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A MATHEMATICAL APPROACH TO THE EVALUATION OF INTERNATIONAL DIVERSIFICATION FOR THE SOUTH AFRICAN INVESTOR

by

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A thesis prepared under the supervision of Professor J.F. Affleck-Graves in partial fulfilment of the requirements for the degree of Master of Science in Operations Research

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C H A P T E R 1

INTRODUCTION

In February 1983 Mr O P F Horwood (1983), South Africa's then Minister of Finance, made the following statement:

> *"With effect from 7 February 1983 exchange control over non-residents will be abolished. This implies the disappearance of the 'financial rand' and of the dual exchange rate system as it has existed in one form or another since exchange control over non-residents was first introduced in South Africa in 1961."*

Mr Horwood cited the main reason for the relaxation of exchange control as *"the recent quite remarkable improvement in South Africa's balance of payments and domestic financial situation."* This was evidenced by the sharp decline in South Africa's balance of payments deficit towards the end of 1982 due to a further decline in imports and a recovery in the gold price, and a substantial net inflow of private sector foreign capital in the form mainly of trade credits and loans. This led to an appreciation of the rand by almost 10 percent in terms of a weighted basket of currencies in the last half of 1982, and rapidly rising net foreign reserves (an increase of R3,6 billion (if valuation judgements are included) in the last half of 1982).

A further consequence has been a renewed acceleration of the annualised and seasonally adjusted rate of increase of the broad money supply, with the rate of increase for 1982 as 17,4 per cent. Long- and short-term interest rates declined sharply, and the Johannesburg Stock Exchange enjoyed considerable share price rises and turnover increases in the last half of 1982.

Since this announcement there has been much speculation on what further steps, if any, would take place in the process of relaxing exchange control.

The Director-General of Finance, Dr Joop de Loor (1983) has been quoted as saying that exchange control in South Africa is to be abolished completely and permanently. However, he added that since the Government had no intention of reversing these relaxations once instituted, it is likely to proceed in steps, sure that each one works before getting to the ultimate objective of a totally unrestrictive flow of funds to and from the country. Thus the abolition will not occur immediately, but rather over the next few years. Dr de Loor spelt out two main preconditions for exchange control relaxation:

(i) Net reserves will have to improve even more and more short-term foreign debt will have to be repaid before further relaxation takes place.

(ii) The new system of a unified rand and freedom for non-residents to withdraw investment funds must have time to settle in, and be subjected to rigorous testing.

The second step in the relaxation process took place on 5 September 1983 when the gold mines gained permission to be paid for their gold in US dollars, and they will be able to hold or exchange these as they see fit. A view expressed by Mr Wim Holtes (1983), Executive Director of the South African Foreign Trade Organisation is that the next probable step will be to allow the large institutions such as the life assurers, pension funds and mining houses to invest a portion of their assets abroad. Initially only long-term investment will be permitted to prevent speculation against the rand. Thereafter companies will be able to make portfolio investments outside of the country, and finally individual investors will be permitted to move their money in and out of South Africa at will.

Mr Holtes sees two main advantages to South Africa in relaxing the foreign money curbs. They are that fluctuations in exchange rates would be smoothed out, and that South Africa's reputation with the international business community would improve.

Initially it is envisaged by leading financiers that there will be a maximum proportion of funds permitted out of the country. Mr Marinus Daling (1983), Senior General Manager of Sanlam, forsees that *"a limit of 5 per cent of total assets would be a healthy maximum."* Financial analysts agree on this figure since institutions have to meet their liabilities in rands and thus it would be inadvisable to invest a significant percentage of their assets abroad.

If South Africans are allowed to invest their funds outside of this country an examination should be made of the alternatives available to them and the profit opportunities that arise from such investments. This thesis attempts to isolate the major markets for investment outside South Africa, and to determine what proportion of a South African investor's capital, if any, should be held in foreign securities under different possible restrictions laid down by the South African Reserve Bank.

The main argument advanced in favour of foreign investment is risk reduction through diversification. This has been shown by Solnik (1974), who states that *"movements in stock prices in different countries are almost unrelated ..... when securities of one country are doing worse than expected, another market is likely to be doing better, hence offsetting the losses. Simply by investing in stocks of different countries, the risk is drastically reduced."* It should be noted that this study employs *ex post* analysis. That is, past data is employed to test a theory. Much has been written on the value of such studies, but it is hoped that by determining those strategies that would have been sensible in the past had certain conditions prevailed, some insights into rational future strategies will be gained. Thus the results of the study will indicate what position an investor should have taken had these proposed relaxations in the present exchange control regulations existed over the time period of the data. For this reason the blocked rand, securities rand or financial rand discounts which existed

from after the Sharpville incident in 1961 when there was an outflow of capital funds and a decline in the gold and foreign exchange reserves, until Mr Horwood's announcement in February 1983, and which allowed foreign investors in South African securities to buy rands at a more favourable rate than that which was commercially available, will not be considered. This will be done despite the fact that they were in existence over the entire time period of the data.

