporate bond spreads as a measure of market sentiment towards riskier asset classes, and the Capacity Utilization as a proxy for the point along the business cycle that the economy is in. The graph in Figure 28 reveals that the SA 1-30-year government bond and swap slopes track each other extremely closely up until the crisis period, when the government bond slope steepens more dramatically than that of the swap curve – despite exhibiting the same trends in movement over time. For the remainder of the sample period, the government bond slope remains the steeper of the 2 curves, with minor dips such that the size of the slope difference narrows. One might presume that such a trend reflects a worsening outlook for the sovereign versus the banking sector around the time of the crisis. This might also reflect the increased supply of longer-term government bonds seen over the sample period that caused the curve to steepen and sell-off for long-term maturity buckets.

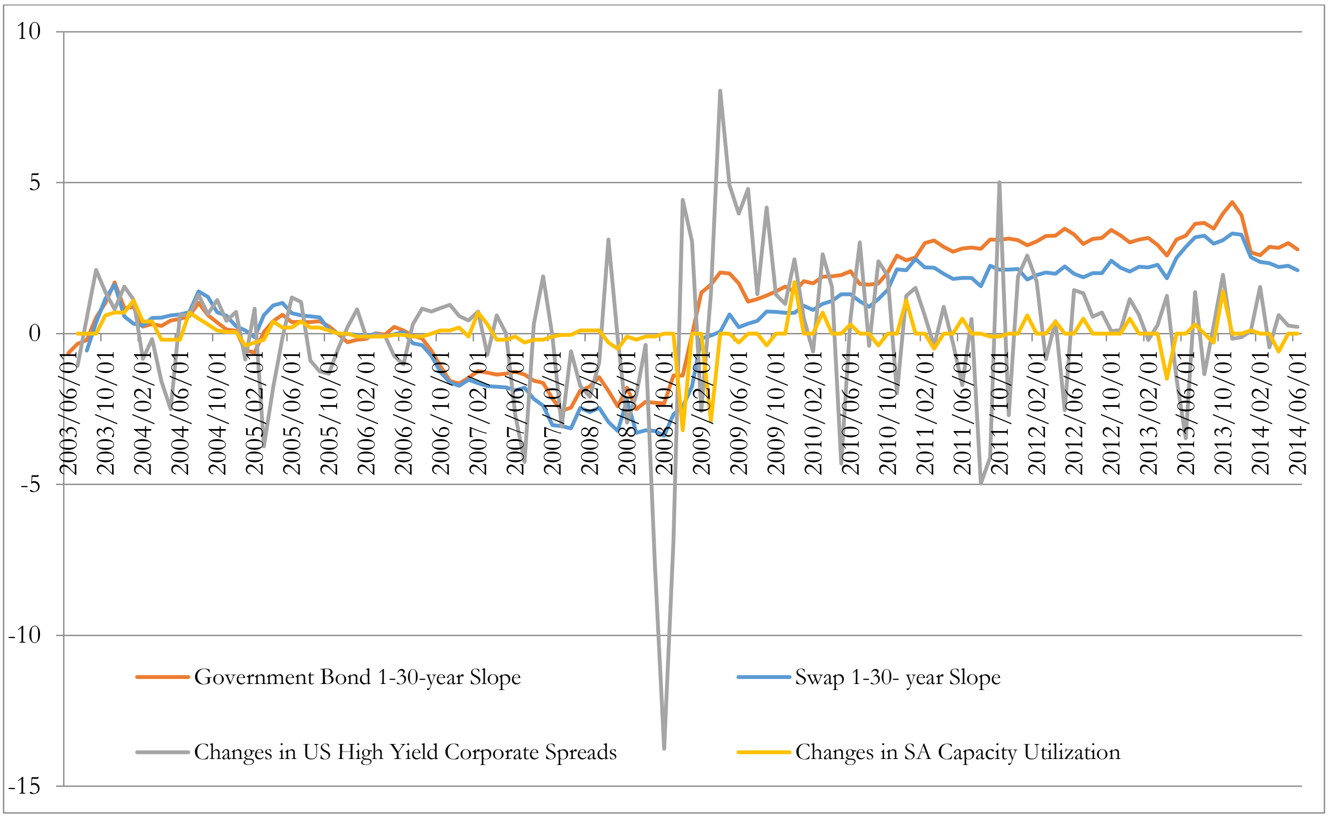


Figure 28: Potential Predictors of the 1 to 30-year SA Swap Curve Slope

Source: Reuters datastream

Examining changes in SA Capacity Utilization reveals that the flattening of the swap / bond slopes over the 2003 to 2008 period was accompanied by small (and frequently positive) increases in the level of productive capacity utilized in the SA economy. The steepening of the slope around the crisis period was precipitated in part by dramatic downward swings in local capacity utilization. Similarly, for the post-crisis period, the steepening of the local swap / bond curves has been associated with a greater number and size of upswings in the US high-yield corporate bond spreads – indicating a sell-off in high yield corporate debt and the weak sentiment of US investors towards riskier asset classes.

5. Results

5.1 Part 1. Nelson Siegel Parameterisation of the South African Sovereign Yield Curve

The regression analysis used assumes λ = 1.37 for annual compounded data, which equates to λ of 16.42 when looking at monthly compounded returns, as per that used by Diebold and Li (2006). Using this factor points to a hump or trough at the 2.5-year point on the yield curve (depending on whether the yield curve was normal or inverted in shape at the time). This is the benchmark level identified as being most appropriate in literature on the topic.

In order to assess whether 2.5 years was an appropriate benchmark for the the South African Government Bond yield curve, the daily yield curve closing levels were assessed for the 2738 trading days falling within the sample period used (18/03/2003 – 28/02/2014). On 52% of these days, the maximum slope of the yield curve had been achieved at the 2-year point, while on 82% of these days, the maximum factor had been achieved by the 3-year point. Additionally, there were no clear trends over time as to when the slope was maximized by 2-years or by 3-years, and instances of each were distributed quite evenly throughout the pre-2008 crisis versus post-2008 crisis time periods. Thus, 2.5 years was deemed an appropriate benchmark.

Nelson Siegel Parameterisation was carried out seperately for a matrix of shorter-dated maturities out to 10 years, as in the work of Diebold and Li (2006), and a longer-dated constellation of maturities, as per Fabozzi et al. (2005) (see Table 3). Certain shorter-dated points were excluded from the analysis (i.e. 9-month zero rates) due to lack of liquidity in the South African yield curve context and a far shorter subsequent history of data. The correlation between the slope and curvature regressors lies within the range exhibited in the parameterisation of Diebold and Li (2006) and Fabozzi et al. (2005) *(r = -0.051 to -0.324).*

Table 3: Constellation of Yield Curve Maturities used in the Nelson Siegel Parameterisations

<u>10-year Nelson Siegel Analysis: Shorter-dated Maturity Buckets</u>
3, 6, 12, 24 months, 3 - 10 years; $\lambda_t = 1.37$, ***r (slope, curvature)= -0.191***
<u>30-year Nelson Siegel Analysis: Longer-dated Maturity Buckets</u>
3 and 6 months, 1, 2, 3, 4, 5, 10, 12, 15, 20, 25 and 30 years;
$\lambda_t = 1.37$, ***r (slope, curvature)= 0.339***

5.1.1 10-year Nelson Siegel Analysis

The results in Figure 29 indicate that while the Level Parameter is largely trendless over the sample period of 2003 to 2014, the slope and curvature parameters exhibit far more volatility – with the slope parameter steepening sharply around the sudden rise in inflation seen in 2003, as well as the 2008 credit crisis period. Note that under the Nelson Siegel framework, the slope parameter represents short-end minus long-end rates (or the "slope inverse"). The most pronounced change in the curvature parameter occurred around the 2008 crisis, when a steeper slope was coupled with a sharp increase in convexity. This points to a less pronounced humpedness around the 2.5-year point, which might have been the result of a decline in inflationary expectations.

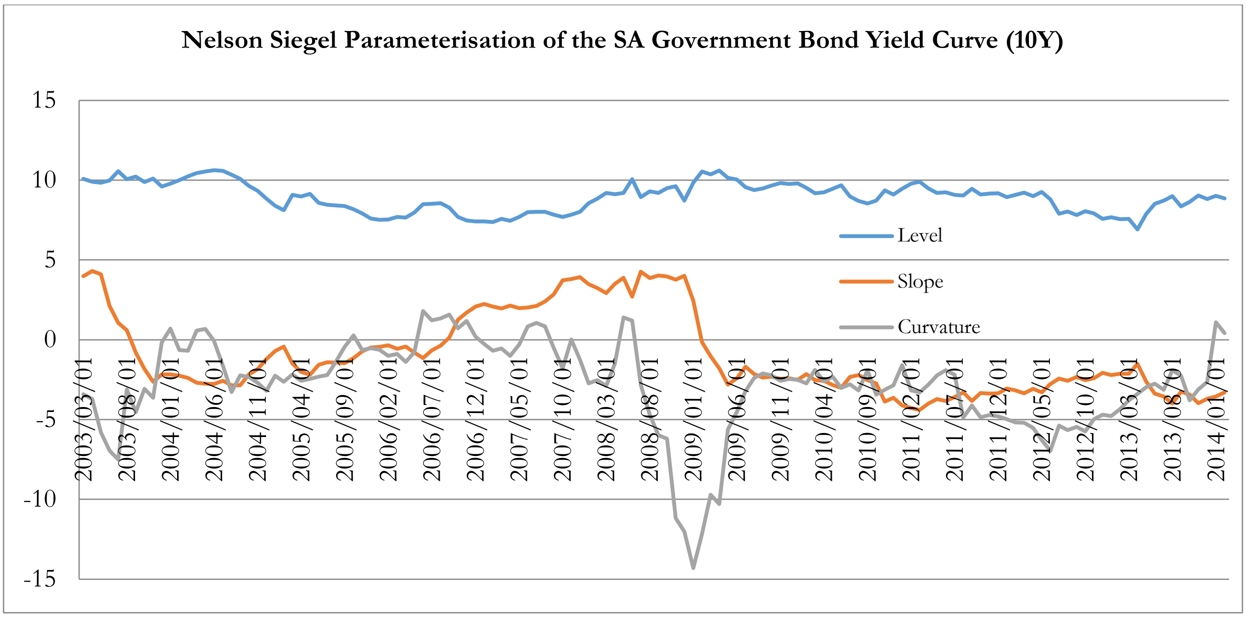


Figure 29: 10-year Nelson Siegel Level, Slope and Curvature Parameter Results

The analysis indicates that the Nelson Siegel parameters are excellent predictors of the 1-year, 5-year, and 10-year points, with an Adjusted R-squared of 99% over the sample period (Table 4).

The fit of the model for each of the overall month-end Government yield curves is very high, with the adjusted R-squared value exceeding 90% in all but 10% of the 132 months falling within the sample period (see Figure 30). The most notable decline in the adjusted R-squared value is a fall to 38% over September month-end of 2006.

Table 4: OLS Regression results of 1, 5, 10-year yields against NS parameters

Method: Least Squares				
Sample: 2003M03 2014M02				
Dependent Variable: _1Y				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
LEVEL	0.99	0.00	457.16	0.00
SLOPE	0.69	0.01	134.90	0.00
CURVATURE	0.26	0.00	54.83	0.00
R-2	0.994	Adj R-2	0.993	
Dependent Variable: _5Y				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
LEVEL	1.00	0.00	1199.94	0.00
SLOPE	0.40	0.00	206.26	0.00
CURVATURE	0.29	0.00	163.05	0.00
R-2	0.998	Adj R-2	0.998	
Dependent Variable: _10Y				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
LEVEL	1.00	0.00	881.68	0.00
SLOPE	0.13	0.00	49.09	0.00
CURVATURE	0.15	0.00	63.38	0.00
R-2	0.992	Adj R-2	0.991	

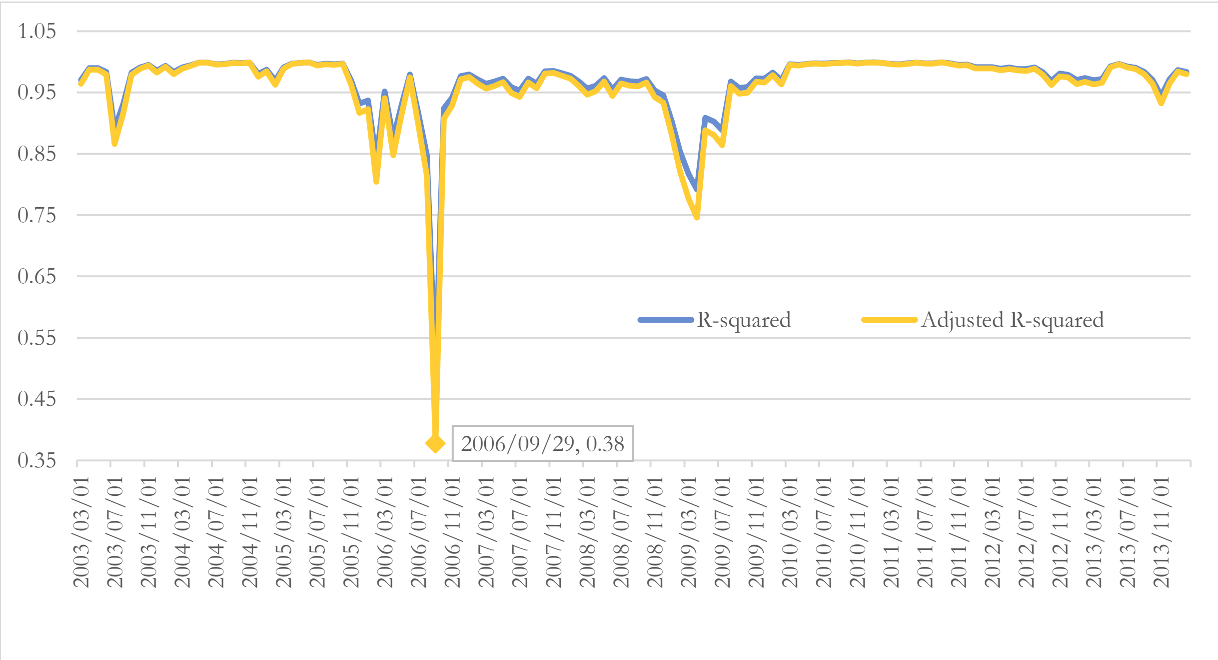


Figure 30: R-Squared and Adjusted R-Squared values for the Nelson Siegel 10-year Model

(3-month to 10-year Yield Curve versus NS Regressors at each month in the sample)

A closer look at the Government bond yield curve on this date reveals that it was reasonably inverted in terms of shape (see Figure 31). In fact, as per Figure 32, this was the first time within the sample period that the slope parameter pushed above the "0" mark, indicating the beginning of the strong flattening seen over the 2006/ 2007 boom years when a negative risk premium was assigned to long-term South African Government debt.

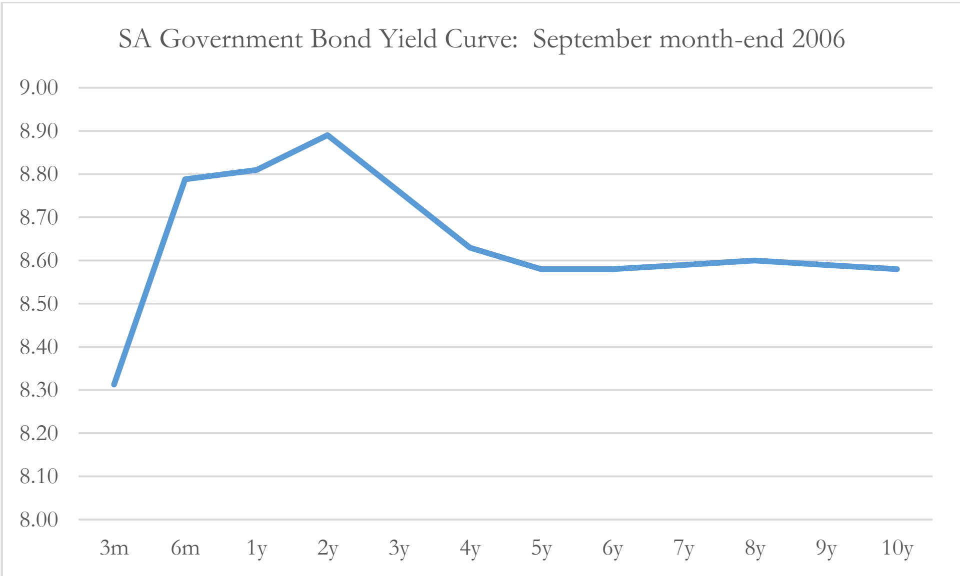


Figure 31: SA Govt Bond Yield Curve as at 29/09/2006

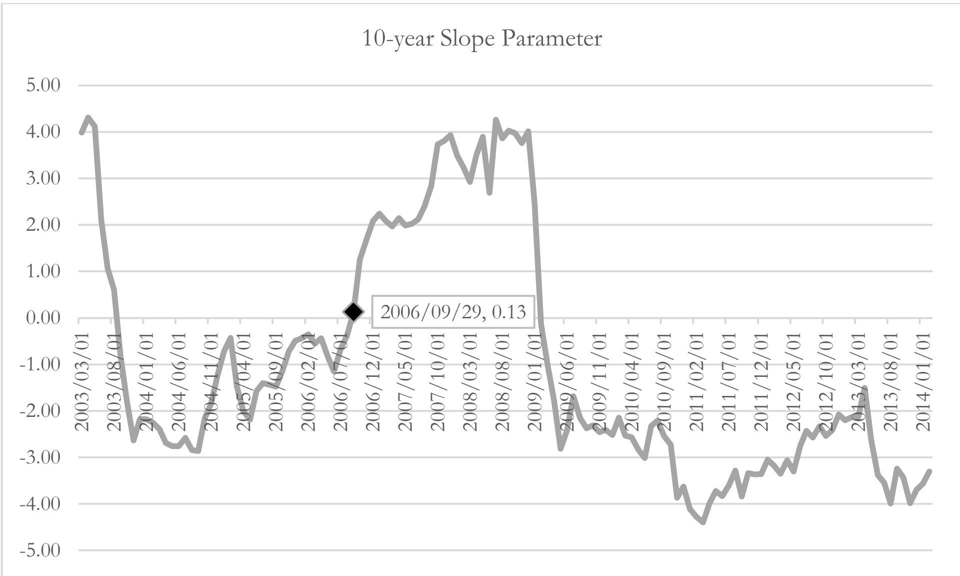


Figure 32: Nelson Siegel 10-year Slope Parameter over time

In terms of the significance of the Level, Slope, and Curvature parameters in explaining the shape of the Government Bond Yield Curve, the Level Parameter is unsurprisingly the best predictor, with statistical significance exhibited at the 1% level in all of the 132 months in the sample period (Figure 33). This is foreseeable as 89.5% of all movements in the yield curve are purported to be the result of an equal and parallel rise in yields across all maturity buckets such that only the Level Parameter is affected (Litterman & Scheinkman, 1991). A further 8.5% of movements is purported to be attributable to changes in slope, with a minor 2% the result of changes in curvature (also known as "butterfly twists") (Litterman & Scheinkman, 1991).

For the Slope parameter, statistical significance at the 1% level is not exhibited over 3 of the month-end dates in the sample (with p>0.05 for 2 separate months within the sample). Unsurprisingly, one of these data points falls over the 2009 crisis period when volatility in the financial markets was rife.

The curvature parameter is somewhat more problematic, with statistical significance (p<0.05) not exhibited in 18% of the month-ends falling within the sample period. That said, the overall fit of the curve is seen to be highly satisfactory.

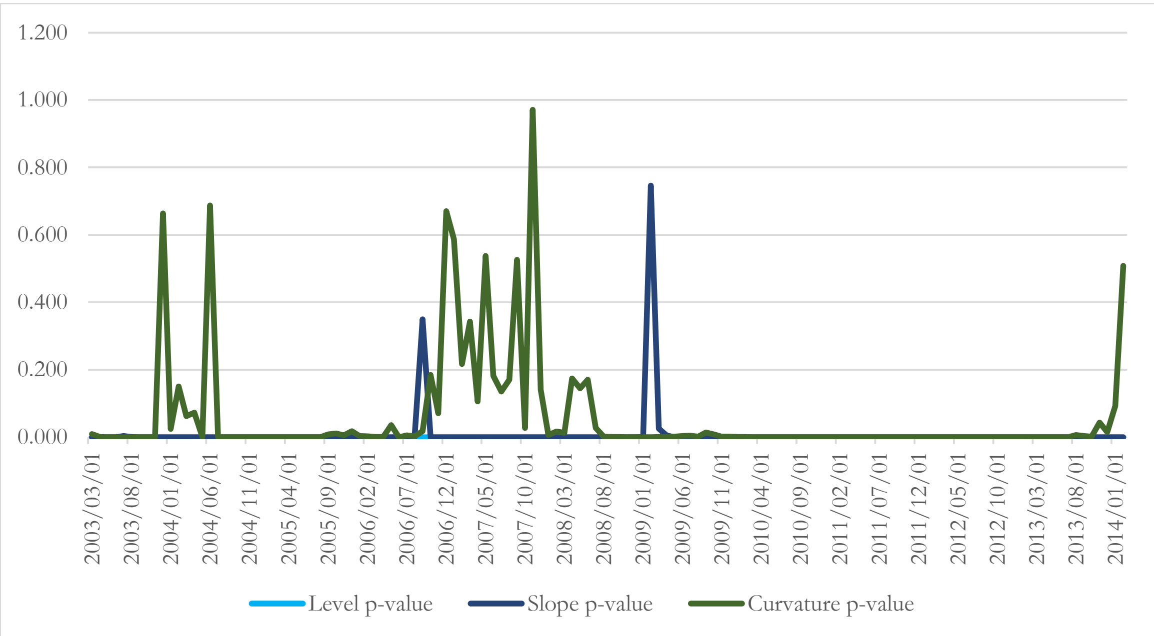


Figure 33: Extracted p-values for the Level, Slope, and Curvature Parameters in the 10-year Nelson Siegel Model

5.1.2 Comparisons with Svensson Approach

While Svensson (1994) does not fix the 2 shape parameters λ_{1t} and λ_{2t}, but rather estimates them at each trade date such that the sum of squared price errors are minimized, it has already been seen that on 82% of the trading days in the sample the maximum factor had been achieved by the 3-year point. Further investigation of the raw data reveals that on 18% of the sample trading days the maximum point of curvature took place at the 5- to 6-year point. This equates to an annuaized λ_{2t}-value of 3.37 (while λ_{1t} is kept at 1.37). The resulting slope and curvature regressors are plotted in Figure 34.

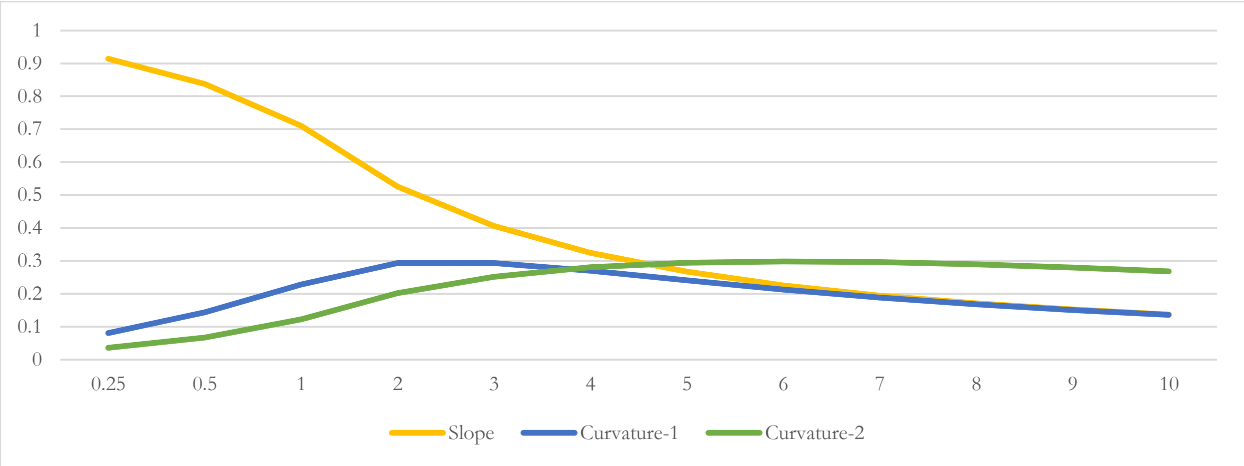


Figure 34: Svensson Model Slope and Curvature Regressors

While the correlation between the slope and the curvature-1 regressors, as well as the curvature 1 and 2 factors, is reasonable at -0.191 and 0.4, the correlation between the slope and the new curvature-2 factor is very high at a value of -0.968 (see Table 5). A test for multicollinearity (correlation) of the beta-parameters in a regression is the Variance Inflation Factors test, with a VIF greater than 5 generally indicating that multicollinearity is present, and a factor of 10 indicating severe multicollinearity (O' Brien, 2007). An excerpt from the Variance Inflation Factors table for 28/02/2014 in Table 6 subsequently reveals that the Centered VIF's for the Nelson Siegel Svensson model are unacceptably high (57.4 and 65.85 for the slope and curvature-2 parameters).

Table 5: Constellation of Maturity Buckets for Nelson Siegel Svensson Model

10-year Nelson Siegel Svensson Analysis: Shorter-dated Maturity Buckets
3, 6, 12, 24 months, 3 - 10 years;

$\lambda_{1t} = 1.37, \quad \lambda_{2t} = 3.37$

r (slope, curvature-1)= -0.191

r (slope, curvature-2)= -0.968; r (curvature-1, curvature-2)= 0.4

Table 6: VIF Test for Nelson Siegel vs Svensson Model, 28/02/2014

Nelson Siegel Variance Inflation Factors

Variable	Coefficient Variance	Uncentered VIF	Centered VIF
C	0.02	15.29	NA
B2	0.02	3.45	1.04
B3	0.36	11.04	1.04

Nelson Siegel Svensson Variance Inflation Factors

Variable	Coefficient Variance	Uncentered VIF	Centered VIF
C	1.05	676.46	NA
B2	1.26	191.01	57.40
B3	1.60	46.05	4.33
B4	12.24	461.51	65.85

One implication of multicollinearity of the parameters is that the coefficient estimates can no longer be reliably interpreted as "the isolated impact of one parameter upon the yield curve while holding the others constant". Given the sizeable relationship between the slope and curvature-2 parameters, their strong dependence on one another confounds this interpretation. Furthermore, the standard errors of the individual beta-coefficients will be magnified such that the presence of Type II Errors will increase (a failure to reject the Null Hypothesis of the significance of the parameters when it is in fact false) (O'

Brien, 2007). Such models are generally deemed to fall prey to the issue of "overfitting" the parameters and to exhibit poor out-of-sample generalizability.

In that vein, the adjusted R-squared values for the Svensson model are seen to be superior to that of the Nelson Siegel model (see Figure 35), with a low of 66% exhibited in September 2006 (previously 38%). The model is also able to better capture the yield curve variation seen around the crisis period when the curve underwent dramatic changes in shape.

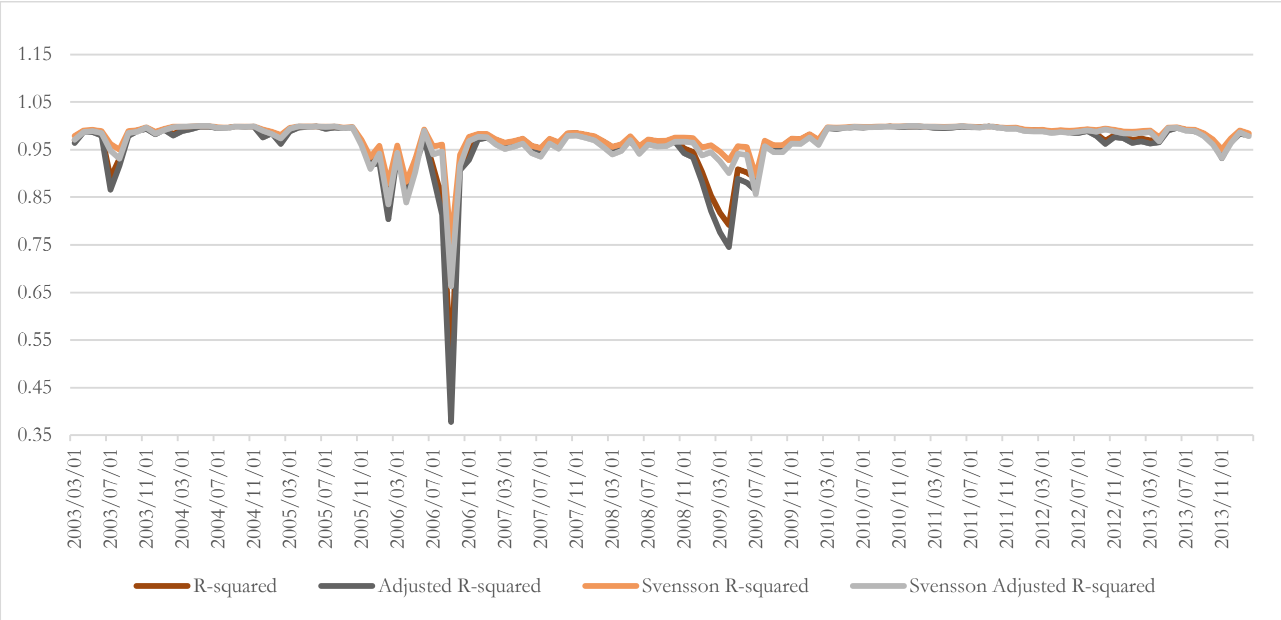


Figure 35: R-Squared for NS versus Svensson Model

As is to be expected given the presence of Multicollinearity in the Svensson model, the p-values of the slope and curvature-2 parameters are much greater than that of the Nelson Siegel model, with the slope parameter not exhibiting significance ($p>0.05$) on 26% of the month-ends within the sample, and the Curvature-2 parameter not exhibiting significance on 61% of the trading periods within the sample (see Figures 36 and 37).

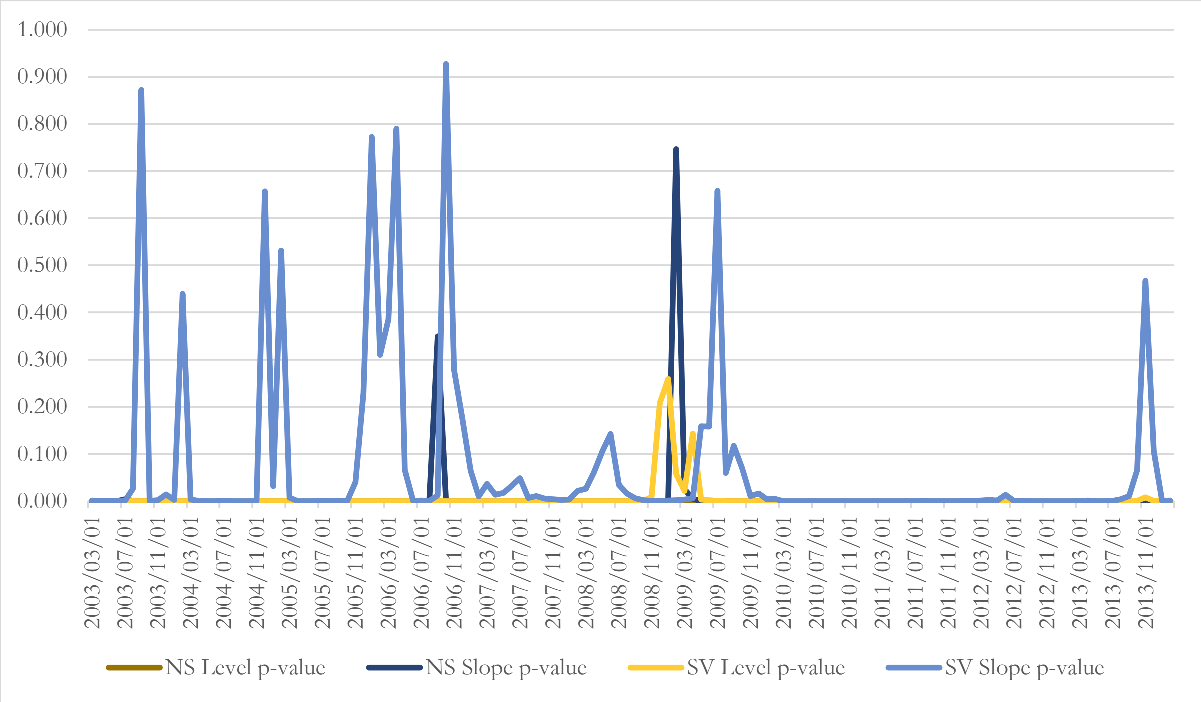


Figure 36: NS vs Svensson Level and Slope p-values

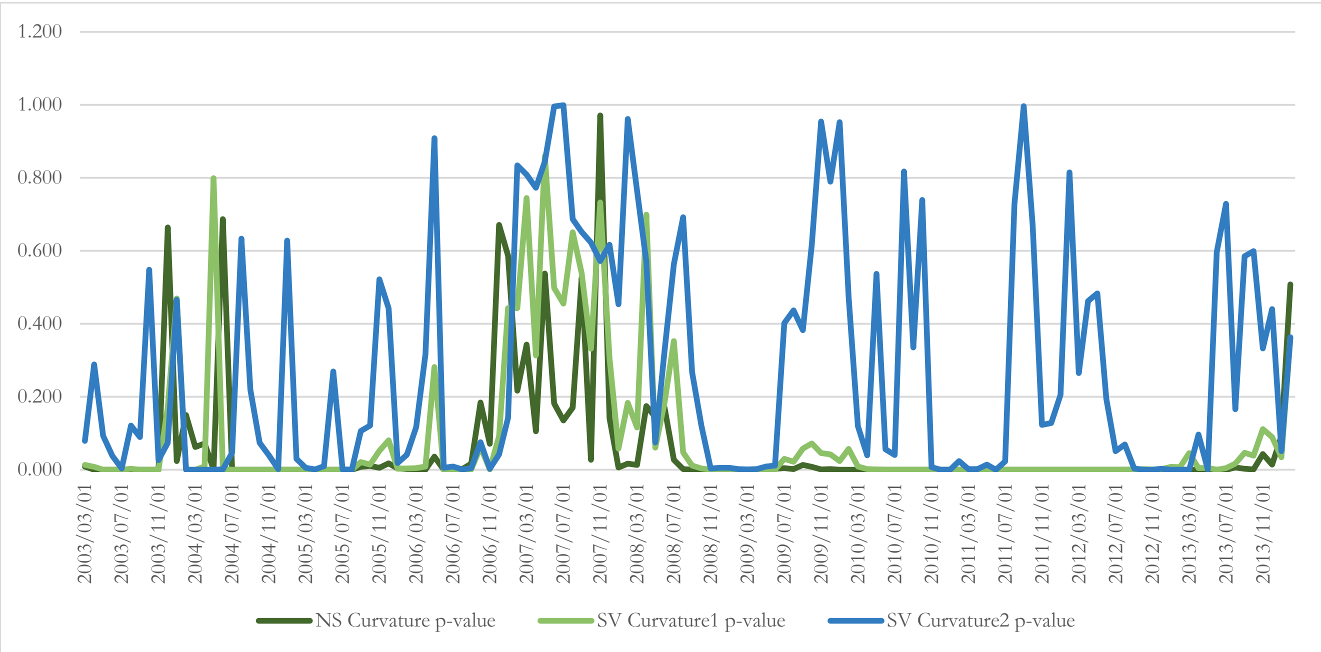


Figure 37: NS vs Svensson Curvature p-values

In order to assess the relative forecast accuracy of the Nelson Siegel and Svensson approaches, a rolling window estimation period is employed such that yield forecasts are generated and back-testing of their accuracy is assessed. Results are obtained by fitting the model over a 48-observation (4-year) moving window estimation period such that the parameter estimates obtained at each separate window are used to forecast yields for 12-months into the future.

An OLS regression was initially performed for each yield maturity bucket (i.e. 1-year, 2-year etc.) over the period:

2003:03 to 2007:03

After this, the second estimation period would begin in 2004:03 (i.e. a lag of 12 months between each window). Forecast estimates are then obtained for each month over the next 1-year period following each regression window, meaning that the final forecast period was from February 2013 to February 2014.

Plotted below are the absolute value of the differences between the Nelson Siegel estimates of the 1-year, 5-year, and 10-year yields against the true level of yields, as well as the same metric for the Svensson forecasts. While the Svensson forecasts offer a marginal improvement to those of the Nelson Siegel estimates of the 1-year yield (as witnessed by a smaller difference from the true yield in Figure 38), it is in the case of the 5-year and 10-year yields that one sees a vast improvement (see Figures 39 and 40). This is unsurprising as the effect of the second curvature parameter at the 5.5-year point should perform better in modelling longer-term yield levels.

