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Model-based teaching and learning of kinematics in an introductory physics course for underprepared students

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A dissertation submitted to the Faculty of Science at the University of Cape Town
in fulfilment of the requirements for the degree of Doctor of Philosophy in Tertiary Physics Education

August 2009

Abstract

This study concerns the application of a model-based approach for problem solving and conceptual understanding, in the context of kinematics, relating to the “foundation” component of an introductory physics course designed for students who are academically and scientifically underprepared. A new method for portraying objects in motion, “freeze frame” representation, was introduced. The particular visual conceptual model was employed as a representational bridge for translating physics information between different modes of representations as well as for eliciting qualitative information. The students’ handling of diverse representations, in particular, their use of freeze frame representations, when attempting kinematics tasks, requiring the generation of qualitative or quantitative solutions, was explored. An investigation was also carried out for the categories of cognitive structures generated by students from educationally disadvantaged backgrounds. An analysis of the data revealed that freeze frame representations support the interpretation and derivation of appropriate qualitative information but play no prominent role as a representational bridge. Moreover, 88% of the sample’s actions, when dealing with the various kinematics tasks, could be captured within four profiles from which it was possible to infer that most of the students’ cognitive constructs were associated with the category of “propositional” mental representations. The consequences of this work, based on the use of the modelling process for the teaching and the learning of physics at an introductory level, are discussed.

Acknowledgements

My heartfelt thanks go to…

- Andy Buffler and Fred Lubben for their advice and guidance in my endeavour to complete the project and present the thesis,

- all the GEPS students (class of 2007 and 2008) who participated in the pilot tests and the main study,

- the National Research Foundation (South Africa) as well as the University of Cape Town for funding the project, and

- my parents for their sustained effort, patience and support.

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1. Introduction

Visualisation plays a key role in the teaching and learning of physics. Zimmermann and Cunningham (1991, page 3) defined visualisation, in the context of learning mathematics, as "the process of forming images (mentally, or with pencil and paper, or with the aid of technology) and using such images effectively for mathematical discovery and understanding." Visualisation may therefore be viewed as the use of internal representations (cognitive constructs) and external representations, for example simulations, graphs and diagrams (Gobert, 2007). The relationship between internal and external representations has been well documented (for example Greca and Moreira, 2000; Schnotz and Bannert, 2003; Rapp, 2007). It is argued that the application of external visualisations promotes the construction of mental models which are crucial elements in the learning process. The development of comprehension occurs only when correspondence exists between the internal representation generated and its external version. However, the link between these two forms of representations is not straightforward. Many factors influencing the particular process have been reported, including the quality of external representations which impact on students' construction of mental models and hence upon their understanding.

An important aspect of external visualisations in teaching and learning is that they "contribute to students' understanding of physical principles by attaching mental images to these ideas" (Cadmus, 1990, page 397). In science education, studies implemented in the area of visualisation have focused principally on external representations. These include students' handling of diagrammatic representations of electric circuits and fields (McDermott and Shaffer, 1992) and images related to energy (Stylianidou *et al.*, 2002) and optics (Colin *et al.*, 2002). The use of specific symbols such as arrows as vectors in mechanics (Flores *et al.*, 2004) and the various purposes and formats of particular representations in biology textbooks (Plessis *et al.*, 2002) have also been investigated. Additionally, students' difficulties in interpreting graphical representations, mainly in the context of kinematics, have been extensively reported (for example McDermott *et al.*, 1987; Goldberg and Anderson, 1989; Beichner, 1994). Most importantly, in physics education, a considerable amount of research work has been implemented to explore the effect of using multiple visual (sequential or parallel) representations on students' problem solving abilities, and for conceptual development. The application of multiple representations has been introduced mainly to shift students' formula-centred problem solving strategy to recognising the involvement, the importance and the benefits of other representational modes, particularly visual representations. Moreover, with the use of a modelling process, students are engaged in portraying, mapping and translating information within and across different representations which are important for meaningful learning to occur (Kozma, 2003; Ainsworth, 2006; Gilbert 2007). Studies have reported on the positive outcome of using multiple representations for conceptual understanding (such as Van Heuvelen and Zou, 2001; Hinrichs, 2005). In contrast, it was revealed that the application of diverse representational modes does not *necessarily* result in an improvement in problem solving performance. Many factors were identified either in the physics context (for example Kohl and Finkelstein, 2005; Rosengrant *et al.*, 2006) or the cognitive domain (such as Kalyuga *et al.*, 1999; Seufert, 2003) for the ineffectiveness of using multiple representations as a problem-solving strategy.

The current study, involving the utilisation of a model-based approach (multiple sequential representations) for teaching and learning, was conceptualised for students who are academically disadvantaged. Of main concern in the present study is the "foundation" component of a special physics course designed for these students (Allie and Buffler, 1998). The central role of visual representations has been exploited in this course for developing the comprehension of the fundamental physics concepts as well as the appropriate physics reasoning associated with the various mathematical expressions.

Of interest in the present work, is the kinematics section in the foundation component of this course. The lecture notes, tutorial tasks and teaching intervention were designed to be representation-rich, with emphasis placed on the modelling process for problem solving and comprehension of kinematics concepts. The particular topic was chosen as a research area because of the students' familiarity with the context from school level. Furthermore, an opportunity was provided to introduce "freeze frame" representations which are based on the idea of stroboscopic photographs. Freeze frame representations were included as an intermediate step for problem solving.

Two theoretical frameworks underpin the current study. The first is the model-based view of physics and physics education. According to Koponen (2007), the "semantic view of scientific theories" is limited and inadequate as it does not provide a proper description regarding the methodology around how a theory and an experiment are linked. Consequently, the role of models functioning as conceptual mediating instruments between an experiment and a theory (Morrison and Morgan, 1999) was adopted. The second framework employed is based on the cognitive work of Johnson-Laird (1983) which provides three classifications for mental representations, namely propositional constructs, mental models and mental images. A comprehension of a situation or process under consideration occurs at the level of mental model which also acts as a medium for making links between the two other forms of internal representations.

The present work investigates students' handling of multiple representations, in particular the use of freeze frame representations when attempting different kinematics tasks. The study also aims at contributing to the body of knowledge concerned with the relationship between students' mental and external representations. Johnson-Laird's (1983) cognitive framework of sense-making is applied in the physics domain for categorising the mental representations of introductory physics students with poor academic and conceptual backgrounds.

2. The role of visualisation in the teaching and learning of physics

2.1 Models and modelling in physics

“Models” and “modelling” are advocated to play vital roles for the learning of science, learning about science and learning how to do science (Justi and Gilbert, 2002b). In fact, it may be argued that one of the goals of science education is to encourage learning at a profound level by providing students with opportunities to be directly involved in both the development and manipulation of their own models (Gobert, 2007). Nowadays, physics education is placing much emphasis on model-based teaching and learning where models and modelling constitute essential elements of the physics enterprise. In general, models are considered as simplified and idealised versions of real world phenomena or processes. They are crucial for communicating, constructing and comprehending scientific knowledge (Harrison and Treagust, 2000). They allow the relationships, the characteristics and the properties of directly inaccessible phenomena to be perceived or imagined (Gilbert, 2007), and may be used for making abstract phenomena observable (Francoeur, 1997), for example the depiction of magnetic and electric field lines. Additionally, models provide the basis for explaining and describing entities or processes, for making predictions (Etkina *et al*., 2006) as well as for interpretation of experimental data (Ryder and Leach, 2000). They are also considered as exploratory or investigative instruments (Morisson and Morgan, 1999; Mathewson, 2005). Moreover, models are “semi-autonomous”, that is, on the one hand they have partial dependence on both a theory and a physical system, and on the other hand they can be partially independent of either the physical system or the theory, which consequently allow them to function as instruments (Morrison and Morgan, 1999). Above all, in order to fulfil these different roles, models act as “a representative rather than a representation of a physical system” (Morrison and Morgan, 1999, page 33). They have the capacity of depicting a version of the physical system itself. Also, by analysing the connections among the features and constituents which compose the structure of the model it is possible to understand the behaviour of the physical system and hence certain of its theoretical aspects when observing demonstrations. Modelling is often referred to as the process of building, applying or revising of models (Justi and Gilbert, 2002a). A more detailed description was provided by Halloun (1996) who specifies the need to equip students with a definite set of rules for dealing with models during the different stages of their manipulation, in particular for their evaluation or validation. Moreover, the definition given by Nersessian (1995) stresses the importance of mental processes during the modelling activity where mention was made about the application of “thought experiment”. Hence, the essential feature of modelling lies in the fact that it is “this process of interpreting, conceptualising and integrating that goes on in model development” that allows learning to occur (Morrison and Morgan, 1999, page 31).

In order to meaningfully engage in the modelling process, recognise its importance and value for both instruction and learning, an understanding of the nature of models is essential (Lehrer *et al.,* 1994). However, studies have shown that students have naïve notions regarding models (Grosslight *et al.,* 1991; Treagust *et al.,* 2002). There is evidence that even science teachers display inconsistent views about models and their applications in science (for example Van Driel and Verloop, 1999). From the study by Smit and Finegold (1995), it was revealed that a link exists between the teachers’ educational background and their views on the nature of models. Furthermore, Justi and Gilbert (2002b) found that during instruction, the modelling process was rarely included and implemented due to the teachers’ lack of skills or interest. According to Justi and Gilbert (2002b) the teachers’ superficial views on the nature of models are translated in their attitude towards and capability of conducting activities based on the modelling process. They are unaware of their students’ views of models and they are of the opinion that their students are not interested in the nature of models and the modelling process. Even if modelling activities were included, the teachers failed to deal with the models generated by their students for learning and understanding of concepts.

2.1.1 Model-based views of physics

In physics, of main interest is the relationship between the theories which constitute the declarative aspects (the physics world) and the experiments which make up the procedural aspects (the real world). It can be claimed that both the enterprise of physics and the teaching of its elements may best be understood through a modelling framework. Figure 1 illustrates a model-based view of physics. The framework is made up of three worlds which are related to one another, namely, the real world revolving around observations, experiments and measurements, the world of physical theories and the world of models.

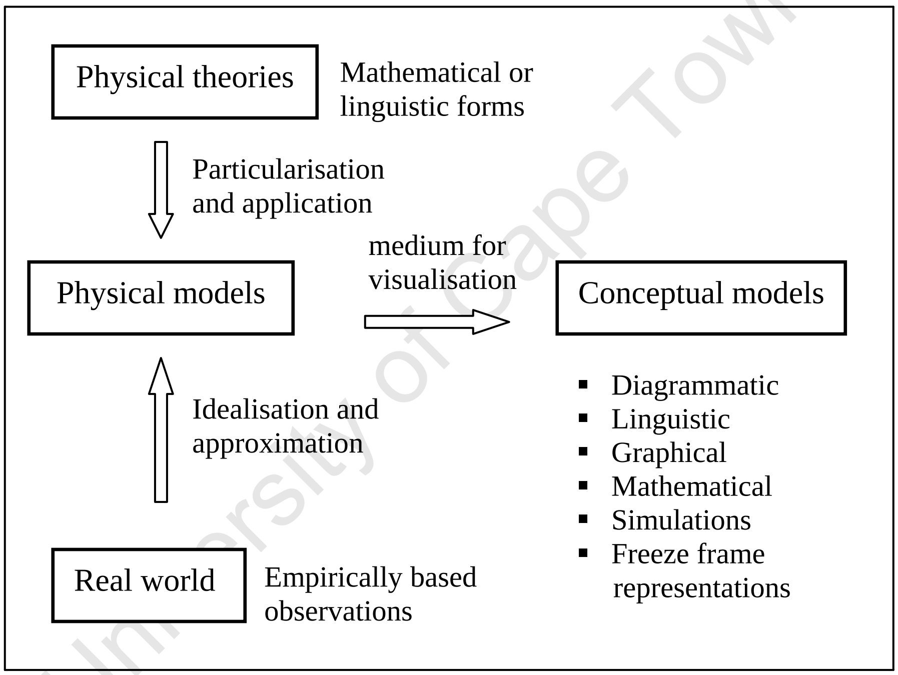


Figure 1. *Framework for a model-based view of physics.*

According to Koponen (2007), explanations of the role of models and modelling and how they are used for linking the physics and the real worlds are philosophically embedded. The author argues that the modelling process in physics is grounded on the “semantic view of scientific theories”. This particular philosophical view is characterised by the idea that a given physical phenomenon can be expressed by a variety of different models all derived or emerging from the same theory (referred to as theoretical models) and obeying the same fundamental idea encompassed by the theory (Morrison, 1999). Models are essentially viewed as mediums to substantiate a theory (Morrison and Morgan, 1999; Koponen, 2007). Moreover, models in physics are mainly considered to be expressed in mathematical forms (Redhead, 1980; Bunge, 1983).

In physics, modelling revolves mainly around constructing predictions from well established theories and making connections between the predictions and experimental data (Koponen and Mäntÿla, 2006). However, the present philosophical underpinning of the physics modelling process is limited and inadequate (Morrison and Morgan, 1999) and many reasons are provided by Koponen (2007) for a need to widen the philosophical view. According to Koponen (2007), there is no clear account of the methodology, currently based on the notion of "constructive realism" and "constructive empiricism", around how a theory and an experiment are related. Constructive realism is grounded on the idea of "similarity" between the model and the real world entity being modelled. However, this notion is not clearly defined, since assumptions and inferences have to be made about the characteristics of the non-observable aspects. Furthermore, for the observable elements, the use of only experimental data for the matching process is inadequate with the result that further suppositions are required. Constructive empiricism is governed by the idea of "empirical adequacy" which specifies that models emerging from theories should have aspects of their structures corresponding to parts of observable real world phenomena. Also, the semantic view of scientific theories does not take into account the semi-autonomous nature of models. Instead of being derived mainly from a particular theory, models "are made up from a mixture of elements, including those from outside the original domain of investigation, that they maintain this partially independent status" (Morrison and Morgan, 1999, page 14). Finally, the notion of modelling as a two-way process between the physics and the real worlds is not considered. In order to cater for all these different aspects in physics modelling and provide an authentic view of the physics enterprise, it is essential to consider the role of models as conceptual mediating instruments between theory (the physics world) and experiment (the real world) (Hughes, 1997; Morrison and Morgan, 1999; Koponen, 2007). The relationship between a theory and an experiment is therefore not a simple, straightforward process and models play a more important role than simply acting as interpretive tools (Portides, 2007).

With reference to Figure 1, the declarative aspect of the physics enterprise constitutes mainly of physical theories which are abstract and external formulations of empirically based observations. The explanations provided by theories are therefore not directly based on processes, events or conditions as they exist or occur in the natural world (Morrison, 1999; Matthews, 2007). Physical theories are manifested either in mathematical or linguistic forms and are semantically blind when not situated within a context (Greca and Moreira, 2002). Therefore, within this framework, physical theories acquire meaning through their application to real world phenomena. However, the connection between a theory and a real world phenomenon is complex with no straightforward agreement between them and hence in order to be able to link the two entities, there needs to exist a technique of matchmaking (Darling, 2002). For the procedural aspect, empirical measurements obtained from observations of a particular physical system are statistically manipulated in order to generate an experimental law or a model of the data which is usually expressed in mathematical or algebraic form (Koponen, 2007). For the declarative aspect of the physics domain, from the high level theory (for the same physical system), a theoretical model emerges and it is in accordance with the principles and conditions of the experiment under consideration. Consequently, the matching process is made possible since the theoretical and empirical models have both been "structured into a mutually compatible form" (Morrison and Morgan, 1999, page 22). The theoretical model and experimental law are compared, and a series of corrections or small changes are possible in both the experimental conditions, and theoretical interpretation and formulation. This bi-directional revision process may lead to the construction of a hierarchy of models until a degree of mutual correspondence is achieved (Koponen, 2007). The theoretical models derived from the theoretical superstructure also encompass aspects of the physical theory which are applied to idealised and approximated versions of a real world phenomenon. Portides (2007) differentiates between the terms "idealisation" and "approximation", where "idealisation" refers to either changing or ignoring aspects of the nature (physical appearance, structure and properties) of a physical system in a given theory or model. On the other hand, "approximation" refers to the simplification of either the nature of certain features of the physical system in a theory or model, or the simplification of the whole theory (or model) resulting in a version which is very close to but does not mirror the physical system.

Physical models may thus be seen to mediate between physical theories and real world phenomena since they possess the particular characteristic of comprising aspects of a physical theory which are applied to an idealised and approximated version of real world phenomena (Greca and Moreira, 2002). Hence, physical models play a central role by allowing the visualisation of the ways in which empirically observed natural phenomena are connected to aspects of physical theories (Koponen and Mäntÿla, 2006). Via physical models, the semantic content of physical theories is highlighted (Greca and Moreira, 2002). The ideas embedded in a theory are evoked and can be fully explored since physical models, by being concrete and external, provide a context for the application of the physical theory. As Jammer (1974, page 11) points out, physical models are "powerful heuristic pictures which in themselves sum up the essential aspects of the theory, so it is possible to visualise with more ease through them the explanatory principles of the theory".

2.2 Models in the teaching and learning of physics

2.2.1 Conceptual models

In science education and even physics education, there is no definite method of how to implement model-based teaching (Justi and Gilbert, 2002a; Gobert and Buckley, 2000). In physics education, various model-based strategies have been designed with the aim of not only developing students' understanding of specific topics but also their conceptions of the nature of models, and to familiarise or instruct them about the modelling process. Examples include Frederiksen *et al.* (1999) who used simulations to develop students' notion of the tentative nature of models in the context of electricity. Hestenes's (1992) framework, made up of three main stages, termed as "model building, ramification and deployment", was applied in the field of learning Newtonian mechanics. Raghaven and Glaser (1995) applied the MARS (Model Assisted Reasoning in Science) curriculum for teaching the concepts of mass and force to high school students. Halloun (1996 and 1998) used a five step framework consisting of "selection, construction, validation, analysis and deployment of models" for attempting textbook tasks which the author termed as "paradigmatic problems". The modelling nature of the physics enterprise is also fore-fronted in textbooks, such as Matter and Interactions (Chabay and Sherwood, 2006) which is widely used in courses designed for introductory physics students.

The examples of model-based strategies in the physics domain highlight the key role of models and modelling for both the declarative and procedural aspects. The right-hand side of the modelling framework in Figure 1 refers to the pedagogical aspects: the models employed for the teaching of the physics discipline are referred to as *conceptual models*. They are didactical versions of physical models generated to explain and communicate scientific knowledge, facilitate discourse comprehension or for the teaching and understanding of the corresponding physical model (Greca and Moreira, 2000 and 2002). Similar to physical models, the conceptual versions can be expressed in various forms such as mathematical, linguistic, simulations and numerical or graphical models which are all abstract in nature, and scale models also referred to as "material artefacts" which are concrete depictions emphasising mainly the physical appearance and structure of the entity being modelled (Harrison and Treagust, 2000). Overall, conceptual models are external, complete, accepted and shared representations (Greca and Moreira, 2000).

As was pointed out earlier, only physical models can be visualised and not physical theories. A given physical model can be portrayed into either mathematical or various visual conceptual forms thus enabling the translation of information and also the exploration of relationships among them. Unlike their mathematical counterparts, the visual conceptual models fore-front the main ideas or concepts of a physical theory, by its application via a physical model, which otherwise remain hidden within the syntactic structures.

2.2.2 Mental models

Gobert and Buckley (2000, page 892) defined model-based learning "as the construction of mental models of phenomena" and model-based teaching as "any implementation that brings together information resources, learning activities, and instructional strategies intended to facilitate mental model-building both in individuals and among groups of learners". Research concerned with teaching and learning in science has drawn significantly from the theoretical cognitive work of Johnson-Laird (1983). Figure 2 depicts the cognitive framework of sense-making.

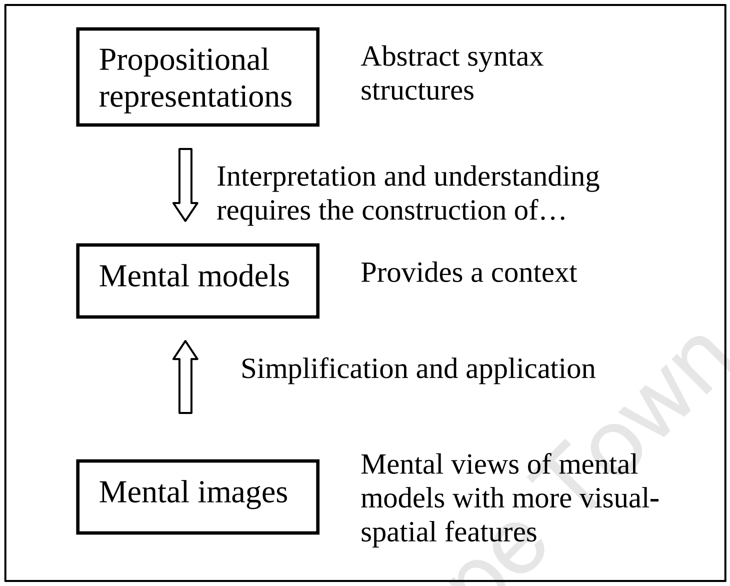


Figure 2. *Johnson-Laird's cognitive framework of sense-making.*

According to Johnson-Laird (1983), there are three main categories of internal (mental) representations, namely propositional constructs, mental models and mental images. These three classifications of internal constructs are personal, abstract, and have different purposes and structures. Propositional representations are constituted of syntactic structures which connect a series of symbols together. The symbols can be words (definitions or formulae), equations or numbers (for example computer language which corresponds to 0s and 1s) which are meaningless and abstract without a context. Mental models, usually generated by perception or imagination, are analogical representations of real world objects, physical events or situations. They are incomplete and unstable and hence subject to change, can either be improved, discarded or replaced with the accumulation of new information, content knowledge or with more exposure and familiarity to the situation. The type of mental model constructed is dependent on the individual's knowledge and experience with the situation under consideration and also on the purpose of the model (Borges and Gilbert, 1999). Mental models are personal and functional only to the person constructing it. The third category of mental representation is the mental image which is a "coherent and integrated representation of a scene or object from a particular viewpoint in which each perceptible elements occurs only once with all such elements being simultaneously available" (Johnson-Laird, 1983, page 147). In short, mental images are internal views of mental models with greater visual spatial features which allow perceptible information to be visualised. The relationship between the three forms of mental structures is dynamic in nature. Sense-making and interpretation of abstract propositional representations occur via mental models. It is at the level of the mental model that the propositional constructs and the mental images are provided with a context for their application thus acting as a support for the comprehension of the situation or process under consideration.

According to Johnson-Laird, an indicator of comprehension is the ability to generate explanations. The author argues that the key factor for comprehension is the construction of a working mental model which allows for inferences and predictions to be made, for adjustments according to these predictions and consequently an understanding of the situation which is manifested by the explanation provided. Therefore, the comprehension of the semantic content of a physical theory together with the resulting physical model requires the construction of a corresponding mental model (working model) of this physical model. If the predictions and explanations emerging from a mental model (internal representation) are consistent with scientific knowledge (usually represented externally via conceptual models) then it can be concluded that an appropriate mental model of the physical model has been built (Greca and Moreira, 2002).

The physics discipline comprises scientific theories. Therefore, teaching of physics should aim at encouraging thought and reasoning processes for developing students' conceptual understanding of these theories thus enabling the application of the concepts to new situations or contexts (Rapp, 2007).

2.2.3 Relationship between external (conceptual models) and internal (mental model) visualisations

To learn with an external visualisation, an internal visualisation of the process or entity under consideration needs to be created (Gobert, 2007). In science education, it is difficult, if not impossible, to have a consistent and definite view of students' mental models (Greca and Moreira, 2000; Coll *et al*., 2005). However, the reasoning employed by students within a specific context can provide an insight about their mental models (Gobert and Buckley, 2000). Students' external manifestations can either take the form of verbal or visual representations, sketches, diagrams or material artefacts (Halloun, 1996; Gobert and Buckley, 2000).

The process of constructing mental models is complex (Clement, 2000) as there is no direct relation between the internal (mental model) and external (conceptual model) representations (Greca and Moreira, 2000). Clement (2000) as well as Greca and Moreira (2000) provide many factors accounting for the difficulty of linking the two types of models during instruction. Students fail to recognise the actual phenomenon or process being represented by the conceptual model. They are of the view that the conceptual model is in fact the real world entity. Their lack of content or background knowledge about a subject matter may result in an inability to interpret a situation or process as highlighted by the given conceptual model. Moreover, since students are used to learning at a surface level, consisting mainly of rote memorisation, it is difficult for them to interpret the model at a deeper level. Sense-making is then limited only to the level of description rather than probing for the explanation provided by the model. In the terminology of Rapp (2007), the students have a superficial cognitive engagement whereby they are poorly motivated to be indulged in profound level of understanding, thought and reasoning processes. Additionally, the ways in which models are presented in textbooks do not provide students with an authentic view around how a process or phenomenon can be represented both, internally and externally, for its manipulation and comprehension before being structured in a scientifically acceptable format. Conceptual models in physics textbooks are often portrayed in mathematical forms which make it difficult to perceive and visualise the corresponding physical model (Greca and Moreira, 2002). The manipulation of mathematical formulation does not promote the construction of mental models but hinders the development of the conceptual understanding of the scientific theory in question. Also, prior knowledge, which is socially and culturally embedded (Greca and Moreira, 1997), is a key factor hindering the construction of appropriate mental models for a given situation. According to Greca and Moreira (2000), when students are confronted with their mental models and the given conceptual model, there is a possibility that the conceptual model is interpreted and understood in terms of the available prior knowledge. Useful and important information obtained from the conceptual model is applied to the mental model thus leading to the creation of models which are neither totally incorrect nor completely consistent with accepted scientific knowledge. Moreover, students can also discard their mental model, accept and learn the new knowledge by rote for exam purposes.

The two frameworks on which the current study is based, the model-based view of physics, which is external in nature, and Johnson-Laird's cognitive model of sense-making have certain similarities as illustrated in Figure 3.

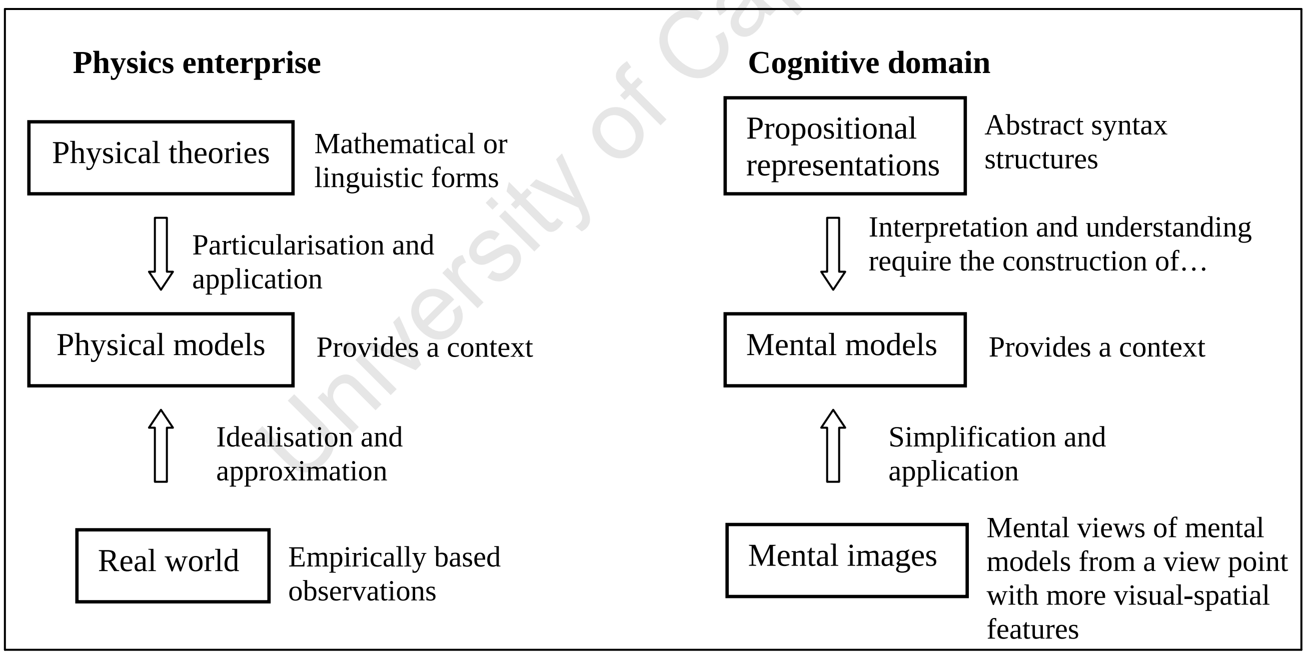


Figure 3. *Similarities between the model-based view of physics and Johnson-Laird's cognitive framework of sense-making.*

Both the physical theories and propositional representations are made up of syntactic structures which are abstract in nature when de-contextualised. In the physics enterprise, the real world consists of elements such as observations, experiments and measurements. In the cognitive domain, mental images are based on experience and observations from actual phenomena or physical events. Visualisation and comprehension of physics principles occurs via physical models. Within this framework, physical models may be viewed to mediate between the physics and the real worlds. In the cognitive domain the process of understanding occurs upon the construction of a mental model which allows visualisation to take place. Mental images, which also enable visualisation, acquire their meaning at the level of mental model which provides a context for their application. Mental models therefore acts as a medium allowing connections between the two other internal structures to be made.

It has been argued by many science educators and researchers that comprehension of a physical model can only take place if there is correspondence between the mental and the conceptual forms of the physical model, particularly the visual conceptual models. In order to support the construction of mental models compatible with scientifically accepted knowledge visualisation tools (external visualisation) may be used for teaching and learning. The importance of these visual aids lies in the fact that they make explanatory mechanisms more visible, which otherwise remain abstract and blurred. During instruction it is common practice to expose students to simulated or real world demonstrations of phenomena or situations to highlight the concepts being studied. Various visualisation tools are employed for teaching specific topics, such as kinematics (for example Simpson *et al.*, 2006; Thornton and Sokoloff, 1990). Even though many benefits were reported for teaching and learning via the application of various visual tools, they may also have a negative impact on students' construction of mental models, and consequently on their understanding (Rapp, 2007). Rapp (2007) argues that the design of these visual aids is of key importance as an erroneous depiction may result in the development of incorrect mental models. The presence of superfluous details may lead to failure in capturing the essential message being communicated if the students are uncertain about which features of the visualisation are to be considered. To ensure sense-making, the development of appropriate mental models, the concepts, ideas of interest should be projected in an organised, consistent and rational way. The relationship between internal and external visualisations is also stressed by Tversky (2007) who mentions the need to consider the two cognitive principles, "congruence" and "apprehension", in the design of external visualisations for the latter to promote comprehension and learning. Briefly, the "congruence principle" refers to the depiction of only the relevant information, that is, only the key ideas are presented without additional information which can act as interferences, while the "apprehension principle" is the projection of information to be conveyed and assimilated in an organised and consistent manner.

The literature reports instructional and problem solving strategies, across different physics contexts, which focus on the application of diverse representations for depicting information. The main idea underlying this particular strategy is similar to that of the modelling process. A particular scientific theory is applied via a physical model (the task itself which are normally presented in linguistic, diagrammatic or graphical forms) which is then expressed in various forms of external representations (conceptual models). This experience results in the formation of internal representations (mental models) and consequently the comprehension of the concepts underlying the theory if there is correspondence between the internal and external constructs. Visualisation, either internal or external, therefore plays a key role in the learning process by enabling students to handle and manipulate a specific representation and also to translate information into different representational forms (Gilbert, 2007).

2.3 Teaching and learning with multiple representations

The main aims of teaching students the use of multiple representations are to shift their problem solving strategy from the utilisation of mathematical formulations, to equip them with the necessary skills for solving problems with diverse forms of representations and to recognise the benefits and importance of qualitative depictions (Dufresne *et al.*, 1997). With multi-representational problem solving strategies students are involved in the whole process of representing and translating information from one representational mode to another and learn to make explicit use of visual representations for qualitative reasoning and understanding (Van Heuvelen, 1991a). Ainsworth (1999) claims that the use of a multitude of external representations supports the development of students' understanding of a situation, process or an idea. It has been further argued that learning and comprehension grounded on multiple representations are more pronounced with the students' direct and active involvement in the construction of the representations and manipulation of the information presented (Cox, 1999; Goldman, 2003; Rapp, 2007). However, the effectiveness of teaching and learning based on multiple representations is influenced both by the design of these external constructs, and students' ability to engage in various cognitive activities when manipulating several representations (Ainsworth, 2006). These cognitive processes include the ability to decode, interpret and derive information from a particular representation, which in turn depends both on the students' familiarity with the structures and syntax inherent to the depiction and on their background or conceptual knowledge about the domain under consideration. Also involved is the ability to choose the appropriate representations (meta-representational skills) for attempting a particular problem, to generate representations of the situation presented in the task (representational competence), to intra and inter relate relevant elements of representations and also to translate information. Van der Meij and de Jong (2006) distinguish between the term "relate" and "translate". The process of relating representations includes the mapping of different elements from various representational modes which present similar information while the process of translating involves transforming information from a given representation to fit other comparable depictions.

2.3.1 Problem solving with sequential multiple representations

Problem solving involving the application of multiple representations may take the form of specific strategies consisting of a series of sequential steps where information provided in a task is first structured in different visual forms before finally being manipulated mathematically. Figure 4 presents an illustration of the application of sequential multiple representations for problem solving.

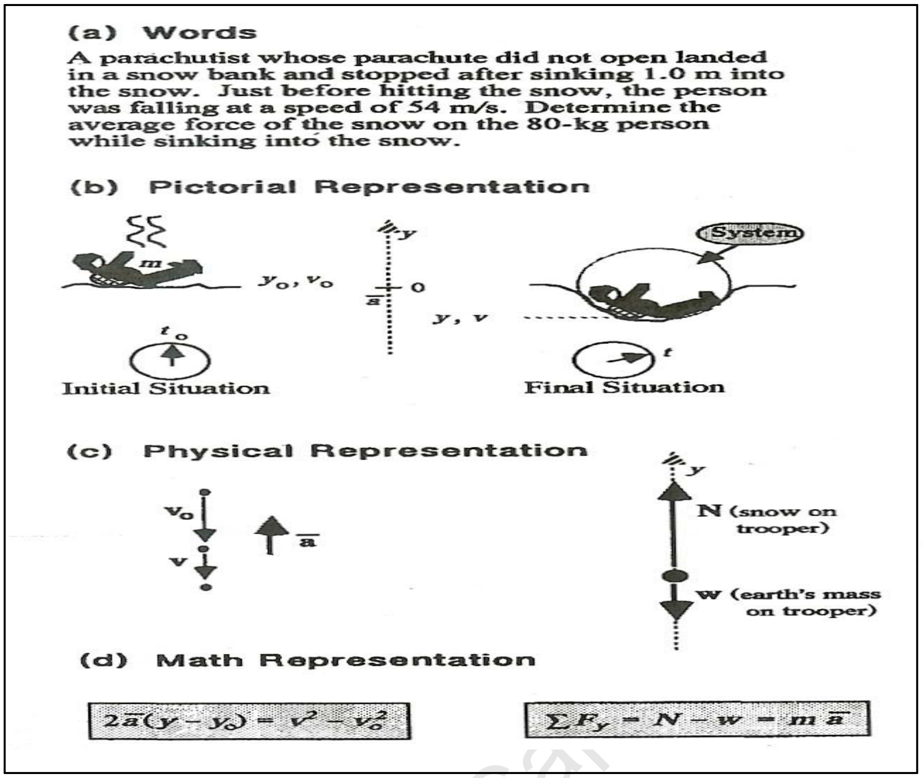


Figure 4. *An example of the use of sequential multiple representations for problem solving (Van Heuvelen, 1991a, page 892).*

Van Heuvelen (1991a) provides a description for this particular technique of handling problems. In general, a pictorial representation for the whole situation described in a problem statement is first constructed. There is a translation from linguistic to pictorial form in which all variables together with their quantitative information (if known) are indicated. A physical representation then follows whereby the pictorial depiction is simplified. The object of interest, referred to as the "system" is often circled and idealised. Moreover, a combination of various physics-based visual tools such as the coordinate systems, vectors, arrows and snapshots may be applied. Depending on the context, kinematics graphs or bar charts (for the work-energy topic) may also be present. In the last stage, the relevant equation is employed and related to the physical representation for generating the appropriate mathematical expression for the situation. The visual representations (pictorial and physical) are qualitative in nature and act as concrete intermediates between the abstract linguistic and mathematical representations. According to Van Heuvelen and Zou (2001), the visual representations allow students to perceive the relationships among variables, to substantiate and ascribe significance to the values obtained from working out an equation hence leading to a better comprehension of the situation presented. It was further argued that in order to develop a deeper understanding of these visual representations and recognise their importance for effective handling of problem tasks, students should be initially exposed to the qualitative depictions instead of their mathematical counterparts.

Various studies have explicitly unpacked the various steps for representing a problem into a variety of representational forms. Examples include the work by Van Ausdal (1988) for drawing motion diagrams in kinematics, Puri (1996) for free body diagrams and, Van Heuvelen and Zou (2001) for using bar charts in the context of work and energy. The application of multiple representations as a problem solving strategy for dealing with context-rich problems during cooperative learning was also explored (for example, Heller *et al.*, 1991a).

<u>2.3.1</u> (a) Effect of teaching sequential multiple representations as a problem solving strategy

Van Heuvelen (1991b) made a comparison of the problem solving strategies and performance of students engaged in a traditional and reformed instructional environment, the Overview Case Study Physics (OCS). For the latter case, diverse representations were extensively used either during instruction or when students attempted paper and pencil tasks. The students were taught how to translate back and forth from one form of representation to another, for example, drawing the visual (pictorial and physical) representations from a linguistic representation of the question or working backwards from the given mathematical expression to generate the visual representations and even frame a problem statement corresponding to the given equations. Tasks where no mention was made about the use of multiple representations, based on work-energy, impulse-momentum, Newton's Second Law and kinematics, were administered to the participants. Upon comparison of the two cohorts, the majority of the students from the reformed environment applied multi-representational problem solving strategies while a larger proportion of the sample from the traditional course directly used and manipulated mathematical formulations. It was also noted that most of the participants from the reformed group had a better performance when various visual representations were applied for working out a value.

For the study implemented by Rosengrant *et al.* (2005), the sample was exposed, in detail, to the procedures for solving problems across various contexts using different representations. Students learned to translate from linguistic to mathematical representations via the application of a free body diagram. Five questions, in multiple choice format without any instruction regarding the application of various representations, were completed. It was found that when attempting the different tasks, a large majority of students drew a free body diagram although they were not prompted or hinted to do so and were aware that no marks would be allocated for the inclusion of the particular visual representation. However, in very few instances a free body diagram was consistently provided across all five questions. The authors hypothesised that the particular visual representation is included only when, according to the students, a task has a high level of difficulty.

The work of Kohl and Finkelstein (2008) provides additional evidence regarding the positive effect of exposing students to the application of multiple representations for problem solving. The population made up of “novices” and “experts” completed tasks based on electrostatics. The results indicate that both categories of participants constructed and used multiple representations for attempting the questions, with the generation of pictorial and physical depictions before formulating the mathematical expressions. According to Kohl and Finkelstein (2008), this outcome emerged either due to the reformed method of instruction where emphasis was consistently and continuously placed on portraying information in different representational modes during the course, or because of the nature of the task with which the students were familiar and had a standard approach.

Contrasting findings were gathered from the study by Van Heuvelen and Zou (2001). A problem solving strategy, based on the use of bar charts, was created for solving problems on work and energy. The students learnt to translate between various representational modes. When probed about the usefulness of the bar chart as an integral step for solving work-energy questions, the participants mostly stated that its presence supported their visualisation of the various processes and they better comprehended the underlying abstract physics ideas in work-energy. However, when provided with open-ended questions, the learners did not always apply diverse representations. Pictorial and mathematical representations were mainly generated. The bar chart was rarely included as by the end of the course, the students were accustomed to the various steps required for handling work-energy problems. Hence, its involvement as an additional step was considered as a waste of time. Instead, the qualitative representation was constructed and manipulated mentally.

<u>2.3.1</u> (b) Factors influencing the effectiveness of using sequential multiple representations as a problem solving strategy

Rosengrant *et al.* (2005) studied a sample of 125 undergraduate physics students' use of free body diagrams as a step for handling tasks with multiple representations. It was revealed that students who employed various intermediate representations for solving a problem, in particular those who drew an appropriate (complete and correct) free body diagram were more successful. The participants who did not apply any mediating representations performed better than those who generated an incorrect free body diagram. The authors surmised that learners who were successful in spite of not including a free body diagram may have generated and applied the diagrammatic representation mentally. The generation of an incorrect free body diagram was interpreted as a lack of understanding of the physics involved.

The study by DeLeone and Gire (2006) supports the outcome regarding the impact of quality of representations on students' problem solving performance. The cohort constituted of 39 students majoring in biology and taking a reformed introductory physics course emphasising on the application of model-based teaching, learning and problem solving. The participants completed a total of seven questions from the topics of dynamics, mechanics, thermodynamics and energy conservation. Students who used intermediate qualitative representations (pictorial, physical and graphical representations) referred to as non-mathematical representations (NMR) were more successful compared to the participants who dealt merely with equations. However, the number of qualitative representations employed did not seem to impact on the success rate. Numerous representations were applied but in many cases the students performed poorly due to their inability to translate the information provided in a problem statement into a qualitative representation. The visual aids are therefore a requirement, but in order to be effective the necessary skills required for their manipulations are of crucial importance.

Further evidence for the correlation between students' task performance and quality of representations generated was provided by Kohl and Finkelstein (2005). Although the sample was exposed to a representation-rich learning environment, no difference in performance was observed between students who employed additional representations when attempting a question and those who did not use various intermediate depictions. These mediating representations vary from being a diagrammatic, graphical or mathematical representation to a linguistic definition. When dealing with a problem posed in mathematical form, most of the students who manipulated only equations performed better than those who included a diagram. Further analysis for this particular finding showed that either an inappropriate diagram was drawn or the diagrammatic representation was incorrectly annotated with physics information which consequently led to a poor performance when solving the problem.

In a separate study, Kohl *et al.* (2007) shows that a considerable variation in students' performance occurs due to either the category of the free body diagram or pictorial representation drawn. The cohort in the study was involved in a reformed introductory physics course with the extensive application of a variety of ways for depicting information. It was observed that substantial enhancement in task performance occurred only with the construction of complete and correct depictions, the appropriate translation of information among the different representations and the improvement was more pronounced when solving harder problems.

Hence, the presence of poor representations, either incomplete (lacking in details) or incorrect, has a negative effect on performance. In order for problem solving based on multiple representations to be effective, of vital importance is the students' fluency with the different representations, their skills in manipulating the representations and translating information between them. The generation of an incomplete or incorrect representation indicates either an inability to understand information presented in a task, poor representation competence, lack of comprehension of the physics involved or inability to coordinate the various aspects within a representation. From the list of scientific abilities compiled by Etkina *et al.* (2006) figures the need for students to be able to represent information in a variety of different representational modes. It was claimed that the students should be able to extract relevant and substantial information from a given representation, use the information for generating other depictions and finally validate the consistency of information among the various representations and make adjustments for any discrepancies.

Reasons provided by students for their application of multiple representations were gathered from the study by Van Heuvelen and Zou (2001). From the students' responses, it was clear that the more experienced students are and the greater their expertise in handling similar problems, the less likely multiple representations will be externalised. The different sequences are memorised and the steps are applied in a more mechanical way.

Additional explanations of why and how students employ multiple representations were provided in a separate study by Rosengrant *et al.* (2006). It was revealed that the "high achievers" consistently included several representations regardless of the level of difficulty of the problem question. These students stated that the application of multiple representations helped them to make sense of the situation and for progressing stepwise from a simple to a more complex diagrammatic representation. In contrast, the "low achievers" either did not include or provided an inappropriate intermediate representation, in this case free body diagram, owing to their failure to interpret, comprehend and translate information from the particular representation. Moreover, students who did not always employ multiple representations claim that diverse representations are applied only if the question is hard and cannot be manipulated mentally.

The distinction between "novice" and "expert" handling of multiple representations during problem solving was investigated at a deeper level by Kohl and Finkelstein (2008). The novice cohort in the study constituted undergraduate science learners exposed to a representation-rich learning environment while the experts were postgraduate students enrolled for a masters degree. A difference was observed in how the two cohorts manipulate the representations and their purpose for including multiple representations. The novice participants applied multiple depictions in a routine and rote manner. They did not have any specific or clear reason for using the strategy except that they were exposed to the particular method during teaching and they had to stick to its usage. Moreover, they did not engage meaningfully with the various depictions generated. The students were able to navigate through the several representations (similar to the experts) but were unsure how to use the depictions for solving the problem. There was random manipulation of the different representations and application of trial and error processes for generating the solution to the problem. The authors concluded that the novice students did not benefit from representing information in several modalities. In contrast, the experts represented information in different representational forms in order to comprehend the problem and the underlying physics ideas. During problem solving they spent more time translating information from linguistic to qualitative depictions which were consequently used for reasoning and sense-making of the situation. They were able to move across the different representations more rapidly, were focused and organised in their manipulation of the different depictions and hence were more successful with the tasks.

The approach, implicit and explicit, used to implement reformed style instruction also impact on students' inclusion of multiple representations (Kohl *et al.,* 2007). The implicit method of instruction involves exposing students to the various forms in which physics concepts can be modelled without prescribing any problem solving steps which must be adhered to. Instead, the students are responsible for and expected to utilise, on their own, various representations during problem solving. In contrast, the explicit approach includes teaching and equipping students with a specific procedure for dealing with problems and the particular strategy is continuously applied across different topics in the course. It was revealed that most of the students instructed with an implicit approach generated multiple representations more often for the simpler and straightforward problems as opposed to the sample from the explicit instructional method. Participants from the latter group applied diverse representations more frequently when attempting problem questions with a higher level of difficulty. They also constructed better quality free body diagram in terms of its completeness and correctness. This particular finding led to reference being made to the "willingness" factor whereby students taught with the explicit approach do not necessarily recognise the worth of applying the whole process, which is detailed and lengthy, for solving a simple and easy task compared to a more difficult one.

Another factor influencing whether diverse representational modes are used for problem solving includes the characteristics of the problem questions (Rosengrant *et al.,* 2007; Kohl *et al.,* 2007). The presence of an existing diagrammatic representation may result in the non-inclusion of an additional simplified diagram. The words used to frame a question and the focus of the problem can also impact on whether several representations are applied for problem solving. The condition, under which the tasks are implemented, during examinations or tutorial sessions, is also a determining factor for the inclusion of multiple representations (Van Heuvelen and Zou, 2001).

To conclude, it can be claimed that the teaching of problem solving based on multi-representational modes does not guarantee that the technique will be applied under any circumstances when attempting problem questions. In addition, many factors have been identified for influencing its effectiveness in terms of enhancing problem task performance. Van Heuvelen and Zou, (2001), page 193 state that "students must understand why they are learning to represent processes in the more qualitative ways and how these qualitative representations can be used to increase their success in quantitative problem solving". A similar claim was made by Dufresne *et al*. (1997) who argued that unless students know how to deal with representations and acknowledge the associated benefits, they will not be applied when attempting questions. Kohl and Finkelstein (2008) reinforced this point of view by arguing that even if reformed-based teaching encourages students to use various representational modes, to benefit from this particular technique, students should learn how to meaningfully handle multiple representations and most importantly understand why they are being applied.

2.3.2 Problem solving with parallel multiple representations

When solving problems with parallel multiple representations, the various representations (either visual or mathematical) emerge directly from the given problem and each mode of depiction is used to directly interpret and attempt the task (Dufresne *et al.* 1997). An illustration of the use of parallel multiple representations problem solving strategy is shown in Figure 5. It is concerned with the context of kinematics whereby the focus of the problem can be obtained by solely manipulating stroboscope photographs, kinematics equations or motion graphs.