In Chapter 2 the main alternative markets for the South African investor's funds are discussed from a general point of view. These include the major international stock exchanges and the world's commodity markets. The various securities chosen from these markets for the empirical studies are introduced.

Chapter 3 discusses the data in more detail, and various forms of summarising the data are presented and analysed. A theoretical discussion of the main parameters involved in selecting securities to create portfolios, and the Markowitz model for portfolio selection is presented in Chapter 4. An initial attempt to choose portfolios for the South African, US and UK investor using the Markowitz model appears in Chapter 5.

A more detailed examination of the range of annual portfolios that were applicable to both the rational investor bound by the current exchange control regulations and his counterpart who is free to exploit the international markets is presented in Chapter 6. Special attention is paid to the selection of an optimal portfolio.

The two sets of portfolios of interest i.e. those that were applicable to an exchange control restricted investor and those that were applicable to an investor with no restrictions placed on his foreign investments, are compared in Chapter 7 to obtain a measure of the cost to the local investor of the current restrictions. This is extended in Chapter 8 to quantify the maximum proportion of an investor's funds that should be allowed to leave the country. Conclusions are drawn in Chapter 9.

All computer-based work was performed on the University of Cape Town's SPERRY 1100 computer.

C H A P T E R 2

ALTERNATIVE INVESTMENT MEDIA AND DATA

2.1 Introduction

"Investors continue to compete in an effort to arrive at superior judgements. The likelihood of being consistently superior is apparently quite small, but the rewards for success can be enormous. Clearly it is a game worth winning, although it may not be a game worth playing."

This quote by Lorie and Hamilton (1973) is typical of the thoughts of many investment analysts, who strive to obtain more information about particular securities and thereby gain an edge on the other investors in the sense that they are better able to value the securities in question. Since the earliest days of trading investors have devoted attention to research into the relative merits and demerits of individual security ventures.

Trading in stocks has occurred since the sixteenth century in the hope of making the investor wealthy. The original investors were people who put up money to finance expeditions to the then unknown East - countries like India, Russia and the East Indies. The investor's reward depended wholly on the success or failure of the expedition to reach its destination and return safely with exotic wares

such as silks, gold, spices and so on, since their payout was a share of the profits gained from the subsequent sale of such goods. The advantage to the merchants was of course the 'sharing' of the risk.

The development of industry resulted in an even greater interest of investors willing to share in the risk for the potentially great rewards. As a result stock market analysts appeared who attempted to evaluate the possible gains to be had from various securities and to advise investors.

In the event of an abolition or relaxation of the current exchange control regulations a South African investor could consider an investment in stocks quoted on the numerous major stock exchanges of the world (including, of course, the Johannesburg Stock Exchange), an investment in bonds, the purchase of one or more commodities quoted on the large international commodity markets or an investment in some non-security asset such as real estate, stamps or art. In the next four sections these major spheres of investment are outlined.

2.2 The International Stock Exchanges

The oldest stock exchange in the world is the Amsterdam Stock Exchange which was established in 1602. The London and New York Stock Exchanges have, however, attracted most attention, mainly due to the rapid industrial development of these two countries in the eighteenth and nineteenth centuries. As a result, most of the research into the

behaviour of such markets and the movement of individual stock prices has emanated from these two countries.

Shares were traded in London as early as 1568, but it was not until 1773 that the London Stock Exchange was opened. Later a large number of provincial exchanges were established in the more important commercial centres. In March 1973 the seven British and one Irish Stock Exchange (in Dublin) were amalgamated into one unified exchange with a single set of rules, and 'floors' in each of the old centres. Security prices throughout the country tend to be the same on any particular day. The London 'floor' is second only to the New York Exchange in its volume of shares traded each year.

In New York, shares were probably traded as early as 1725, but it was only in 1792 that the first formal organisation of the New York Stock Exchange took place. This has now grown into by far the largest and most important market for common stocks in the world. Figures show that this exchange alone handles over two thirds of the market value of all shares in the United States. This amounted to some 17.5 billion shares listed in 1927 listings of common and preferred stock in 1426 companies in 1971.

Almost every industralised and most developing countries today have a stock exchange, and these exchanges play an important part in the economic life of these nations. Armstrong (1936) writes about stock exchanges *"the stock exchange as an institution has been evolved by time and*

perfected by experience ..... . It is the Citadel of Capital, the Temple of Values. It is the axle on which the whole financial structure of the Capitalistic System turns. It is the Bazaar of human effort and endeavour, the mart where man's courage, ingenuity and labour are marketed."