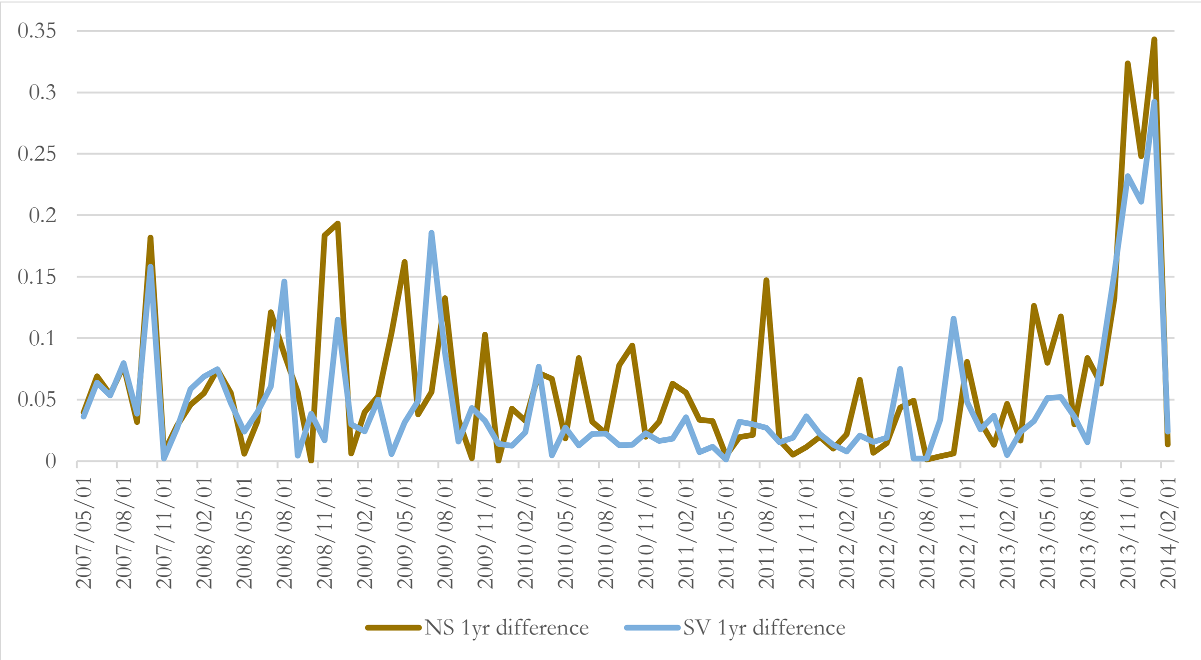


Figure 38: Absolute Value of Predicted versus Actual 1-year yields

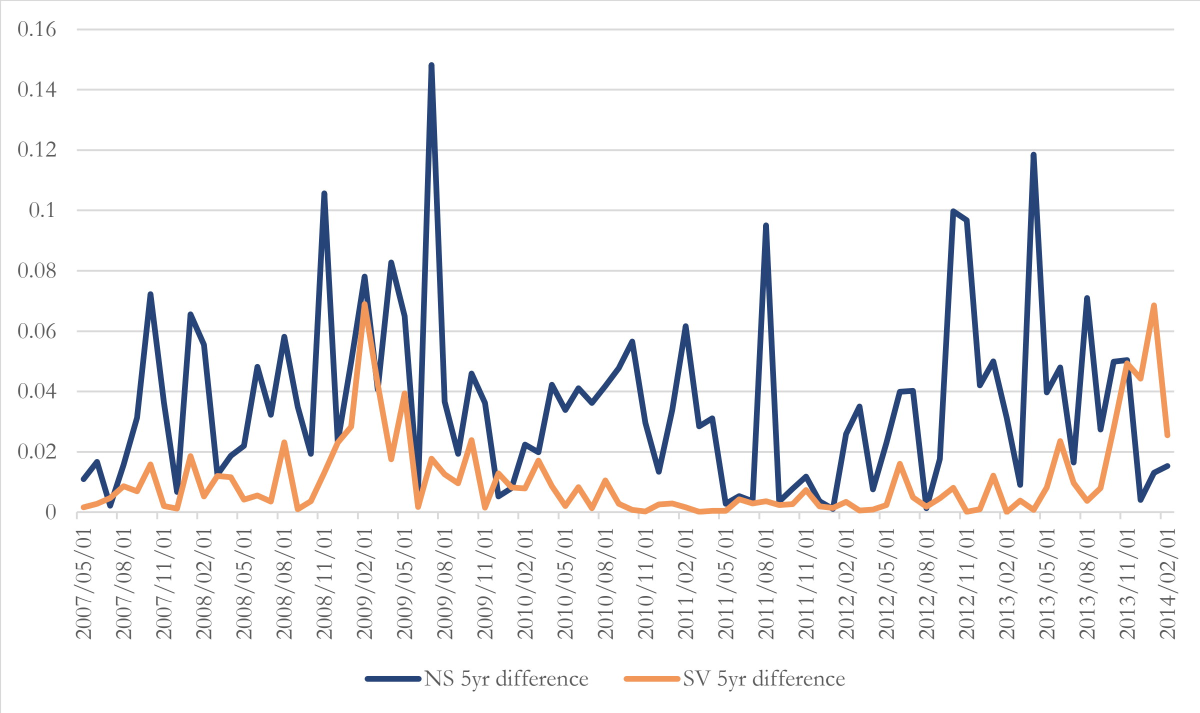


Figure 39: Absolute Value of Predicted versus Actual 5-year yields

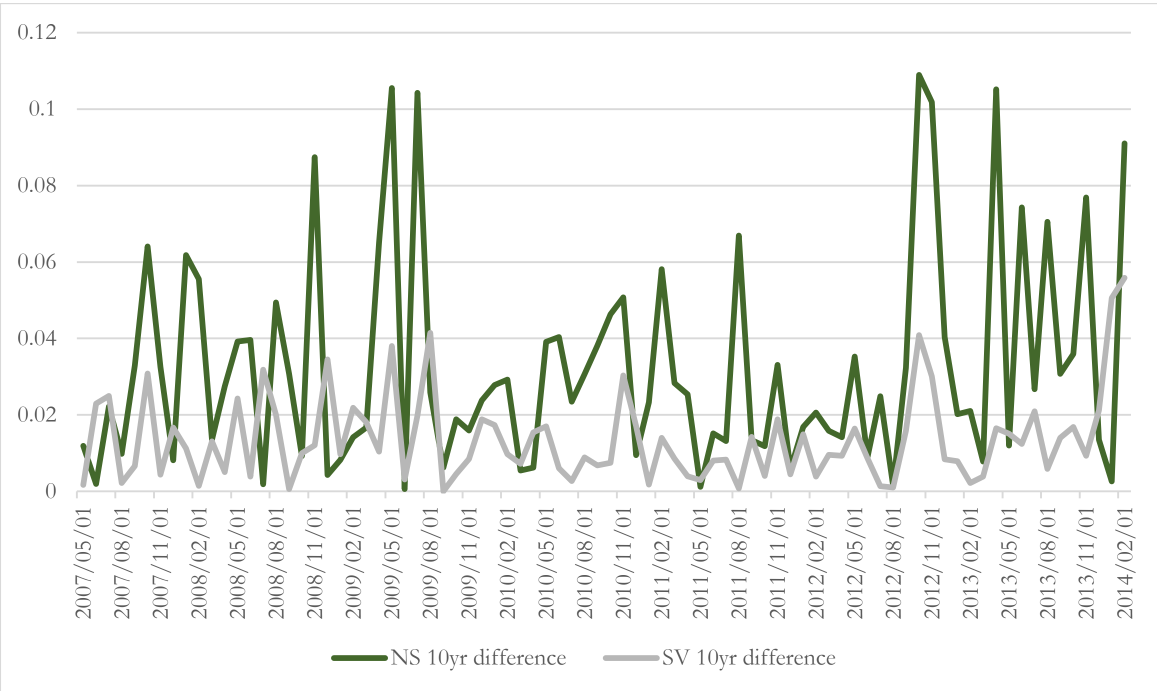


Figure 40: Absolute Value of Predicted versus Actual 10-year yields

Ultimately, the relative performance of the Nelson Siegel versus Svensson Model is assessed by calculating the Root Mean Square Error (RMSE) of the forecasts for each of the yield maturity buckets as per the below:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\,Forecasted\ Yield - \ Actual\ Yield)^2}$$

Where:

n = the number of months in the forecast sample from May 2007 to February 2014

The results indicate that the Svensson Model RMSE is lower (more superior) than that of the Nelson Siegel Model for all maturity buckets from 1-year out to 10-years with the exception of the 2-year and 7-year maturities (see Figure 41). That said, one finds that the RMSE for both models falls within the range of 0.01 to 0.09 for all maturities – and thus occupies quite a tight range.

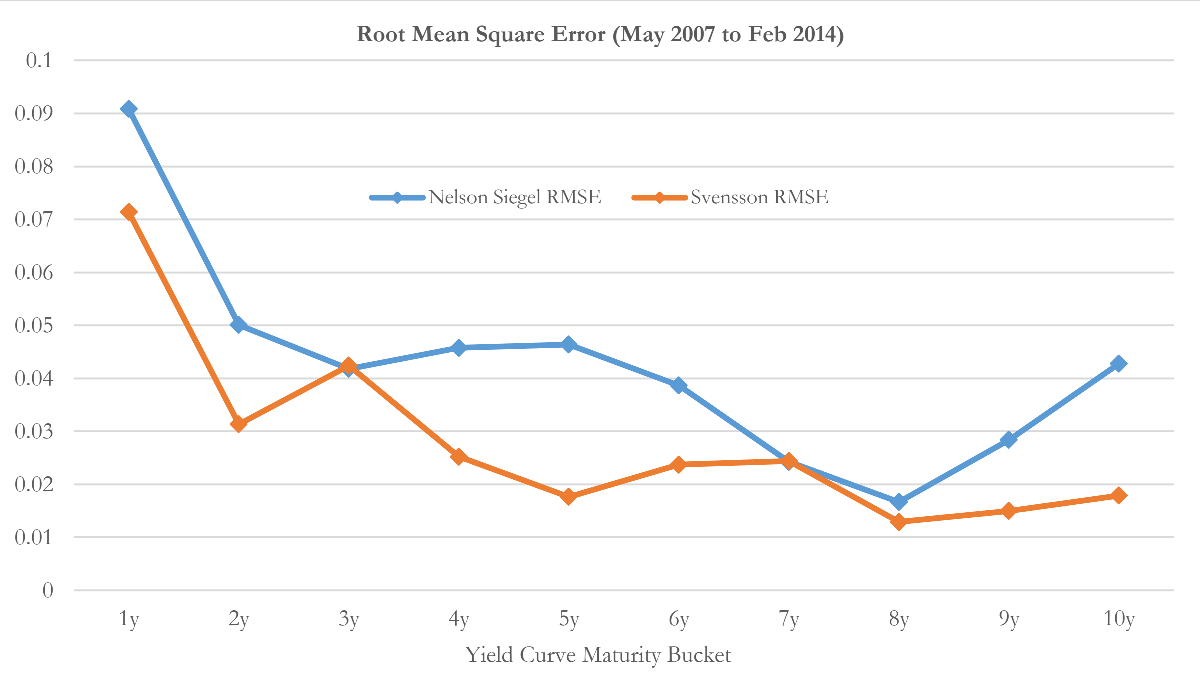


Figure 41: Nelson Siegel vs Svensson RMSE (by maturity bucket)

Such a result points to only a small degree of improvement from the Nelson Siegel to the Nelson Siegel – Svensson model. This idea of it being only a marginal enhancement is reinforced when one compares this result to the more sizeable outcome of Laurini and Hotta's (2010) study. The authors find in their investigation of the Brazilian term structure of interest rates for the period 2004 to 2006 that the addition of a second "Svensson" curvature parameter vividly improves the Root Mean Squared Forecast Error of the model from 36.61 to 17.05 – an increase in model accuracy that might warrant the inclusion of this 4th parameter in future analysis.

Thus, given this only marginal improvement in forecast accuracy, the economic simplicity of interpretation surrounding 3 yield curve parameters, and the introduction of a high degree of multicollinearity in the Svensson model, the analysis in the remainder of this research will utilize the 3-factor Nelson Siegel Model.

5.1.3 30-year Nelson Siegel Analysis

As per Figure 42, the results of the 30-year Nelson Siegel Parameterisation reveal similar trends over time in the yield curve parameters to that of the 10-year analysis. What becomes apparent, however, is that capturing the yield curve in its entirety from the 3-months out to 30-year points worsens the model fit (see Figure 43).

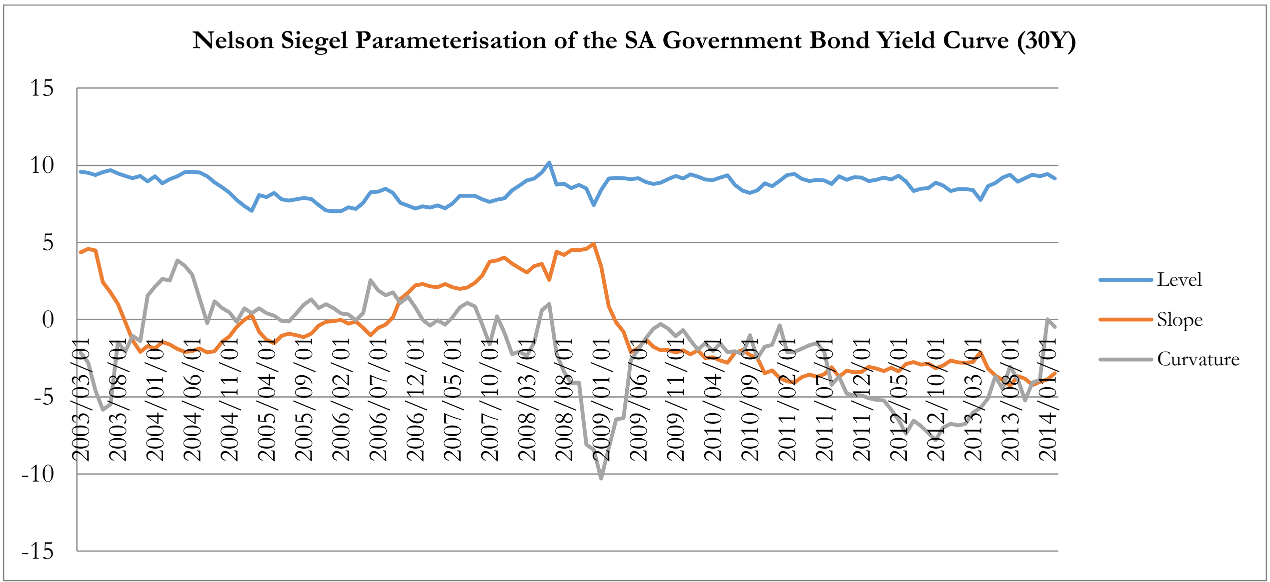


Figure 42: 30-year Nelson Siegel Level, Slope and Curvature Parameter Results

The Adjusted R-squared now spans from 99% down to -15% over the sample period – with certain yield curve realizations in the early 2005 and mid-2006 period being so poorly captured by the model that a negative proportion of variation is explained. That said, only 28% of the month-ends sampled have an adjusted R-squared of below 80%.

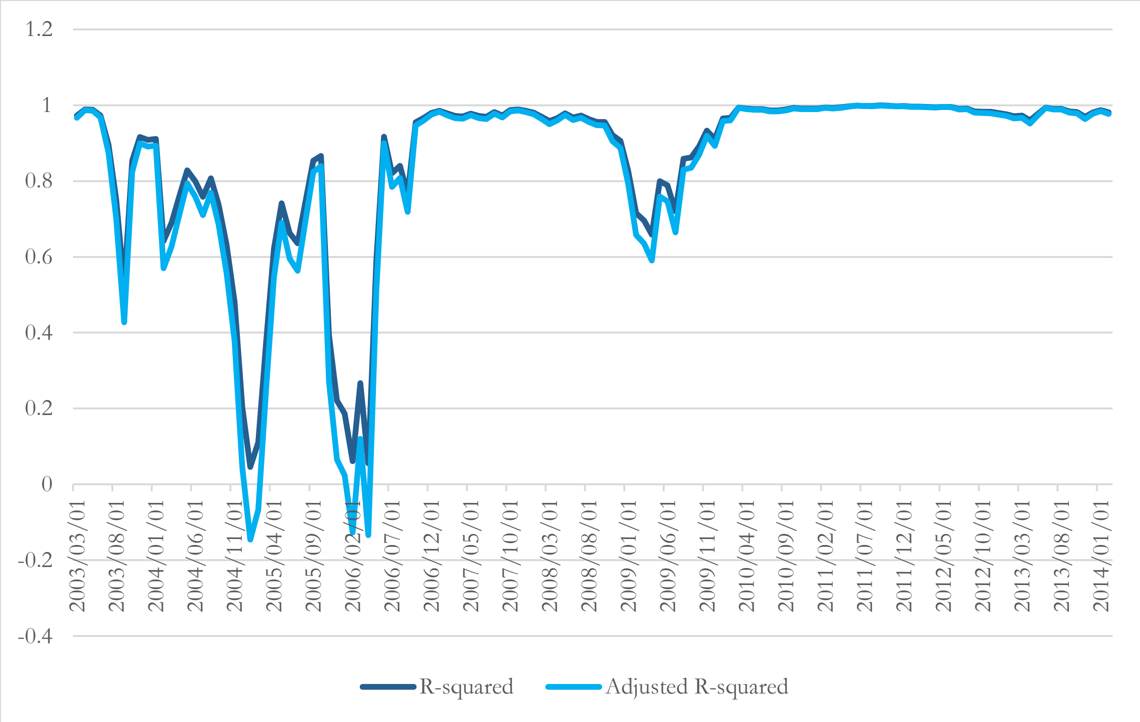


Figure 43: R-Squared and Adjusted R-Squared values for the 30-year Nelson Siegel Model

Similarly, in 10% of the months sampled the slope coefficient is not statistically significant (p>0.05), and in 36% of the time periods one finds that the curvature parameter is similarly insignificant (see Figure 44).

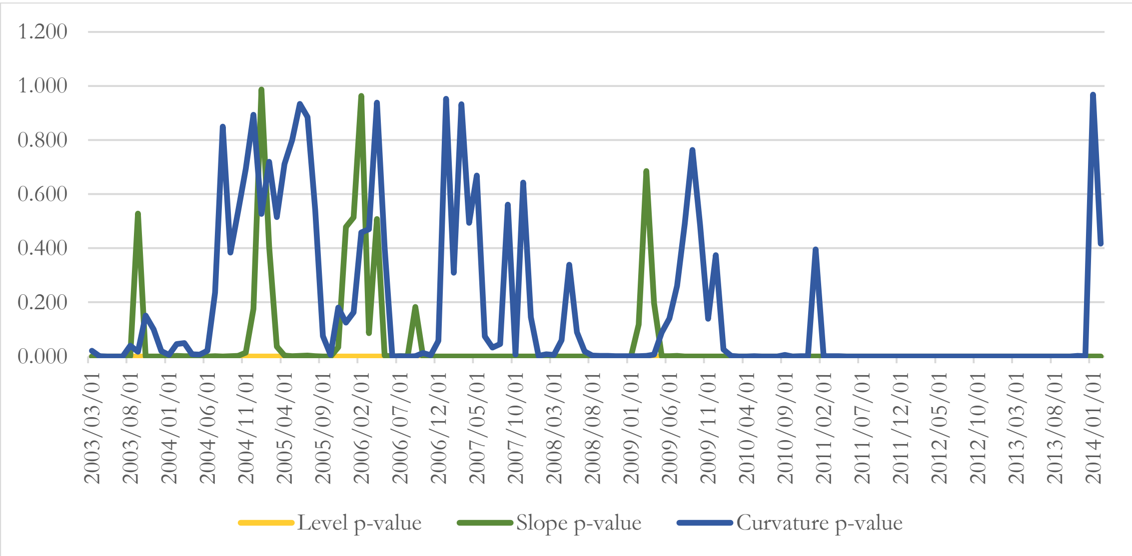


Figure 44: Extracted p-values for the Level, Slope, and Curvature Parameters in the 30-year Nelson Siegel Model

5.1.4 Parameter Comparisons: 10 versus 30-year Analysis

The results indicate that the Level parameter in the 1st and 2nd analysis roughly corresponds to 10-year and 30-year yields respectively, or the stipulated long-end of the curve (see Figures 45 and 46).

Investigating the trends in the 1-year, 5-year, and 10-year areas of the Government Bond yield curve reveals that in the early stages of the sample period in 2003 the yield curve was inverted such that 1-year yields traded above 5-year yields – which were also greater than 10-year yields in turn. By 2004 this area of the yield curve had normalized such that the 10-year point traded at the greatest risk premium. In 2006 the situation reverted to an inverted shape, and in the post-crisis period up to the present day the 1 – 10-year area of the yield curve has traded in a "normal" term structure.

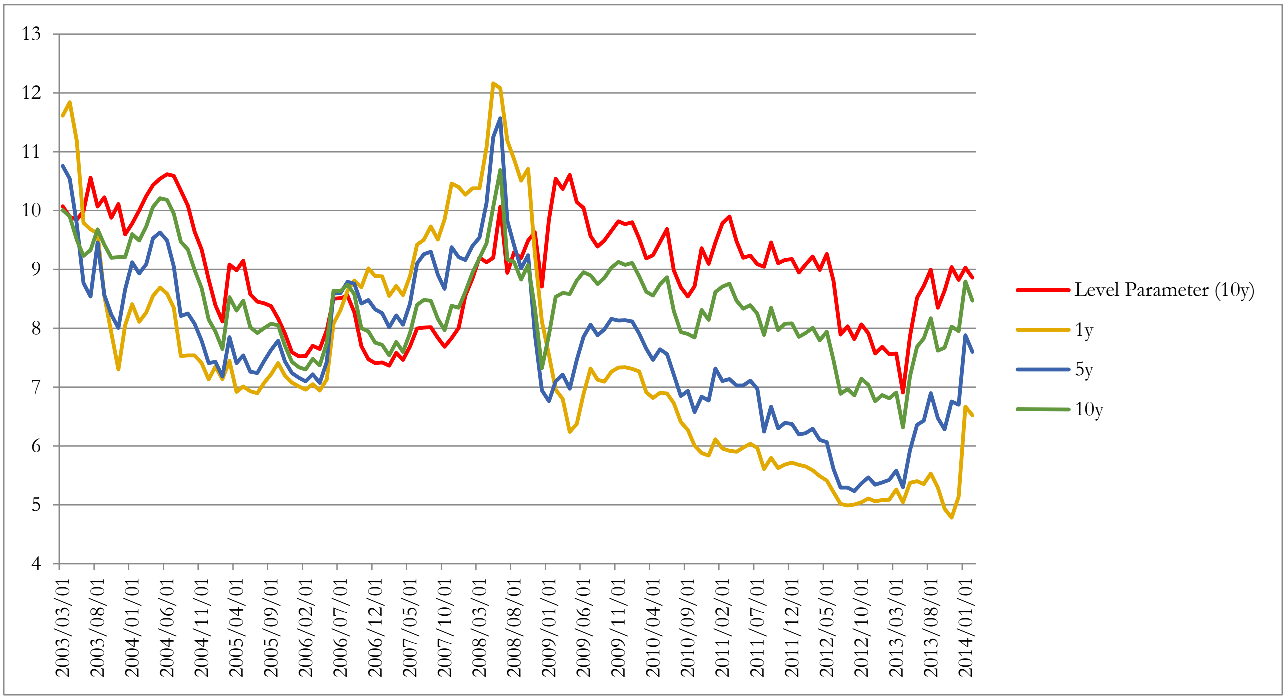


Figure 45: 10-year NS Level Parameter vs 1, 5, 10-year yields

Investigating the 10 to 30-year area of the yield curve reveals that in the entire pre-crisis period, 30-year yields were actually trading below 10 and 20-year yields due to the inversion of the yield curve. Around the 2008 period, 20 and 30-year yields rose to match those of the 10-year point, and in the post-crisis period the yield curve steepened even further to resemble the "normal" yield curve structure.

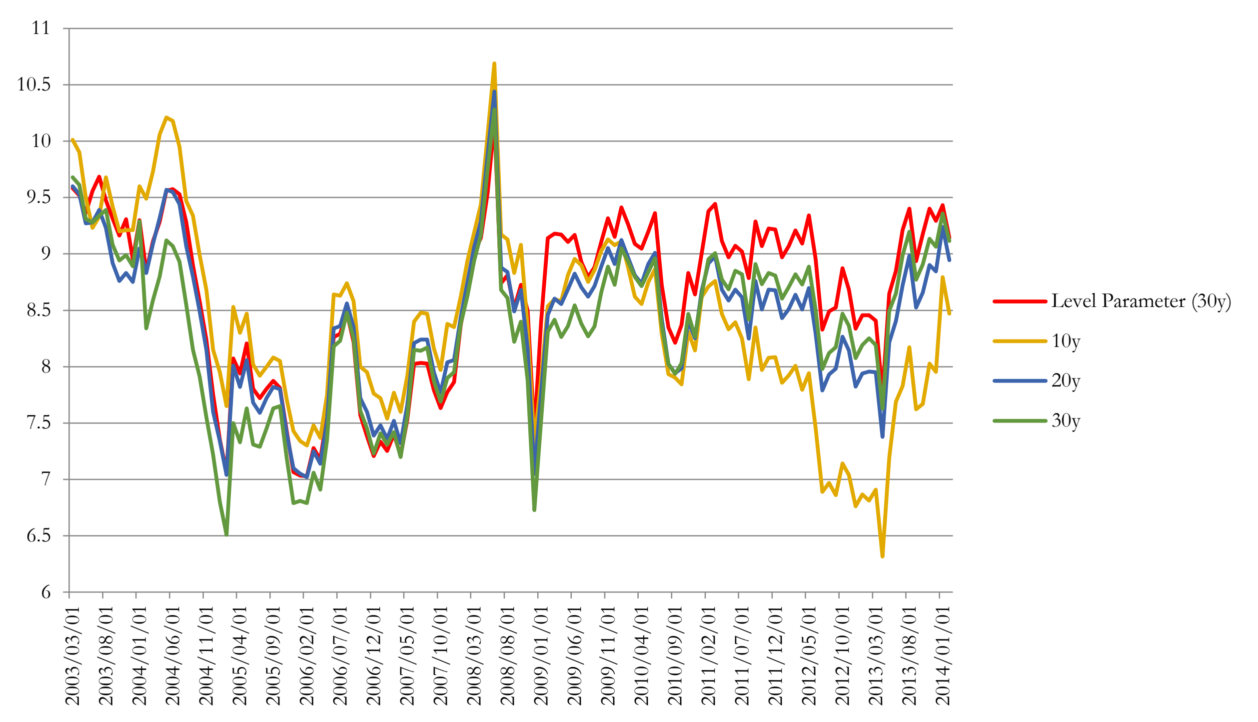


Figure 46: 30-year NS Level Parameter vs 10, 20, 30-year yields

This highlights that there are different dynamics and market forces at play in the 1 – 10-year versus 10 – 30-year areas of the yield curve. The 1 – 10-year area is seen as far more liquid, meaning that market players should encompass a range of categories – namely, market speculators and traders, "Fast Money" Hedge Funds, "Real-Money" Investment Management houses, and certain Pension Funds (Kaminsky, Lyons, & Schmukler, 2001). This area of the curve is far more representative of short-term interest rate expectations and is often more reactive to short-term market shocks before they filter into the long-end of the curve over time.

The short-term area of the yield-curve is thus presumably well explained by expectations of the inflation rate, with the initial 2003 inversion proceeded by a dramatic decline in the inflation rate, and the 2006 – 2008 inversion followed by another decline in inflation from its high of 12.4% year-on-year in 2008 (see Figure 47).

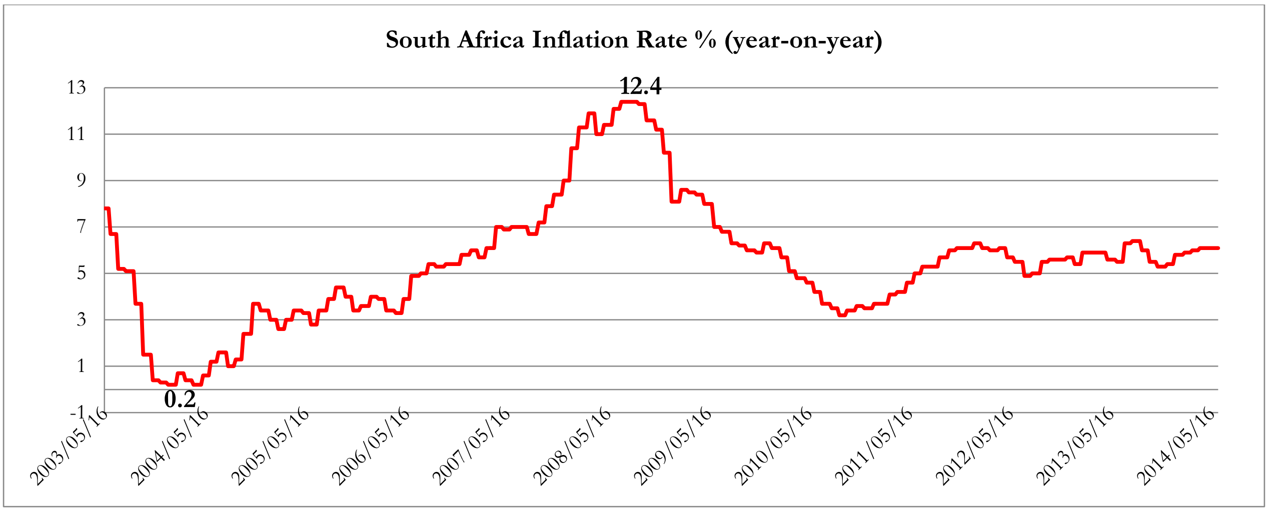


Figure 47: SA Inflation Rate (% y/y), 2003 - 2014

The 10-year to 30-year area is presumably less well explained by these market forces, and it can be seen that despite fluctuating inflation over the sample period, the 10-year to 30-year yields exhibited an inverted term structure shape pre-crisis, and then transformed to a normal yield-curve shape in the post-crisis period (see Figure 46). One might hypothesise that the market forces at play at this longer-end of the yield curve are more likely to be long-term growth expectations, the country risk premium needed to compensate investors for holding a nation's long-term debt, and demand/ supply dynamics. Such an assumption would tie up to the worsening SA foreign investor climate rating and increased debt issuance seen after the crisis period in the earlier Data Exploration.

As per Figure 48, the 30-year versus 10-year Nelson Siegel parameters follow roughly the same trends over the sample period. Similarly, the slope parameters follow the same trend and are seen to fall quite dramatically over 2003 - 2004 and the post-crisis 2008 period – which signifies a steepening of the yield curve under the Nelson Siegel framework (see Figure 49). While it has been seen that the 1 – 10-year section of the curve follows different trends to the 10 – 30-year space, the second Nelson Siegel Parameterisation encompasses the 3-month to 30-year points, and thus includes the earlier sections of the yield curve in its slope factor. Thus, this analysis does not capture the "long-term only" yield curve factors.

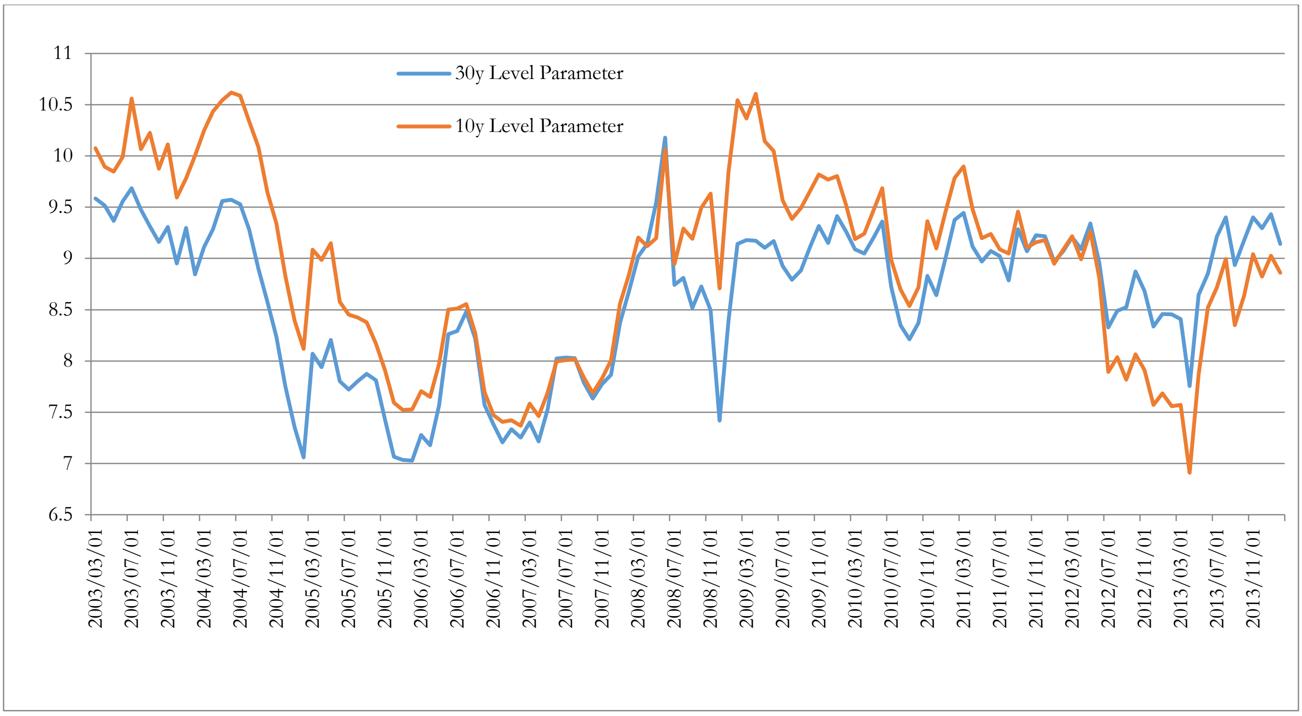


Figure 48: 10 versus 30-year NS Level Parameters

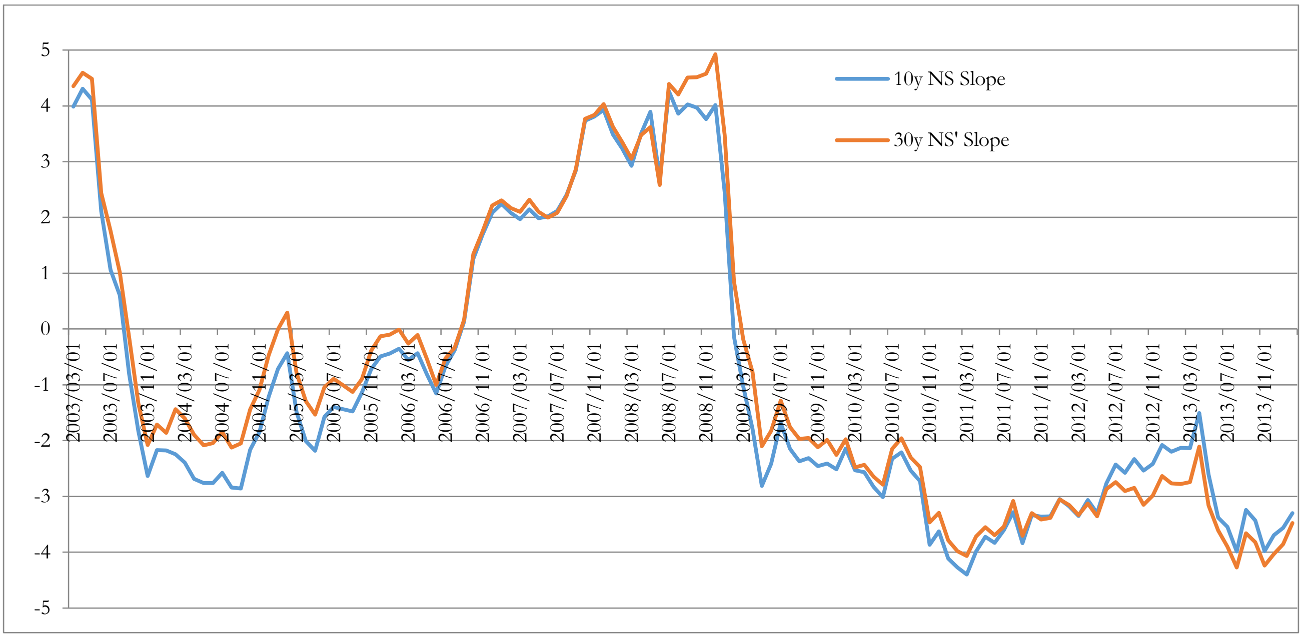


Figure 49: 10 versus 30-year NS Slope Parameters

The curvature factor represents the degree of concavity of the curve. For the 10-year analysis, the 2.5-year point was selected as the time to maturity τ at which the point of maximum convexity or concavity was reached. As per Figures 50 and 51, an increase in the curvature parameter represents the fact that the curve is approaching a humped / concave shape – as seen on 30/05/2008 (10-year curvature factor =

1.41) and 31/01/2014 (10-year curvature factor = 1.1). A concave yield curve is associated with an expectation of rising short-term interest rates and inflation. Notice also how short-term borrowing rates blew out to much greater levels and dislocated from the rest of the curve around the 2008 – 2009 crisis period (see Figure 51)

Contrastingly, the point of minimum concavity (maximum convexity) over the sample period is seen in the Government Bond yield curve on 30/01/2009 (10-year curvature factor = -14.3), with a local minimum on 29/06/2012 (10-year curvature factor = -6.97) (see Figures 50 and 51).

While the degree of concavity /convexity is more pronounced for the 10 versus 30-year Nelson Siegel curvature parameters at different points over the sample period, ultimately these series follow the same trends – as seen in Figure 50.

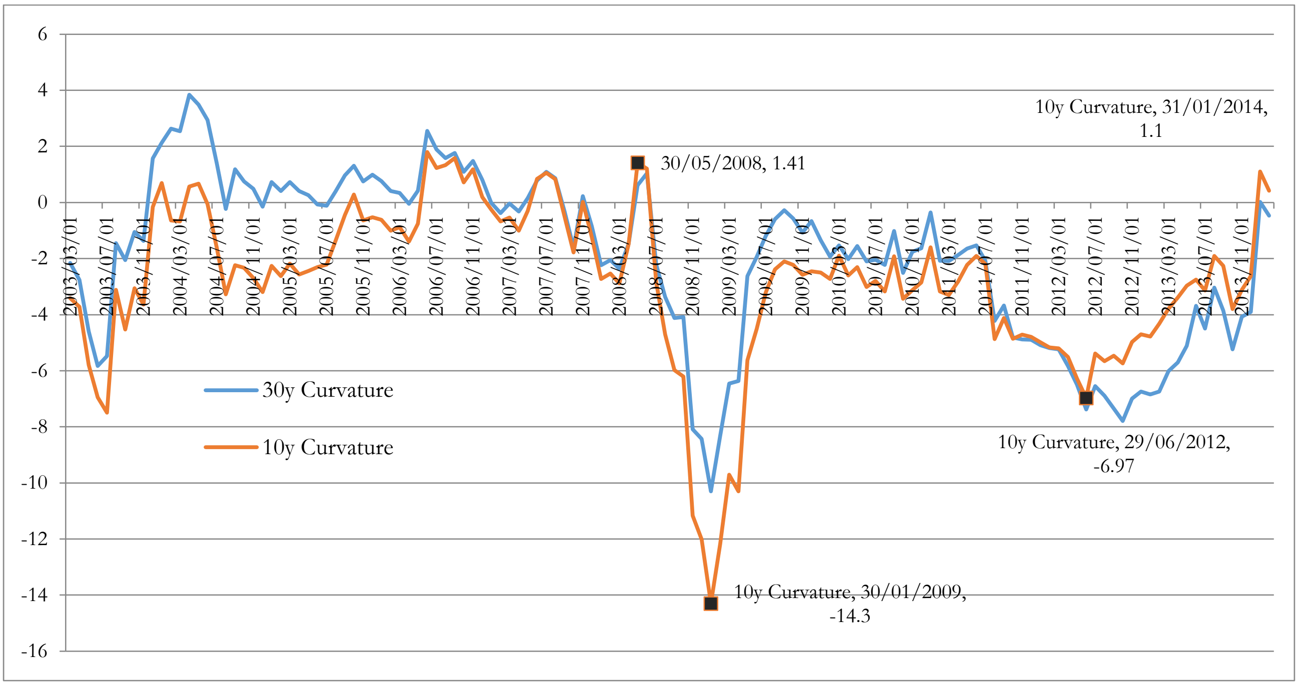


Figure 50: 10 versus 30-year NS Curvature Parameters

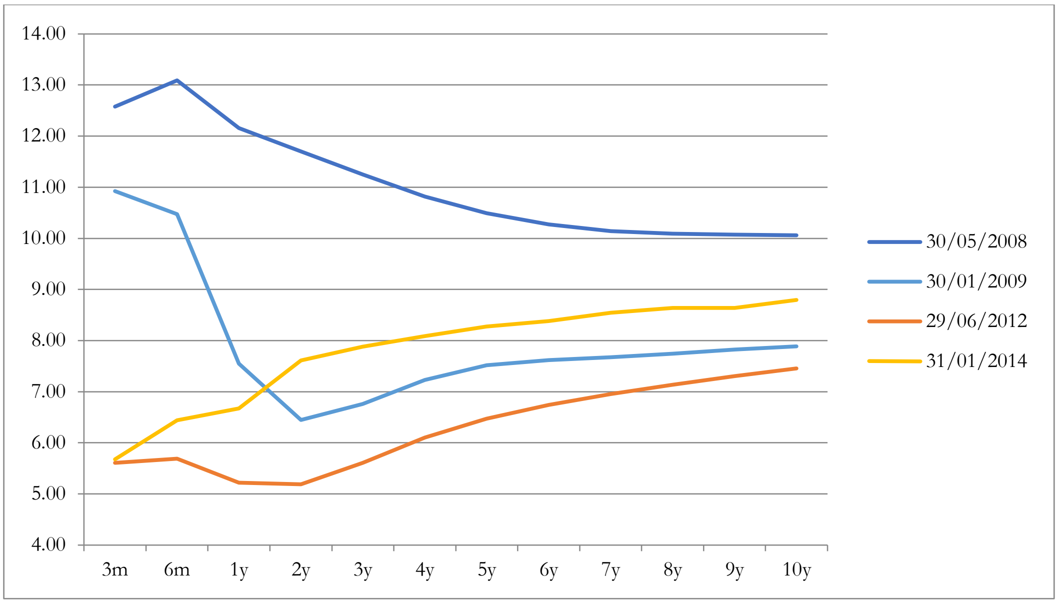


Figure 51: 10-year SA Yield Curve at 30/05/2008, 30/01/2009, 29/06/2012, 31/01/2014

What becomes apparent from the results and the similarity in the 10 versus 30-year slope and curvature trends is that in order to capture the distinct slope movements of the long-end of the yield curve (10 – 30-year area), the short-end tenors must be excluded from the analysis. This might also improve the weak adjusted R-squared values exhibited in the above 30-year analysis. Thus, a third Nelson Siegel Parameterisation analysis is conducted for the 10- to 30-year area of the yield curve.