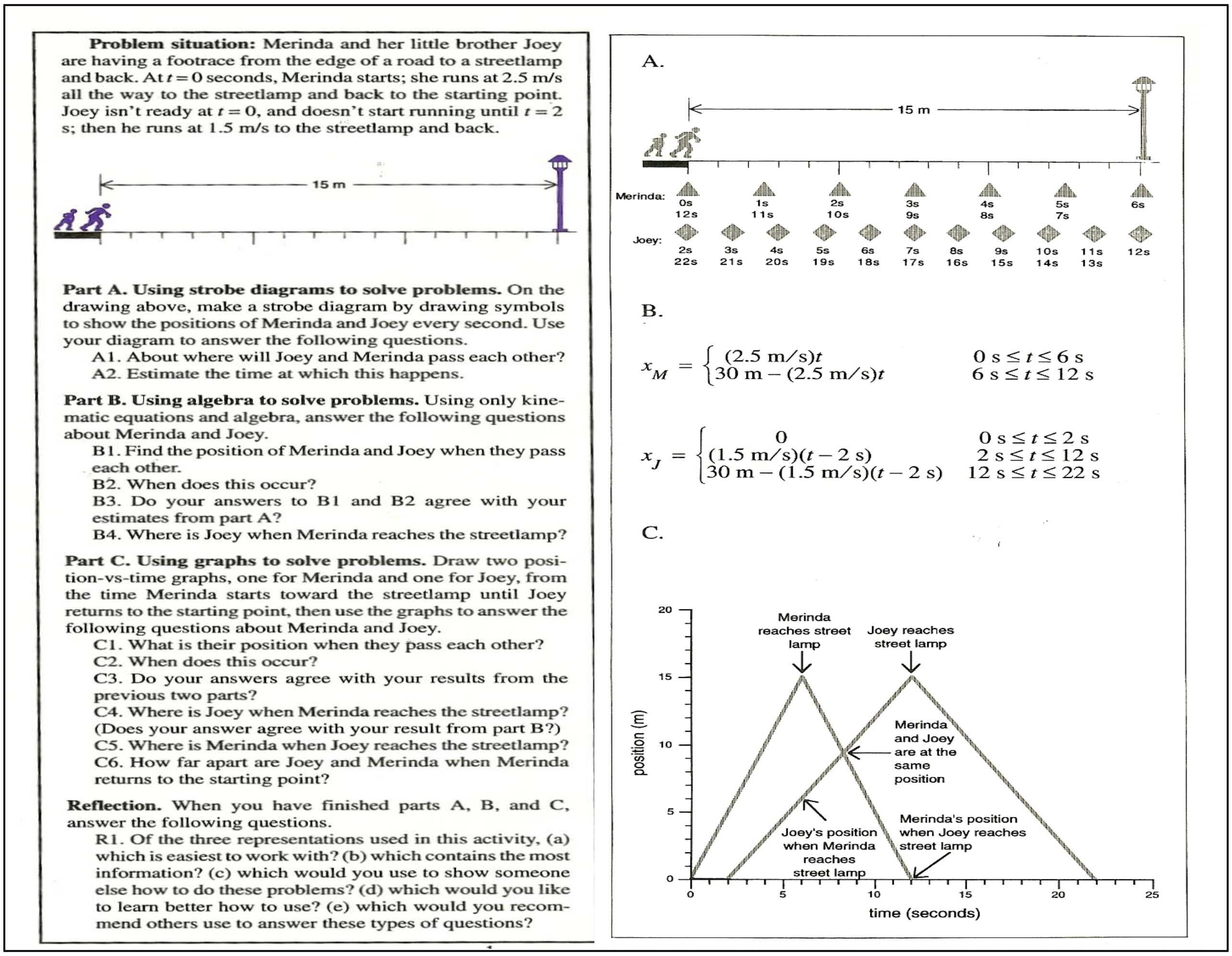


Figure 5. *An example of the use of parallel multiple representations for problem solving (Dufresne et al. 1997, page 274-275).*

The majority of research exploring the effect of teaching and using parallel multiple representations as a problem solving strategy have been implemented in the domain of cognitive science. These studies have reported mixed outcomes, both beneficial and detrimental, when diverse forms of representations are simultaneously used for learning or problem solving.

2.3.2 (a) Effect of teaching parallel multiple representations as a problem solving strategy

A recent example of a study demonstrating the benefits of teaching students to deal with information presented in a variety of concurrent representational modes is that by Eilam and Poyas (2008). Cards consisting of information, based on cellular phones, posed in both linguistic and visual forms (pictures, charts, graphs and tables of data) were utilised. One group of students in the study dealt with only the linguistic representation while the other cohort was provided with both linguistic and visual representations. It was found that participants who were familiar with handling diverse representations tend to use more cards when attempting their homework tasks and hence had a better performance as opposed to the population who were used to manipulating a single representation. Moreover, they were better able to recall, apply and transfer the given information to new situations. Hence, there is indication that the use of parallel multiple representations results in better understanding of the underlying ideas within a situation and knowledge are acquired more efficiently and significantly.

Earlier studies include Mayer and Gallini (1990), and Mayer and Anderson (1992) who compared the quality of answers generated for problems requiring the transfer of information. Explanations regarding the operation of various processes (the functioning of a pump or a brake) were presented in written form only, in linguistic or verbal representation together with the associated visual representations which were either dynamic computer simulations or static. These studies indicated that students exposed to various representations provided more detailed, meaningful and inventive responses compared to those dealing with a single representation. One explanation provided for the positive effect of using multi-representational modes (linguistic and visual) is based on the dual coding theory. According to Mayer (2003), the simultaneous presence of a variety of depictions allows frequent mental interactions and manipulations to take place. The relevant information from either forms of representation is first identified and a mental model of the linguistic and visual representations is constructed. The mental models generated for the linguistic and visual depictions are different since these two forms of representations are processed separately in different areas of the working memory (Mayer, 1997). The two mental models are then related to each other by a mapping process together with the inclusion of any existing knowledge resulting in the formation of an integrated mental model which is used to attempt recall and transfer tasks.

Positive outcomes on the application of parallel multiple representations were also highlighted in the physics domain from the study implemented by Stelzer *et al.* (2009) where the effect of using multimedia and traditional textbook on introductory physics students' learning of electricity and magnetism was explored. The sample was made up of three groups of students. One cohort was exposed to computer-based learning materials consisting of dynamic visual representations simultaneously accompanied with verbal representation while the two other groups were presented with either static (pictures and text) version of the multimedia materials or with textbook problems. It was revealed that, compared to the two other cohorts, students dealing with the computerised version of the materials had a better performance on conceptual questions and, acquired as well as retained information more efficiently.

However, it was also reported that the application of several representations at the same time may have a detrimental effect on students' performance. Ainsworth *et al.* (2002) advocated that the use of a single representation involving all the relevant and required information can have greater benefits compared to the application of multiple representations. An example of a study exemplifying this claim is that implemented by Chandler and Sweller (1992). The participants were provided with learning materials where information was presented either in diagrammatic form with brief explanations integrated onto the diagram, or as two separate representations, linguistic and diagrammatic. It was revealed that students from the single (all the required information provided onto the diagram) representation group, performed better for their tests as opposed to those learning information presented in two representational forms. One reason put forward for the failure of using parallel multi-representational problem solving strategy is grounded on the notion of cognitive load theory. According to Chandler and Sweller (1992), in order to process the two disparate representations, aspects of interest in the linguistic representation have to be retained in the working memory while simultaneously searching for the corresponding relevant facets in the visual representation. In other words, students dealing with multiple representations need to "split their attention" between the separate representations to gather the relevant information, mentally integrate and process them in order to make sense of the situation. These numerous memory activities are claimed to lead to cognitive overload and hence poor performance.

<u>2.3.2</u> (b) Factors influencing the effectiveness of using parallel multiple representations as a problem solving strategy

Many factors have been identified leading to the ineffectiveness of using multi-representational modalities concurrently for learning and problem solving purposes. The major inhibitor is the inability to relate and translate information within and among representations which is of key importance for improved performance (for example Tabachneck *et al*., 1994; Kozma, 2003; Seufert, 2003; Ainsworth, 2006).

Kozma (2003) compared how expert chemists and novice chemistry students deal with various representations. It was revealed that the experts were better able to relate the several features within a particular representation and were more fluent in shifting across the different forms of representations which were interpreted according to the relevant concepts. In contrast, when presented with multiple representations, the novice students often attended to the representations on an individual basis. They were incapable of transforming information from one representational form to another. Their linguistic representations were mainly in the form of a description based on direct observations (termed as surface features) rather than providing an explanation for the concept involved. During experiments, they failed to make the link between the real world phenomena and its various representational forms as used in the chemistry context.

A difference was also observed in how experts (postgraduate students) and novices (undergraduate learners) relate different representations in the physics domain (Kohl and Finkelstein, 2008). The participants were provided with several tasks in kinematics requiring the grouping of comparable depictions presented in the form of texts, graphs and snapshots. The novices always started with either the linguistic representation or snapshot as the basis for combining the depictions. In addition, they only shifted to grouping another set of depictions upon the completion of the current one and rarely revisited their solutions. The experts varied their starting points and move back and forth among the several representations for the different tasks. However, contrasting with the study by Chi *et al.* (1981) and, Kozma and Russell (1997), it was found that similar to the experts, the novices also focused on the underlying physics ideas for combining the representations. They did not concentrate on superficial features such as the shapes of graphs or key words as criteria for the grouping.

Van der Meij and de Jong (2006) have compared the effect of sequencing different representations and simultaneously depicting information in various modes on students' performance for tasks requiring them to make linkages and translations between different forms of representations. The sample was provided with computer-based physics learning materials on the topic of moments which were presented in visual (pictorial, diagrammatic and data tables), symbolic (mathematical formulation) and linguistic forms. The results obtained indicate that the basic knowledge and skills to deal with the specific physics topic were acquired. However, none of the methods of depicting information succeeded in enhancing students' ability to relate and translate information. The learners concentrated mainly on the given instructions which guide them through the tasks while failing to probe deeper into understanding the different representations as well as the connections between them. The mapping and translation processes were performed in a mechanical way, by rote memorisation. Also, the students did not willingly relate representations unless they were prompted to perform the operation.

Seufert (2003) designed and applied the "directive" and "non-directive" approaches for teaching the process of mapping and translating information among several representations in the chemistry context. The explicit or "directive" approach guides the learners to identify those elements of relevance among the representations which need to be related. For the implicit or "non-directive" approach, the students were only encouraged to search, on their own, for the appropriate constituents and their connections among the representations. Learning materials about a particular chemistry process were presented in a combination of linguistic and visual representations. Tasks requiring connections to be made between representations by recollecting and transferring learnt information were completed. The findings revealed that the “directive approach” cohort had the best performance on “recall tasks”. However, in terms of the cohort’s level of prior knowledge, classified as low, medium and high, it was found that students with moderate level of prior knowledge benefited the most with either form of instruction, in particular with the explicit approach. The detailed guidance reduces cognitive load by acting as support prompting students to use and attend to only the relevant parts from their prior ideas which are merged with related aspects of learnt knowledge. The combined information is then applied to the current situation for successful connections to be made among representations. An enhancement in performance was observed even for students with high level of prior knowledge. However, for learners with low level of existing knowledge, both approaches impact negatively on their ability to recollect information and perform the mapping process. One explanation put forward for the particular outcome is that both forms of instructional approach emphasise conceptual understanding in order to inter and intra relate aspects of representations. In contrast, the non-guidance, low level prior knowledge group had a better performance as the focus may have been only on rote memorisation of main ideas.

Learners with low existing knowledge or novice students refer either to the most discernible feature or the surface structures as the basis for classifying tasks or representations, unlike the “experts” or learners with high level of prior knowledge who concentrate mainly on the main ideas on which the problems or depictions are underpinned (Chi *et al*., 1981; Kozma and Russell, 1997). Consequently, the novices’ superficial-level focus hinders their ability to navigate across different representations and make linkages that went beyond the surface displays (Kozma, 2003). In addition, the implicit or explicit teaching of the process for relating and translating information has a harmful impact on their task performance as these instructional strategies also emphasise promoting comprehension of the underlying principles in order to make connections within and across representations (Seufert, 2003).

Another cause for the failure of problem solving strategy based on the concurrent inclusion of diverse representations is associated with the quality of the external visual representations. Schnotz and Bannert (2003) administered two tasks, dealing with time differences between countries, to a sample of 60 students. For one cohort, the questions were set in linguistic form only while the other two groups completed the same problems presented with a combination of linguistic and visual representations. The projections of information from the visual depictions for the latter two groups were different. The outcomes of the study indicate that depicting information in diverse forms does not necessarily results in an improved performance. In-depth analysis revealed that the failure of using multiple representations to enhance task performance was due to certain features in the visual representations interfering with the students' construction of mental models and hence with their task performance. It was argued that the design of an external visualisation should be specific to the purpose of the task which otherwise hinders the construction of the required mental model. The explanation provided for the relationship between internal and visual external depiction is grounded on the notion of "analogical structure mapping" (page 153). The sense-making of a visual external representation requires the construction of mental models which encompass the semantic content. The development of mental models occurs by mapping corresponding elements between the external visualisation and its internal version. Linkages are made between the visual aspect (surface features) of the external visualisation and the constructed mental image while the spatial features are related to the semantic elements which constitute the mental model.

The importance of the design of external visualisations and their impact on mental model construction was also stressed by Rapp (2007) and Tversky (2007). Excess information or irrelevant details included in an external visualisation may lead to the construction of an inappropriate mental model. These details may distract students from the essence of the depiction, from the main idea being conveyed and also make it difficult to retrieve crucial information.

Another explanation provided for the impact of external visual depictions on internal constructs is grounded on the cognitive load theory where the design of external constructs has an effect on the load of information which is processed in working memory (Kalyuga *et al.*, 1999). From the study by Kalyuga *et al.* (1999), it was revealed that excessive or additional depictions of the same information in multiple forms, referred to as "redundancy of information", interfere with students' task performance. Learners provided with three representation modes, visual, linguistic and verbal representations did not perform any better than those exposed to a combination of visual and linguistic representations. The reason provided for the poor performance is that there is a need to map and integrate the same information, presented in different versions, from multiple sources resulting in an overload of cognitive activities in the working memory whose potentiality is restricted. The study also indicates that the core for enhancement of task performance lies in the quality of visual displays rather than the numerous and diverse representational modes to which students are exposed.

2.3.3 Comparison of findings on the application of sequential and parallel multiple representations as problem solving strategies

One of the main differences between sequential and parallel depiction of multiple representations for problem solving is that in the former, diverse representations are used as intermediate support for attempting a task while in the latter, the various representation modes are used directly for solving the problems (Dufresne *et al.*, 1997). Irrespective of whether the problem solving strategy is concerned with sequentially or simultaneously depicting information in multiple forms, both techniques are proved to be beneficial only under certain conditions. Of primary importance is the ability to make linkages and translate information among representations, two processes which are considered to be cognitively demanding and challenging for students, especially those with low prior content knowledge. When applying various representations sequentially, poor quality intermediate representations result due to the lack of coordination among the different elements or structures within the depiction (Kohl *et al.* 2007), and a failure to translate information from a given physical model into a qualitative conceptual model (DeLeone and Gire, 2006). These findings can be compared with that of Van der Meij and de Jong (2006) and Kozma (2003) on students' failure to relate and translate information when parallel multiple representations is used for problem solving. Consistent outcomes emerged at primary (Ainsworth *et al.*, 2002) and secondary (Yerulshamy, 1991) levels in the context of mathematics. The influence of prior knowledge was observed for the case where information is depicted in multiple forms and sequentially (Rosengrant et *al.*, 2006). Students with high level of prior knowledge consistently use diverse representations regardless of the complexity of the tasks, while those with low level of prior knowledge either generate poor quality or prefer not to include intermediate representations due to their inability to understand and interpret these depictions. The effect of previous knowledge was also highlighted from the study by Seufert (2003) on the use of parallel multiple representations. Additionally, the work of Stern *et al.* (2003) in the context of economics further supports the crucial role of prior knowledge for the effective application of parallel multiple representations as a problem solving strategy. Finally, the correlation between quality of external representations and task performance was revealed by both sequential and simultaneous depiction of representations. For the former, it was found that the number of qualitative representations employed did not necessarily result in success with attempting the tasks because of the poor quality student-generated representations (DeLeone and Gire, 2006; Kohl *et al.*, 2007; Rosengrant *et al.*, 2005). For the latter case, Schnotz and Bannert (2003), Rapp (2007) and Tversky (2007) argue that external visualisations may impact on the quality of mental model constructed while Kalyuga *et al.* (1999) claim that the quality of external representation can result in cognitive load and hence poor task performance.

2.3.4 Multiple representations for conceptual understanding

It is useful to explore the ways in which a variety of representations may be used for developing students' conceptual understanding. Hinrichs (2005) employed the Modeling Instruction curriculum for developing introductory physics students' understanding of the concept of Newton's Third Law. The programme emphasises the application of multiple representations, in particular the use of system schema which acts as a "conceptual bridge" between the pictorial representation and the free body diagram. The system schema provides a more concrete depiction of the interactions between objects hence supporting reasoning and processing of the qualitative information. The Force Concept Inventory Test was completed before and after the teaching intervention. A substantial improvement was noted in the students' understanding of Newton's Third Law after instruction. In addition, another group of students were taught the particular concept using the Workshop Physics programme which also involves the use of multiple representations. A comparison was then made between the two samples' level of comprehension. It was found that the Modeling Instruction group outperformed the Workshop Physics cohort. The students taught with the former programme had a better understanding of the ideas underlying Newton’s Third Law. The difference was attributed to emphasis being laid on the application of system schema in the Modeling Instruction curriculum unlike the Workshop Physics programme where no mention was made about the particular representation.

The outcomes from the study by Van Heuvelen (1991b) strengthen the evidence obtained regarding the effect of using a variety of representations on students’ conceptual understanding during problem solving. Two groups of students were involved in the study, namely those instructed with conventional method of teaching and learners who are familiar with handling a variety of representational modes. The findings revealed that students from the reformed group had a better comprehension of the concepts involved for the topic under consideration (work-energy, impulse-momentum, Newton’s Second Law and kinematics) and consequently had a better task performance.

Problem tasks referred to as Jeopardy Problems were designed by Van Heuvelen and Maloney (1999), with the questions structured either in mathematical, diagrammatic or graphical forms. The information presented in a given representational mode needs to be converted into other forms of depictions followed by the generation of a problem statement for the situation. The tasks were based on Newton’s Second Law, electricity, First Law of thermodynamics and kinematics. The efficacy of Jeopardy Problems in developing conceptual understanding in mechanics and enhancing students’ ability to manipulate mathematical formulations in conjunction with the appropriate physics reasoning was investigated. The sample consisted of honours engineering students who completed two tests, the Mechanics Baseline Test (MBT) and Force Concept Inventory Test. For the MBT, made up of questions which had to be solved for a quantitative solution, an understanding of the physics ideas underlying the mathematical expressions was reflected. For the FCI test, which is based mainly on qualitative reasoning, it was found that a comprehension of the various mechanics concepts has been attained. According to Van Heuvelen and Maloney (1999), the translation of information between the several representations requires an understanding of the mathematical formulations and the qualitative depictions (graphs and diagrams). On the one hand, the syntactic structures are therefore substantiated and ascribed with meaning instead of being viewed as abstract symbols to be manipulated in a rote manner. On the other hand, the qualitative depictions need to be decoded and interpreted in order to portray the derived information into other representational forms.

In the study implemented by Seufert (2003), students learnt a particular concept from the chemistry context by being explicitly (directive approach) or implicitly (non-directive approach) taught to make linkages among the various representations. The sample had different level of existing knowledge about the situation under consideration, classified as low, moderate and high. It was revealed that irrespective of the students' level of prior knowledge, those from the "directive" and "non-directive" groups performed better on the comprehension tasks compared to learners who received no guidance about mapping information during the learning process. In-depth analysis revealed that students with medium level of prior knowledge exposed to either forms of instructional methods had a considerably better performance on the comprehension tasks. Even learners with a low degree of existing knowledge benefited from the "directive" and "non-directive" approach. They outperformed the counterparts from the "no-instruction" cohort. However, for the group with a high level of previous knowledge, no difference was observed in their tasks competence regardless of whether guidance was provided or not. The strategy they were exposed to during the learning process was ignored as they already had an understanding of the required concepts. The application of multiple representations for learning therefore yields positive results only if students are able to map the relevant elements and translate information between various depictions.

In a similar line of research, Bodemer *et al.* (2005) provided the sample in their study with learning materials dealing with kinematics. It was found that students who had hands-on involvement with the mapping and translation of information during the learning process had a better grasp of the underlying physics ideas presented by a situation. They were better able to solve problem tasks requiring translation among motion graphs and from graphical to mathematical form as well as for constructing kinematics graphs from a linguistic representation.

To summarise, the various studies described above have reported on the positive effect of using multiple representations to enhance comprehension of underlying ideas in different contexts. Multiple representations were applied either during teaching or when solving paper and pencil tasks with emphasis placed on the mapping and the translation processes which are crucial for comprehension and hence, for learning to take place.

2.4 Visual tools for developing conceptual understanding and graphing skills in kinematics

The effect of applying multiple representations for problem solving and conceptual understanding in a variety of contexts was discussed in the previous sections. It is now useful to discuss the literature dealing with students' difficulties in handling kinematics graphs, and the teaching strategies designed for conceptual development and enhancement of graphing skills in kinematics.

2.4.1 Students' handling of kinematics graphs

McDermott *et al.* (1987) identifies 10 difficulties faced by students when dealing with motion graphs. These difficulties were classified into two main groups. The first category is concerned with the failure in relating the relevant aspect of a graph to the required kinematics concepts. These include the abilities to

- recognise that the area under a velocity-time graph yields the displacement;
- distinguish between the height and slope of lines on a graph;
- interpret and compare objects' velocities from position-time graphs having different shapes, either a straight line or a curve;
- transform the information depicted by one form of graphical representation to another;
- relate the descriptive information to its corresponding features depicted on the graphical representation.

The second classification of difficulties, associated particularly with visualisation, includes the abilities to

- construct graphical representations which correspond to an object's actual motion as observed in real world. Often the shapes of position-time graphs drawn are similar to the path followed by an object;
- graphically represent qualitative information, such as acceleration and velocity, associated with real world situations where an object is shown to speed up in a direction opposite to its motion, slow down or to undergo a change in direction;
- differentiate between the shape of motion graphs for position, velocity and acceleration for a given motion;
- differentiate between instantaneous and average quantities;
- meaningfully connect plotted points for the different graphical representations. Students seem not to understand that a discrete point depicts an object's motion at a particular instant of time while the continuous motion, represented by a line or a curve, stands for the object's motion over a period of time.

Beichner (1994) designed and used the Test of Understanding Graphs in Kinematics (TUG-K) questionnaire to identify the difficulties faced by secondary and college students when interpreting kinematics graphs. It was revealed that when shifting among graphical representations for position, velocity and acceleration, the shapes were depicted to be the same indicating that the students do not discriminate among these three variables. A reason provided for the portrayal of similar shape graphs is the students viewing graphical depictions as pictures for a given situation rather than a formal method of representing motion. Mokros and Tinker (1987) reported that the sample (7^{th} and 8^{th} grade learners) in their study generated graphs with shapes which are similar to the path of the motion. When interpreting a graphical representation, sections of the graph, the shape of which corresponds to that of the path for the motion were matched. Furthermore, from the work of Simpson *et al.* (2006) it was found that when translating from linguistic to graphical form, each motion description was considered separately for drawing the corresponding shape, resulting in the whole graphical representation reflecting a discontinuous motion.

In contrast to the studies on interpretation of kinematics graphs, Goldberg and Anderson (1989) explored students' ability to construct motion graphs. College students' difficulties in dealing with graphs depicting negative values for velocity were highlighted. One of the tasks consisted of making observations of a ball rolling down and then up a V shape path, verbally describing the motion and finally drawing the corresponding velocity-time graph. The majority of the students were able to describe the motion. However, only a few of them succeeded in graphically depicting the velocity of the ball for the whole situation. In most cases, the shape of the graph portraying motion down the slope was appropriately drawn. For the section of the ball moving up the slope, the graph's shape was portrayed to be similar to the path followed by the ball. In addition, velocity-time graphs with negative values were interpreted in terms of magnitude only. A positive slope was associated with an increase in velocity even if it lies in the negative part of the coordinate system and extending to the positive area. The point where the curve meets the time axis was not recognised to represent instantaneous zero velocity. The main reason provided for students' inability to understand the concept of negative values for velocity is concerned with the meaning ascribed to the term "velocity" and "negative quantity". In the everyday context, velocity is usually referred to in terms of its magnitude, that is, the speed. This particular notion may be transferred to the physics context. The words "negative quantity" understood in terms of "reducing a quantity" hinders the students' notion of negative velocity as changing from a value less than zero to a value above zero. They are unable to conceive the idea of velocity being a value smaller than zero as according to the students if an object has velocity it means that it is in motion and if it is at rest, it has zero velocity.

2.4.2 Instructional strategies in kinematics using visual tools

A number of instructional strategies have been designed, based on the application of visual tools, for developing students' conceptual understanding in kinematics and to enhance their abilities in handling kinematics graphs. The importance of the visual devices lies in the fact that they "can contribute to students' understanding of physics concepts by attaching mental images to these concepts" (Escalada and Zollman, 1997, page 467).

The instructional strategy designed by Rosenquist and McDermott (1987) utilises easily available instruments such as ticker tape and stop watch. The students' understanding of instantaneous velocity according to the "limit" concept was developed by observing and making measurements between the spacing of the dots. As time interval decreases, the distance between the dots appears to be constant indicating that the object is undergoing uniform motion and hence its limit has been attained. To enhance kinematics graphing skills, students learned to translate information between real world situations and graphical representations. On one hand, two kinematics graphs of similar shape were provided. The motion portrayed by each graph needs to be reproduced in real world situation. On the other hand, a particular motion was generated using ticker tape from which measurements were made to plot the motion graphs. Therefore, students learned to link different segments between motion graphs and also from graphical representations to the actual motion and vice-versa. It was recognised that information for the same motion can be portrayed by different shapes. Also, even if the shapes of the graphs are the same, the motion they depict may be different.

Alternatively, the strategies used to remedy students' difficulties in dealing with kinematics graphs may be more sophisticated, involving the use of computer software, for example, WorkShop Physics (Laws, 1996) and RealTime Physics (Sokoloff *et al*., 1994). The computer packages, known as Microcomputer-based laboratory (MBL) tools, are used to generate real-time graphs. MBL tools play a crucial role in enhancing students' conceptual understanding and graphing skills in kinematics and its success is advocated to lie in its real-time nature (Brasell, 1987; Thornton and Sokoloff, 1990). These devices "take advantage of the strong response of the human visual perception system to objects moving in the visual field" (Beicher, 1996, page 1273). MBL tools allow two forms of visual motion display to be viewed simultaneously and hence support students' ability to mentally make the connection between the actual motion and its graphical representation (Brasell, 1987; Mokros and Tinker, 1987). When presented with a real world motion and its graphical version, direct comparison can be made between these two forms of representations. A change onto the graphical representation prompts students to attend to these specific aspects and link the detected changes to the real world situation (Brasell, 1987). The "co-occurrence" feature of MBL devices enables students to better deal with graphical depictions of abstract concepts such as acceleration and velocity, and to retain the mental image of a physical situation while different graphs of motion are being displayed (Brungardt and Zollman, 1995). Furthermore, it helps to dissipate the confusion between the concept of slope and height of lines on a graph, and also reduce the mistake associated with viewing motion graphs as pictures for a situation rather than a formal tool for representing measurements and conveying information (Mokros and Tinker, 1987). Thorton and Sokoloff (1990) investigated the effect of using MBL tools on students' kinematics conceptual understanding. Students were presented with tasks requiring predictions which consequently led to discussions about the situation under consideration. The sample's motion was the source for data collection when translating between real world motion and, the graphical representations. A comparison between students involved with MBL activities and those who attempted paper and pencil tasks and performed traditional laboratory activities shows that for the latter case there was no substantial gain in understanding of kinematics concept.

The ways in which MBL activities helped in developing students' comprehension of kinematics concepts were explored by Russell *et al.* (2003). Tasks which include prediction, observation and explanation, were administered to the participants who have superficial kinematics knowledge and limited experience with handling motion graphs. The students were directly involved in creating motions and simultaneously observing the graphical displays. The study revealed that an in-depth level approach to learning was adopted. An attempt was made to extract more knowledge beyond what was presented and more importance was given to the conceptual rather than the procedural aspect of the task. It was argued that MBL activities promote conceptual change by providing concrete and immediate evidence which may either support or counteract the students' existing kinematics knowledge. The simultaneous depiction of the actual motion and the graphical depiction prevent mental overload with the result that learners are better able to analyse, interpret the graphs and memorise any information they portray. The consistent presence of the visual display encourages students to frequently refer to it for gathering information about the motion graphs which may consequently lead to the retention of information over a long period of time. Mental templates for the shapes of graphs and their corresponding real world situations may be formed which are recollected later on when presented with a similar physical event.

The effectiveness of MBL tools in improving kinematics graphing skills was also demonstrated by Mokros and Tinker (1987). The motion of a toy car, controlled by middle school students together with their own motion was the origin for the data. The considerable progress noted in the sample's ability to handle kinematics graphs was alluded to the kinaesthetic feature of the device. The fact that the students were directly involved in manipulating the motion may have been one of the factors which led to an improvement in graphing skills. The author commented that "the students bring a unique level of understanding to the graph when the data comes from an experiment towards which students feel a sense of ownership" (page 381).

The kinaesthetic component associated with MBL devices was also claimed by Beichner (1990) to be the major factor responsible for their effectiveness and success. One group of students involved in the study was presented with simulations of videotaped images of real world motions. Data were gathered beforehand but graphed at the same time as the simulated motions progress. The students did not have any direct engagement in the creation and control of motion. Another cohort was engaged in traditional laboratory activities. The participants were provided with stroboscopic photographs of an object undergoing projectile motion from which measurements were made and motion graphs were plotted manually. The results indicate that there was no significant difference in the learners' abilities, from either cohort, to deal with kinematics graphs. The simultaneous viewing of animated copies of real world motions and the corresponding graphical depictions yielded no substantial positive outcomes.

More recently, Simpson *et al.* (2006) designed kinematics tasks based on the extensive use of the computer software, ToonTalk, for improving students' kinematics conceptual understanding and handling of motion graphs. The activities were mainly concerned with relating motion, as described, to the corresponding graphical representations. They were structured around the motion of a rocket which was controlled by the students who decide and enter their own values for the variables. The data collected from the generated motion were graphed in real-time. After completing one of the tasks, a change was observed in the students' comprehension of kinematics concepts and ability to deal with kinematics graphs. It was recognised that graphs of similar shapes with the same variables along the axes do not necessarily portray the same motion, and a graphical representation is not a literal depiction of the physical situation. Simpson *et al.* (2006) stated that of major importance in the instructional method is its kinaesthetic aspect.

In addition to investigating the consequences of using an MBL unit on motion during one class session, Brasell (1987) also compared the effect of graphing data in real and delayed time on students' kinematics graphing skills. In delayed-time graphing, data are collected at the same time that motion is taking place but the graphical representation is presented at a later stage. The students' own movements, walking to and fro at various speeds and in different directions, were used as the source of data. It was revealed that the short period of time in which the students were in contact with the MBL tool results in an improvement in their ability to deal with kinematics graphs. When provided with paper and pencil tasks, they were able to either draw graphs of motion which correspond to the linguistic description of the situation or describe the motion as depicted in graphical form. However, students from the delayed-time group did not make substantial progress in their graphing skills. Apart from data collection, analysis and interpretation of the kinematics graphs, the actual motion has to be recalled before making the link with the graphical version. These various stages may have led to extra mental processing of information which is beyond the students' mental ability. Another reason for poor development of kinematics graphing skills is the students' lack of technique for dealing with information before the generation of the graph. They do not engage in memorising and recalling the motion as it occurred in real world. Even if they engaged in the cognitive process, they failed to mentally store information. For making the connection between these two representations, information from real world situations must be shifted to either long term or short term memory where for its retention frequent repetition is required until the graphical representation can be viewed.

The effect of using delayed and simultaneous-time graphing on students' kinematics graphing skills was also explored by Brungardt and Zollman (1995). An interactive videodisc instrument consisting of video displays of sports situations was utilised. Data were gathered by tracing the position of the images from individual frames. They were then captured onto spreadsheet and processed to generate the relevant graphs. No significant difference in kinematics graphing skills was observed between the delayed and simultaneous-time cohort. The main reason for the particular outcome was explained by the absence of the kinaesthetic feature. The students were only presented with recorded video displays of real world motion and the corresponding graphical representations which they had to analyse.

The study implemented by Beichner (1996) indicates that the extent to which visual tools are used and the degrees to which students are in contact with these devices have an effect on their ability to interpret motion graphs. It was found that a substantial improvement in motion graphs interpretation resulted for students who had direct involvement with manipulating the device ("VideoGraph" software). Those who only observed demonstrations during a single day session performed no better than students involved with traditional laboratory activities. The condition under which the visual tool was more successful and effective was when the learners were engaged, very often and over a long period of time, with both hands-on laboratory work and lecture demonstrations.

Alternatively, Shaffer and McDermott (2005) employed vector representations for developing students' notion of the difference between the concept of velocity and acceleration. The students were presented with stroboscopic photographs for the motion of an object and were explicitly guided through the use of vectors for understanding and deriving qualitative information. Problem tasks requiring the application and transfer of the learnt materials to new situations were attempted. Most importantly, qualitative information portrayed by vector representations need to be mapped to the corresponding information presented in linguistic, diagrammatic and graphical representational modes. It was found that the majority of the students displayed an understanding of the physics presented in the situation and the difference between the concept of velocity and acceleration was recognised.

To summarise, various instructional strategies grounded on visualisation were designed to deal with the topic of kinematics. These methods of instruction were implemented mainly during laboratory activities and visual devices, with different level of sophistication, were used as mediators for translating information about motion from the real world to the physics world. MBL tools were shown to be more effective, irrespective of the extent to which they were integrated in laboratory activities, with their success attributed to the kinaesthetic feature. In contrast, the instructional method designed by Shaffer and McDermott (2005) was used for paper and pencil tasks and emphasises the application of vector representations.

3. Context of study

3.1 Educational background of sample in the study

The current study was implemented with students enrolled for the General Entry for Programmes in Science (GEPS) which forms part of a four year structured Bachelor of Science degree programme at the University of Cape Town. These students are usually from socio-economically disadvantaged backgrounds, speak English as their second or third language, and are mainly from a race group categorised as African and Coloured. In addition, they have a poor academic background (Allie and Buffler, 1998) as a result of being from disadvantaged schools where there is a scarcity of basic educational facilities and resources. These schools are often not equipped with proper laboratory facilities. The students therefore have superficial or no prior experience with practical work and many of them have their first contact with experimental activities at tertiary level. In general, they have experienced mediocre mathematics and science teaching at high school, and their learning is characterised principally by rote memorisation. They often fail to manipulate simple algebraic expressions and are trained to use prescribed "recipe-type" techniques for solving various categories of problem tasks.

Typically, for their matriculation year, the students register for six subjects. These include a combination of subjects from social science (history or geography), pure science (chemistry and physics combined, biology and mathematics) and humanities (English, Afrikaans or another South African language). However, they usually have low grades for the matriculation examination for the different science subjects, with symbols among D to F which represent marks ranging from 59% to 30% (Allie and Buffler, 1998).

At the University of Cape Town, science students from disadvantaged schools have the possibility to enrol for the GEPS programme. Admission to both the normal three year Bachelor of Science degree (BSc) programme and to the four year BSc programme is based on the total number of points derived from the students' matriculation results (Allie and Buffler, 1998) and on condition of at least 50% in mathematics. Students who obtain marks of 80% or above for each subject are admitted to the regular three year degree. Target applicants whose matriculation results correspond to between 47 and 36 points are offered a place on the GEPS programme.

3.2 The GEPS programme

Students joining the GEPS programme take a total of four half-year courses in the first academic year. Mathematics is compulsory and a selection of three courses from Computer Science, Chemistry, Earth System Science and Physics is made. To gain entry to second year, a minimum of three of these courses have to be passed. Of main importance is that the GEPS programme caters for the students' poor academic background. The course curricula are different from the normal introductory courses as they aim at providing the students with the necessary science and mathematics knowledge to cope with and complete a BSc degree.

3.2.1 Structure of the introductory physics course for the GEPS programme

Students enrolled for the GEPS programme complete the introductory physics course, which is calculus-based, over a period of three semesters. In contrast, the introductory physics course in the three year BSc programme spans over two semesters (a single academic year). Figure 6 provides an overview of the structure of the introductory physics programme as taken by the GEPS students at the University of Cape Town.

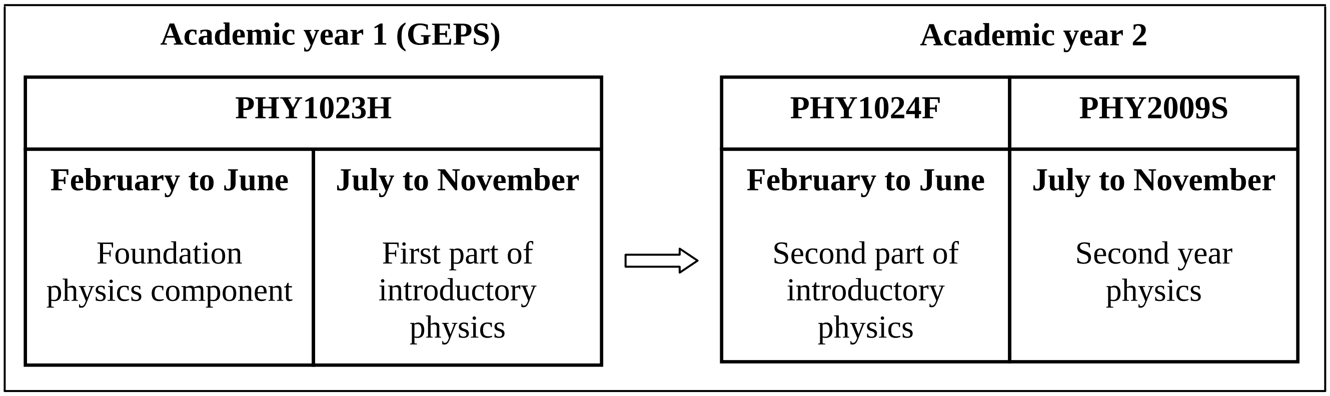


Figure 6. *Structure of the physics course stream for students starting in the GEPS programme.*

For the first academic year, the students join the GEPS physics course, PHY1023H. In the first semester (from February to June) they are exposed to the "foundation" component of the course and in the second semester to the first part of the introductory physics course (from July to November) which is calculus-based. Successful completion of the PHY1023H course allows students to attempt PHY1024F, in the first semester of their second academic year, which features the second half of the introductory physics course.

3.2.2 The "foundation" component of the GEPS physics course

The PHY1023H course is made up of three constituents, namely, laboratory activities, report writing (communication skills), and a theoretical lecture-based component (Allie and Buffler, 1998). The main aim of the theoretical aspect of the foundation component of PHY1023H is to familiarise students with fundamental physics concepts as well as equip them with the necessary tools and skills for handling first year calculus-based physics. The expectation is that students become fluent in the application and manipulation of various physics and mathematical tools. They should be able to perform a range of mathematical operations and most importantly to provide explanations of the physics principles underlying physical systems. The syllabus was constructed by grouping according to similarity in required skills, those tools which occur within a wide range of contexts, and which are deemed relevant and crucial for dealing with an introductory physics programme. The content emphasises students' exposure to various mathematical procedures and the development of skills for using these tools. A diversity of physics concepts are taught together with relevant mathematical tools. Skills development takes place via their application across several physics contexts. The benefit of the foundation syllabus is that students are made aware that tools and skills acquired in a specific context are transferable to a multitude of topics. The tools and skills, as applied across various contexts, which compose the theory component of the first semester GEPS physics syllabus, are shown in Table 3-1.

As illustrated, the “foundation” component of the GEPS physics course provides ample exposure to a variety of mathematical as well as physics tools which are applied over a range of contexts. The rectangular coordinate system is used to describe points in space leading to the concept of position and displacement vectors which are further explored in the topic of kinematics. For example, the manipulation of vectors including their multiplication, addition and subtraction, and the construction of vector diagrams, is performed in contexts such as Newton’s Second law, Coulomb’s Law, relative motion, conservation of linear momentum and torque. The construction and interpretation of physics visual tools which include motion graphs, freeze frame and vector representations are also introduced. Freeze frame representations, which are based on the idea of stroboscopic photographs, are used to describe the motion of an object the positions of which are recorded at equal time periods apart. Moreover, throughout the course a lot of emphasis is placed on the modelling aspect of physics which is introduced via the topic of the travelling wave equation. When solving problems, students are encouraged to draw pictorial representations of the situations and include qualitative depictions such as free body diagrams, vector diagrams, freeze frame or vector representations. In short, the focus is principally on depicting information in a multiple representational forms with mathematical expressions being the end product of the modelling process.

Table 3-1: Tools and skills of the GEPS physics "foundation" course.

Tools	Physics context	Abilities / Outcomes / Skills
1. Rectangular coordinate system in 2D and introduction to vectors	Displacement, Coulomb's Law, Newton's Second Law, Newton's Laws of Universal Gravitation, conservation of linear momentum and average quantities.	• Represent and understand the notion of position vectors as being relative to a set of coordinate system. • Translate from polar to component form and vice versa.
	Kinematics	• Unpack notion of position vectors by applying vector equation for motion in 1D to various situations.
2. Vector representations	Fundamental forces of nature	• Identify and represent the magnitude and direction of all significant contact forces acting on a system. • For non-contact forces, interpret the notation $\vec{\mathbf{F}}_{21}$ in the equations $\vec{\mathbf{F}}_{21} = k\dfrac{q_1\ q_2}{r_{12}^{\ 2}}(\hat{r}_{12})$ and $\vec{\mathbf{F}}_{21} = G\dfrac{m_1 m_2}{r_{12}^{\ 2}}(\hat{r}_{12})$ .Depict the force vectors for like and unlike charges and between two masses.
	Relative velocity	• Interpret the notations for relative velocities (for example $\vec{\mathbf{v}}_{\mathbf{AO}}$, $\vec{\mathbf{v}}_{\mathbf{BO}}$, $\vec{\mathbf{v}}_{\mathbf{BA}}$) and depict the velocity vectors within a reference frame.
	Kinematics	• Portray the magnitude and direction of velocity and acceleration for different situations. Use the vector representation to interpret and extract implicit physics information.
3. Addition and subtraction of vectors	Position and displacement.	• Depict the difference between position and displacement vectors as well as displacement and total displacement vectors. • Use notion of head to tail method for vector addition to generate equation relating position and displacement vectors.
	Relative velocity	• Depict velocity vectors within a reference frame. • Use notion of head to tail method for vector addition to generate equations relating the velocity vectors.
	Newton's Second Law, Law of Universal Gravitation, Coulomb's Law, conservation of linear momentum and average	• Portray situation in polar form within a coordinate system. • Apply component method of vector addition or subtraction.

	quantities.	
4. Multiplication of vectors using the dot and cross product	Work done by a force to move an object	• Describe the physical meaning of the dot product. • Apply the equation $W = \vec{\mathbf{F}} \bullet \Delta\vec{\mathbf{r}} = \lvert\vec{\mathbf{F}}\rvert \bullet \lvert\Delta\vec{\mathbf{r}}\rvert \cos\theta$ to each of the identified forces acting on the system in a diagrammatic representation. • Recognise that θ is the angle between the tails of the displacement and force vector. • Solve for dot product using component method. • Recognise that dot product of a vector with itself is 1 and with another vector is 0.
	Rotational motion – torque	• Describe the physical meaning of cross product. • Apply the equation $\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \lvert\vec{\mathbf{r}}\rvert \, \lvert\vec{\mathbf{F}}\rvert \sin\theta$ to each of the identified forces acting on the system in a diagrammatic representation. • Recognise that θ is the angle between the tails of the displacement and force vector. • Use the right hand screw rule from displacement to force vector for direction of the torque vector. • Solve for cross product using component method. • Recognise that cross product of a vector with itself is 0 and with another vector is either 1 or -1.
5. Integration	Motion, force-displacement and pressure-volume graphs	• Describe integration as determining the area under a graph. • Perform integral operation on mathematical functions. • Determine area under various graphical shapes and solve the problem using integrals.
6. Differentiation	Average and instantaneous quantities: position and displacement vectors, velocity, acceleration and force vectors.	• Recognise differentiation as determining the slope of functions. • Perform differential operation on mathematical functions. • Distinguish between and evaluate average and instantaneous quantities using mathematical and graphical representations.
7. Graphs of motion	Kinematics	• Recognise that area under velocity-time and acceleration-time graphs yield displacement and velocity respectively. • Recognise that the gradients of position-time and velocity-time graphs yield velocity and acceleration respectively. • Interpret and elicit qualitative information depicted by different shapes of motion graphs. • Translate information among motion graphs. • Use motion graphs to generate quantitative solutions.
8. Freeze frame	Kinematics	• Portray motion and hence abstract physics

representations		• Interpret the visual representation to derive implicit physics information and explain motion. • Use the visual tool as a representational bridge for translating and depicting information in different representational forms.
9. Modelling process	Waves	Use the relationship between the sine function (in radians and degrees) and 1D wave generated on an oscillating string to • describe the physics underlying the mathematical expressions $y(x, t_{\text{fixed}})$ = $A\sin\left[\left(\frac{x}{\lambda}\right)2\pi\right]$ and $y(x_{fixed,}\ t)$ = $A\sin\left[\left(\frac{t}{T}\right)2\pi\right]$. • describe the notion of initial conditions and hence the role of phase constant. • interpret, understand and manipulate the equation, $y(x, t) = A\sin\left[2\pi\left(\frac{x}{\lambda}\right) - 2\pi\left(\frac{t}{T}\right) + \phi\right]$, for a travelling wave.
	Conservation of linear momentum, torque, work done, average quantities, Newton's Law of Universal Gravitation and Coulomb's Laws.	• Generate a pictorial representation followed by a vector diagram as an intermediate and simplifying step to which the general mathematical model is applied.
	Newton's Second Law	• Depict linguistic information in pictorial form followed by a free body diagram to which the vector equations, $\sum \vec{\mathbf{F}}_{\mathbf{x}} = m\ \vec{\mathbf{a}}_{\mathbf{x}}$ and $\sum \vec{\mathbf{F}}_{\mathbf{y}} = m\ \vec{\mathbf{a}}_{\mathbf{y}}$, apply.
	Kinematics	• Generate pictorial, diagrammatic and freeze frame representations. • Apply kinematics equations to both the diagrammatic and qualitative depiction to formulate the mathematical model for the situation. • Depict motion via freeze frame representations which is used to translate among motion graphs.

3.2.3 Teaching and learning with "freeze frame" representations in the representation-rich kinematics course of the foundation component of the GEPS physics programme

Of particular interest for this study is the introduction of kinematics in the foundation component (highlighted in Table 1) which was structured with the aim of exposing students to the different forms in which kinematics information can be presented. Teaching strategies place particular emphasis on a variety of visual representations which include diagrams, graphs, vector and "freeze frame" representations. The central role of the visual depictions for reasoning, conceptual sense-making and interpretation of physics situations is fore-grounded. Students are taught explicitly to visually unpack problems posed in different representational modes, namely, linguistic, diagrammatic and graphical formats when generating qualitative and quantitative solutions. Activities in the course are designed to guide students in solving problems by using a variety of visual conceptual models and translating information between them.

3.2.3 (a) Use of "freeze frame" representations in kinematics teaching and learning

In kinematics, a diagrammatic representation usually portrays an object at its initial and final positions (two definite instances of time), from which it is difficult to visualise the object's motion. However, moving bodies can be visually depicted by tracing their path (continuous motion). Alternatively, objects in motion can be modelled and externally visualised by using "freeze frame representations" which are based on the idea of stroboscopic photographs. Freeze frame representations can therefore be considered as an intermediate between the two other visual modes of representing objects in motion.