For the purpose of this study composite indices of both the New York Stock Exchange and the London Stock Exchange were employed. The Standard and Poor's Composite Index of 500 shares on the New York Stock Exchange and the Financial Times UK Actuaries Index consisting of 594 shares on the London Stock Exchange are indices which reflect the behaviour of these two markets and both indices are market capitalisation type indices. Other exchanges were not considered for reasons of data availability and relative unimportance. Furthermore almost every empirical study in the literature has been performed on one or both of these two exchanges, and the South African investor would thus be most likely to direct his attentions to these markets with which he is somewhat familiar, rather than for other world markets for which he has no 'feeling'.

2.3 The South African Share Market

A stock exchange was established in South Africa in November 1884 by one Benjamin Woollan, a year after gold was discovered on the Witwatersrand. This discovery caused many small businesses to spring up and a necessity for the

formation of the Johannesburg Stock Exchange (JSE). Today investors can trade in any of the 412 stocks* quoted on this exchange.

An attempt to select shares from the universe of all 412 stocks quoted on the JSE would involve the collection of an enormous amount of data, not all of which was available, and an unjustifiably vast amount of computer time. However, the JSE Actuaries Indices have been constructed so that each of the 34 sector indices has a continuous price history from January 1965. Figure 2.1 shows the structure of the sectoral and composite indices with the percentage contribution that each index made to its immediately superior composite index at the end of June 1980. It was decided to employ the first level composite indices as the universe of 'securities' available. Thus the following JSE Actuaries Indices were used:

JSE All Gold Index
JSE Coal Index
JSE Diamonds Index
JSE Metals and Minerals Index
JSE Mining Financials Index
JSE Financial Index
JSE Industrial Index.

Thus the JSE may be regarded as a 7 'security' market, where each 'security' is in fact a portfolio of similar shares aggregated into a composite index. The results in future chapters do not suffer from a loss of generality because of this approach.

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* As at 29 January 1982.

JSE ACTUARIES INDICES *

		% to Immediate Index					
1.	Gold - Rand	7,5%					
2.	Gold - Evander	4,7%	All				
3.	Gold - Klerksdorp	20,1%	Gold	67,4%			
4.	Gold - OFS	24,5%			All		
5.	Gold - W Wits	43,2%			Mining	48,4%	
6.	Coal			6,5%			
7.	Diamonds			16,5%			
8.	Platinum	51,5%	Metals and				
9.	Copper, Tin, Others	48,5%	Minerals	9,6%			
10.	Mining Holding			27,1%	Mining		
11.	Mining Houses			72,9%	Financials	20,0%	
12.	Inv. Trusts	13,1%					
13.	Insurance	13,0%					
14.	Property	11,3%	Financial	16,0%			
15.	Banks	62,6%					All
16.	Ind. Holding	19,5%					Shares
17.	Beverages	7,4%					100%
18.	Building	5,3%					
19.	Chemicals	22,1%					
20.	Clothing	2,1%			Industrial		
21.	Electrical	2,8%			&	31,6%	
22.	Engineering	6,0%			Financial		
23.	Fishing	0,5%					
24.	Food	5,3%					
25.	Furniture	2,8%	Industrial	84,0%			
26.	Motors	1,6%					
27.	Paper, Packaging	5,7%					
28.	Pharmaceutical	0,7%					
29.	Printing	0,4%					
30.	Steel	3,0%					
31.	Stores	6,2%					
32.	Sugar	4,0%					
33.	Tobacco	3,0%					
34.	Transport	1,6%					

* The indices have been expressed excluding Property Trusts which only entered in 1976.

Figure 2.1 The structure of sectoral and composite indices of the JSE

Data on all the composite indices were available from the JSE Public Relations Department Publication (1978), and these indices reflect the behaviour of the total market.

2.4 The Commodity Markets - Development and Characteristics

The trading of commodities has accompanied the growth of civilisation since its earliest periods. Although the ancient Greeks and Romans traded in commodities, the development of modern trading practices is best traced to medieval Europe. As early as the tenth century Venice had emerged as the major trading centre of Europe, dealing predominantly in luxury goods such as spices and cloths. Western Europe traded mostly in necessities, including furs, timber and honey. Trade between the two regions can be likened to today's situation between the more developed and less developed nations.

About the eleventh century the medieval 'fair' came into being, and this caused the number as well as the quantity of commodities traded to increase rapidly. Early fairs were held in the districts of Champagne and Flanders and were organised to function on a regular basis in existing market towns. The fair of Champagne, for example, consisted of six fairs spread throughout the year, each lasting from one to two months, and rotated among the four market towns of Lagny, Bar, Provins and Troyes.