5.1.5 10 to 30-year Nelson Siegel Analysis

As per Table 7, a λ_t-value of 9 is chosen as this maximizes the curvature component when time to maturity τ = 15-years. The 15-year point was selected as the time to maturity τ at which the point of maximum convexity or concavity has been reached as this was shown to be the case in 63% of the trading days within the sample period.

Note that in the 10 – 30-year period, the level of curvature is only very slight, and it is highly probable that this portion of the yield-curve could more effectively be modelled by a straight line (see Figures 52 and 53).

Table 7: Constellation of Maturities for 10-to-30-year NS Analysis

Long-end only Maturity Buckets:
10, 12, 15, 18, 20, 22, 25 and 30 years;
$\lambda_t = 9$, ***r (slope, curvature)= 0.395***

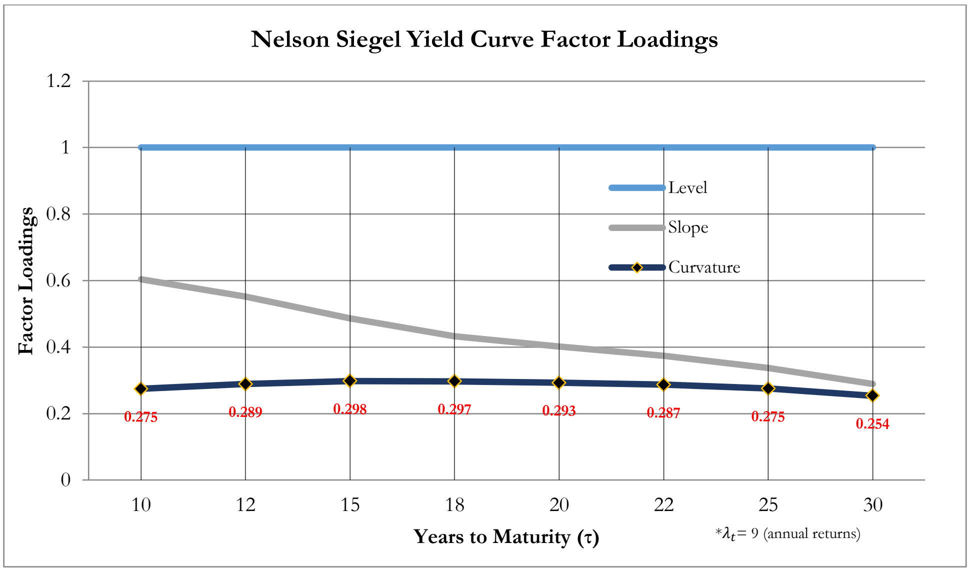


Figure 52: Factor Loadings for 10-to-30-year NS Analysis

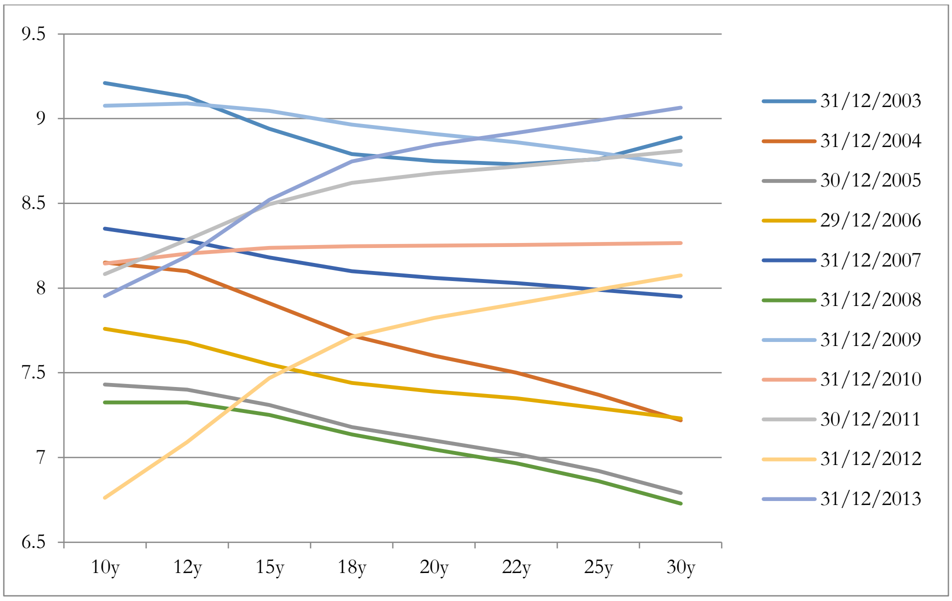


Figure 53: 10-to-30-year Yield Curve at year-end sample period dates

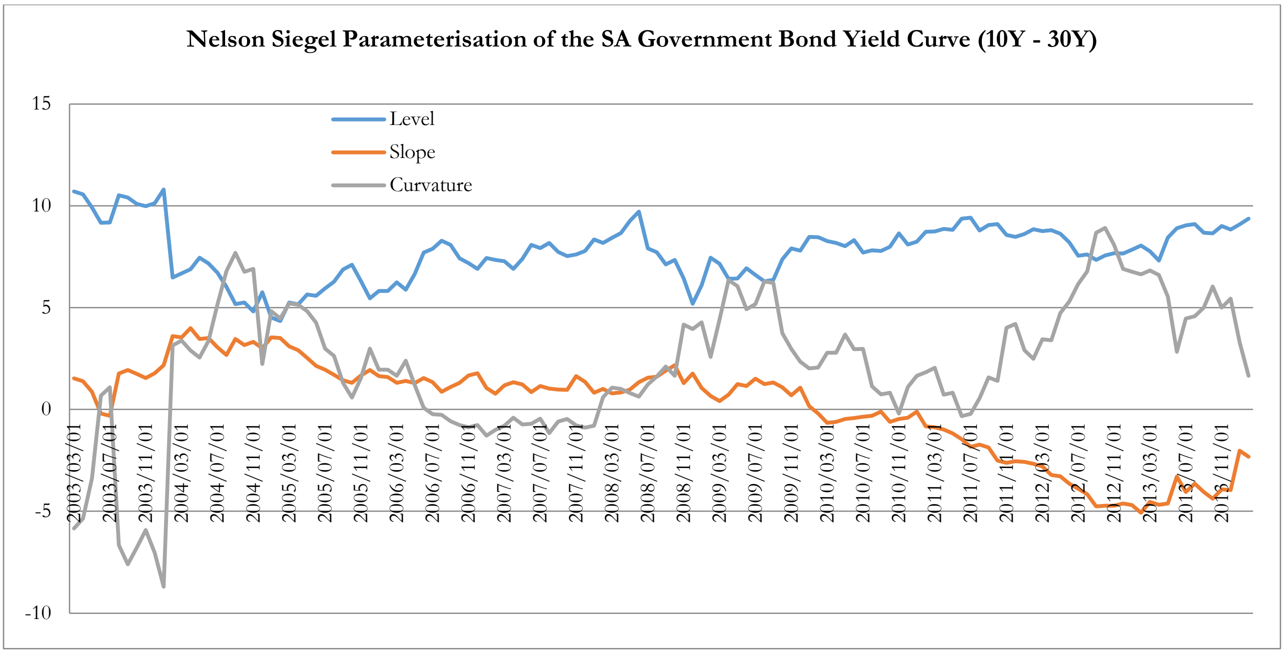


Figure 54: 10 to 30-year Nelson Siegel Level, Slope and Curvature Parameter Results

The results in Figures 54 and 55 indicate that the Level Parameter is roughly equal to 30-year yield levels. Furthermore, the 10-30-year slope is now quite distinct from the 3-month to 10-year slope (see Figure 56). While the 3-month to 10-year slope falls quite dramatically around 2003 and 2008 as inflationary and market-crisis expectations give way to rampant curve steepening, the 10-30-year slope seems to steepen reasonably steadily over the sample period as the foreign climate investor rating for South Africa worsens, credit rating downgrades occur, and the Government issues increasing amounts of debt relative to GDP.

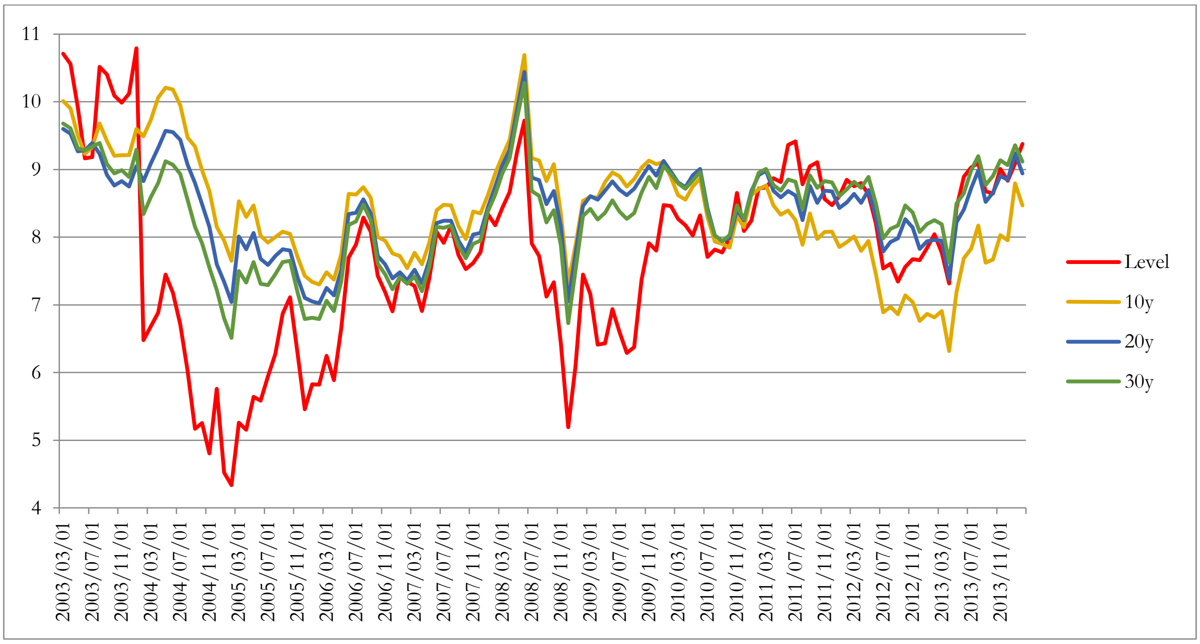


Figure 55: 10- to 30-year NS Level Parameter vs 10, 20, 30-year yields

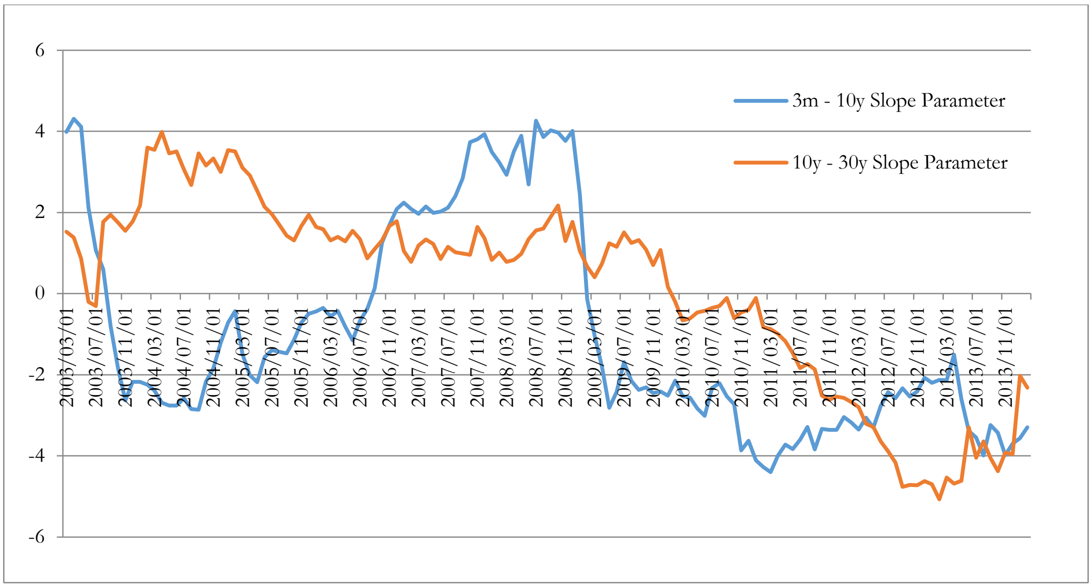


Figure 56: 3m-to-10-year vs 10-to-30-year Slope Parameters

The model fit (seen in Figure 57) has also now improved from the earlier 3-month to 30-year analysis, and one might surmise that this is the result of honing in on a more specific part of the yield curve. Only in 4.5% of the time periods sampled does the adjusted R-Squared fall below 80%, and even when it does the minimum value is a far more respectable 31%. The slope coefficient is now only insignificant ($p>0.05$) for 2 months within the sample, while the curvature parameter is statistically insignificant at the 5% level for 14 of the 132 months sampled – an improvement from the 48 months (or 36%) seen in the 3-month to 30-year analysis (see Figure 58).

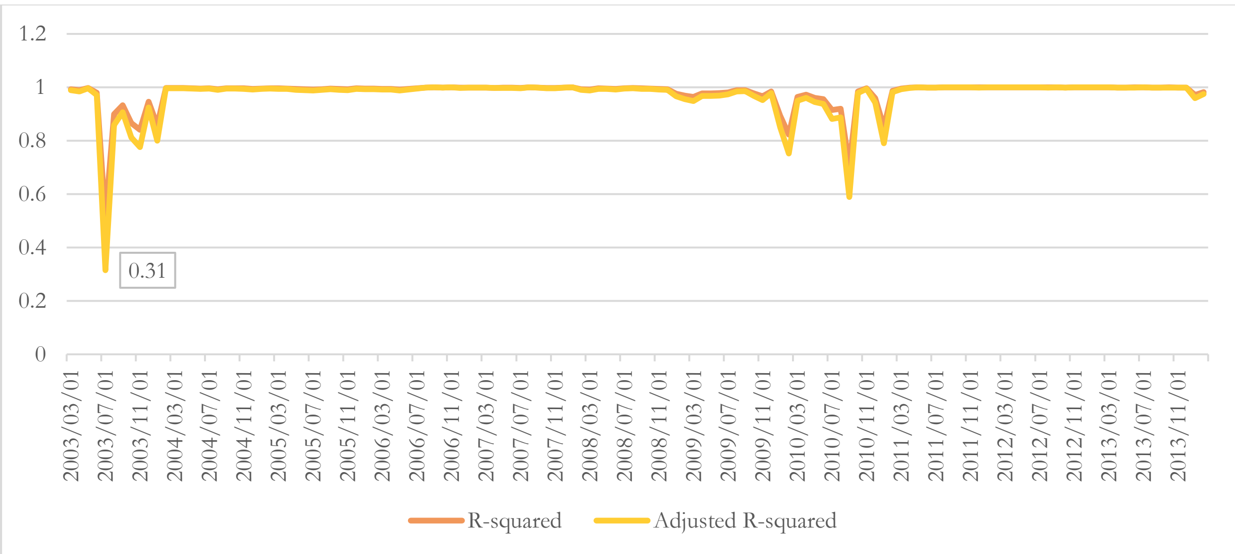


Figure 57: R-Squared and Adjusted R-Squared values for the 10 to 30-year Nelson Siegel Model

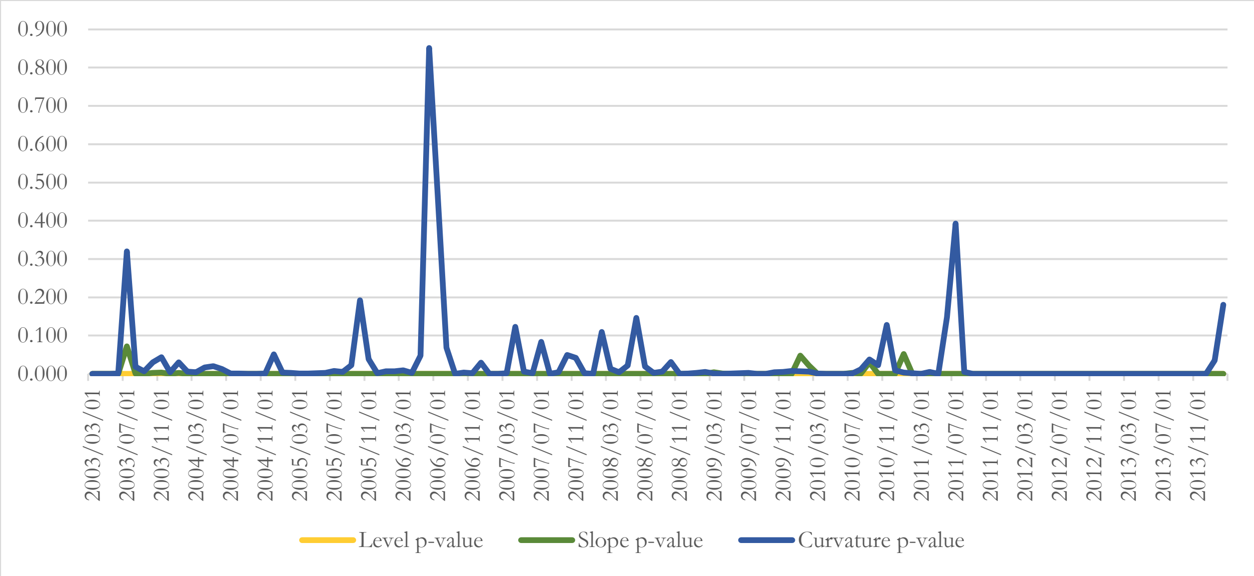


Figure 58: Extracted p-values for the Level, Slope, and Curvature Parameters in the 10- to 30-year Nelson Siegel Model

5.2 Part 2. Ordinary Least Squares Regression Model of the Predictors of the Yield Curve Parameters

5.2.1 Predictors of the 10-year Level Parameter

The results of the OLS Regression for the 10-year Level parameter of the Government bond curve over the period Q2 2003 – Q4 2013 reveals that 54% of the variation in this parameter is explained by changes in the Real Oil Price, JP Morgan Emerging Markets Bond Index, and US Government Bond Index (see Table 8). This endorses the research of Longstaff et al. (2007) and Baek, Bandopadhyaya, and Du (2005) in that local South African economic fundamentals are not seen to significantly predict the level of the sovereign yield curve. Instead, "market shock" variables (the real oil price) and measures of global risk sentiment (as measured by the JP Morgan Emerging Markets Bond Index) can adequately explain fluctuations in SA yields. Thus, for a 100bp increase in the JP Morgan EMBI spread level, the 10-year SA Government level parameter increases by 60bps on average. Similarly, the level of US Government Bonds is seen to be more important than that of local in-country fundamentals, with a sell-off in US yields on the JP Morgan US Government Bond Index associated with SA yields being marked higher in sympathy.

Table 8: OLS Regression Model for 10-year Level Parameter

Dependent Variable: 10-year Level Parameter
Method: Ordinary Least Squares Regression
Sample: 2003Q2 - 2013Q4

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
C	-0.026	0.071	-0.363	0.719
Real Oil Price	0.052	0.011	4.779	0.000
JP Morgan EMBI	0.006	0.001	5.336	0.000
US Govt. Bond Index	0.058	0.013	4.528	0.000
R-squared	0.54	Adjusted R-squared		0.50

Coefficient Test for Multicollinearity

The Variance Inflation factor Test results in Table 9 indicate that Multicollinearity is not a concern in the current model. The predictor variable with the largest uncentered VIF is the JP Morgan Government Bond Index, and the test indicates that the standard error of the coefficient for this variable would be 1.32x larger than if there was zero correlation with the other explanatory variables in the analysis (which is within the rough upper limit of 2.3x that is utilized for the VIF test).

Table 9: Variance Inflation Factors Test for 10-year Level Parameter Model

Variance Inflation Factors
Sample: 2003Q2 2013Q4

Variable	Coefficient Variance	Uncentered VIF	Centered VIF
C	0.005	1.038	NA
Real Oil Price	0.000	1.207	1.179
JP Morgan EMBI	0.000	1.746	1.733
US Govt. Bond Index	0.000	1.554	1.551

Residual Diagnostics: Tests of Autocorrelation, Heteroskedasticity, and Normality of the Residuals

Analysis of the residuals for normality in Figure 59 indicates that this assumption is met (Jarque-Bera statistic = 0.257, p = 0.88), and the Breusch-Pagan-Godfrey Test for Heteroskedasticity similarly reveals that heteroskedasticity (non-constant error variance) is not an issue for this model (see Table 10).

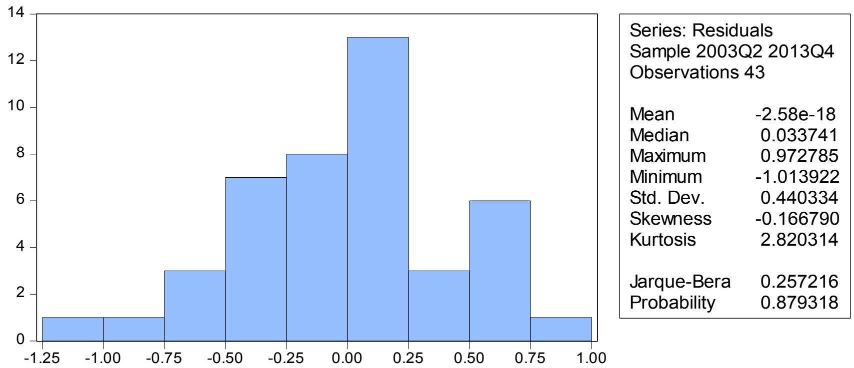


Figure 59: Jarque-Bera Test of Normality of Residuals (10-year Level Parameter Model)

Table 10: Heteroskedasticity Test (10-year Level Parameter Model)

Heteroskedasticity Test: Breusch-Pagan-Godfrey

F-statistic	0.887	Prob. F(3,39)	0.456
Obs*R-squared	2.747	Prob. Chi-Square(3)	0.432
Scaled explained SS	2.06	Prob. Chi-Square(3)	0.561

The Breusch-Godfrey Test of Serial Correlation in Table 11 reveals that autocorrelation of the residuals is a non-issue.

Table 11: Autocorrelation Test (10-year Level Parameter Model)

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.249	Prob. F(2,37)	0.781
Obs*R-squared	0.572	Prob. Chi-Square(2)	0.751

Stability of Beta-Parameter Estimates

Fabozzi et al. (2005) caution that the inclusion of many economic variables in a stepwise Ordinary Least Squares Regression procedure may lead to a high in-sample R-squared but very poor out-of-sample generalizability. In order to ascertain the stability of the Beta-coefficients over the sample period, the pre-crisis (Q2 2003 – Q2008) and post-crisis (Q3 2008 – Q4 2013) periods are analysed separately (See Tables 12 and 13).

While the model fitted over the entire period (utilizing the Real Oil Price, JP Morgan EMBI, and US Government Bond Index) explains roughly 70% of the variation in the Level Parameter in the post-crisis period, it only account for 30% of the variation in the pre-crisis (2003 – 2008) period. Furthermore, only the JP Morgan explanatory variable is significant in the pre-crisis model.

Table 12: OLS Regression Model for 10-year Level Parameter (2003 Q2: 2008Q2)

Dependent Variable: 10-year Level Parameter
Method: Ordinary Least Squares Regression
Sample: 2003Q2 - 2008Q2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.056	0.149	0.376	0.712
Real Oil Price	0.018	0.029	0.597	0.558
JP Morgan EMBI	0.007	0.002	3.095	0.007
US Govt. Bond Index	0.025	0.026	0.948	0.357
R-squared	0.408	Adjusted R-squared		0.304

Table 13: OLS Regression Model for 10-year Level Parameter (2008 Q3: 2013Q4)

Dependent Variable: 10-year Level Parameter
Method: Ordinary Least Squares Regression
Sample: 2008Q3 - 2013Q4

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.027	0.082	0.333	0.743
Real Oil Price	0.060	0.011	5.287	0.000
JP Morgan EMBI	0.007	0.001	5.414	0.000
US Govt. Bond Index	0.071	0.014	5.073	0.000
R-squared	0.738	Adjusted R-squared		0.694

Using Stepwise Regression, the fit of the pre-crisis model considerably improves to roughly 60% if one models the pre-crisis period using the JP Morgan EMBI Index and the SA Government Budget Deficit to GDP Ratio (see Table 14). Baldacci and Kumar (2010) confirm in their analysis of developed country and Emerging market yields that for the period 1980 – 2008, the size of the Budget Deficit is positively correlated with the yields of long-term Government Bonds. In the present research, too, results indicate that a 10% increase in the Budget Balance to GDP ratio (i.e. indicative of a move towards a surplus) is associated with a 100bp decrease in the 10-year yield parameter.

Table 14: Improved OLS Regression Model for 10-year Level Parameter (2003 Q2: 2008Q2)

Dependent Variable: 10-year Level Parameter
Method: Ordinary Least Squares Regression
Sample: 2003Q2 - 2008Q2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.106	0.092	1.150	0.265
JP Morgan EMBI	0.006	0.002	2.962	0.008
SA Budget Balance to GDP	-0.104	0.030	-3.402	0.003
R-squared	0.618	Adjusted R-squared		0.575

Furthermore, what these results show is that the market forces at work in the pre-crisis period appear to be distinct from those at play since 2008, with a heavier focus on in-country creditworthiness and solvency in the 2003 – 2008 period. An explanation for this finding is that the percentage of Foreign ownership of Domestic Debt has increased by more than threefold since 2008 (see Figure 60), and as a result of this phenomenon South African Government debt is far more reactive to global market sentiment and far more susceptible to foreigners pulling out of the market than was previously the case. This idea is confirmed in the IMF's (2013) Country Report on South Africa. As previously discussed, such a finding has important ramifications for the level of contagion risk amongst the various asset classes in the market and the success of investment managers in using South African Government debt for portfolio diversification purposes.

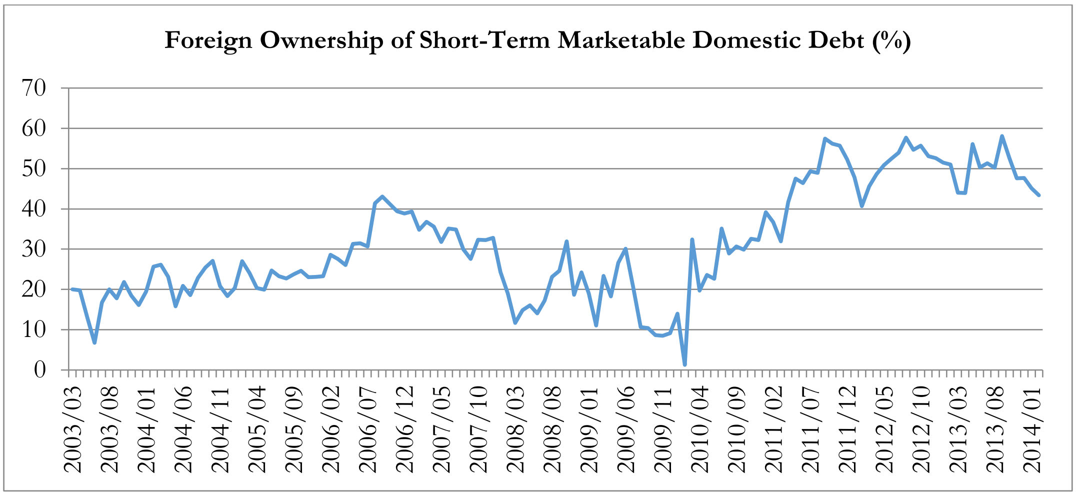


Figure 60: Foreign Ownership of Short-term SA Government Bonds

5.2.2 Predictors of the 10-year Slope Parameter

A similar model emerges for predictors of the 10-year slope parameter as that seen for the Level parameter, with increases in the real oil price, US Government Bond Index yield, and JP Morgan EMBI sovereign spreads leading to a steepening of the slope of the SA Government Bond yield curve (see Table 15). However, a superior 75% of variation in the slope is captured by these variables, as well as the level of 3-month JIBAR. This is unsurprising, as if 3-month rates are marked higher it would make sense that the slope of the curve (i.e. the Long-end minus the Short-end) should flatten. This also highlights that the market forces that determine short-end JIBAR rates (inflation and interbank borrowing) are ultimately different to those at play in the longer-end 10-year area, which appear to capture global market sentiment, price shocks, and US Government bond trends.

Table 15: OLS Regression Model for 10-year Slope Parameter

Dependent Variable: 10-year Slope Parameter
Method: Ordinary Least Squares Regression
Sample: 2003Q2 - 2013Q4

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
C	0.057	0.093	0.608	0.547
3-month JIBAR	0.256	0.028	9.275	0.000
Real Oil Price	-0.066	0.014	-4.652	0.000
US Govt. Bond Index	-0.081	0.017	-4.843	0.000
JP Morgan EMBI	-0.007	0.002	-4.678	0.000
R-squared	0.78	Adjusted R-squared		0.757

Coefficient Test for Multicollinearity

The Variance Inflation factor Test results (see Table 16) indicate that Multicollinearity is again not a concern in the current model. The predictor variable with the largest uncentered VIF is the JP Morgan Government Bond Index, and the test indicates that the standard error of the coefficient for this variable would be 1.39x larger than if there was zero correlation with the other explanatory variables in the analysis (which is within the rough upper limit of 2.3x that is utilized for the VIF test).

Table 16: Variance Inflation Factors Test for 10-year Slope Parameter Model

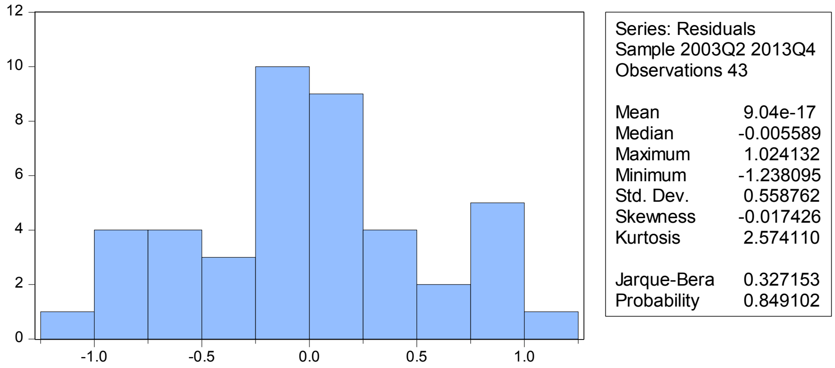
Variance Inflation Factors
Sample: 2003Q2 2013Q4

Variable	Coefficient Variance	Uncentered VIF	Centered VIF
C	0.008742	1.089290	NA
3-month JIBAR	0.000764	1.254942	1.172044
Real Oil Price	0.000201	1.215847	1.187740
US Govt. Bond Index	0.000279	1.590616	1.587673
JP Morgan EMBI	2.36E-06	1.951687	1.937440

Residual Diagnostics: Tests of Autocorrelation, Heteroskedasticity, and Normality of the Residuals

Analysis of the residuals for normality indicates that this assumption is met (Jarque-Bera statistic = 0.327, p = 0.849), and the Breusch-Pagan-Godfrey Test for Heteroskedasticity and Breusch-Pagan test for autocorrelation similarly reveals that heteroskedasticity (non-constant error variance) and autocorrelation (dependence of residuals on lagged error values) is not an issue for this model (See Table 17).

Table 17: Residual Diagnostic Tests for 10-year Slope Parameter



Heteroskedasticity Test: Breusch-Pagan-Godfrey

F-statistic	1.252005	Prob. F(4,38)	0.3057
Obs*R-squared	5.007088	Prob. Chi-Square(4)	0.2866
Scaled explained SS	3.077660	Prob. Chi-Square(4)	0.5449

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.207203	Prob. F(2,36)	0.8138
Obs*R-squared	0.489352	Prob. Chi-Square(2)	0.7830

Weighted OLS Regression Model

As per Figure 61, the 10-year Slope Parameter experiences a vivid decline around late-2008 / early 2009 owing to the sudden steepening of the yield curve around the time of the financial crisis and the gloomy long-term outlook for economic growth. Using a weighting system in order to decrease the weighting on these quarters reveals that the fitted model is still highly significant, although the adjusted R-squared value notably decreases from 75% to 65% (see Table 18).

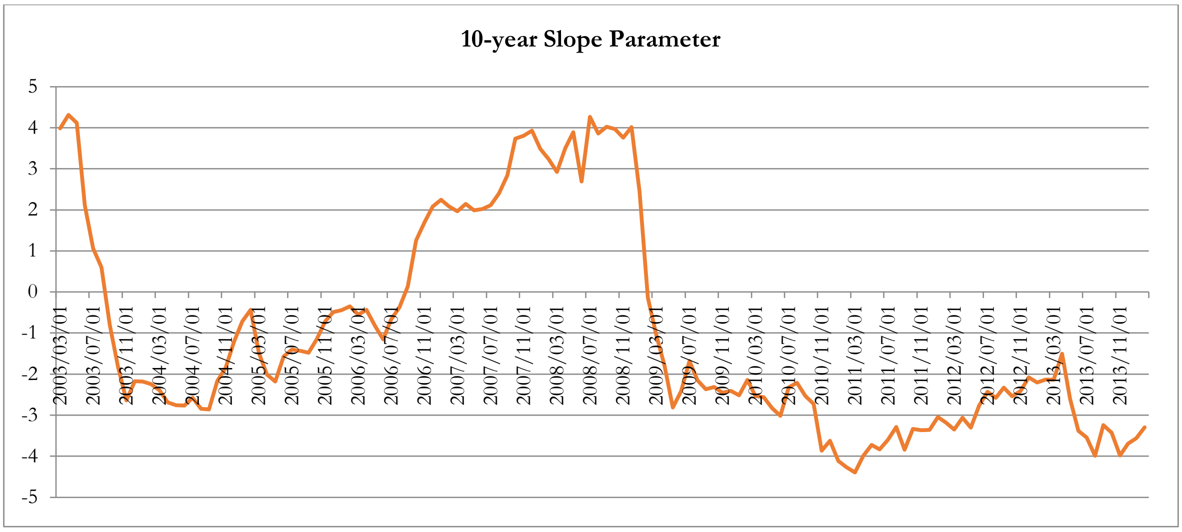


Figure 61: 10-year NS Slope Parameter

Table 18: Weighted OLS Regression Model for 10-year Slope Parameter

Dependent Variable: 10-year Slope Parameter
Method: Ordinary Least Squares Regression
Sample: 2003Q2 - 2013Q4
Weight type: Inverse standard deviation (EViews default scaling)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.067	0.086	0.787	0.436
3-month JIBAR	0.218	0.029	7.620	0.000
Real Oil Price	-0.056	0.013	-4.165	0.000
US Govt. Bond Index	-0.073	0.015	-4.739	0.000
JP Morgan EMBI	-0.007	0.001	-4.444	0.000
Weighted Statistics				
R-squared	0.686	Adjusted R-squared		0.653

Stability of Beta-Parameter Estimates

The Beta-parameters for the slope factor appear to be somewhat more stable than those of the 10-year level parameter, with the adjusted R-squared for the pre versus post-crisis period in Table 19 climbing from 67% to 81.5% (previously 30% to 70%). That said, the Real Oil Price (a proxy for global shocks and risk sentiment) is again no longer a significant predictor of the slope of the yield curve in the 2003 – 2008 period model.

Table 19: OLS Regressions for 10-year Slope Parameter (2003Q2: 2008Q2 and 2008Q3: 2013Q4)

Dependent Variable: 10-year Slope Parameter
Method: Ordinary Least Squares Regression
Sample: 2003Q2 - 2008Q2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.140	0.205	-0.683	0.504
3-month JIBAR	0.279	0.049	5.701	0.000
Real Oil Price	-0.003	0.041	-0.073	0.943
US Govt. Bond Index	-0.058	0.033	-1.753	0.099
JP Morgan EMBI	-0.009	0.004	-2.370	0.031
R-squared	0.739	Adjusted R-squared		0.674

Dependent Variable: 10-year Slope Parameter
Method: Ordinary Least Squares Regression
Sample: 2008Q3 - 2013Q4

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.04	0.14	0.31	0.76
3-month JIBAR	0.26	0.04	5.96	0.00
Real Oil Price	-0.08	0.02	-4.95	0.00
US Govt. Bond Index	-0.09	0.02	-4.45	0.00
JP Morgan EMBI	-0.01	0.00	-4.57	0.00
R-squared	0.851	Adjusted R-squared		0.815

While the 2003 – 2008 model provides a good model fit of the Slope parameter (R-squared = 74%), as has been seen before a better model fit (R-squared = 87%) arrives from removing the global sentiment indicators of JP Morgan EMBI and the Real Oil price, and instead using the SA Budget Balance to GDP Ratio as an indicator of in-country creditworthiness and solvency (see Table 20). An 11% increase in this ratio (towards a budget surplus) causes the slope of the curve to flatten by 100bps – indicating a lower risk premium being assigned to long-term government debt due to the sovereign's enhanced creditworthiness.

Table 20: Improved OLS Regression Model for 10-year Slope Parameter (2003 Q2: 2008Q2)

Dependent Variable: 10-year Slope Parameter
Method: Ordinary Least Squares Regression
Sample: 2003Q2 2008Q2

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
C	-0.149	0.106	-1.407	0.179
3-month JIBAR	0.228	0.027	8.616	0.000
SA Budget Balance to GDP	0.109	0.034	3.224	0.005
US 3-month Deposit Rate	0.061	0.016	3.869	0.001
US Govt. Bond Index	-0.104	0.027	-3.857	0.001
R-squared	0.868	Adjusted R-squared		0.835

5.2.3 Predictors of the 10-year Curvature Parameter

Unlike the overlap in predictors for the Level and Slope Parameters, the 10-year Curvature parameter has a very distinct set of predictors. As discussed, this factor measures the degree of concavity of the yield curve, with a more concave or "humped" shape pointing to rising short-term inflation/ interest rate expectations. Once again, as per Table 21, the level of US Government Bonds is seen as being highly significant, with a rise in US-Treasury yields being associated with a more concave SA yield curve. One might postulate that rising yield levels in the US indicate increased inflationary expectations (leading to a sell-off in fixed interest debt).

Conversely, rising yields across the US BAA-rated corporate bond index (an indicator of poorer sentiment towards riskier credits) is associated with a more convex shape in the SA yield curve. Similarly, a decrease in the credit rating of the SA sovereign is associated with a more convex yield curve shape. A decline in these variables points to a market perception of the deterioration in creditworthiness of lower-investment grade issuers, which might be the case not only when in-house fundamentals are poor, but when the business cycle is pointing towards a period of low growth – which would affect riskier corporate and EM debt issuers.