Figure 7 shows a real stroboscopic photograph depicting the motion of two balls from the same height, one which is dropped and the other projected horizontally.

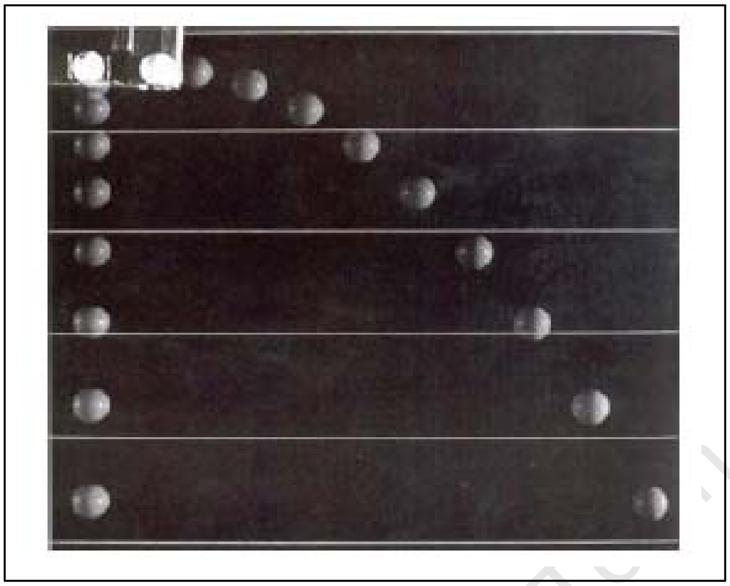


Figure 7. *A stroboscopic photograph of two balls having different motion.*

As the balls are released a stroboscopic camera captures their positions at equal intervals of time apart. The positions are captured individually on separate frames which when superimposed onto one another result in the formation of a composite picture. The resulting photographs transform the balls' (continuous) flight-path into a discrete representation. As the time interval is reduced, more photographs of the motion of the ball are taken. The spacing between the positions of the ball becomes smaller resulting in a more "continuous" description of the motion. If a line is drawn joining the individual images from the separate frames, then the path of the ball can be traced. The usefulness of depicting moving bodies via freeze frame representations lies in the fact that the continuous motion is held still or appears to be "frozen" for the different instances of time. Of crucial importance is the spacing between the individual frames which conveys particular physics information, such as the velocity and acceleration of the body.

Freeze frame representations may also be referred to as "snapshots" or "photographs" and are used in many physics textbooks, for example University Physics by Reese (2000), and Young and Freedman (1996) as well as Physics for Scientists and Engineers by Knight (2003). However, freeze frame representations are typically not used in these books as one of the intermediate steps for problem solving. For the foundation component of the GEPS physics programme, freeze frame representations are integral to solving kinematics tasks, either for generating quantitative or qualitative solutions. It is assumed that the application of freeze frame representations may result in an understanding of the physics ideas underlying a situation as an opportunity is provided for the students to also focus on those descriptions or depictions in the problem which encapsulate the physics information.

Lecture materials were designed to introduce students, in the foundation physics course, to the notion of freeze frame representations. During lectures, the teaching sequence often included the real world demonstration of a particular situation. Occasionally simulations were also employed. The object's continuous motion as observed in real world was initially portrayed by drawing its pathline. Its positions as taken at equal time periods apart are then drawn. Thus, from observations of the continuous motion, a discrete (freeze frame) representation emerged. Figure 8 presents aspects of the lecture notes dealing with depicting motion of objects visually and highlighting the distinction between continuous and discrete motion depiction.

1. Discrete depiction for projectile motion.

2. If we join up all the dots, we get the **pathline** (worldline) of the object.

3. The time between each photograph is Δt.

4. As we make Δt smaller and smaller, and hence take more photographs along the object's path, then we approach a "continuous" movie.

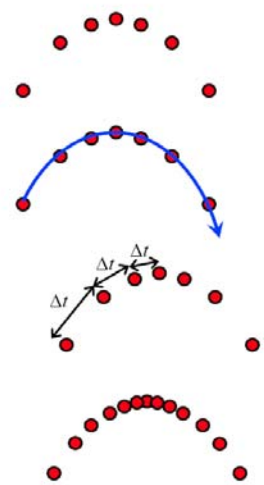


Figure 8. *Discrete and continuous visual depictions of an object in motion (kinematics lecture notes, page 17).*

Students' ability to portray motion using freeze frame representations and unpack information from the particular visual conceptual model was developed. Different situations of motion, both in one and two dimensions were presented, as shown in Figure 9.

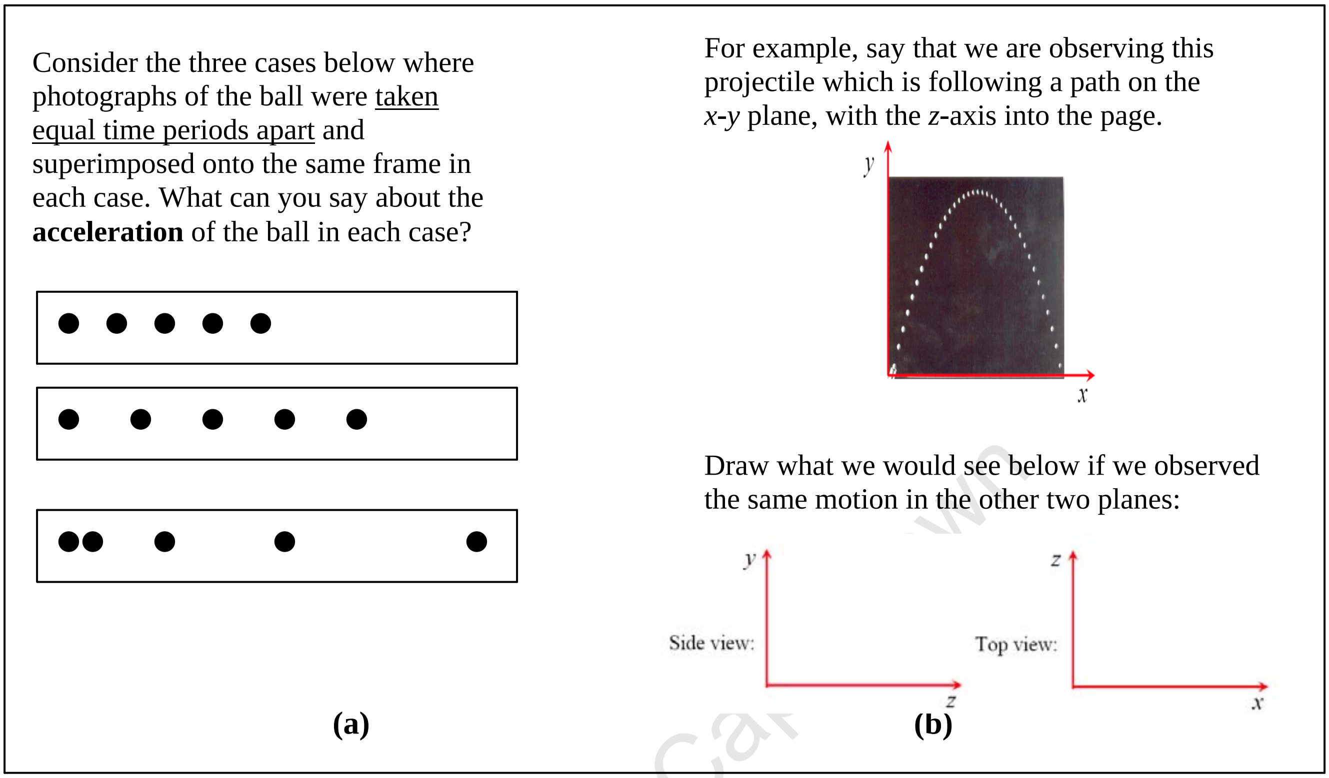


Figure 9. *Examples of motion in one and two dimensions (kinematics lecture notes, page 18 and 27).*

Particular emphasis was placed on the importance and meaning of the spacing between the photographs to obtain maximum physics information for a given situation. For example, from Figure 9(a), the students were introduced to the idea that the depictions of constant spacing and increasing spacing between the photographs indicates that the ball is moving with constant velocity (zero acceleration), and increasing velocity (constant acceleration) respectively. Moreover, they learned that although the spacing between the photographs is shown to remain the same in the first two frames, the ball is moving with a higher constant velocity in the second frame since in this case, the spacing is larger. For motion in two dimensions, such as projectile and circular motion, the path and freeze frame representations were sketched from different perspectives followed by an interpretation of the physics information. From Figure 9(b), the photographs of the ball as viewed from the top (in the *z*-*x* plane) and sideways (in the *y*-*z* plane) are drawn. The top view results in the portrayal of the constant spacing between the photographs while the side view yields a depiction of a dropping object with an increase in spacing between the photographs. Consequently, the students are introduced to components of velocity and acceleration, in the *x*-*y* plane, for projectile motion. They learned that the *x*-component of velocity remains the same (thus absence of acceleration) while its *y*-component changes as there is the presence of acceleration due to gravity.

Additionally, the students were presented with tasks requiring the use of multiple visual representations of motion. These tasks aim at highlighting the difference between continuous and discrete motion depiction, developing students' fluency in drawing freeze frame and vector representations for different situations, as well as their ability to interpret and derive information from these qualitative depictions. Figure 10 illustrates one of the tasks, presented in the lecture notes, used for developing students' skills in dealing with the various visual modes of representing moving bodies.

A wooden block slides across a table (with friction) at an initial speed v_0 m s^{-1} (at $t_i = 0$) and comes to rest at t_f. On the diagram below, the block is shown at times $t_i = 0$ and t_f.

(a) Draw the freeze frame representation of the motion of the block.

(b) Draw in a vector next to each position of the block to indicate the magnitude and direction of the velocity of the block.

(c) What can you say about the acceleration of the block?

(d) Draw in the continuous path of the block from its start to end position.

(e) What can you say about the shape of the path?

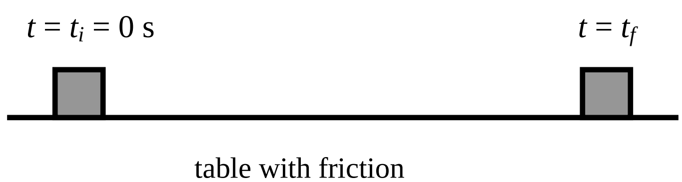


Figure 10. *Example of a task for developing students' skill with freeze frame representations (kinematics lecture notes, page 25).*

A diagrammatic representation indicating the object's initial position, at time $t = 0$ s, and final position at time $t = t_f$ s is provided. Students are required to fill in the object's freeze frame representation between its initial and final position. Vector representations are also included. In addition to the discrete motion depiction, the object's continuous motion is represented by drawing its path. The different visual representations are interpreted for the extraction of physics information, in this case the magnitude and direction of acceleration of the object.

<u>3.2.3</u> (b) Teaching the use of multiple representations in the kinematics course of the foundation component of the GEPS physics programme

The application of visual representations, in particular freeze frame representations, in the kinematics context is illustrated in Figure 11. It shows the different visual depictions to which information presented by a linguistic and graphical physical model can be translated when generating either a graphical or mathematical conceptual model.

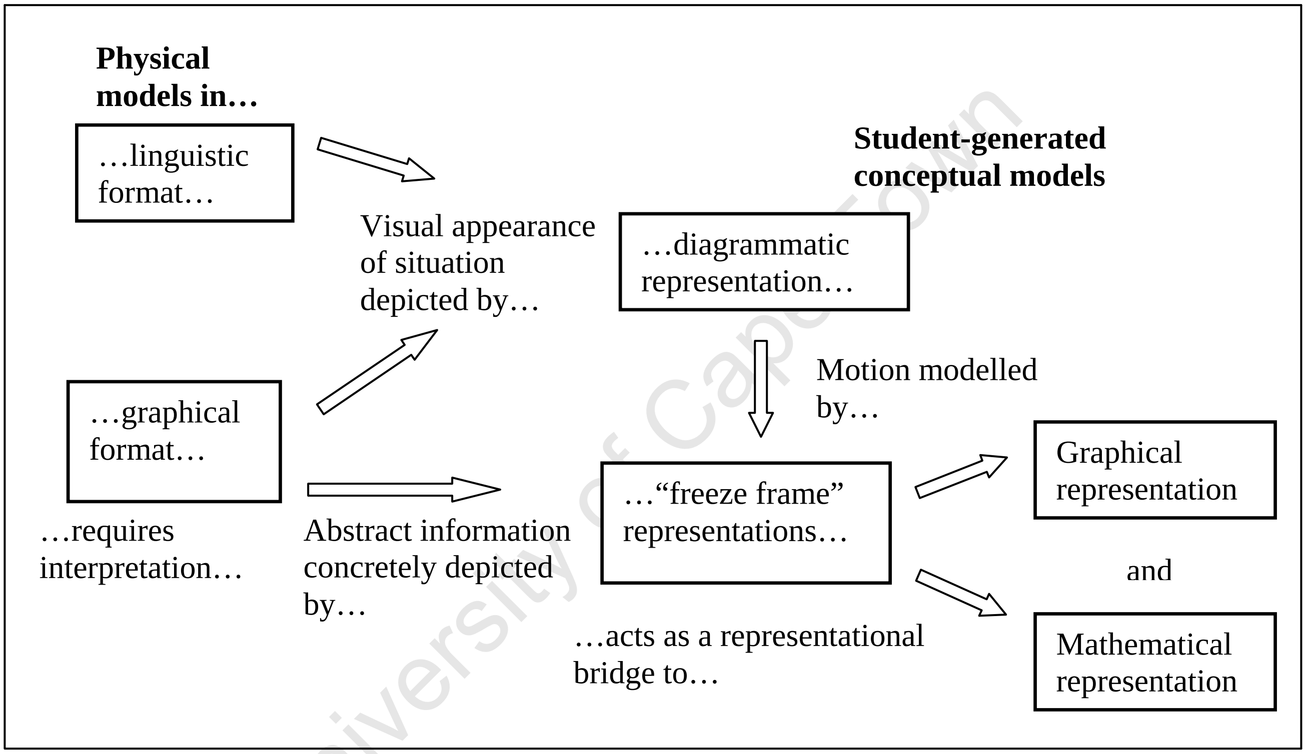


Figure 11. *Teaching the use of multiple representations in the kinematics context of the foundation course.*

In teaching the representation-rich kinematics course for the "foundation" aspect of the GEPS physics programme, the first step in solving problems posed in linguistic and graphical forms is to generate a diagrammatic representation to model the visual appearance of the situation. Students are taught to translate information from the abstract linguistic description or graphical depiction to a pictorial format where physics-based visual tools such as the coordinate system and arrows indicating the directions of motion are applied. The diagrammatic representation therefore makes the overall situation more concrete and understandable and mainly depicts quantitative information. However, a diagram usually depicts objects in motion at two instants of time, the initial and final positions, from which it is difficult, for students involved in the GEPS programme, to derive any implicit qualitative information. Freeze frame representations are consequently introduced and used to model the motion of objects. The particular visual representation is a means of concretely depicting the abstract physics information which subsequently supports the students' visualisation and sense-making of the implicit information. Vector representations are also included which further aid the interpretation and extraction of information about the different variables involved in a physical model. Students learned to transform the gathered information into other representational modes, either mathematical (quantitative) or graphical (qualitative). Thus, in the particular kinematics course, freeze frame representations are used for eliciting qualitative information, and as a representational bridge for translating information in graphical and mathematical forms.

A typical example of task requiring the use of multiple representations is shown in Figure 12. The task forms part of one of the examples in the lecture notes. It was used for teaching students the various steps required before the manipulation of a mathematical expression. In this particular problem, the physical model is presented in a linguistic format and requires the generation of a quantitative solution. The students are initially requested to unpack the situation by considering both the quantitative and qualitative information. A linguistic representation in the form of an explanation for the motion is generated. It requires highlighting the physics information underlying the descriptions "immediately hits the brakes", "truck coming directly towards you at a constant speed of 40 km h^{-1}" and "truck smashes into you". The construction of a complete diagrammatic representation follows. The two vehicles approaching each other as well as their meeting point are depicted. Freeze frame representations are then drawn to portray the motion of both vehicles. It should correspond to the diagrammatic representation in terms of the vehicles' depicted initial and final positions. Velocity vectors are included at each position of the vehicles. The depiction is then interpreted in order to derive further physics information, in this case, the magnitude and direction of the acceleration for each vehicle. The concrete qualitative representation is further used as a bridging tool where information is translated into more abstract graphical formats, namely position-time and velocity-time graphs, for both vehicles.

You are driving at a speed of 60 km h^{-1} and see a truck 20 m ahead coming directly towards you at a constant speed of 40 km h^{-1}. If you immediately hit the brakes and your car starts to slow down at 8.0 m s^{-2}, how long is it before the truck smashes into you? Choose the origin at the car with the $\hat{\mathbf{i}}$ unit vector pointing to the right.

(a) What can you say about the motion of the car and the truck? Use words!
(b) Sketch the physical situation. Draw the car and the truck at their initial and final positions.
(c) Sketch the freeze frame representation for the car and the truck
(d) Draw in vectors at each of the seven positions to indicate the direction and magnitude for the velocity of the car and the truck.
(e) What is the acceleration of the car and the truck?
(f) Sketch, on the same set of axes, **velocity-time graphs** for the car and the truck. Indicate clearly the point where the truck and the car will meet.
(g) Sketch, on the same set of axes, **position-time graphs** for the car and the truck. Indicate clearly the point where the truck and the car will meet.
(h) **Use your graphs** to determine the time before the truck smashes into the car.
(i) Now **use the equations of motion** to calculate the time (and the position of the collision).
(j) Write down the conditions (in terms of velocity and acceleration) under which there will be no collision between the car and the truck, i.e. the car and the truck will just touch for an instant before moving apart

Figure 12. *Example of a task requiring multiple representations (kinematics lecture notes, page 66-68).*

In order to generate the quantitative answer, two methods are employed. On the one hand, the motion graphs are used. The problem is solved using qualitative reasoning which includes applying the notion of area under the motion graphs. On the other hand, the general kinematics equations are applied to both the diagrammatic (for quantitative information) and freeze frame (for qualitative information) representations. Finally, the graphical representations are interpreted to explore and comprehend further concepts within the situation.

Therefore, the kinematics context of the "foundation" physics course emphasises mainly on students' conceptual development through the use of multiple visual models with particular focus on the application of freeze frame representations.

3.3 Pre-instruction kinematics test

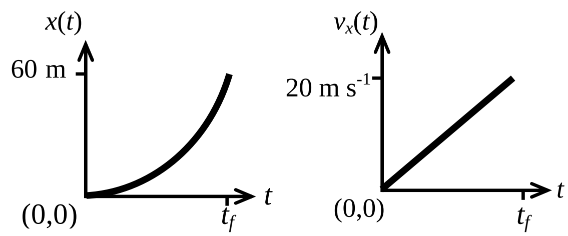
In the year 2008 a total of 179 students were registered for the GEPS physics course. Before being introduced to the representation-rich kinematics component of the course, students completed a kinematics pre-test. They were provided with 6 tasks requiring the generation of quantitative solutions and graphical representations. These tasks were completed, individually, as class works given at the end of the 45 minutes lecture period. Students who attended the lecture automatically completed the given problem. Hence, the number of students attempting the various tasks differs. The problems were posed in diagrammatic, linguistic and graphical formats. The pre-test aimed to investigate the strategies used by GEPS students for solving problems requiring a quantitative answer. It also explored GEPS students' ability to interpret, derive and translate information, mainly qualitative, into different representational forms.

3.3.1 Generation of mathematical formulations from different formats of physical model

Three problem questions, as shown in Figure 13, were concerned with the generation of a quantitative solution.

Question 1: A truck travelling at 60 m s^{-1} in a straight line slows down until it reaches a speed of 40 m s^{-1} while covering a distance of 70 m. It then maintains this speed for a further 10 m. Determine the time taken by the truck to complete the whole journey. Show all your steps clearly.

Question 2: Shown are the graphs of x versus time, and v_x versus time for the motion of a car along a horizontal straight road. Determine the time for the car to travel a distance of 60 m, i.e, determine t_f. Show all your steps clearly.



Question 3: The diagram describes the motion of a ball:

Determine the total displacement of the ball. Show all your steps clearly.

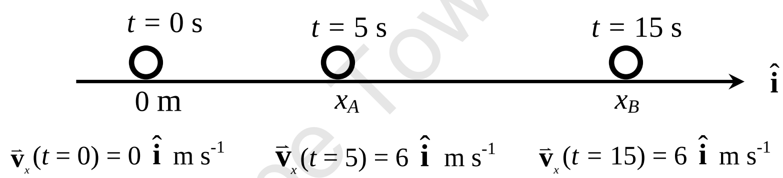


Figure 13. *Tasks administered as pre-tests requiring the generation of quantitative solutions.*

When solving for a value from a linguistic physical model (Question 1, Figure 13), 60% (103 out of 172) of the students applied mathematical expressions as the main step for attempting the problem. From the scripts of this sub-group it is evident that the primary focus is on the quantitative information. Often, a list of all the given and required values was written down, together with the relevant formulae. Moreover, the mathematical expression employed did not take qualitative information into account. The simplest equation relating the three variables (speed, distance and time) was often applied. Examples of responses which typify these descriptions are shown in Figure 14:

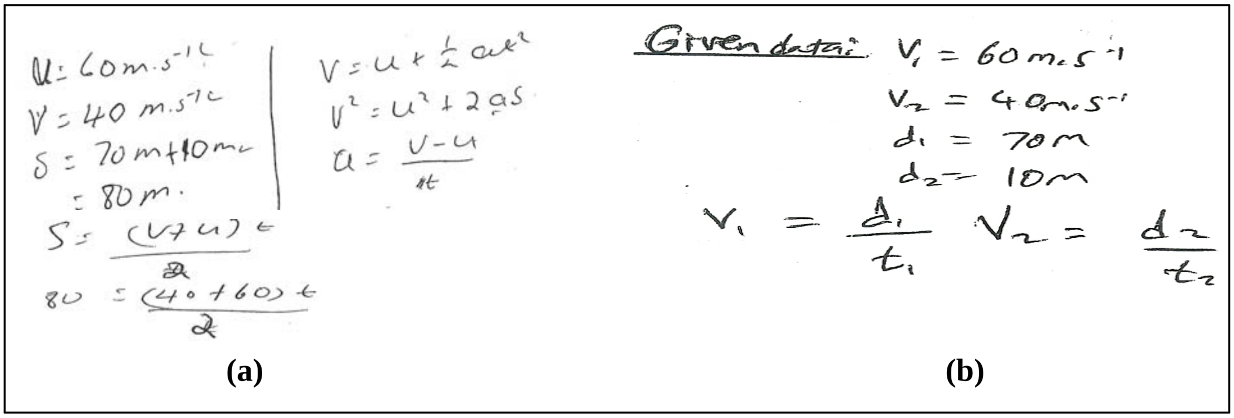


Figure 14. *Students' responses illustrating the use of equations only.*

Around 40% (69 out of 172) of the sample provided visual representations such as graphs and diagrams of varying quality. Only 5 of these students were successful with the task. The relevant motion graph for the situation was generated and employed for solving the problem. For the remaining instances, velocity-distance instead of velocity-time graphs was drawn as shown in Figure 15 (a). The diagrams were in the form of rough sketches (Figure 15(b)) or incomplete translations of the problem statement (Figure 15 (c)).

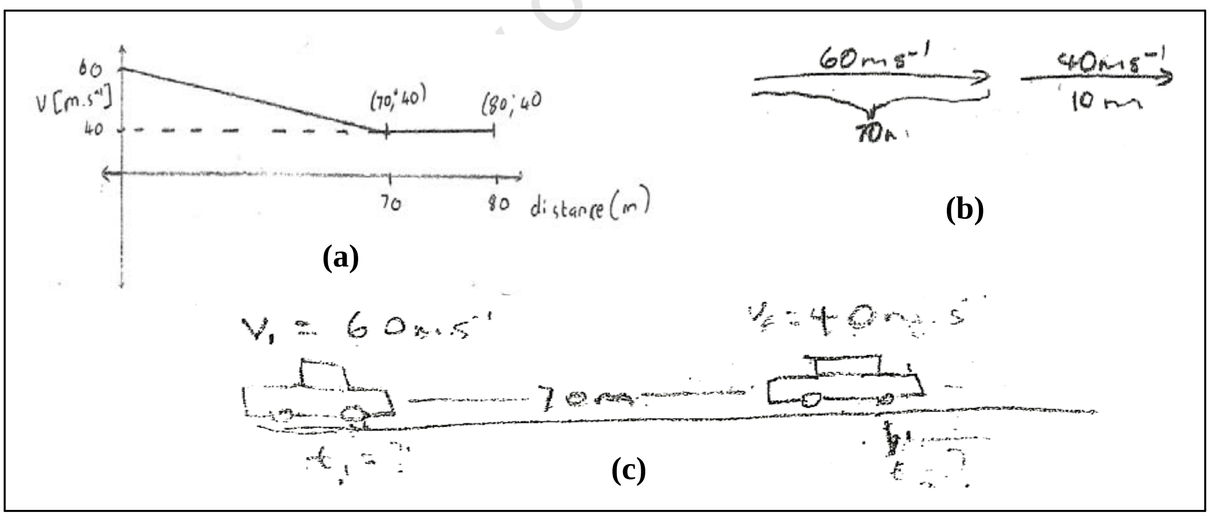


Figure 15. *Illustrations of the categories of student-generated visual representations.*

The visual representations are provided to have a better idea of the situation in terms of which numerical values correspond to which section of the situation. Hence, the substitution of values into the various mathematical formulae was facilitated. In addition, the visual and mathematical representations were completely dissociated.

When presented with a task posed in graphical form (Question 2, Figure 13), 95% (156 out of 165) of the sample used an equation with the qualitative information either irrelevant or not considered. Examples of mathematical expressions constructed by the students are presented in Figure 16.

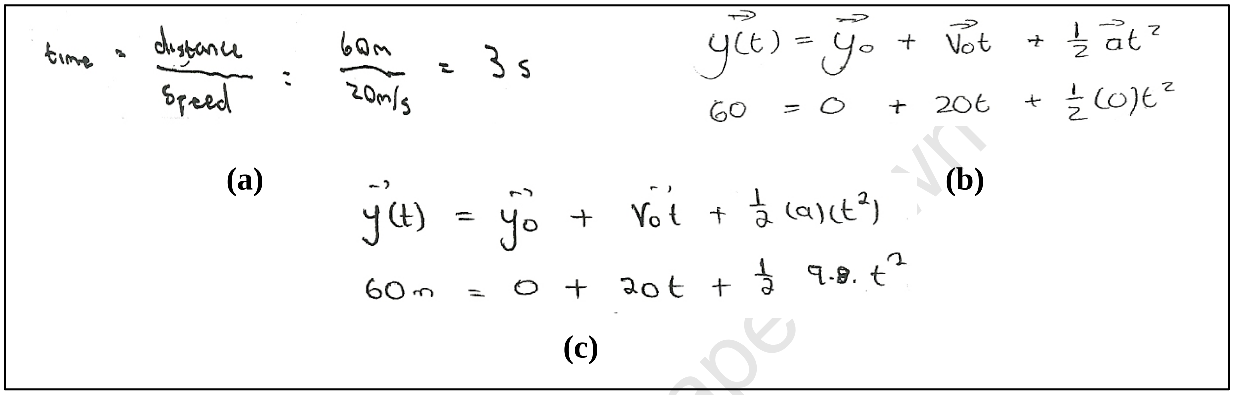


Figure 16. *Examples of student-generated mathematical representations.*

Only 3% (5 out of 165) of the sample made an attempt to solve the problem using qualitative reasoning (graphical method) with two of these students being successful. A graph and an equation with no link between them were provided in 2 instances. In another 2 cases, equations which account for qualitative information were used and corresponded to the physical model.

A similar pattern was observed when dealing with the problem question structured in diagrammatic form (Question 3, Figure 13). A large proportion, 79% (133 out of 167), of the cohort employed equations involving only the given and required quantitative information. Around 8% (13 out of 167) of the students used mathematical formulae focusing on qualitative information also but which were inconsistent with the physical model. An attempt was made by 10% (16 out of 167) of the sample to solve the problem qualitatively with 10 students successfully completing the task. Figure 17 provides an example of response involving the use of qualitative reasoning for solving the problem.

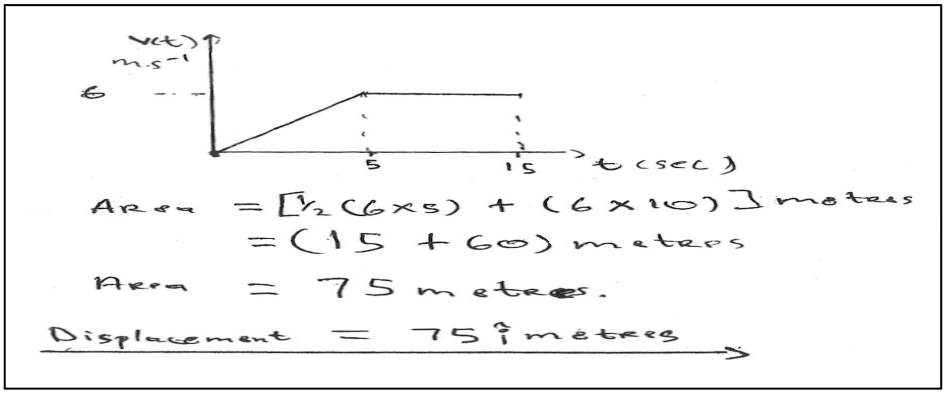


Figure 17. *An example of student's response using the graphical method.*

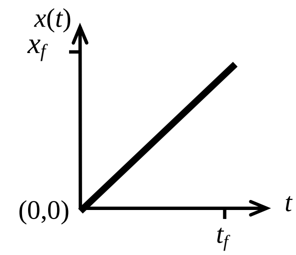
The remaining 3% (5 out of 167) of the students generated graphs and equations with no discernible association and the equations did not account for qualitative information.

The outcome, in terms of the sample's emphasis on quantitative information, mathematical expressions and hence formula-centred problem solving strategy, is consistent with those studies which highlighted no change in students' problem solving strategy after exposure to conventional methods of instruction from an introductory physics course (for example, Heller and Reif, 1984; Halloun and Hestenes, 1987). Additionally, previous work exploring the effectiveness of the application of reformed-based (the use of multiple representations) as opposed to traditional method of instruction have reported that most of the students from the latter category tend to mainly focus on and manipulate equations during problem solving (such as Van Heuvelen, 1991b; Rosengrant *et al.,* 2006).

3.3.2 Generation of graphical conceptual model from different formats of physical model

Figure 18 presents the three tasks dealing with the translation of information to graphical form.

Question 4: Consider the graph showing the magnitude of the position as a function of time for an object moving on a horizontal track. Sketch the corresponding graphs for the object of (i) v_x versus time, and (ii) a_x versus time.



Question 5: A toy car is moving along a horizontal track in a straight line. It starts from rest and speeds up uniformly until it reaches a speed of v m s^{-1}. On the axes, sketch graphs for the motion of the toy car for (i) x versus time (ii) v_x versus time, and (iii) a_x versus time.

Question 6: The diagram shows the motion of a box:
Sketch graphs for the motion of the box of (i) x versus time (ii) v_x versus time, and (iii) a_x versus time.

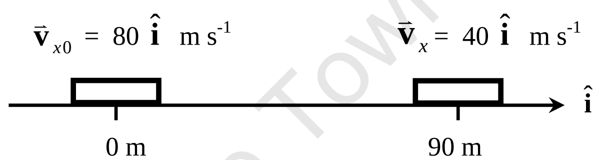


Figure 18*. Tasks administered as pre-tests requiring the construction of graphical representations.*

For the three written exercises, the vast majority of the students failed to generate the appropriate graphical representations. Around 16% (26 out of 161) of the sample were successful with the task presented in graphical form (Question 4, Figure 18). For the problem questions with a linguistic (Question 5, Figure 18) and a diagrammatic (Question 6, Figure 18) format, a respective 32% (55 out of 174) and 6% (10 out of 163) of the sample were able to generate the required motion graphs.

In most cases, an inability to interpret, derive and translate information is displayed. The students failed to recognise that the physics information portrayed by the different graphs drawn conflict with one another and with the physical model. Irrelevant graphical representations were also generated due to lack of graphing skills. In particular position-time graphs give considerable difficulties since they are more abstract, dealing with positively and negatively sloped curves. Examples of responses provided for the problem question with a graphical, linguistic and diagrammatic format highlighting the difficulties described are shown in Figure 19(a), 19(b) and 19(c) respectively.

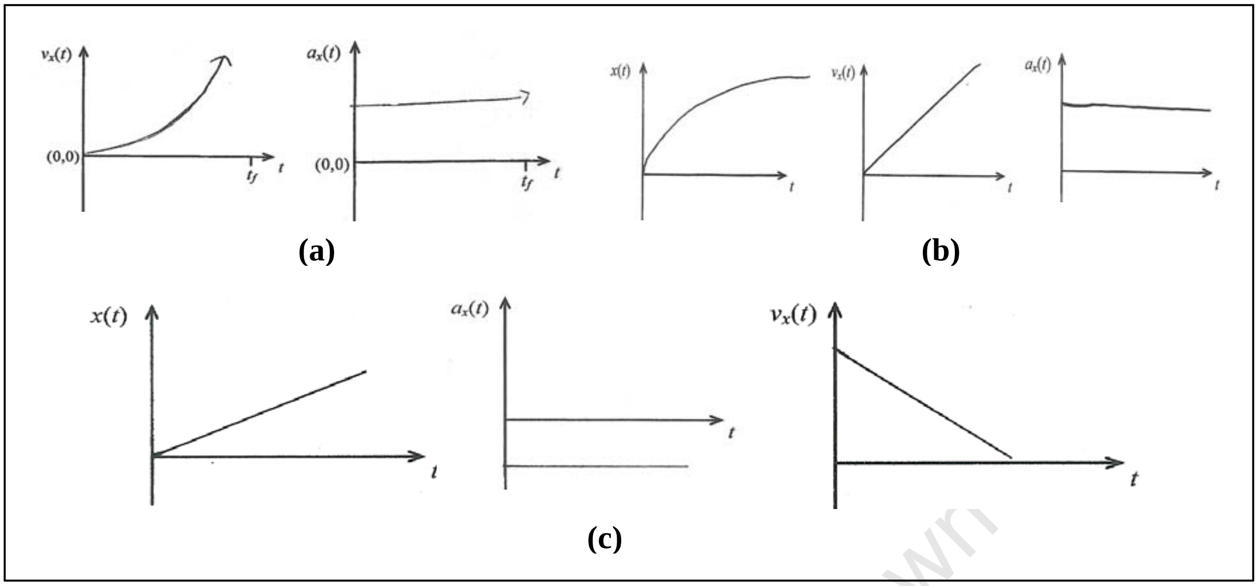


Figure 19. *Examples of student-generated graphical representations.*

Previous studies implemented on students' handling of kinematics graphs have reported on their poor graphing skills. In particular, students fail to interpret motion graphs and are unable to translate information in graphical form. In most of these studies, the cohort has a good academic and scientific background and was either from secondary level (such as Beichner, 1994; Simpson *et al.*, 2006) or tertiary level (for example, Goldberg and Anderson, 1989). In addition, the work by McDermott *et al.*, (1987) also involved undergraduate students taking a "preparatory physics course". The findings emerging from the current work on university students from educationally disadvantaged backgrounds are in line with those of the previous studies.

<u>3.3.3</u> GEPS students' mental representations and ability to translate information in kinematics

From the strategies used by the GEPS students to attempt tasks requiring the formulation of mathematical solutions it can be claimed that the importance of diagrams for problem solving is not recognised and qualitative information holds secondary importance. Physics information presented by the shape of a graph, in linguistic or diagrammatic forms, either conveys no meaning or is misinterpreted. Equations are applied at random and values are filled in regardless of their relevance. There is no understanding of the relationship between the symbols and the quantitative information, no sense-making of the physical model, as the students' main aim is to manipulate the symbolic representation in order to obtain the required final value. The strategies employed by the large majority of the GEPS students indicate that they think in terms of equations and prioritise the symbolic representations. According to Johnson-Laird's (1983) cognitive framework, the students' mental representations can be categorised as "propositional". Only a minimal percentage of students, 7% (10 out of 167), 2% (4 out of 165) and 3% (5 out of 172) constructed an appropriate mental model for the tasks presented in diagrammatic, graphical and linguistic forms respectively, requesting the generation of a quantitative solution. It should be pointed out that the successful students solved the problem by using mainly qualitative reasoning instead of applying kinematics equations. Factors that may hinder the construction of appropriate mental models after exposure to the representation-rich kinematics course include the students' approach to learning, based mainly on rote memorisation, as well as prior experience with the kinematics context in terms of instruction and problem-solving procedure. Their views of the insignificant role of visual representations in the physics domain, their prioritisation of mathematical expressions and quantitative information may affect their generation of appropriate mental models.

The students' mastery in translating information into different forms, from abstract linguistic or graphical representation, and the concrete diagrammatic representation to mathematical form is undeveloped. The inability to translate information among various representational modes was also observed when generating motion graphs. The majority of the students constructed irrelevant graphical representations due to their lack of techniques and skills in manipulating kinematics graphs. They fail to interpret, understand, and represent the derived information in other forms of representations.

3.4 Research questions for the present work

Most of the instructional strategies designed for developing kinematics conceptual understanding and enhancing graphing skills have been employed during laboratory activities (procedural aspects) on students from good educational backgrounds. The present study is implemented during the process of solving paper and pencil tasks (declarative aspects) with students who are academically disadvantaged, have superficial kinematics conceptual knowledge and formula-based problem solving strategies. The main concern is to equip these students with those techniques and procedures which will both develop their kinematics conceptual understanding and improve their problem-solving strategies. The students are therefore familiarised with the notion of multiple representations. Their hands-on experience with information depiction in diverse representational modes may result in the generation and manipulation of their own conceptual models. They are involved in the translation and integration of information among the several representations which are crucial processes for learning. In addition, freeze frame representations are included as an intermediate step for solving paper and pencil kinematics problems. It acts as a support in the interpretation, derivation and translation of information when generating motion graphs, mathematical and linguistic representations from different formats of physical models. Moreover, the students' external representations provide insights into the category of mental representations.

The effect of the application of diverse representations, in particular the role of using freeze frame representations on students' kinematics conceptual understanding and problem solving performance is explored. The research questions designed for the study are:

1. What is the effect of using freeze frame representations on students' generation of graphical conceptual models from diagrammatic, graphical and linguistic conceptual models?
2. What is the effect of using freeze frame representations on students' generation of mathematical conceptual models from graphical and linguistic conceptual models?
3. What is the effect of using freeze frame representations on students' generation of linguistic conceptual models from mathematical and graphical conceptual models?

4. What are the categories of cognitive constructs that students generate when dealing with different representations of physical models?

4. Method

4.1 Evaluation research

The current study utilises a mixture of quantitative and qualitative research methods, supporting an evaluative approach. Evaluation research in education is "the process of making judgements about the merit, value, or worth of educational programs" (Gall *et al.*, 2007, page 559). The main difference between evaluation and research is that the former aims at generating decisions while the latter contributes to broadening the body of knowledge about a particular phenomenon (Cohen *et al*., 2007). Both of these purposes underlie this study.

Within the Context-Input-Process-Product (CIPP) evaluation model (Stufflebeam *et al.*, 2000) the current work involves a combination of process and product evaluation. It spans over an extended period of time. It is also concerned with exploring whether the intended goals of the program have been achieved and this will consequently inform decision-making regarding its application. Moreover, the study includes formative together with summative evaluation (Verma and Mallick, 1999) whereby an assessment of the intervention is performed during and after its implementation.

For the present study, a model-based approach is applied for supporting the development of appropriate mental models in kinematics with a particular focus on the use of freeze frame representations. The research questions constituting the evaluation component of the study explore the quality of student-generated conceptual models when attempting tasks with and without freeze frame representations. Students' handling of the particular visual representation when dealing with problem questions posed in different representational forms and requiring the generation of either qualitative or quantitative solutions is considered. An investigation is made in terms of whether the presence of freeze frame representations in a task results in its application as an intermediate for translating information to graphical or mathematical forms. The use of freeze frame representations for unpacking qualitative kinematics information is also explored. By considering the consistency among the various forms of conceptual model together with their correspondence to the physical model, it is possible to assess the extent to which the objectives of the intervention have been achieved. The outcomes from these particular research questions will facilitate decision-making with regards to changes which can be made to the model-based approach for teaching the topic of kinematics. The research aspect deals with understanding the link between mental representations and external representational models. The students' actions when attempting tasks presented in an open-ended and structured (presence of freeze frame or multiple representations) format are of interest. The category and quality of mental representations constructed are therefore gathered. Also, of main concern are the possible reasons underlying the students' actions when handling problem questions involving multiple representations.

The major drawback of evaluative research is that the outcomes may not be generalisable (Gall *et al.,* 2007). In other words, the findings have a low comparability and transferability (Lincoln and Guba, 1985). Usually, evaluative research studies aim to improve a specific intervention and are implemented within a particular context (Bennett, 2003). Hence, the results obtained may not be validly applied to other situations. To cater for the particular drawback, the context of the current study is extensively described providing ample information for judging the transferability of the findings. Moreover, the interest of evaluation research is to measure the extent to which the aims and objectives of a program have been attained. The focus is principally on learning outcomes and therefore the current work is an example of a single-case design for evaluating the response to an intervention (Riley-Tillman and Burns, 2009). The assessment of the intervention is performed by comparing the expected learning outcomes with those gathered from the actual evaluative study (Gall *et al.,* 2007). However, the disadvantage of objectives-based evaluation lies in the evaluators' awareness of the program's goals which may interfere in their assessment of the intervention. The tendency to adopt the prescribed intended behavioural objectives may lead to disregarding other important aspects of the evaluation mainly exploring the negative consequences of applying the intervention. In the present study, the researchers are familiar and well-versed with the course content and terminology, the context of the intervention as well as its intentions or goals. Consequently, the possibility of emphasising particular behavioural objectives is countered.

4.2 Design of instruments for the main study

A total of 24 paper and pencil kinematics tasks were created of which 6 items were administered before the teaching intervention. The remaining 18 problem questions were completed during and after the intervention. They were designed for answering the different research questions constructed for the study. The tasks formed part of class works, tests and the end of first semester examination. The complete set of tasks administered during and after instruction is presented in Appendix B. For five of the tasks, the construction of graphical representations from diagrammatic, graphical and linguistic conceptual models was required. These data address research question 1. A total of eight questions dealt with generating a mathematical formulation from problem questions posed in diagrammatic, graphical and linguistic forms. The solution of research question 2 will draw on the responses provided for the tasks structured in graphical and linguistic forms. In one problem question with a linguistic format, both graphical and mathematical conceptual models were requested. The responses provided were also used to answer research questions 1 and 2. Four questions were concerned with the formulation of a linguistic representation (written explanation) for the information presented in mathematical and graphical forms. The responses therefore provide evidence for answering research question 3. Research question 4, concerned with the sample's categories of mental representations, is addressed by considering the various tasks presented for the first three research questions in addition to the problem tasks with a diagrammatic format requiring the generation of mathematical representations.

For each research question, the problem tasks are set in different formats which can be either non-directive or directive. In the former case, the tasks have an open-ended format. The problem questions are presented without any prescribed steps to be followed for generating the required conceptual model. Instead, the students have to design their own strategies and decide on the representations to be included in the problem solution. In contrast, a "directive" task specifically guides the students through the different steps to solve the problem with the inclusion of freeze frame representations.

The non-directive format is employed for tasks aiming at investigating students' use of multiple modes of representation, in particular the spontaneous use of freeze frame representations as an integral step for problem solving. When no guidance is provided it is therefore possible to capture the quality of mathematical or graphical conceptual models generated. A comparison is then made with the type of mathematical or graphical representations constructed from problems which explicitly request the inclusion of freeze frame representations.

Figure 20 illustrates a non-directive kinematics task posed in linguistic form.

Train A and train B are moving towards each other on a flat horizontal track. Train B is initially 36 m ahead of train A, and moves with a constant speed of 4 m s^{-1} throughout the motion. Train A is initially moving at 16 m s^{-1}, and slows uniformly at a rate of 2 m s^{-1}. Determine the position at which the two trains smash into each other, relative to the initial position of train A. Show all your steps clearly.

Figure 20. *Example of a non-directive task intended to capture students' use of visual representations.*

The task is structured in linguistic form and requires the generation of a quantitative solution. The particular case explores whether kinematics equations are directly manipulated or if visual depictions such as diagrammatic or freeze frame representations are initially drawn and then used for generating the mathematical formulation. Students' strategies for solving the problem provide insight into the various mental representations used. The quality of the students' conceptual models is also of interest. Quantitative information (such as values for positions and velocities) together with the physics information (for example direction of acceleration for decreasing velocity, absence of acceleration for constant velocity and direction of the vehicles' motion) presented in the mathematical model are considered. The analysis takes account of the characteristics of the diagram provided (if any), ranging from a rough sketch, the depiction of only part of the motion to the external visualisation of the whole situation. If freeze frame representations are included, its consistency with the qualitative information as presented by the physical model and with the mathematical representation of the solution is considered.

Another example of a non-directive task is shown in Figure 21. Here, the information is presented in graphical form.

Consider the a_x versus time graph for an object moving in a straight line. At $t = 0$, the object is at the origin moving a velocity of v_0 in the positive x-direction. Sketch the corresponding graphs of (i) x versus time, and (ii) v_x versus time, for the motion of the object. Label the graphs clearly.

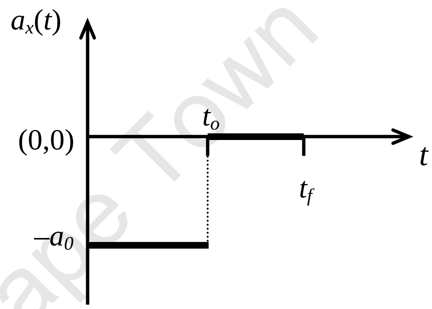


Figure 21. *Example of a non-directive task for exploring the quality of graphical representations generated.*

The physical model is presented in the form of an acceleration-time graph. An interpretation of the shape of the graph together with the meaning of zero acceleration and the negative sign associated with the acceleration ($-a_0$) is required. Ideally, the information derived from the graphical representation needs to be depicted via freeze frame representations before being transformed into motion graphs for position and velocity. The interest lies in the quality of the two kinematics graphs generated, in terms of their correspondence to the physical model. A comparison is then made with the strategies used for solving problems which explicitly request the inclusion of freeze frame representations before constructing the motion graphs and hence the quality of the student-generated graphical representations.

The directive format is applied when exploring the effect of using freeze frame representations as a representational bridge for constructing motion graphs or mathematical expressions, and for unpacking qualitative information. It is also employed for investigating the students' problem-solving strategies and hence performance when requested to construct different modes of representations. Of interest is whether the different representational forms are linked and used for generating the linguistic, mathematical and graphical conceptual models and hence the analysis also focuses on their quality.

Figure 22 is an example of a directive task. The item investigates whether freeze frame representations support the interpretation of information as portrayed by the mathematical (kinematics equation) physical model.

The equation of motion for a ball moving along a horizontal track is

$$v_x\,(\hat{\mathbf{i}}) = v_{x0}\,(\hat{\mathbf{i}}) + a_x\,(-\hat{\mathbf{i}})\,t$$

(a) Draw a "freeze frame" representation for the motion of the ball from its initial to final positions.

Initial position of ball

Final position of ball

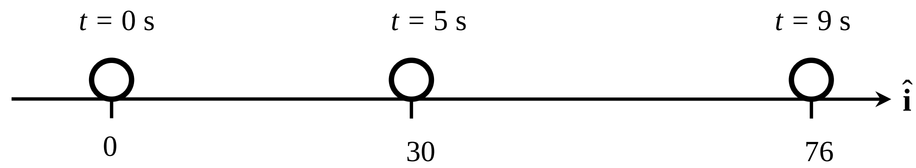
(b) Write down (in words) everything you can say about the motion of the ball.