These fairs became well established and certain trading

institutions and practices became standardised. Merchant associations were formed in which all merchants, foreign and domestic, co-operated with local government authorities to establish the dates and places of the fairs. Similarly, a code of conduct was drawn up which ensured ethical trading and this was enforced by a 'fair-court'. Written contracts of exchange, letters of credit, agreements on grading of merchandise etc were also covered in this code. Forward trading and postponed payments also appear to have been in existence.

The range of commodities continued to expand. Goods reaching the fairs from southern Europe were the more exotic, ranging from spices to oranges, aprictos, cotton or silk. Goods travelling from northern Europe and Germany were more substantial, including timber, grain, wool, cloth, potash, silver and iron. The commodity which came to dominate the trade was wool cloth. This became so important that the centre of its production, Bruges, also became the major commodity trading centre of Western Europe.

In the fifteenth century the local fairs declined in importance as industrialisation set in, and the major cities had urbanized to such an extent that they could provide permanent trade in most commodities. Bruges was overtaken as the leader in the cloth trade by Antwerp, but during the Reformation the major trading activity shifted to Amsterdam.

As cities began to grow in size and commercial importance, commodity trading began to take place in special markets known as 'bourses' in Europe and 'exchanges' in

England. These were meeting places where buyers and sellers could trade commodities and merchandise throughout the year. The exchanges generated so much financial activity that the Royal Exchange was opened in London by Elizabeth I in 1570. Within a hundred years London had become the commercial and financial centre of the world. The Royal Exchange never, however, supported the buying or selling of any negotiable securities. Such an exchange only began later in London.

As economic conditions improved and the volume of trading on the London exchanges increased, dealers began to specialise in the trade of individual commodities. Furthermore, forward transactions gained in importance and provided the market with the useful service of risk coverage. Risk had always been present, but this increased as markets became more distant. No protection was offered against the prices which a merchant might have to pay or charge for commodities purchased or sold. The forward contract fixed the price at the time of the deal which the buyer would have to pay even though delivery may only be months hence.

As dealers specialised the commodity markets split up but still tended to concentrate near the London docks where ships would unload their cargoes. These individual exchanges, taken together, became known as the London Commodity Exchange. Later, however, several exchanges moved from this location, for example the Corn Exchange and the Metal Exchange.

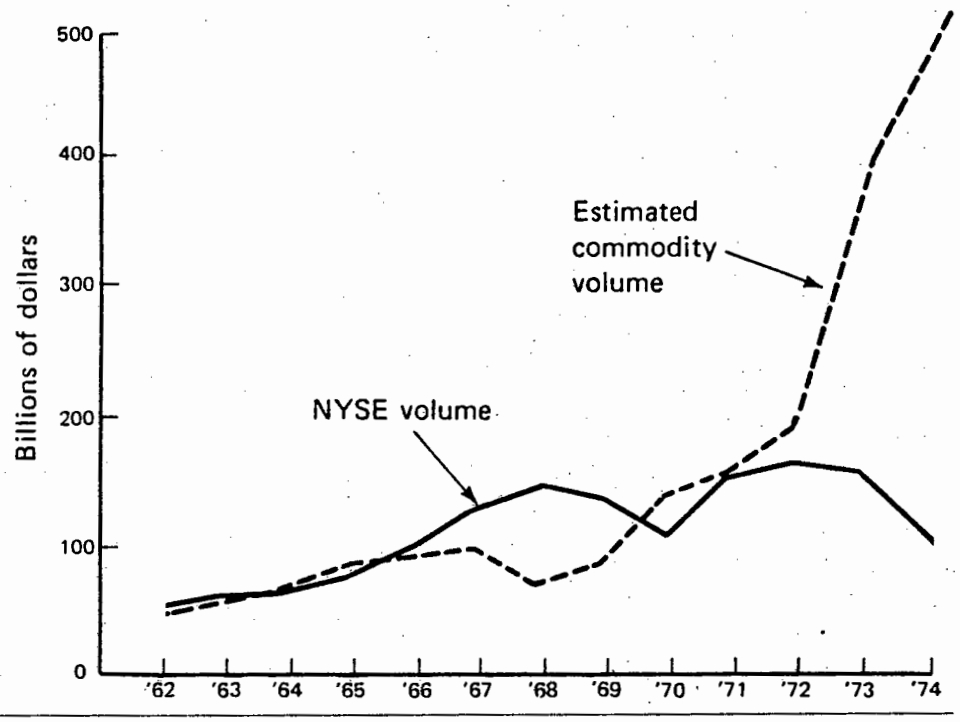
In the United States large central markets sprang up during the nineteenth century in the larger cities like Chicago, New York, New Orleans and St Louis. New York grew quickly, being on the main shipping route between the cotton plantations of the South and the mills in the North. As the port of New York grew trade in other commodities grew as well and an international commodity exchange was established on Wall Street. Chicago in the Mid-West became the agricultural centre, and the Chicago Board of Trade was established in 1848. The US commodity markets introduced the futures contract. Originally, a futures contract was simply a contract for the delivery of a specified quantity of a certain grade of commodity at an agreed price at a named future date, the price for immediate delivery being known as the 'spot' price. This in time led to the buying of 'futures'. If the spot price increased in the time period between the buying of a future and receiving the commodity, the purchaser would be better off. If the spot price decreased the purchaser would lose. The result is a steadying of prices for the buyers, who are in effect insuring against price fluctuations which are more frequent and wider in extent for raw material than for manufactured goods (Labys and Granger (1970)). For a futures market to exist in a commodity, it should be homogeneous and capable of being graded. There should also be an uncertain and competitive supply-demand relationship for the commodity, and numerous producers and users of the commodity. Futures trading began in Chicago in 1865 and by 1930 all major commodity markets dealt in these contracts. Table 2.1 shows the major commodity futures markets and the commodities they deal mainly in.