In either case (deteriorating solvency or poor economic growth), one would not expect inflationary concerns, which are generally associated with periods of rising GDP, to be an issue. This might then point to a more convex shape to the SA yield curve.

Finally, the Barclays South Africa Inflation-Linked Bond Index measures market expectations concerning inflation across the yield curve. An increase in these Real (inflation-adjusted) Total Returns points to rising inflation, which in turn is associated with a more concave yield curve shape.

Table 21: OLS Regression Model for 10-year Curvature Parameter

Dependent Variable: 10-year Curvature Parameter
Method: Ordinary Least Squares Regression
Sample: 2003Q2 - 2013Q4

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
C	-0.534	0.366	-1.461	0.152
US Govt. Bond Index	0.287	0.040	7.132	0.000
US BAA Corp Bond Index	-0.406	0.104	-3.904	0.000
SA Credit Rating	-0.377	0.135	-2.794	0.008
SA Inflation Bond Index	0.559	0.267	2.093	0.043

R-squared	0.597	Adjusted R-squared	0.554

Weighted OLS Regression Model

The 10-year Curvature Parameter exhibits a sudden fall around late-2008 / early 2009 as the shape of the yield curve moves from being concave to suddenly convex, which is in part due to declining inflationary expectations but also as the result of increases in short-term borrowing rates owing to the drying up of liquidity in the markets. The relative weights on these quarters are thus decreased in the regression seen in Table 22, and results reveal that the fitted model is still highly significant, with the adjusted R-squared declining only slightly from 55.4% to 50.5%.

Table 22: Weighted OLS Regression Model for 10-year Curvature Parameter

Dependent Variable: 10-year Curvature Parameter
Sample: 2003Q2 - 2013Q4
Weight type: Inverse standard deviation (EViews default scaling)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.551	0.361	-1.524	0.136
US Govt. Bond Index	0.287	0.044	6.497	0.000
US BAA Corp Bond Index	-0.436	0.122	-3.570	0.001
SA Credit Rating	-0.387	0.132	-2.926	0.006
SA Inflation Bond Index	0.533	0.262	2.037	0.049
		Weighted Statistics		
R-squared	0.552	Adjusted R-squared		0.505

Coefficient Test for Multicollinearity

Despite the fact that the US BAA Corporate Bond Index and SA Credit Rating might both be presumed to proxy for market sentiment towards risky credits, and the fact that global inflationary conditions might similarly cause both US government bonds to sell off and the total returns of the SA Inflation-linked bond index to rise, these series are distinct enough that multicollinearity is a non-issue (see Table 23).

Table 23: Variance Inflation Factors Test for 10-year Curvature Parameter Model

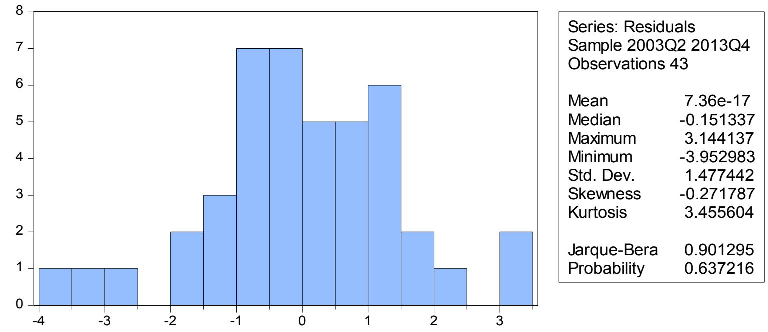
Variance Inflation Factors
Sample: 2003Q2 2013Q4

Variable	Coefficient Variance	Uncentered VIF	Centered VIF
C	0.134	2.382	NA
US Govt. Bond Index	0.002	1.323	1.320
US BAA Corp Bond Index	0.011	1.337	1.333
SA Credit Rating	0.018	1.075	1.074
SA Inflation Bond Index	0.071	2.658	1.211

Residual Diagnostics: Tests of Autocorrelation, Heteroskedasticity, and Normality of the Residuals

Similarly, as per Table 24, the residual diagnostics tests are met for the curvature-parameter model.

Table 24: Residual Diagnostic Tests for 10-year Curvature Parameter



Heteroskedasticity Test: Breusch-Pagan-Godfrey

F-statistic	2.228	Prob. F(4,38)	0.084
Obs*R-squared	8.170	Prob. Chi-Square(4)	0.086
Scaled explained SS	7.834	Prob. Chi-Square(4)	0.098

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	1.33	Prob. F(2,36)	0.276
Obs*R-squared	2.97	Prob. Chi-Square(2)	0.227

5.2.4 Predictors of the 30-year Level and Slope Parameter

Models of the 30-year Level and 10-30-year Slope Parameters have little explanatory power and suffer from Beta Parameter instability over the sample period. This indicates that the Long-end of the curve is quite distinct from the short and intermediate term area, and a separate set of market forces are responsible for its movements. Furthermore, this is a relatively illiquid area of the curve with the main market players presumably being Pension Funds and Liability Management houses who are interested in hedging their long-term liability exposure. Thus, it is unsurprising that it is not particularly sensitive to the economic forces tested.

The 30-year Level Parameter Model for the entire sample period already gives an early indication that distinct market forces are at play in this area of the curve (see Table 25). While increases in the real oil price are once again associated with a sell-off in long-term bond yields, this area of the curve also appears to be associated with the amount of South African Government debt in issue relative to GDP – with a 10% increase in this ratio leading to a 28-bp increase in long-term yields. Given the glut of issuance taking place in this area of the curve over recent years, one might presume that increases in Government-Debt-to-GDP have largely been the result of increased back-end bond issuance and a relative oversupply in this area of the curve.

Furthermore, what this area of the curve appears to tap into long-term growth expectations in the global economy, as seen by the US Real GDP explanatory variable. A 0.12% increase in US inflation-adjusted GDP growth is associated with a 100bp increase in back-end yields, presumably as a result of expected increases in inflation. Risk-sentiment towards riskier credits and the health of the corporate sector are also captured via the US BAA-Corporate Bond Index, with a 24bp sell-off in yields associated with a 100bp sell-off in 30-year SA Government yield levels.

Table 25: OLS Regression Model for 10-to-30-year Level Parameter

Dependent Variable: 30-year Level Parameter
Method: Ordinary Least Squares Regression
Sample: 2003Q2 - 2013Q4

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
C	-0.032	0.134	-0.237	0.814
Real Oil Price	0.078	0.021	3.705	0.001
Gross Govt. Debt to GDP	0.282	0.097	2.898	0.006
US Corp. Bond Index	0.242	0.053	4.535	0.000
US Real GDP Growth	0.122	0.059	2.091	0.043
R-squared	0.50	Adjusted R-squared		0.447

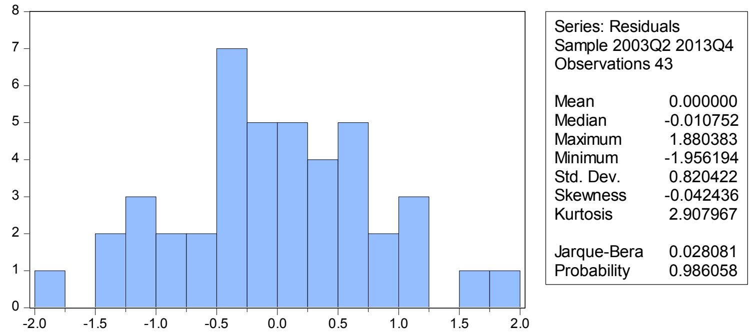
While the assumptions around Multicollinearity, Heteroskedasticity, Autocorrelation and Normality of the Residuals are met (see Tables 26 and 27), ultimately this model has very poor predictive power for the pre- versus post-crisis period and suffers from lack of generalizability.

Table 26: Variance Inflation Factors Test for 10-to-30-year Level Parameter Model

Variance Inflation Factors
Sample: 2003Q2 2013Q4

Variable	Coefficient Variance	Uncentered VIF	Centered VIF
C	0.018	1.039	NA
Real Oil Price	0.000	1.255	1.226
Gross Govt. Debt to GDP	0.009	1.156	1.149
US Corp. Bond Index	0.003	1.138	1.135
US Real GDP	0.003	1.147	1.147

Table 27: Residual Diagnostic Tests for 10 – to- 30-year Level Parameter



Heteroskedasticity Test: Breusch-Pagan-Godfrey

F-statistic	1.524	Prob. F(4,38)	0.215
Obs*R-squared	5.946	Prob. Chi-Square(4)	0.203
Scaled explained SS	4.430	Prob. Chi-Square(4)	0.351

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.195	Prob. F(2,36)	0.824
Obs*R-squared	0.460	Prob. Chi-Square(2)	0.795

A model with better generalizability is simply that of the Real Oil price (representing market shocks) and the US Corporate Bond Index (representing the health of the US Corporate sector and perceptions of creditworthiness). While overall this model only has 30% predictive power as per the Adjusted-R-squared value in Table 28, the percentage of explained variation swells to 71% in the post-crisis period -- once again indicating that global sentiment has outstripped local fundamentals in predicting SA Government yields in recent years (see Table 29).

Table 28: Improved OLS Regression Model for 30-year Level Parameter

Dependent Variable: 30-year Level Parameter
Method: Ordinary Least Squares Regression
Sample: 2003Q2 2013Q4

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.005	0.150	-0.034	0.973
Real Oil Price	0.076	0.022	3.497	0.001
US Corp. Bond Index	0.184	0.057	3.208	0.003
R-squared	0.328	Adjusted R-squared		0.294

Table 29: OLS Regressions for 10- to- 30-year Level Parameter (2003Q2: 2008Q2 and 2008Q3: 2013Q4)

Dependent Variable: 30-year Level Parameter
Method: Ordinary Least Squares Regression
Sample: 2003Q2 2008Q2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.032	0.235	0.135	0.894
US Corp. Bond Index	0.415	0.121	3.440	0.003
R-squared	0.384	Adjusted R-squared		0.351

Dependent Variable: 30-year Level Parameter
Method: Ordinary Least Squares Regression
Sample: 2008Q3 2013Q4

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.087	0.117	0.742	0.467
Real Oil Price	0.103	0.014	7.219	0.000
US Corp. Bond Index	0.141	0.039	3.634	0.002
R-squared	0.741	Adjusted R-squared		0.713

As per Table 30, the 10 – 30-year slope is even less inclined to prediction, with a poor Adjusted R-squared value of 26% for the fitted model. As per the 10-year slope parameter, increases in short-end rates (as indicated by 3-month JIBAR) cause the slope to flatten. Furthermore, the Quarterly lagged South African Inflation rate is seen to predict the slope in this back-end of the curve such that a 0.36% increase in inflation is associated with a 100-bp steepening of the slope. Lagged values are used because the release of South Africa's CPI Index is a backward-looking variable that operates on a quarterly lag.

Table 30: OLS Regression Model for 10-to 30--year Slope Parameter

Dependent Variable: 30-year Slope Parameter
Method: Ordinary Least Squares Regression
Sample: 2003Q2 2013Q4

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
C	-0.086	0.126	-0.682	0.499
3-month JIBAR	0.110	0.038	2.888	0.006
South Africa Inflation rate (-1)	-0.357	0.091	-3.909	0.000
R-squared	0.300	Adjusted R-squared		0.265

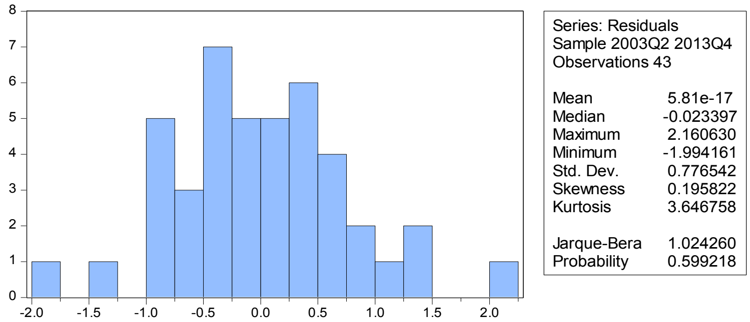
There are various issues with this model around OLS regression assumptions. Firstly, as per Table 31, the residual terms suffer from heteroskedasticity (non-constant variance). Using White-adjusted standard errors solves this issue and reveals that that the model is still significant (see Table 32), however, as per

Table 33, it is found to only be able to explain changes in the 10-30-year slope in the pre-crisis portion of the sample period – which was incidentally when a strong decline in inflation in early 2004 caused the slope to flatten dramatically.

Table 31: VIF and Residual Diagnostic Tests for 10 – to- 30-year Slope Parameter

Variance Inflation Factors
Sample: 2003Q2 2013Q4

Variable	Coefficient Variance	Uncentered VIF	Centered VIF
C	0.016	1.080	NA
3-month JIBAR	0.001	1.307	1.221
SA Inflation rate (-1)	0.008	1.222	1.221



Breusch-Godfrey Serial Correlation LM Test:

F-statistic	1.506	Prob. F(2,38)	0.235
Obs*R-squared	3.158	Prob. Chi-Square(2)	0.206

Heteroskedasticity Test: Breusch-Pagan-Godfrey

F-statistic	4.216	Prob. F(2,40)	0.022
Obs*R-squared	7.486	Prob. Chi-Square(2)	0.024
Scaled explained SS	8.572	Prob. Chi-Square(2)	0.014

Table 32: White-Adjusted OLS Regression Model for 10-to-30-year Slope Parameter

Dependent Variable: 30-year Slope Parameter
Method: Ordinary Least Squares Regression
Sample: 2003Q2 2013Q4
White heteroskedasticity-consistent standard errors & covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.086	0.107	-0.802	0.427
3-month JIBAR	-0.357	0.131	-2.725	0.010
South Africa Inflation rate (-1)	0.110	0.053	2.081	0.044

R-squared	0.300	Adjusted R-squared	0.265
Prob(Wald F-statistic)	0.020	Wald F-statistic	4.348

Table 33: OLS Regression Model for 10-to-30-year Slope Parameter (2003Q2: 2008Q2)

Dependent Variable: 30-year Slope Parameter
Method: Ordinary Least Squares Regression
Sample: 2003Q2 2008Q2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.059	0.218	-0.269	0.791
South Africa Inflation rate (-1)	-0.307	0.116	-2.635	0.016
R-squared	0.268	Adjusted R-squared		0.229

Given the poor predictability of the Quarterly Model, a Regression on monthly data is performed in Table 34 (using those explanatory variables for which monthly data is available, and to the exclusion of such data series as Quarterly GDP). Despite drilling down into an even greater level of depth in the data, the results are similar to that of the quarterly analysis, with South Africa's Inflation Rate (lagged) being the only significant and reasonably stable predictor of the 10-30-year slope. Furthermore, the R-Squared for this model falls significantly from 30 to 15% explanatory power. Predictors of this area of the curve thus remain somewhat illusive, although literature on the topic asserts that while the long-end of the sovereign curve is driven by the long-term growth outlook and inflation, it is far less liquid and thus less likely to capture "minute-to-minute" market expectations given the fewer market players who engage in this area (Kempf, Korn & Homburg, 2012). For South Africa's case, the data exploration section revealed that the slope seemed to steepen reasonably steadily over the sample period along with worsening foreign investor climate ratings and a greater degree of long-term government debt. Clearly, long-term inflationary expectations also play a role.

Table 34: OLS Regression Monthly Model for 10-to-30-year Slope Parameter

Dependent Variable: 30-year Slope Parameter
Method: Ordinary Least Squares Regression
Sample: 2003M07 2014M02

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.012	0.054	0.222	0.825
SA Inflation rate (-3)	-0.021	0.006	-3.523	0.001
R-squared	0.158	Adjusted R-squared		0.146

5.3 Part 3. Dynamic Latent Factor Approach to the SA Government Yield curve

5.3.1 Yields-Only Model

State Parameter Estimates for A matrix

The optimal maximum likelihood estimates of the "A transition matrix" defined in the Methodology section for the "yields-only" dynamic latent factor approach are presented below in Table 35. The results reveal that the yields-only coefficients for the SA Government Bond yield curve are significant at the 5% level, with lagged coefficients of 0.90, 0.94, and 0.92 for the Level, Slope, and Curvature parameters. A statistically significant mean value is only found for the Level and Curvature parameters, indicating that these series are somewhat more mean-reverting than the slope parameter – which has the highest standard deviation. The only off-diagonal significant parameter estimate is the minor relationship between the (short-term) slope parameter and the (intermediate-term) lagged curvature parameter – in which we find that an increase in the concavity ("humped-shape") of the yield curve at time t-1 is significantly associated with a small 10 basis-point flattening of the short-term slope at time t. This may be the result of the specific overlap in time buckets of these forces.

Table 35: A transition matrix Results, Yields-Only Model[1]

Yields-Only Model Parameter Estimates	L_{t-1}	S_{t-1}	C_{t-1}	μ
L_t	**0.90**	0.01	0.01	**8.82**
	0.03	*0.01*	*0.01*	*0.26*
S_t	-0.05	**0.94**	**0.10**	-0.47
	0.05	*0.02*	*0.02*	*1.06*
C_t	-0.12	-0.03	**0.92**	**-2.10**
	0.11	*0.04*	*0.04*	*0.67*

[1] Significant coefficients are found in red font, and standard deviations are italicized.

White Noise Error terms and Measurement Disturbances

As seen in the Methodology section, the Q matrix contains optimal estimates of the 3 white noise error terms η_t and the 3 off-diagonal covariances (see Table 36). All 3 of the variance terms and 2 of the off-diagonal covariance terms are significant, and the Likelihood Ratio and Wald Tests for diagonality both strongly reject the hypothesis that the Q matrix is diagonal. This is in line with the non-restriction on the diagonality of the Q matrix – thus allowing for market shocks affecting the yield level, slope, and curvature to be correlated. It is clear from the below that shocks increasing the level parameter simultaneously steepen the slope and produce a more concave curve shape on average.

Table 36: Q white noise matrix Results (Yields-Only Model) and Tests for Diagonality[2]

Estimated Q matrix

	L_t	S_t	C_t
L_t	**0.12**	**-0.14**	0.03
	0.01	*0.02*	*0.03*
S_t		**0.24**	**-0.08**
		0.03	*0.04*
C_t			**1.08**
			0.14

Tests for diagonality Q matrix

	Test Statistic	P-value
Likelihood ratio	**136.15**	0.00
Wald	**62.79**	0.00

The optimal value for the state parameter λ is found to be 0.071, which is similar to Diebold and Li's (2006) use of the value 0.0609, as well as Diebold et al.'s (2006) optimal λ-value estimate of 0.077.

[2] Significant coefficients are found in red font, and standard deviations are italicized.

The optimal mean estimates of the 12 measurement disturbances ϵ_t, corresponding to each of the 12 yield maturity buckets, are presented in Table 37. The average residuals (measured in basis points) are very small and the average standard deviations are similarly as small, indicating an excellent model fit for the 12 yield maturities considered. This also points to the success of the Kalman filter in estimating yield forecasts and standard deviations, while the Gaussian likelihood function of the model is maximized using the Broyden–Fletcher–Goldfarb–Shanno (BFGS) iterative procedure to converge to optimal estimates of the earlier-mentioned model parameters (i.e. the A transition matrix, λ etc.). Exceptions to this assertion are the 3-month and 8-year mark, which exhibit a reasonably large mean and standard deviation value. Diebold et al. (2006) find a similarly high standard deviation values at very high and low maturities. The volatility of the 3-month point, which should be most receptive to surprize interest rate changes in the policy rate as determined by the South African Reserve Bank, contributes to the poor model fit of this parameter. Furthermore, for some of the sample period, the SA Government "benchmark bond" was the R157 (maturity date: 15/09/2015), which undoubtedly saw the most in nominal amount of trading and thus exhibited the highest subsequent volatility. By the start of the 2008 crisis, there was roughly 8 years until maturity on the R157 (see Figure 62). This possibly explains the poor model fit for the 8-year period given the sudden (and unexpected) market-shock related sell-off in the R157 seen around this time.

Table 37: Summary Statistics for yield measurement disturbances ϵ_t, Yields-Only Model

Yield Maturity	Mean	Standard Deviation	t-statistic	P-value
3m	**-28.32**	**20.99**	**-1.35**	**0.18**
6m	-2.06	0.13	-16.29	0.00
1y	-2.14	0.13	-16.43	0.00
2y	-2.27	0.13	-18.01	0.00
3y	-3.36	0.13	-26.28	0.00
4y	-5.18	0.14	-38.20	0.00
5y	-7.97	0.33	-24.22	0.00
6y	-7.35	0.14	-52.25	0.00
7y	-7.85	0.13	-59.98	0.00
8y	**-28.16**	**36.62**	**-0.77**	**0.44**
9y	-7.09	0.13	-56.43	0.00
10y	-5.59	0.12	-44.74	0.00

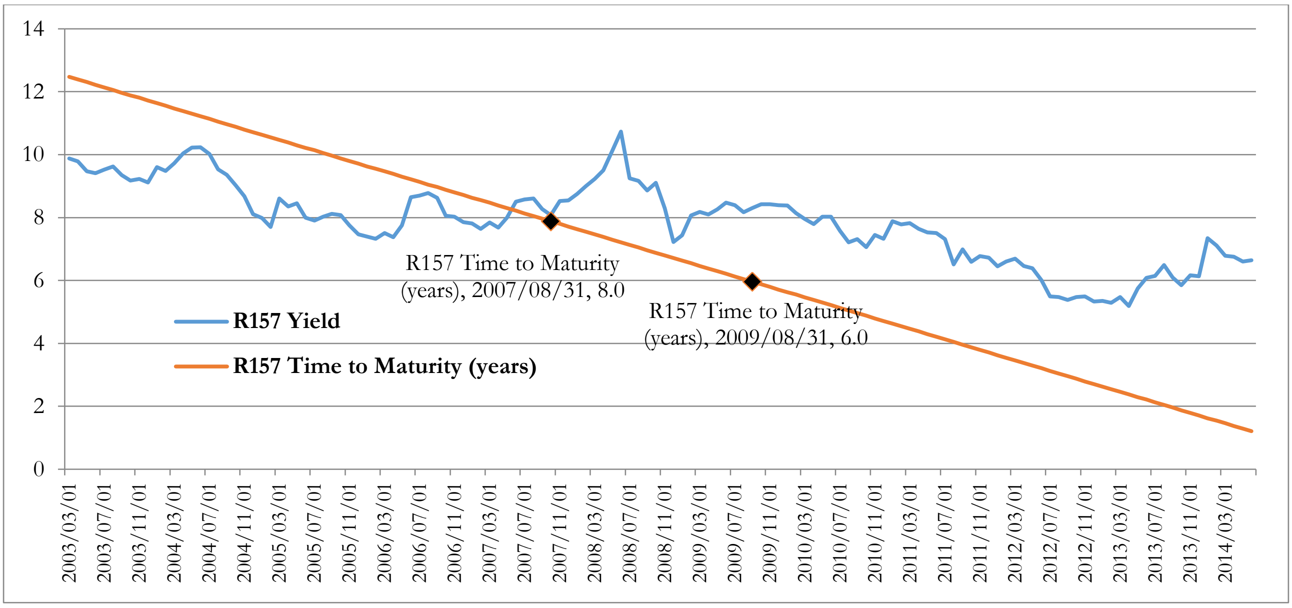


Figure 62: R157 Historical Yield and Time to Maturity (years)

Source: Bloomberg

5.3.2 Yields-Macro Model

State Parameter Estimates for A matrix

As discussed in the Methodology section, the Broyden–Fletcher–Goldfarb–Shanno (BFGS) iterative procedure is used so that the earlier-mentioned model parameters (i.e. the A transition matrix, λ etc.) converge to optimal estimates. For the “Yields-Macro” Model, this does not initially take place after even 5000 iterations. A closer look at the data reveals that while all series fit the VAR (1) Autoregressive model structure assumed in this section, the SA Deposit rate has a “near-unit root” at 2 lags (see Table 38). Using first differences for this data series solves the issue of non-convergence via the BFGS algorithm.

Table 38: Augmented Dickey-Fuller Test, SA Deposit Rate

Null Hypothesis: Depo Rate has a unit root		
Exogenous: Constant		
Lag Length: 2 (Automatic - based on SIC, maxlag=12)		
	t-Statistic	Prob.*
Augmented Dickey-Fuller t-stat	-2.68	0.08

As per the yields-only model, the lagged coefficients for the Nelson Siegel Beta parameters enter the model as significant predictors of the curvature parameters – although, as per Table 39, the coefficients are now in the region of 3 to 10 basis-points smaller given the presence of other macro-factor predictors.

With the exception of changes in the Deposit Rate, the yields and macro-factors all exhibit significant persistence over time and are significantly associated with their own lagged values. Looking at the macro factors in particular, one sees that an increase in the lagged-level of Capacity Utilization in the economy precipitates a rise in inflation. This makes sense in the context of economic theory as capacity utilization, a measure of the degree to which the economy uses up its productive capacity, is seen as an indicator of aggregate demand and a strong predictor of rising future inflation. The strongest predictor of rising inflation in this model (as per the coefficient size) is that of a lagged rise in inflation, as well as increases in the SA deposit rate. This represents the South African Reserve Bank's attempt to combat escalating inflation by increasing the policy rate via the mechanism of monetary policy.

This all said, a **decrease** in lagged inflation is seen to be what precipitates the rise in Capacity Utilization in the first place (i.e.$CU_t = -0.07\ (INFL_{t-1}\ )$). This essentially captures the complex nature of the business cycle and the fact that lower inflation serves to increase consumer wealth and stimulate aggregate demand – which then contributes to future rising inflation as the cycle continues.

As discussed, Diebold et al. (2006) do not only consider one-way or "unidirectional" macroeconomic relationships (i.e. "macro-to-yield" relations), but instead focus on the potential biodirectional interaction loops that exist between the yield curve and the economy. They thus make the assumption that each in some way informs the other and that the yield curve contains valuable information about the future state of the economy. From this yield-to-macro perspective, one notes that an increase in the concavity of the yield curve (potentially reflecting the market's expectation and pricing in of rising inflation) tends to be associated with 5-unit increase in Capacity Utilization in the following month on average. Similarly, a 1% (100- basis point) increase in the slope parameter precipitates an 8 basis-point (0.08%) increase in the SA inflation rate in the following month.

Furthermore, increases in the slope and concavity and decreases in the Level parameter tend to be followed by increases in the SA deposit rate. This indicates the SARB's combatting of price inflation as captured in the yield curve's earlier pricing in of a more humped shape. Interestingly, the 10-year level component points to declining yields, but the rising slope points to a decline in short-term rates versus those in the back-end. This captures the idea that there is a dislocation between short-term deposit rates and the long-term (10-year) level of bond yields. The Level component's coefficient appears to be in the "wrong direction" as one might expect a rise in yields to signify a selling off of bond levels in expectation of greater inflation, which then predicates an interest rate hike. The rationale for this negative Level coefficient may be multifold, and could also capture the idea that changes in the deposit rate often catch the market unawares as evidenced by their lower pricing of long-term yields in the previous month. The lower level of short-term versus long-term yields may also indicate that even though the market is anticipating a rise in inflation, that they do not believe that the SARB will act on this by hiking rates in the near future, perhaps due to inherent consumer weakness in the economy or due to the lack of perceived credibility of the SARB.

Table 39: A transition matrix Results, Yields-Macro Model

Yields-Macro Model Parameter Estimates

	L_{t-1}	S_{t-1}	C_{t-1}	CU_{t-1}	$INFL_{t-1}$	$Depo_{t-1}$	μ
L_t	**0.83**	**0.06**	0.02	**-0.08**	**-0.04**	0.05	**8.71**
	0.03	*0.02*	*0.01*	*0.01*	*0.01*	*0.09*	*0.17*
S_t	**0.13**	**0.84**	**0.06**	**0.11**	**0.11**	0.14	-0.51
	0.05	*0.02*	*0.02*	*0.02*	*0.01*	*0.12*	*0.95*
C_t	-0.18	**-0.07**	**0.91**	0.07	0.02	**-0.58**	**-2.33**
	0.11	*0.03*	*0.04*	*0.04*	*0.03*	*0.29*	*0.86*
CU_t	-0.02	0.01	**0.05**	**0.93**	**-0.07**	0.23	**82.71**
	0.05	*0.03*	*0.02*	*0.02*	*0.02*	*0.14*	*1.19*
$INFL_t$	-0.04	**-0.08**	0.02	**0.08**	**1.03**	**0.47**	**6.44**
	0.05	*0.02*	*0.02*	*0.02*	*0.02*	*0.12*	*0.96*
$Depo_t$	**-0.08**	**-0.10**	**0.06**	**0.04**	**0.05**	0.14	-0.02
	0.03	*0.01*	*0.01*	*0.01*	*0.01*	*0.07*	*0.04*

Looking at more traditional "macro-to-yields" relationships, the results indicate that an increase in the level of long-term yields (a government bond sell-off) is precipitated by decreases in capacity utilization and the inflation rate. These macro conditions create an atmosphere of a weakened economy that is well below its maximum productive capacity. While we have seen that a steeper slope parameter precipitates rising inflation in South Africa, the reverse cannot be said – and rising inflation in fact precipitates a flatter yield curve slope. Again, the reasons here may be two-fold. What this might highlight is that the yield curve and market players' expectations (as exhibited in their pricing of the yield curve) are **forward-looking** and don't necessarily tie up to current or lagged macro-conditions. Alternatively, the flatter slope may be the result of a sharp pricing in of a rate hike as evidenced by a rising 3-month JIBAR point (thus causing the overall curve to be flatter given that the short-end is the first to respond to the expected hike).

In a similar vein, a rising curvature parameter is precipitated by **lagged decreases** in the deposit rate, which might have then given rise to inflationary fears priced in for the intermediate future (i.e. in roughly 1.5-years' time, as per the optimal λ-value). However**, increases** in the deposit rate tend to occur after the market has priced in an increase in the concavity of the yield curve one month earlier. This indicates that the market players in the government bond arena and the SARB are both watching the same key economic indicators and that increases in inflation and capacity utilization and a subsequent pricing in of a more concave yield curve shape are naturally followed by the SARB's response of raising the deposit rate one month later. The estimation of such bidirectional relationships in Diebold et al.'s (2006) model assists in mapping out these complex feedback relationships between the yield curve and the macroeconomy.

White Noise Error terms and Measurement Disturbances

As per the yields-only model, the estimated Q matrix of white noise error terms η_t and the off-diagonal covariances are significant and the Wald and Likelihood Ratio tests of diagonality indicate that market shocks tend to be correlated in terms of their effect on the f_t state vector of Nelson Siegel yield curve parameters and macroeconomic variables (see Table 40).

Furthermore, as per Table 41, the yields-macro model exhibits similar means and standard deviations for the ϵ_t measurement disturbance terms and a similarly satisfactory model fit. The optimal estimates of the yields are quite stable across both models, which indicates (as per the findings of Diebold et al. (2006) and

Krishnan et al. (2007)) that the forecast accuracy is not necessarily improved by the addition of macro-factors.

Table 40: Q white noise matrix Results (Yields-Macro Model) and Tests for Diagonality

Estimated Q matrix

	L_t	S_t	C_t	CU_t	$INFL_t$	$Depo_t$
L_t	**0.13**	**-0.16**	**0.10**	**0.04**	**0.05**	**0.0**
	0.01	*0.01*	*0.02*	*0.01*	*0.01*	*0.0*
S_t		**0.25**	**-0.10**	-0.02	**-0.04**	**0.06**
		0.01	*0.03*	*0.01*	*0.01*	*0.00*
C_t			**1.10**	**-0.12**	**0.09**	**0.11**
			0.13	*0.04*	*0.04*	*0.02*
CU_t				**0.23**	-0.01	0.02
				0.03	*0.02*	*0.01*
$INFL_t$					**0.19**	**0.02**
					0.02	*0.01*
$Depo_t$						**0.06**
						0.00

Tests for diagonality Q matrix

	Test Statistic	P-value
Likelihood ratio	**710.50**	0.00
Wald	**2086.22**	0.00

ϵ_t

Table 41: Summary Statistics for yield measurement disturbances , Yields-Macro Model

Yield Maturity	Mean	Standard Deviation	t-statistic	P-value
3m	**-24.46**	**18.13**	**-1.35**	**0.18**
6m	-2.07	0.12	-16.83	0.00
1y	-2.13	0.12	-17.34	0.00
2y	-2.27	0.12	-18.18	0.00
3y	-3.35	0.13	-26.38	0.00
4y	-5.15	0.14	-38.11	0.00
5y	-7.95	0.34	-23.28	0.00
6y	-7.37	0.14	-52.15	0.00
7y	-7.87	0.13	-61.14	0.00
8y	**-28.32**	**41.99**	**-0.67**	**0.50**
9y	-7.10	0.12	-56.89	0.00
10y	-5.60	0.12	-45.28	0.00

5.4 Part 4. Linking the SA Asset Classes: An investigation of Equity sectors and their relationship with the Currency and Yield Curve Parameters

5.4.1 Links between the Currency and Government Yield Curve Parameters

Due to the issue of multicollinearity between the Rand currency term and SA Government Bond yield curve parameters, the following Monthly regression in Table 42 was performed of the USDZAR exchange rate against the yield curve parameters. This was done in order to establish the association between these variables and to extract movements in USDZAR not explained by the Government Bond yield curve (as represented by the residual terms).

Table 42: OLS Regression of USDZAR vs 10-year NS Parameters

Dependent Variable: USDZAR
Method: Least Squares
Sample: 2003M06 2014M02

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.185	0.168	1.097	0.275
10-year Level	3.781	0.643	5.880	0.000
10-year Slope	1.223	0.418	2.924	0.004
10-year Curvature	0.292	0.132	2.206	0.029
R-squared	0.238	Adjusted R-squared		0.220

Results indicate that all Nelson Siegel yield curve parameters can significantly explain fluctuations in the Rand, with increases in the level of the yield, a flattening of the slope, and a more concave curvature component associated with sell-offs in the currency. This indicates that Government bond yield levels typically sell-off in tandem with the currency. This may in part be the result of risk-off sentiment towards South African assets, but also as a result of the fact that **foreign selling** of local-currency bonds would require proceeds from the sale to be converted to foreign currency – thus meaning that Rands would need to be sold.

In terms of the slope, a flatter slope may be associated with lower long-end inflationary expectations, which is somewhat counterintuitive as lower inflation should lead to less erosion of the Rand value and a subsequently stronger currency. However, as seen in earlier analysis, a flatter 10-year slope parameter is often the case due to increases in 3-month JIBAR and short-end rates. Thus, expected short-term inflationary expectations would be concomitant with rand weakness. As per the curvature parameter, a more concave or "humped" shape points towards rising short-term inflation (generally out to 2.5-years as per the chosen value of maturity τ), which in turn is associated with rand weakness. These parameters explain about 20% of currency fluctuations.

5.4.2 Links between Equity Indices and Government Yield Curve Parameters

Next, the Total Returns for the following Johannesburg Stock Exchange (JSE) Derivative indices are regressed against the All Share Index total returns, adjusted monthly compounded USDZAR returns, and changes in the Level, Slope, and Curvature of the 10-year Government Bond Yield curve.

- FINI15 (Financials Index)
- FINDI30 (Financials and Industrials Index)
- INDI25 (Industrials Index)
- RESI10 (Resources Index)

The results in Table 43 indicate that the All Share Index total returns are a significant predictor of total returns for all of the derivative indices, with the Resources Index exhibiting the highest beta-sensitivity to this variable such that for a 1% increase in the monthly ALSI Return, the Resources Index experiences a 1.46% increase in total returns. This is unsurprising as Resources companys (minings, metals, and energy) dominate the JSE in terms of market capitalization.

Table 43: Regression Results of JSE Indices vs ALSI, USDZAR, 10-year NS Parameters – July 2003 to February 2014 (128 Months)

Dependent Variable	β ALSI	β ZAR	β LEVEL	β SLOPE	β CURVATURE	R-squared
JSE FINI	0.673	-0.136	-2.042	0.061	-0.395	66%
JSE FINDI	0.730	-0.078	-0.989	0.304	-0.110	77%
JSE INDI	0.737	-0.062	-0.741	0.285	-0.004	74%
JSE RESI	1.461	0.155	1.840	-0.080	0.175	84%

After separating out variation in the local currency not explained by the Government yield curve factors, only the Financials and Resources Index are significantly predicted by fluctuations in the Rand. As per Barr and Kantor (2005) and Barr, Kantor, and Holdsworth (2007), these indices fit the classification system of *Rand Play* and *Rand Leverage* shares. Rand/ Dollar decreases in monthly returns (i.e. a higher USDZAR rate) cause the share returns of Rand Play companies in the Financial Index to decrease in tandem (with a sensitivity factor of -0.136). This is unsurprising as these companies are almost entirely SA-based and almost all costs and revenues are incurred locally in Rands. For the Rand Leverage Resources index, companies offer protection from a weak rand via an average share return increase of 0.15 per unit decrease in the Rand/Dollar monthly return. This is a result of the fact that these company's shares benefit from generating income abroad and are affected positively by weakness in the rand because they profit from the dollar price of what they sell.

In terms of the 10-year Level and Curvature Parameters, the FINI15, FINDI30 and INDI25 all exhibit the same trends. Decreases in 10-year Government Bond Levels and a more convex curvature parameter all lead to increases in the monthly total returns of the indices. Thus, these equity sectors are positively correlated with local Government Bond prices such that gains in debt prices (lower yields) are associated with increased share returns. Furthermore, a more convex curve shape (indicating low near-term inflationary expectations and a stronger rand) is seen as positive for South African corporates in the Financials and Industrials space.

However, not all of these relationships are significant. The Financials Index is the most highly sensitive to the Level of the curve (with a coefficient of -2.04), with subsequent increases in Government Bond yields affecting the rate at which banks can finance themselves as the majority of fixed rate bank-debt issued is marked at a spread over the Government bond curve. While this pricing methodology is true of all SA corporates, bank financing levels are especially important to their revenues so as to maximize the spread between that of deposit rates paid out to customers and in-house financing. The Financials Index is also the only of these 3 indices to react significantly to the curvature factor ($p<0.001$) and to exhibit sensitivity to short-term inflation expectations. Contrastingly, the FINDI and INDI indices have statistically significant relationships only with the Level Parameter.

Unsurprisingly, the Resources Index offers a hedge against both Government bond and rand weakness, with a 100bp (1-unit) increase in the 10-year Level parameter associated with a 1.84 unit increase in total share returns on average. This is undoubtedly due to the associated weakness in the rand, lower local factory and production costs incurred, as well as relative increases in dollar-revenue. Interestingly, this index is also sensitive with respect to the Curvature Parameter ($p = 0.06$), with a more concave yield curve shape and increases in near-term inflationary expectations associated with increases in total share returns (undoubtedly due to the associated rand weakness).

None of the JSE Derivative Indices is significantly related to the 10-year slope.

Differences in 10 versus 30-year Yield Curve Relationships

The analysis is repeated for the 10 to 30-year matrix of maturities so as to assess if the longer-end of the curve's relationship with the currency and equity sectors exhibits different trends. As per Table 44, the relationships between the currency and 30-year yield curve factors hold, and the model has similar explanatory power (roughly 20%). Notably, the coefficients are slightly more significant. The coefficients for the Slope and Curvature parameters are slightly larger (1.37 and 0.75 versus 1.22 and 0.29), suggesting that the Rand may be more sensitive to long-term inflationary expectations than shorter-term.