Figure 22. *Example of a directive task requiring freeze frame representations for unpacking qualitative information.*

The question requires the application of freeze frame representations to concretely depict the qualitative information. An interpretation of the mathematical model is required, in particular the meaning ascribed to the unit vectors which is consequently portrayed in visual form. Freeze frame representations may also be annotated with acceleration and velocity vectors. The visual representation is then applied for deriving further physics information presented by the physical model. A linguistic representation (written response) is generated. In analysing the solution, consideration is given as to whether it is presented in the form of a description or an explanation. In addition, the quality and completeness of the physics information is assessed in terms of the magnitude and direction of velocity together with acceleration, and the direction of motion of the ball for the case of a decreasing velocity.

For the case shown in Figure 23 freeze frame representations are used as a representational bridge for generating motion graphs unlike the task shown in Figure 22 where it was applied for eliciting qualitative information from a mathematical conceptual model.

The motion of a ball rolling along a straight road is described below:



$\vec{\mathbf{v}}_x(t = 0) = 6\ \hat{\mathbf{i}}\ \text{m s}^{-1}$ $\vec{\mathbf{v}}_x(t = 5) = 10\ \hat{\mathbf{i}}\ \text{m s}^{-1}$ $\vec{\mathbf{v}}_x(t = 9) = 4\ \hat{\mathbf{i}}\ \text{m s}^{-1}$

(a) Draw the "freeze frame" representation for the motion of the object from its initial to final positions

(b) Sketch graphs of (i) x versus time, and (ii) v_x versus time for the motion of the ball. Label the graphs clearly.

Figure 23. *Example of a directive task with freeze frame representations used as a representational bridge.*

A translation from the diagrammatic physical model to the graphical conceptual model is required with freeze frame representations as the mediator. While the diagrammatic representation portrays the position of the ball at three different instances, freeze frame representations by definition taken at regular time intervals apart, provide a more complete version of the ball's motion. In addition, vector representations may be included. The position and velocity graphs for the physical model need to be drawn by applying freeze frame representations. The instrument probes whether the presence of freeze frame representations in the task results in its application for constructing the graphical representations. The quality of the generated kinematics graphs is also of interest.

Using the responses from the "directive" items, it is possible to capture the translation of information between the various forms of conceptual models, in particular to and from freeze frame representations. In addition, the directive items allow the identification of a student's understanding of the physical model by considering the consistency of the depicted information among the various representational modalities. Of crucial importance is that non-directive and directive formats provide instances where conceptual and physical models are in conflict. They allow the identification of the cause for constructing an inappropriate conceptual model. Thus, the reasons governing the errors made when generating mathematical or graphical conceptual models for the different tasks can be explored.

Table 4-1 provides an overview of the purpose of the tasks administered during and after instruction. The tasks are grouped according to the research questions they are associated with.

Table 4-1: Purpose of kinematics tasks and their associated research questions.

Research questions	Tasks	Purpose of tasks
1. What is the effect of using freeze frame representations on students' generation of graphical conceptual models from diagrammatic, graphical and linguistic conceptual models?	Class work 15 Class test question 1 Class test question 5 Class work 10 Class work 9 June class test question 10	To compare the quality of motion graphs constructed when handling tasks with and without freeze frame representations. In other words, the application of freeze frame representations for constructing graphical representations consistent with the physical model is investigated.
2. What is the effect of using freeze frame representations on students' generation of mathematical conceptual models from graphical and linguistic conceptual models?	Class work 14 Class test question 7 Class work 13 Class test question 4 Class work 16 June class test question 10	To explore whether the presence of freeze frame representations leads to the construction of better quality mathematical expressions. The students' performance when attempting the same task in a directive (presence of multiple representations) and non-directive format is also compared.
3. What is the effect of using freeze frame representations on students' generation of linguistic conceptual models from mathematical and graphical conceptual models?	Class work 8 Class test question 1 Class works 7 Class work 12	To investigate whether the inclusion of freeze frame representations in a task results in the formulation of a written response with physics information related to the physical model. A comparison is then made with responses gathered from tasks without freeze frame representations.

4. What are the categories of cognitive constructs that students generate when dealing with different representations of physical models?	Eighteen tasks were considered excluding non-directed tasks concerned with translation to graphical representations.	To classify the kinds of mental representations constructed by students when handling kinematics tasks with different representational modes requiring the generation of qualitative or quantitative solutions.

4.3 Administration of instruments

The data collection process spans over the full period of five weeks that kinematics was taught within the course (see Appendix C), from the second week of April to mid May and includes the PHY1023H course examination in June 2008. Of the 24 tasks, 16 problems were completed as class works, six tasks were included in a regular test paper and two problems questions formed part of the end of first semester examination paper. As mentioned earlier, a total of six of the 24 instruments were administered as pre-tests.

At least three class works were completed per week. Students were requested to keep away their lecture notes and any other materials relevant to the topic. They were required to fill in their name and student number for identification during later analysis. The class works constituted mainly of short tasks, such as sketching of motion graphs and providing written explanations for a given physical model. These problems were completed in 5 to 10 minutes. No marks were allocated for their completion. The questions which form the class works were selected to be short and non-directive in order not to disrupt the time scheduled for covering the lecture materials. On Thursdays, the given class work was considered as a "class test". The students were provided with 15 to 20 minutes for its completion and the allocated marks contributed to the final assessment. The sample size may vary between 131 to 178 students for the different tasks. The test and examination lasted for 45 minutes and 2 hours respectively. Hence the majority of the directive tasks were included as test and examination questions.

Instruments investigating particular aspects of the research questions were administered either during the same week of, or early next week following the introduction of the relevant part of the kinematics topic. For example, one week after being familiarised with the use of freeze frame representations and while learning to generate graphical and mathematical representations, students completed tasks requiring the construction of motion graphs and the formulation of linguistic representation. The data collection planner can be viewed in Appendix C.

4.4 Method of interpretational analysis

The process of interpretation is the core of qualitative research. It applies to the study design, the selection of the context for its implementation, determining the type of data to be collected as well as to the data analysis (Peshkin, 2000). Interpretation is also involved when ascribing meaning to the outcomes of an inquiry. It plays a particularly crucial role for the sense-making of actions or processes (Cohen *et al.*, 2007). Peshkin (2000, page 9) defines interpretation as "the act of imagination and logic. It entails perceiving importance, order, and form in what one is learning that relates to the argument, story, narrative that is continually undergoing creation".

The method of interpretational analysis was used for organising the data obtained from the 24 instruments designed for the study. Gall *et al*. (2007, page 466) refers to the particular analysis procedure as "the process of examining data closely in order to find constructs, themes, and patterns that can be used to describe and explain the phenomenon being studied".

For tasks comprising only one form of generated representation (a graphical, mathematical or linguistic conceptual model), 30 randomly chosen scripts were used to formulate "statements" succinctly describing the characteristics of the representation provided. The responses, from all the scripts, were then clustered according to the description they correspond to (that is the existing statement) or give rise to the formulation of a new statement. During this process, statements were refined and the distinctions among them were highlighted. Similar statements were grouped together thus leading to the creation of categories. Within each category, sub-categories may be developed fore-grounding additional and in-depth details associated with the representation provided. All the scripts were then reconsidered in order to check the category and sub-category classification of the specific representation. Frequencies for each description category were compiled.

If responses show more than one representation mode within the same task, such as freeze frame together with graphical, mathematical or linguistic representations, then the analysis was divided into segments. Starting with 30 scripts, chosen at random, statements were generated for the first representation which is usually visual in nature, either freeze frame or diagrammatic representations. Within each classification of the initial representation, the characteristics of the subsequent representational modes provided were described. The responses provided by each student were then grouped according to their correspondence to the overall representational description. During this process the descriptions were improved and new ones emerged to adequately capture the students' responses. The differences between the various descriptions were fore-grounded leading to the construction of categories. All the scripts were then revisited for verifying the categorisation of the students' responses.

If a specific inappropriate graphical shape was drawn by less than five students then it was classified as "uncodeable". This particular category is reported as it represents motion graphs which are inconsistent with the physical model and hence comprises those students who either failed to interpret and derive information from the given representational mode or were unable to depict information about motion graphically. Moreover, in certain instances a variety of inappropriate shapes were drawn. The frequencies under the category "uncodeable" may add up to 20 responses, a cluster size that cannot be ignored.

For tasks dealing with the formulation of a mathematical model, the types of errors, a combination of qualitative and quantitative mistakes or either one of them, are of interest. Hence, if a particular category of an incorrect mathematical expression was provided by less than five students its characteristics were described instead of clustering them under the heading "uncodeable".

To illustrate the application of the method of interpretational analysis, the task presented in Figure 20 is considered. The data emerging from the particular case are organised as shown in Figure 24.

	Descriptive statements	Total
1	**Presence of equations only**	
1.1	Relevant variables for the notations and the associated directions. Absence of acceleration for constant velocity and direction of acceleration for decreasing velocity is included.	4
1.2	Train B velocity direction is ignored	16
1.3	Direction of acceleration for decreasing velocity ignored, and	2
1.3.1	train B velocity direction is ignored	10
2	**Presence of equation and diagram**	
2.1	*Diagram in the form of a rough sketch. Mathematical model includes…*	
2.1.1	relevant variables for the notations and the associated directions. Absence of acceleration for constant velocity and direction of acceleration for decreasing velocity is included.	3
2.2	*Diagram includes an axis and depicts part of the motion. The vehicles at their respective initial position are shown. Mathematical model includes…*	
2.2.1	relevant variables for the notations and the associated directions. Absence of acceleration for constant velocity and direction of acceleration for decreasing velocity is included	16
2.2.2	train B velocity direction is ignored	12
2.2.3	Direction of acceleration for decreasing velocity ignored, and	4
	- train B velocity direction is ignored and its initial position is – 36 m / train A position is 36 m / both trains initial position is 36 m.	4
2.3	*Diagram includes an axis and depicts the whole motion. The vehicles' initial and meeting positions are shown. Mathematical model includes…*	
2.3.1	relevant variables for the notations and the associated directions. Absence of acceleration for constant velocity and direction of acceleration for decreasing velocity is included.	30
2.3.3	Direction of acceleration for decreasing velocity ignored, and	
	- train B velocity direction is ignored	7
2.4	*Diagram includes an axis and depicts the vehicles to move in the same direction. Their initial and meeting points are shown. The mathematical model includes…*	

Figure 24. *Main categories of data emerging from the non-directive task shown in Figure 20 to identify various conceptual models (Full version of results can be viewed in Appendix E, June class test question 12).*

The scripts were first sorted according to whether several or only one form of representation, that is, a mathematical expression was used. For the latter, descriptions were developed to highlight the features of the mathematical representation, resulting in different categories (1.1 to 1.3). The descriptions focused on the values for the corresponding symbolic notations such as positions and velocities. The qualitative information was also considered. This included the student's consideration of the direction associated with the vehicles' velocity and position, the direction of acceleration for decreasing velocity and the absence of acceleration for constant velocity.

Subsequently, responses with two representational modes, namely diagrammatic and mathematical representations, were analysed in segments. Starting with the visual depiction, categories for the diagram were developed. They ranged from being a rough sketch, portrayal of part or the whole motion to an irrelevant depiction of the situation (categories 2.1 to 2.4). Within each category of diagrammatic representation, the mathematical expressions formulated were described. Sub-categories may cater for further details within the mathematical formulation. For example, category 2.2.3 includes responses that, in addition to ignoring the direction of acceleration for decreasing velocity in the mathematical model, do not include the velocity direction and, values for one or both vehicles' positions were inappropriate.

From the descriptions for both the diagrammatic and mathematical representations, the errors made by the students could be identified. The nature of the mistakes was fore-grounded enabling inferences on the underlying reasons for the construction of a conceptual model inconsistent with the physical model. For example, mathematical expressions involving inappropriate quantitative information may be explained by the absence of a diagrammatic representation, the category of the particular visual representation which may be either incomplete or unsuitable for the situation, or the misinterpretation of the diagram drawn. Qualitatively inappropriate mathematical formulations (such as absence of acceleration for decreasing velocity, inclusion of acceleration for constant velocity and ignoring the direction of acceleration) may be attributed to the inability to interpret and derive information from the qualitative descriptions. It should be noted that the rare application of freeze frame representations as an integral step for problem solving was a determinant feature of the task.

To facilitate comparison across tasks for each student, alphanumeric coding schemes were developed. They aim at capturing more broadly the category of the representations generated. Graphical and freeze frame representations were coded in terms of their correctness or incorrectness. Additionally, inappropriately drawn motion graphs were coded according to the category of the mistake. The codes allocated for diagrammatic representations take into consideration whether the diagram is complete and correct, depict part of the motion, is inappropriate for the situation or is in the form of a rough sketch. For tasks dealing with the generation of a quantitative solution, the coding scheme was designed to capture the category of mistakes presented in the mathematical model, whether it is qualitative or quantitative in nature or both. The strategy employed for attempting open-ended tasks posed in linguistic form was also classified in terms of presence of equations only, or the inclusion of diagram or sketch. Student-generated linguistic conceptual models were coded according to whether an explanation with the inappropriate or appropriate physics information was formulated or qualitative information was not interpreted. From these codes it is possible to capture consistency among the different conceptual models generated as well as with the physical model. Moreover, the codes allocated for the students' actions when dealing with the different representations are used to infer the category of mental representations constructed by the students for the different tasks. The quality of the mental model, in terms of its correspondence (or lack of) may also be determined. The complete version of the coding scheme is presented in Appendix D.

However, it is possible that for some of the questions, the responses can be generated by rote memorisation, mainly after teaching has taken place and with practice in handling tasks of similar nature. Consequently, the validity of the data interpretation may be reduced.

<u>4.5</u> Validity and Reliability

Validity may be referred to as "a demonstration that a particular instrument in fact measures what it purports to measure" (Cohen *et al.*, 2007, page 133). The term reliability may be defined as "the extent to which other researchers would arrive at similar results if they studied the same case using exactly the same procedures as the first researcher" (Gall *et al.*, 2007, page 477). The present study is both qualitative and quantitative in nature. Usually, in qualitative research the researcher acts as the "measuring instrument" (Gall *et al.*, 2007, page 458), is directly engaged in different phases of the research (Hitchcock and Hughes, 1989) and plays a particularly crucial role during data collection and analysis. Consequently credibility and trustworthiness of the data and their interpretation are key elements of qualitative research (Patton, 2002).

<u>4.5.1</u> Research validity

<u>4.5.1</u> (a) Modifications due to piloting

A series of pilot tests was performed with students enrolled for the GEPS physics programme during the first semester of the 2007 academic year. A total of 120 students were registered for the course. The pilot cohort's educational background together with the nature and contents of the course was similar to that described for the present study. The main purposes of the pilot activities were:

- to improve on the teaching materials regarding the use of freeze frame representations in teaching kinematics concepts;
- to check the validity of the instruments with respect to the research questions;
- to check the validity of the instruments with respect to students' interpretation of the tasks;
- to refine and focus the research questions;
- to check the feasibility and validity of the strategies for data collection; and also
- to check the validity of the method of analysis for the purpose of the study.

The pilot students were introduced to the notion of freeze frame representations as an independent topic before its application in the kinematics context. Different situations of motion, both in one and two dimensions were dealt with. Freeze frame representations were used as an intermediate step when translating between motion graphs and for generating mathematical expressions from tasks presented in linguistic form. Tutorials guided the students explicitly through the various representational transformations.

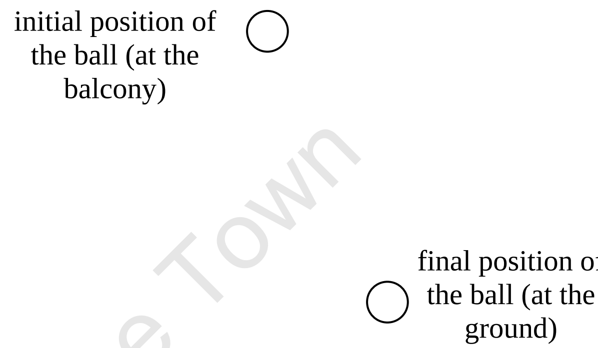
For the current study, some modifications to the teaching sequence and materials were implemented for pedagogical reasons. The sample was introduced to freeze frame representations immediately within the context of kinematics but with motions limited mainly to one dimension. Illustrations of freeze frame representations from textbooks were also included in order to emphasise its importance in kinematics. The translation of information from a graphical to a mathematical conceptual model via the use of both diagrammatic and freeze frame representations was taught explicitly. The teaching strategy now included weekly class works on aspects of kinematics to which students were exposed during the week. They served the dual purpose of data collection and formative assessment of the students' kinematics knowledge. In terms of tutorials, unlike the pilot cohort, the students in the main study were presented with a combination of tasks in directive and non-directive formats posed in different representational forms.

The data collection instruments were also modified. A set of 17 written exercises was completed by the pilot sample. The instruments were constructed according to broad initial research questions formulated. They dealt with the application of visual elements in the foundation component of the GEPS physics course and teaching the skills associated with their manipulation for improving understanding of the underlying physics concepts. A total of 13 pilot instruments were presented in a directive format while the remaining four tasks were non-directive in nature. In order to address the research questions fully, the final set of tasks was more balanced with eight problem questions being directive in nature and 16 tasks with a non-directed format (of which six tasks were used as pre-tests).

A total of 10 pilot instruments focused on depicting motion using freeze frame representations, unpacking qualitative information and generating a linguistic representation highlighting the physics information. An illustration for such task is shown in Figure 25.

A ball is thrown **vertically upwards** from a balcony which is at a certain height above the ground. The ball rises to a maximum height and then drops past the balcony to the ground.

Photographs of the ball are taken at its start (just after it leaves the balcony) and end position (just before it reaches the ground). On the same diagram below, add in what you will see if **5** more photographs had been taken between these two positions. The photographs were taken at equal intervals of time apart.



On your diagram above, draw in **vectors** to indicate the magnitude and direction of the (i) velocity and (ii) acceleration of the ball **at each of the seven positions**. Label the vectors clearly.

Now use your diagram to describe the motion of the ball for this situation.

Figure 25. *An example of a directive pilot task with application of freeze frame representations for unpacking physics information.*

The remaining pilot tasks, as shown by the example provided in Figure 26, explored the ways in which students in the GEPS physics programme handle problems in kinematics, specifically how freeze frame representations are employed in solving the graphical and mathematical aspects of kinematics tasks.

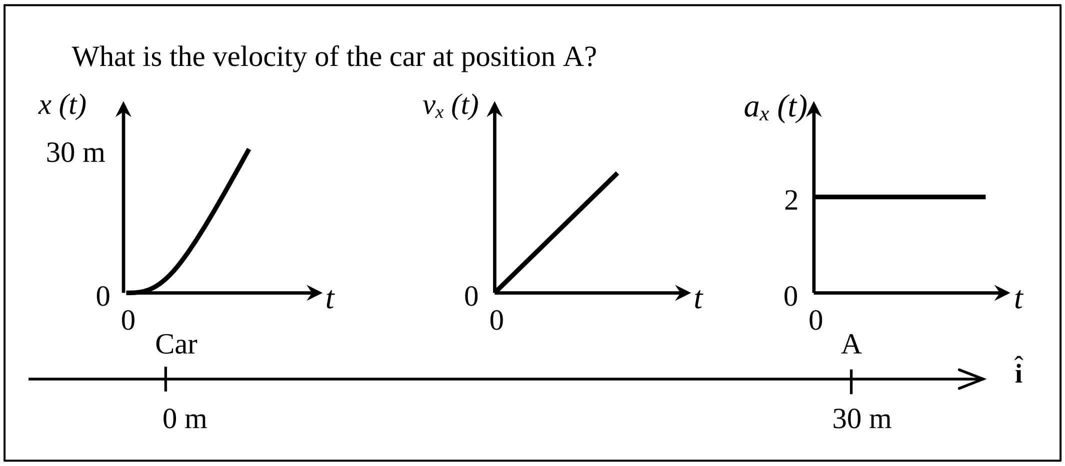


Figure 26. *An example of a non-directive pilot task requiring the generation of mathematical formulations from visual models.*

Based on the outcomes obtained from the various pilot tasks, the broad research questions were refined to explore particular aspects on the application of freeze frame representations in the kinematics context. A distinction was made between using freeze frame representations for unpacking physics information and as a representational bridge for translating information among various forms of representations. Research questions were introduced to identify representational translations potentially hindering students' problem solving performance as well as for exploring the sample's categories of mental representations based on their handling of diverse external representations.

Upon amendments of the research questions additional written exercises were designed to fill in the gaps for collecting a full spectrum of data, that is, presenting a physical model in linguistic, diagrammatic and graphical forms. The students' strategies when dealing with these different modes of representations for generating graphical and mathematical conceptual models are thus explored in directive and non-directive tasks. A number of the tasks used for the current study were adapted from the pilot instruments. For example, the question shown in Figure 25 requires the expression of qualitative information through freeze frame representations and subsequently in a linguistic representation. This particular task structure was applied to design items investigating the effect of freeze frame representations for eliciting qualitative information from mathematical and graphical conceptual models. Moreover, using the structure of the non-directive task presented in Figure 26, questions were created with a graphical and diagrammatic format exploring the effect of using freeze frame representations for generating a mathematical conceptual model. Additionally, two of the directive pilot tasks were refined in order to bring out the required translation more explicitly. On the one hand, some of the questions which comprise the pilot task were omitted as they interfered with the students' responses. Consequently, only freeze frame representations were included as the intermediate step in the newly designed item (see class work 10 in Appendix B). On the other hand, the sequencing of the guidance provided was re-ordered and some of the questions constituting the pilot written exercise were either re-framed or not included in the final version (see question 10 from June class test in Appendix B).

4.5.1 (b) Content and external validity

Content validity may be referred to as "the instrument must show that it fairly and comprehensively covers the domain or items it purports to cover" (Cohen *et al.*, 2007, page 137). Of the 24 written tasks involved in the main study, four tasks were adapted from existing problems designed for the foundation component of GEPS physics course. These four tasks included class works 11, 13, 16, and question 10 from the June examination paper. The remaining items were newly created based on the research questions. The final set of the problems was obtained after several rounds of peer reviewing by two other researchers for the inclusion of all main aspects of kinematics. In addition, issues around the language and words used to frame the items were considered. The vocabulary was chosen to be simple as the students speak English as their second language. Particular emphasis was placed on the use of technical physics terms such as "speed up or slow down uniformly", "velocity", "displacement" and "distance". The peer validation also included a critique of the relevance of quantitative and qualitative information in the tasks. Attention was also given to the symbolic representation employed to represent acceleration, initial and final positions or velocities. Uniformity across tasks and consistency with what the students were exposed to during lectures was verified. Additionally, the various tasks were checked for their relevance to particular research questions and grouped according to the research question they are associated with.

According to Cohen *et al.,* (2007, page 136) "external validity refers to the degree to which the results can be generalized to the wider population, cases or situations". Since the study dealt with a very specific student type and undergraduate physics course, the findings are not intended to be generalisable. However, in order to facilitate readers to judge the transferability of findings, the context in which the main study was implemented was extensively described. Particular attention was placed on the educational background of the students, and the nature and purpose of the course. The context of the pilot tests, in terms of the sample's academic background, the wide use of multiple representations for presenting learning materials, teaching the contents and solving tutorial problems in kinematics, was similar to the current study. Ample information is therefore provided for assessing the extent to which findings obtained from the particular study is applicable to other cases implemented under either similar or different conditions.

4.5.1 (c) Validity of the research process

From the pilot study, it was apparent that during the administration of the class works, in certain instances the students communicated with their peers. However, for the main study, during the data collection of class works, utmost efforts have been made, within the realistic context of a teaching session, to make students respond individually.

Since the study explores problem solving as employed by individual students, data were not collected from tutorial work. During tutorial, the students work in groups of three and there is the intervention of tutors. Hence, a faithful reflection of the students' own responses is not guaranteed. In addition, 45 minutes are available for a tutorial session with a total of four questions to be attempted across topics. In the majority of cases, the selected problem for analysis was either left incomplete or not attempted.

Some of the data were collected during the learning process in the form of class works for formative assessment while others were obtained at a much later summative stage, that is, during tests and examination. Time constraints also resulted in the majority of the tasks involved in formative assessment to be non-directive in nature. Where there was a potential tension between learning and research, the learning process was prioritised. The formative data obtained form the class works were too valuable to be omitted. They provide useful insights into the students' handling of the different tasks thus allowing improvements regarding the strategies to be employed for teaching the different contents in kinematics. The fact that the data are gathered at different stages (class works, tests and examinations) of the study, from the same sample and using the same data collection method enables the process of triangulation. Comparison can be made regarding the students' processes or actions when attempting the various written exercises during learning or at a much later stage where it is assumed that familiarity and assimilation of the necessary concepts have been achieved.

One of the reasons for collecting data via the application of written problem tasks is to eliminate the possibility of bias on the part of one of the researchers who also taught the kinematics topics. Even though the researcher was involved with the design of the instruments there was no interference with the analysis of students' responses. Data collection using observation during lectures is unsuitable because of the interest in individual student's problem solving strategies and mental representations. In case of an interview study, the researcher/interviewer, who also lectures the course, may prejudice students' responses since the interview questions themselves may be prone to changing the students' notions or ideas about particular issues and guiding them towards understanding the concepts. Consequently, the questions asked are not fit for gathering the reasons underlying the students' actions when attempting the various tasks. In addition, the application of interviews is inappropriate as it is not practical to perform either individual or group interviews on a population of 179 students involved in the study. Time constraint and the unavailability of students are also factors which contributed to not conducting interviews.

One factor that may reduce the validity of the data is the students' perception that some of the tasks results will not be marked, especially the class works. Consequently less importance may have been given to the class works which may not fully reflect students' abilities as their level of engagement may have been low. However, of the 16 items administered as class work, a total of five problem questions were completed on Thursdays where the tasks were considered as “class tests”. These exercises were marked and the allocated grade contributed to the final assessment.

4.5.2 Research reliability

The research reliability of this study is increased due to the fact that for most of the research questions several different tasks were used as the source of data. By referring to Table 4-1, for example, when exploring the effect of using freeze frame representations for unpacking physics information a total of four tasks was considered. They were presented in graphical and mathematical forms from which written responses were generated. Additionally for investigating the effect of using freeze frame representations for generating graphical conceptual model, data were obtained from six tasks. They were posed in a variety of representational modes namely, linguistic, diagrammatic and graphical forms. For each of the research questions triangulation between different data sources will be used, thus increasing the reliability of the findings.

The responses generated by the students for at least three of the tasks were independently coded by two different researchers. The codes allocated from the coding scheme were then compared and discussed, and an inter-coder agreement of 82% was obtained.

5. Results

In this chapter, the findings are presented according to the four research questions. The first two research questions explore the effect of using freeze frame representations for generating graphical and mathematical conceptual models from tasks posed in a variety of representational modes. The third research question deals with the consequence of using freeze frame representations for unpacking qualitative information from tasks with mathematical and graphical formats. For the first two research questions the interest lies in the quality of student-generated graphical (visual) and mathematical (quantitative) conceptual models with or without the application of freeze frame representations. In contrast, for the third research question, the students were presented with a physical model structured in mathematical and graphical forms with the main concern being in their ability to elicit qualitative information from these two representational modes. Finally, the outcomes from research question 4, concerned with profiles for the categories of mental representations for introductory physics students with poor academic background, are outlined. With the exception of the last research question, the data from non-directive and directive tasks are presented for groups of students who attempted paired tasks with and without freeze frame representations. For each pair of tasks, an overview is provided of the differences between responses to these written exercises for individual students. The detailed version of the results, for the whole cohort, can be viewed in Appendix E.

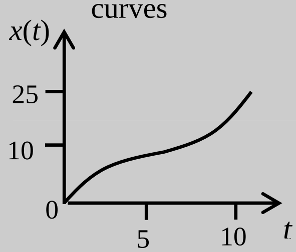
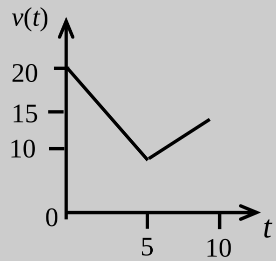
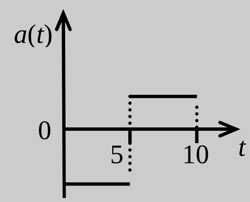
5.1 Research question 1: The effect of using freeze frame representations on generating graphical conceptual models from diagrammatic, graphical and linguistic conceptual models

For this particular aspect of the study 6 tasks were involved: class work 15, class test question 5 and class work 9 were presented in diagrammatic, graphical and linguistic form respectively, without freeze frame representations. Class test question 6, class work 10 and June class test question 10 were directive in nature, with the physical model posed in diagrammatic, graphical and linguistic form respectively. The quality of the graphical conceptual models generated for tasks with and without freeze frame representations was compared. For the different sections presented below, the shaded part of the tables highlights responses which were considered appropriate for the task.

5.1.1 (a) Non-directive and directive tasks presented in diagrammatic form

The two problem questions considered in this section can be viewed on page 196 (with non-directed format) and on page 201 (with directed format). Table 5-1 provides an overview of the different categories of graphical representations provided by the students for the non-directive task presented in diagrammatic form (class work 15). Less than half of the cohort (45%, 69 out of 153) were able to translate information depicted by the diagrammatic representation into the form of motion graphs for position, velocity and acceleration. Around 28% (44 out of 153) of the sample generated only two appropriate (mainly velocity and acceleration) kinematics graphs. Only one correct motion graph was produced by 20% (29 out of 153) of the cohort, mostly the velocity-time graph (in 25 cases).

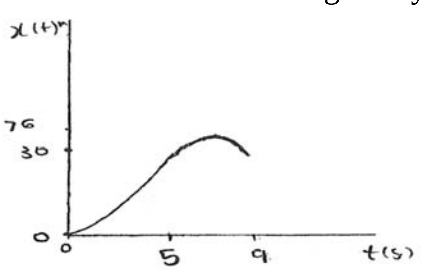
Table 5-1: Categories of student-generated graphical representations for the non-directive task posed in diagrammatic form (n = 153).

	Position-time graph	**Velocity-time graph**	**Acceleration-time graph**	**Total (%)**
1. All three graphs consistent with physical model	Negatively and positively sloped curves 	Negatively and positively sloped lines 	Horizontal lines for acceleration in negative and positive direction 	**69 (45)**
2. Two graphs consistent with physical model	Positively and negatively sloped curves / positively sloped line followed by Gaussian-like shape / one or three stages of motion / uncodeable	*Consistent with physical model*	*Consistent with physical model*	42 (27)
	Consistent with physical model	*Consistent with physical model*	Horizontal lines for two positive accelerations / uncodeable	2 (1)
3. One graph consistent with physical model	Positively and negatively sloped curves / positively sloped line followed by Gaussian-like shape/ one or three stages of motion / uncodeable	*Consistent with physical model*	Horizontal lines for two positive accelerations / one or three stages of motion / uncodeable	25 (17)
	One stage of motion	Motion in one or 3 stages / uncodeable	*Consistent with physical model*	3 (2)
	Consistent with physical model	One stage of motion	Three stages of motion	1 (1)
4. No consistency	Inconsistent with physical model	Inconsistent with physical model	Inconsistent with physical model	11 (7)
			Total	**153 (100)**

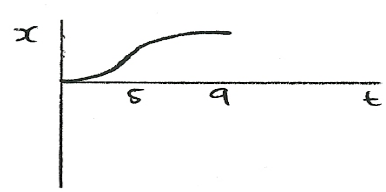
For 7% (11 out of 153) of the students, none of the graphical representations drawn was related to the physical model. In particular, the position-time graph was inappropriately drawn. Most commonly, positively and negatively sloped curves were provided. Alternatively, a positively sloped line followed by a Gaussian-like shape was generated. The shapes of these graphs indicate that the students did not distinguish between the qualitative information presented by curves with negative and positive slopes, and may lack skills for drawing position-time graph. An inappropriate acceleration-time graph was generated owing to the inability to derive the particular information from the diagrammatic physical model.

Therefore, when freeze frame representations were not included in the task, around 55% (84 out of 153) of the students were unable to draw three motion graphs corresponding to the physical model. One may infer that the main reasons for presenting inappropriate kinematics graphs include the lack of skills in depicting motion graphically, particularly position-time graphs, as well as the inability to derive information from the diagrammatic task format, mainly the acceleration of the object.

Table 5-2 presents the various categories of freeze frame and graphical representations generated for the directive task with a diagrammatic format (class test question 6). Around 43% (66 out of 153) of the sample provided freeze frame and graphical representations considered suitable for the task. The spacing between the object's positions was indicated to increase and then decrease. Positively and negatively sloped curves were drawn for the position-time graph corresponding to lines with positive and negative slopes for the velocity-time graph. For another 43% (65 out of 153) of the cohort, although freeze frame representations corresponded to the physical model, either one or none of the motion graphs was appropriately drawn. The majority (62) of these students generated an incorrect position-time graph. Most commonly, for the second stage of motion, either a Gaussian-like shape curve was drawn or the negatively sloped curve was extended horizontally, as shown:

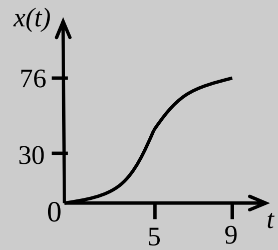
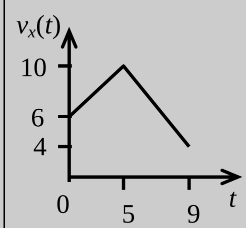


(Student 14)



(Student 10)

Table 5-2: Categories of student-generated freeze frame and graphical representations for the directive task expressed in diagrammatic form (n = 153).

	Freeze frame representation	**Position-time graph**	**Velocity-time graph**	**Total (%)**
1. Both conceptual models consistent with physical model	 Increase and decrease in spacing	Positively and negatively sloped curves 	Positively and negatively sloped lines 	**66 (43)**
2. Freeze frame representation consistent with one graph		Positively sloped curve followed by Gaussian-like shape / positively and negatively sloped curves extending horizontally / one stage motion / Gaussian / uncodeable	*Consistent with physical model*	48 (32)
		Consistent with physical model	Motion in one stage / uncodeable	3 (2)
3. Only freeze frame representation consistent with physical model		Positively sloped curve followed by Gaussian-like shape / positively and negatively sloped curves extending horizontally / one stage motion / Gaussian / uncodeable	Motion in one or three stages / Gaussian / uncodeables	14 (9)
4. Only one or both graphs consistent with physical model	Decrease and increase in spacing / increase or no change in spacing for both stages of motion / depiction of one stage of motion	*Consistent with physical model*	*Consistent with physical model*	2 (1)
		Positively sloped curve followed by Guassian-like shape / motion in one stage / Gaussian / uncodeable	*Consistent with physical model*	12 (8)
5. No consistency	Inconsistent with physical model	Inconsistent with physical model	Inconsistent with physical model	8 (5)
			Total	**153 (100)**

The inverse was observed for 9% (14 out of 153) of the students. Both or one of the graphical representations were appropriately drawn even if freeze frame representations conflicted with the qualitative information presented by the diagrammatic representation. For 5% (8 out of 153) of cohort, none of the visual conceptual models presented was in line with the physical model.

Therefore, no major difference was noted in the proportion of students constructing the appropriate graphical representations for the non-directive (45%) and the directive (43%) tasks with a diagrammatic format. For the directed problem question, discrepancies were noted between the two forms of visual conceptual models. Mostly, although freeze frame representations were appropriately drawn either only one of the motion graphs, mainly velocity-time graph, or none of the graphs generated was consistent with the physical model. For a separate sub-group, either both motion graphs or only the velocity-time graph was correctly drawn while freeze frame representations conflicted with the physical model. The students still have difficulties in drawing position-time graphs. The particular graphical representation may have been generated from memory of the shape of the graph which was apparent in cases where for the second stage of motion, either the negatively sloped curve was extended horizontally or a Gaussian-like shape was provided to correspond to a decrease in velocity. Additionally, confusion was displayed between curves with positive and negative slopes. Alternatively, the generation of an inappropriate motion graph for position may be due to the students' lack of skills in depicting information about position graphically.

5.1.1 (b) Comparison of the categories of motion graphs drawn by the same student for the non-directive and directive tasks posed in diagrammatic form

The categories of graphical representations generated by the same student for the two tasks were compared. The results are shown in Table 5-3.

Table 5-3: Comparison of the categories of graphical representations generated by the same student for the directive and the non-directive tasks presented in diagrammatic form (n = 153).

			Non-directive task				
	Categories of freeze frame represent-ations	**Categories of graphical represent-ations**	All graphs consistent with physical model	Two graphs consistent with physical model	One graph consistent with physical model	No graph consistent with physical model	**Total (%)**
Directive task	Freeze frame represent-ations consistent with physical model	Both graphs consistent with physical model	41 (27)	18 (12)	4 (2)	3 (2)	66 (43)
		One graph consistent with physical model	19 (12)	19 (12)	11 (8)	2 (1)	51 (33)
		No graph consistent with physical model	4 (2)	4 (2)	4 (3)	2 (2)	14 (9)
	Freeze frame represent-ations inconsistent with physical model	Both graphs consistent with physical model	1 (1)	1 (1)	0 (0)	0 (0)	2 (2)
		One graph consistent with physical model	3 (2)	0 (0)	7 (5)	2 (1)	12 (8)
		No graph consistent with physical model	1 (1)	2 (1)	3 (2)	2 (1)	8 (5)
		Total	69 (45)	44 (28)	29 (20)	11 (7)	**153 (100)**

The majority of the students who generated the appropriate motion graphs for the non-directive task also presented two forms of visual conceptual models corresponding to the physical model for the directive task (41 in 69). The data also indicate that less than half of the students who were unsuccessful with the non-directive task were able to draw the correct kinematics graphs when freeze frame representations are included in the problem question.

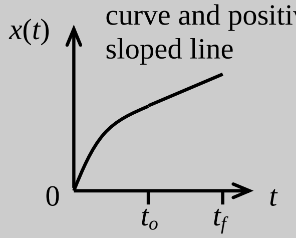
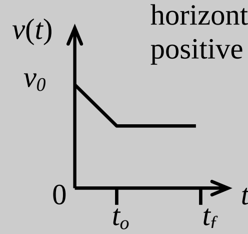
Firstly, the 18 out of 44 students who presented two appropriate kinematics graphs for the non-directive task provided a combination of correct freeze frame and graphical representations, while 23 in 44 of them generated either only one or no appropriate motion graphs although freeze frame representations were suitably drawn. Moreover, a small proportion of 4 out of 29 students who constructed only one correct kinematics graph for the non-directive task generated the appropriate freeze frame and graphical representations, compared to 15 in 29 of them providing the correct freeze frame representations together with either only one or no appropriate motion graphs. Additionally, a mere 3 out of 11 students without any suitably drawn graphical representation for the non-directive task produced two forms of correct visual conceptual models, while 4 in 11 of them provided appropriate freeze frame representations along with one or no correct kinematics graphs.

Two-tailed tests for differences between proportions, performed on the data in Table 5-3, at 5% significance level, for students who were unsuccessful with the non-directive task yielded z-values of 2.50 and -2.50. These values indicate that there is a significant difference in the proportion of students with appropriate and inappropriate motion graphs and, freeze frame representations corresponding and conflicting with the physical model. When a similar test is performed on the data for the whole cohort, z-values of 3.60 and -3.60 were obtained indicating that a significant link exists between freeze frame representations categorised as consistent and inconsistent with the physical model and the category of graphical representations generated.

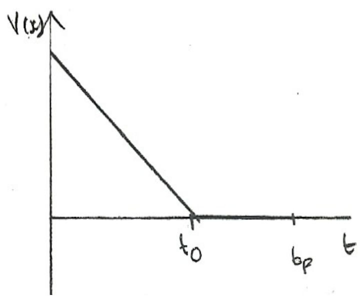
5.1.2 Non-directive and directive tasks presented in graphical form

The task with a non-directive format is presented on page 191 while the directive problem question can be viewed on page 201.Table 5-4 provides an overview of student-generated graphical representations from the task presented in graphical form without the presence of freeze frame representations (class test question 5).

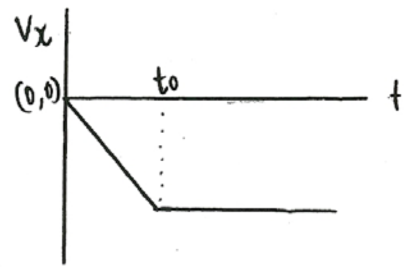
Table 5-4: Combinations of graphical representations for position and velocity provided for non-directive task with a graphical format (n = 158).

	Position-time graph	**Velocity-time graph**	**Total (%)**
1. Both motion graphs consistent with physical model	Negatively sloped curve and positively sloped line 	Negatively sloped and horizontal line in positive direction 	**59 (38)**
2. Only one motion graph consistent with physical model	*Consistent with physical model*	Negative slope and horizontal line along x = 0 / motion in negative direction / change in direction of motion / negatively and positively sloped lines	26 (16)
	Positively sloped curve and line / negatively sloped curve and horizontal line / motion in one stage / uncodeable	*Consistent with physical model*	13 (8)
3. No consistency	Positively sloped curve and line / negatively sloped curve and horizontal line / motion in one stage / uncodeable	Negative slope and horizontal line along x = 0 / motion in negative direction / change in direction of motion / negatively and positively sloped lines / motion in one stage / uncodeable	60 (38)
		Total	**158 (100)**

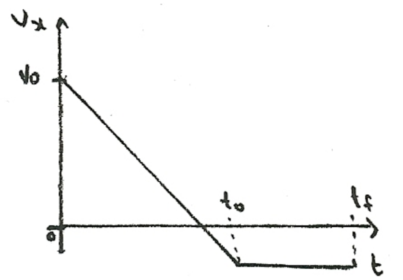
Around 38% (59 out of 158) of the cohort displayed an understanding of the situation and the physics concepts portrayed by the acceleration-time graph (physical model). In most (86) instances, an inappropriate velocity-time graph was drawn mainly due to the students' inability to derive information from the shape of the acceleration-time graph and ignoring or misinterpreting the negative sign associated with acceleration. The "negative" acceleration was understood either to indicate that motion is taking place in a negative direction or to reflect a change in the direction of motion or velocity. Examples of students' responses which typify these mistakes are shown below:



(Student 20)



(Student 97)

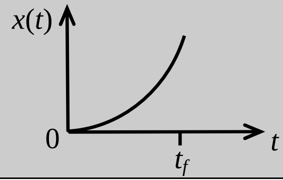
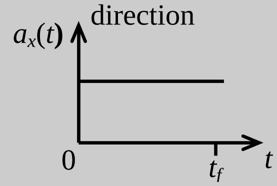


(Student 119)

Inappropriate position-time graphs were provided owing to the students' inability to derive the particular information. It was noted that they were mainly unable to distinguish between qualitative information depicted by curves with positive and negative slopes. Alternatively, their lack of skills in portraying information about position graphically may have resulted in the construction of incorrect position-time graphs.

The various categories of freeze frame and graphical representations generated for the directive task with a graphical format (class work 10) are shown in Table 5-5. For the directed task, the vast majority (75%, 118 out of 158) of the cohort generated the appropriate freeze frame and graphical representations. Discrepancies were also noted between the two forms of visual conceptual models. Suitably drawn freeze frame representations together with one or both graphs being inappropriate were provided by 5% (8 out of 158) of the students. The inverse, a combination of incorrect freeze frame representations and both or one of the motion graphs consistent with the physical model were presented by 16% (26 out of 158) of the cohort. However, the presence of freeze frame representations in the task may not fully explain the improvement in the quality of motion graphs generated. Compared to the non-directive written exercise, the particular directive task has a physical model with a lower level of difficulty, depicting one of the basic shapes for kinematics graphs. Also, the large proportion of students providing two forms of correct visual conceptual models may be explained, in part, as a result of learning taking place. Hence, the quality of motion graphs provided, for both tasks, by individual student was not compared.

Table 5-5: Categories of student-generated freeze frame and graphical representations for the directive task with a graphical format (n = 158).

	Freeze frame representation	**Position-time graph**	**Acceleration-time graph**	**Total (%)**
1. Both conceptual models consistent with physical model	 Increase in spacing	Positively sloped curve 	Horizontal line in a positive direction 	**118 (75)**
2. Freeze frame representation and one graph consistent with physical model		*Consistent with physical model*	Horizontal line along x = 0 / positively sloped line	1 (1)
		Negatively sloped curve / motion in 2 stages	*Consistent with physical model*	5 (3)
3. Freeze frame representation consistent with physical model		Positively sloped line	Positively sloped curve	2 (1)
4. Both or only one graph consistent with physical model	No change in spacing / decrease in spacing	*Consistent with physical model*	*Consistent with physical model*	18 (11)
		Consistent with physical model	Horizontal line along x = 0	6 (4)
		Negatively sloped curve / positively sloped line	*Consistent with physical model*	2 (1)
5. No consistency	No change in spacing / decrease in spacing	Horizontal line / negatively sloped curve or line	Horizontal line along x = 0 / horizontal line in negative direction	6 (4)
			Total	**158 (100)**

5.1.3 (a) Non-directive and directive tasks presented in linguistic form

The non-directed and the directed problem question attempted for this section are presented on page 190 and page 203 respectively. The various combinations of graphical representation for position, velocity and acceleration drawn for the non-directive task presented in linguistic form (class work 9) are summarised in Table 5-6. A mere 26% (45 out of 170) of the students were able to interpret the qualitative description and translate the information into the form of kinematics graphs for position, velocity and acceleration. Around 42% (71 out of 170) of the cohort presented two appropriate motion graphs. The students were mainly (in 62 cases) unable to draw the position-time graph. Most often, for the second stage of motion, either a Gaussian-like shape curve was depicted or the negatively sloped curve was extended horizontally. The presence of these particular shapes indicates the ability to interpret the physical model but failure to portray information regarding the object's position graphically. The students also failed to understand the qualitative information conveyed by lines or curves with positive and negative slopes for position-time graphs. Either two consecutive lines with a positive slope were drawn or positively and negatively sloped curves were provided. Around 14% (23 out of 170) of the sample generated only one appropriate motion graph, mostly (in 16 cases) the velocity-time graph. A combination of all three inappropriate kinematics graphs was presented by 18% (31 out of 170) of the cohort.

Thus, for the non-directive task structured in linguistic form, the large majority (74%, 125 out of 170) of the students generated graphical representations which were not related to the physical model. An inappropriate motion graph, mainly position-time graph, was constructed due to the students' lack of skills in graphically depicting the particular information as well as the inability to understand the qualitative information portrayed by lines or curves with positive and negative slopes. Moreover, an incorrect acceleration-time graph is generated owing to the failure to interpret and derive the particular information from the qualitative descriptions.

Table 5-6: Categories of graphical conceptual models generated for the non-directive task with a linguistic format (n = 170).