	Barley	Broilers Iced	Cattle (Live)	Citrus	Cocoa	Coffee	Copper	Corn	Cotton	Cottonseed Oil	Eggs (& Frozen)	Fishmeal	Flaxseed	Grain Sorghums	Hides	Hogs (Live)	Lead	Lumber	Mercury	Molasses	Oats	Palladium and Platinum	Plywood	Pork Bellies (Frozen)	Potatoes	Propane	Rapeseed	Rubber	Rye	Silver	Soybeans	Soybean Oil	Soybean Meal	Sugar	Tin	Wheat	Wool & Tops	Zinc
Chicago Board of Trade		•	•					•													•		•						•	•	•	•	•				•		
Chicago Mercantile Exchange			•								•					•		•						•	•													
Citrus Assoc. of N.Y. Cotton Exch.				•																																		
Commodity Exchange Inc., N.Y.							•								•		•		•							•		•		•					•			•	
Kansas City Board of Trade								•						•																					•			
London Cocoa Terminal Market Assoc.					•																																	
London Coffee Terminal Market						•																																
London Commodity Exchange																															•							
London Corn Trade Assn.	•							•																														
London Metal Exchange						•											•												•					•		•		•
London Sugar Terminal Market Ass'n.																																	•					
London Wool Terminal Market Ass'n.																																				•		
Minneapolis Grain Exchange														•																					•			
N.Y. Cocoa Exchange					•																																	
N.Y. Coffee & Sugar Exchange						•														•														•				
N.Y. Cotton Exchange									•																														
N.Y. Mercantile Exchange																						•	•		•													
N.Y. Produce Exchange										•		•																											
Paris Commodity Exchange																																	•	•				
Rubber Trade Ass'n. London																												•										
Sydney Greasy Wool Futures Market																																				•		
Winnipeg Grain Exchange	•		•										•								•					•		•									
Wool Assoc. of the N.Y. Cotton Exchange																																				•		

Table 2.1 The major commodity futures markets

Source: *How to Buy and Sell Commodities* (New York: Merrill Lynch, Pierce, Fenner & Smith, January 1970), p.55.

Commodity trading has increased substantially in recent years, and new commodities have been traded : gold was traded in its raw form for the first time in 1975. Figure 2.2 shows the estimated dollar volume of trading in commodities in the USA compared to the trading volume of shares on the New York Stock Exchange for the period 1962 to 1974.



Note: NYSE data are calendar year; commodity data are fiscal year.
Sources: *New York Stock Exchange 1974 Fact Book* (New York: NYSE, June 1974), p. 73; *SEC Statistical Bulletin* (Washington, D.C., U.S. Government Printing Office, February 1975), p. 137; and Association of Commodity Exchange Firms.

Figure 2.2 New York Stock Exchange and Commodity Trading Volumes, 1962-1974

In commodity trading the size of the contract is standardised. For example, most grain contracts in the United States are denominated in 5000 bushel units, whereas live cattle are sold by the 40 000 pounds of cattle and pork belly (unsliced bacon) by the 36 000 pounds of pork bellies. Furthermore the grade of the commodity is standardised. Due to certain seasonal characterisatics the delivery

may take place only at certain times of the year. For example, on the US markets wheat is delivered in July, September, December, March and May. July is the main harvest month for winter wheat, September for spring wheat; December is the last month of navigation on the Great Lakes, March is the first month of navigation; and May is the final month before the new crop harvest begins. Crops are seasonal and thus when the new crop is first marketed related prices will be lowest for the year; when the stocks of the crop are lowest just before harvest, prices will be highest. Certain commodities, like the metals, do not exhibit this seasonal tendency, and supply and demand depend mainly on government programs and policy.