Table 44: OLS Regression of USDZAR vs 30-year NS Parameters

Dependent Variable: USDZAR
Method: Least Squares
Sample: 2003M06 2014M02

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.116	0.169	0.686	0.494
30-year Level	2.793	0.488	5.720	0.000
30-year Slope	1.372	0.431	3.181	0.002
30-year Curvature	0.752	0.183	4.101	0.000
R-squared	0.215	Adjusted R-squared		0.196

The relationships between the JSE Derivative Indices and the 30-year yield curve parameters show that these factors are in many cases better able to explain equity fluctuations (see Table 45). In fact, the significance and size of the coefficients for the USDZAR term have now been reduced as a result of the improved explanatory power of the yield curve parameters. The direction of the relationships for the Level and Curvature parameters remains the same such that a sell-off in yields and a more concave shape to the yield curve (higher inflation hump out at the 15-year point) is associated with a reduction in total returns for the FINI, FINDI, and INDI (and an increase in total returns for the RESI).

However, while previously in the 10-year analysis the slope was not significant for any of the indices, the 10-30 year slope is now significant for the FINI, FINDI, and RESI. A steepening of the 10-30-year curve increases the total returns of the FINI and FINDI indices, and reduces the total returns of the RESI Index. While we found in the 10-year analysis that the slope was well predicted by 3-month JIBAR, market shock factors, and global sentiment towards the EM's, the 10-30-year slope does not seem to capture global risk sentiment. Rather, it is weakly explained by inflation and, according to literature, is perhaps better explained by long-term growth prospects for the SA economy. Estrella and Hardouvelis (1991) find that a 100bp increase in the long-term slope of the US Government bond curve is associated with a real GNP growth increase of 3% one year later. The results below indicate that a steeper 10-30-year slope and greater expected long-term economic growth is associated with positive total returns for the FINI and FINDI, and negative returns for the RESI.

Table 45: Regression Results of JSE Indices vs ALSI, USDZAR, 30-year NS Parameters – July 2003 to February 2014 (128 Months)

Dependent Variable	β ALSI	β ZAR	β LEVEL	β SLOPE	β CURVATURE	R-squared
JSE FINI	0.677	-0.099	-2.546	-1.419	-0.644	66%
JSE FINDI	0.755	-0.052	-1.409	-0.685	-0.418	77%
JSE INDI	0.772	-0.043	-1.039	-0.388	-0.380	74%
JSE RESI	1.425	0.124	2.141	1.006	0.661	84%

5.4.3 Links between Individual JSE Top40 Shares and Government Yield Curve Parameters

Analysis of the individual JSE Top 40 shares in Table 46 indicates that apart from the All Share Index, the 10-year Level Parameter is most able to predict changes in share returns – with 20 of the shares in the Top 40 arena displaying statistical significance ($p<0.05$) with regards to the Level Beta coefficient.

The **USDZAR coefficient**, on the other hand, is only a significant explanatory variable for 9 of the shares on the Top 40 (see Figure 63). The majority of these shares fall under the Resources counter and react positively to weakness in the rand due to their Rand Leverage classification (Anglogold Ashanti, Goldfields, and Sasol). The share with the largest significant Beta coefficient is British American Tobacco (β ZAR = 0.56), whose headquarters are in London and whose financial reporting currency is the GBP. Contrastingly, of the Rand Play companies, it is mainly the Financials / Banking sector that dominates the analysis in terms of those shares with significant Rand-coefficients. That said, Shoprite is most significantly sensitive to rand weakness over the sample period (β ZAR = -0.32).

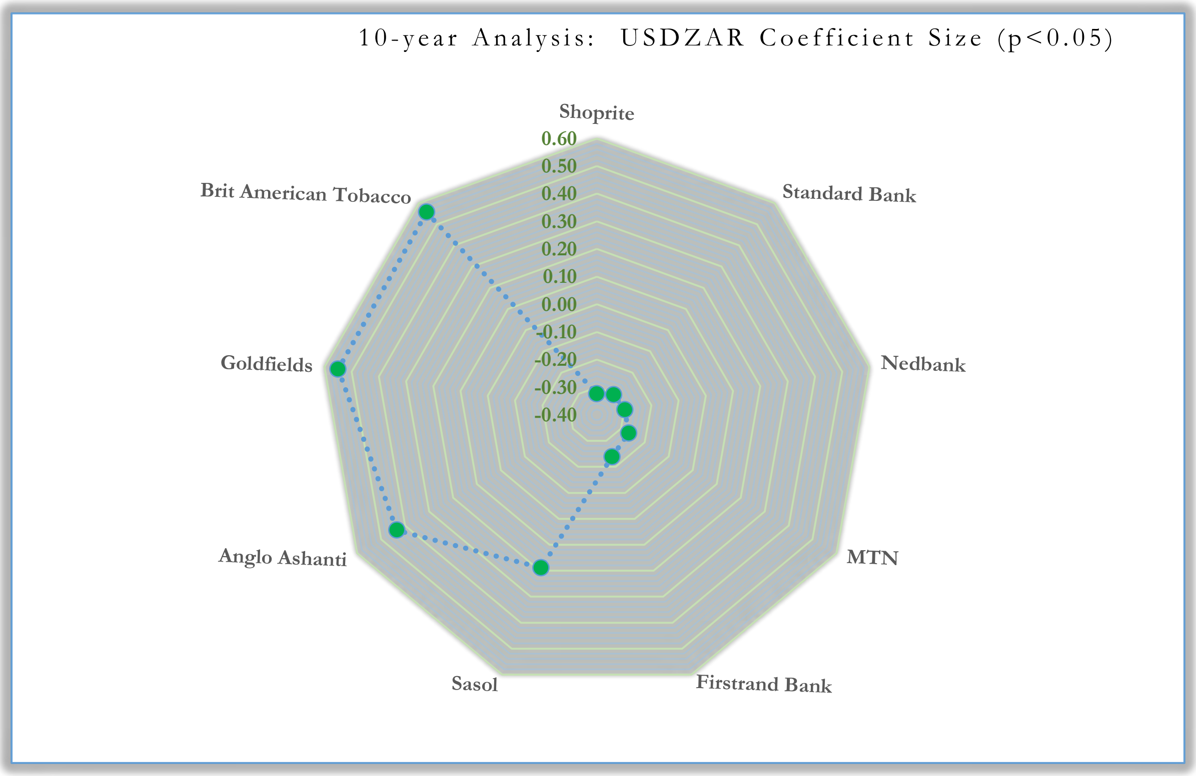


Figure 63: USDZAR significant Coefficients for individual shares

Table 46: OLS Regression Results for Individual Top40 Shares, 10-year NS Parameters[3]

10-yr Curve Analysis	β ALSI	β ZAR	β LEVEL	β SLOPE	β CURVATURE	R-squared
Food						
Shoprite	0.257	-0.324	-1.859	-0.298	-0.342	15.1%
Tiger Brands	0.409	0.131	-0.742	0.463	0.096	21.0%
Retail						
Truworths	0.407	-0.225	-4.375	-0.859	-0.715	33.4%
Massmart	0.359	-0.217	-4.046	-0.991	-0.472	24.7%
Woolworths	0.735	-0.129	-2.893	-0.523	-1.042	41.0%
Consumerables						
SAB Miller	0.726	0.070	0.907	0.507	0.100	35.6%
Steinhoff	0.872	-0.054	-0.985	0.423	0.448	34.9%
Naspers	0.938	-0.155	-1.690	-0.309	-0.045	43.6%
Brit American Tobacco	0.449	0.557	2.442	1.570	0.063	27.2%
Richemont	1.091	0.368	2.038	0.902	0.868	36.3%
Resources						
Sasol	1.173	0.189	2.091	0.373	0.351	60.2%
BHP Billiton	1.507	0.147	1.632	0.208	0.164	68.6%
Anglo Platinum	1.528	-0.068	0.329	-1.172	0.233	44.1%
Anglo American	1.732	0.044	3.545	1.124	0.411	69.6%
African Rainbow	1.622	-0.121	2.278	0.746	0.169	46.9%
Assore	1.042	-0.144	1.960	2.586	0.684	31.7%
Exxaro	1.167	-0.369	2.077	0.530	0.265	40.8%
Goldfields	0.799	0.550	-0.561	-1.855	-0.315	16.7%
Mondi	1.347	-0.343	-1.447	-2.016	-0.157	44.7%
Anglo Ashanti	0.840	0.434	-0.049	-1.566	0.126	17.3%
Impala Platinum	1.384	-0.240	-2.131	-2.203	-0.252	47.8%
Kumba Iron Ore	1.249	-0.247	-0.641	-1.231	-0.045	42.0%
Medical care						
Aspen	0.457	-0.181	-3.107	-1.536	-0.502	16.7%
Medicare	0.458	0.156	-1.196	-0.814	-0.285	18.0%
Life Insurance						
Old Mutual	1.002	-0.164	-0.598	0.498	-0.065	44.0%
Sanlam	0.560	-0.179	-2.851	-0.591	-0.409	43.0%
Discovery	0.384	0.062	-2.855	-0.910	-0.059	21.2%
Property						
Intu	0.555	0.174	1.307	1.675	0.426	22.0%
Growthpoint	0.290	-0.036	-4.174	-0.806	-0.922	45.9%
Industrials						
Imperial	0.798	-0.220	-3.784	-1.636	-0.365	34.5%
Bidvest	0.679	-0.015	-1.196	0.444	-0.256	39.6%
Remgro	0.612	-0.021	-1.611	-0.276	-0.379	54.3%
TeleComs						
MTN	0.674	-0.267	-2.745	0.018	0.016	44.0%
Vodacom	0.702	-0.132	-1.735	-1.794	-0.332	24.8%
Financials						
Investec Ltd	0.915	-0.006	-1.989	-0.248	-0.189	39.6%
Standard Bank	0.620	-0.305	-2.658	-0.260	-0.527	45.2%
Firstrand Bank	0.757	-0.239	-2.912	-0.078	-0.869	54.5%
Absa	0.526	-0.179	-3.606	-0.886	-0.620	41.1%
Investec PLC	0.946	-0.022	-1.891	-0.471	-0.166	39.3%
Nedbank	0.440	-0.297	-1.916	0.929	-0.286	36.6%

[3] Significant coefficients are found in red font

In terms of sensitivity to the **Level Parameter**, it is the Rand Play Retailers and Financials who display the most significant sensitivity to this factor (see Figure 64). Retailers like Truworths and Massmart are the most sensitive to the level of 10-year yields (β Level = -4.4 and -4.0), as well as Growthpoint Properties (β Level = -4.2). This means that for a 1% increase in yield levels, share returns will decrease by 4% on average. The profitability of the Retail sector is related to the level of yields and interest rates through the vehicle of consumer spending, with rising interest rates pointing to a weaker consumer. The Level of yields and interest rates is thus found to be significant for the 3 major clothing retailers (Truworths, Massmart, and Woolworths) but not for the Food sector (Shoprite and Tiger Brands). This is due to the fact that their products fall into the "basic necessities" category, and are thus less demand elastic to price changes that occur as a result of rising interest rates and inflation, as well as deteriorating consumer wealth.

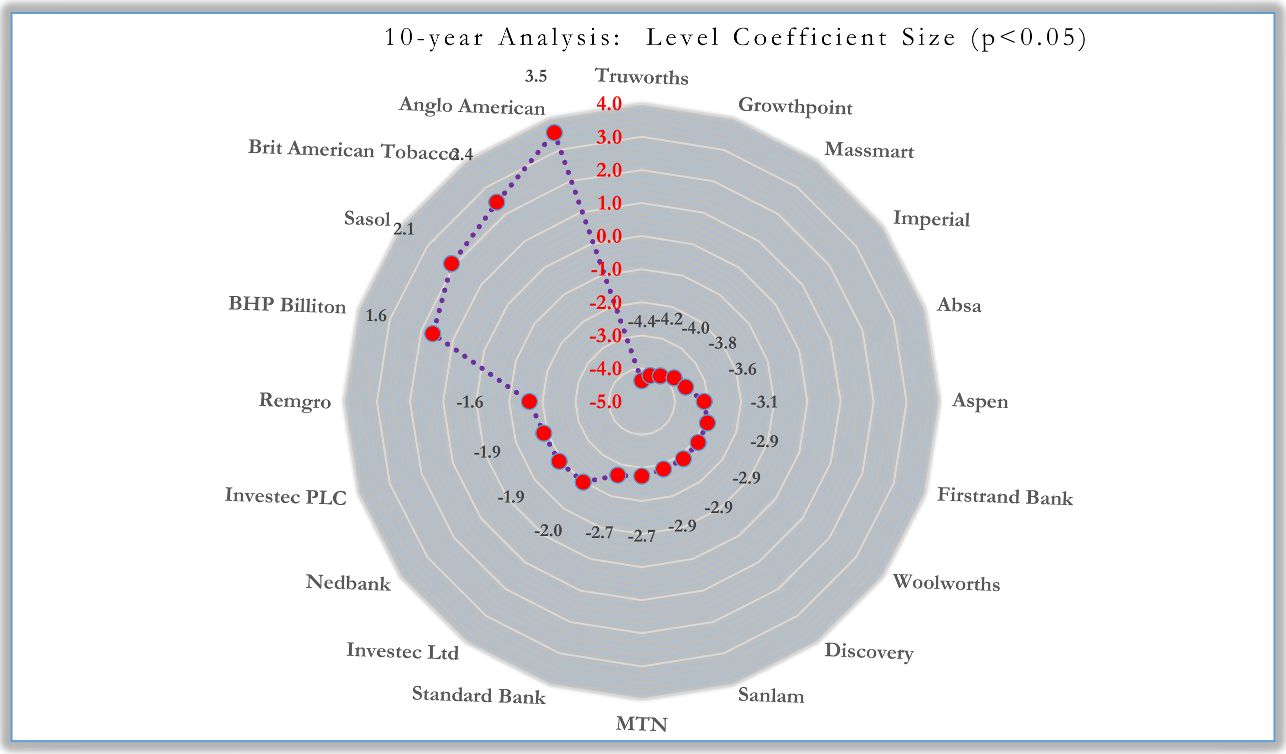


Figure 64: 10-year Level Parameter significant Coefficients for individual shares

For the banking sector, too, rising yields affect their ability to fund themselves at levels lower than those offered on consumer deposits. Such findings as to this arena's strong sensitivity to long-term bond yields and interest rates is confirmed in such papers as that of Beirne, Caporale, and Spagnolo (2009), who investigate the European Financial Sector Markets. Of the Financials, Absa and Firstrand have the most sensitive Level-coefficients (-3.6% and -2.9%), and the total share returns of these banks is also very well explained by the fitted model at an R-squared of 41% and 55%.

Of the major Rand Leverage Resources companies, Anglo American, Sasol and BHP Billiton have very large and significant level coefficients (3.5, 2.1, and 1.6) and exhibit the highest degree of model fit, with R-squared values of 60 – 70%. For these mining, metals and energy companies, an increase in bond yields and SA interest rates, and subsequent weakening of the rand, is highly positive for share returns.

Looking at the 10-year **Slope Parameter**, the earlier analysis performed on the Derivative Indices revealed that none of the slope coefficients were statistically significant. Here too, results for the 10-year slope parameter are very mixed (see Figure 65).

Of the entire Top 40 share universe, only 6 shares are significantly sensitive to the slope of the 10-year Government Bond curve. 3 of these shares belong to the Resources sector, with Assore and Anglo American's share returns increasing by 2.6% and 1.1% on average for a 100bp flattening of the 10-year yield curve, and Impala Platinum's total share returns decreasing by 2.2% on average for the same move.

As seen in the earlier OLS regression analysis in Part 2 regarding the predictors of the yield curve, a flatter 10-year slope is associated with a higher 3-month JIBAR rate, which may be indicative of short-term rises in interest rates and weakness in the rand. This should be positive for the Rand leverage Resources counters. The 10-year slope is also found to be associated with global risk sentiment towards EM's, with a flatter slope indicating a risk-positive attitude. This might encourage investment in such JSE sectors as the resources arena, with poor news out of the mining sector (strikes, riots, and above-inflation wage increase demands) often associated with a steeper SA yield curve slope. That said, Impala Platinum operates in a different manner with regards to the slope and appears to exhibit increased share returns for a steeper slope, as per Imperial Holdings – which is classified as a diversified Industrial Rand Play share (Barr, Kantor & Holdsworth, 2006). As such, Imperial's share returns increase when bond yields are lower and the 10-year slope is steeper (possibly due to a lower JIBAR rate and inflationary expectations).

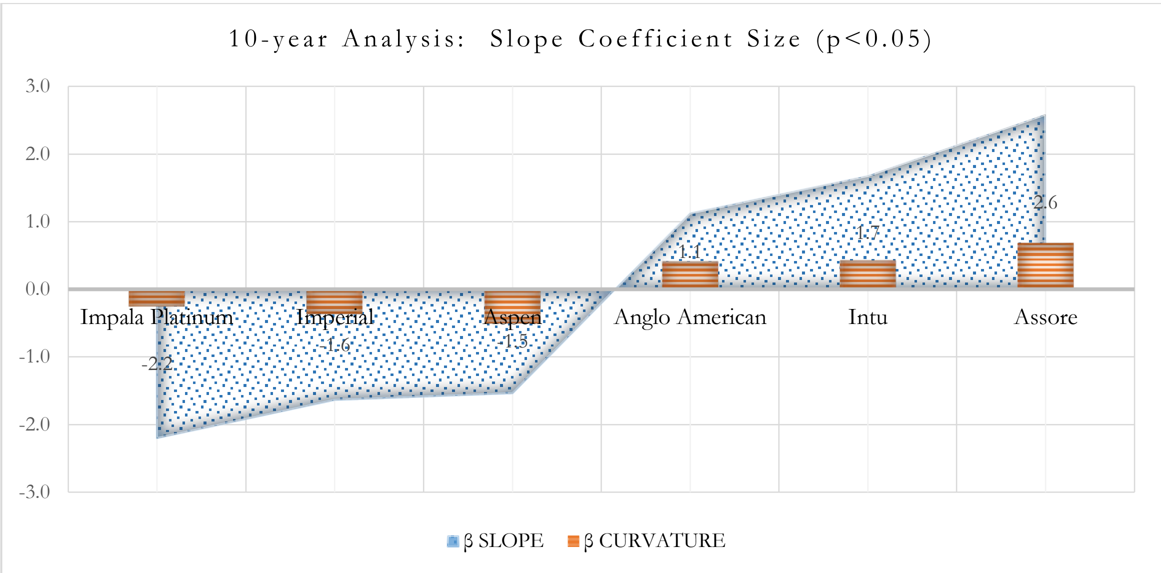


Figure 65: 10-year Slope Parameter significant Coefficients for individual shares (with curvature coefficient)

The results for the **Curvature coefficient** in Figure 66 are far more consistent, with 16 of the share returns on Top 40 companies being significantly explained by this variable. The General Retailers (Woolworths, Truworths, Massmart), Financials (Absa, Standard Bank, Firstrand) and assorted other Rand Play shares (e.g. Growthpoint Properties) all experience increases in total share returns for movements in the curve shape towards greater convexity – which points to lower near-term inflation and a strong Rand.

Of the Rand Leverage / Hedge shares, Richemont has the highest curvature coefficient, with a 1% increase in curvature around the 2 – 3-year point on the Government Bond yield curve associated with a 0.87% increase in share returns on average.

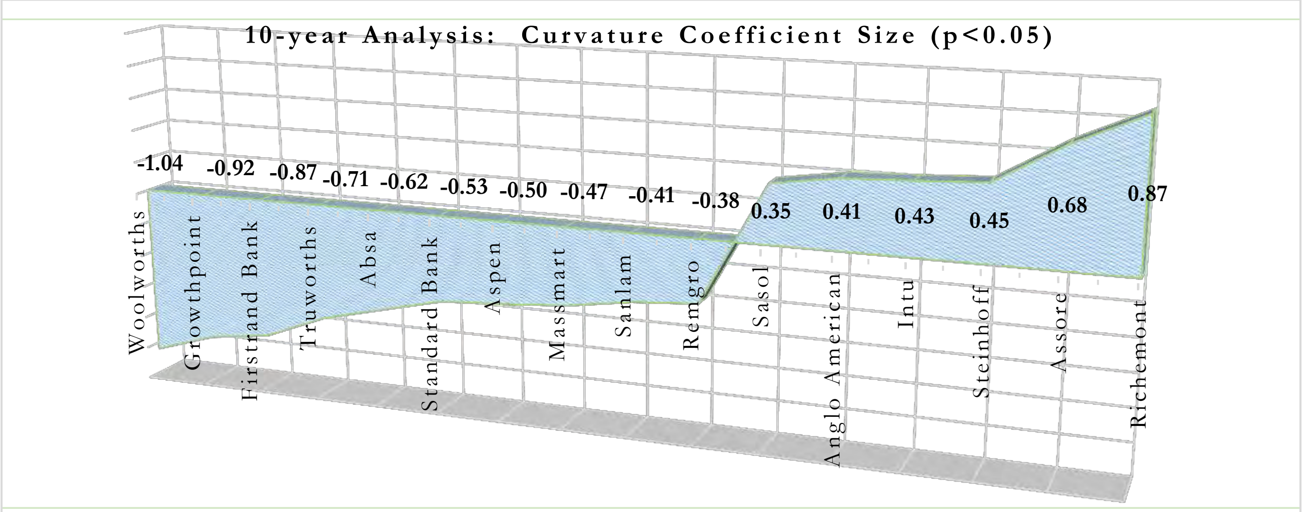


Figure 66: 10-year Curvature Parameter significant Coefficients for individual shares

Interesting to note is that the only share that exhibits significance with respect to all 3 10-year yield curve parameters and has a fitted model with high explanatory power (R-squared = 70%) is that of Anglo American. The addition of the yield curve parameters into the model actually nullifies the significance of the USDZAR coefficient term – meaning that fluctuations in the rand that are not explained by changes in the Government yield curve parameters do not significantly explain changes in Anglo's share returns.

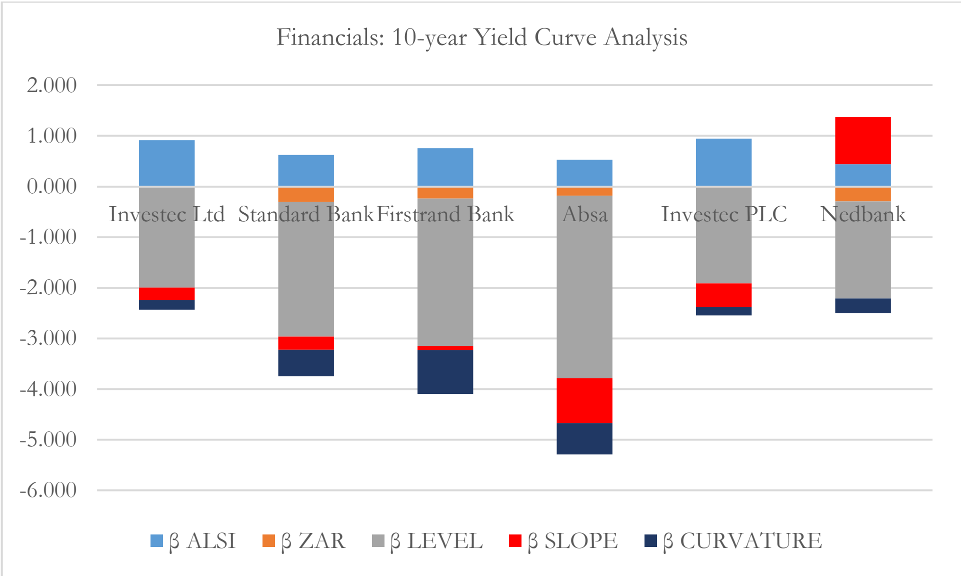
Differences in 10 versus 30-year Yield Curve Relationships

The individual Top 40 share analysis is repeated for the 30-year Level, Slope, and Curvature Parameters, with results indicating an increase in the number of shares with significant Level coefficients (from 20 to 25), Slope coefficients (6 to 15), and Curvature coefficients (16 to 18) (see Table 47).

Table 47: OLS Regression Results for Individual Top40 Shares, 30-year NS Parameters

30-yr Curve Analysis	β ALSI	β ZAR	β LEVEL	β SLOPE	β CURVATURE	R-squared
Food						
Shoprite	0.252	-0.294	-2.038	-0.277	-0.492	16.3%
Tiger Brands	0.492	0.154	-1.049	-0.630	-0.561	19.6%
Retail						
Truworths	0.423	-0.174	-4.496	-1.505	-1.201	35.9%
Massmart	0.394	-0.204	-3.559	-1.378	-1.062	25.2%
Woolworths	0.631	-0.111	-3.657	-3.356	-1.146	39.8%
Consumerables						
SAB Miller	0.726	0.071	0.527	0.008	0.065	35.8%
Steinhoff	1.007	-0.038	-0.886	0.585	-0.557	34.8%
Naspers	0.979	-0.159	-1.493	-0.632	-0.765	46.3%
Brit American Tobacco	0.497	0.636	1.168	-0.561	-0.102	29.1%
Richemont	1.266	0.398	1.570	1.239	-0.380	37.7%
Resources						
Sasol	1.177	0.172	2.063	1.151	0.479	60.5%
BHP Billiton	1.485	0.138	1.624	0.647	0.593	69.0%
Anglo Platinum	1.506	-0.150	1.942	0.437	0.410	43.9%
Anglo American	1.711	0.056	2.826	1.916	0.967	71.1%
African Rainbow	1.583	-0.126	1.747	0.976	0.503	47.0%
Assore	1.168	-0.063	0.199	1.013	0.069	26.3%
Exxaro	1.126	-0.368	2.115	-0.399	0.151	44.0%
Goldfields	0.734	0.461	1.087	0.253	0.312	12.0%
Mondi	0.951	-0.455	2.499	-1.466	0.296	52.7%
Anglo Ashanti	0.842	0.353	1.862	1.683	0.589	15.0%
Impala Platinum	1.330	-0.334	0.113	-0.481	0.044	45.6%
Kumba Iron Ore	1.183	-0.308	0.805	-0.669	0.139	42.5%
Medical care						
Aspen	0.431	-0.195	-2.165	-1.137	-0.567	15.0%
Medicare	0.434	0.145	-0.854	-0.818	-0.353	17.0%
Life Insurance						
Old Mutual	1.016	-0.153	-0.930	-0.742	-0.202	43.3%
Sanlam	0.570	-0.157	-2.707	-1.349	-0.677	44.0%
Discovery	0.463	0.038	-1.793	-0.203	-0.661	20.0%
Property						
Intu	0.645	0.218	0.205	0.583	0.041	17.4%
Growthpoint	0.261	0.007	-4.548	-2.874	-1.225	47.4%
Industrials						
Imperial	0.819	-0.217	-2.664	-1.016	-0.696	34.3%
Bidvest	0.695	0.047	-2.052	-1.057	-0.380	42.3%
Remgro	0.588	-0.013	-1.618	-0.982	-0.266	55.0%
TeleComs						
MTN	0.773	-0.249	-2.507	-0.600	-0.777	42.5%
Vodacom	0.657	-0.217	0.260	-0.397	0.320	25.3%
Financials						
Investec Ltd	0.946	0.005	-1.896	-1.332	-0.582	40.0%
Standard Bank	0.608	-0.262	-3.092	-1.617	-0.798	45.3%
Firstrand Bank	0.705	-0.172	-3.949	-2.177	-0.875	53.7%
Absa	0.527	-0.144	-3.559	-1.355	-0.929	42.7%
Investec PLC	0.968	-0.020	-1.544	-1.228	-0.450	40.0%
Nedbank	0.480	-0.226	-3.199	-1.484	-0.921	33.2%

A closer look at the Financials shares in Figure 67 reveals that both the size and significance of the slope coefficients has markedly increased now that the 10-30-year slope is utilized in the analysis. The slope coefficients for this sector are now all significant, whereas previously this was not the case for even one bank. For Nedbank, too, the slope coefficient is now negative as per the other banks (previously it was positive for the 10-year slope) – indicating that a steepening of the 10-30-year curve increases total share returns on average. While the 10-30-year curve was somewhat difficult to predict, literature asserts that a steepening is indicative of rising long-term growth prospects (Kempf, Korn & Homburg, 2012).



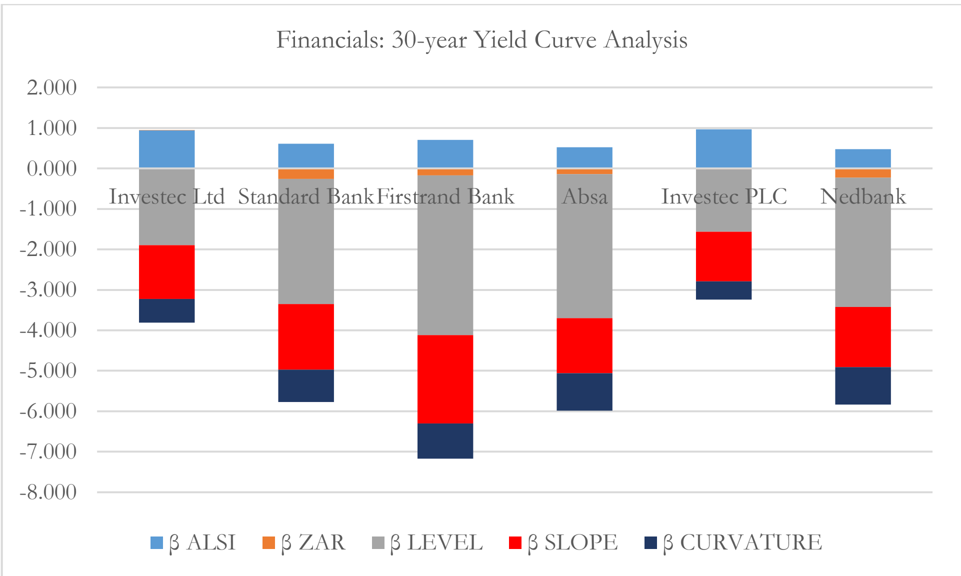
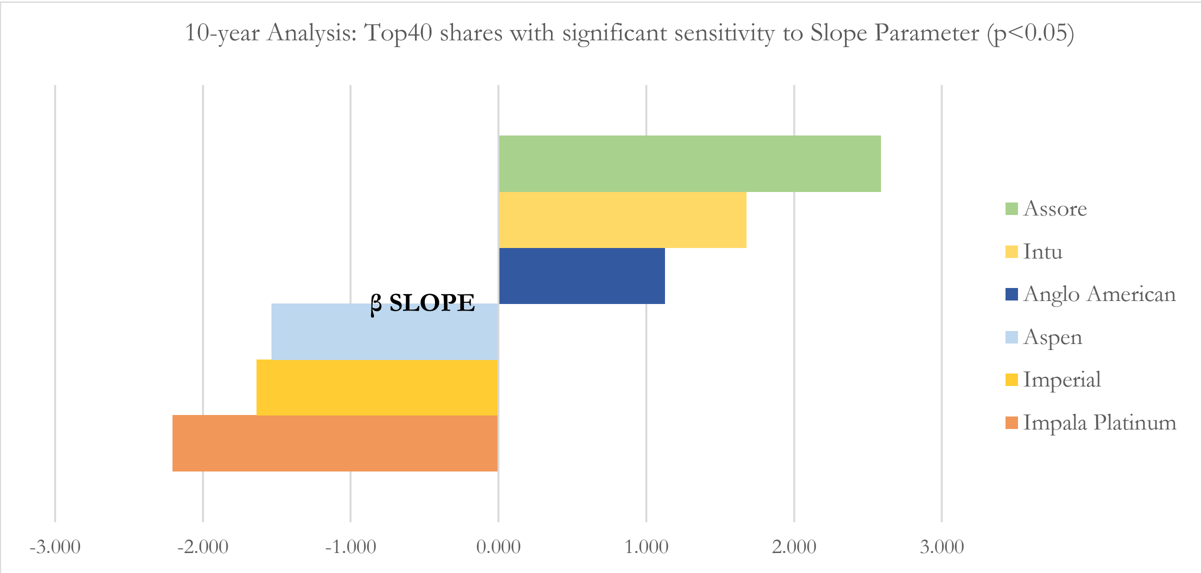


Figure 67: 10 vs 30-year Model Results for the Financials sector (Coefficient Size)

As mentioned, the number of shares in the Top 40 with significant slope coefficients has increased from 6 to 15. Furthermore, as per Figure 68, far greater consistency is seen in the direction of relationships. Of the Rand Leverage Resources shares, only Anglo American and Sasol have positive (significant) slope coefficients. The other 13 shares consist of Rand Play companies (mainly Financials, Industrials, and General Retailers) and all exhibit negative slope coefficients. Thus, in the case of Woolworths (which has the largest slope coefficient) a 100bp steepening of the 10-30-year slope is associated with a 3.4% increase in share returns on average.



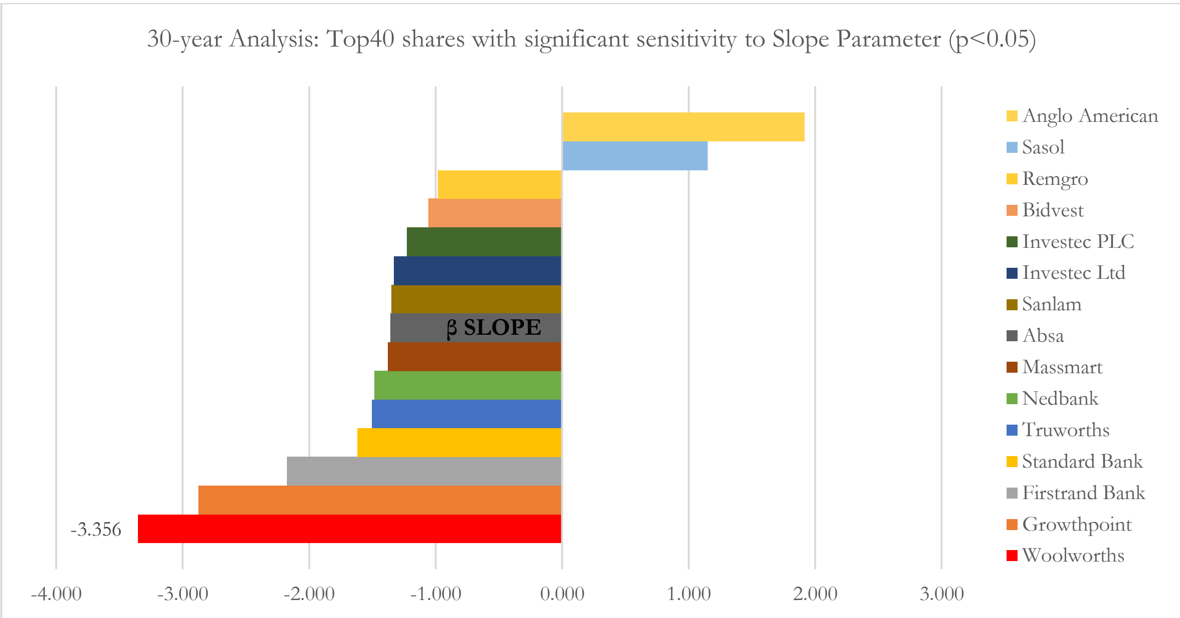


Figure 68: 10 vs 30-year Model Results for significant Slope Parameter Coefficients

5.4.4 10-year Beta Parameter Stability: Moving Estimation Window Forecasts

Results were also obtained by estimating the model over a 48-observation (4 year) moving window estimation period such that the parameter estimates obtained at each separate window could be used to forecast share returns for each of 12-months into the future.

An OLS regression was initially performed for each share over the period:

2003:06 to 2007:06

After this, the second estimation period would begin in 2004:06 (i.e. a lag of 12 months between each window) until the final regression period of 2009:06 to 2013:06. Forecast estimates were then obtained for each month over the next 1-year period following each regression window, meaning that the final forecast period was from July 2013 to July 2014.

After generating a sample of fitted share returns for each Top 40 company by forecasting after each of the window periods, these fitted values were brought together so as to assess the overall model fit for this rolling window regression. In Appendix A (Section 8.1), scatterplots of the fitted versus observed share returns over the entire forecast period are presented, an example being of Absa's share returns presented below.

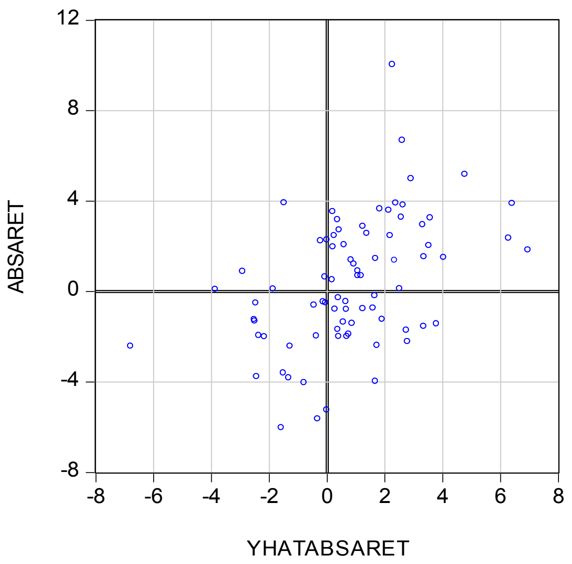


Figure 69: Actual vs Predicted Share Returns for Absa Bank Ltd

A measurement of the goodness of fit of the fitted share returns is the proportion of observed-fitted pairs falling within the 1st and 3rd quadrants. This gives an indication of the proportion of share returns whose 'sign' was correctly interpreted (i.e. whether positive or negative returns were yielded).

Table 48: Goodness of Fit Tests for 10-year Model

10-year analysis	Proportion Forecasts in 1st & 3rd Quadrant	Proportion Forecasts with residuals > (2*SE)
Food		
Shoprite	65%	97%
Tiger Brands	68%	98%
Retail		
Truworths	64%	98%
Massmart	63%	96%
Woolworths	77%	92%
Consumerables		
SAB Miller	71%	97%
Steinhoff	65%	95%
Naspers	78%	97%
Brit American Tobacco	77%	98%
Richemont	78%	100%
Resources		
Sasol	83%	95%
BHP Billiton	78%	95%
Anglo Platinum	72%	97%
Anglo American	75%	93%
African Rainbow	71%	93%
Assore	76%	95%
Exxaro	77%	99%
Goldfields	55%	96%
Mondi*	N/A	N/A
Anglo Ashanti	55%	97%
Impala Platinum	67%	96%
Kumba Iron Ore	60%	94%
Medical care		
Aspen	57%	95%
Medicare	66%	95%
Life Insurance		
Old Mutual	66%	92%
Sanlam	76%	96%
Discovery	63%	96%
Property		
Intu	58%	96%
Growthpoint	79%	96%
Industrials		
Imperial	65%	93%
Bidvest	79%	95%
Remgro	76%	98%
TeleComs		
MTN	76%	93%
Vodacom*	N/A	N/A
Financials		
Investec Ltd	74%	93%
Standard Bank	65%	96%
Firstrand Bank	75%	96%
Absa	68%	98%
Investec PLC	71%	92%
Nedbank	72%	93%

*Certain shares did not have a long enough history of data in order to fit moving-window estimation periods

This is a somewhat crude measure of goodness of fit, and one can determine from the plot of fitted versus observed Absa share returns that although certain returns in the 1st quadrant were correctly predicted by the fitted values as yielding positive returns, there is quite a large scale of difference between the magnitude of the predicted and observed returns. In order to assess this goodness-of-fit criterion, residuals were thus calculated and the standard error for each share determined. The proportion of residuals falling within 2 standard errors of the actual share return was then calculated as a second fit measure. These statistics are summarized in Table 48 for the case of the 48-window moving observation period.

Evaluation of Table 48 reveals that the proportion of fitted and observed share returns falling into the 1st quadrant or the 3rd quadrant is relatively high. Anglo Ashanti, Aspen, Goldfields, and Intu Properties have the lowest proportion (55 – 58%) falling into these quadrants.