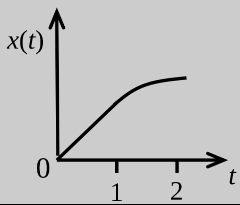
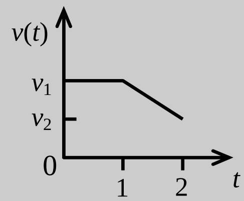
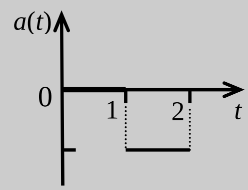
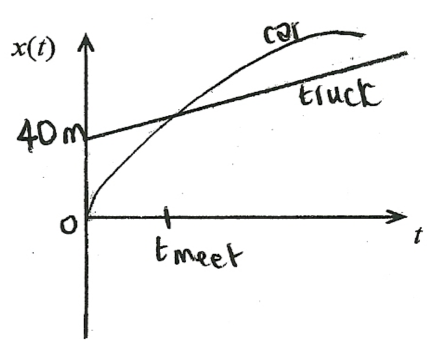
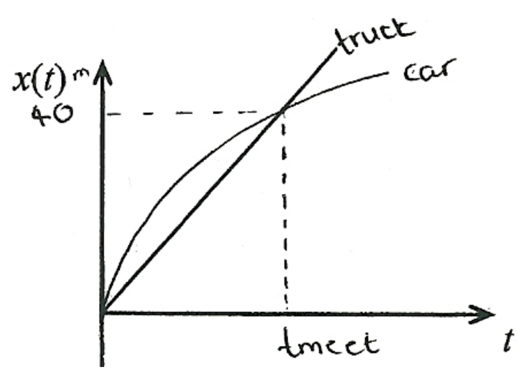
<table>
<tr><th></th><th>Position-time graph</th><th>Velocity-time graph</th><th>Acceleration-time graph</th><th>Total (%)</th></tr>
<tr><td>1. All three graphs consistent with physical model</td><td>Positively sloped line and negatively sloped curve
</td><td>Horizontal and negatively sloped lines
</td><td>Horizontal lines along x axis and in negative direction
</td><td>45
(26)</td></tr>
<tr><td rowspan="3">2. Two graphs consistent with physical model</td><td>Positively sloped line and Gaussian-like shape / positively sloped line and curve extending horizontally / positively and negatively sloped curves / two positively sloped lines / one stage motion / uncodeable</td><td>Consistent with physical model</td><td>Consistent with physical model</td><td>62
(36)</td></tr>
<tr><td rowspan="2">Consistent with physical model</td><td>Consistent with physical model</td><td>Horizontal lines in positive and negative directions / horizontal line along x axis and negatively sloped line</td><td>5
(4)</td></tr>
<tr><td>Positively and negatively slope lines / uncodeable</td><td>Consistent with physical model</td><td>4
(2)</td></tr>
<tr><td rowspan="3">3. Only one graph consistent with physical model</td><td>Consistent with physical model</td><td>Inconsistent with physical model</td><td rowspan="2">Inconsistent with physical model</td><td>3
(1)</td></tr>
<tr><td rowspan="2">Inconsistent with physical model</td><td>Consistent with physical model</td><td>16
(10)</td></tr>
<tr><td>Inconsistent with physical model</td><td>Consistent with physical model</td><td>4
(3)</td></tr>
<tr><td>4. No consistency</td><td>Inconsistent with physical model</td><td>Inconsistent with physical model</td><td>Inconsistent with physical model</td><td>31
(18)</td></tr>
<tr><td></td><td></td><td></td><td>Total</td><td>170
(100)</td></tr>
</table>

Table 5-7 highlights the categories of student-generated freeze frame and graphical representations when presented with task structured in linguistic form (June class test question 10). Less than half of the cohort, 20% (35 out of 170), generated the appropriate freeze frame and graphical representations. The spacing between the positions of the car was drawn to decrease while it was depicted to remain constant for the truck's motion. Moreover, the vehicles were portrayed at their respective initial positions together with the meeting point. Around 23% (39 out of 170) of the sample constructed freeze frame representations corresponding to the physical model in terms of qualitative information only together with the appropriate motion graphs (category 5).

Regardless of whether freeze frame representations take into account quantitative information (depiction of the vehicles' initial and meeting positions), only one correct kinematics graph was provided by 41% (71 out of 170) of the students (combination of categories 2 and 6). In most (69) cases, the position-time graph was inappropriately drawn. A Gaussian-like shape graph was provided for the car's motion reflecting either the lack of skills for drawing motion graph for position or rote memorisation of the shapes of graphs. A positively sloped curve was also drawn for the car's motion indicating the failure to distinguish between the qualitative information conveyed by curves with positive and negative slopes. Additionally, an incorrect graphical representation for position was presented owing to either lack of skills to graphically depict the two vehicles' meeting point or ignoring quantitative information regarding the vehicles' initial position. In the former case, the meeting point was portrayed as intersecting instead of tangential. In the latter case, both vehicles were shown to have the same starting position of 0 m. The visual depictions below typify these two categories of position-time graphs:

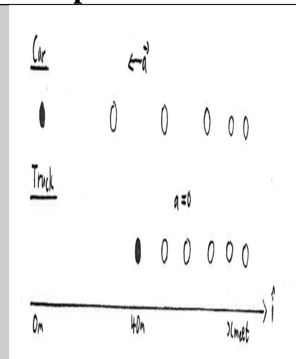
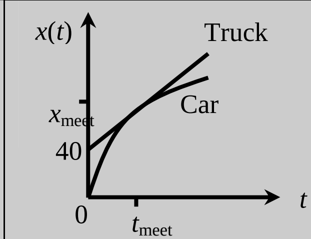
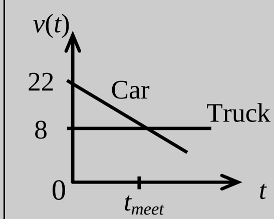
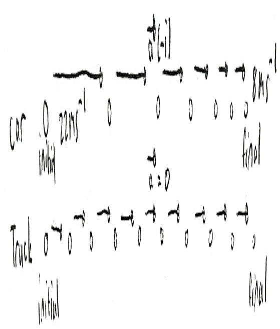


(Student 152)



(Student 52)

Table 5-7: Categories of student-generated freeze frame and graphical representations for the directive task structured in linguistic form (n = 170).

	Freeze frame representations	Position-time graph	Velocity-time graph	Total (%)
1. Both conceptual models consistent with physical model				**35 (20)**
2. Freeze frame and one graphical representation consistent with physical model		Gaussian shape or positive slope curve for car / correct graphs' shape with either meeting point not tangential or both vehicles' initial position is 0 m / uncodeable	*Consistent with physical model.*	14 (8)
3. Only freeze frame representations consistent with physical model			Positive slope and horizontal line for car and truck / uncodeable	9 (5)
4. Freeze frame representations consistent with qualitative information only		Positive slope curve and line for car and truck.	Positive slope line and horizontal line for car and truck.	4 (3)
5. Both graphs consistent with physical model		*Consistent with physical model*	*Consistent with physical model*	39 (23)
6. Only one graph consistent with physical model		*Consistent with physical model*	Inconsistent with physical model	2 (1)
		Inconsistent with physical model	*Consistent with physical model*	55 (32)
7. Only graphs consistent with physical model	Increase and no change in spacing for car's and truck's motion / No change in spacing for both vehicles. Quantitative information may be considered.	*Consistent with physical model*	*Consistent with physical model*	4 (3)
8. One graph consistent with physical model		*Consistent with physical model*	Positive slope and horizontal line for car and truck	1 (1)
		Meeting point is not tangential / uncodeable	*Consistent with physical model*	2 (1)
9. No consistency		Inconsistent with physical model	Inconsistent with physical model	5 (3)
			Total	**170 (100)**

Although freeze frame representations were consistent with the qualitative description (categories 3 and 4), inappropriate position-time and velocity-time graphs were presented by 8% (13 out of 170) of the cohort. Freeze frame representations for one of the vehicles' motion not corresponding to the qualitative information described by the physical model were provided by 8% (12 out of 170) of the sample. An increase or no change in spacing for the car's motion was depicted. However, the required motion graphs for position and velocity were generated by 4 of these students while in 3 instances, only one kinematics graph, particularly velocity-time graph, was correctly constructed. For the remaining 5 students neither forms of conceptual models was in agreement with the physical model.

Hence, no substantial difference was recorded in the percentage of students generating a suitable mental model for the directive (20%) and the non-directive (26%) tasks. It was also noted that the required motion graphs were provided although freeze frame representations were related only to the qualitative description. Moreover, both or only one of the motion graphs (mainly position-time graph) drawn was inappropriate but freeze frame representations may correspond, qualitatively and quantitatively, to the physical model. Additionally, although freeze frame representations conflicted completely with the physical model, one or both motion graphs were appropriately drawn. It was also found that the students' lack of skills for depicting the vehicles' meeting point graphically contributed to the construction of an incorrect position-time graph.

5.1.3 (b) Comparison of the categories of graphical representations drawn by the same student when dealing with the non-directive and the directive tasks structured in linguistic form

Table 5-8 provides an overview of the results gathered upon comparing the categories of graphical representations generated by the same student for the two tasks which highlight two different situations. The table presents frequencies and in parentheses the percentages.

Table 5-8: Comparison of the categories of graphical representations generated by the same student for the directive and the non-directive tasks presented in linguistic form (n = 170).

	Directive task		**Non-directive task**				
	Categories of freeze frame representations	**Categories of graphical representations**	All graphs correct	Two graphs correct	One graph correct	No graphs correct	**Total (%)**
Directive Task	Freeze frame representations consistent with physical model	All graphs consistent with physical model	10 (6)	10 (6)	6 (3)	9 (5)	35 (20)
		One graph consistent with physical model	6 (3)	7 (4)	1 (1)	0 (0)	14 (8)
		No graph consistent with physical model	1 (1)	4 (2)	1 (1)	3 (1)	9 (5)
	Freeze frame representations consistent with physical model in terms of qualitative information only.	All graphs consistent with physical model	11 (6)	19 (12)	4 (2)	5 (3)	39 (23)
		One graph consistent with physical model	15 (8)	25 (14)	5 (3)	12 (8)	57 (33)
		No graph consistent with physical model	0 (0)	3 (2)	1 (1)	0 (0)	4 (3)
	Freeze frame representations inconsistent with physical model	All graphs consistent with physical model	1 (1)	0 (0)	3 (2)	0 (0)	4 (3)
		One graph consistent with physical model	0 (0)	1 (1)	2 (1)	0 (0)	3 (2)
		No graph consistent with physical model	1 (1)	2 (1)	0 (0)	2 (1)	5 (3)
		Total	45 (26)	71 (42)	23 (14)	31 (18)	**170 (100)**

Upon consideration of students who were successful with the non-directive task, no substantial difference was noted between the proportion of respondents with the correct (10+11 in 45) and incorrect (both or only one) kinematics graphs (6+1+15 out of 45) for the directive problem question although freeze frame representations corresponded to the qualitative description. The data also indicate that less than half of the students with inappropriate kinematics graphs for the non-directive task generated the required motion graphs when freeze frame representations are included in the written exercise. Firstly, the 39 out of 94 students who presented either two or only one appropriate kinematics graph for the non-directive task provided the correct motion graphs irrespective of whether freeze frame representations are consistent with only the qualitative information. A difference of 6% exists with the proportion of students constructing inappropriate (both or only one) kinematics graphs (47 out of 94). Furthermore, 14 out of 31 students with no correct motion graphs for the non-directive task drew freeze frame representations corresponding qualitatively to the physical model of the directed task together with the appropriate graphical representations. No major difference was noted with the proportion of the students presenting incorrect motion graphs (15 out of 31). Overall, it can be claimed that only 6% (10 out of 170) of the cohort was successful with both tasks. For the directive problem question in addition to drawing the relevant motion graphs, freeze frame representations corresponded both to the qualitative and quantitative information highlighted by the physical model.

5.1.4 Main findings for research question 1

It was found that positive outcomes emerged from the directed problem questions with a diagrammatic and graphical format. For the tasks posed in diagrammatic form, the data for the whole cohort (Tables 5-1 and 5-2) indicate that there is no major difference in success rate between the non-directive (45%) and the directive (43%) tasks. However, the data comparing individual students (Table 5-3) show that most (61%, 42 in 69) of the students who were successful with the non-directive problem question generated the appropriate motion graphs for the directed task and around 31% (26 in 84) of them who failed in drawing the required graphs for the non-directive task were able to do so for the directed written exercise. For the problem question with a graphical format, a higher percentage (75%, Table 5-5) of students was successful with the directed task compared to the non-directed one (38%, Table 5-4). However, the large percentage of respondents being able to generate the required motion graphs for the particular task may be explained by the simplistic nature of the physical model rather than the use of freeze frame representations. In contrast, for the written exercise structured in linguistic form (Table 5-8), the data show that the presence of freeze frame representations plays no crucial role in the generation of appropriate motion graphs. Within each category of kinematics graphs presented for the non-directed task, no substantial difference was noted between the proportion of students with appropriate and inappropriate motion graphs although freeze frame representations corresponded to the qualitative description.

From statistical analysis, a substantial link was found to exist between the quality of freeze frame and graphical representations constructed for the written exercise with a diagrammatic format. However, it cannot be claimed that the generation of correct freeze frame representations results in the construction of appropriate kinematics graphs. Firstly, only around 30% of the students who failed to draw the required motion graphs for the non-directive task presented a combination of correct freeze frame and graphical representations. Also, most (59%) of the students who were successful with the non-directed problem question provided a combination of correct freeze frame and graphical representations for the directed task. Moreover, non-correspondence between the two forms of visual conceptual model was observed. On the one hand, for about 43% (65 in 153) of the sample, one or both graphs drawn were inappropriate but freeze frame representations were related to the physical model. On the other hand, to a lower extent (9%, 14 in 153), freeze frame representations conflicted with the qualitative information but one or all the motion graphs generated were related to the physical model. It is possible that the students are unable to depict information using freeze frame representations. The motion graphs may also have been generated by rote memorisation where there is a tendency to associate particular shapes of graphs with one another. For the category of consistency between the two forms of visual conceptual model, it cannot be ascertained that freeze frame representations act as a simplifying step for the extraction of qualitative information and its translation in graphical form. It is possible that the students are able to generate the depictions independently of each other. The only conclusion that can be made is that a comprehension of the physical model is displayed. Hence, it can be claimed that the presence of freeze frame representations in tasks requesting the generation of graphical representations does not guarantee its meaningful application and consequently does not necessarily promote the construction of an appropriate mental model for the situation presented.

5.2 Research question 2: The effect of using freeze frame representations on generating mathematical conceptual models from graphical and linguistic conceptual models

Six tasks were involved for the particular research question. Class work 14 and class test question 7 were posed in graphical form with a non-directive and directive format respectively. Class work 13 (without freeze frame representations) and class test question 4 (with freeze frame representations) were structured in linguistic form. The remaining two tasks, class work 16 and June class test question 10, were also presented in linguistic form. They both described the same situation but have different formats. The students had to design their own procedures for solving class work 16 while step-wise instructions were provided for June class test question 10. The nature of the mathematical expressions formulated from the various forms of conceptual model was of particular interest. The final answer was not considered. A comparison was made for the quality of mathematical models generated from tasks with and without freeze frame representations. In the sections which follow, the shaded part of the tables represents responses which were deemed appropriate for the problem question.

5.2.1 (a) Non-directive and directive tasks presented in graphical form

Page 195 and page 202 highlight the non-directive and the directive written exercise structured with a graphical format. Table 5-9 outlines the various categories of mathematical expressions formulated for the non-directive task presented in graphical form (class work 14). None of the students was successful with the particular task. A mathematical representation with only inappropriate qualitative information was presented by 45% (53 out of 117) of the sample while 47% (42+13 out of 117) of the students constructed a mathematical model which was both qualitatively and quantitatively incorrect. Qualitative mistakes ranged from ignoring the direction of acceleration for decreasing velocity (negatively sloped curve) in the mathematical formulation, including acceleration for constant velocity, disregarding acceleration for decreasing velocity and using 9.8 m s^{-2} as the acceleration for the vehicle moving in a straight horizontal line. The misinterpretation of the notation v_{yo} as the final velocity also constituted mistakes which are qualitative in nature.

Table 5-9: Categories of mathematical representations generated for the non-directive task with a graphical format (n = 117).

Mathematical representation	Total (%)
1. Direction of acceleration for decreasing velocity (negative slope curve) ignored. Acceleration may be included for constant velocity. Acceleration may be taken as 9.8 m s^{-2} for one or both stages of motion.	53 (45)
2. Application of instantaneous time / initial or final position for second stage of motion is 0 m / initial velocity for first stage of motion is 0 m s^{-1}. Presence of acceleration for constant velocity / ignoring direction of acceleration for decreasing velocity / absence of acceleration for both stages of motion.	42 (36)
3. Motion considered as a whole. Either misinterpretation of notation or initial velocity is 0 m s^{-1}. Presence of acceleration for whole motion or acceleration is 9.8 m s^{-2}.	13 (11)
4. Use of equation focusing only on quantitative information.	6 (5)
5. Problem solved graphically. Area under position-time graph yields velocity.	3 (3)
Total	**117 (100)**

Quantitative errors included the application of instantaneous instead of time interval for a change in position and use of inappropriate values for the vehicle's positions as well as initial velocity. A qualitatively and quantitatively incorrect mathematical model as formulated by a student is:

$$\vec{x}(t) = \vec{x}_0 + \vec{V}_{x0}t + \tfrac{1}{2}\vec{a}t^2$$
$$20 = 0 + 0(2) + \tfrac{1}{2}\vec{a}(2)^2$$
$$\vec{x}(t) = \vec{x}_0 + \vec{V}_{x0}t + \tfrac{1}{2}\vec{a}t^2$$
$$60 = 20 + 20(6) + \tfrac{1}{2}\vec{a}(6)^2$$

(Student 32)

In the particular mathematical formulation, for the first stage of motion, the vehicle's initial position and velocity were considered to be 0 m and 0 m s^{-1} respectively. Acceleration was included for constant velocity (shape of graph depicts a linear increase in position) and direction of acceleration for decreasing velocity (negatively sloped curve) was ignored.

Additionally, the position-time graph was understood (in 13 instances) to indicate one stage of motion, as illustrated by:

$$\begin{aligned} x(t) &= x_0 + v_{0x}(t) + 1/2(g)t^2 \\ 60 &= 0 + v_{0x}(8) + 1/2(9.8)(82) \end{aligned}$$

(Student 76)

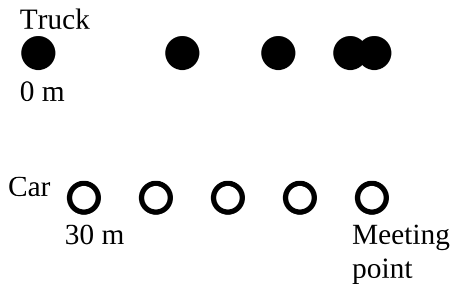
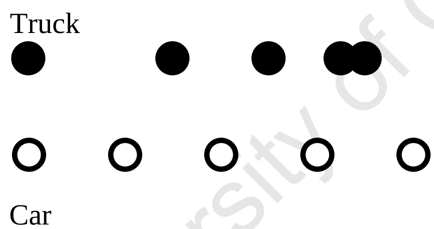
Initial and final positions were taken to be 0 m and 60 m respectively while 9.8 m s^{-2} was used for the vehicle's acceleration.

The mathematical expression relating speed, distance and time (focusing only on quantitative information) was utilised by 5% (6 out of 117) of the cohort. The remaining 3% (3 out of 117) students solved the problem qualitatively. However, an inappropriate reasoning was employed with the notion that area under the position-time graph yields velocity.

Therefore, none of the students succeeded in constructing an appropriate mathematical model. Factors leading to the particular outcome include the incomplete translation of information from the negatively sloped curve. Even if the shape of the graph was appropriately interpreted the direction of acceleration for decreasing velocity was not included in the mathematical formulation. The inability to elicit information portrayed by the graph (with application of 9.8 m s^{-2}, presence and absence of acceleration for constant and decreasing velocity respectively) also resulted in the construction of an incorrect mathematical expression. The misinterpretation of the situation as well as quantitative information depicted by the graphical physical model also contributed to the formulation of an inappropriate mathematical representation.

The various categories of student-generated freeze frame and mathematical representations for the directive task with a graphical format (class test question 7) are shown in Table 5-10

Table 5-10: Categories of freeze frame and mathematical representations generated for the directive task presented in graphical form (n = 117).

	Freeze frame representations	Mathematical representation	Total (%)
1. Only freeze frame representations consistent with physical model.		Direction of acceleration for decreasing velocity is ignored. Acceleration for one of the vehicles is 9.8 m s^{-2} / At meeting point, velocity is either not mentioned or is 0 m s^{-1}.	25 (21)
		Direction of acceleration for decreasing velocity ignored. At meeting point, velocity is 0 m s^{-1} and position is 0 m.	2 (1)
2. Freeze frame representations consistent with qualitative information only.		Required mathematical model generated.	1 (1)
		Direction of acceleration for decreasing velocity ignored. Acceleration may be included for constant velocity.	23 (20)
		Absence of acceleration for decreasing velocity / acceleration is 9.8 m s^{-2}.	11 (9)
		Direction of acceleration for decreasing velocity ignored. At meeting point velocity is not mentioned / 0 m s^{-1}.	48 (41)
		Direction of acceleration for decreasing velocity is ignored. Acceleration included for constant velocity. At meeting point, velocity and position is 0 m s^{-1} and 0 m respectively.	3 (3)
3. No consistency with physical model	No change and increase in spacing for car and truck.	Velocity at meeting point is not mentioned. Acceleration is consistent with freeze frame representations.	3 (3)
		Velocity at meeting point is 0 m s^{-1} and acceleration is 9.8 m s^{-2}	1 (1)
		Total	**117 (100)**

Although freeze frame representations corresponded to the qualitative information depicted by the graphical physical model, 91% (25+23+11+48 out of 117) of the students constructed a mathematical expression which was qualitatively inappropriate. The direction of acceleration for decreasing velocity was not included in the mathematical expression. The equation was also formulated with presence of acceleration for constant velocity, absence of acceleration for decreasing velocity or the application of 9.8 m s^{-2} for acceleration. The vehicles' velocity when they reach the same position were either considered as 0 m s^{-1} or not mentioned. Mathematical representations which were additionally quantitatively inappropriate were formulated by 4% (2+3 out of 117) of the students, with the vehicles' final position (at meeting point) taken as 0 m. Only one student in the sample generated the correct mathematical model together with freeze frame representations agreeing with the physical model in terms of qualitative information only (vehicles' initial and meeting positions are not taken into account).

An increase in spacing between the positions of the truck was drawn by 4% (4 out of 117) of the sample. In 3 cases, the vehicle's acceleration in the mathematical formulation, indicated to be in a positive direction, was consistent with freeze frame representations while the remaining one student applied 9.8 m s^{-2} for acceleration.

Hence, no substantial improvement was noted in the proportion of students constructing the appropriate mathematical representation (1% versus 0%) for the directed task. Although freeze frame representations were related to the qualitative information depicted by the physical model, the mathematical expression generated was mainly qualitatively inappropriate. Qualitative mistakes which emerged from the non-directive written exercise persist, principally ignoring direction of acceleration for decreasing velocity, including acceleration for constant velocity and use of 9.8 m s^{-2}. The students were also unable to derive physics information regarding the vehicles' velocity when they are at the same position. The failure to interpret and understand the depiction of meeting point on the motion graphs for position and velocity is thus reflected. It also indicates that freeze frame representations consistent (qualitatively and quantitatively) with the physical model were either not understood or ignored when constructing the mathematical expressions.

5.2.1 (b) Comparison of the categories of mathematical representations formulated by the same student for the non-directive and the directive tasks expressed in graphical form.

Table 5-11 highlights the categories of mathematical models formulated by the same student when dealing with the non-directive and directive written exercises posed in graphical form. For the non-directive task, categories 1 and 5 (from Table 5-9) were grouped to constitute qualitatively inappropriate mathematical expressions while classifications 2, 3 and 4 comprised qualitatively and quantitatively incorrect mathematical models.

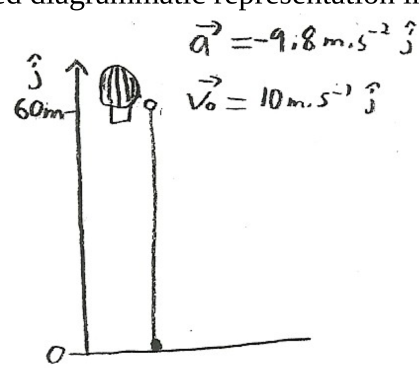
Table 5-11: Comparison of the categories of mathematical representations generated by the same student for the directive and the non-directive tasks presented in graphical form (n = 117).

			Non-directive task		
	Categories of freeze frame representations	**Categories of mathematical representations**	Inconsistent qualitative information	Inconsistent qualitative and quantitative information	**Total (%)**
Directive task	Freeze frame representations consistent with physical model	Inconsistent qualitative information	15 (13)	10 (8)	25 (21)
		Inconsistent qualitative and quantitative information	0 (0)	2 (1)	2 (1)
	Freeze frame representations consistent in terms of qualitative information only	Consistent with physical model	1 (1)	0 (0)	1 (1)
		Inconsistent qualitative information	39 (33)	43 (37)	82 (70)
		Inconsistent qualitative and quantitative information	0 (0)	3 (3)	3 (3)
	No consistency with physical model	Inconsistent qualitative information	1 (1)	3 (3)	4 (4)
		Total	56 (48)	61 (52)	**117 (100)**

The data in Table 5-11 indicate that a large proportion of students providing a mathematical representation with incorrect qualitative information for the non-directive task still formulated a qualitatively inappropriate mathematical model although freeze frame representations corresponded to the physics information presented by the physical model (54 in 56). Moreover, the 56 out of 61 students generating a mathematical expression which was qualitatively and quantitatively unsuitable for the non-directed written exercise provided a qualitatively incorrect mathematical formulation for the directed task independently of the category of freeze frame representations. It was also noted that 53 in 61 of these students provided freeze frame representations agreeing with the qualitative information depicted by the physical model. Therefore, the inclusion of freeze frame representations in the problem question with a graphical format does not lead to an improvement in the quality of mathematical expressions formulated. Moreover, there is no link between the category of freeze frame and mathematical representations generated. Around 91% (107 in 117) of the students with an inappropriate mental model for the non-directed task presented a qualitatively incorrect mathematical expression together with freeze frame representations agreeing with the qualitative information portrayed by the shapes of the graphs.

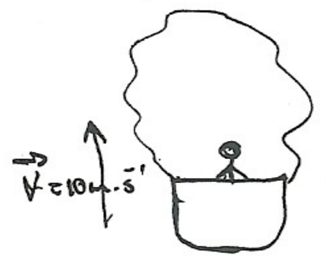
5.2.2 (a) Non-directive and directive tasks presented in linguistic form

The two problem tasks posed in linguistic form attempted for this section are provided on page 194 (non-directive in nature) and on page 200 (with a directed format). Table 5-12 summarises the procedures employed when solving the non-directive written exercise presented in linguistic form (class work 13) as well as the categories of mathematical representations produced. Half of the cohort produced mathematical representations using various kinematics equations with or without a supporting sketch while the other half made use of a diagram. An illustration of student-generated diagrammatic representation includes:



(Student 32)

The diagram depicts the whole situation and involves the application of the coordinate system. It is also clearly labelled with given and derived information and is integral to solving the problem. In contrast, a sketch, as shown below, is an incomplete depiction of the situation which is partially annotated with given information. It is comprehensible and functional only to the person constructing it.



(Student 156)

A mathematical model corresponding to the linguistic physical model was formulated by 35% (47 out of 133) of the sample, 31 of whom included a diagrammatic representation and the remaining 16 a rough sketch at most.

Table 5-12: Categories of mathematical representations constructed for the non-directive task posed in linguistic form (n = 133).

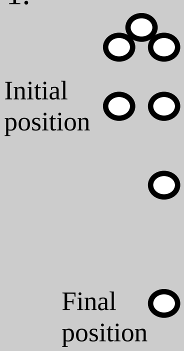
Strategy used	**Mathematical representation**	**Total (%)**
1. Presence of a diagram	Required mathematical model generated.	31 (23)
	Inconsistencies in direction of variables.	10 (8)
	Misinterpretation of notation in kinematics equations.	4 (3)
	Initial and final position is 0 m and 60 m / initial velocity is 0 m s^{-1}.	21 (16)
2. Application of equations only / presence of rough sketch	Required mathematical model generated.	16 (12)
	Inconsistencies in direction of variables.	17 (13)
	Misinterpretation of notation in kinematics equations.	2 (1)
	Initial and final position is 0 m and 60 m or both positions are 60 m / initial velocity is 0 m s^{-1}.	32 (24)
	Total	**133 (100)**

Regardless of the strategy employed for solving the problem, a mathematical expression with inappropriate qualitative information (inconsistencies in direction of variables) was constructed by 21% (10+17 out of 133) of the cohort. A quantitatively incorrect mathematical formulation was given by 40% (21+32 out of 133) of the students. Inappropriate values were used for the initial and final position of the object as well as its initial velocity. Also, in certain (6) instances, the notation v_{yo} was misunderstood as the final instead of the initial velocity.

Hence, an inappropriate mathematical expression (either qualitatively or quantitatively) was mainly presented when kinematics equations were manipulated with or without the presence of a sketch (37%, 49 out of 133) compared to 24% (31 out of 133) of students who used a diagram. In contrast, more students formulated the required mathematical model when a diagrammatic representation was included (23%, 31 out of 133) compared to its absence in the task (12%, 16 out of 133).

The various characteristics of visual and mathematical representations presented for the directive task posed in linguistic form (class test question 4) are presented in Table 5-13. A diagram was included by 41% (57 out of 133) of the cohort although the particular visual representation was not requested in the task. Regardless of the strategy employed to attempt the problem, 34% (44 out of 133) of the sample generated the appropriate freeze frame together with mathematical representations. However, discrepancies were also noted. For 9% (12 out of 133) of the students, although freeze frame representations were appropriately drawn the mathematical formulation was incorrect. Either only qualitative errors (inconsistencies in direction of variables) or both qualitative and quantitative (no mention of velocity at maximum height and inappropriate values for positions) mistakes occurred. The inverse was observed for 32% (43 out of 133) of the cohort. No freeze frame was portrayed at maximum height. Equal spacing between the freeze frames for part or the whole motion was drawn. The motion of a dropping object was presented. A depiction of the object in horizontal motion was also provided. However, the correct mathematical expression was generated.

Table 5-13: Categories of visual and mathematical conceptual models provided for the directive task presented in linguistic form (n = 133).

<table>
<tr><th colspan="2">Visual representations</th><th></th><th></th></tr>
<tr><th>Freeze frame representation</th><th>Diagrammatic representation</th><th>Mathematical representation</th><th>Total (%)</th></tr>
<tr><td rowspan="6">1.
</td><td rowspan="3">Presence of a diagram</td><td>Required mathematical model generated.</td><td>18 (13)</td></tr>
<tr><td>Inconsistencies in direction of variables.</td><td>4 (3)</td></tr>
<tr><td>No mention of velocity at maximum height and final position is 0 m or initial and final positions are 7 m. Direction of variables may be inconsistent.</td><td>2 (1)</td></tr>
<tr><td rowspan="3">Rough sketch / no diagram</td><td>Required mathematical model generated.</td><td>26 (21)</td></tr>
<tr><td>Inconsistencies in direction of variables.</td><td>1 (1)</td></tr>
<tr><td>No mention of velocity at maximum height and final position is 0 m.</td><td>5 (4)</td></tr>
<tr><td rowspan="6">2. No depiction of freeze frame at maximum height / equal spacing between freeze frames for different stages of motion / depiction of horizontal motion or dropping object</td><td rowspan="3">Presence of a diagram</td><td>Required mathematical model generated.</td><td>17 (12)</td></tr>
<tr><td>Inconsistencies in direction of variables.</td><td>3 (2)</td></tr>
<tr><td>Velocity at maximum height is not mentioned / is 7 m s^{-1} while initial velocity is 0 m s^{-1}. Final position is 0 m or initial and final position is 0 m and 7 m respectively. Direction of variables may be inconsistent.</td><td>13 (10)</td></tr>
<tr><td rowspan="3">Rough sketch / no diagram</td><td>Required mathematical model generated.</td><td>26 (20)</td></tr>
<tr><td>Inconsistencies in direction of variables.</td><td>6 (4)</td></tr>
<tr><td>No mention of velocity at maximum height and final position is 0 m. Direction of variables may be inconsistent.</td><td>12 (9)</td></tr>
<tr><td></td><td></td><td>Total</td><td>133 (100)</td></tr>
</table>

Freeze frame and mathematical representations conflicting with the physical model were presented by 25% (34 out of 133) of the students. The mathematical expression was mainly (in 25 cases) qualitatively and quantitatively inappropriate. It was also noted that when the students made no reference to the velocity of the object at maximum height, the equation relating position, velocity and acceleration was commonly equated to zero to determine the time taken by the object to reach its highest position. Overall, more students were successful with the directive (65%) compared to the non-directive task (35%).

5.2.2 (b) Comparison of the categories of mathematical representations provided by the same student for the non-directive and the directive tasks presented in linguistic form

The categories of mathematical models constructed by the same student for the two tasks are presented in Table 5-14. For the directive written exercise, mathematical formulations conflicting with both the qualitative and quantitative information were grouped under the heading "inconsistent". The classification "direction is inconsistent" was not taken into account in the particular analysis as the origin of the mistake is associated with diagrammatic representation which was either misinterpreted or not drawn. The strategies used by the students to attempt the tasks were not considered since the section is concerned with investigating the effect of using freeze frame representations on the quality of mathematical representations generated.

A high proportion of students with the correct mathematical formulation for the non-directive task also provided a suitable mathematical model for the directive problem question regardless of the category of freeze frame representations (34 in 47). However, only 18 in 47 of these students provided appropriate freeze frame together with mathematical representations. The data also indicate that more than half of the students who failed to construct a suitable mathematical expression for the non-directive problem question were able to do so for the directed task. The 53 out of 86 students who presented an inappropriate (qualitatively or quantitatively) mathematical formulation for the non-directive written exercise formulated the correct mathematical model for the directive task irrespective of the quality of freeze frame representations. It was also noted that for only 26 in 86 of these students, both forms of conceptual model corresponded to the physical model.

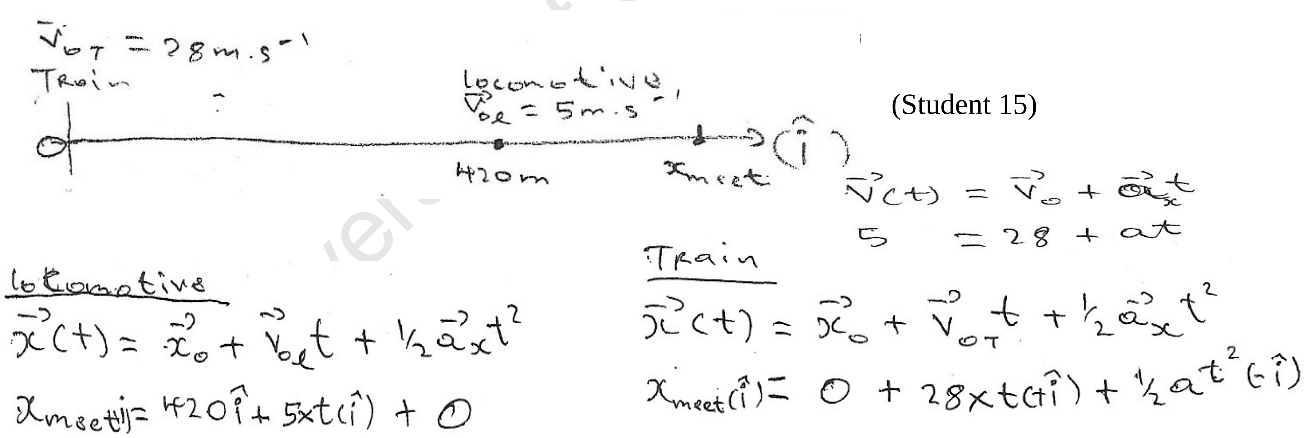
Table 5-14: Comparison of the categories of mathematical representations generated by the same student for the directive and the non-directive tasks presented in linguistic form (n = 133).

Directive task		**Non-directive task**			
Categories of freeze frame representations	**Categories of mathematical representations**	Consistent with physical model	Inconsistent qualitative information	Inconsistent quantitative information	**Total (%)**
1. Consistent with physical model	Consistent with physical model	18 (13)	8 (7)	18 (13)	44 (33)
	Direction is inconsistent	1 (1)	4 (3)	0 (0)	5 (4)
	Inconsistent	4 (3)	3 (2)	0 (0)	7 (5)
2. Inconsistent with physical model	Consistent with physical model.	16 (12)	7 (5)	20 (15)	43 (32)
	Direction is inconsistent	1 (1)	4 (3)	4 (3)	9 (7)
	Inconsistent	7 (5)	7 (5)	11 (9)	25 (19)
	Total	47 (35)	33 (25)	53 (40)	**133 (100)**

For students who were unsuccessful with the non-directive task, two-tailed tests for differences between proportions performed on the data in Table 5-14, at the 5% significance level, excluding the category "direction is inconsistent", yielded z-values of 2.76 and -2.76. These values indicate that a significant difference in proportion exists between the category of mathematical representations constructed and freeze frame representations agreeing and conflicting with the physical model. However when a similar test was performed on students who were successful with the non-directive written exercise, z-values of 0.96 and -0.96 were obtained indicating that there is no significant link between mathematical model categorised as appropriate and inappropriate and the quality of freeze frame representations generated. The test for difference in proportion was also performed on the data for the whole cohort. A relationship was found to exist between the category of mathematical model formulated and freeze frame representations corresponding and conflicting with the physical model, with *z*-values of 2.80 and -2.80.

5.2.3 (a) Non-directive and directive tasks describing the same situation

The non-directive and the directive version of the same task posed in linguistic form are presented on page 197 and page 203 respectively. The categories of mathematical model generated as well as the strategies employed for attempting the non-directive task with a linguistic format (class work 16) are highlighted in Table 5-15. Only one student in the sample was successful with the task and, as an illustration of a complete response, the work is shown below. The appropriate mathematical representations were formulated and a diagram portraying the whole motion (the vehicles at their initial and meeting points) was provided. The student generated the following combination of visual and mathematical representations:



(Student 15)

Irrespective of the strategy employed to solve the problem, a mathematical expression with inappropriate qualitative information was generated by 36% (52 out of 145) of the sample. The qualitative mistakes ranged from ignoring the direction of acceleration for decreasing velocity, including acceleration for constant velocity, the velocity at meeting point either not mentioned, considered as 0 m s^{-1} or 28 m s^{-1} to the direction of motion for one of the vehicles being inappropriate.

Table 5-15: Categories of mathematical expressions generated for the non-directive task describing the motion of two vehicles in a straight horizontal line (n = 145).

Strategy used	Mathematical representation	Total (%)
1. Diagram depicts whole motion	Required mathematical representation generated.	1 (1)
	Direction of acceleration for decreasing velocity ignored. Velocity at meeting point may not be mentioned / 0 m s^{-1}.	3 (2)
2. Diagram depicts part of the motion	Direction of acceleration for decreasing velocity ignored. Acceleration may be included for constant velocity. At meeting point, velocity is not mentioned or taken as 0 m s^{-1}.	19 (14)
	Use of inappropriate values for vehicles' positions. Direction of acceleration for decreasing velocity ignored. Acceleration may be included for constant velocity. - velocity at meeting point not mentioned / 0 m s^{-1} / 28 m s^{-1}.	12 (8) 25 (17)
3. Diagram depicts an irrelevant situation	Direction of acceleration for decreasing velocity ignored. Velocity of one vehicle is in the - $\hat{\mathbf{i}}$ direction. Velocity at meeting point may not be mentioned / 0 m s^{-1}.	9 (6)
4.Application of equations only / rough sketch	Direction of acceleration for decreasing velocity ignored. Acceleration may be included for constant velocity. At meeting point, velocity is not mentioned or taken as 0 m s^{-1}.	21 (14)
	Use of inappropriate values for vehicles' positions. Velocity at meeting point not mentioned / 0 m s^{-1} / 28 m s^{-1}. Direction of acceleration for decreasing velocity ignored. Acceleration may be included for constant velocity.	55 (38)
	Total	**145 (100)**

A mathematical model with incorrect qualitative and quantitative information was provided by 63% (92 out of 145) of the cohort. An example of student's response includes:

$$\underline{\text{min}}: X(t) = X_0 + V_0 t + \tfrac{1}{2}at^2$$
$$420 = 0 + 28t + \tfrac{1}{2}at^2$$

$$\underline{\text{locomotive}}: X(t = X_0 + V_0 t + \tfrac{1}{2}at^2$$
$$420 = 0 + 5t + \tfrac{1}{2}at^2$$

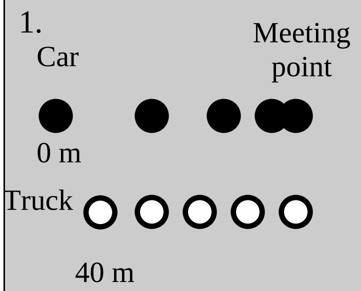
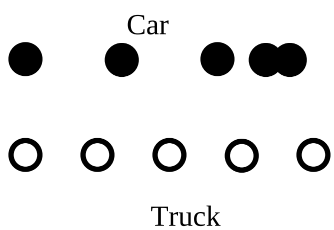
(Student 155)

In the particular student-generated mathematical model, the direction of acceleration for decreasing velocity was ignored (the train's motion) while acceleration was included for constant velocity (locomotive's motion). No mention was made of the vehicles' velocity when they reach the same position (at meeting point). Moreover, inappropriate values were used for the both vehicles' final positions and the locomotive's initial position.

Therefore, the large majority (99%, 144 out of 145) of the cohort generated an incorrect mathematical representation for the particular task. Just over half of the students (52%, 76 out of 145) solved the problem by manipulating equations with or without a rough sketch while around 48% (69 out of 145) of the cohort included a diagram. However, in most (65) cases the diagrammatic representation was inconsistent with the situation described, being either incomplete (depicting part of the motion) or irrelevant. On the whole an incorrect mathematical model (either only qualitatively or both qualitatively and quantitatively) was generated mainly when manipulating equations with or without a supporting sketch (52%, 76 in 145) compared to when a diagram was included (47%, 68 out of 145). It was also noted that, in particular, when the velocity at meeting point was not referred to, inappropriate values were used for the vehicles' positions for solving the problem.

The characteristics of the mathematical expressions and the two forms of visual representations provided for the directive version of the same problem are presented in Table 5-16.

Table 5-16: Categories of visual and mathematical representations generated for the directive task describing the motion of two vehicles in a straight horizontal line (n = 145).

Visual representations		**Mathematical representation**	**Total (%)**
Freeze frame representations	Diagrammatic representation		
1. 	Depiction of whole motion.	Required mathematical model generated.	1 (1)
		Direction of acceleration for decreasing velocity ignored,	31 (21)
		- velocity at meeting point not mentioned / 22 m s^{-1}	11 (8)
		Inappropriate qualitative information. Final position is either 0 m or 40 m.	4 (3)
	Depiction of part of the motion.	Direction of acceleration for decreasing velocity ignored.	2 (1)
2. 	Depiction of whole motion.	Required mathematical model generated.	1 (1)
		Direction of acceleration for decreasing velocity ignored,	29 (20)
		- velocity at meeting point is 0 m s^{-1} / 22 m s^{-1}	4 (3)
		Inappropriate qualitative information. Final position is either 0 m or 40 m.	6 (4)
	Depiction of part of the motion.	Direction of acceleration for decreasing velocity ignored,	14 (9)
		- velocity at meeting point not mentioned / 0 m s^{-1}	7 (4)
		Final position either 0 m or 40 m and inappropriate qualitative information.	10 (8)
	Inappropriate depiction of situation.	Direction of acceleration for decreasing velocity ignored,	1 (1)
		- direction of motion in opposite direction	4 (3)
		- final position is 40 m. Final velocity not mentioned / 0 m s^{-1}	9 (6)
3. Increase and no change in spacing for car and truck. Quantitative information may be considered.	Depiction of whole motion.	Direction of acceleration consistent with freeze frame depiction	5 (3)
		- velocity at meeting point is 22 m s^{-1}	2 (1)
		- vehicles' final position is 0 m	1 (1)
	Depiction of part of the situation.	Direction of acceleration consistent with freeze frame representations. Velocity at meeting point is 0 m s^{-1} or 22 m s^{-1}.	2 (1)
	Inappropriate depiction of situation.	Car's final position is 40 m and acceleration consistent with freeze frame representations.	1 (1)
		Total	**145 (100)**

Although freeze frame representations corresponded to the qualitative description, when the diagrammatic representation portrayed the whole motion, a mathematical model involving qualitative mistakes was mostly (52%, 75 out of 145) formulated. A similar trend was observed in cases where only part of the motion (the vehicles depicted at their initial positions) was drawn, with qualitative mistakes prevailing (14%, 23 out of 145) in the mathematical expressions. At meeting point, the velocity of the vehicles was either not mentioned or values such as 0 m s^{-1} or 22 m s^{-1} were utilised. The direction of acceleration for decreasing velocity was ignored and acceleration may be included for constant velocity. A qualitatively and quantitatively inappropriate mathematical representation together with freeze frame representations related to the qualitative description was presented by 21% (29 out of 145) of the cohort. The incorrect quantitative information may be due to the category of diagram constructed, being either irrelevant or incomplete.

Around 7% (11 out of 145) of the sample drew an increase in spacing between the positions for the car. The mathematical representations generated were incorrect owing to the qualitative information regarding the car's acceleration which was inappropriately interpreted. The non-reference of the velocity of the two vehicles when they are at the same position as well as the construction of an irrelevant diagram also contributed to the formulation of an inappropriate mathematical expression.

Therefore, no substantial difference was noted in the proportion of students generating a suitable mathematical representation for the non-directive (1 in 151) and the directive (2 in 151) problem questions. Qualitative mistakes which emerged for the non-directive version of the task still prevail. However, a decrease was noted in the proportion of students providing a qualitatively and quantitatively inappropriate mathematical expression for the directed task (21% versus 63%). In contrast, the proportion of students generating a qualitatively incorrect mathematical model was found to increase (75% versus 36%).

<u>5.2.3</u> (b) Comparison of the categories of mathematical representations generated by the same student for the non-directive and the directive tasks describing the same situation

The classifications of mathematical representation formulated by the same student when handling the non-directive and the directive versions of the same task are presented in Table 5-17. For the directive task, diagrams depicting part of the motion, vehicles approaching each other or the two vehicles meeting at the truck's initial position were considered unsuitable for the situation. A mathematical model was classified as "inconsistent" if inappropriate qualitative and quantitative information were reflected in the kinematics equation. The strategies employed for attempting the non-directive task were not taken into account in the analysis.

Only one student in the sample consistently presented the required mathematical formulation for both tasks. A mere 1 in 52 students with a qualitatively inappropriate mathematical model for the non-directed task provided the correct mathematical expression for the directed problem question. A substantial proportion of these students still generated a mathematical formulation which was qualitatively incorrect independently of the quality of freeze frame representations (48 out of 52). It was also noted that for 46 (17+28+1) of them freeze frame representations agreed with the qualitative description. None of the students who presented a mathematical representation which was qualitatively and quantitatively inappropriate for the non-directive problem question was successful with the directive task. Regardless of the category of freeze frame representations drawn, the vast majority of these students (64 in 92) generated a qualitatively incorrect mathematical formulation. Moreover, for 57 of these students freeze frame representations generated were related to the qualitative description.

Table 5-17: Comparison of the categories of mathematical representations generated by the same student for the directive and the non-directive tasks posed in linguistic form (n = 145).

Directive task		**Non-directive task**			
Visual representations	**Mathematical representations**	Consistent with physical model	Inconsistent qualitatively	Inconsistent	**Total (%)**
1. Consistent with physical model.	Consistent with physical model	1 (1)	0 (0)	0 (0)	1 (1)
	Inconsistent qualitatively	0 (0)	17 (12)	25 (17)	42 (29)
	Inconsistent	0 (0)	0 (0)	4 (3)	4 (3)
2. Diagram may be consistent with physical model. Freeze frame representations consistent in terms of qualitative information only.	Consistent with physical model	0 (0)	1 (1)	0 (0)	1 (1)
	Inconsistent qualitatively	0 (0)	28 (19)	31 (21)	59 (40)
	Inconsistent	0 (0)	3 (2)	22 (15)	25 (17)
3. Freeze frame representations consistent with physical model. Diagram depicts part of the motion	Inconsistent qualitatively	0 (0)	1 (1)	1 (1)	2 (2)
4. Only diagram consistent with physical model.	Inconsistent qualitatively	0 (0)	2 (1)	5 (3)	7 (4)
	Inconsistent	0 (0)	0 (0)	1 (1)	1 (1)
5. No consistency with physical model.	Inconsistent qualitatively	0 (0)	0 (0)	2 (1)	2 (1)
	Inconsistent	0 (0)	0 (0)	1 (1)	1 (1)
	Total	1 (1)	52 (36)	92 (63)	**145 (100)**

Moreover, the data reveal that with the presence of a diagrammatic representation the percentage of students presenting a quantitatively incorrect mathematical model decreases. In contrast, the proportion of students constructing a mathematical expression conflicting with only the qualitative information increases. Upon consideration of the category 1 in Table 5-17, a decrease from 20% (29 in 145) to 3% (4 in 145) was recorded in the proportion of students with a qualitatively and quantitatively inappropriate mathematical expression for the non-directive and directive task respectively. However, an increase from 12% (17 in 145) to 29% (42 in 145) was noted in the percentage of students generating a qualitatively incorrect mathematical model when dealing with the non-directive and directive problem question respectively. A similar trend was observed when categories 2 and 3 are considered together. For the non-directive task, a qualitatively and quantitatively inappropriate mathematical expression was provided by 36% (53 in 145) of the cohort compared to 17% (25 out of 145) of the students for the directive task. An increase from 21% (31 out of 145) to 40% (59 out of 145) was observed in the proportion of students presenting a qualitatively inappropriate mathematical representation when dealing with the non-directive and directive version of the problem question.