Apart from the seasonal tendency of commodities, Robichek, Cohn and Pringle (1972) point out that there are also very large year-to-year variations in return. They also calculated the correlation coefficient between various commodity futures and other investment media such as common stocks and bonds, and found that these correlations were on the whole low and their signs were almost equally divided between positive and negative values. They conclude that *"the absence of significant positive correlation of returns between commodity futures and other investment media suggests that investment in commodity futures may provide some opportunity for portfolio diversification."*

The commodities quoted on the major commodity markets

of the world can be broadly divided into three groups:

- metals
- raw materials
- foodstuffs

Table 2.2 below shows the commodities chosen from each of the above three groups for empirical study in this thesis, as well as the units in which they are commonly quoted.

Commodity	Units
Metals	
lead	£/ton
tin	£/ton
zinc	£/ton
silver	p/troy oz*
aluminium	£/ton
antimony	£/ton
copper	£/ton
nickel	£/ton
platinium (official price)	£/troy oz*
platinium (free price)	£/troy oz*
gold	$/troy oz*
Raw materials	
wool	p/kg
cotton	£/ton
Foodstuffs	
sugar	£/ton
wheat	£/ton
maize	£/ton

Table 2.2 List of commodities chosen, and associated commonly-quoted units.

Key: $ = US Dollars; £ = British Pounds Sterling;
p = British pence = 0.01 £; *troy oz = 31,10348 gm.

The commodity prices are the London quotations, except the cotton price which was quoted in US c/lb until mid-August 1971, after which the quotations were in UK £/ton, and the gold price which is traditionally quoted in US $/troy oz. The cotton series was corrected so that all prices were in UK £/ton.

2.5 Corporate and Government Bonds

A bond or debenture is a form of fixed-interest debt issued by most governments and many corporations. The holder receives a fixed set of cash payoffs - an annual or semi-annual interest payment at the coupon rate, and the face value of the bond at maturity. The holder has no say in the running of the issuing corporation. The maturity of bonds may range between 3 months and 35 years.

Many studies have been undertaken to determine the risk and returns associated with bonds. All these studies have shown that yields on bonds tend to be much less than those on stocks, but the associated risks are much less. Amling (1965) concludes his study by saying "*it is apparent that the more speculative securities* [c.f. stocks] *provide the highest rate of return and variation, and therefore the risk is correspondingly high.*" Ibbotson and Sinquefield (1977) showed that over the period 1926 to 1976 the average annual rates of return on short-term US Treasury bills, long-term US government bonds and corporate bonds were 2.5 per cent,

3.5 per cent and 4.2 per cent respectively. By comparison the average annual rates of return on US stocks over the same period was 11.3 per cent. The reason why government debt is so safe (and hence offers low return) is well put by Roll and Buckley (1961) when they say (about US bonds) "..... *the obligation of the United States have been considered prime investments. Moody's Investors Service* rates them Aaa - top quality with maximum safety .... . The United States has never defaulted on its debt, and there appears to be no reason to expect any break in this tradition.*"

In a study on optimal international asset allocation by American institutions, Solnik and Noetzlin (1983) showed that the major stock markets on which institutional investors trade generated compound annual gains ranging between 10% and 25%, broadly outperforming bond markets. However, associated with these large gains are correspondingly large risks - something not associated with government bonds if they are held to maturity. This thesis is only considering risky investments and for this reason bonds will not be included in the universe of securities.

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*Moody's Investor Service is a firm supplying statistical information regarding various investments to enable investors to assess their prospects. A bond rated Aaa is judged to be of the best quality with the smallest degree of risk. They are generally referred to as "gilts".

2.6 Other Investments

Apart from common stocks, bonds and commodities, the investor can invest in many other forms of non-security assets, such as real estate, stamps, art or antiques. There are several problems when investing in these assets since it is often extremely difficult to value each piece of land, each painting etc. since not all pieces of land or all paintings are identical. Furthermore transaction costs tend to be relatively much higher than is the case for common stocks and thus the asset must normally be held for a longer period of time than is true for stocks. Not only is the return on these non-security assets difficult to calculate since the asset must usually be sold to produce a final accounting, but non-monetary returns are often very important. Winjum and Winjum (1974) put it this way: *"Although art objects do not provide an annual cash flow, the psychic income derived from possessing them can be enormous. The day-to-day experience of living with fine objects is extremely rewarding.* [The] *psychic rewards ..... [may] become more important than* [the] *original objective of capital appreciation."*

Postage stamps appear to be a good hedge against inflation and in recent years have shown rapid appreciation. Studies on US stamps in recent years (see Shepherd (1972)) indicate that they have appreciated by over 10 per cent annually since the early 1960's, and by 5 per cent annually since 1949. Rare stamps performed even better, with average

annual returns close to 50 per cent. Although recent rates of return have been high, the long-term historical rates of return are not especially high relative to the return earned on common stocks during the same period. However, the positive rates of return do indicate potential profit in stamp collecting. Examples of stamps commanding high prices include the one-of-a kind British Guiana 1c Magenta of 1856 sold for $280 000 to a syndicate of businessmen, and the US Air Mail 1918 24c Inverts which command $47 000 a piece.