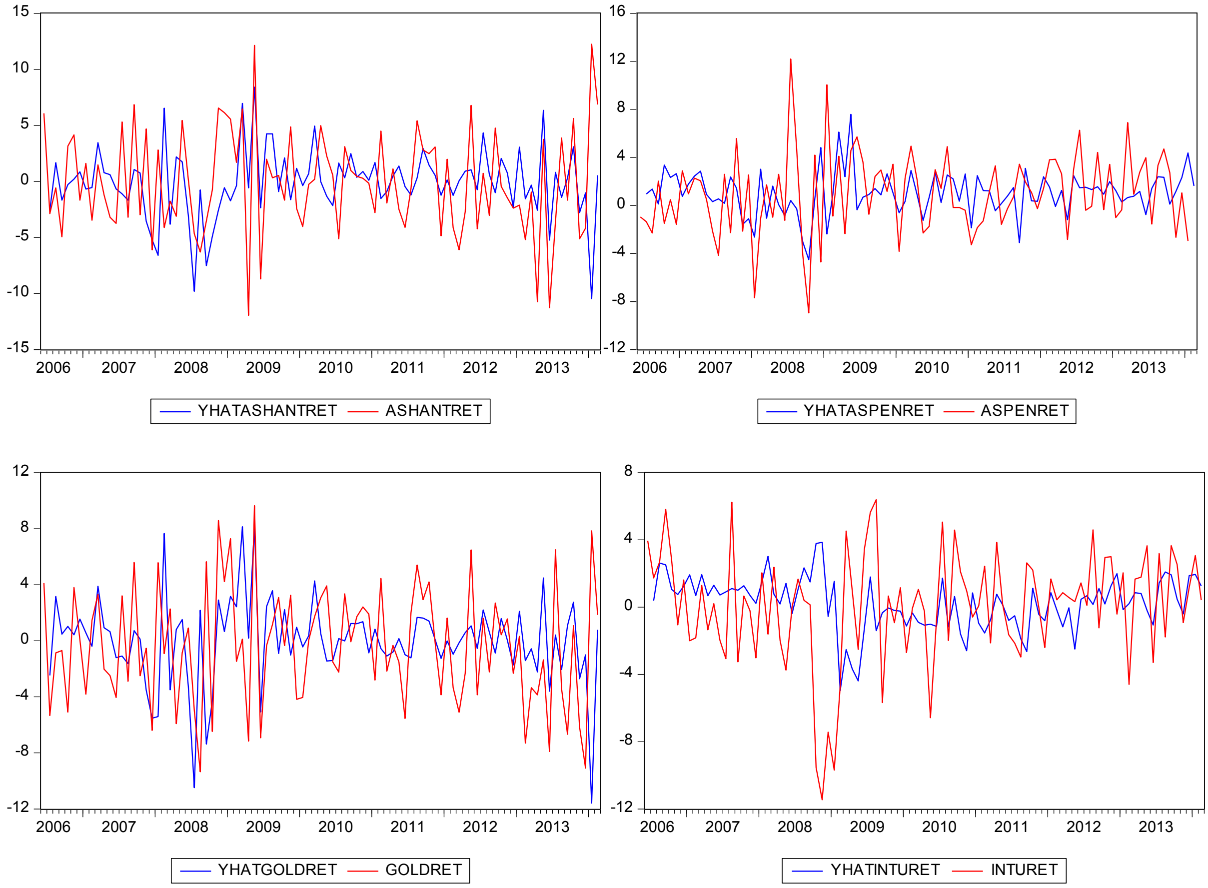


Figure 70: Predicted vs Actual Share Returns; Anglogold Ashanti, Aspen, Goldfields, Intu Properties

A closer look at these potentially problematic shares in Figure 70 reveals that these shares are reasonably well predicted by the fitted model values. The reasons for the relative poorness of the goodness of fit statistic (proportion of fitted-observed pairs falling into the 1st and 3rd quadrant) is shown in the line graph to be due to the inability to accurately predict the sudden stock market crash during the credit crisis period (late 2008/early 2009). Volatility in this period was rife and share returns are shown to sink lower than almost any other point in the sample period. It is thus unsurprising that prediction was so poor over this period due to the presence of market shock factors that are not well explained by fundamentals like the USDZAR or preceding levels of the ALSI and SA Government yields circa 2007.

In the case of Anglogold Ashanti, one also sees an inability of the fitted model to predict the sudden share price upturn in the last few months of the dataset. This rise was the result of in-company fundamental factors that the fitted model was unable to predict – namely, an increase in the mine's gold production and the efficiency of a cost-cutting program implemented by management.

As has been discussed, however, this goodness of fit estimate is somewhat crude and is best supplemented with other information. An evaluation of the proportion of residuals falling within a 2x standard error confidence interval indicates that on the whole, the fitted models for the Top40 satisfy this fit criterion extremely well, with proportions vacillating within the 90% + range.

5.4.5 30-year Beta Parameter Stability: Moving Estimation Window Forecasts

While the significance of certain explanatory coefficients (mainly the Slope parameter) improved when the 30-year Nelson Siegel Model was used, the degree of model fit using a 48-month rolling-window estimation period is very similar, with no notable differences (see Table 49).

In Appendix A (Section 8.2), scatterplots of the fitted versus observed share returns over the entire forecast period are presented.

Table 49: Goodness of Fit Tests for 30-year Model

30-year analysis	Proportion Forecasts in 1st & 3rd Quadrant	Proportion Forecasts with residuals > (2*SE)
Food		
Shoprite	61%	97%
Tiger Brands*	75%	95%
Retail		
Truworths	66%	98%
Massmart	65%	97%
Woolworths	77%	95%
Consumerables		
SAB Miller	72%	97%
Steinhoff	70%	97%
Naspers	74%	97%
Brit American Tobacco*	75%	100%
Richemont*	82%	98%
Resources		
Sasol	85%	97%
BHP Billiton	79%	96%
Anglo Platinum	72%	97%
Anglo American	77%	97%
African Rainbow	71%	95%
Assore	75%	95%
Exxaro*	76%	99%
Goldfields	54%	95%
Mondi*	N/A	N/A
Anglo Ashanti	53%	97%
Impala Platinum	67%	98%
Kumba Iron Ore*	70%	95%
Medical care		
Aspen	60%	96%
Medicare	68%	97%
Life Insurance		
Old Mutual	66%	96%
Sanlam	77%	92%
Discovery	60%	97%
Property		
Intu	59%	95%
Growthpoint	79%	97%
Industrials		
Imperial	65%	93%
Bidvest	74%	95%
Remgro*	85%	98%
TeleComs		
MTN	73%	96%
Vodacom*	N/A	N/A
Financials		
Investec Ltd	74%	96%
Standard Bank	66%	98%
Firstrand Bank	73%	96%
Absa	67%	95%
Investec PLC	67%	95%
Nedbank	74%	95%

5.5 Part 5. Nelson Siegel Parameterisation of the SA swap rate curve: Implementation of Systematic Trading Strategies

5.5.1 Nelson Siegel Model Results and Fit

Using Fabozzi et al.'s (2005) matrix of swap curve maturity buckets in Table 50, Nelson Siegel Parameterisation is performed for the period August 2003 to June 2014. As per this pivotal study, the value for λ_t is fixed at 3, indicating that the maximum point of concavity across the swap curve is reached by a swap-maturity of 6 years. The authors choose this value of λ_t as it maximizes model fit while still producing a satisfactorily low level of correlation between the slope and curvature regressors (***r = -0.324).*** This point of concavity is also seen to be appropriate for the SA swap rate curve, and in 88% of the months within the sample period the point of maximum concavity is reached at the 5- to 7-year maturity bucket.

Table 50: Fabozzi et. al.'s (2005) Constellation of Swap Maturities

Fabozzi et al.'s (2005) Maturity Buckets:
3 and 6 months, 1, 2, 3, 4, 5, 7, 10, 15, 20 and 30 years;
$\lambda_t = 3$, ***r = -0.324***
(Annaert, Claes, Ceuster, & Zhang, 2013).

The South African swap rate curve exhibits similar trends to that of the Government Bond Yield curve over time, with a steepening of the slope parameter seen over the crisis period (see Figure 71). This enhances one's confidence in the potential predictive ability of the Government bond yield curve slope as an explanatory variable in the following analysis. As per Figures 72 and 73, the level parameter is seen to best approximate the level of 30-year yields, and the slope parameter most closely fits the 1-year to 20-year slope (calculated simply as the difference between 20-year and 1-year yields) and the 1-year to 10-year slope. The 10-year to 30-year slope is quite trendless and poorly described by the slope parameter, which obviously has implications for the curve steepener / flattener positions in the systematic trading model (namely, the 1-/ 10-year, 1-/ 20-year, and 10-/30-year positions).

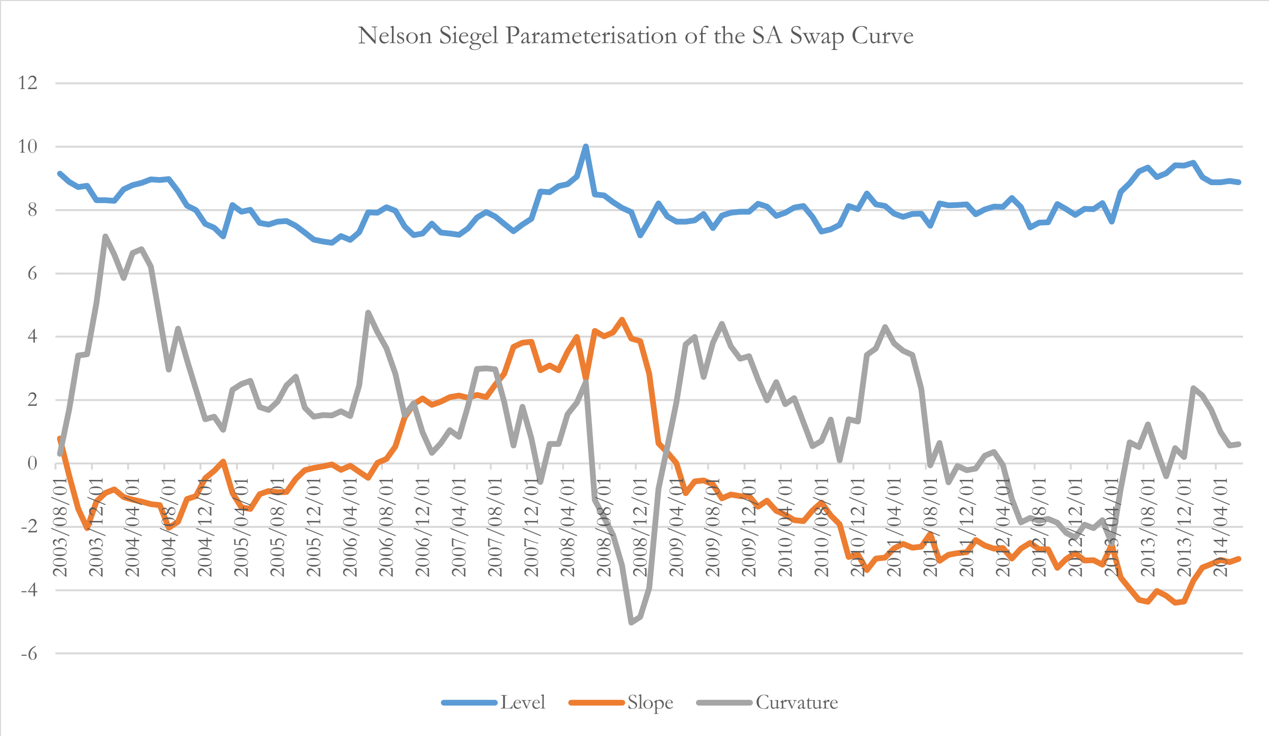


Figure 71: Extracted NS Level, Slope, and Curvature Parameters for the SA Swap Curve

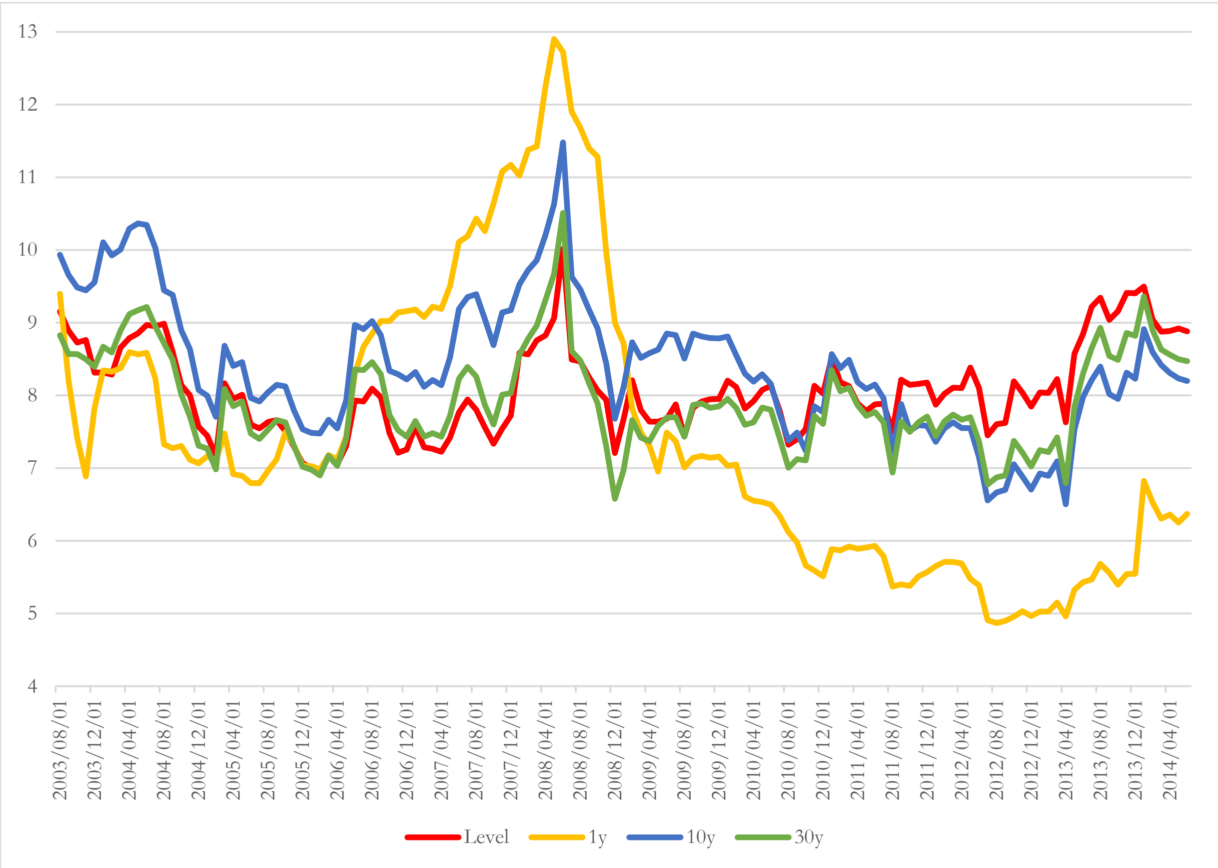


Figure 72: Swap Level Parameter vs 1, 10, 30-year swap rates

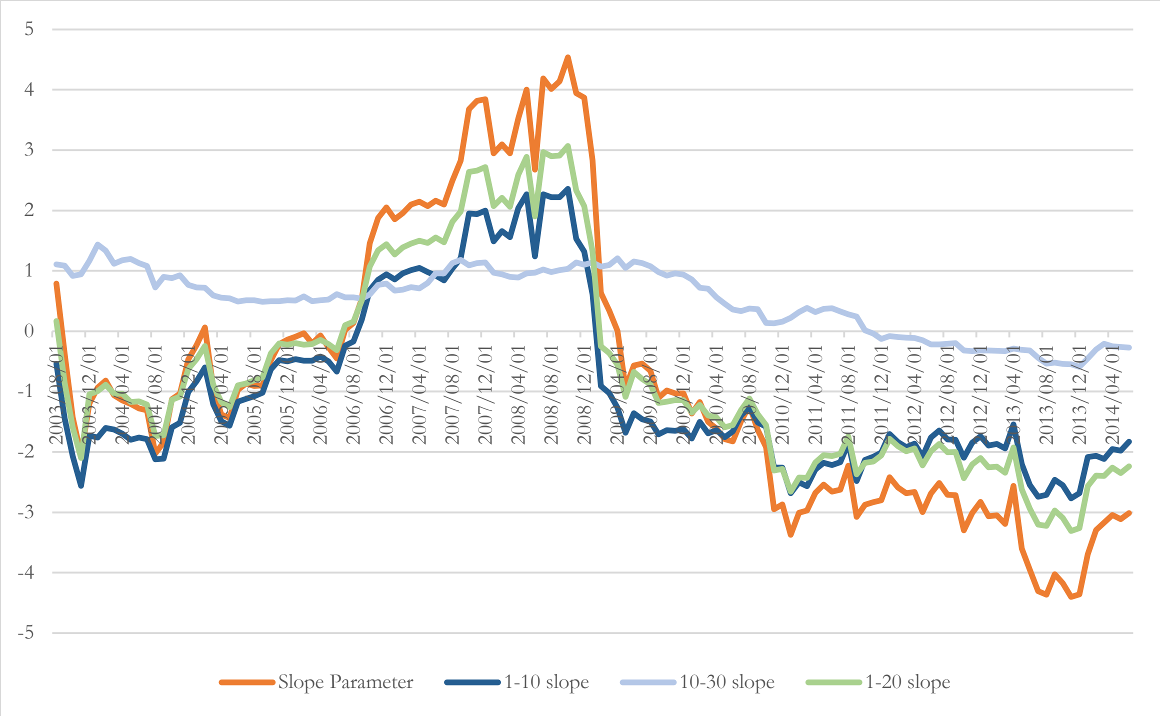


Figure 73: Swap Slope Parameter vs 1-10, 1-20, 10-30 slopes

The curvature parameter is seen to be far more difficult to describe in comparison to the level and slope parameters – with the 1/10/30 and 1/5/10 butterflies in Figure 74 most closely approximating this factor. That said, these trading strategies do not satisfactorily capture the volatility in the curvature factor.

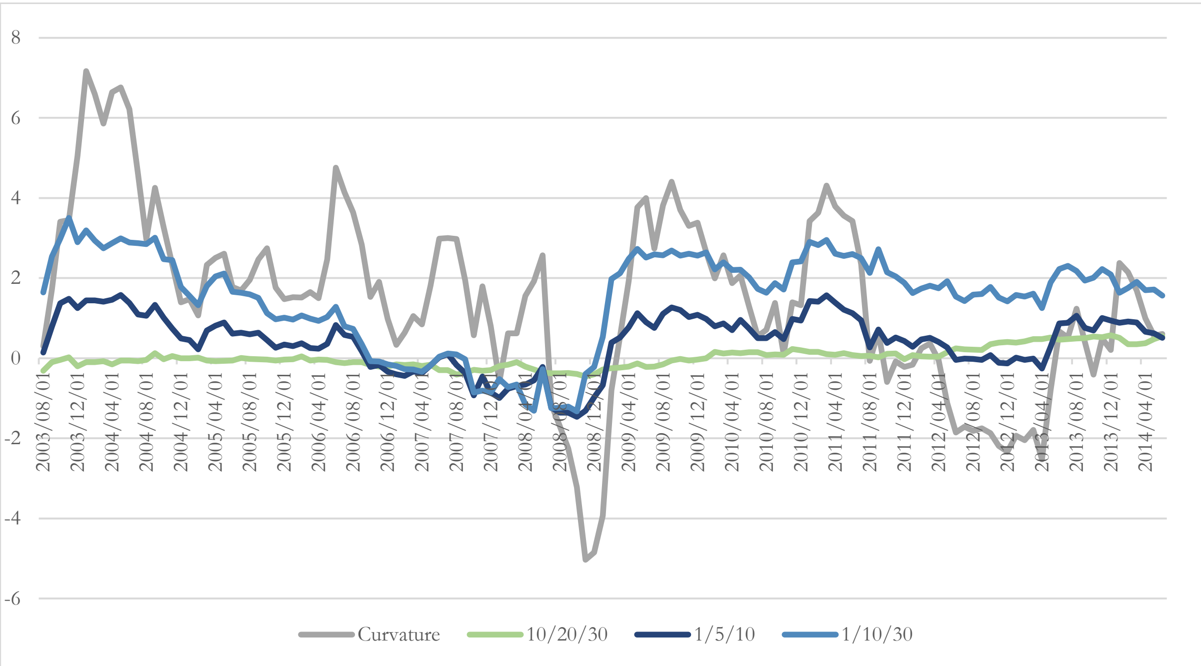


Figure 74: Swap Curvature Parameter vs 1/5/10, 1/10/30, 10/20/30 butterflies

Looking at the Nelson Siegel model fit in Figure 75, the adjusted R-Squared value is less than 75% in 15% of the time periods sampled. At an all-time low, an adjusted R-squared value of -16% is exhibited in early 2009 as the result of sudden and dysfunctional fluctuations in the curve during the crisis. The slope parameter is found to be insignificant ($p>0.05$) in 12% of all time periods over the 2003 to 2014 range (see Figure 76). In the case of the curvature parameter, 36% of the month-end swap curve term structures cannot be significantly explained by this factor. This is unsurprising as approximately 2% of all yield curve movements can be attributed to butterfly twists (Litterman & Scheinkman, 1991). Such a finding sheds light on the inability of various butterfly strategies to closely approximate the curvature parameter.

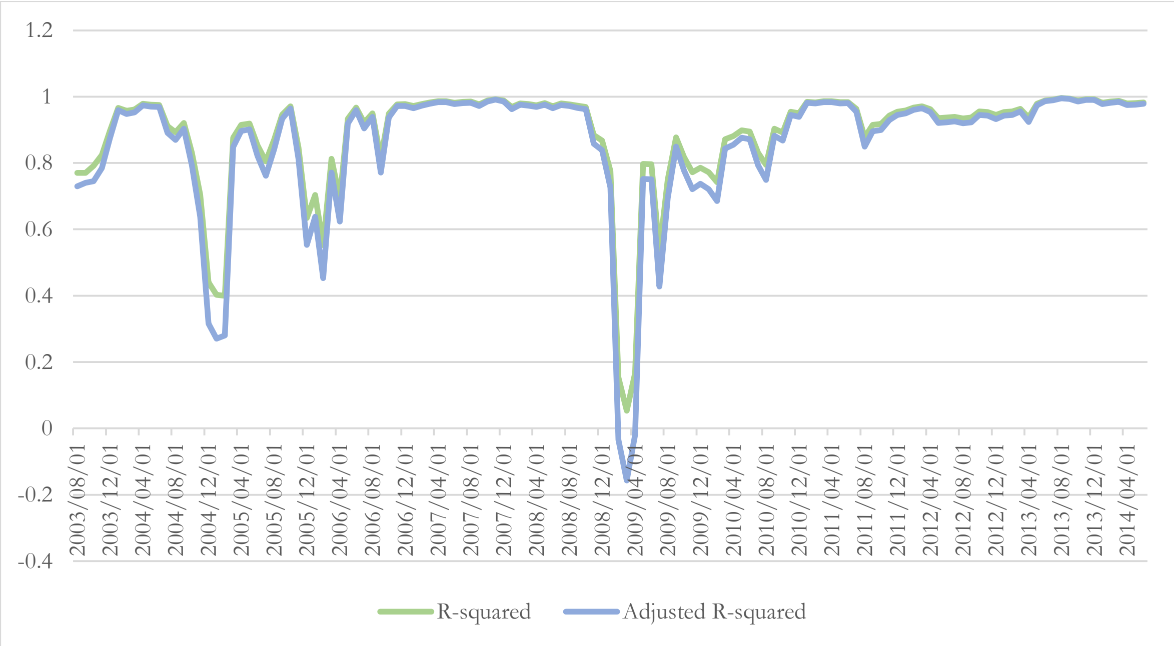


Figure 75: R-Squared and Adjusted R-Squared values for the Nelson Siegel Swap Model

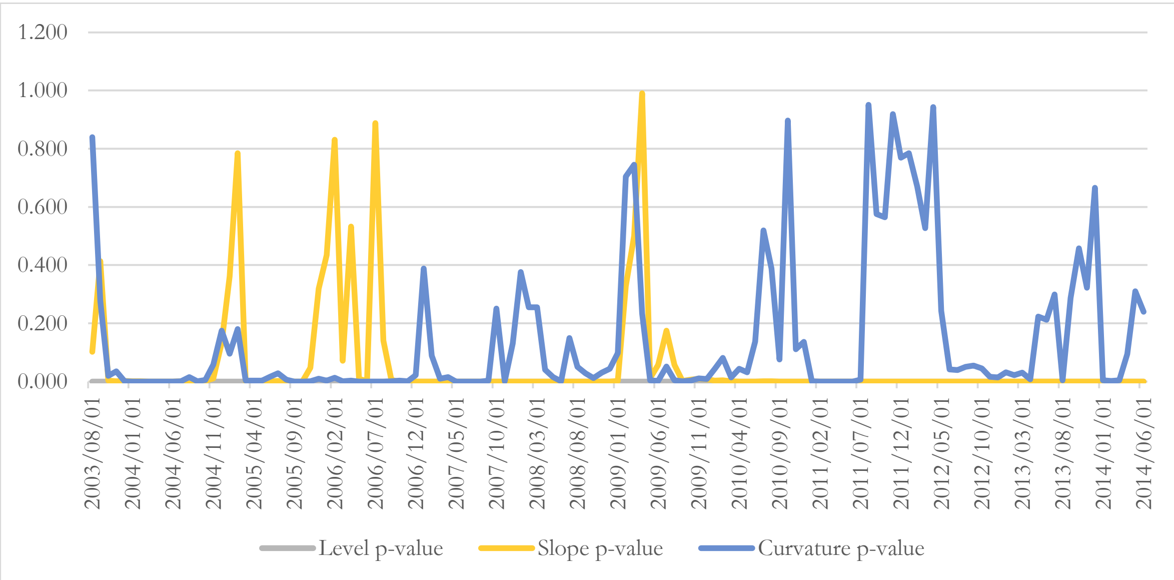


Figure 76: P-values for the Nelson Siegel Swap Model

5.5.2 Swap Parameter Model Construction

As per the Dickey-Fuller test results in Table 51, the swap curve level parameter is the only parameter that is a stationary series that does not have a unit root. This is also the only parameter for which it is not possible to construct a model that meets the criteria outlined in the Methodology section (i.e. maximum of 4 significant explanatory variables, R-squared value of minimum 20% and adjusted R-squared of minimum 10%, significance of variables in the 3-year model as well as over the preceding 12-months). This result concurs with the findings of Fabozzi et al. (2005), who similarly are unable to model the level parameter satisfactorily at the monthly level.

Table 51: Augmented Dickey Fuller Test for Swap Level Parameter

Null Hypothesis: Level Parameter has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.37	0.014
Test critical values:	1% level	-3.481	

*MacKinnon (1996) one-sided p-values.

Far more satisfactory results are obtained for the slope parameter, although at no period is more than one model obtained that meets the specified model criteria and adequately explains this yield curve parameter. Thus, the Bayesian averaging of classical estimates (BACE) used in Fabozzi et al.'s (2005) paper cannot be adopted in the present research given the absence of a constellation of satisfactory models at any one period.

Instead, what emerges is 4 distinct monthly models over the sample period of 2003 to 2014 (see Table 52). The first of these models is fit to the period June 2003 to June 2006, with a prediction then obtained for July 2007. This model satisfies criteria 1 and 2 for the next 11 monthly rolling window estimation periods until 2004M08 : 2007M08 (i.e. minimum R-squared value of 20% and adjusted R-squared of 10%, significance of variables in the 3-year model as well as over the preceding 12-months). At this point a new model is selected, which is then utilized for the next 8 "steps" forward until the model criteria are no longer met.

2 Models emerge in the pre-crisis period (yielding slope parameter predictions for September 2007 to May 2008) and 2 in the post-crisis period (yielding slope parameter predictions for March 2009 to June 2014).

Predictions cannot be obtained for the period June 2008 to February 2009 due to an inability to construct satisfactory models that meet the specified model criteria. The sharp movements in the slope parameter over this time period are characteristic of the widespread asset volatility in the crisis period and are ultimately the result of market shock factors, thus meaning that they cannot be effectively explained by fundamental factors.

Table 52: Fitted Swap Slope Parameter Monthly Models

Slope Parameter Models

1.) Pre-Crisis Model 1 [11-Steps]

Sample Start: 2003M06 2006M06

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.13	0.09	-1.53	0.14
3m JIBAR	0.43	0.17	2.58	0.02
SA Banks Dividend Yield	-1.08	0.24	-4.45	0.00
Oil	-0.05	0.01	-3.29	0.00
SA Inflation Index	0.09	0.03	3.03	0.01
R-squared	0.54	Adj R-2	0.48	

Sample End: 2004M08 2007M08

R-squared	0.20	Adj R-2	0.10

2.) Pre-Crisis Model 2 [8-Steps]

Sample Start: 2004M09 2007M09

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.05	0.05	1.17	0.25
Govt Slope	-0.79	0.18	-4.36	0.00
1yr USDZAR Forward	0.47	0.16	3.02	0.00
CAPUTIL	0.52	0.20	2.68	0.01
R-squared	0.41	Adj R-2	0.36	

Sample End: 2005M04 2008M04

R-squared	0.21	Adj R-2	0.13

3.) Post-Crisis Model 3[36-Steps]

Sample Start: 2006M02 2009M02

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.29	0.13	2.34	0.03
CAPUTIL	0.53	0.19	2.79	0.01
Govt Slope	-1.92	0.53	-3.60	0.00
Lag (Level)	1.16	0.38	3.02	0.01
3m JIBAR	-1.98	0.71	-2.77	0.01
R-squared	0.35	Adj R-2	0.27	

Sample End: 2009M02 2012M02

R-squared	0.32	Adj R-2	0.23

4.) Post-Crisis Model 4[26-Steps]

Sample Start: 2009M03 2012M03

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.06	0.06	-1.09	0.28
Govt Slope	-0.48	0.21	-2.23	0.03
Lag (Level)	0.53	0.21	2.50	0.02
Lag(Curv)	0.07	0.00	2.00	0.04
CAPUTIL	-0.06	0.00	2.03	0.03
R-squared	0.22	Adj R-2	0.13	

Sample End: 2011M05 2014M05

R-squared	0.26	Adj R-2	0.17

The model building process reveals that in the 2003 to 2007 period, the 3-month JIBAR rate is best able to predict changes in the slope parameter, with an increase in this rate being associated with significant curve flattening. Similarly, a greater average dividend yield for the South African banking sector of the JSE is associated with a steeper swap curve. Fabozzi et al. (2005) assert that rising dividend yields point to the fact that dividends are falling more slowly than share price, which is in turn indicative of banking stock weakness and a greater risk premium attached to this sector. Similarly, oil price shocks steepen the swap curve owing to knock-on effects on inflation and market risk. Higher returns on the Barclays SA Inflation-linked bond index are also associated with a steeper swap curve.

Over the 2004 to 2008 period, swap curve steepening is associated with steepening in the Government bond yield curve (pointing to the interrelatedness of these interest rate asset classes). Rising 1-year forward USDZAR expectations and levels of Capacity Utilization also contribute to a steeper swap curve (again, presumably as the result of inflationary effects).

Interestingly, the post-crisis period yields more enduring models (36-steps and 26-steps forward for models 3 and 4) that place greater focus on lagged values for the Level and Curvature parameters, as well as coincident changes in the government slope. This may indicate a stronger emphasis placed on "trending" and contagion risk within the market as opposed to fundamental factors (banking sector weakness, inflationary variables).

Table 53: Fitted Swap Curvature Parameter Monthly Models

Curvature Parameter Models

1.) Pre-Crisis Model 1 [7-Steps]					**2.) Pre-Crisis Model 2 [6-Steps]**				
Sample Start: 2003M10 2006M06					**Sample Start: 2004M05 2007M05**				
Variable	Coefficient	Std. Error	t-Statistic	Prob.	Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.00	0.15	0.00	1.00	C	-0.01	0.13	-0.05	0.96
VIX	0.19	0.07	2.84	0.01	VIX	0.14	0.07	2.04	0.05
3m JIBAR	-0.88	0.44	-2.02	0.05	CAPUTIL	-1.08	0.56	-1.95	0.05
Lag(Curv)	0.20	0.10	2.00	0.06	3m JIBAR	-1.50	0.65	-2.31	0.03
R-squared	0.33	Adj R-2	0.26		R-squared	0.25	Adj R-2	0.18	
Sample End: 2004M04 2007M04					**Sample End: 2004M10 2007M10**				
R-squared	0.20	Adj R-2	0.12		R-squared	0.20	Adj R-2	0.13	
3.) Post-Crisis Model 3[36-Steps]					**4.) Post-Crisis Model 4[23-Steps]**				
Sample Start: 2006M02 2009M02					**Sample Start: 2009M03 2012M03**				
Variable	Coefficient	Std. Error	t-Statistic	Prob.	Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.06	0.19	0.33	0.75	C	-0.09	0.19	-0.50	0.62
US Corp HY	0.15	0.06	2.60	0.01	VIX	-0.11	0.03	-3.19	0.00
Govt Slope	0.89	0.45	1.96	0.05	JPM EMBI	-0.02	0.00	-2.27	0.05
					SA Inflation Index	0.06	0.00	2.10	0.04
R-squared	0.22	Adj R-2	0.18		R-squared	0.27	Adj R-2	0.20	
Sample End: 2009M02 2012M02					**Sample End: 2011M02 2014M02**				
R-squared	0.24	Adj R-2	0.19		R-squared	0.19	Adj R-2	0.11	

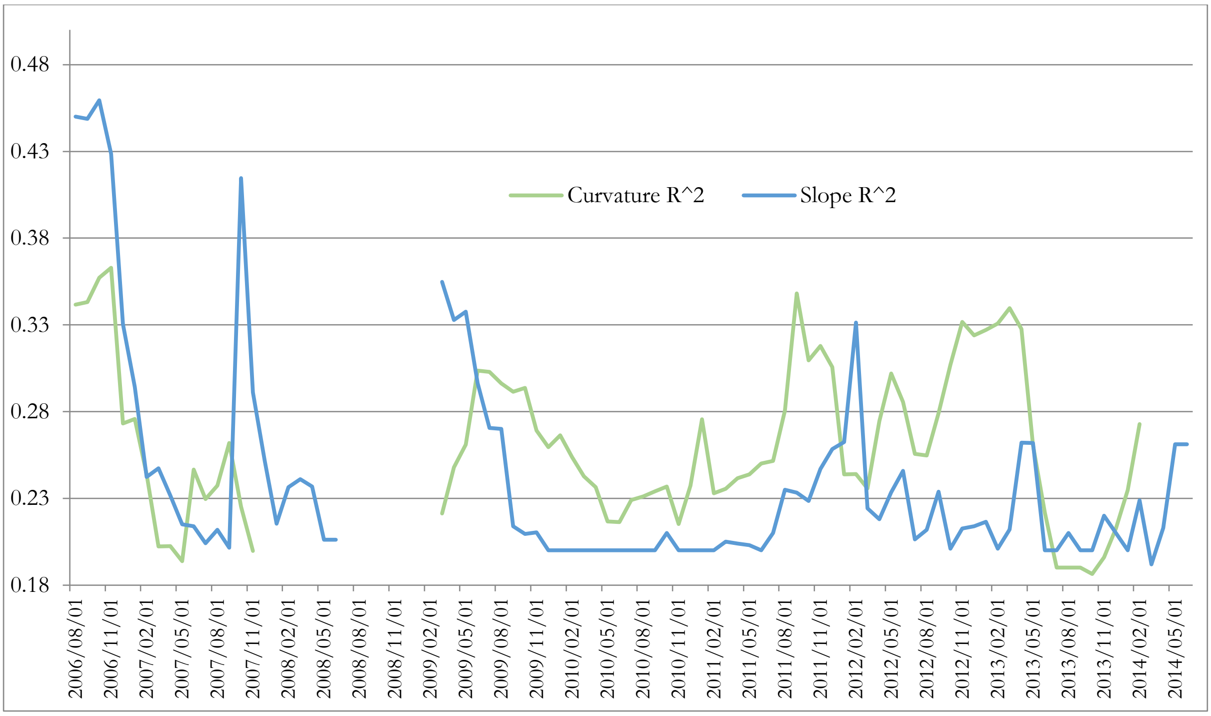


Figure 77: Slope and Curvature Models -- R-Squared Values

For the curvature parameter, too, 4 distinct modelling periods emerge in Table 53, albeit the number of forward-steps and date ranges do differ slightly. This is somewhat unsurprising given the degree of overlap (***r = -0.324)*** in the slope and curvature Nelson Siegel regressors. The pre-crisis curvature modelling process, however, breaks down as early as November 2007 (previously May 2008 for the slope parameter) and is also unable to produce satisfactory models for the most recent periods (April 2014 onwards). The R-squared values under the curvature and slope parameter models are reasonably similar for the distinct modelling periods (see Figure 77).

A closer look at the types of variables that enter the curvature parameter models reveal that they are mainly related to market shock and global risk sentiment factors –namely; the Volatility Index, JP Morgan Emerging Markets Bond Index, and US High-Yield corporate bond index. This already points to an inherent lack of fundamental-based predictability of this swap curve component.

5.5.3 Automated Swap Trading Strategies

Monthly-Return Trading Models

Using the predicted values for the Slope and Curvature parameter models, 1-month yield curve steepener / flattener positions and butterfly twists are entered into, as outlined in the Methodology section. The decision to ignore the carry component of return is reinforced by the fact that a roughly equal number of flatteners versus steepeners are entered into over the entire trading period (40: 47) and a roughly equal number of payer versus receiver butterfly positions are put on (39: 36).

As per Fabozzi et al.'s (2005) structure, each of the forecasted movements in the slope or curvature parameter is categorized under the number of standard deviations in size from a "neutral view" of zero change. These standard error categories are summarized in Table 54, with a "category 3" standard error classification meaning that the forecasted change is more than 1x standard deviation away from a neutral view.

Table 54: Standard Error Categories for Forecast Values

Std Error Categories	Range
0	(0 - 0.2)*(std deviation)
1	(0.2 - 0.5)*(std deviation)
2	(0.5 - 1.0)*(std deviation)
3	(>1.0)*(std deviation)

Slope Parameter Results

The hit rates of the slope parameter indicate the number of successfully predicted 1-month movements in the slope (i.e. steepen / flatten). Similar hit rates are obtained to those of Fabozzi et al.'s (2005) 67 – 71% range (see Table 55). Interestingly, while the highest hit rate is obtained under "category 3" standard deviations for 1/10 and 1/20 curve steepeners, the 10/30 trading strategy exhibits the highest hit rate under the "category 0" standard deviation category (which is a maximum of 0.2x standard deviations from a neutral view). The 10/30 strategy also exhibits some of the highest hit rates despite the trendless movements in this series that were noted earlier. That said, hit rates do not necessarily equate to trading returns.

Table 55: Hit rates for Swap Slope Trading Strategies

Hit rates

Std Dev	1/10	1/20	10/30
0	62%	38%	**66%**
1	43%	43%	64%
2	50%	**56%**	61%
3	**64%**	**55%**	36%

A look at the gains in basis-point movement terms of the 3 slope trading strategies in Table 56 reveals that the greatest gains are garnered from the 1/20 and 1/10 trading strategies, which concurs with the earlier-seen ability of these steepener positions to approximate the slope parameter.