5.2.4 Main findings for research question 2

The data revealed that the presence of freeze frame representations in 2 of the 3 problem questions does not lead to an enhancement in the quality of mathematical representation formulated. For the directed tasks with a graphical and a linguistic format (June class test question 10), a respective 1% and 2% of the sample were able to generate the correct mathematical representation (Tables 5-9 and 5-16). Moreover, the data comparing individual students show that the vast majority (around 98%) of them providing an incorrect mathematical model for the non-directive task were still unsuccessful with the directed problem question (Tables 5-11 and 5-17). Additionally, no relationship exists between the category of freeze frame and mathematical representations for the two particular directed tasks. None of the students with an inappropriate mental model for the non-directive tasks presented a combination of freeze frame and mathematical representations corresponding to the physical model.

In contrast, for the directive task describing the situation of a vertically propelled object (class test question 4), a high proportion of students (65%, 87 in 133) constructed the appropriate mathematical representation regardless of the category of freeze frame representations (Table

5-13). Upon comparison of individual students (Table 5-14) it was found that 72% (34 in 47) of the students with the appropriate mathematical expression for the non-directive task were also successful with the directive written exercise while 62% (53 in 86) of the students who failed to formulate the correct mathematical model for the non-directive problem question were able to do so for the directive task. A link was found to exist between the quality of freeze frame and mathematical representations constructed. However, the generation of appropriate freeze frame representations does not necessarily result in the formulation of the required mathematical expression. From Table 5-14, for students who were successful with the non-directed task, a difference of 4% was noted between the proportion of students constructing the appropriate mental model (38%, 18 in 47) for the directed problem question and those providing the correct mathematical expression although freeze frame representations were inappropriately drawn (34%, 16 in 47). Moreover, 30% (26 in 86) of the respondents with an inappropriate mental model for the non-directed task constructed the appropriate freeze frame and mathematical representations for the directive written exercise. A negligible difference of 1% was noted with those students providing the correct mathematical representation but inappropriate freeze frame representations (27 in 86). Therefore, the positive outcome emerging from the particular task may be due to the nature of the physics underlying the situation rather than the use of freeze frame representations. Qualitative information required for solving the problem, the object's velocity at maximum height, may be memorised by rote.

The data also indicate that although freeze frame representations are appropriately drawn the derived information is not included in the mathematical expression. The direction of acceleration for decreasing velocity was ignored, acceleration was indicated for constant velocity while it was absent for decreasing velocity, vehicles' velocity at meeting point was inappropriate or not mentioned and 9.8 m s^{-2} was employed for vehicles moving in a horizontal line. Further evidence of the insignificant role of freeze frame representations was obtained from the task posed in linguistic form (class test question 4). Although the students failed to depict qualitative information visually, the required mathematical representation was constructed. They have no understanding of the physics involved and appeal to rote memorisation of qualitative information for generating the mathematical representations. In addition, when the directive (June class test question 10) and the non-directive (class work 16) versions of the same problem were compared, a shift from mathematical representations which are quantitatively and qualitatively incorrect to qualitatively inappropriate mathematical expressions was noted.

Therefore, for the particular aspect of the study, it can be claimed that freeze frame representations have no substantial role in the construction of a mathematical representation consistent with the given physical model. The students do not fully and meaningfully engage with the visual conceptual model, basically concerned with qualitative information. Freeze frame representations may help in recognising the presence or the absence of acceleration but are not employed as an intermediate step for translating qualitative information to mathematical form.

5.3 Research question 3: The effect of using freeze frame representations on generating linguistic conceptual models from mathematical and graphical conceptual models

Four tasks were considered for this section, namely class work 8 and class test question 1 which were posed in mathematical form and are non-directive and directive respectively, and class works 7 and 12 were presented in graphical form with and without freeze frame representations respectively. A comparison was made for the quality of linguistic representations (written responses) generated by individual students when freeze frame representations are present or not included in the task. For the various sections presented below, the shaded part of the tables highlights explanations which were considered appropriate for each physical model.

5.3.1 (a) Non-directive and directive tasks presented in mathematical form

The non-directive and the directive problem question posed in mathematical form are shown on page 189 and page 198 respectively. Table 5-18 provides an overview of the categories of linguistic representations provided for the tasks presented in mathematical form with and without freeze frame representations.

Table 5-18: Categories of linguistic representations generated from tasks with and without freeze frame representations posed in mathematical form (n = 159).

		Directive task			
Categories of linguistic representations	**Non-directive task**	Decrease in spacing	Increase in spacing	No change in spacing	Depiction of 2 stages of motion
1. Acceleration is in the - $\hat{\mathbf{i}}$ direction / opposite to direction of motion. Velocity decreases (in the $\hat{\mathbf{i}}$ direction).	71 (45)	102 (64)	1 (1)	0 (0)	0 (0)
2. Derived information for acceleration is inconsistent with physical model	9 (6)	19 (12)	1 (1)	0 (0)	0 (0)
3. Misinterpretation of the physical model	18 (11)	4 (3)	3 (1)	9 (6)	3 (1)
4. Description of the physical model	61 (38)	15 (9)	1 (1)	1 (1)	0 (0)
Total	159 (100)	140 (88)	6 (4)	10 (7)	3 (1)

For the task without freeze frame representations (class work 8), around 45% (71 out of 159) of the sample formulated a written statement highlighting the appropriate qualitative information as shown by the responses below:

> The object is moving in the positive $\hat{\mathbf{i}}$ direction but experiences it slowing down. Therefore, the acceleration is in the opposite direction. Velocity is also in the positive $\hat{\mathbf{i}}$ direction. (Student 50)

> The object was slowing down. Acceleration is in opposite direction to the motion. Velocity is decreasing / slowing down. The object is moving in the direction $\hat{\mathbf{i}}$. (Student 120)

An explanation based on physics information conflicting with the given mathematical formulation was provided by 6% (9 out of 159) of the students. Qualitative information regarding the object's acceleration was inappropriate, as exemplified in the following:

> The object is moving with velocity which is in the positive $\hat{\mathbf{i}}$ direction but to the - $\hat{\mathbf{i}}$ direction of the acceleration. I would expect that the velocity of the object is decreasing. The acceleration will be decreasing. (Student 149)

> Object is moving in decreasing velocity in the $\hat{\mathbf{i}}$ direction as it's slows down acceleration increases in the negative ($\hat{\mathbf{i}}$) direction of the velocity. (Student 126)

The physical model was misinterpreted by 11% (18 out of 159) of the students. Either the meaning of the unit vectors associated with the variables was not grasped, or the equation was understood to indicate two stages of motion, as represented by the following quotes:

> Displacement is occurring in the $\hat{\mathbf{i}}$ direction. Acceleration is constant in the negative $\hat{\mathbf{i}}$ direction. The velocity is in the $\hat{\mathbf{i}}$ direction which means velocity is increasing. (Student 64)

> It is accelerating in the negative $\hat{\mathbf{i}}$ direction. It is slowing down (velocity decreases) and moving backward. (Student 84)

> The object's initial velocity increases in the positive $\hat{\mathbf{i}}$ direction and then it decreases. This tells us that the acceleration of the object first accelerates in the $+\hat{\mathbf{i}}$ direction because of the direction of the initial velocity and then the object accelerates in the opposite direction ($-\hat{\mathbf{i}}$) because the velocity decreases. (Student 154)

For 38% (61 out of 159) of the cohort, as shown in the quotations provided next, no qualitative information was derived. On the one hand, the variables included in the equation were identified. On the other hand, the apparent information presented by the kinematics equation was described.

> $x(\hat{\mathbf{i}})$ – final displacement in the $\hat{\mathbf{i}}$ direction; $x_o(\hat{\mathbf{i}})$ – initial displacement in the $\hat{\mathbf{i}}$ direction; $v_{xo}(\hat{\mathbf{i}})$ – initial velocity in the $\hat{\mathbf{i}}$ direction; t – the time at that point; a_x – the acceleration at that time in the $-\hat{\mathbf{i}}$ direction. (Student 96)

> The object is moving along the x-axis. The acceleration is in the (-$\hat{\mathbf{i}}$) direction. The initial position is in the positive $\hat{\mathbf{i}}$ direction. The velocity is at the positive $\hat{\mathbf{i}}$ direction. The object is moving in a straight line. (Student 137)

Thus, for the task with a non-directive format just over half (55%, 88 out of 159) of the students failed to generate a linguistic representation with the appropriate physics information. Within this cluster, normally a description rather than an explanation of the mathematical model was presented. A sub-group interpreted the situation appropriately in terms of the object's velocity but was unable to derive information about its acceleration. Another group of students misinterpreted the significance of the unit vectors in the kinematics equations.

Table 5-18 also shows that 64% (102 out of 159) of the students constructed a suitable mental model for the situation when presented with the directive version of the task (class test question 1). Their responses showed correspondence between freeze frame representations and the written explanation. A decrease in spacing was portrayed. The quotations below typify the category of explanation provided:

> The ball is travelling along a horizontal plane with a decrease in velocity due to an acceleration in the opposite direction of the motion. (Student 175)

> The velocity of the ball is decreasing since the acceleration of the ball is in the (-$\hat{\mathbf{i}}$) direction which moves in the opposite direction with the ball's velocity. (Student 39)

For 15% (19+4 out of 159) of the sample, the given explanation was not in agreement with the physical model even if freeze frame representations were appropriately drawn. In part, the derived information conflicted with the qualitative information shown by the mathematical physical model. Alternatively, the meaning conveyed by the unit vectors was misunderstood or the equation was interpreted to indicate two stages of motion. These responses include:

> The velocity of the ball decreases in the (+$\hat{\mathbf{i}}$) direction, seen by the length of the velocity vectors. Since velocity is decreasing, acceleration is increasing in the in the (-$\hat{\mathbf{i}}$) direction. (Student 135)

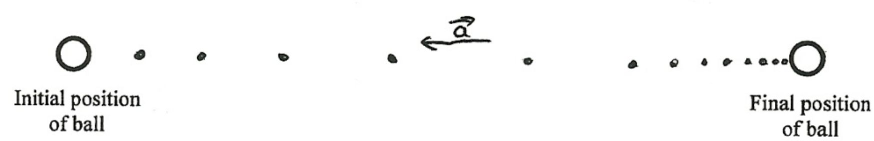
The ball is accelerating in the opposite direction. It has a velocity that is increasing. (Student 52)

The ball initially moves with an increasing velocity accelerating in the $+\hat{\mathbf{i}}$ direction. Then it starts again to move with a decreasing velocity accelerating in the $-\hat{\mathbf{i}}$ direction until it reaches its final position then stops. (Student 40)

Discrepancies between freeze frame representations and the physical model existed for 12% (3+9+3+4 out of 159) of the sample. Freeze frame representations ranged from an increase or no change in spacing to the portrayal of two stages of motion. However, in 15 instances, consistency prevailed between the explanation and the incorrect freeze frame representations. A sample of written statements together with freeze frame representations which characterise this category of student responses is provided:

The ball accelerates in the $\hat{\mathbf{i}}$ -direction constantly until it reaches its final position. The velocity increases because of constant acceleration. (Student 106)

The ball is moving with constant velocity. The acceleration of the ball is zero. (Student 142)



The object accelerate in the $\hat{\mathbf{i}}$ direction then it slows down and accelerate in the $-\hat{\mathbf{i}}$ direction. Velocity increases at time interval then it decrease. (Student 147)

Regardless of the category of freeze frame representations, 11% (17 out of 159) of the students presented a linguistic representation in the form of a description, such as:

> The initial velocity is in the $\hat{\mathbf{i}}$ direction. The acceleration is in the negative direction. (Student 123)

> The ball travels with a velocity v towards the $\hat{\mathbf{i}}$ axis and the acceleration is in the opposite direction that is - $\hat{\mathbf{i}}$ axis. (Student 162)

Overall, for the directive task, a higher proportion of students (64% versus 45%) provided written responses which are in line with the physical model thus indicating that more students construct an appropriate mental model and have an understanding of the given situation. The percentage of students merely describing the mathematical model also decreased (from 38% to 11%). Factors contributing to the formulation of an inappropriate explanation include the inability to derive physics information (object's acceleration) from freeze frame representations and the misinterpretation of the kinematics equation which was reflected by the freeze frame representations drawn.

5.3.1 (b) Comparison of the categories of linguistic representations generated by the same student for the directive and the non-directive tasks with mathematical conceptual model

The quality of the linguistic representations provided by the same student for the directive and non-directive tasks was compared. The results are presented in Table 5-19, in which it can be seen that around 34% (54 out of 159) of the sample consistently generated an explanation with the required qualitative information irrespective of the task's format. Thus, the vast majority of those students who formulated an appropriate written response for the non-directive task also provided a combination of correct freeze frame and linguistic representations for the directive problem question (54 of the 71 responses). Table 5-19 also indicates that the inclusion of freeze frame representations in the task helped more than half of those students who failed to provide a suitable linguistic representation in the non-directive written exercise. A proportion of 18 out of 27 students who presented an incorrect explanation for the non-directive task were successful after the use of freeze frame representations. Additionally, although a substantial proportion of students who merely described the physical situation for the non-directive task formulated an inappropriate explanation (12+9 out of 61) for the directive version of the problem question, a much larger proportion (30 out of 61) provided an appropriate freeze frame and linguistic representation.

Table 5-19: Comparison of the categories of linguistic representations generated by the same student for the directive and the non-directive tasks presented in mathematical form (n = 159).

			Non-directive task			
Directive task	**Categories of freeze frame representations**	**Categories of linguistic representations**	Consistent with physical model	Inconsistent with physical model	Description of physical model	**Total (%)**
	Consistency between freeze frame and physical model	Consistent with physical model	54 (34)	18 (11)	30 (19)	102 (64)
		Inconsistent with physical model	5 (3)	6 (4)	12 (8)	23 (15)
		Description of physical model	7 (4)	0 (0)	8 (5)	15 (9)
	Inconsistency between freeze frame and physical model	Consistent with physical model	0 (0)	0 (0)	1 (1)	1 (1)
		Inconsistent with physical model	4 (3)	3 (2)	9 (4)	16 (9)
		Description of physical model	1 (1)	0 (0)	1 (1)	2 (2)
		Total	71 (45)	27 (17)	61 (38)	**159 (100)**

5.3.2 (a) Non-directive and directive tasks presented in graphical form

Class work 7 (with a non-directive format, page 188) and class work 12 (directive in nature, page 193) were posed in graphical form and asked for written statements (linguistic representations). The non-directive task involves a velocity-time graph depicting a decrease in velocity as time changes. Information presented about velocity is directly interpreted thus supporting the derivation of information concerned with acceleration where reference has to be made to both its magnitude and direction. For the directive problem question, a motion graph for position was presented indicating a linear increase in position as time changes. Position-time graphs are more abstract in nature. The students are less familiar in handling the particular graphical representation. Therefore, derived information regarding velocity and acceleration is not straightforward requiring in depth interpretation of the position-time graph. Table 5-20 summarises the various categories of student-generated linguistic representations.

Table 5-20: Categories of linguistic representations generated from tasks with and without freeze frame representations posed in graphical form (*n* = 128).

		Directive task	
Categories of linguistic representations	**Non-directive task**	Equal spacing	Increase in spacing
1. Velocity decreases and object moves with uniform deceleration or (constant / constant negative) acceleration is opposite to the direction of motion / in a negative direction.	37 (29)	-	-
2. As above but application of ambiguous terms	11 (9)	-	-
3A. Velocity is constant (due to constant increase in position / spacing between individual positions is constant) and hence acceleration is zero.	-	87 (69)	2 (1)
3B. Object is moving with constant velocity (lack explanation).	-	17 (13)	2 (1)
4. Derived information for velocity and / or acceleration is inconsistent with physical model.	30 (23)	3 (2)	0 (0)
5. Misinterpretation of physical model.	14 (11)	0 (0)	15 (13)
6. Description of the physical model.	36 (28)	0 (0)	2 (1)
Total	128 (100)	107 (84)	21 (16)

For the task with a non-directive format, only 29% (37 out of 128) of the sample were able to interpret the graph and derive the appropriate qualitative information as highlighted by:

> At time t = 0s the object is at v_{xo}. The object then slows down until its velocity is zero so it stops at time t_f. The a is in the opposite direction of the motion from t = 0 to t_f .(Student 148)

> The object's velocity is decreasing constantly. The acceleration of the object is constant in the negative direction. (Student 106)

> The object moves slower and slower as time passes by. The object travels with uniform deceleration. (Student 77)

The ambiguous terms "acceleration is negative" was applied by 9% (11 out of 128) of the students. The meaning attributed to these words cannot be ascertained. They may refer either to a decrease in acceleration, or to the acceleration being opposite to the direction of motion:

> The object is slowing down until it stops. It also has a negative acceleration meaning that as time goes by its acceleration decreases. The object starts at position v_{xo} and stops at origin. (Student 5)

> The object is slowing down, velocity is decreasing with time, moving with a negative acceleration. (Student 41)

A poor understanding of the concepts depicted by the physical model (category 4) was displayed by 23% (30 out of 128) of the cohort. The graphical representation was appropriately interpreted to indicate a decrease in velocity. However, information about the object's acceleration, as portrayed by the quotations that follow, was not related to the graphical depiction:

> This motion is between a certain time interval. It's from an initial velocity to final velocity which is 0 m s^{-1}. The velocity is decreasing from the beginning. The acceleration is also decreasing because of the negative slope. (Student 81)

> The velocity decreases with time. The object has a constant acceleration. It's initial velocity is v_{xo}. (Student 166)

> The object is moving at a decreasing velocity which is constant and its acceleration is zero. (Student 150)

Around 11% (14 out of 128) of the students misinterpreted the physical model. On the one hand, the qualitative information depicted by the graphical representation was considered to indicate constant velocity in negative direction. The following explanations were provided:

> The object moves in a constant velocity but in the negative direction. (Student 113)

> Velocity is constant in opposite direction (negative). The acceleration is decreasing. (Student 145)

On the other hand, the situation presented by the physical model was misunderstood to reflect the motion of vertically propelled object instead of motion in a straight horizontal line. A student gave the following written explanation:

> The object move in the constant velocity when it is thrown up and then decelerate on its way down. (Student 33)

A description of the physical model, as shown by the examples of responses below, was presented by 28% (36 out of 128) of the respondents. Typically, no mention is made of acceleration.

> The object is decelerating i.e decreasing velocity with time, until the velocity is 0 (zero). It stops at time t_f. (Student 102)

> The object is slowing down from a velocity of v_{xo} until it reaches a final velocity (0) i.e it stops. It takes t_f seconds for this to happen. (Student 156)

> Over a period of time the velocity is decreasing constantly (61)

Overall, the large majority of the students, 71% (91 out of 128) generated a linguistic representation considered unsuitable for the non-directive task. The failure to derive physics information (particularly about the object's acceleration) and misinterpretation of the physical model (velocity to be in a negative direction) led to the formulation of inappropriate explanations. Moreover, the tendency to provide a description of the physical model resulted in an incorrect and incomplete written statement.

In contrast, for the directive task a large proportion of students, 82% (104 out of 128), generated freeze frame representations and explanations corresponding to the graphical physical model. No change in spacing was portrayed and the following explanations were given:

The object starts from a position (initial position) x_o when $t = 0$. The object position increases constantly to time t_f. Because the position increases constantly therefore the velocity is constant and acceleration is zero. (Student 58)

The object is moving at a constant velocity for a time "t_f" seconds. Therefore the acceleration is zero. (Student 7)

For 2% (3 out of 128) of the students, discrepancy exists between the appropriate visual depiction (freeze frame representations) and information concerning acceleration. One student wrote:

Acceleration is constant, velocity is positive in the $\hat{\mathbf{i}}$ direction. (Student 145)

Around 16% (21 out of 128) of the cohort depicted an increase in spacing between the freeze frames. However, in 4 cases, as shown by the written statement below, the qualitative information highlighted in the explanation was related to the physical model.

An object is moving along a horizontal track in a straight line from an initial position x_o. The object is moving at constant velocity thus the acceleration is zero. (Student 124)

In 15 instances, the explanation provided for the object's velocity was in agreement with the inappropriate physics information portrayed by freeze frame representations. The responses presented below are illustrative:

The object is accelerating positively from point x_o metres. It is speeding up gradually. (Student 41)

An object start moving from position x_o when time is zero. From its time interval, an object increases its speed and accelerated along the way. (Student 20)

Only 2 students in the cohort (with inappropriate freeze frame representations) formulated a description for the situation presented by the motion graph:

The object has an increasing position as time increases and this starts at a position x_o away from the origin. (Student 4)

Overall, with the inclusion of freeze frame representations in the task, a higher proportion of students (82% versus 29%) constructed an appropriate mental model and demonstrated an understanding of the graphical physical model. They succeeded in expressing given and derived information presented by the graphical model in two representational modes, namely visual (freeze frame representations) and linguistic (written responses) forms. A decrease (from 28% to 1%) was also noted in the percentage of students describing the physical model. However, the misinterpretation of the physical model to indicate an increase in velocity as reflected by freeze frame representations influenced the quality of the explanation provided. The inability to derive information from freeze frame representations resulting in the formulation of an inappropriate explanation was also observed but to a lower extent.

5.3.2 (b) Comparison of the categories of linguistic representations generated by the same student for the directive and the non-directive tasks with a graphical format

The quality of the linguistic representations provided by the same student for the directive and non-directive tasks was compared. The results are summarised in Table 5-21.

On the whole, 25% (32 out of 128) of the students were able to provide an appropriate explanation for both tasks. Thus, most of the students who generated a suitable linguistic representation for the non-directive task also provided the appropriate freeze frame representations and a written response for the directed task (32 of the 37 responses). The data also indicate that with the inclusion of freeze frame representations in the task, four out of five students who failed to provide the correct explanation for the non-directive task were able to do so for the directed problem question. Firstly, 44 out of 55 students who presented inappropriate written responses for the non-directive task provided the required explanation and freeze frame representations for the directive problem question. Moreover, the 28 out of 36 students who described the physical model for the non-directive task formulated an appropriate explanation and drew freeze frame representations which are related to the physical model of the directive written exercise.

Table 5-21: Comparison of the categories of linguistic representations generated by the same student for the directive and the non-directive tasks presented in graphical form (n = 128).

			Non-directive task			
Directive task	**Categories of freeze frame representations**	**Categories of linguistic representations**	Consistent with physical model	Inconsistent with physical model	Description of physical model	**Total (%)**
	Consistency between freeze frame and physical model	Consistent with physical model	32 (25)	44 (34)	28 (23)	104 (82)
		Inconsistent with physical model	1 (1)	1 (1)	1 (1)	3 (3)
		Description of physical model	0 (0)	0 (0)	0 (0)	0 (0)
	Inconsistency between freeze frame and physical model	Consistent with physical model	0 (0)	2 (1)	2 (1)	4 (2)
		Inconsistent with physical model	3 (2)	8 (6)	4 (3)	15 (11)
		Description of physical model	1 (1)	0 (0)	1 (1)	2 (2)
		Total	37 (29)	55 (42)	36 (29)	**128 (100)**

5.3.3 Main findings for research question 3

The data indicate that there are similarities in the patterns of response for the tasks posed in a mathematical and graphical format. With the inclusion of freeze frame representations in the problem questions, an increase in success rate was noted. The proportion of students that formulated an explanation based on derived information consistent with the underlying physical model of the problem increased from around one in three, to two in three of the sample (Tables 5-18 and 5-20). The data comparing individual students show that the generation of appropriate freeze frame representations coincides with the formulation of a suitable written explanation. For both sets of written exercises (Tables 5-19 and 5-21), more than half of the students constructing an inappropriate mental model for the non-directive task provided a combination of freeze frame and linguistic representations corresponding to the physical model, reflecting an understanding of the situation. In the case of the problem question posed in graphical form, this improvement applied to even about four out of five of these students. Moreover, it was found that for a small proportion of students (ranging between 7% and 11%), the use of freeze frame representations tend to reduce their understanding of the physical model (Tables 5-19 and 5-21). An appropriate mental model was constructed for the non-directed task but when handling the directed task an incorrect mental model was generated. Therefore, it can be claimed that the application of freeze frame representations aids in the derivation of appropriate qualitative information and hence helps in the construction of a suitable mental model and promotes conceptual understanding.

However, discrepancies were also observed between freeze frame and linguistic representations particularly for the task with a mathematical format. This suggests that an inappropriate mental model was constructed and that superficial understanding of the physical model prevails. Around 24% of the students provided the required visual depiction (freeze frame representations) but the written response was not related to the mathematical physical model. Only a few (2%) students provided this particular combination for the task structured in graphical form. It is possible that these students failed to meaningfully use freeze frame representations for eliciting qualitative information. They may lack skills in interpreting and extracting information from the particular visual conceptual model. Also, freeze frame representations may have been drawn by rote memorisation.

A respective 12% and 16% of the students produced an unsuitably drawn freeze frame representations for the task posed in mathematical and graphical form mainly due to misinterpretation of the qualitative information presented by the physical model. However, consistencies were noted between the two forms of student-generated conceptual models thus indicating the application of freeze frame representations for generating the explanation. Very few students in the sample (1% and 2% for the task structured in mathematical and graphical form respectively) generated an appropriate explanation although freeze frame representations were not in agreement with the mathematical or graphical physical model. These students may lack skills in depicting motion using freeze frame representations. In these cases, freeze frame representations may also have been ignored when formulating the explanation.

<u>5.4</u> Research question 4: The categories of cognitive constructs generated by the students when dealing with different representations of physical models

The categories of mental representations constructed by the students when handling kinematics tasks structured in different representational modes requiring the generation of qualitative (linguistic and graphical representations) and quantitative (mathematical) solutions were explored by designing four profiles. The term "profile" may be understood as a set of descriptors assembled to represent the variation in the actions of students when dealing with the various tasks (Ibrahim *et al.,* 2009). By relating the profiles to Johnson-Laird's (1983) cognitive framework of sense-making, it is possible to infer whether the students work at the level of the mental image, propositional mental representation or construct a working model (either appropriate or inappropriate). According to Johnson-Laird (1983), propositional representations comprise syntactic structures which connect a series of symbols together. Mental models are analogical representations of real world objects or events, while mental images are mental views having visual spatial features. A level of comprehension is attained only upon the construction of a mental model, which also acts as a medium for making links between propositional representations and mental images. Greca and Moreira (1997) applied the cognitive framework in the physics domain, when solving problems, to classify the mental representations of the sample in their study. The students categorised with propositional representations only mechanically manipulated formulae and used abstract definitions without understanding the underlying physics principles. Students classified as "mental imagers" were unable to deal with the mathematical formalism but appealed to their generated visualisations to reason about the problem task. Those who constructed a mental model emphasised on understanding the situation as well as on identifying the physics ideas presented in a problem and constructed diagrammatic representations before applying mathematical formulations.

For the current study, a total of 18 problem questions, with and without freeze frame representations, were considered. Non-directive tasks requiring the generation of graphical representations (pages 81, 112, 184, 190, 196, 201) were not used for the analysis as it cannot be ascertained whether a particular student constructed a mental image or a mental model (appropriate or inappropriate). The correct and even incorrect motion graphs may have been drawn by either rote memorisation of the shapes of the graphs or by the translation of (appropriate or inappropriate) information. For a particular student reference was made to the codes assigned for the strategies used to attempt the different problem tasks and the quality of external representations generated. For the directive written exercises, additionally, the correspondence between the various student-generated conceptual models was taken into account. Descriptions were then formulated summarising the student's overall actions for the different tasks, thus leading to a profile. The same process was repeated for all 179 students in the sample, allowing for the descriptors to be improved in an iterative way. It was found that 88% (157 out of 179) of the cohort's actions could be captured within one of the four profiles shown in Table 5-22. The profile allocation was repeated for all the 179 students, and 8 students were shifted to other profiles. A total of 12 students did not complete more than 6 tasks which made it difficult to classify their actions within one of the profiles.

Around 22% (35 out of 157) of the sample was classified as using appropriate mental models. These students are described by their focus on the translation of information presented in linguistic forms to diagrammatic representations before applying mathematical formulations which are linked to their diagrams. Qualitative reasoning plays a crucial role during problem solving. For these students there is evidence that consideration was made to both qualitative and quantitative information (derived and given) in the mathematical expressions used, and explanations of the physics concepts rather than descriptions of the given conceptual models were formulated. Moreover, the qualitative information used is mostly appropriate with the physical models of the problem questions. When dealing with multiple representations, more students (23 in 35) consistently generated different forms of conceptual models agreeing with each other and the physical model. Qualitative information portrayed by freeze frame representations was translated in the form of an explanation highlighting the appropriate physics concepts. Correspondence also exists between freeze frame and graphical representations. For a separate sub category (12 in 35) although freeze frame representations were appropriately drawn, incorrect graphical and linguistic representations were mainly generated. However, for both groups of students, discrepancies were noted between freeze frame and mathematical representations and there was rote memorisation of physics information.

Table 5-22: *Profiles for introductory physics students' problem-solving requiring the use of multiple representations.*

Cognitive structure	Features of overall strategy used	Interpretation of representational mode	Translation between multiple representations	Total
Appropriate mental model	Diagram drawn is related to a mathematical model. Both qualitative and quantitative information used generally agree with the physical model.	An explanation highlighting physics concepts agreeing with the physical model is formulated.	Freeze frame representations consistently agree with other forms of generated conceptual model.	23
			Freeze frame representations consistently conflict with other forms of generated conceptual model.	12
Inappropriate mental model	Diagram drawn is related to a mathematical model. Quantitative and qualitative information used mostly do not agree with the physical model.	An explanation highlighting physics concepts not agreeing with the physical model is formulated.	Freeze frame representations consistently agree with other forms of generated conceptual model.	4
			Freeze frame representations consistently conflict with other forms of generated conceptual model.	21
Propositional	Prioritisation of symbolic representations with pattern matching of given and required quantitative information.	Only surface level features of physical model are discussed.	Freeze frame representations consistently agree with other forms of generated conceptual model.	16
			Freeze frame representations consistently conflict with other forms of generated conceptual model.	78
Mental image	Diagram drawn may not be related to a mathematical model. Quantitative and qualitative information used mostly do not agree with the physical model.	Only surface level features of physical model are discussed.	Freeze frame representations consistently conflict with other forms of generated conceptual model.	3
			Total	**157**

The 16% (25 out of 157) students categorised with an inappropriate mental model are characterised by their tendency to include a diagram for the situation described which is then related to the mathematical representations employed. Although qualitative reasoning is of key importance during problem solving, it was mostly inappropriate. Explanations highlighting physics information conflicting with the physical model were also formulated. When engaged in the modelling process, most students (21 in 25) generated representations which are mainly at variance with each other. Qualitative information depicted by freeze frame representations corresponds to the physical model but kinematics graphs were mostly incorrectly drawn and explanations with inappropriate physics concepts were provided. Only 4 students with inappropriate mental models consistently generated conceptual models corresponding to each other as well as to the physical model. However, for both groups of students, qualitative information presented by freeze frame representations was not translated to the mathematical formulations and physics information was memorised by rote.

Most (60%, 94 out of 157) of the students in the study were classified as using propositional mental representation. Students with the particular cognitive structure are characterised by their emphasis on apparent information as well as on the symbolic or structural aspect of representations. Priority was given to the direct manipulation of symbolic representations (equations) with pattern matching of given (quantitative) information and subordination of qualitative information which often needs to be derived. The mathematical or graphical representation used to pose a problem was described by referring to its surface displays or the situation it presents instead of eliciting qualitative information. When presented with multiple representations, a minority (16 in 94) of these students were able to generate different forms of conceptual models which are in agreement with each other. In most cases (78 out of 94) discrepancies exist between freeze frame and mathematical together with graphical representations. Physics information were memorised by rote and kinematics graphs were drawn by pattern matching of the shapes of the graphs. However, correspondence was noted between freeze frame and linguistic representations. The visual conceptual model supports the generation of explanations with physics information corresponding to the physical model.

A minority (2%, 3 out of 157) of the sample was found to operate at the level of mental image. This category of students is described by their tendency to include diagrammatic representations which may however not be linked to the mathematical formulations used. Qualitative information derived was mainly inconsistent with the physical model. The representation employed to structure a problem was described or the situation it presents was highlighted instead of unpacking qualitative information. During the modelling process, the various representations generated were in conflict with each other. Discrepancies were noted between freeze frame, linguistic, graphical as well as mathematical representations. A description instead of an explanation was formulated, kinematics graphs were drawn by pattern matching of the shapes of the graphs and physics information was memorised by rote.

6. Discussion and conclusion

The present study appeals to two theoretical frameworks: a model-based view of physics and Johnson-Laird's (1987) cognitive framework of sense-making. As discussed in chapter 2, these two frameworks, illustrated in Figure 27, have common characteristics.

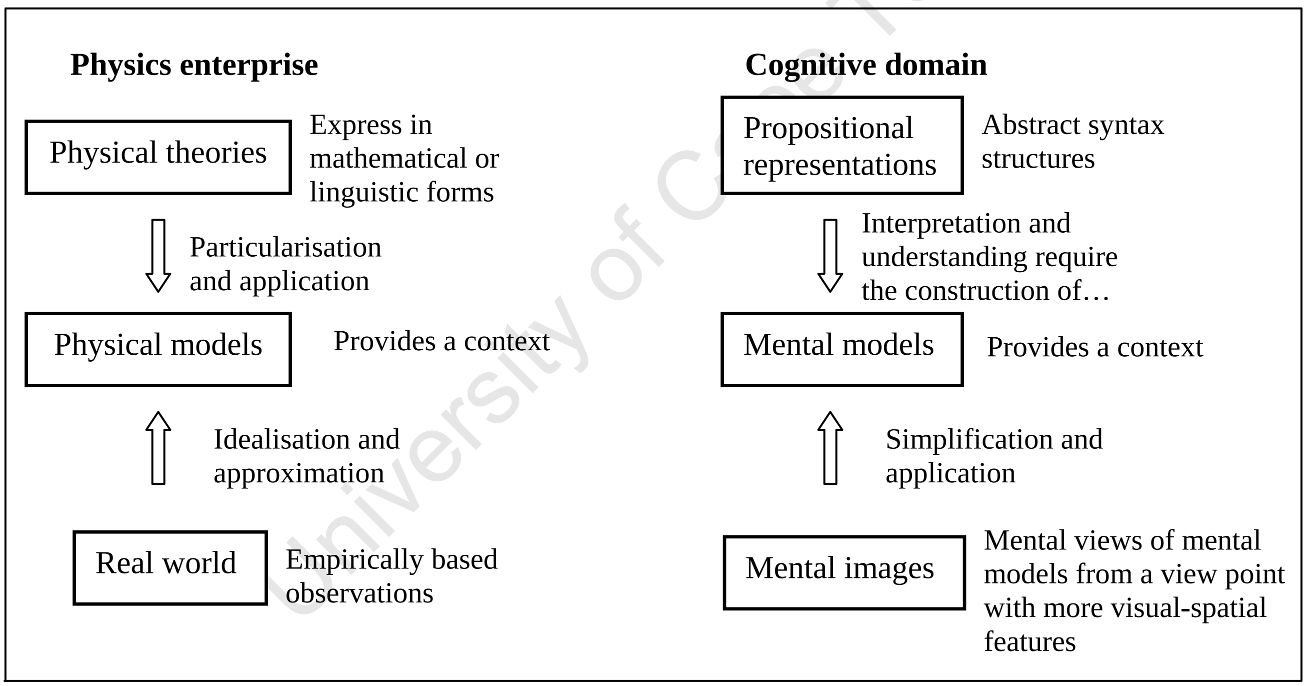


***Figure 27**: Framework for model-based view of physics and Johnson-Laird's cognitive framework of sense-making*

Physical theories and propositional mental representations constitute abstract syntactic structures, while real world and mental images are both based on experience and observation of actual phenomena. When not integrated within a context, these particular components, from both the physics and cognitive domain, are often meaningless. In the physics enterprise, the process of modelling occurs at the level of physical model which provides a context for the application of physical theories. Via physical models it is therefore possible to visualise and understand the physics principles underlying the physical theories. Within this framework, physical models can also be viewed to mediate between the real world and the physics world. In the cognitive domain, mental models allow the visualisation and comprehension of a situation or process under consideration. Mental models provide a context for the application of propositional representations and mental images and allow links to be made between them. Therefore, in the cognitive domain, modelling takes place at the level of the mental model.

The findings discussed in the first three sections (research questions 1 to 3) which follow pertain to the declarative aspects of the physics enterprise which is external in nature. The outcomes reported in section 6.1.4 (research question 4) are concerned with the cognitive domain. The students' categories of mental representations are inferred by relating their actions, when handling the various kinematics tasks, to the cognitive framework of sense-making.

6.1 Discussion of main outcomes of the study

6.1.1 Effectiveness of using multiple representations on students' problem-solving performance

The data indicate that the application of multiple representations as a problem solving strategy does not necessarily result in an improvement of the students' problem solving performance. Upon comparison of the same task posed in linguistic form but with different formats (question 10 from June class test and class work 16), a negligible difference was observed in the proportion of students providing the appropriate mathematical representation for the non-directive (1 in 145) and directive (2 in 145) versions of the written exercise. Moreover, for 3 of the 5 directive tasks (presence of freeze frame representations), less than half (around 43%) of the students were successful when generating graphical representations, while a negligible (around 1%) percentage of the students was able to formulate a suitable mathematical model.

The outcome is in line with findings from the work of Kohl and Finkelstein (2005) and DeLeone and Gire (2006), where it was found that the number of representations employed, sequentially, during problem solving did not impact on success rate. Moreover, consistent results emerged from studies performed in the area of cognitive science. The ineffectiveness of parallel multiple representations as a problem solving strategy was explained mainly on the grounds of "redundancy of information" (Kalyuga *et al.*, 1999) or the "split attention effect" (Chandler and Sweller, 1992) both governed by the cognitive load theory, and students' inability to relate and translate information (for example, Tabachneck *et al.*, 1994).

6.1.2 Effect of freeze frame representations for generating graphical and mathematical conceptual models

Positive outcomes emerged from the directed problem questions concerned with the construction of graphical models. For 2 of the 3 directed tasks (posed in diagrammatic and graphical forms) requiring the generation of kinematics graphs, it was found that the success rate was higher compared to when the written exercises are non-directive in nature (from Tables 5-3 and 5-5). In contrast, for only 1 of the 3 directed tasks dealing with the formulation of a mathematical model (class test question 4 with a linguistic format) a higher proportion of students was successful (from Table 5-13) compared to when freeze frame representations were not included in the task. However, the improvement in problem-solving performance noted for these 3 tasks cannot be solely attributed to the use of freeze frame representations. For the problem question dealing with the translation of information among motion graphs (class work 10), the simplistic nature of the shape of the graph, an increase in velocity as time changes, provided for the physical model may be the cause for the enhancement in performance. The class work was administered a week immediately after the students were taught to translate among graphs. They may be accustomed to the given shape of graph as well as the qualitative information involved. The improvement in performance noted for the task posed in diagrammatic form (class test question 6) may be due to its completion as a class test. At this stage, it is possible that the appropriate skills for drawing motion graphs have been acquired and conceptual understanding has been attained. In addition, it may be that the students formed mental templates for the shapes of graphs which are associated with one another. For the written exercise with a linguistic format concerned with the motion of a vertically propelled object and requiring a quantitative answer, the physics information (object's velocity at maximum height) may be memorised by rote and hence the higher success rate. Moreover, the majority of the directive tasks were completed as class tests and examination questions due to time constraint for formative assessment. Hence, it is also possible that the students performed better due to learning taking place rather than the consequence of using freeze frame representations.

Statistical analyses (tests for difference in proportion) show that, for two tasks, there is a link between the quality of freeze frame and graphical representations (class test question 6) as well as between the category of freeze frame and mathematical representations (class test question 4). However, it cannot be claimed that the construction of freeze frame representations which are qualitatively consistent with the physical model under consideration will result in the generation of appropriate graphical or mathematical models. On the one hand, a small proportion of students (around 1 in 3) with an inappropriate mental model for the non-directed written exercise posed in diagrammatic and linguistic form concerned with the generation of graphical and mathematical representations respectively presented a combination of correct freeze frame and graphical (from Table 5-3) or mathematical (from Table 5-14) representations when handling the directed tasks. On the other hand, it was noted that most (41 in 69) of the students constructing the correct mental model for the non-directed problem question with a diagrammatic format dealing with the generation of motion graphs presented a combination of appropriate freeze frame and graphical representations (Table 5-3). Moreover, among the students with the correct mental model for the non-directive task with a linguistic format requiring the generation of mathematical formulations, no major difference was observed in the proportion of students providing the appropriate mathematical expression and generating freeze frame representations corresponding (18 in 47) or conflicting (16 in 47) with the physical model (Table 5-14).

Consistency between freeze frame representations and either mathematical or graphical conceptual models reflects that an understanding of the physics involved in a situation is attained. However, the appropriate mathematical or graphical representations may have been generated independently of freeze frame representations. Moreover, discrepancies were observed which principally indicate that freeze frame representations and either mathematical or graphical conceptual models were considered on an individual basis. For directive tasks requiring the generation of motion graphs, on the one hand, inappropriate freeze frame representations were drawn but correct kinematics graphs were provided. On the other hand, the graphical representations were incorrect although freeze frame representations corresponded to the qualitative information presented by the physical model. The students were still unable to derive physics information principally about position and depict this particular information graphically. Confusion between qualitative information depicted by curves with positive and negative slopes persists. The lack of skills in drawing position-time graphs prevails with the particular motion graph generated mainly by memorisation of the shapes of graphs. Therefore, contrasting with studies in the field of kinematics where visualisation tools, mainly MBL devices, were used for enhancing graphing skills (for example Brasell, 1987; Simpson *et al.*, 2006), the utilisation of freeze frame representations was unsuccessful in supporting students' ability to draw kinematics graphs.

For directive tasks dealing with the formulation of mathematical expressions, mistakes which are primarily qualitative in nature were recorded although freeze frame representations are related to the qualitative information described or depicted by the physical model. The direction of acceleration for decreasing velocity was ignored, acceleration was included for constant velocity and 9.8 m s^{-2} was used as the acceleration for vehicles moving along a straight horizontal line. The students were also unable to derive information concerned with the velocity of objects when they reach the same position. Also, rote memorisation of physics information was apparent in instances where the required mathematical expression was provided but freeze frame representations were inappropriately drawn. Further evidence of freeze frame representations being disregarded was obtained when the two written exercises with different formats but same linguistic physical model were compared (class work 16 and June class test question 10). A shift from qualitatively and quantitatively incorrect mathematical models for the non-directive task to mainly qualitatively inappropriate mathematical expressions for the directive version of the problem question was observed.

It was also noted that when presented with non-directive tasks structured in linguistic form requesting for a value, an inappropriate mathematical representation was provided mainly when equations, with or without supporting sketches, were manipulated as opposed to when a diagrammatic representation was included. Additionally, regardless of whether the problem question is directive or non-directive in nature, the link was mainly between the diagrammatic and mathematical conceptual model. The category of diagram drawn (depiction of part of the motion or an irrelevant visualisation of the situation) was found to impact on the quality of the mathematical model generated in terms of quantitative information. The particular finding is consistent with the results gathered from studies investigating the effectiveness of using sequential multiple representations as a problem solving strategy. It was reported that the presence of an incorrect or incomplete visual depiction (such as diagrammatic representation, free body diagram) has an effect on the students' problem solving performance (for example, Rosengrant *et al.,* 2005; Kohl and Finkelstein, 2005; Kohl *et al.,* 2007). In the cognitive domain, the effect of the quality of external visualisation on students' construction of mental models was reported (Schnotz and Bannert, 2003; Rapp, 2007). Moreover, it was found that the students' inability to derive qualitative information (for either non-directive or directive tasks) led to the construction of a mathematical model which was both qualitatively and quantitatively incorrect. For instance, for class work 11 (posed in diagrammatic form requiring the generation of a value) concerned with the motion of a vertically propelled object, it was noted that the students failed to recognise and derive information for the object's velocity at maximum height. Consequently, either two sets of equations for different stages of motion were formulated and equated, or the equation of motion from the point of release to maximum height was constructed and set up to zero in order to determine the time taken for the object to reach its highest position. Additionally, for class work 16 and June class test question 10, both describing the motion of two vehicles and meeting at a certain point, when the students made no mention of the vehicles' final velocity upon reaching the same point, inappropriate values for positions (mainly final position) were used for solving the problem. Therefore, in cases where a diagram was provided, the particular visual representation does not seem to play a crucial role in the formulation of an appropriate mathematical model. It should be pointed out that the particular strategy was employed mainly when an incomplete diagram (depicting the objects at their initial positions only) was drawn.

6.1.3 Effect of freeze frame representations for eliciting qualitative information

It was revealed that freeze frame representations were ineffective as a bridge for translating qualitative information between diverse modes of representations. However, the study indicates that freeze frame representations did support the unpacking of qualitative information presented in mathematical and graphical forms. With the presence of freeze frame representations, the students' focus is shifted from describing the surface features of the conceptual model or the possible situation presented by the given representational mode, to explaining the concept involved. From Table 5-19, 38% v/s 11% formulated a description for the non-directive and directive tasks posed in mathematical form while 29% v/s 2% (Table 5-21) described the physical model for the non-directive and directive problem question with a graphical format. Moreover, the qualitative information highlighted in the explanation is related to the physical model (45% v/s 64% for the non-directed and directed written exercise with a mathematical format, Table 5-19, and 29% v/s 82% for the non-directive and directive problem question presented in graphical form, Table 5-21). The possible influence of learning leading to improved performance may be ruled out in this case as one of the directive tasks was completed during formative assessment (class work 10). It is assumed that the positive effect of freeze frame representations is related to the tasks dealing with the derivation of information and its expression in linguistic form instead of its translation into a visual or mathematical representation. Consequently, the need for mapping and depicting the same information, mainly in various visual modes is not required.

6.1.4 Profiles for the categories of mental representations for students in the GEPS physics programme

The construction of the profiles was based on the students' actions when dealing with the different tasks presented in the first three research questions concerned with students' ability to generate and handle various modes of external representations. Johnson-Laird's (1983) cognitive model of sense-making was applied in the physics domain, at the level of physical model, and related to the profiles for categorising the cohort's mental representations. The designed profiles therefore allow inferences to be made for the sample's categories of cognitive structures. The data suggest that most (60%) of the students registered for the GEPS physics course operate at the level of propositional mental representations even after exposure to the representation-rich kinematics course, while only 2% of the students have their mental representations associated with the category of "mental image". With reference to the cognitive framework shown in Figure 27, these two classifications of students do not construct a working model. Rather they manipulate the symbolic or the generated visual representations in isolation and hence display no understanding of the physical model. The predominance of propositional mental representation was also revealed from the work by Greca and Moreira (1997) which involved first year engineering students attempting problem questions based on electricity and magnetism. For the current study, kinematics tasks, requiring the generation of quantitative or qualitative solutions, were solved by students from educationally disadvantaged backgrounds.