"[Art and antiques] *are a good investment compared with other investments, including real estate and the stock market*" according to Rush (1961). The rates of return on these investments have generally been good, although risk and return problems are clearly present. The Times-Sotheby Index (an art index prepared by the London Times in conjunction with one of England's leading auction houses) for 1950 to 1970 showed that most categories of art were able to show a performance superior to that of the average mutual fund. This may be due to the fact that the supply of genuine antiques and old paintings is fixed, yet demand has increased. Furthermore, museums have bought or been donated art pieces, and so in reality there exists a "*limited and declining supply and increasing demand - the almost perfect investment situation*" according to Stevenson and Jennings (1976).

The investment in real estate is attractive since equity is built up over a number of years as the mortgage is

repaid and it is very likely that the property will appreciate in value since new land is not being produced. Furthermore there may be significant tax advantages for these investors. However, again it may be difficult to calculate rates of return from real estate ownership since there are many non-quantifiable benefits that may result, such as the psychological and personal satisfaction of owning land and possibly a house. Since more purchases require an outside source of financing, real estate investors should be aware that during times of economic recession when outside money is not freely available, property prices may slump dramatically.

There are also marketability problems associated with art and property investments. That is, it is not easy to establish the exact market price of each item at each point in time, and it may be extremely difficult to sell a particular item at a particular point in time.

For the purpose of this thesis the non-security assets will be ignored as investment media. The reason for this is the heterogeneous character of these investments which makes it extremely difficult to value each item. Furthermore the psychological return from owning a non-security asset is unquantifiable.

2.7 Summary

Table 2.3 displays the 25 securities chosen as

Security			
JSE Coal Index	South African Stocks	International Stocks	All Securities
JSE Diamond Index			
JSE All Gold Index			
JSE Metals & Minerals Index			
JSE Mining Financials Index			
JSE Financials Index			
JSE Industrial Index			
Standard & Poor's '500' Index	Foreign Stocks		
Financial Times UK Actuaries Index			
Lead	Metals	Commodities	
Tin			
Zinc			
Silver			
Aluminium			
Antimony			
Copper			
Nickel			
Platinum (Official Price)			
Platinum (Free Price)			
Gold			
Wool	Soft Commodities		
Cotton			
Sugar			
Wheat			
Maize			

Table 2.3 25 Securities chosen as investment alternatives for funds at risk

investment alternatives for funds at risk in this thesis.

Although other investment alternatives exist they have been excluded from this study because they are either risk-free investments (for example, bonds) or they are non-security assets and their heterogeneous nature makes it extremely difficult to value them.

It should be noted that the intention of the thesis is not to be an exhaustive study of all possible investment media. Rather it is merely intended to provide an indication of the potential benefits that might exist for the South African investor should foreign exchange restrictions be lifted. It is also hoped to provide an indication of what percentage of funds the South African investor would on average invest outside the Republic of South Africa given the abolition of foreign exchange control.

C H A P T E R 3

AN INITIAL INVESTIGATION OF THE DATA

3.1 Introduction

Investment in foreign as well as South African securities involves dealing with widely differing types of securities (common stock, bullion, raw metals and soft commodities, for example), markets, trading procedures and currencies.

Thus the South African investor who is considering divesting part or all of his funds outside of South African securities would like to know how each security has performed in the recent past. Furthermore any foreign stocks and/or commodities the South African investor purchases must be paid for with foreign currency. For example, if he wishes to buy copper (quoted in £/ton) or buy securities quoted on the London Stock Exchange, he will have to first exchange his South African rands into UK pounds before the purchase can take place. Therefore the floating exchange rates between rands and pounds, and rands and dollars is of great importance.

In this chapter these factors are considered more closely. Section 2 discusses the subject of the currency factor on the relative performance and volatility of each of

the securities in the study. Finally, in section 3 the statistical technique of multidimensional scaling is applied to the data and the resulting display examined.

3.2 Performance, Volatility and the Currency Factor

The 25 securities under study as investment alternatives appear in Table 3.1, together with performance and risk figures calculated from month-end data for the period February 1965 to January 1980 (180 months in all). This data was all expressed in local currencies.

The total performance of each security is the percentage gain of that security over the entire period. This is defined as

$$\text{Total Performance} = \frac{\text{Price}_{\text{January 1980}} - \text{Price}_{\text{February 1965}}}{\text{Price}_{\text{February 1965}}} \times 100$$

The compound annual return of each security (expressed in percent per annum) is defined as

$$r = \left(\sqrt[15]{\frac{\text{Price}_{\text{January 1980}}}{\text{Price}_{\text{February 1965}}}} - 1 \right) \times 100$$

The 15th root is taken, since the 180 months between February 1965 and January 1980 constitute exactly 15 years. The domestic risk for each security is defined as the standard deviation of the annual returns over the entire 15 year period.