Table 56: Positive basis point gains in slope trading strategies

Std Error	1/10 bp move	1/20 bp move	10/30 bp move
0	0.11	-0.14	-0.52
1	0.21	2.31	0.32
2	0.17	0.80	-0.24
3	2.18	0.92	0.04
Total bp move	2.67	3.90	-0.40

Finally, a look at hard-and-fast percentage trading returns over the entire trading period of August 2006 to June 2014 reveals that the automated trading algorithms were most successful for the 1/20 trading strategy with a 43% total return (see Table 57). This was closely followed by the 1/10 slope positions (40% total return), and there is a significant lag between these trading profits and those of the 10/30 strategy (3.1% total return). The "zero standard error category" actually produced negative returns in the case of the 10/30 slope strategy.

Table 57: Trading Returns for Slope Trading Strategies

Std Error	1/10 returns	1/20 returns	10/30 returns
0	9.23	7.63	-1.60
1	1.01	2.21	1.19
2	18.75	20.81	2.06
3	11.08	12.50	1.41
Total Returns	40.08	43.15	3.07

An investigation of the returns by year reveals that 2007, 2008, and 2009 yielded negative annual returns under even the most successful 1/10 and 1/20 trading strategies (see Table 58). This indicates that despite the fact that the model was suspended for much of the crisis period due to an inability to obtain models of the prescribed fit and significance, the entire period around the credit crisis yielded poor results and did not lend itself to accurate prediction. This serves to caution potential investors as to the hazards of using automated trading strategies or "black box trading rules" in times of extreme market volatility when fundamental relationships break down.

The annualized returns adjust for specific years within the sample in which 12-months of trading returns were not available due to model suspension and the absence of full-year data (i.e., Data for 2014 only existed out to June at the time the research was conducted).

Table 58: Annual and Annualized Trading Returns for Swap Slope Trading Strategies

Annual Returns

	1/10 returns	1/20 returns	10/30 returns
2006	11.3	13.2	1.9
2007	-3.9	-3.7	0.1
2008	-9.6	-11.9	-2.3
2009	-2.9	-6.6	-3.7
2010	7.4	8.1	0.7
2011	3.1	0.8	-1.2
2012	7.5	11.2	3.7
2013	13.1	13.0	-0.1
2014	14.0	17.8	3.8
Total	40.1	42.0	3.1

Annualized Returns

	2/10 returns	2/20 returns	10/30 returns
2006	27.1	31.7	4.6
2007	-3.9	-3.7	0.1
2008	-19.1	-23.8	-4.7
2009	-3.9	-8.8	-4.9
2010	7.4	8.1	0.7
2011	3.1	2.0	-1.2
2012	7.5	11.2	3.7
2013	13.1	13.0	-0.1
2014	33.6	42.7	9.1
Total	64.9	72.5	7.5

A look at the volatility of monthly returns by year in Table 59 reveals that while the 1/20 slope strategy was the most profitable, it also exhibited the highest standard deviation of trading returns.

Table 59: Volatility of Trading Returns for Swap Slope Trading Strategies

Volatility of Returns

	1/10 vol	1/20 vol	10/30 vol
2006	2.3	2.7	0.6
2007	2.5	2.6	0.7
2008	4.5	4.9	0.8
2009	2.8	3.4	0.7
2010	3.5	3.8	0.9
2011	3.3	3.1	0.7
2012	2.7	2.8	0.6
2013	3.2	3.0	0.4
2014	6.1	6.5	0.8
Total	30.8	32.7	6.3

This has implications for the Sharpe Ratio, which measures the ability of the yearly return to produce an excess over and above the average benchmark (cash) return, per unit of risk taken on for the year. Fabozzi et al.'s (2005) average Sharpe Ratios for 2x leveraged positions over their sample period of 1994 to 2003 were 0.6 to 2.01 for their bets on changes in the slope. In the present research, the average Sharpe Ratios range from -1.2 to 1.8, with the highest ratio clearly belonging to the 1/20 trading strategy (see Table 60).

Table 60: Sharpe Ratios for Swap Slope Trading Strategies

Sharpe Ratio

	1/10	1/20	10/30
2006	10.9	11.1	4.6
2007	-2.2	-2.1	-2.1
2008	-4.7	-5.2	-7.8
2009	-2.0	-3.1	-8.8
2010	1.6	1.7	-1.1
2011	0.4	0.1	-4.2
2012	2.2	3.4	3.2
2013	3.6	3.8	-4.1
2014	5.2	6.3	9.7
Total	15.0	15.9	-10.7
Average	1.7	1.8	-1.2

Curvature Parameter Results

The butterfly twist curvature strategies in Table 61 exhibit higher hit rates than the slope strategies (41 – 82% versus 38 to 66%), although the overall returns over the sample period are paltry in comparison to those of the slope trades (-2.34 to 11.76% versus 3.1 to 42%). This is unsurprising given that butterfly changes in the yield curve account for maybe 2% of all term structure movements (Litterman & Scheinkman, 1991). This is further anticipated given the earlier-seen poor ability of the butterfly strategies to approximate the Nelson Siegel curvature parameter extracted from the swap data.

Table 61: Hit Rates, basis-point gains, and Trading Returns for Swap Curvature Trading Strategies

Hit Rates

Std Error	1/5/10	1/10/30	10/20/30
0	41%	45%	52%
1	**69%**	48%	52%
2	27%	**73%**	**82%**
3	57%	57%	71%

Std Error	1/5/10 bp move	1/10/30 bp move	10/20/30 bp move
0	0.38	-1.01	0.19
1	2.23	1.82	0.39
2	-1.48	-0.73	0.08
3	0.77	0.86	0.10
Total	1.90	0.94	0.76

Std Error	1/5/10 returns	1/10/30 returns	10/20/30 returns
0	-0.50	-16.46	2.61
1	28.64	16.97	4.39
2	-27.32	-12.61	2.26
3	10.95	9.76	0.75
Total Returns	11.76	-2.34	10.02

Interestingly, despite the fact that the 10/20/30 butterfly strategy was largely trendless and had a poor fit with the curvature parameter movements, it yields far better, albeit weakly positive, trading returns in comparison to the more volatile 1/5/10 and 1/10/30 butterfly strategies – whose volatility is almost double that of the 10/20/30 butterfly. Annualizing the results of the 1/5/10 and 1/10/30 strategies magnifies the negative returns obtained in January and February 2014 (see Table 62).

Table 62: Annual Returns, Annualized Returns, and Volatility of Returns for Swap Curvature Trading Strategies

Annual Returns

	1/5/10 returns	1/10/30 returns	10/20/30 returns
2006	3.9	-0.1	1.1
2007	-3.1	4.5	-0.9
2009	3.1	-1.1	2.5
2010	2.2	-0.7	4.5
2011	3.5	-4.1	1.2
2012	3.2	-2.0	0.3
2013	-6.9	0.2	0.9
2014	-5.7	-4.0	0.3
Total	0.2	-7.2	10.0

Annualized Returns

	1/5/10 returns	1/10/30 returns	10/20/30 returns
2006	9.3	-0.1	2.6
2007	-3.4	4.9	-1.0
2009	3.4	-1.2	2.8
2010	2.2	-0.7	4.5
2011	3.5	-4.1	1.2
2012	3.2	-2.0	0.3
2013	-6.9	0.2	0.9
2014	-34.2	-24.2	2.0
Total	-22.9	-27.1	13.4

Volatility

	1/5/10 returns	1/10/30 returns	10/20/30 returns
2006	1.9	1.9	0.4
2007	2.8	2.7	0.7
2009	2.7	2.4	0.6
2010	3.2	3.6	0.7
2011	4.3	3.6	0.8
2012	2.3	2.6	0.9
2013	4.8	3.3	0.5
2014	2.3	5.2	1.4
Total	24.2	25.4	6.1

Fabozzi et al. (2005) achieve negative Sharpe Ratios of -0.05 to -0.4 on average for their 2x leveraged butterfly swap positions based on the curvature parameter – indicating an inability of this automated trading strategy to beat the average annual return on cash. Similarly here, average Sharpe ratios of 0 to -1.7 are obtained (see Table 63).

Table 63: Sharpe Ratios for Swap Curvature Trading Strategies

Sharpe Ratio

	1/5/10	1/10/30	10/20/30
2006	4.1	-2.2	2.2
2007	-1.9	2.5	-3.9
2009	0.6	-0.8	1.7
2010	0.2	-0.3	3.7
2011	0.4	-1.3	-0.8
2012	0.6	-1.0	-1.6
2013	-1.8	0.6	-1.7
2014	-15.7	-1.6	0.1
Total	-13.5	-4.0	-0.3
Average	-1.7	-0.5	-0.0

Quarterly Return Trading Models

An alternative explanation for the weak returns exhibited under the butterfly curvature trading strategy is that the curvature parameter is a medium or "intermediate-term" factor, whereas the slope parameter is by definition a short-term factor (Diebold & Li, 2006). As such, while the slope parameter yields positive trading returns for monthly positions, it is possible that the curvature parameter is better suited to quarterly trading positions.

As such, the above analysis is altered such that quarterly forecasts are utilized to enter into trading positions for 3-monthly periods. As per Table 64, while the returns for the slope steepener/ flattener positions perform far more poorly, the butterfly curvature returns are much improved. While annual returns for the entire trading period previously fell between -7.2 to 10% for the monthly-position trading model, they now range from 13.7 to 36.8% under the quarterly-position trading model. This reinforces the assertion that the market forces at play that determine the convexity of the yield curve are more medium-term in nature than the short-term slope parameter.

Table 64: Quarterly Trading Model Returns, Volatilities, and Sharpe Ratios (Swap Slope and Curvature Strategies)

Annual Returns	2/10	2/20	10/30	1/5/10	1/10/30	10/20/30
2006	-5.0	-10.8	-3.5	5.5	5.0	1.3
2007	-7.7	-11.1	-3.3	3.8	5.8	0.8
2008	6.7	12.7	4.9			
2009	3.4	10.1	2.5	7.5	5.1	2.4
2010	12.5	19.5	9.3	2.2	4.2	2.7
2011	-2.6	-3.5	3.5	6.8	7.2	3.1
2012	1.8	3.5	2.8	-8.1	-3.9	6.2
2013	0.7	11.6	2.3	22.9	16.3	-0.1
2014	-10.0	-14.9	-4.5	-3.7	-6.9	-2.7
Total	-0.3	17.1	14.2	**36.8**	**32.8**	**13.7**

Volatility	2/10	2/20	10/30	1/5/10	1/10/30	10/20/30
2006	1.8	5.4	2.1	2.8	1.7	1.0
2007	2.5	3.1	1.6	3.6	4.5	1.0
2008	0.5	2.2	1.6			
2009	0.5	3.0	1.1	3.3	2.4	0.7
2010	3.8	5.4	1.8	2.9	4.7	0.9
2011	6.2	4.4	2.7	5.7	6.6	0.4
2012	2.3	1.6	0.9	3.3	2.3	1.4
2013	2.0	2.4	1.2	5.7	2.3	0.7
2014	6.3	9.2	2.6	3.3	4.5	1.7
Total	25.8	36.7	15.5	30.7	28.9	7.7

Sharpe Ratio	2/10	2/20	10/30	1/5/10	1/10/30	10/20/30
2006	-3.8	-2.3	-2.4	1.3	1.9	-0.4
2007	-3.8	-4.1	-3.1	0.6	0.9	-0.9
2008	10.6	5.1	2.0			
2009	3.5	2.8	0.8	1.7	1.4	1.1
2010	2.9	3.3	4.3	0.2	0.5	1.1
2011	-0.7	-1.2	0.7	0.9	0.8	3.5
2012	0.0	1.1	1.3	-3.0	-2.5	3.3
2013	-0.5	4.2	0.5	3.7	6.5	-2.5
2014	-1.9	-1.8	-2.4	-1.6	-1.9	-2.6
Total	6.4	7.1	1.5	3.9	7.8	2.5
Average	0.7	0.8	0.2	**0.5**	**1.0**	**0.3**

While the volatility of the quarterly-position butterfly trading strategies increases in comparison to the monthly model, the Sharpe Ratios for the butterfly strategies now yield positive returns on average and are able to outperform the benchmark cash return. Average Sharpe Ratios for the butterfly strategies fall within the 0.3 to 1.0 range and are well above the negative ratios achieved under the present research and Fabozzi et al.'s (2005) monthly trading models. The best performer is the 1/10/30 quarterly butterfly position, which undoubtedly captures the full extent of the curvature in the yield curve. While it may be tempting to continue such analyses for longer-term maturities, Fabozzi et al.'s (2005) justification for ignoring the carry aspect of returns was in part due to the short-term monthly holding period of their trades. Thus, to hold such positions for semi-annual or yearly holding periods would dilute the accuracy of results given that the carry component of return would become magnified and potentially erode reported trading profits.

6. Discussion

6.1 Nelson Siegel Yield Curve Factorization Method and SA Government Bond Predictors

The fit of the Nelson Siegel model and Level, Slope, and Curvature factors to the SA Yield Curve in Part 1 is seen to be highly satisfactory, with adjusted R-squared values exceeding 90% in 90% of all of the 132 months within the sample period. While for the majority of months one finds all of the yield curve parameters to contain significant predictive power in explaining the shape of the Government term structure, the curvature factor lends the least significant proportion of variability to the explained variation of the model. This finding is in line with earlier assertions that the majority of changes in the yield curve are the result of parallel shifts in the Level factor, with only 8.5% and 2% of movements attributable to the slope and curvature respectively (Litterman & Scheinkman, 1991).

The utilized value for λ_t of 0.0609 (equating to a maximum concavity at 2.5-years) in Part 1 is found to fit the curvature of the SA government bond yield curve very well. In fact, the optimal value for λ_t is found to be 0.071 via the BFGS iterative algorithm and the Kalman filter procedure in Part 3. That said, a potential drawback related to this finding is that the parameter estimates in this section, such as that for λ_t, will naturally be biased based on the starting values used when initializing the Kalman filter (Annaert et al., 2013).

For the periods within the sample that the model fit substantially declines, the addition of a 4th "Svensson parameter", representing a 2nd curvature factor at the 5.5-year point, remedies the issue. Despite this model having better predictive power and out-of-sample generalizability (as witnessed via marginally lower Root Mean Squared errors during the back-testing procedure), it also suffers from multicollinearity between the slope and second curvature factor.

Initially, an assemblage of 18 economic, fundamental, and market-based predictors of the Government yield curve are considered based on the findings of such studies as Sy (2002), Mehl (2009), Min et al. (2003) and Fabozzi et al. (2005). The findings of the Quarterly OLS Models fitted in Part 2 confirm that since the post-crisis 2008 period it is largely global market-sentiment related variables and shock factors, such as investor attitude towards Emerging markets, the Real Oil price, and US government bond trends, that inform the relative level and slope of the SA Government curve. In the 2003 to 2008 period, the findings reveal that it is the SA Budget Balance (as a proportion of national GDP) that holds the greatest predictive power when explaining the Government curve's level and slope on a quarterly basis. This finding confirms the hypothesis arrived at in such studies as Longstaff et al. (2007) and Baek, Bandopadhyaya, and Du (2005). These studies assert that with the growing globalization of the investment community, international capital flows are increasingly being put to work in emerging markets. With the rising number of foreign holders of EM domestic debt, it is global market sentiment towards riskier assets and issuers (such as EM's and poorly rated corporates) that determines the shape of local EM government curves, as opposed to in-country fundamentals and the relative "health" of the sovereign. Under this frameowrk, a sudden global risk-off sentiment in the markets is more likely to cause a sell-off in emerging market yields than might previously have been the case due to the fact that a larger base of investors would be selling assets – meriting a larger rise in yields. More clought is given to such a theory when one considers the rise in foreign ownership of SA government debt that occurred over the 2003 to 2014 sample period.

Contrastingly, the SA government curvature parameter is best explained on the Quarterly Level by a slightly more distinct subset of economic indicators that include South Africa's Credit Rating and Inflation rate expectations. The relative humpedness and concavity of the Government bond curve is theorized to be an intermediate-term factor that taps into the extent to which an inflationary hump and medium-term risk premium is priced into the curve. Despite these findings, global market sentiment towards both US Government debt and US poorer-rated corporates is still seen to play a role in determining this curvature parameter for SA.

Adopting the state space modelling and optimization procedures of Diebold et al. (2006), a monthly biodirectional model of the latent yield curve factors is fitted in Part 3. This monthly model outlines that the most persistent predictors of the SA yield curve factors are their own lagged values. It also allows complex interrelated "yields-to-macro" and "macro-to-yields" relationships to be mapped out. More traditional "macro-to-yields" findings involve the fact that a weaker SA economy, as evidenced by declining productive capacity utilization and lower inflation, leads to a significant sell-off in the longer-term level of SA Government yields on average in the following month. These changes (falling inflation and capacity utilization) also precipitate a steepening of the Government curve. This is indicative of lower rates being priced into the near-term due to the gloomy outlook for growth, as well as expectations of greater relative long-term risk premia.

Similarly, a more concave hump in the yield curve is priced in on average when the SA Deposit rate has been cut by the SARB in the previous month. This may indicate the market's concerns about impending inflationary growth given the low level of interest rates and subsequent increase in consumer spending and economic activity in the medium term.

On the "yields-to-macro" front, the fitted model highlights that the yield curve is an important source of information that comprises of market predictions as to expected future changes in the macro-economy. In this light, a rise in the slope factor and more concave pricing of the yield curve is followed by increases in the SA Deposit rate – presumably as the SARB combats rising inflation. Furthermore, lagged increases in the slope parameter and a rising level of capacity utilization are typically followed by growth in the SA year-on-year inflation rate in the following month. Interestingly, a **decrease** in lagged inflation is seen to be what precipitates this rise in Capacity Utilization in the first place. This essentially captures the complex nature of the business cycle and the fact that lower inflation serves to increase consumer wealth and stimulate aggregate demand such that the market prices in a steeper curve in the coming months as productive capacity rises. This then contributes to future rising inflation and triggers future SARB hikes of the policy rate as the cycle continues.

6.2 The relationship between South African Equities, Bonds, and the Rand

The first asset relationship outlined in Part 4 of the research highlights that a sell-off in Government Bond yields, a flatter slope (presumably due to higher short-end rates), and a more concave yield curve structure are associated with a sell-off in the Rand . This is ostensibly due to the vehicle of inflation and its erosion of government bond yields and relative value in the currency, however, another more direct relationship does exist that explains this connection. As the number of foreign holders of SA domestic debt rises, selling in SA Government bonds would necessitate that the ZAR premium received from the sale must be converted out of Rands into a foreign currency like the US-dollar (i.e. concomitant rand selling and weakness).

After removing the portion of USDZAR exchange rate fluctuations that is associated with Government Bond activity, what becomes apparent is that the Level of 10-year SA Government Bonds is far better able to explain changes in Monthly Share returns on the JSE than is the "residual" currency fluctuation movements. Predictably, the major SA Retailers and Financials are the most sensitive in terms of their share returns to the Level parameter of SA Government debt, seemingly due to the effect of rising interest rates on consumer luxury spending and the ability of banking institutions to fund themselves at a lower rate than their fixed-interest consumer deposit pay-outs. For many of the Rand Leverage Mining companies, a sell-off in the bond market is unsurprisingly positive for monthly share returns, apparently due to the effect of rising inflation on currency weakness. This would lead to subsequent lower in-house mining costs and higher dollar profits from gold and other mined products.

The effect of the Government bond curvature component (which moves in fashion with rising SA inflation on average) is also a useful classification tool by which to categorize the Rand Play versus Leverage JSE companies – with an increase in concavity being Rand Play "negative" and Rand Leverage "positive".

While the 3-month to 10-year slope parameter only significantly explains 6 of the JSE Top40 shares and is somewhat incongruous in its classification system, it is the 10-to-30-year slope factor that significantly explains the returns of 15 of the Top40 shares. This long-term yield curve slope factor was found to be difficult to predict in the Quarterly OLS Regression Models in Part 2 of this research. This is somewhat foreseeable as this less-liquid portion of the Government yield curve encapsulates long-term growth trends that might not effectively be captured or modelled at the quarterly level (Kempf, Korn & Homburg, 2012). Furthermore, fewer market players trade in this area of the yield curve and it is presumed to be habituated by long-term Liability Managers and Pension Funds, thus making it less attuned to the "noise" of shorter-term changes in economic factors. Increases in this long-term slope parameter and alleged increases in the long-term SA growth outlook are associated with increasing share returns for such Rand Play companies as the SA Retailers, Financial and Banking institutions, and the Industrials and Property companies. Conversely, share return losses are exhibited by some of the major Resources counters (Anglo American and Sasol).

This highlights that while the 10-year level of the Government yield curve is the strongest predictor of SA Equity share returns (apart from the All Share Index monthly returns), it is the long-term 10-to-30-year slope of the curve and growth outlook for SA that is next best suited to share return classification.

6.3 The efficacy of Automated Interest Rate Trading Strategies for the SA Market

Systematic Trading strategies are implemented for the South African interest rate swap curve as per the seminal work of Fabozzi et al. (2005). Notably, this analysis is carried out in the SA swap market as opposed to the Government bond yield curve as, even though the **Generic** bond curve is analysed for the purposes of this research, in reality the SA Government bonds have set maturity dates. This complicating factor means that one cannot take a view on the level of "10-year yields" over time, but must rather invest in a set-maturity bond whose maturity falls most closely around the 10-year mark. Over time, however, such a trade will not represent a market view on 10-year yields but on some nearer-maturity time bucket – thus obscuring the analysis.

The findings of this section indicate that a constellation of swap parameter explanatory models cannot be formed at each point (month-end) within the sample period. Only one explanatory model at most exists at a given point in time that adequately explains the swap curve parameters as per the model fit guidelines stipulated in the Methodology section. This means that Fabozzi et al.'s (2005) adoption of Bayesian Averaging of Classical Estimates (BACE) cannot be utilized. In line with the findings of this seminal work, the swap level parameter cannot be effectively modelled to the degree of model significance and fit required. This is perhaps somewhat unsurprising as the Level parameter is the only factor asserted by Diebold and Li (2006) to represent a "Long-term" factor, and thus may not be well captured via a monthly regression model. The Slope and Curvature Factors are purported by the authors to signify short- and intermediate-term parameters.

As per their classification, the Slope and Curvature Trading Strategies (as manifested through swap steepeners / flatteners and butterfly swaps) yield the highest trading returns when implemented for holding periods of one month and one quarter respectively (i.e. short and intermediate holding period returns). In both cases the average benchmark cash return is outperformed over the sample trading period. A possible drawback of using quarterly holding periods is that the accuracy of results is somewhat distorted given that the carry component of return would become magnified and potentially erode reported trading profits as one holds positions for longer time periods. A direction for future research into automated SA swap trading strategies would involve the inclusion of the carry component of return, as well as the investigation of longer-term holding periods that would be of greater relevance and interest to "long-term outlook" portfolio managers. Such strategies could also look at slowly building positions over time given model forecasts – as opposed to the more rudimentary approach of holding a position for strictly one month or quarter. Stop losses would also ideally be worked into the trading strategy, as well as signals as to the "optimal" time at which to take profit on a given position.

Another important finding of this section involves the fact that the dynamic modelling process completely breaks down as one enters the 2008 crisis period when economic fundamentals and market sentiment variables are no longer able to reliably predict the swap parameters to the degree required by the model fit criteria. This highlights that systematic trading strategies are problematic when used during economic crises and periods of extreme market volatility. For the monthly curve steepener/ flattener positions traded based on swap slope predictions, the annual returns over 2007, 2008, and 2009 are negative despite the trading model being suspended from May 2008 to March 2009 (during which no trading positions were entered into). The quarterly curvature model, despite delivering lower annual returns and Sharpe Ratios on average versus the monthly slope strategies, performs better over the 2007 to 2009 period. It is able to maintain positive trading returns with lower average volatility around the crisis period in comparison to the monthly trading models, which undoubtedly capture more of the market "noise" seen over this period.

7. References

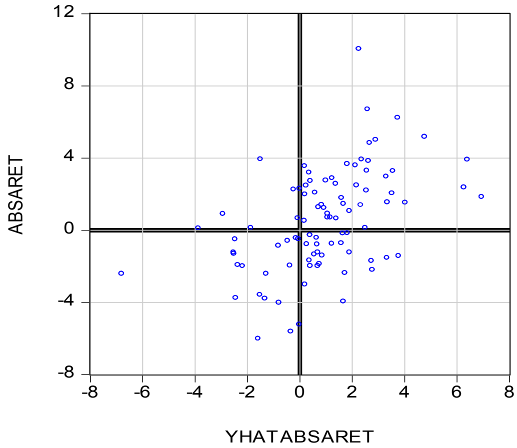
1. Alam, M. M., & Uddin, M. G. S. (2009). Relationship between interest rate and stock price: empirical evidence from developed and developing countries. *International journal of business and management*, *4*(3), P43.
2. Annaert, J., Claes, A. G., De Ceuster, M. J., & Zhang, H. (2013). Estimating the spot rate curve using the Nelson–Siegel model: A ridge regression approach. *International Review of Economics & Finance*, *27*, 482-496.
3. Asiedu, E. (2006). Foreign direct investment in Africa: The role of natural resources, market size, government policy, institutions and political instability. *The World Economy*, *29*(1), 63-77.
4. Baek, I. M., Bandopadhyaya, A., & Du, C. (2005). Determinants of market-assessed sovereign risk: Economic fundamentals or market risk appetite? *Journal of International Money and Finance*, *24*(4), 533-548.
5. Barberis, N. (2000). Investing for the long run when returns are predictable. *The Journal of Finance*, *55*(1), 225-264.
6. Barr, G. D. I., & Kantor, B. S. (2005). The impact of the rand on the value of the Johannesburg Stock Exchange. *Journal for Studies in Economics and Econometrics*, *29*(2)
7. Barr, G. D. I., Kantor, B. S., & Holdsworth, C. G. (2007). The effect of the rand exchange rate on the JSE Top-40 stocks--An analysis for the practitioner. *South African Journal of Business Management*, *38*(1), 45-58.
8. Beirne, J., Caporale, M., & Spagnolo, N. (2009). Market, Interest Rate and Exchange Rate Risk Effects on Financial Stock Returns: A GARCH-M Approach. *Quantitative and Qualitative Analysis in Social Sciences*, *3*(2), 44-68.
9. Bicksler, J., & Chen, A. H. (1986). An economic analysis of interest rate swaps. *The Journal of Finance*, *41*(3), 645-655.
10. Bliss, R., Sener, E., Erdogan, G., & Ahi, E. (2011). Robust Term Structure Estimation. *Istanbul Risk Management Laboratory*. http://i-rml.org/sites/default/files/Robust%20Term%20Structure%20Estimation_0.pdf
11. Caceres, C., Guzzo, V., & Segoviano, M. A. (2010). *Sovereign spreads: Global risk aversion, contagion or fundamentals?*. International Monetary Fund, Monetary and Capital Markets Department.
12. Cox, J. C., Ingersoll Jr, J. E., & Ross, S. A. (1985). A theory of the term structure of interest rates. *Econometrica: Journal of the Econometric Society*, 385-407.
13. Diaz Weigel, D., & Gemmill, G. (2006). What drives credit risk in emerging markets? The roles of country fundamentals and market co-movements. *Journal of International Money and Finance*, *25*(3), 476-502.

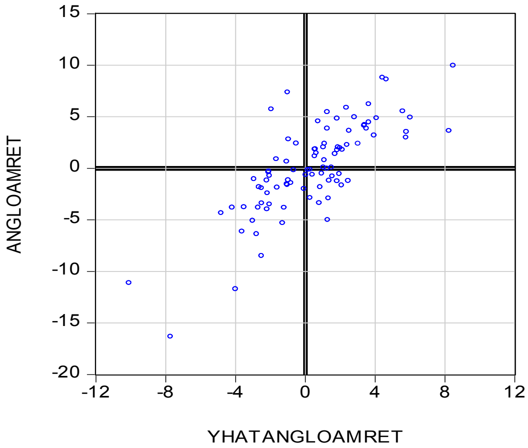
14. Diebold, F. X., & Li, C. (2006). Forecasting the term structure of government bond yields. *Journal of econometrics*, *130*(2), 337-364.
15. Diebold, F. X., Rudebusch, G. D., & Boragan Aruoba, S. (2006). The macroeconomy and the yield curve: a dynamic latent factor approach. *Journal of econometrics*, *131*(1), 309-338.
16. Doppelhofer, G., Miller, R. I., & Sala-i-Martin, X. (2000). *Determinants of long-term growth: A Bayesian averaging of classical estimates (BACE) approach* (No. w7750). National bureau of economic research. http://www.nber.org/papers/w7750
17. Erb, C. B., Harvey, C. R., & Viskanta, T. E. (1999). New perspectives on emerging market bonds. *The Journal of Portfolio Management*, *25*(2), 83-92.
18. Estrella, A., & Hardouvelis, G. A. (1991). The term structure as a predictor of real economic activity. *The Journal of Finance*, *46*(2), 555-576.
19. Fabozzi, F. J., Martellini, L., & Priaulet, P. (2005). Predictability in the shape of the term structure of interest rates. *The Journal of Fixed Income*, *15*(1), 40-53.
20. Harvey, A. C. (1993). *Time series models* (Vol. 2). New York: Harvester Wheatsheaf.
21. IMF Staff Report (2013). South Africa Staff Report for the 2013 Article IV Consultation. *IMF Country Report*, *13*(303), 1 - 59. https://www.imf.org/external/pubs/ft/scr/2013/cr13303.pdf
22. Kaminsky, G., Lyons, R., & Schmukler, S. (2001). *Mutual fund investment in emerging markets: An overview* (pp. 157-185). Springer
23. Kandel, S., & Stambaugh, R. F. (1996). On the Predictability of Stock Returns: An Asset-Allocation Perspective. *The Journal of Finance*, *51*(2), 385-424.
24. Kempf, A., Korn, O., & Uhrig-Homburg, M. (2012). The term structure of illiquidity premia. *Journal of Banking & Finance*, *36*(5), 1381-1391.
25. Khomo, M. M., & Aziakpono, M. J. (2007). Forecasting recession in South Africa: a comparison of the yield curve and other economic indicators. *South African Journal of Economics*, *75*(2), 194-212.
26. Krishnamurthy, A. (2009). *How debt markets have malfunctioned in the crisis.* (No. w15542). National Bureau of Economic Research. http://www.nber.org/papers/w15542
27. Krishnan, C. N. V., Ritchken, P., & Thomson, J. B. (2007). *On Forecasting the Term Structure of Credit Spreads* (No. 0705). Federal Reserve Bank of Cleveland.
28. Kumar, M. S., & Baldacci, E. (2010). *Fiscal deficits, public debt, and sovereign bond yields*. International Monetary Fund.
29. Laurini, M. P., & Hotta, L. K. (2010). Bayesian extensions to diebold-li term structure model. *International Review of Financial Analysis*, *19*(5), 342-350.
30. Litterman, R. B., & Scheinkman, J. (1991). Common factors affecting bond returns. *The Journal of Fixed Income*, *1*(1), 54-61.
31. Liu, J., Longstaff, F. A., & Mandell, R. E. (2002). *The market price of credit risk: An empirical analysis of interest rate swap spreads* (No. w8990). National Bureau of Economic Research. http://www.nber.org/papers/w8990

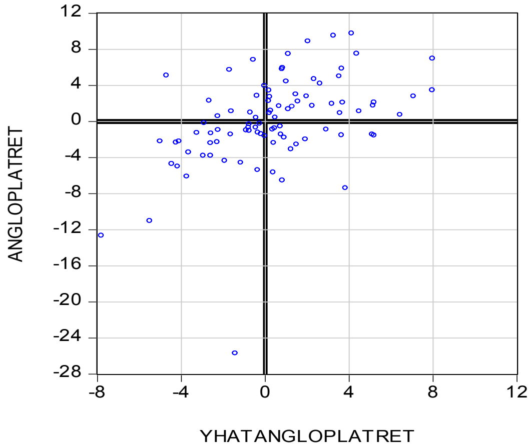
32. Liu, J., Longstaff, F. A., & Mandell, R. E. (2006). The market price of risk in interest rate swaps: the roles of default and liquidity risks*. *The Journal of Business*, *79*(5), 2337-2359.
33. Longstaff, F. A., Pan, J., Pedersen, L. H., & Singleton, K. J. (2007). *How sovereign is sovereign credit risk?* (No. w13658). National Bureau of Economic Research.
34. Mangani, R. (2011). Monetary policy, structural breaks and JSE returns. *Investment Analysts Journal*, (73), 27-35.
35. Mehl, A. (2009). The yield curve as a predictor and emerging economies. *Open Economies Review*, *20*(5), 683-716.
36. Min, H. G., Lee, D. H., Nam, C., Park, M. C., & Nam, S. H. (2003). Determinants of emerging-market bond spreads: Cross-country evidence. *Global Finance Journal*, *14*(3), 271-286.
37. O'brien, R. M. (2007). A caution regarding rules of thumb for variance inflation factors. *Quality & Quantity*, *41*(5), 673-690.
38. Peiris, S. J. (2010). *Foreign Participation in Emerging Markets' Local Currency Bond Markets (EPub)* (No. 10-88). International Monetary Fund.
39. South African Reserve Bank (2014, July 20). *General Terms and Conditions of SARB Switch Auctions*. Retrieved from https://www.resbank.co.za/Markets/Domestic/MarketDocuments/Pages/General-Terms-and-Conditions-of-SARB-Switch-Auctions.aspx
40. Svensson, L. E. (1994). *Estimating and interpreting forward interest rates: Sweden 1992-1994* (No. w4871). National Bureau of Economic Research.
41. Sy, A. N. (2002). Emerging market bond spreads and sovereign credit ratings: reconciling market views with economic fundamentals. *Emerging Markets Review*, *3*(4), 380-408
42. Woglom, G. (2003). How has inflation targeting affected monetary policy in South Africa?. *South African Journal of Economics*, *71*(2), 198-210.