The strategies employed by students with mental representations classified as mental image and propositional when attempting the various directed and non-directed problem questions can be compared to those of the "novice" students reported by Chi *et al.* (1981) in the physics domain and Kozma (2003) in the chemistry context. It was found that "novice" students concentrated mainly on superficial features, formulated descriptions based on direct observations and handled multiple representations on an individual basis. Moreover, the current study reveals that when engaged in the modelling process, students with cognitive structure categorised as propositional and mental image mostly generated diverse representations conflicting with each other. In principal, freeze frame representations play no crucial role in the generation of motion graphs and mathematical models. These students rely mainly on rote memorisation of physics information and pattern matching of the shapes of graphs for generating mathematical and graphical models respectively. A comparison can also be made with the results gathered from the study by Seufert (2003). It was revealed that the application of either the non-directive or the directive approach for teaching students with low prior content knowledge to map and translate information had a negative impact on their problem solving performance, as opposed to those who did not receive any instruction. One possible reason put forward for the outcome is that the students who did not receive any guidance may have focused on rote memorisation of ideas for attempting the tasks and hence had a better performance. In contrast those students who were taught with either category of instructional approach need to have an understanding of the concepts involved in order to be able to make links between representations.

Less than half (38%) of the cohort could be categorised as having constructed a mental model of any sort. These students are identified by their emphasis on relating symbolic representations to the diagrams drawn and on translating information between various modes of representations. Therefore, they are able to internally visualise the physical model under consideration. The generation of a mental model also provides a context for making links between the syntactic and structural aspect of the physical model. Moreover, if the students' mental model corresponds to scientifically accepted knowledge, reflected by the consistency between the various forms of conceptual models generated, then it was inferred that an appropriate mental model is constructed and there was comprehension of the physical model. Hence, the present analysis suggests that only 22% of the sample in the current study has an understanding of the various kinematics concepts highlighted in the different tasks. For students categorised with an appropriate mental model, freeze frame together with graphical or linguistic representations generated correspond to the qualitative information presented by the physical model. In contrast, for the 16% students with mental representations associated with the category of inappropriate mental model, discrepancies were noted between the various forms of conceptual model provided. Although freeze frame representations were in line with the qualitative information presented by the physical model, the motion graphs drawn and written explanations formulated were mostly inappropriate. However, regardless of the students' categorisation of mental representations, freeze frame and mathematical representations generated were in conflict.

6.1.5 Possible reasons for the ineffectiveness of using freeze frame representations as a representational bridge

The ineffectiveness of using freeze frame representations as a mediator when translating to mathematical representations may be a consequence of the students' prioritisation for the manipulation of symbolic representations and quantitative information. For problem questions where a value has to be determined, freeze frame representations, concerned with qualitative information, are consequently considered separately from mathematical expressions. The discrimination between qualitative and quantitative information was also apparent in cases where objects' respective initial and final positions were not taken into account when drawing freeze frame representations. Moreover, in the majority of cases, it was observed that unit vectors were not included in the mathematical models formulated.

The students' tendency to continue learning by rote memorisation, a technique employed at school level, may also have resulted in freeze frame representations not playing a prominent role as a representational bridge. It was noted that the students failed to visually depict the physics involved in a situation but succeeded in constructing mathematical expressions with the correct qualitative information. This condition was most apparent from class test question 4 (structured in linguistic form and requiring a quantitative solution). Freeze frame representations for the situation of a vertically propelled object were inappropriately drawn but the object's velocity at maximum height was correctly taken to be zero in the mathematical formulation. For tasks dealing with drawing motion graphs, reference may have been made to memorisation of the shapes of graphs for position, velocity and acceleration which are associated with one another for different situations. The required kinematics graphs were generated although freeze frame representations did not correspond to the qualitative information presented by the physical model. Rote memorisation of shapes of graphs was more apparent when position-time graphs were drawn for the case of an object moving with decreasing velocity. Either a Gaussian-like shape was provided or no distinction was made between curves with negative and positive slopes. For situations involving two stages of motion, either the negative slope curve was extended horizontally or a shape similar to the Gaussian was drawn for the position-time graph to portray a decrease in velocity for the second part of the motion.

Freeze frame representations may also have been ignored by the students as a result of their inability to identify and relate specific aspects between freeze frame and graphical representations conveying similar information. From studies implemented in the field of cognitive science dealing with the effectiveness of using parallel multiple representations, the inability to map and translate information within and across representations was found to be the major factor affecting problem-solving performance (Kozma, 2003; Ainsworth 2006). Although these two processes were explicitly taught no enhancement was recorded in problem-solving performance (for example Van der Meij and de Jong, 2006) as the mapping and translation processes were not willingly employed unless the students were prompted to do so and if utilised they were often performed in a routine and rote manner. Also, from the study by Seufert (2003), it was found that the mapping and translation processes are cognitively demanding and challenging for students with low prior content knowledge. The sample involved in the current study is academically and scientifically underprepared. The students have poor prior conceptual knowledge in kinematics. Even after the teaching intervention where they were explicitly taught to visually unpack problems structured in different representational forms, use various visual conceptual models and translate information between them, the students still had a poor problem-solving performance.

The inefficacy of using freeze frame representations as a representational bridge may also be a consequence of the students' surface level engagement with the particular visual representation. As reported by Kohl and Finkelstein (2008), the "novice" students in their study dealt with multiple representations in a routine manner and were uncertain how to use the different representations for solving a problem. When attempting paper and pencil tasks, the sample in the current study handled freeze frame representations superficially, limited to its depiction, annotation with vector representations and derivation of information regarding mainly the presence and the absence of acceleration. The construction of freeze frame representations was done in a mechanical way. For the foundation component of the GEPS physics programme, in the kinematics context, there are three basic standard motions, namely constant, increasing and decreasing velocity which by the end of the course the students are accustomed with. They become fluent in depicting these three motions using freeze frame representations but are still unable to generate the corresponding graphical representations when applied in different situations. It was also noted that for cases describing or depicting motion of two objects with different initial positions and meeting at the same point, freeze frame representations drawn did not take into account these quantitative information. If considered, the students failed to interpret the meaning conveyed by the portrayal of two freeze frames at the same position in terms of the objects' final velocity.

Van Heuvelen and Zou (2001) reported that students' level of experience with handling tasks in a particular topic and the conditions under which a problem question is attempted, that is, during tutorial sessions or examination influence the application of multiple representations. However, for the present study, no difference was observed in the students' use and handling of freeze frame representations irrespective of whether the written tasks were attempted as class tests, class works or as examination questions. Their experience with freeze frame representations and handling of similar type of problem questions seem not to impact on how freeze frame representations were dealt with. Regardless of whether the tasks (non-directive or directive) were completed while learning is taking place or at a later stage during examination or test, freeze frame representations were disregarded.

Therefore, the students' priority for manipulating quantitative information and symbolic representations, which according to Johnson-Laird's (1983) cognitive framework of sense-making can be categorised as propositional mental representation, may have had an effect on their use of freeze frame representations. Moreover, their approach to learning, level of engagement with freeze frame representations and inability to relate information between the particular representations and other forms of conceptual models (mathematical or graphical) may have resulted in the students not using freeze frame representations meaningfully as a representational bridge. Additional inhibitors identified from prior work, in the physics domain, probing the reasons behind the ineffectiveness of using multiple representations as a problem solving strategy, were mainly external in nature. These include the students' actions when attempting different types of tasks such as the quality of visual representations generated, the characteristics of the problem questions as well as the method employed for exposing students to the notion of multiple representations.

<u>6.2</u> Limitations of the study

The reasons formulated for the ineffectiveness of using freeze frame representations as a representational bridge were mostly based on inference and the literature review. The design of the instruments and consequently the type of data collected do not specifically allow the identification of the factors for disregarding the particular visual conceptual model. The students were not probed or interviewed about their actions when attempting the non-directive and directive tasks. However, there was no need for validation of inferences of written work through interviews because of continuous interactions between researchers and respondents during interactive class sessions and tutorials.

<u>6.3</u> Implications for teaching

The study shows that with the intervention of the representation-rich kinematics course, around 38% of the sample worked with a mental model of some sort compared to a negligible (between 2% and 7%) percentage of students upon joining the course. It may be argued that the modelling approach designed for handling the topic of kinematics, teaching the use of diverse representations with particular focus on freeze frame representations, has been beneficial. As described in the context section (chapter 3), the students involved in the study are from educationally disadvantaged backgrounds. By relating their actions, when handling the kinematics tasks administered as pre-tests, to the cognitive framework shown in Figure 27, it was possible to infer that these students' mental representations are predominantly propositional. They are exposed to the notion of modelling for the first time in the course. In the foundation component of the GEPS physics programme, the students are provided with the opportunity to generate and manipulate different modes of representations, present information using models which are either qualitative (such as free body diagram, graphical, freeze frame and vector representations) or quantitative (for example diagrammatic and mathematical representations) in nature and learn to unpack each form of representation although it is difficult for the majority of the learners to relate and translate information among them.

It is also advocated that the integration of freeze frame representations in the kinematics course of the foundation component of the GEPS physics programme is of key importance. The presence of the particular visual conceptual model in a task results in the students' engagement with qualitative information depicted or described by the physical model which is otherwise ignored. The modelling of motion in more concrete form via freeze frame representations facilitates its external visualisation and hence supports the interpretation and derivation of physics information. The presence or the absence of acceleration is recognised, and the magnitude and direction of acceleration together with velocity, annotated onto freeze frame representations, are consistent with the physics information presented by the physical model. It was also found that a higher proportion of students formulated explanations highlighting the appropriate qualitative information from the mathematical or graphical conceptual model compared to when freeze frame representations were not included in the task.

However, improvements need to be implemented in terms of teaching the use of freeze frame representations and students' level of engagement with the particular visual conceptual model. When constructing graphical representations it is not sufficient to engage with freeze frame representations only in terms of its interpretation, annotation with velocity and acceleration vectors and the application of the derived information for drawing the motion graphs. The particular strategy seems to appeal to memorisation of shapes of graphs according to the derived information which constitute principally of velocity and acceleration. From the results obtained, irrespective of the representational mode of the physical model, it was found that a higher percentage of the students were unable to generate position-time graphs, compared to motion graphs for velocity and acceleration. In certain instances, it was evident that an attempt was made to construct graphical representations for acceleration and position from the velocity-time graph which was first drawn. Therefore, it can be claimed that freeze frame representations were not integrally involved in the whole process. Students should be equipped with skills which will allow them to draw position-time graph from freeze frame representations. From the study by Rosenquist and McDermott (1987) which also involved students who are academically disadvantaged, ticker tape was applied as the mediator between the real world and the physics world. The spacing between the dots generated by the ticker tape was measured and used to draw the motion graphs. A similar strategy can be employed for the case of freeze frame representations. The notion of plotting the distance between the spacing of the object from freeze frame representations onto the position-time graphs for generating the required shape can be introduced. Once the graphical representation for position is drawn, it can be used together with the information derived from freeze frame representations to construct the corresponding velocity-time and acceleration-time graphs. Hence, the possibility of ignoring freeze frame representations and appealing to rote memorisation of visual representations may be reduced.

For problem questions dealing with quantitative solutions, it was observed that in very rare cases unit vectors were included in the mathematical expressions formulated. From freeze frame representations drawn, the absence or presence of acceleration together with its direction either for an increasing or decreasing velocity was recognised. However, the information is not fully included in the mathematical model. The notion that the task requires a quantitative solution may have resulted in the non-inclusion of unit vectors which are qualitative in nature. It may be argued that it makes no difference whether unit vectors are present or not included in the mathematical expression as the value obtained for acceleration will be negative although its direction is not specified. Moreover, even if quantitative information for position, velocity or time need to be determined and unit vectors are not included the required value will be obtained if the mathematical formulations are appropriately manipulated. However, if this particular strategy is encouraged, then the whole purpose of including freeze frame representations is defeated. The students involved in the study have their cognitive structure categorised as "propositional" and hence the demarcation between qualitative and quantitative information when dealing with mathematical representations is reinforced. There should be uniformity in the presentation of qualitative information be it in the form of mathematical or visual (graphical) representation. Motion graph for acceleration drawn for the case of decreasing velocity takes into account both the magnitude (which is shown to be constant) and the direction (depicted on the negative side of the coordinate system). If the direction is disregarded then the acceleration-time graph is considered to be inappropriate. Even if it is consequently employed for solving for a quantitative answer, the value obtained will be incorrect. Therefore, the same should be applied when dealing with mathematical expressions. Differences in the presentation and manipulation of qualitative information when dealing with mathematical and graphical representations may lead students to not acknowledging the benefits and purpose of employing freeze frame representations.

From the students' test and examination scripts gathered during piloting, it was observed that visual representations such as free body diagrams and diagrammatic representations tend to be spontaneously employed as an integral part of problem solving after instruction. This particular action may be explained by the consistent presence of these visual conceptual models across different contexts such as electrostatics, force, momentum and relative velocity which comprise the foundation component of the GEPS physics course. A similar strategy can be applied for the case of freeze frame representations for the students to recognise its importance and benefits. Another topic where the particular visual conceptual model can be included is in the context of waves. Freeze frame representations in the wave context may be more readily accepted since the students are exposed to the particular topic only at tertiary level. Hence, there is no interference of prior experience in terms of instruction and their solving of similar types of problems.

Students' views about what constitute physics, their previous instructions about kinematics, and the strategies they were taught for solving problems may have an effect on their adaptation to the representation-rich kinematics course. It may be difficult to shift their learning and problem solving techniques from rote memorisation, formula-centred strategies to the application of a model-based approach within a span of one month. The particular process may be facilitated by making the students aware of why applying the modelling strategy is beneficial to their own learning. To encourage the utilisation of diverse modes of representations, in particular freeze frame representations, incentives can be provided in terms of mark allocation with priority given to the presence as well as the application of different representations for problem solving.

6.4 Future work

Improvements in the design and methodological aspect of the study can be implemented in at least four ways. To answer research questions concerned with the effectiveness of using freeze frame representations, non-directive and directive tasks were selected for their equivalence of the physics ideas involved, but they could be selected and drafted also for a closer similarity in the level of cognitive demand and qualitative reasoning required. Thus, the possibility of extraneous variables influencing students' responses may be reduced. Additionally, instead of the whole sample completing non-directive and directive tasks at different times, the sample can be divided in two cohorts each completing one format of the problem question each time. Thus the data collection method is controlled for the effect of learning of kinematics concepts and representational skills over time. After all, a simultaneous administration of non-directive and directive tasks as well as the equal distribution of both problem formats to be completed as class works (formative assessment) and class tests / examination questions (summative assessment) will limit the influence of learning on the outcomes of the study. Finally, in addition to collecting data via paper and pencil tasks, selective individual interviews could have been conducted. They allow the possibility of probing deeper into the reasoning underlying the students' actions when handling the various tasks hence providing validation of the data interpretation.

Future research work on the application of freeze frame representations can be implemented in the context of waves. Unlike the kinematics topic, the wave context in the foundation component of the GEPS physics course is mainly qualitative in nature with the rare application of wave equations for generating quantitative solution. Moreover, there is no standardised physics information for motion which the students tend to memorise in different representational form, either graphically or numerically. The students also have no prior experience with the particular topic. Freeze frame representations can be used as an integral and intermediate step for introducing and teaching different concepts such as the principle of superposition, differentiating between the motion of a pulse and particle along a string, for highlighting the differences between standing and travelling waves, developing the notion about phase constant and also to understand the physics underlying the wave equations. An opportunity will be provided for the students to have an in-depth involvement with freeze frame representations by drawing the shapes and specific positions of pulses or waves at different instants of time. The various freeze frames need to be decoded and interpreted for deriving information, either qualitative or quantitative, which may consequently be expressed in linguistic or graphical form. In addition to emphasising the declarative aspects for conceptual development, laboratory activities can be designed highlighting some of the concepts involved. An investigation can be made around how students with learning based mainly on rote memorisation, mental representation categorised as propositional and no prior experience with the topic of waves, deal with freeze frame representations in a context requiring principally qualitative solutions. The relationship between their mental representation and their handling of diverse qualitative representations can be explored. In addition, the effectiveness of freeze frame representations for developing conceptual understanding in the wave context can be evaluated. A comparison can therefore be made with the outcomes gathered from the present study in kinematics and explanations can be generated for any similarities or differences in findings.

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Appendix A

Tasks administered as pre-tests

University of Cape Town
Department of Physics

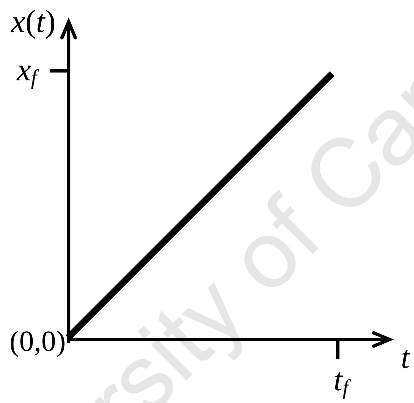
PHY1023H

Class work 1

Name: ____________________________ **Student number:** __________________

Answer the question below on this page. Use a pen.
Do not look at any other student's answers.

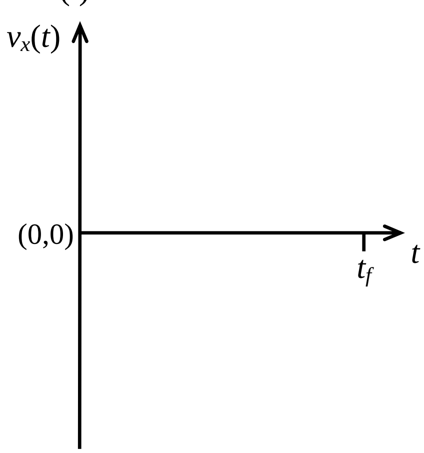
Consider the graph below showing the magnitude of the position as a function of time for an object moving on a horizontal track.



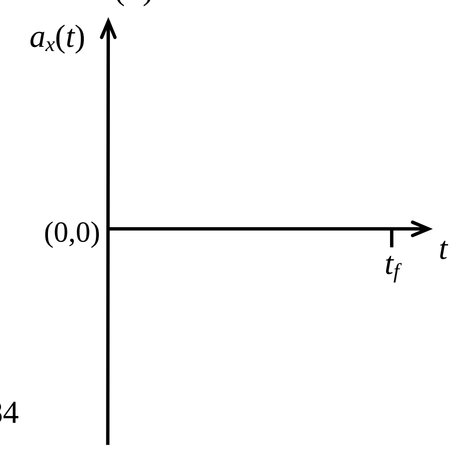
Sketch the corresponding graphs for the object of

(i) v_x versus time, and
(ii) a_x versus time

(i)



(ii)



University of Cape Town
Department of Physics

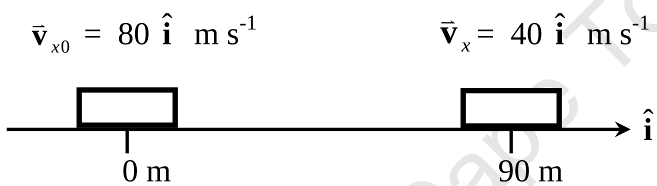
PHY1023H

Class work 2

Name: ______________________________ **Student number:** ____________________

Answer the question below on this page. Use a pen.
Do not look at any other student's answers.

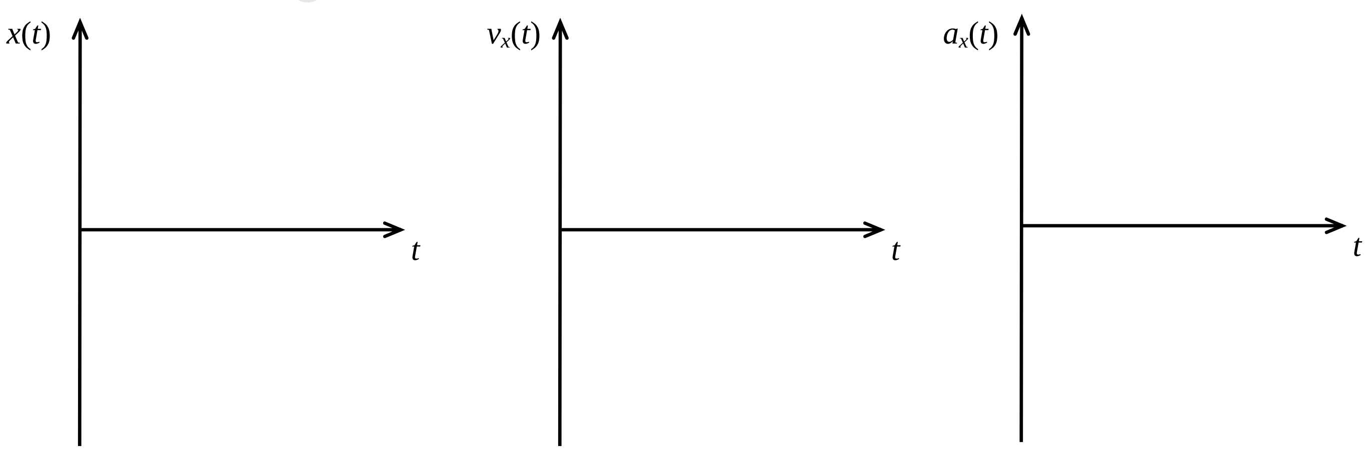
The diagram below shows the motion of a box:



On the axes below, sketch graphs for the motion of the box of
(i) x versus time
(ii) v_x versus time, and
(iii) a_x versus time

Label your graphs clearly.

(i) (ii) (iii)



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Class work 3

Name: ___________________________ **Student number:** _________________

Answer the question below on this page. Use a pen.
Do not look at any other student's answers.

A truck travelling at 60 m s^{-1} in a straight line slows down until it reaches a speed of 40 m s^{-1} while covering a distance of 70 m. It then maintains this speed for a further 10 m.
Determine the time taken by the truck to complete the whole journey.
Show all your steps clearly.

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Class work 4

Name: ___________________________ **Student number: _________________**

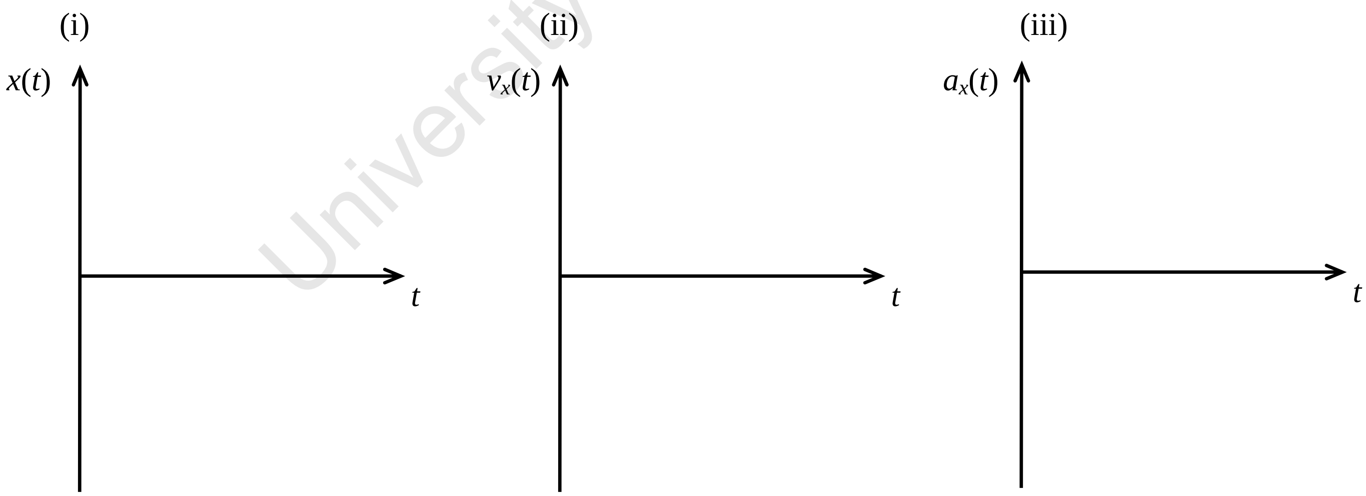
Answer the question below on this page. Use a pen.
Do not look at any other student's answers.

A toy car is moving along a horizontal track in a straight line. It starts from rest and speeds up uniformly until it reaches a speed of v m s^{-1}.

On the axes below, sketch graphs for the motion of the toy car of

(i) x versus time
(ii) v_x versus time, and
(iii) a_x versus time.

Label your graphs clearly.



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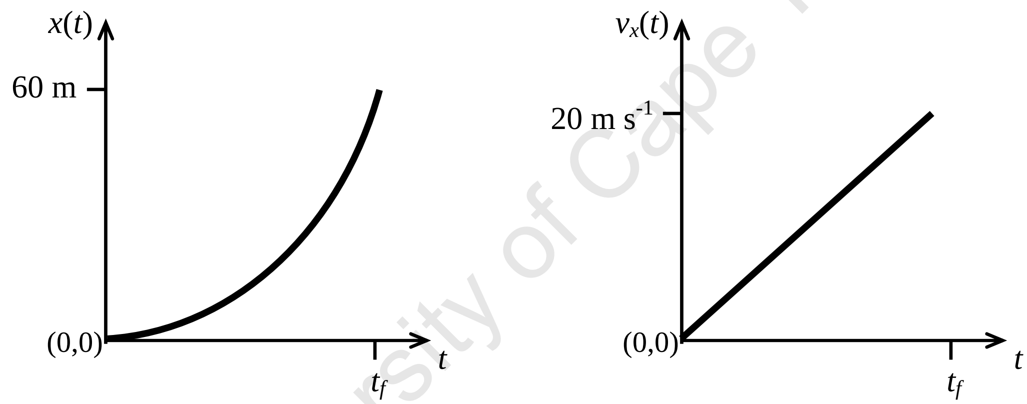
Class work 5

Name: ____________________________ Student number: __________________

Answer the question below on this page. Use a pen.
Do not look at any other student's answers.

Shown below are graphs of x versus time, and v_x versus time, for the motion of a car along a horizontal straight road:



Determine the time for the car to travel a distance of 60 m, i.e. determine t_f.
Show all your steps clearly.

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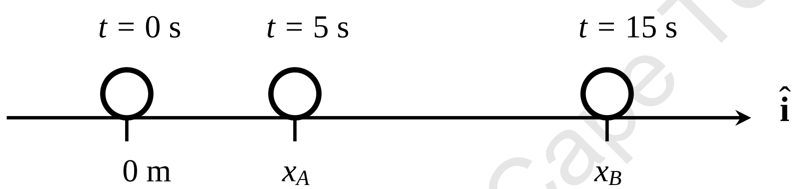
Class work 6

Name: ______________________________ **Student number:** ____________________

Answer the question below on this page. Use a pen.
Do not look at any other student's answers.

The diagram below describes the motion of a ball:



$\vec{\mathbf{v}}_x(t = 0) = 0\ \hat{\mathbf{i}}\ \text{m s}^{-1}$ $\quad \vec{\mathbf{v}}_x\ (t = 5) = 6\ \hat{\mathbf{i}}\ \text{m s}^{-1}$ $\quad \vec{\mathbf{v}}_x\ (t = 15) = 6\ \hat{\mathbf{i}}\ \text{m s}^{-1}$

Determine the total displacement of the ball. Show all your steps clearly.

Appendix B

Tasks administered during and after the intervention

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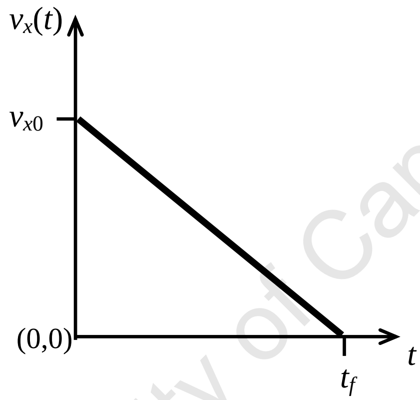
Class work 7

Name: ___________________________   **Student number: _________________**

Answer the questions below on this page.   Use a pen.
Do not look at any other student's answers.

Below is a graph of v_x versus time for an object moving in a straight line.



Write down (in words) everything you can say about the motion of the object from this graph.

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Class work 8

Name: ___________________________   Student number: _________________

Answer the question below on this page.   Use a pen.
Do not look at any other student's answers.

The equation of motion obtained for an object moving in a straight line is given by

$$x(\hat{\mathbf{i}}) \;=\; x_0(\hat{\mathbf{i}}) + v_{x0}(\hat{\mathbf{i}})\,t + \tfrac{1}{2}\,a_x(-\hat{\mathbf{i}})\,t^2$$

Write down (in words) everything you can say about the motion of the object from this equation.

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Class work 9

Name: ___________________________ Student number: _________________

Answer the question below on this page. Use a pen.
Do not look at any other student's answers.

A truck is travelling along a straight road. It maintains a speed of v_1 m s^{-1} from $t = 0$ s to $t = 1$ s and then slows down uniformly until a speed of v_2 m s^{-1} is reached at $t = 2$ s.

Sketch graphs for the motion of the truck of

(i) x versus time
(ii) v_x versus time, and
(iii) a_x versus time.

Label the graphs clearly.

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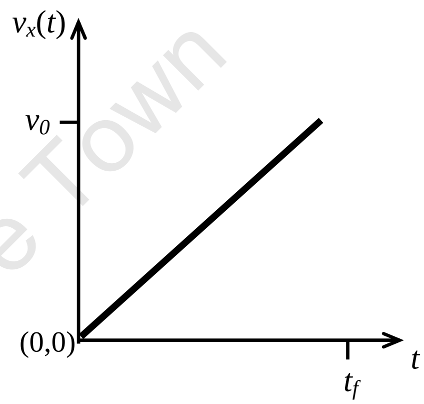
PHY1023H

Class work 10

Name: ____________________________ **Student number:** __________________

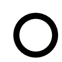
Answer the question below on this page. Use a pen.
Do not look at any other student's answers.

Consider the v_x versus time graph for the motion of an object moving on a horizontal track in a straight line.



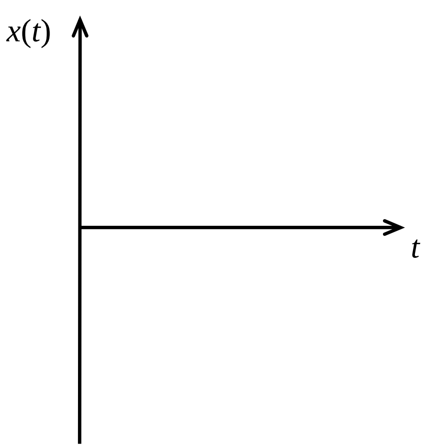
(a) Complete the "freeze frame" representation for the motion of the object from its initial to final positions:

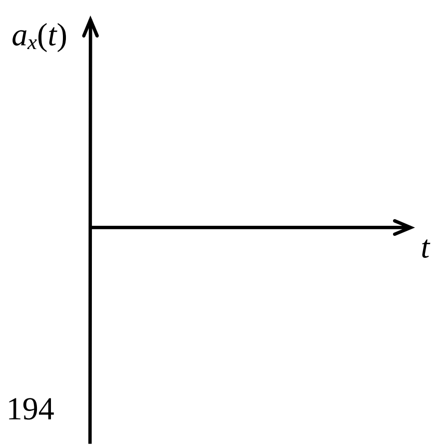
Initial position of object



Final position of object

(b) Sketch the x versus time, and a_x versus time, graphs for the object.





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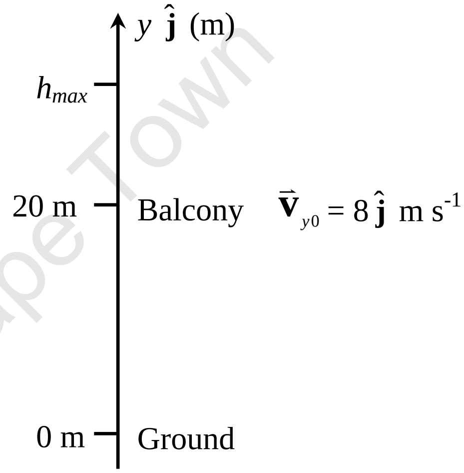
Class work 11

Name: ______________________________ **Student number:** ____________________

Answer the question below on this page. Use a pen.
Do not look at any other student's answers.

Consider the diagram alongside for the motion of a ball which is thrown from a balcony at a height 20 metres above the ground:



Determine the maximum height h_{max} reached by the ball. Show all your steps clearly.

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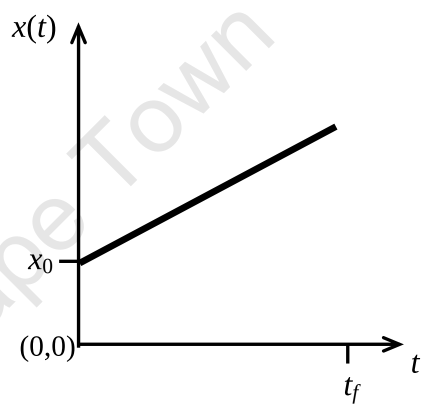
Class work 12

Name: ____________________________ **Student number:** __________________

Answer the question below on this page. Use a pen.
Do not look at any other student's answers.

Consider the graph of the magnitude of the position as a function of time for an object moving along a horizontal track in a straight line.



(a) Complete the "freeze frame" representation for the motion of the object from its initial to final positions.

Initial position of object

Final position of object

(b) Write down (in words) everything you can say about the motion of the object.

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Class work 13

Name: ___________________________ Student number: __________________

Answer the question below on this page. Use a pen.
Do not look at any other student's answers.

You are in a hot air balloon which is rising at a speed of 10 m s^{-1}. When you are 60 m above the ground you drop a ball over the side.

With what velocity does the ball hit the ground? Show all your steps clearly.

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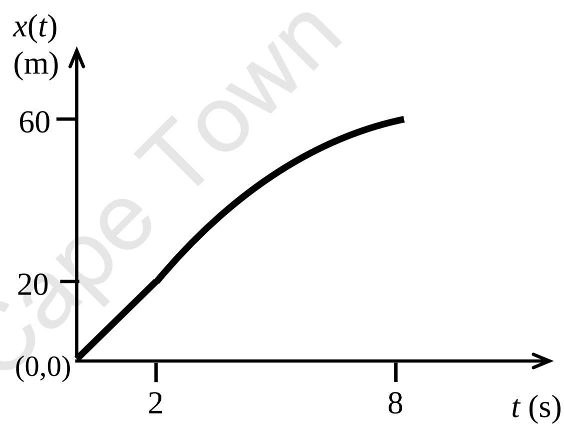
Class work 14

Name: ___________________________ **Student number:** _________________

Answer the question below on this page. Use a pen.
Do not look at any other student's answers.

The motion of car travelling along a straight road is described by the graph alongside:



Determine the velocity of the car when it is at 60 m. Show all your steps clearly.

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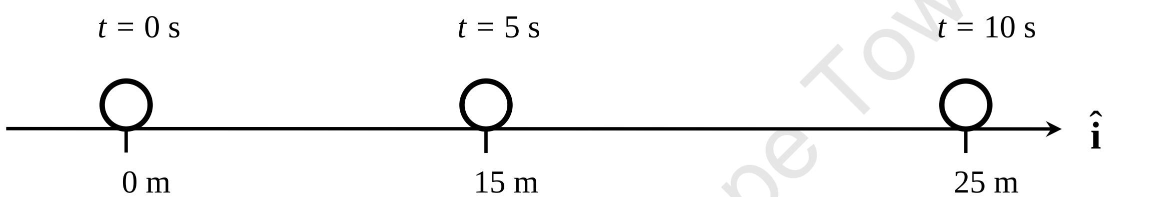
Class work 15

Name: ______________________________ **Student number:** ____________________

Answer the question below on this page. Use a pen.
Do not look at any other student's answers.

The diagram below describes the motion of a ball along a straight horizontal line:



$\vec{\mathbf{v}}_x(t = 0) = 20\ \hat{\mathbf{i}}\ \text{m s}^{-1}$ $\quad$ $\vec{\mathbf{v}}_x(t = 5) = 10\ \hat{\mathbf{i}}\ \text{m s}^{-1}$ $\quad$ $\vec{\mathbf{v}}_x(t = 10) = 15\ \hat{\mathbf{i}}\ \text{m s}^{-1}$

Sketch graphs for the ball's motion of

(i) x versus time
(ii) v_x versus time, and
(iii) a_x versus time.

Label the graphs clearly.

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Class work 16

Name: ___________________________ Student number: _________________

Answer the question below on this page. Use a pen.
Do not look at any other student's answers.

A train and a locomotive are travelling in the **same direction** along the same track. The train is travelling at 28 m s^{-1} when the driver of the train sees a locomotive 420 m ahead of him and immediately hits the brakes. The locomotive is travelling at 5 m s^{-1} throughout its motion.

Determine the magnitude and direction of acceleration of the train in order for it to **just touch** the locomotive. Show all your steps clearly.

Before you start to answer this question, explain the significance of the phrase "just touch" … how does this information help you?

Surname: **Initials:** **Student number:**

University of Cape Town : Department of Physics

PHY1023H

Class Test 2

19 May 2008

Time: 45 minutes **Full marks: 35**

Write your name in the box above and your student number on each page.

All rough work and answers should be written on this question paper.

Acceleration due to gravity = 9.8 ms^{-2} towards centre of the earth.

This test has 6 pages ... Check your copy now.

- - - - - - - - - - - - - - -

QUESTION 1

The equation of motion for a ball moving along a horizontal track is

$$v_x\,(\hat{\mathbf{i}}) \;=\; v_{x0}\,(\hat{\mathbf{i}}) + \;a_x\,(-\hat{\mathbf{i}})\;t$$

(a) Draw a "freeze frame" representation for the motion of the ball from its initial to final positions.

Initial position of ball

Final position of ball

(b) Write down (in words) everything you can say about the motion of the ball.

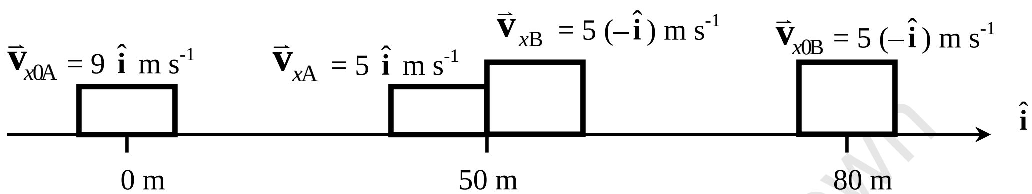
PHY1023H Class test 2

Student number:

QUESTION 2

Below is the diagram showing the motion of two blocks, **A** and **B**.

Block **A** starts from x = 0 m and block **B** starts from x = 80 m. They meet at position x = 50 m.



(a) Draw "freeze frame" representations for the motion of the two blocks. Label your diagrams clearly.

(b) Determine the magnitude and direction of acceleration of block **A**. Show all your steps clearly.

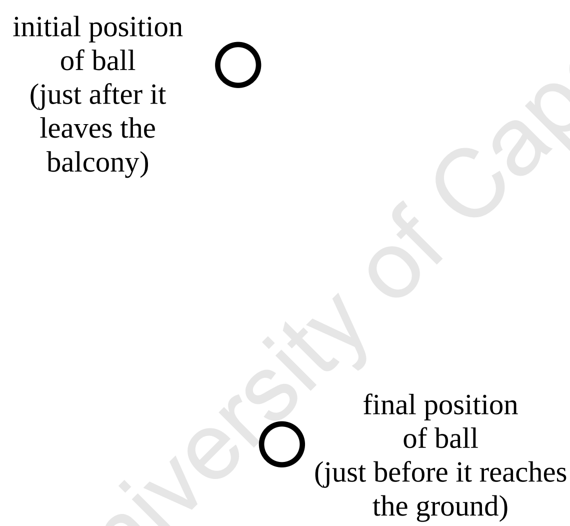
PHY1023H Class test 2

Student number:

QUESTION 4

A ball is thrown vertically upwards at 12 m s^{-1} from the balcony which is 7 m above the ground. The ball rises to a maximum height and then drops past the balcony to the ground.

(a) Photographs of the ball are taken at its start position (**<u>just after</u>** it leaves the balcony) and its end position (**<u>just before</u>** it reaches the ground). Draw the "freeze frame" representation for the motion of the object from its initial to final positions.



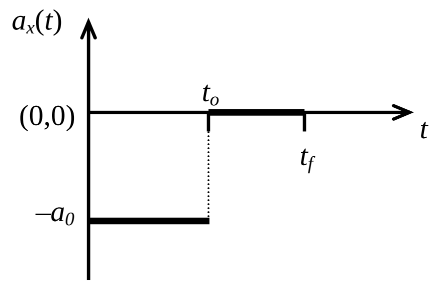
(b) Determine the maximum height reached by the ball. Show all your steps clearly.

Student number:

QUESTION 5

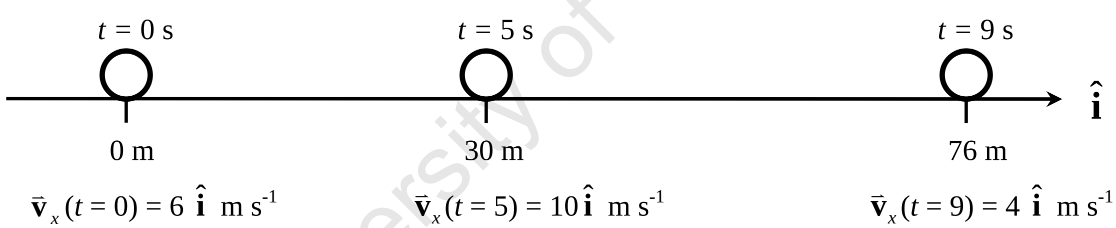
Consider the a_x versus time graph for an object moving in a straight line.
At $t = 0$, the object is at the origin moving with a velocity of v_0 in the positive x-direction.

Sketch the corresponding graphs of
(i) x versus time, and (ii) v_x versus time, for the motion of the object.
Label the graphs clearly.



QUESTION 6

The motion of a ball rolling along a straight road is described below:



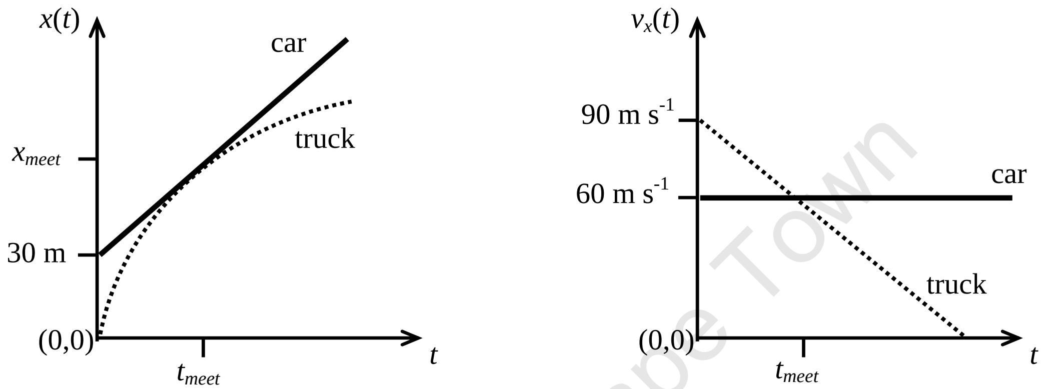
(a) Draw a possible "freeze frame" representation for the motion of the object from its initial to final positions.

(b) Sketch graphs of (i) x versus time, and (ii) v_x versus time, for the motion of the ball. Label the graphs clearly.

Student number:

QUESTION 7

The *x* versus time and v_x versus time graphs for the motion of a car and a truck are shown below. Both vehicles are travelling in the **same direction** and along a straight road.



(a) Draw, one below the other, "freeze frame" representations for the motion of the car and the truck. Label your diagrams clearly.

(b) Determine the time t_{meet} at which the car and the truck meet. Show all your steps clearly.

QUESTION 10

A car and a truck are travelling in the **same** direction. The car is travelling at 22 m s^{-1} when the driver sees a truck 40 m ahead of him and immediately hits the brakes. The acceleration of the car is such that the car ends up **just touching** the truck, which is travelling at 8 m s^{-1} throughout the incident.

In the questions below, choose the origin of your coordinate system to be at the initial position of the car with the $\hat{\mathbf{i}}$-unit vector pointing to the right.

(a) Sketch the physical situation and then draw the coordinate axis on your diagram. Indicate the initial and final positions of the car and the truck. Label the diagram clearly.

(b) Sketch, one below the other, the "freeze frame" representation for the car and the truck.

(c) Explain to Bugs what information about the motion of the car and / or the truck is being provided to you when you read the phrases below. Each answer should include one of more of the following words: *"position"; " velocity"; "acceleration"*.

"immediately hits the brakes" :

"the truck is travelling at 8 m s^{-1} throughout" :

"the car ends up **just touching** the truck" :

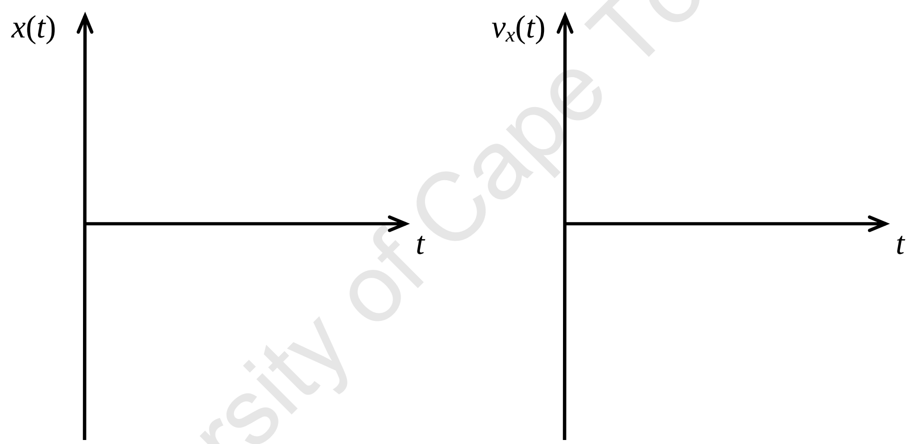
(d) What is the initial velocity of the car?

(e) What is the final velocity of the car?

(f) What is the inital velocity of the truck?

(g) What is the final velocity of the truck?

(h) On the axes below,sketch graphs of *x versus time* and v_x *versus time* for the car and the truck. Use the same set of axes for both the car and the truck. Label the graphs and indicate clearly on each graph the time at which the car and the truck meet.



(i) Determine the magnitude and direction of the acceleration of the car. Show all your steps clearly.

QUESTION 12

Train A and train B are **moving towards each other** on a flat horizontal track. Train B is intially 36 m ahead of train A, and moves with a constant speed of 4 m s^{-1} throughout the motion. Train A is intially moving at 16 m s^{-1}, and slows uniformly at a rate of 2 m s^{-2}.

Detemine the position at which the two trains smash into each other, relative to the initial position of Train A. Show all your steps clearly.

Appendix C

Data collection planner

Month / Date	Teaching	Data collection
April 7th	Average / instantaneous quantities / rate of change	Pre test: class work 1
8		
9		Pre test: class work 2
10		Pre test: class work 3
11	Tutorial	
14	End of rate of change / skills with drawing and interpreting photographs	Pre test: class work 4
15		Pre test: class work 5
16		
17		Pre test: class work 6
18	Tutorial	
21	Translation from graph to graph / motion in a straight line	class work 7
22		
23		class work 8
24		class work 9
25	Tutorial	
29	Graph to maths / free fall.	class work 10
30		
May 5th	Free fall / motion of two bodies	class work 11
6		
7		class work 12
8		class work 13
9	Tutorial	
12	Motion of two bodies / multiplication of vectors	class work 14
13		class work 15
14		
15		class work 16
16	Tutorial	
19	Class test 2	Class test question 1
		Class test question 3
		Class test question 4
		Class test question 5
		Class test question 6
		Class test question 7
20	Multiplication of vectors	
21		
30th May	June examination	June class test question 10
		June class test question 12

Appendix D

Coding schemes

(a) Graphical representations

G00: Not attempted
G01: Cannot be coded

P10: Correct position-time graph
P20: Incorrect position-time graph

V10: Correct velocity-time graph
V20: Incorrect velocity-time graph

A10: Correct acceleration-time graph
A20: Incorrect acceleration-time graph

(b) Freeze frame representations

F00: Not attempted
F10: Correct freeze frame representations
F20: Incorrect freeze frame representations

(c) Diagrammatic representations

D10: Presence of axis and depiction of whole situation
D11: Absence of axis and depiction of whole situation

D20: Presence of axis and depiction of part of the motion
D21: Absence of axis and depiction of part of the motion

D30: Diagram in the form of a rough sketch

D40: Diagram is inconsistent with situation presented

(d) Linguistic representations

L10: Explanation with appropriate physics information
L20: Explanation with inappropriate physics information
L30: Linguistic representation in the form of a description

(e) Categories of mistakes in mathematical expression

M10: Mistake is qualitative in nature
M11: Qualitative mistake associated with velocity
M12: Qualitative mistake associated with acceleration
M13: Qualitative mistake associated with direction
M14: Qualitative mistake associated with velocity and acceleration
M15: Qualitative mistake associated with velocity, acceleration and direction

M20: Mistake is quantitative in nature

M30: A combination of qualitative and quantitative mistakes

M40: Misinterpretation of notation
M41: Misinterpretation of notation together with qualitative mistakes
M42: Misinterpretation of notation together with quantitative mistakes

(f) Strategy used to attempt the problem

Use of…
S10: equations only
S20: diagrams and equations
S30: graphs only
S40: graphs and equations
S50: graph and diagram
S60: diagram and freeze frame representations
S70: freeze frame representations only

Appendix E
Full version of results

1. Generation of linguistic conceptual model from mathematical and graphical conceptual model

(a) Class work 8

	Descriptive statements	Total
1	**Appropriate interpretation of equation and derivation of information**	
1.1	Acceleration in $-\hat{\mathbf{i}}$ direction. Velocity decreases.	13
1.2	Acceleration is opposite to direction of motion / velocity. Velocity decreases.	10
1.3	Motion and/or velocity are in $\hat{\mathbf{i}}$ direction. Acceleration is in $-\hat{\mathbf{i}}$ / opposite to direction of velocity / motion. Velocity decreases.	22
1.4	Object moves from position x_0 to x in $\hat{\mathbf{i}}$ direction. Velocity has same direction. Acceleration is in $-\hat{\mathbf{i}}$ / opposite direction / opposite to direction of motion / velocity. Velocity decreases.	23
1.5	Acceleration is in $-\hat{\mathbf{i}}$ direction. Velocity decreases. Initial velocity is greater than zero. The displacement between the positions decreases with time.	1
1.6	Velocity decreases in $\hat{\mathbf{i}}$ direction.	3
2	**Inappropriate derivation of information.**	
2.1	Acceleration is opposite to direction of velocity. Velocity and acceleration decreases.	2
2.2	Position and velocity are in $\hat{\mathbf{i}}$ direction. Acceleration is in $-\hat{\mathbf{i}}$ direction and decreases	5
2.3	Velocity is decreasing in $\hat{\mathbf{i}}$ direction. Acceleration increases in $-\hat{\mathbf{i}}$ direction.	2
3	**Misinterpretation of equation**	
3.1	Acceleration is in $-\hat{\mathbf{i}}$ direction. Direction of motion changes and velocity decreases	6
3.1.1	Acceleration and initial velocity is zero. Motion is in $-\hat{\mathbf{i}}$ direction.	1
3.2	Acceleration is in $-\hat{\mathbf{i}}$ direction. Velocity increases as it is in $\hat{\mathbf{i}}$ direction	3
3.3	Velocity increases as it is in $\hat{\mathbf{i}}$ direction and then it slows down as the acceleration is in negative direction.	8
4	**No derivation of information**	
4.1	Identification of notations and directions of variables in equation	23
4.2	Description of motion of object in terms of position, velocity and acceleration together with the given directions.	39
	Total	**161**

(b) Class test question 1

	Descriptive statements	Total
1	**Freeze frame representations depict a decrease in spacing**	
1.1	*Appropriate interpretation of equation and derivation of information*	
1.1.1	Acceleration in -$\hat{\mathbf{i}}$ direction. Velocity decreases	29
1.1.2	Acceleration is opposite to direction of motion / velocity. Velocity decreases.	26
1.1.3	Motion and/or velocity in $\hat{\mathbf{i}}$ direction. Acceleration is in -$\hat{\mathbf{i}}$ / opposite to direction of velocity / motion. Velocity decreases.	32
1.1.4	Acceleration in -$\hat{\mathbf{i}}$ direction. Velocity decreases in $\hat{\mathbf{i}}$ direction. Spacing between individual positions decreases with time.	4
1.1.5	Velocity decreases in the $\hat{\mathbf{i}}$ direction	20
1.2	*Inappropriate derivation of information*	
1.2.1	Acceleration is in the -$\hat{\mathbf{i}}$ direction. Velocity increases	7
1.2.2	Acceleration increases in opposite / -$\hat{\mathbf{i}}$ direction. Velocity decreases in $\hat{\mathbf{i}}$ direction	3
1.2.3	Acceleration is in the - $\hat{\mathbf{i}}$ direction. Velocity and acceleration decreases	10
1.3	*Misinterpretation of physical model*	
1.3.1	Velocity increases in $\hat{\mathbf{i}}$ direction while acceleration increases in -$\hat{\mathbf{i}}$ direction	1
1.3.2	Velocity increases and then it slows down as the acceleration is in -$\hat{\mathbf{i}}$ direction.	5
1.3.3	Velocity decreases and there is a change in direction of motion	1
1.4	*No derivation of information*	
1.4.1	Description of the motion according to the variables	17
2	**Freeze frame representations depict an increase in spacing**	
2.1	*Appropriate interpretation of equation and derivation of information*	
2.1.1	Acceleration in -$\hat{\mathbf{i}}$ direction. Velocity decreases.	1
2.2	*Derived information consistent with freeze frame representations*	
2.2.1	Velocity increases	4
2.3	*Inappropriate derivation of information*	
2.3.1	Acceleration increases in -$\hat{\mathbf{i}}$ direction and velocity decreases	1
2.4	No derivation of information	1
3	**Freeze frame representations depict no change in spacing with time**	
3.1	*Derived information consistent with freeze frame representations*	
3.1.1	Velocity is constant and acceleration is zero	9
3.2	No derivation of information	1
4	**Freeze frame representations depict two stages of motion**	
4.1	*Derived information consistent with freeze frame representations*	
4.1.1	Velocity increases and then decreases	2
4.1.2	Velocity decreases and then remains constant	1
	Total	**175**

(c) Class work 7

	Descriptive statements	Total
1	**Appropriate interpretation of graph and derivation of information**	
1.1	Velocity decreases and (constant) acceleration is opposite to the direction of motion / in a negative direction	25
1.2	Velocity decreases with a constant negative acceleration	13
1.3	Velocity decreases and object moves with uniform deceleration	3
1.4	Gradient is negative, hence	
1.4.1	acceleration is opposite to direction of motion and velocity decreases	1
1.4.2	velocity decreases, acceleration is opposite to direction of motion and area under graph yields the displacement	1
1.4.3	velocity decreases	4
2	**Inappropriate derivation of information**	
2.1	Velocity decreases and acceleration is constant	10
2.2	Both velocity and acceleration decreases	15
2.3	Velocity decreases constantly and acceleration is zero	4
2.4	Gradient is negative hence velocity and acceleration decreases	3
2.5	Spacing between positions decreases and hence	
2.5.1	velocity and acceleration decreases	2
2.5.2	velocity decreases and acceleration is constant	1
3	**Use of ambiguous words**	
3.1	Spacing between positions decreases, velocity decreases and acceleration is negative	3
3.2	Velocity decreases and acceleration is negative	9
4	**Misinterpretation of physical model**	
4.1	Constant velocity and hence zero acceleration	1
4.2	Constant velocity in negative direction, and…	2
4.2.1	acceleration is zero	1
4.3	Direction of motion changes	1
5	**Misinterpretation of situation presented by physical model**	10
6	**No derivation of information**	
6.1	Use of the word decelerate for referring to a decrease in velocity	28
6.2	Velocity is decreasing / object slowing down	24
	Total	**161**

(d) Class work 12

	Descriptive statements	**Total**
1	**Freeze frame representations drawn with equal spacing**	
1.1	*Appropriate derivation of information*	
1.1.1	Constant velocity and hence zero acceleration	73
1.1.2	Constant increase in position / spacing between individual positions is constant / equal. Velocity is constant and acceleration is zero	13
1.1.3	Slope of position-time graph is constant and hence…	1
	- velocity is constant and acceleration is zero	1
	- velocity is constant	1
	- velocity is constant and displacement between individual position is constant	
1.1.4	Object moving with constant velocity	20
1.2	*Inappropriate derivation of information*	
1.2.1	Both velocity and acceleration is constant	2
1.2.2	Slope of position-time graph is constant and hence velocity and acceleration is constant	1
2	**Freeze frame representation drawn with an increase in spacing**	
2.1	*Appropriate derivation of information*	
2.1.1	Constant velocity and hence zero acceleration	2
2.2	*Derived information consistent with freeze frame representations*	
2.2.1	Velocity increases,	10
	- presence of constant acceleration	2
2.3	*Inappropriate derivation of information from freeze frame representations*	
2.3.1	Velocity and acceleration increases	1
2.3.2	Velocity is constant and acceleration increases	1
2.3.3	Velocity increases and acceleration is zero	1
2.4	*No derivation of information*	
2.4.1	Description of position-time graph	2
	Total	**131**

2. Generation of graphical conceptual model from diagrammatic conceptual model

(a) Class work 2

Position-time graph	Velocity-time graph	Acceleration-time graph	Total
Negatively sloped curve	Negatively sloped line	Horizontal line in negative part of coordinate system	10
		Motion in two stages / horizontal line in positive part of coordinate system / uncodeable	3
	Uncodeable	Horizontal line in negative part of coordinate system	1
Negatively sloped line	Negatively sloped line	Horizontal line in negative part of coordinate system	2
	Positively sloped line / Horizontal line in positive direction / Positively sloped curve / Motion in two stages / Uncodeable	Positively sloped line / horizontal line in positive part of coordinate system / Motion in two stages / uncodeable	13
Positively sloped line	Negatively sloped line	Horizontal line in negative part of coordinate system	17
		Horizontal line in positive part of coordinate system	30
		Negatively sloped line / positively sloped line / increasing negative slope / motion in two stages / uncodeable	22
	Increasing negative slope	Horizontal line in negative part of coordinate system	2
	Positively sloped line / Increasing negative slope / Horizontal line in positive direction / Motion in two stages	Motion in two stages / horizontal line in positive part of coordinate system / negatively sloped line / positively sloped line / Increasing negative slope / uncodeable	18
Positively sloped curve	Negatively sloped line	Horizontal line in negative part of coordinate system	6
	Positively sloped line	Horizontal line in positive part of coordinate system / uncodeable	4
	Increasing negative slope	Horizontal line in positive part of coordinate system	3
		Increasing negative slope / Positively sloped line / Uncodeable	4
No change in position	Negatively sloped line	Horizontal line in negative part of coordinate system	3
	Increasing negative slope / positively sloped line	Horizontal line in positive part of coordinate system / increasing negative slope / motion in two stages / negative slope line / Uncodeable	16
Uncodeable	Negatively sloped line	Horizontal line in negative part of coordinate system	2
	Increasing negative slope / Positively sloped line / Motion in two stages	Motion in two stages	3
		Horizontal line in negative part of coordinate system	1
		Motion in two stages / negatively sloped line	3
		Total	**163**

(b) Class work 15

Position-time graph	Velocity-time graph	Acceleration-time graph: Horizontal lines in negative and positive directions	Two horizontal lines in positive direction	One or three stages of motion	Cannot be coded	Total
Negatively and positively sloped curves	Negatively and positively sloped lines	72	1	0	1	74
	One stage of motion / Cannot be coded	1	0	1	0	2
Positively and negatively sloped curves	Negatively and positively sloped lines	4	0	3	1	8
Positively sloped line and negatively sloped curve		3	0	1	1	5
One or three stages motion	Negatively and positively sloped lines	19	4	10	4	37
	Cannot be coded	1	0	1	2	4
	One or three stages of motion	2	0	12	1	15
Cannot be coded	Negatively and positively sloped lines	16	1	0	1	18
	Total	118	6	28	11	**163**

(c) Class test question 6

Freeze frame representation	Velocity-time graph	Position-time graph	Total
Increase and decrease in spacing	Positively and negatively sloped lines	Positively and negatively sloped curves	70
		Positively sloped curve followed by a curve at tip of graph	21
		Positively and negatively sloped curves extending horizontally	9
		Depiction of one stage of motion / Gaussian / Uncodeables	21
	Motion in one or three stages / Gaussian / uncodeables	Positively and negatively sloped curves	3
		Positively sloped curve followed by a curve at tip of graph	2
		Depiction of three / one stage of motion / Gaussian / uncodeables	14
		Positively and negatively sloped curves extending horizontally	1
Decrease and increase in spacing	Positively and negatively sloped lines	Positively and negatively sloped curves	1
		Positively sloped curve followed by a curve at tip of graph	2
		Depiction of one stage of motion	1
	Uncodeables	Uncodeables	1
Increase in spacing	Positively and negatively sloped lines	Uncodeables	1
No change in spacing		Positively and negatively sloped curves	1
Freeze frame representations depict one stage of motion	Positively and negatively sloped lines	Positively and negatively sloped curves	2
		Positively sloped curve followed by a curve at tip of graph	4
		Depiction of one stage of motion / Gaussian / uncodeable	6
	Positively sloped line / Negatively slope line / motion in 3 stages / Gaussian / uncodeables	Positively sloped curve / Negatively sloped curve / Positively sloped curve followed by a curve at tip of graph / Positively sloped line / Depiction of one stage of motion	10
	Negatively sloped curve	Positively and negatively sloped curves	1
		Total	**171**

3. Translation among graphical conceptual models

(a) Class work 1

	Position-time graph							
Acceleration-time graph	Horizontal line in positive direction	Positively sloped line	Two stages of motion	Positively sloped curve	Negatively sloped line	Negatively sloped curve	Cannot be coded	**Total**
Horizontal line on *t* axis	26	2	0	0	0	0	1	29
Horizontal line in positive direction	9	58	5	7	5	4	4	92
Positively sloped line	3	2	1	5	0	0	1	12
Negatively sloped curve	0	5	0	0	1	0	1	7
Negatively sloped line	2	2	0	0	1	0	2	7
Cannot be coded	4	2	0	0	2	1	5	14
Total	44	71	6	12	9	5	14	**161**

(b) Class test question 5

Velocity-time graph	Position-time graph: Negatively sloped curve and positively sloped line	Positively sloped curve and line	Negatively sloped curve and horizontal line	One of stage motion / Gaussian shape	Cannot be coded	Total
Negative slope and horizontal line in positive direction	63	3	4	5	1	76
Negative slope and horizontal line in negative direction	7	1	0	2	4	14
Negative slope with horizontal line along $x = 0$	10	0	3	3	2	18
Positive slope and horizontal line	3	2	0	3	3	11
Positive slope with a change in direction and horizontal line	2	0	0	1	2	5
Negatively and positively sloped lines	2	0	0	1	4	7
Horizontal and negatively sloped line with change in direction	2	0	1	1	1	5
One stage of motion / cannot be coded	2	1	1	11	11	26
Negative slope with a change in direction and horizontal line	1	0	1	5	0	7
Total	92	7	10	32	28	**169**

(c) Class work 10

Freeze frame representation	Position-time graph	Acceleration-time graph: Horizontal line in positive direction	Horizontal line in negative direction	Horizontal line along $x = 0$	Positively sloped curve	Positively sloped line	Total
Increase in spacing with time	Positively sloped curve	124	0	1	0	1	126
	Negatively sloped curve	3	0	0	0	0	3
	Positively sloped line	0	0	0	2	0	2
	Motion in 2 stages	2	0	0	0	0	2
No change in spacing with time	Positively sloped curve	16	0	6	0	0	22
	Negatively sloped curve	1	0	0	0	0	1
	Positively sloped line	1	0	0	0	0	1
	Horizontal line	0	0	4	0	0	4
Decrease in spacing with time	Positively sloped curve	2	0	0	0	0	2
	Negatively sloped curve	1	0	0	0	0	1
	Negatively sloped line	1	0	0	0	0	1
	Decreasing negative slope	0	1	0	0	0	1
	Total	151	1	11	2	1	**166**

4. Generation of graphical conceptual model from linguistic conceptual model

(a) Class work 4

Position-time graph	Velocity-time graph	Acceleration-time graphs: Horizontal line in positive direction	Horizontal line along time axis	Positively sloped line	Different stages of motion / uncodeables	Total
Positively sloped curve	Positively sloped line	55	2	0	3	60
	Different stages of motion	6	0	1	4	11
Positively sloped line	Positively sloped line	16	0	2	5	23
	Horizontal line in positive direction	2	4	1	1	8
	Different stages of motion / uncodeable	17	0	1	9	27
	Positively sloped curve	4	0	2	0	6
Negatively sloped curve	Positively sloped line	3	0	0	1	4
	Different stages of motion	2	0	0	1	3
Horizontal line in positive direction	Positively sloped line	2	1	1	2	6
	Horizontal line in positive direction	0	0	2	0	2
	Different stages of motion	0	0	1	0	1
Negatively sloped line	Positively sloped line	4	0	0	0	4
	Horizontal line in positive direction	1	0	0	0	1
	Different stages of motion	0	0	2	0	2
Different stages of motion / uncodeable	Different stages of motion	2	0	0	7	9
	Positively sloped line	4	1	0	1	6
	Horizontal line in positive direction	0	0	1	0	1
	Total	118	8	14	34	**174**

(b) Class work 9

Position-time graph	Velocity-time graph	Acceleration-time graph: Zero and in negative direction	Positive and negative direction	Zero and linear decrease	Positive direction and zero	One or three stages / uncodeable	Total
Positively sloped line and negatively sloped curve	Constant and linear decrease	45	2	3	0	0	50
	Linear increase and decrease	2	0	0	0	0	2
	3 stages depiction / uncodeable	2	0	1	0	2	5
Positively and negatively sloped curves	Constant and linear decrease	4	0	0	3	0	7
	Linear increase and decrease	0	2	0	0	0	2
	Cannot be coded	0	2	0	1	2	5
Positively sloped line with curve at tip of graph	Constant and linear decrease	33	1	1	0	0	35
	Linear increase and decrease	1	0	0	0	1	2
	Linear increase and constant	0	0	0	0	1	1
	3 stages depiction / cannot be coded	0	1	0	0	1	2
Two positively sloped lines	Constant and linear decrease	3	0	1	0	1	5
	3 stage depiction / cannot be coded	1	0	0	0	2	3
One or three stages of motion / uncodeable	Constant and linear decrease	13	2	5	0	2	22
	Linear increase and constant	1	2	0	4	1	8
	Linear increase and decrease	0	1	0	0	2	3
	3 stages depiction / uncodeable	1	1	2	0	5	9
Positively sloped line and negatively sloped curve extending horizontally	Constant and linear decrease	9	0	0	0	0	9
	Linear increase and decrease	0	0	0	1	1	2
	Total	115	14	13	9	21	**172**

(c) June class test question 10

Freeze frame representation	Velocity-time graph	Position-time graph	Total
Depict vehicles' initial and meeting positions and a decrease and no change in spacing for car and truck	Negatively sloped and horizontal lines for car and truck motion	Negatively sloped curve and positively sloped line for car and truck. Meeting point is the tangent of the two graphs.	36
		Gaussian drawn and is tangential to positively sloped line for truck	3
		Positively sloped line and negatively sloped curve for truck and car. Meeting point is not tangential	8
		Positively sloped curve and line for car and truck	3
		Cannot be coded	2
	Positively sloped and horizontal lines for car and truck	Positively sloped curve and line for car and truck	2
		Positively sloped curve and line for car and truck. Initial position is 0 m for both vehicles	1
		Cannot be coded	1
	Cannot be coded	Cannot be coded	3
Depict a decrease and no change in spacing for car and truck. Quantitative information is not considered	Negatively sloped and horizontal lines for car and truck motion	Negatively sloped curve and positively sloped line for car and truck. Meeting point is the tangent of the two graphs.	39
		Gaussian drawn and is tangential to positively sloped line for truck	9
		Positively sloped line and negatively sloped curve for truck and car. Meeting point is not tangential.	11
		Positively sloped curve and line for car and truck.	9
		Negatively sloped curve and positively sloped line for car and truck and both initial positions are 0 m	20
		Cannot be coded	8
	Positive slope and horizontal lines for car and truck	Negatively sloped curve and positively sloped line for car and truck. Meeting point is the tangent of the two graphs.	2
		Positively sloped curve and line for car and truck.	2
		Negatively sloped curve and positively sloped line for car and truck. Initial position for both vehicles is 0 m	1
		Gaussian drawn and is tangential to positively sloped line for truck	1
	Cannot be coded	Negatively sloped curve and positively sloped line for car and truck and both initial positions are 0 m / cannot be coded	5
Depict an increase and no change in	Negatively sloped and horizontal	Negatively sloped curve and positively sloped line for car and truck. Meeting point is the tangent of the two graphs.	2

spacing for car and truck. Quantitative information may be considered	lines for car and truck	Positively sloped line and negatively sloped curve for truck and car. Meeting point is not tangential.	1
		Cannot be coded	1
	Positively sloped and horizontal lines for car and truck	Positively sloped curve and line for car and truck	3
		Negatively sloped curve and positively sloped line for car and truck. Meeting point is the tangent of the two graphs.	1
	Cannot be coded	Positively sloped curve and line for car and truck / cannot be coded	2
Depicts no change / increase in spacing for both vehicles	Negatively sloped and horizontal lines for car and truck motion	Negatively sloped curve and positively sloped line for car and truck. Meeting point is the tangent of the two graphs.	2
		Total	**178**

5. Generation of mathematical conceptual model from diagrammatic conceptual model

(a) Class work 6

	Descriptive statements	**Total**
1	**Use of equations**	
1.1	Equation focuses on given and required quantitative information with no consideration for qualitative information.	133
1.2	*Equation considers qualitative information with…*	
1.2.1	acceleration either 0 m s^{-2} or 9.8 m s^{-2} for the whole motion	8
1.2.2	recognising presence of acceleration which is inappropriately determined	5
2	**Use of graphs**	
2.1	Velocity-time graph with required shape. Area under graph gives displacement.	11
2.2	Velocity-time graph with inappropriate shape. Area under graph gives displacement.	5
3	**Presence of graphs and equations with no association between them. Equations focus on given and required quantitative information and…**	
3.1	graphs for one stage of motion are provided	3
3.2	velocity and acceleration graphs with required shape are provided	2
	Total	**167**

(b) Class work 11

	Descriptive statements	Total
1	**Equation focuses on given and derived quantitative and qualitative information with…**	
1.1	required values for initial and final velocity (at maximum height), acceleration and initial position together with the associated directions	57
1.2	inconsistencies in direction of variables	10
1.3	absence of direction of acceleration. Initial and final position is h_{max} and 20 m respectively	2
1.4	initial and final velocity (at maximum height) is 0 m s^{-1} and 8 m s^{-1} respectively, and	5
1.4.1	absence of direction for acceleration	2
1.4.2	absence of direction for acceleration and initial position is 0 m instead of 20 m	3
2	**No mention of velocity at maximum height (qualitative information) with…**	
2.1	*equation relating maximum height, acceleration, initial velocity and position equated to zero and…*	
2.1.1	appropriate directions for the variables	22
2.1.2	inconsistencies in directions of variables	13
2.2	*two equations are set up (balcony to maximum height and either from balcony to ground or ground to maximum height) which are equated and…*	
2.2.1	appropriate quantitative information and the associated directions for motion from balcony to maximum height	23
2.2.2	inconsistencies in direction of variables in the equation from balcony to maximum height	9
2.3	equation for motion to maximum height made up of appropriate quantitative information and the associated directions. Time taken for object to reach maximum height is obtained by using the relationship between speed, time and distance.	5
3	**Use of irrelevant equation(s).**	6
	Total	**157**

(c) Class test question 2

	Descriptive statements	Total
1	**Freeze frame representations depict a decrease and no change in spacing for block A and B. The blocks' initial and meeting positions are shown. It may be annotated with velocity and acceleration vectors. Mathematical model includes…**	
1.1	*appropriate magnitude and directions for position, velocity and acceleration for constant velocity,*	
1.1.1	direction of acceleration for decreasing velocity included.	3
1.1.2	direction of acceleration for decreasing velocity is ignored	75
1.2	Lack of knowledge for solving problem	2
2	**Freeze frame representations depict a decrease and no change in spacing for block A and B. It does not indicate blocks' initial and meeting points. It may be annotated with velocity and acceleration vectors. Mathematical model includes…**	
2.1	appropriate magnitude and directions for position, velocity and acceleration for constant velocity. Direction of acceleration for decreasing velocity is ignored	56
2.2	presence of acceleration for constant velocity and direction of acceleration for decreasing velocity ignored	1
2.3	application of irrelevant equations / lack of knowledge for solving problem	4
3	**Freeze frame representations do not indicate blocks' initial and meeting points. Either an increase and decrease in spacing for block B and A are depicted, or the inverse. Mathematical model does not…**	
3.1	include direction of acceleration for decreasing velocity	5
4	**Freeze frame representations do not indicate blocks' initial and meeting points. It depicts an increase and no change in spacing for block A and B. Mathematical model includes…**	
4.1	magnitude and direction of accelerations consistent with freeze frame representations.	3
5	**Freeze frame representations depict initial and meeting positions and an increase and decrease in spacing for block A and B. In the mathematical model…**	
5.1	direction of acceleration for decreasing velocity is ignored	2
5.2	acceleration for block A is 0 m s^{-2} / 9.8 m s^{-2}	2
6	**Freeze frame representations depict initial and meeting positions with either an increase and no change in spacing for block A and B or increase in spacing for both blocks. Mathematical model includes…**	
6.1	magnitude and direction of acceleration consistent with freeze frame representations.	3
7	**Freeze frame representations depict either decrease or no change in spacing for both blocks. Mathematical model includes…**	
7.1	direction of acceleration for decreasing velocity ignored/magnitude of acceleration inconsistent with freeze frame representations	3
	Total	**159**

6. Generation of mathematical conceptual model from graphical conceptual model

(a) Class work 5

	Descriptive statements	Total
1	**Use of equations**	
1.1	Shape of graph understood to indicate presence of acceleration. Quantitative information is related to the relevant notations in the equation	2
1.2	Equation focuses on given and required quantitative information with no consideration for qualitative information	104
1.3	Misinterpretation of notations $x(t)$ and x_0 or v_{x0} in equation. Acceleration is either 0 m s^{-2} or 9.8 m s^{-2} for increasing velocity	36
1.4	Acceleration is 9.8 m s^{-2}	3
1.5	Use of irrelevant equations.	13
2	**Use of graphical method**	
2.1	Area under velocity-time graph yields displacement	4
2.2	Gradient under the graphs yields time	1
3	**Presence of graphs and equations with no link between them. Equation does not consider qualitative information and…**	
3.1	area under velocity-time graph yields displacement	1
3.2	area under the velocity-time graph yields time	1
	Total	**165**

(b) Class work 14

	Descriptive statements	Total
1	**Use of equations**	
1.1	*Graph's shape depicts zero and presence of acceleration for the first and second stage of motion respectively. No mention is made about the direction of acceleration for a negative slope curve*	49
1.1.1	use of instantaneous instead of time interval for second stage of motion	13
1.1.2	for second stage of motion initial position is 0 m and use of instantaneous time	2
1.1.3	misinterpretation of velocity notation in equation	3
1.2	*The situation is considered as a whole with…*	
1.2.1	acceleration as 9.8 m s^{-2} and notation v_{x0} as the final velocity	7
1.2.2	initial and final positions as 0 m and 60 m and initial velocity is 0 m s^{-1} / notation v_{x0} is considered as the final velocity	10
1.3	Acceleration is 9.8 m s^{-2} for both or one stage of motion	12
1.4	*Initial velocity is 0 m s^{-1}, use of instantaneous time for second stage of motion and…*	
1.4.1	direction of acceleration for negative slope curve ignored	4
1.4.2	presence of acceleration for constant velocity with no mention of direction of acceleration for negative slope curve	22
1.5	*Presence of acceleration for constant velocity, no mention of direction of acceleration for negative slope curve and…*	4
1.5.1	instantaneous time used for second stage of motion.	6
1.6	Acceleration is 0 m s^{-2} for both stages of motion. Instantaneous time is used for second stage of motion	2
1.7	Equation focuses on given and required information with no consideration of qualitative information	12
2	**Graphical method**	
2.1	Area under position-time graph yields the velocity	4
	Total	**150**

(c) Class test question 7

	Descriptive statements	Total
1	**Freeze frame representations depict a decrease and no change in spacing for truck and car. It may be annotated with velocity and acceleration vectors. Vehicles' initial and meeting points are shown. Mathematical model includes…**	
1.1	acceleration as 9.8 m s^{-2} and no mention of vehicles' final velocity at meeting point	5
1.2	presence of acceleration for constant velocity and 0 m s^{-1} for velocity at meeting point	3
1.3	*direction of acceleration for decreasing velocity ignored and…*	5
1.3.1	no mention of vehicles' velocity at meeting point	6
1.3.2	velocity at meeting point is 0 m s^{-1}	6
1.4	direction of acceleration for decreasing velocity ignored. Vehicles' position and velocity at meeting point are 0 m and 0 m s^{-1} respectively	3
1.5	lack of knowledge how to solve problem	1
2	**Freeze frame representations depict a decrease and no change in spacing for truck and car. It may be annotated with velocity and acceleration vectors. Vehicles' initial and meeting points are not shown. Mathematical model includes…**	
2.1	appropriate magnitude and direction for position, velocity, acceleration for constant and decreasing velocity	1
2.2	*direction of acceleration for decreasing velocity ignored and…*	22
2.2.1	no mention of vehicles' velocity at meeting point	26
2.2.2	Velocity at meeting point is 0 m s^{-1}	32
2.3	acceleration is 9.8 m s^{-2} / absence of acceleration for decreasing velocity	7
2.4	presence of acceleration for constant velocity and direction of acceleration for decreasing velocity ignored	9
2.5	inappropriate values for initial and final positions. Presence of acceleration for constant velocity and direction of acceleration for decreasing velocity ignored. Vehicles' velocity at meeting point is 0 m s^{-1}	5
2.6	Lack of knowledge how to solve problem	4
3	**Freeze frame representations depict no change and increase in spacing for truck and car. It may be annotated with velocity and acceleration vectors. Vehicles' initial and meeting positions are not shown. Mathematical model does not indicate velocity at meeting point and includes…**	
3.1	magnitude and direction of acceleration correspond to freeze frame representations	4
3.2	acceleration is 9.8 m s^{-2}	1
	Total	**140**

7. Generation of mathematical conceptual model from linguistic conceptual model

(a) Class work 3

	Descriptive statements	Total
1	**Use of equations only**	
1.1	Equation focuses on given and required quantitative information with no consideration for qualitative information	90
1.2	Acceleration is 9.8 m s^{-2} or 10 m s^{-2}	13
2	**Use of equations and diagram with no link between them. In the equation used consideration is made to quantitative information only. Diagram…**	
2.1	depicts part of the motion and is annotated with given information. Displacement instead of individual positions is shown	15
2.2	depicts the whole situation and is annotated with given information. Displacement instead of the individual positions is shown.	15
2.3	includes an axis with depiction of the whole situation. Displacement instead of the individual positions is shown.	3
2.4	includes an axis with the depiction of part of the motion. It is annotated with given information.	2
2.5	is in the form of a rough sketch.	9
3	**Use of graphical method**	
3.1	Velocity-time graph with relevant shape. Area under graph yields displacement.	5
3.2	Velocity-distance graph with relevant shape. Area under graph yields time.	20
	Total	**172**

(b) Class work 13

	Descriptive statements	Total
1	**Use of equations only**	
1.1	Equation consists of relevant values for initial and final positions, initial velocity and acceleration together with the associated directions	7
1.2	Inappropriate values for initial and final positions which are either 0 m and 60 m or both 60 m. Notation v_{y0} may be misinterpreted as the final velocity	19
1.3	Value for initial velocity is 0 m s^{-1}	3
1.4	Inconsistencies in directions of variables	9
1.5	Application of inappropriate equations	4
2	**Presence of equation and diagram**	
2.1	*Diagram in the form of a rough sketch with…*	
2.1.1	relevant values for initial and final positions, initial velocity and acceleration together with the associated directions	10
2.1.2	inappropriate values for initial and final positions which are either 0 m and 60 m or both 60 m. Notation v_{y0} may be misinterpreted as the final velocity	7
2.1.3	value for initial velocity is 0 m s^{-1}	6
2.1.4	misinterpretation of v_{y0} as the final velocity	2
2.1.5	inconsistencies in directions of variables	11
2.1.6	use of inappropriate equations / lack of knowledge how to solve problem	3
2.2	*Diagram includes an axis. Arrows may be present to indicate direction of motion and / or acceleration due to gravity. Displacement instead of individual positions may be shown. Diagrammatic and mathematical model are linked with…*	
2.2.1	relevant values for positions, initial velocity and acceleration together with the associated directions	34
2.2.2	initial velocity is 0 m s^{-1}	11
2.2.3	inconsistencies in direction of variables	8
2.2.4	initial and final positions are both 60 m / 0 m and 60 m	9
2.2.5	misinterpretation of v_{y0} as the final velocity	4
2.2.6	application of irrelevant equations	1
2.3	*Diagram does not include an axis. Displacement, directions of motion and acceleration are shown. Mathematical model includes…*	
2.3.1	inappropriate values for initial and final positions	3
2.3.2	inconsistencies in directions of variables	2
2.3.3	initial velocity is 0 m s^{-1}	1
2.3.4	application of irrelevant equations	1
	Total	**155**

(c) Class work 16

	Descriptive statements	Total
1	**Use of equations only**	
1.1	*Direction of acceleration for decreasing velocity ignored and…*	11
1.1.1	velocity at meeting point is either 0 m s^{-1} or not mentioned	6
1.1.2	presence of acceleration for constant velocity and no mention of velocity at meeting point	2
1.2	*Direction of acceleration for decreasing velocity ignored, train's and locomotive's initial and final positions are either both 0 m and 420 m or 420 m and 0 m and 0 m and 420 m respectively and…*	11
1.2.1	velocity at meeting point is either not mentioned or is 28 m s^{-1} while initial velocity as 0 m s^{-1}	27
1.2.2	presence of acceleration for constant velocity and no mention of velocity at meeting point	7
1.3	Locomotive's initial and final position is either 0 m and 420 m or 420 m and 0 m with velocity at meeting point either not mentioned or is 28 m s^{-1} while initial velocity as 0 m s^{-1}	3
1.4	Lack of knowledge how to solve problem / use of irrelevant equations	3
2	**Presence of diagram and equations**	
2.1	*Diagram is in the form of a rough sketch. Mathematical model includes…*	
2.1.1	direction of acceleration for decreasing velocity ignored	1
	- velocity at meeting point either not mentioned or is 28 m s^{-1} while initial velocity is 0 m s^{-1}	2
2.1.2	Direction of acceleration for decreasing velocity ignored, train's and locomotive's initial and final positions are either both 0 m and 420 m or 420 m and 0 m and 0 m and 420 m respectively. Velocity at meeting point is not mentioned / is 0 m s^{-1} / is 28 m s^{-1}	10
	- presence of acceleration for constant velocity	1
2.2	*Diagram includes an axis and depicts part of the motion. The displacement instead of the vehicles' individual position may be shown. The mathematical model includes…*	
2.2.1	direction of acceleration for decreasing velocity ignored	6
	- velocity at meeting point is either not mentioned or is 0 m s^{-1}	12
	- presence of acceleration for constant velocity and velocity at meeting point is not mentioned / is 0 m s^{-1} / 28 m s^{-1} while initial velocity of 0 m s^{-1}	3
2.2.2	Direction of acceleration for decreasing velocity ignored, train's and locomotive's initial and final positions are either both 0 m and 420 m or 420 m and 0 m and 0 m and 420 m respectively	11
	- velocity at meeting point is either not mentioned / is 0 m s^{-1} / is 28 m s^{-1}	22
	- presence of acceleration for constant velocity	4
	- presence of acceleration for constant velocity and velocity at meeting point is either not mentioned or is 0 m s^{-1}	3
2.2.3	Application of irrelevant equations	2

2.3	*Diagram includes an axis and depicts the whole motion. Mathematical model includes…*	
2.3.1	Direction of acceleration for decreasing velocity included, absence of acceleration for constant velocity and appropriate quantitative information for the notations	1
2.3.2	direction of acceleration for decreasing velocity ignored	1
	- velocity at meeting point is either not mentioned or is 0 m s^{-1}	2
2.4	*Diagram includes an axis and depicts the vehicles approaching each other. Mathematical model includes…*	
2.4.1	direction of acceleration for decreasing velocity ignored. Locomotive's direction of velocity in $-\hat{\mathbf{i}}$ direction, and	5
	- velocity at meeting point is 0 m s^{-1}	4
	Total	**160**

(d) June class test question 12

	Descriptive statements	**Total**
1	**Use of only equations**	
1.1	Relevant variables for the notations and the associated directions. Absence of acceleration for constant velocity and direction of acceleration for decreasing velocity is included.	4
1.2	*Train B velocity direction is ignored, and*	16
1.2.1	initial position is 0 m	1
1.3	*Direction of acceleration for decreasing velocity ignored, and*	2
1.3.1	train B velocity direction is ignored	10
1.3.2	initial position for train A is 36 m an train B velocity direction is ignored	3
1.4	Application of inappropriate equations / lack of knowledge how to solve problem	4
2	**Use of equations and diagrams**	
2.1	*Diagram is in the form of a rough sketch. Mathematical model includes…*	
2.1.1	relevant variables for the notations and the associated directions. Absence of acceleration for constant velocity and direction of acceleration for decreasing velocity is included.	3
2.1.2	train B velocity direction is ignored	3
2.1.3	direction of acceleration for decreasing velocity ignored, and	1
	- train B velocity direction is ignored	3
	- train B velocity direction is ignored and initial position is in negative direction	1
2.2	*Diagram includes an axis and depicts part of the motion. The vehicles at their respective initial position are shown. Mathematical model includes…*	
2.2.1	relevant variables for the notations and the associated directions. Absence of acceleration for constant velocity and direction of acceleration for decreasing velocity is included	16
2.2.2	train B velocity direction is ignored, and	12

	- train A initial and final position is 0 m and 36 m, train B initial position is -36 m	2
2.2.3	direction of acceleration for decreasing velocity ignored, and	4
	- train B velocity direction is ignored and its initial position is – 36 m / train A position is 36 m / both trains initial position is 36 m	4
	- train B initial position is – 36 m	2
	- train B velocity direction ignored	4
2.2.4	Either final position for both trains is 0 m or train B initial position is – 36 m	5
2.2.5	Application of inappropriate equations / lack of knowledge how to solve problem	4
2.3	*Diagram includes an axis and depicts the whole motion. Mathematical model includes…*	
2.3.1	relevant variables for the notations and the associated directions. Absence of acceleration for constant velocity and direction of acceleration for decreasing velocity is included.	30
2.3.2	train B velocity direction is ignored, and	13
	- train B initial and final position is 0 m and 36 m	1
2.3.3	direction of acceleration for decreasing velocity ignored, and	7
	- train B velocity direction is ignored	7
2.3.4	train B initial position is 0 m / 36 ($-\hat{\mathbf{i}}$) m	4
2.4	*Diagram includes an axis and depicts the vehicles to move in the same direction. Their initial and meeting points are shown. The mathematical model includes…*	
2.4.1	relevant variables for the notations and associated directions according to the diagram. Direction of acceleration for decreasing velocity may be ignored.	6
	Total	**172**

(e) Class test question 4

	Descriptive statements	Total
1	**Presence of freeze frame and diagrammatic representations. Diagram includes an axis depicting the whole situation. It may be annotated with both given and derived information.**	
1.1	*Freeze frame representations depict a decrease in spacing from initial position to maximum height followed by an increase in spacing from maximum height to final position. Mathematical model includes…*	
1.1.1	values for variables and their associated directions consistent with choice of origin and direction for + $\hat{\mathbf{j}}$ respectively. Velocity at maximum height is 0 m s^{-1}	18
1.1.2	inconsistency in direction of variables	4
	- initial and final positions are 7 m and no mention of velocity at maximum height	1
1.1.3	no mention of velocity at maximum height and equation is equated to zero	2
1.1.4	use of irrelevant equations	4
1.2	*Freeze frame representations involve (i) no depiction of freeze frame at maximum height (ii) equal spacing for different stages or the whole motion (iii) a depiction for motion of a dropping object (iv) a depiction for horizontal motion. Mathematical model includes…*	

1.2.1	values for variables and their associated directions consistent with the choice of origin and direction for + $\hat{\mathbf{j}}$ respectively. Velocity at maximum height is 0 m s^{-1}	19
1.2.2	inconsistency in direction of variables	3
	- initial and final positions are 0 m and 7 m with no mention of velocity at maximum height	1
1.2.3	velocity at maximum height is not mentioned and mathematical expression equated to zero, and	8
	- inconsistency in direction of variables	3
1.2.4	Initial and final velocities are 0 m s^{-1} and 7 m s^{-1}. Initial position is 0 m	1
2	**Presence of rough sketch and freeze frame representations.**	
2.1	*Freeze frame representations indicate a decrease in spacing from initial position to maximum height followed by an increase in spacing from maximum height to final position. Mathematical model includes…*	
2.1.1	values for variables and their associated directions consistent with the choice of origin and direction for + $\hat{\mathbf{j}}$ respectively. Velocity at maximum height is 0 m s^{-1}	4
2.2	*Freeze frame representations involve (i) no depiction of freeze frame at maximum height (ii) a depiction for motion of a dropping object. Mathematical model includes…*	
2.2.1	values for variables and their associated directions consistent with the choice of origin and direction for + $\hat{\mathbf{j}}$ respectively. Velocity at maximum height is 0 m s^{-1}	6
2.2.2	no mention of velocity at maximum height and equation is equated to zero	4
3	**No diagram drawn. Presence of freeze frame representations only**	
3.1	*Freeze frame representation indicates a decrease in spacing from initial position to maximum height followed by an increase in spacing from maximum height to final position. Mathematical model includes…*	
3.1.1	values for variables and their associated directions consistent with the choice of origin and direction for + $\hat{\mathbf{j}}$ respectively. Velocity at maximum height is 0 m s^{-1}	29
3.1.2	no mention of the velocity at maximum height and mathematical expression equated to zero	5
3.1.3	inconsistency in direction of variables	1
3.1.4	Application of irrelevant equations	2
3.2	*Freeze frame representations include (i) presence of equal spacing between freeze frames for part or the whole motion (ii) no depiction of freeze frame at maximum height (iii) a depiction of an increase followed by a decrease in spacing until the maximum height is reached (iv) depiction for the motion of a dropping object or horizontal motion. Mathematical model includes…*	
3.2.1	values for variables and their associated directions consistent with the choice of origin and direction for + $\hat{\mathbf{j}}$ respectively. Velocity at maximum height is 0 m s^{-1}	24
3.2.2	no mention of velocity at maximum height and mathematical expression equated to zero	9
3.2.3	inconsistencies in directions of variables	6
	- no mention of velocity at maximum height and equation equated to zero	3
3.2.4	Application of irrelevant equations	2
4	**Presence of only diagrammatic representation indicating the direction of + $\hat{\mathbf{j}}$, the initial and final positions. Mathematical model includes…**	

4.1	no mention is made of velocity at maximum height and mathematical expression is equated to zero.	1
5	**Absence of diagrammatic and freeze frame representation.**	
5.1	The required mathematical expression is generated.	2
	Total	**162**

(f) June class test question 10

	Descriptive statements	**Total**
1	**Diagram includes an axis depicting the whole motion.**	
1.1	*Freeze frame representations depict a decrease and no change in spacing for car and truck. The vehicles' initial and meeting positions are shown. Mathematical model includes…*	
1.1.1	appropriate values for variables together with their associated directions	1
1.1.2	direction of acceleration for decreasing velocity ignored	34
	-velocity at meeting point is 0 m s^{-1} / 22 m s^{-1} while initial velocity is 8 m s^{-1} or 0 m s^{-1} / not mentioned	11
	-initial and final positions are 0 m and 40 m	2
	- velocity at meeting position not mentioned / 0 m s while initial velocity is 22 m s. Car's and truck's initial and final positions are 40 m and 0 m, and 0 m and 40 m respectively	2
1.1.3	Lack of knowledge how to solve problem / use of irrelevant equation	2
1.2	*Freeze frame representations depict a decrease and no change in spacing for car and truck. Quantitative information is not considered. Mathematical model includes…*	
1.2.1	appropriate values for variables together with their associated directions	1
1.2.2	direction of acceleration for decreasing velocity ignored	31
	- velocity at meeting point is 0 m s^{-1} / 22 m s^{-1} while initial velocity is 8 m s^{-1} or 0 m s^{-1} / not mentioned	6
	-presence of acceleration for constant velocity	2
	- velocity at meeting point not mentioned with initial and final positions either 40 m and 0 m or 0 m and 40 m	6
1.2.3	use of irrelevant equations / lack knowledge how to solve problem	3
1.3	*Freeze frame representations depict an increase and no change in spacing for the car and truck. Quantitative information may not be considered. Mathematical model includes…*	
1.3.1	direction of acceleration corresponds to freeze frame representations	3
	- initial and final velocity is either 0 m s^{-1} and 22 m s^{-1} or 0 m s^{-1} and 8 m s^{-1}	2
	- no mention of velocity at meeting point. Car and truck initial and final positions are 40 m and 0 m, and 0 m and 40 m	1
1.4	*Freeze frame representations depict no change in spacing for car and truck. Quantitative information is not considered. Mathematical model includes…*	

1.4.1	initial and final position for car is 0 m and 40 m with presence of acceleration	1
2	**Diagram includes an axis and depicts part of the motion, vehicles at their initial positions.**	
2.1	*Freeze frame representations depict a decrease and no change in spacing for car and truck. The vehicles' initial and meeting positions are shown. Mathematical model includes…*	
2.1.1	direction of acceleration for decreasing velocity ignored	2
2.2	*Freeze frame representations depict a decrease and no change in spacing for car and truck. Quantitative information is not considered. Mathematical model includes…*	
2.2.1	direction of acceleration for decreasing velocity ignored	14
	- velocity at meeting point not mentioned / is 0 m s^{-1} / 22 m s^{-1} while initial velocity is 8 m s^{-1}	6
	- velocity at meeting point not mentioned. Initial and final positions are 0 m and 40 m	10
	- initial and final positions are 0 m and 40 m	2
2.2.2	Lack of knowledge how to solve problems / use of irrelevant equation	2
2.3	*Freeze frame representations depict an increase and no change in spacing for car and truck respectively. Quantitative information is not considered. Mathematical model includes…*	
2.3.1	direction of acceleration corresponds to freeze frame representations. Final velocity is 22 m s^{-1} or 0 m s^{-1}	4
3	**Diagram includes an axis with the depiction of the meeting point at the truck's initial position**.	
3.1	*Freeze frame representations depict a decrease and no change in spacing for car and truck. Quantitative information is not considered. Mathematical model includes…*	
3.1.1	direction of acceleration for decreasing velocity ignored	1
	- velocity at meeting point not mentioned / 0 m s^{-1}. Car's final position is 40 m. Truck's initial position can either be 40 m or 0 m and final position is 40 m.	12
3.2	*Freeze frame representations depict an increase and no change in spacing for the car and truck. Mathematical model includes…*	
3.2.1	direction of acceleration consistent with freeze frame representations. Car final position is 40 m	1
4	**Diagram includes an axis and depicts the vehicles approaching each other.**	
4.1	*Freeze frame representation depicts the vehicles' initial and meeting points with a decrease and no change in spacing for car and truck. Mathematical model includes…*	
4.1.1	direction of acceleration for decreasing velocity ignored and direction of velocity in opposite direction is included	4
	Total	**166**