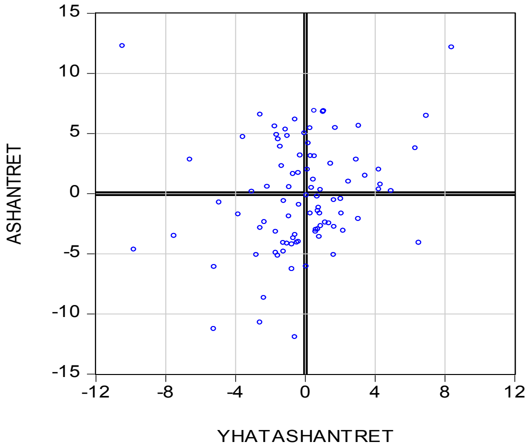
8. Appendix A

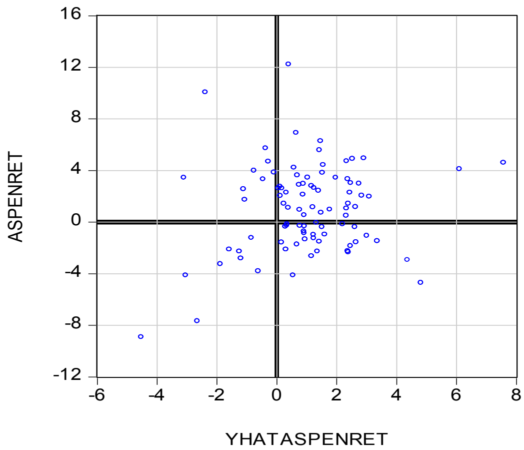
8.1 Scatterplot of Predicted versus Actual Share Returns for JSE /10-year Government Bond Yield Curve Parameter Model

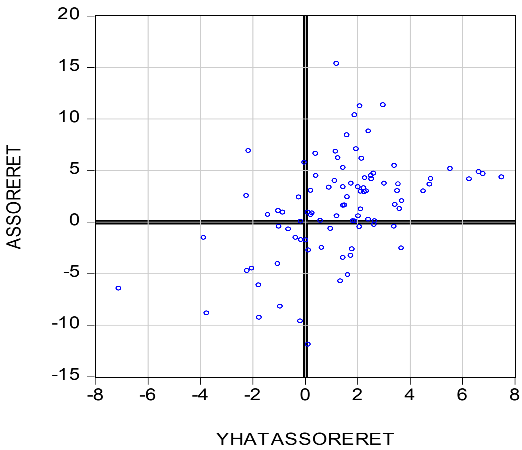


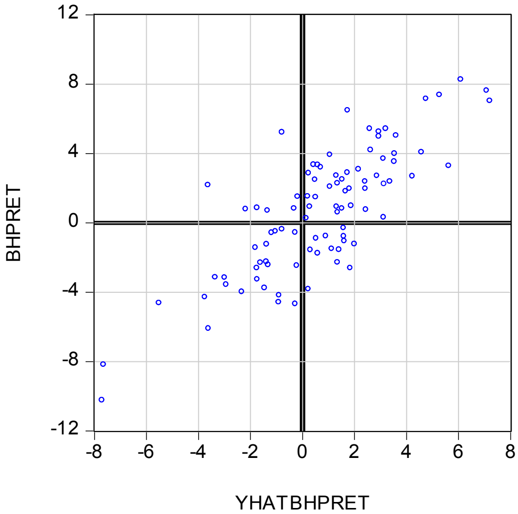


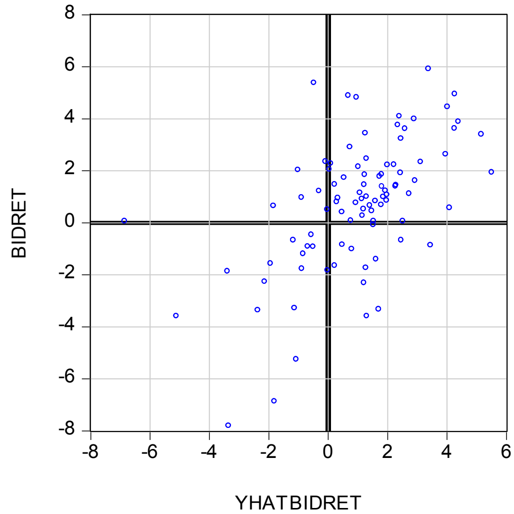


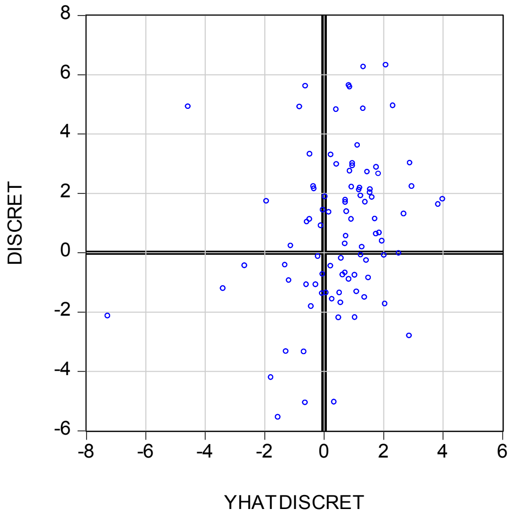


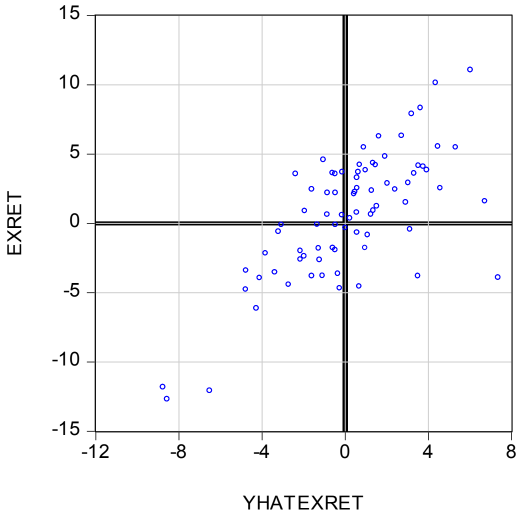


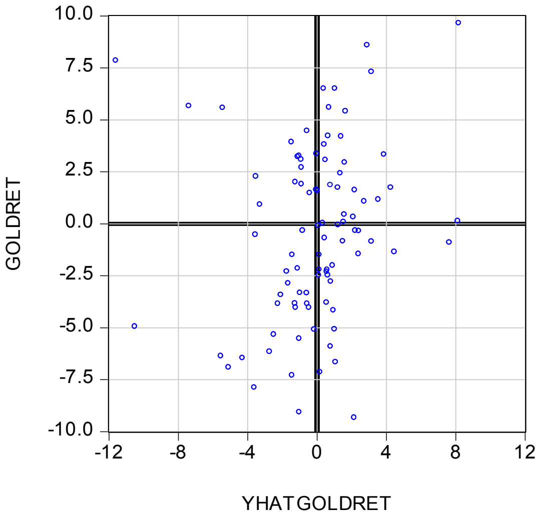


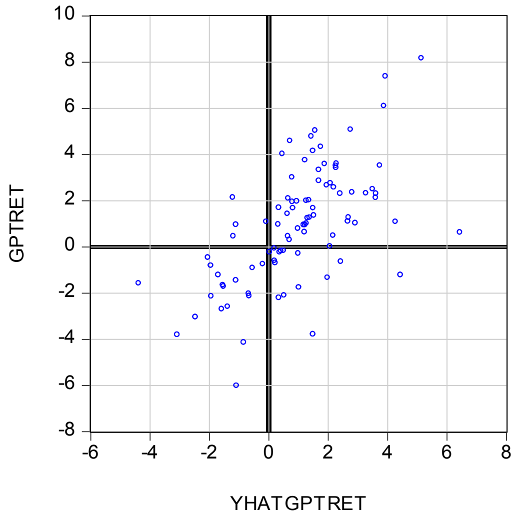

BHPRET
YHATBHPRET

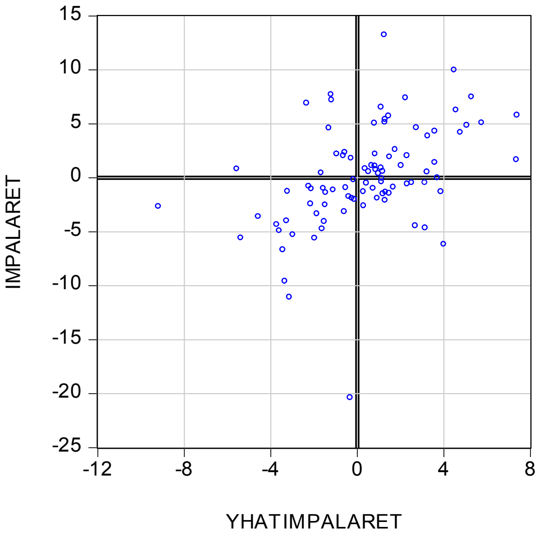

BIDRET
YHATBIDRET

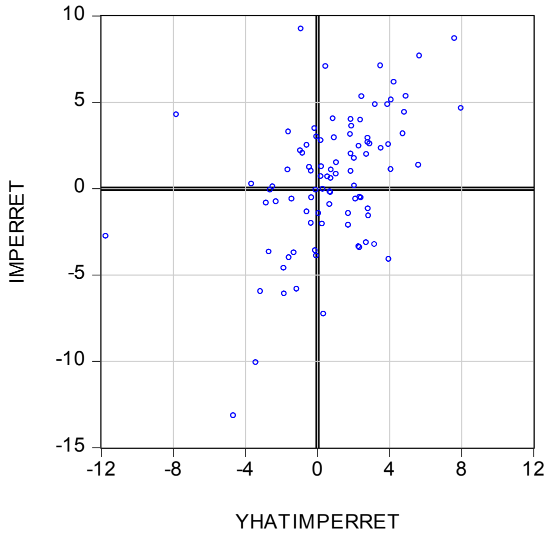

DISCRET
YHATDISCRET

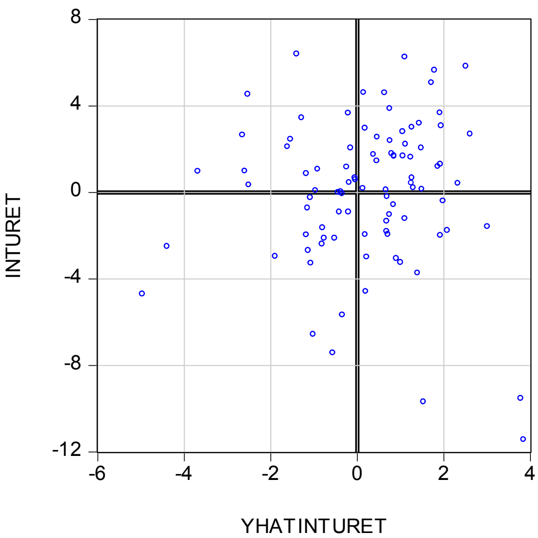

EXRET
YHATEXRET

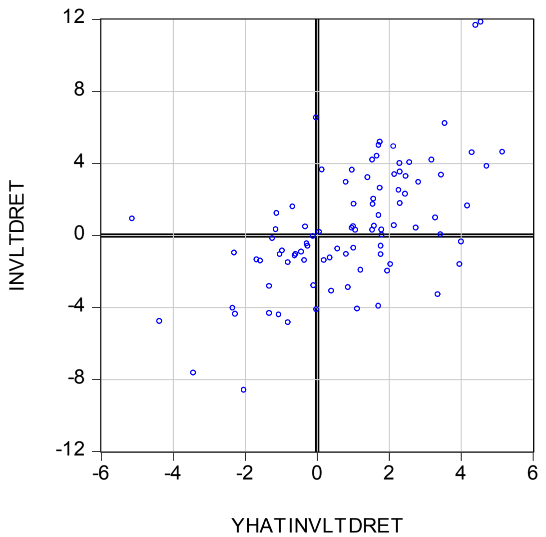

GOLDRET
YHATGOLDRET

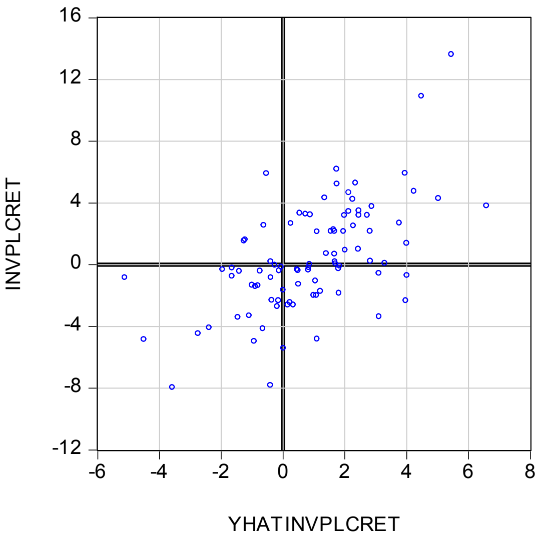

GPTRET
YHATGPTRET

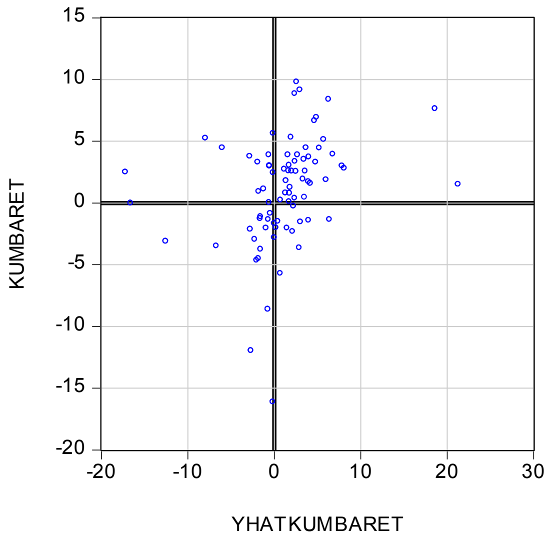

IMPALARET
15
10
5
0
-5
-10
-15
-20
-25
-12
-8
-4
0
4
8
YHATIMPALARET

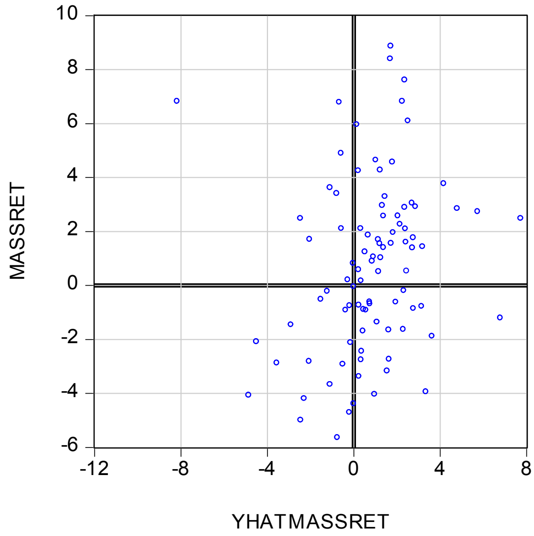

IMPERRET
10
5
0
-5
-10
-15
-12
-8
-4
0
4
8
12
YHATIMPERRET

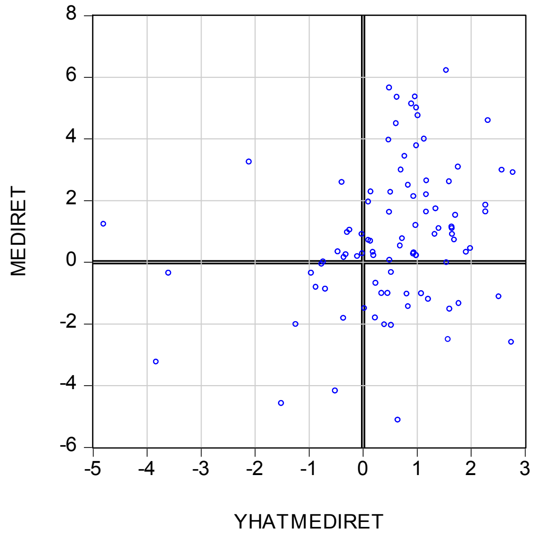

INTURET
8
4
0
-4
-8
-12
-6
-4
-2
0
2
4
YHATINTURET

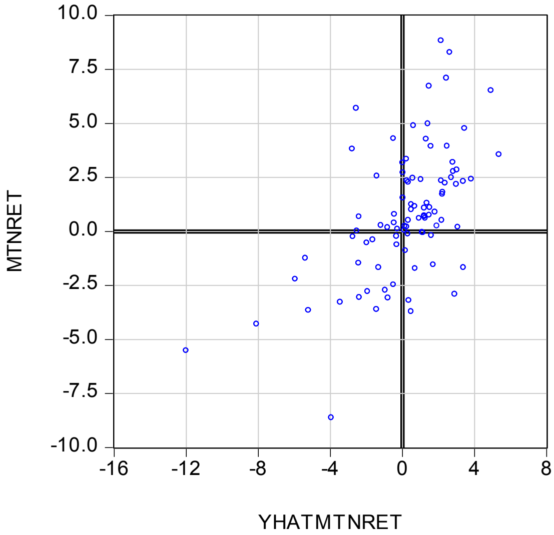

INVLTDRET
12
8
4
0
-4
-8
-12
-6
-4
-2
0
2
4
6
YHATINVLTDRET

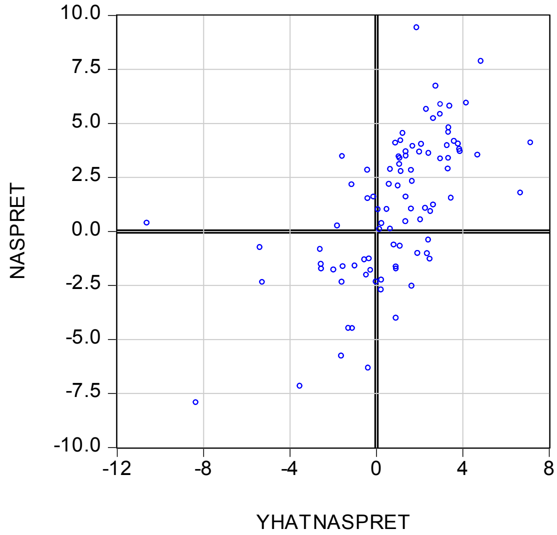

INVPLCRET
16
12
8
4
0
-4
-8
-12
-6
-4
-2
0
2
4
6
8
YHATINVPLCRET

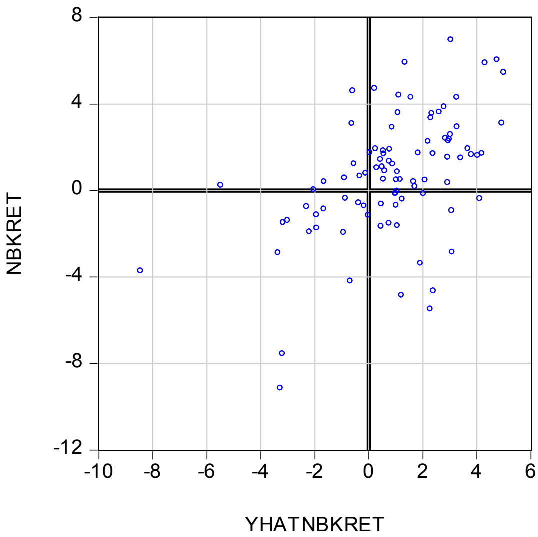

KUMBARET
15
10
5
0
-5
-10
-15
-20
-20
-10
0
10
20
30
YHATKUMBARET

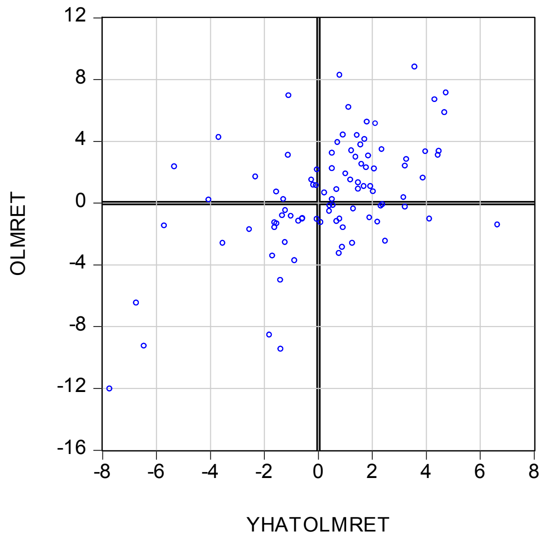

10
8
6
4
2
0
-2
-4
-6
MASSRET
-12
-8
-4
0
4
8
YHATMASSRET

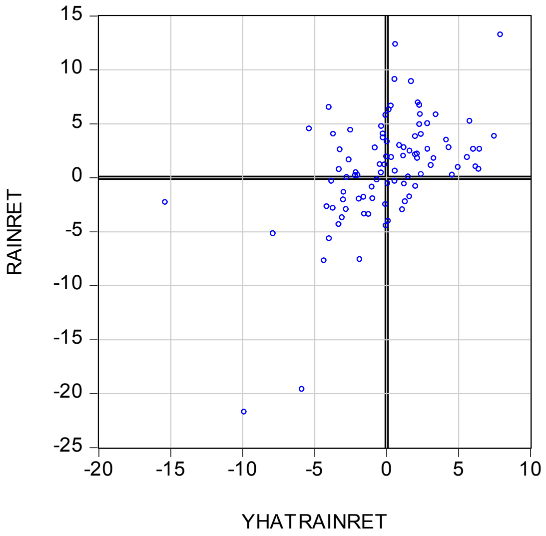

8
6
4
2
0
-2
-4
-6
MEDIRET
-5
-4
-3
-2
-1
0
1
2
3
YHATMEDIRET

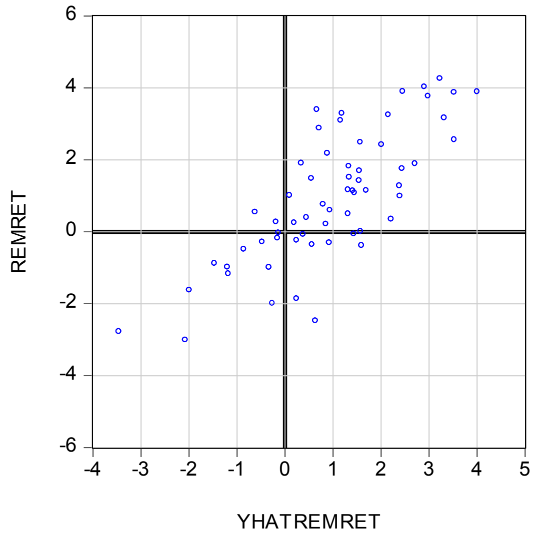

10.0
7.5
5.0
2.5
0.0
-2.5
-5.0
-7.5
-10.0
MTNRET
-16
-12
-8
-4
0
4
8
YHATMTNRET

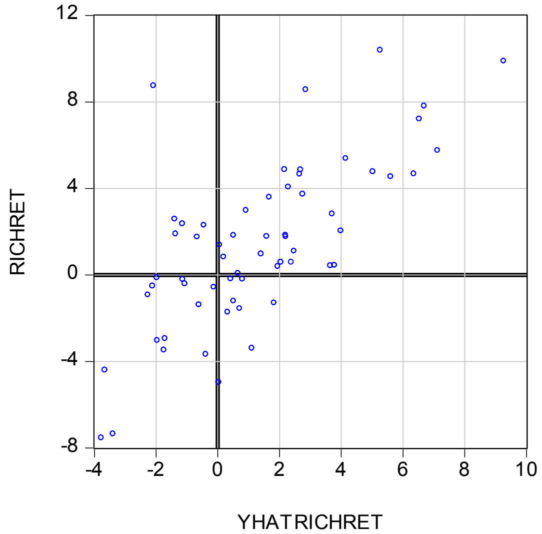

10.0
7.5
5.0
2.5
0.0
-2.5
-5.0
-7.5
-10.0
NASPRET
-12
-8
-4
0
4
8
YHATNASPRET

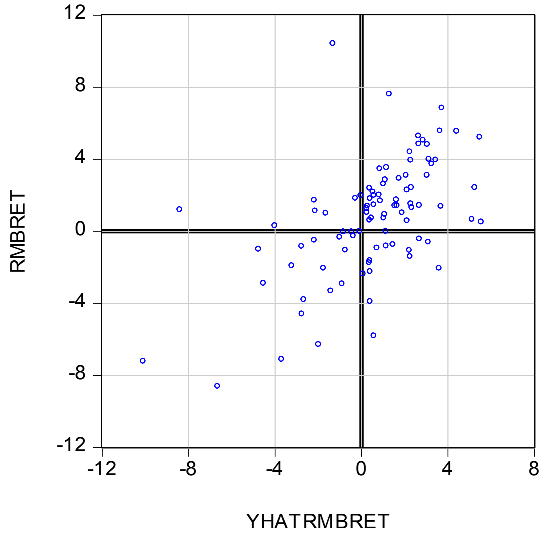

8
4
0
-4
-8
-12
NBKRET
-10
-8
-6
-4
-2
0
2
4
6
YHATNBKRET

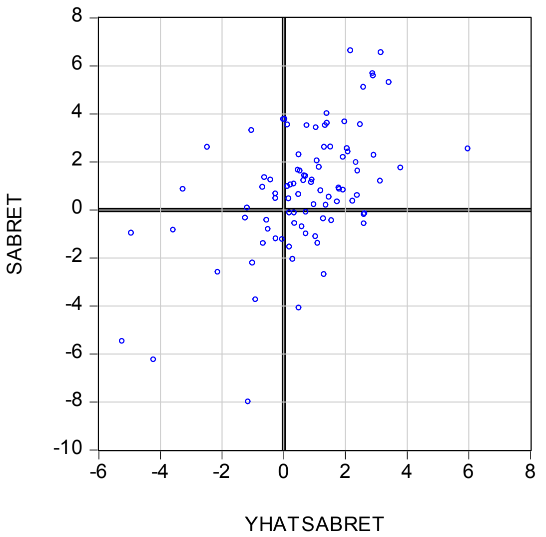

12
8
4
0
-4
-8
-12
-16
OLMRET
-8
-6
-4
-2
0
2
4
6
8
YHATOLMRET

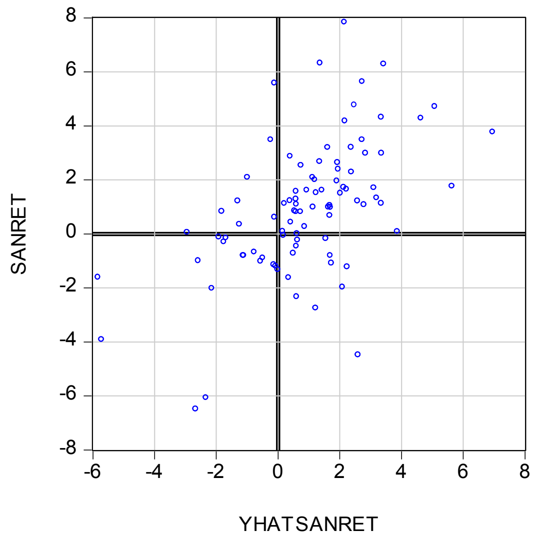

RAINRET
15
10
5
0
-5
-10
-15
-20
-25
-20 -15 -10 -5 0 5 10
YHATRAINRET

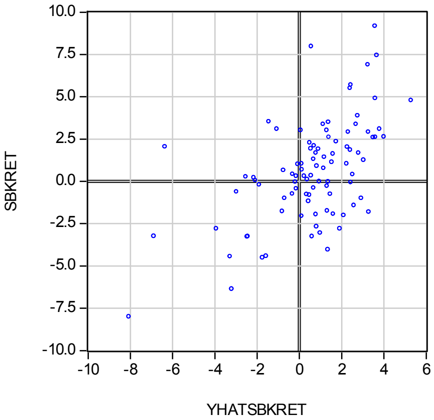

REMRET
6
4
2
0
-2
-4
-6
-4 -3 -2 -1 0 1 2 3 4 5
YHATREMRET

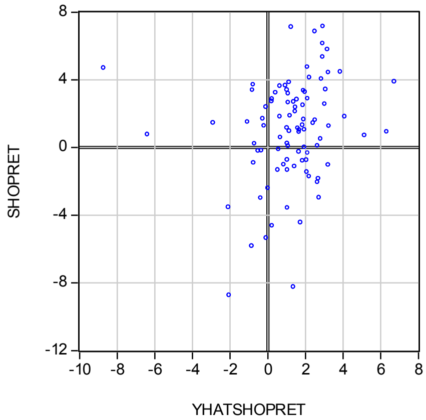

RICHRET
12
8
4
0
-4
-8
-4 -2 0 2 4 6 8 10
YHATRICHRET

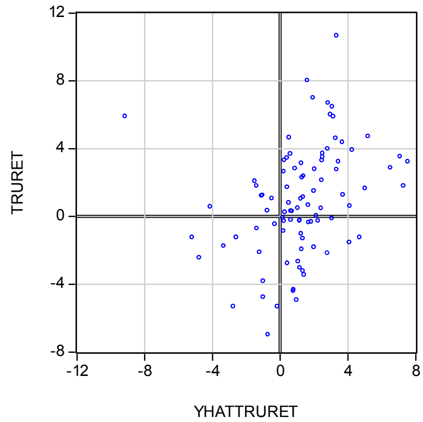

RMBRET
12
8
4
0
-4
-8
-12
-12 -8 -4 0 4 8
YHATRMBRET

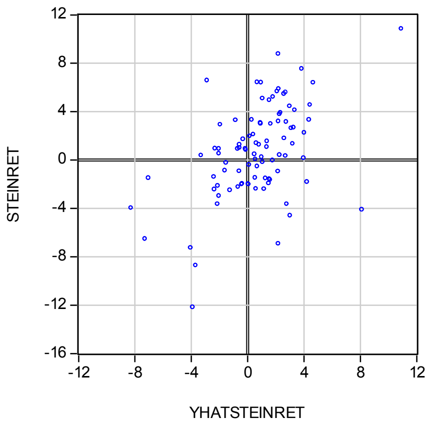

SABRET
8
6
4
2
0
-2
-4
-6
-8
-10
-6 -4 -2 0 2 4 6 8
YHATSABRET

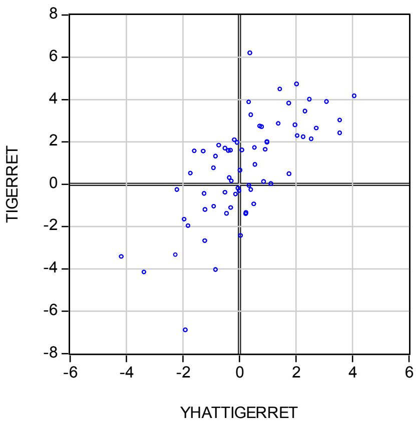

SANRET
8
6
4
2
0
-2
-4
-6
-8
-6 -4 -2 0 2 4 6 8
YHATSANRET

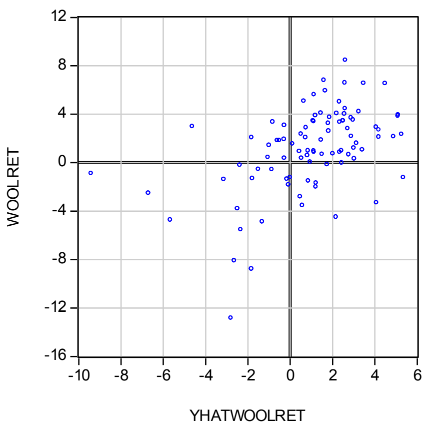

SBKRET
10.0
7.5
5.0
2.5
0.0
-2.5
-5.0
-7.5
-10.0
-10 -8 -6 -4 -2 0 2 4 6
YHATSBKRET

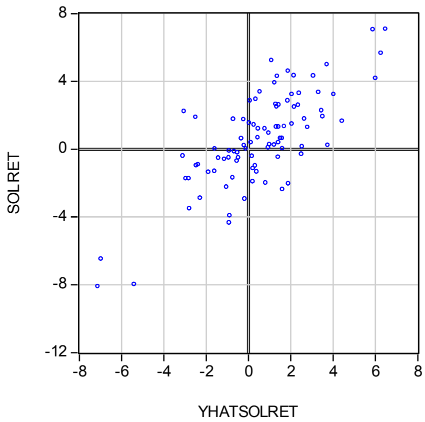

SHOPRET
8
4
0
-4
-8
-12
-10 -8 -6 -4 -2 0 2 4 6 8
YHATSHOPRET


TRURET
12
8
4
0
-4
-8
-12 -8 -4 0 4 8
YHATTRURET

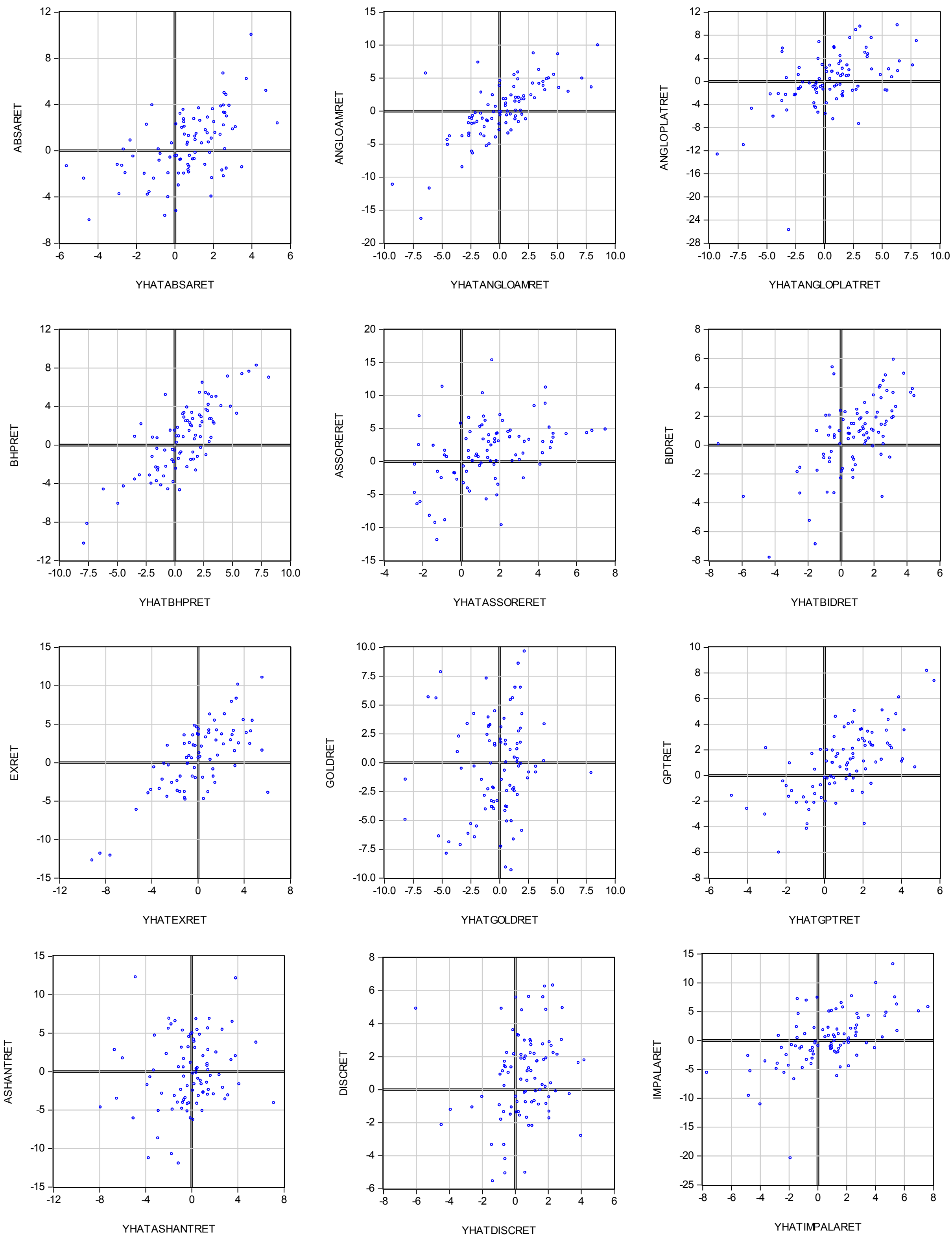

STEINRET
12
8
4
0
-4
-8
-12
-16
-12 -8 -4 0 4 8 12
YHATSTEINRET

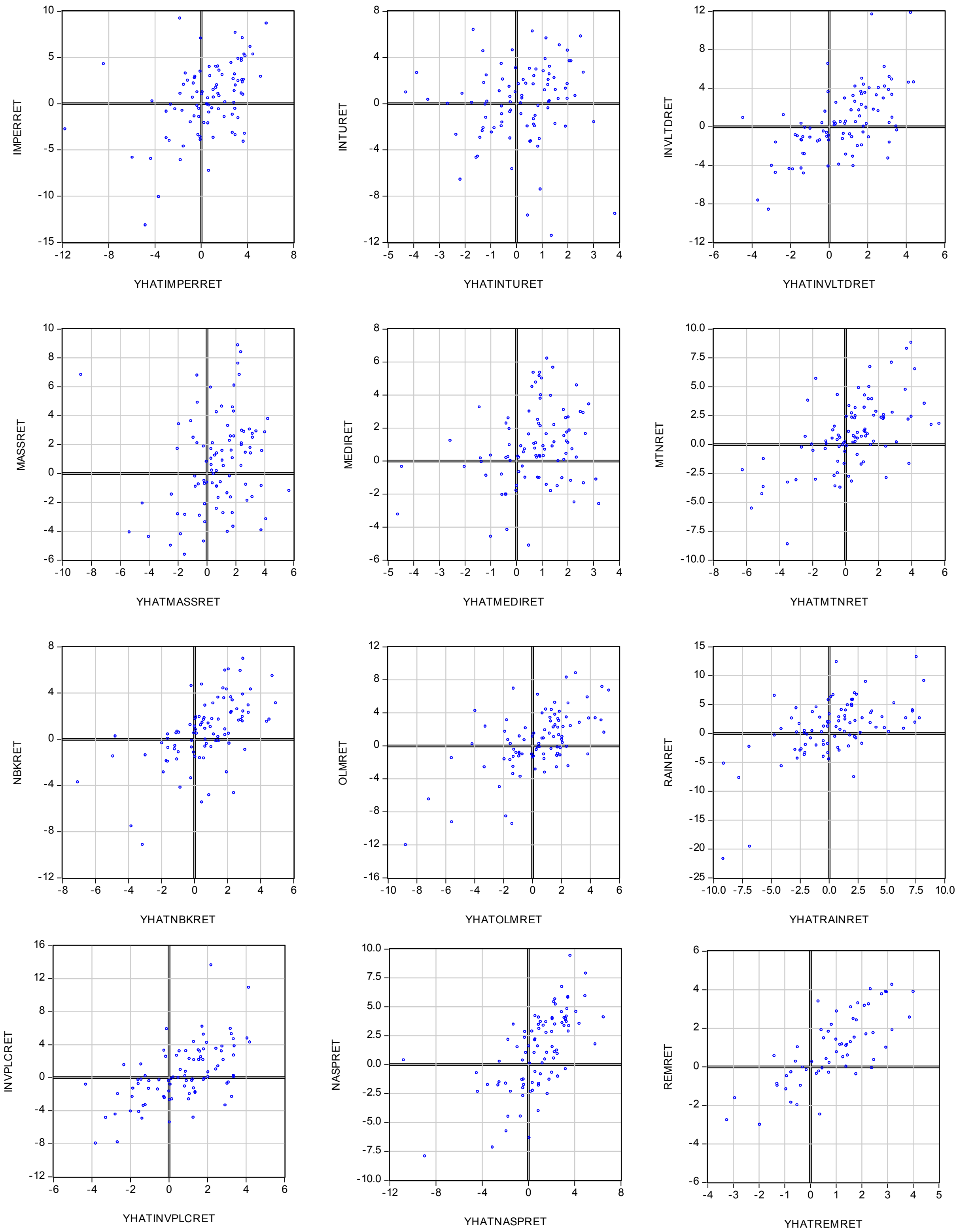

TIGERRET
8
6
4
2
0
-2
-4
-6
-8
-6 -4 -2 0 2 4 6
YHATTIGERRET

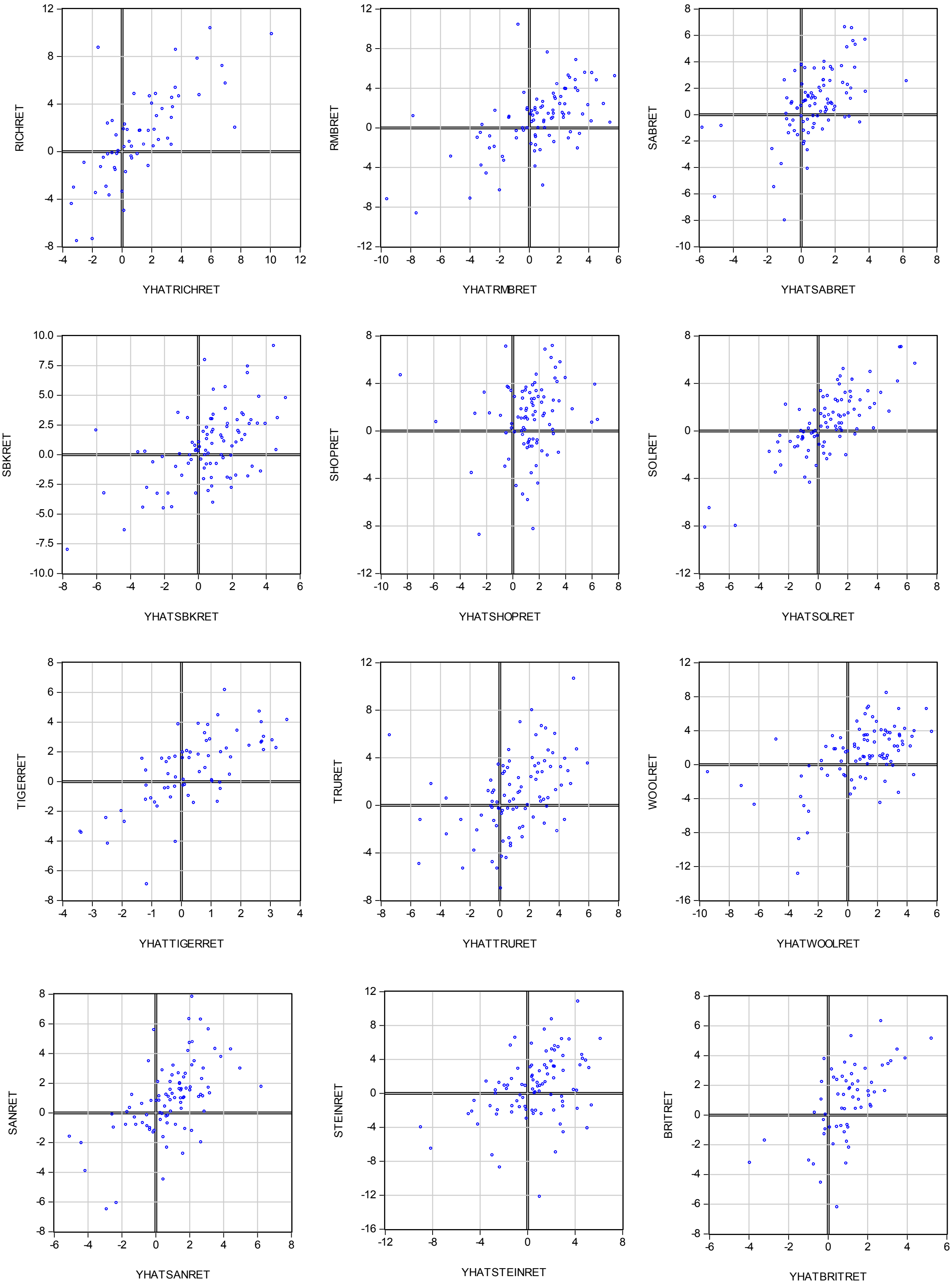

WOOLRET
12
8
4
0
-4
-8
-12
-16
-10 -8 -6 -4 -2 0 2 4 6
YHATWOOLRET


SOLRET
8
4
0
-4
-8
-12
-8 -6 -4 -2 0 2 4 6 8
YHATSOLRET

8.2 Scatterplot of Predicted versus Actual Share Returns for JSE /30-year Government Bond Yield Curve Parameter Model




IMPERRET
YHATIMPERRET
INTURET
YHATINTURET
INVLTDRET
YHATINVLTDRET
MASSRET
YHATMASSRET
MEDIRET
YHATMEDIRET
MTNRET
YHATMTNRET
NBKRET
YHATNBKRET
OLMRET
YHATOLMRET
RAINRET
YHATRAINRET
INVPLCRET
YHATINVPLCRET
NASPRET
YHATNASPRET
REMRET
YHATREMRET


RICHRET
YHATRICHRET
RMBRET
YHATRMBRET
SABRET
YHATSABRET
SBKRET
YHATSBKRET
SHOPRET
YHATSHOPRET
SOLRET
YHATSOLRET
TIGERRET
YHATTIGERRET
TRURET
YHATTRURET
WOOLRET
YHATWOOLRET
SANRET
YHATSANRET
STEINRET
YHATSTEINRET
BRITRET
YHATBRITRET