The domestic risk and compound annual return are expressed in percent per annum while the total performance is

security	local currency	total performance (%)	compound annual return (% pa)	ranking of return	domestic risk (% pa)
JSE coal	SA Rand	393,57	11,23	13	22,08
JSE diamonds	SA Rand	480,70	12,44	6	31,06
JSE all gold	SA Rand	420,69	11,63	10	32,48
JSE mets. & mins.	SA Rand	417,71	11,58	11	29,32
JSE min. fin.	SA Rand	437,50	11,86	7	20,94
JSE fin.	SA Rand	153,61	6,40	22	35,74
JSE indust.	SA Rand	136,25	5,90	24	21,50
S & P	US $	33,09	1,92	25	14,85
UK Actuaries	UK £	139,06	5,98	23	27,07
lead	UK £	234,44	8,38	17	32,59
tin	UK £	519,08	12,92	5	21,93
zinc	UK £	221,05	8,09	18	40,70
silver	UK p	1366,96	19,61	2	46,66
aluminium	UK £	303,06	9,74	14	13,19
antimony	UK £	436,40	11,85	8	32,70
copper	UK £	197,14	7,53	19	38,10
nickel	UK £	397,82	11,29	12	16,34
platinum (OP)	UK £	428,67	11,74	9	14,00
platinum (FP)	UK £	631,96	14,19	4	32,97
gold	US $	1812,10	21,74	1	34,36
wool	UK p	166,06	6,74	21	30,66
cotton	UK £	181,78	7,15	20	34,87
sugar	UK £	738,30	15,23	3	45,96
wheat	UK £	250,11	8,71	16	19,91
maize	UK £	256,96	8,85	15	27,67

Table 3.1 Risks and Returns (in local currencies) of 25 securities, February 1965 to January 1980

expressed as a percentage. Also included in the table are the rankings in terms of returns of each of the 25 securities. From these figures it can be seen that the best returns have been achieved by investing in gold, silver, sugar and platinum (bought at the free price). However these securities (as well as zinc, copper and South African financial shares) carry the largest risks.

This information is not of great relevance to the South African investor as all the returns and risks in Table 3.1 are calculated from prices expressed in local currencies. More meaningful figures for the South African investor would be those calculated when the prices are all expressed in South African rands. Each price series was multiplied by the appropriate exchange rate. Table 3.2 displays performance and volatility figures for all 25 securities in rand terms for the same 15 year period, February 1965 to January 1980.

The total performance and compound annual return for each security are defined in precisely the same way as in Table 3.1, as is the domestic risk. Also included in Table 3.2 is the total risk, defined as the standard deviation of the annual returns of each security over the entire 15 year period, as well as the exchange risk, which is the standard deviation of the returns of the dollar- and pound-exchange rates. Both these measures are displayed as percent per annum.

security	total performance (%)	compound annual return (% pa)	exchange gain (% pa)	total risk (% pa)	domestic risk (% pa)	exchange risk (% pa)
JSE coal	393,57	11,23	0,00	22,08	22,08	0,00
JSE diamonds	480,70	12,44	0,00	31,06	31,06	0,00
JSE all gold	420,69	11,63	0,00	32,48	32,48	0,00
JSE mets. & min.	417,71	11,58	0,00	29,32	29,32	0,00
JSE min. fin.	437,50	11,86	0,00	20,94	20,94	0,00
JSE financial	153,61	6,40	0,00	35,74	35,74	0,00
JSE industrial	136,25	5,90	0,00	21,50	21,50	0,00
S & P	52,22	2,84	-1,05	20,48	14,85	8,16
UK act. index	121,20	5,44	0,67	31,46	27,07	10,05
lead	209,46	7,82	0,67	25,24	32,59	10,95
tin	472,85	13,34	0,67	18,07	21,93	10,95
zinc	197,07	7,53	0,67	33,26	40,70	10,95
silver	3157,85	26,14	0,67	19,51	46,66	10,95
aluminium	272,96	9,17	0,67	15,63	13,19	10,95
antimony	396,34	11,27	0,67	37,09	32,70	10,95
copper	174,95	6,98	0,67	30,93	38,10	10,95
nickel	360,64	10,72	0,67	12,19	16,34	10,95
platinum (OP)	389,19	11,16	0,67	7,42	14,00	10,95
platinum (FP)	577,30	13,60	0,67	22,06	32,97	10,95
gold	2087,04	22,84	-1,05	19,57	34,36	8,16
wool	168,61	6,81	0,67	29,13	30,66	10,95
cotton	160,73	6,60	0,67	33,43	34,87	10,95
sugar	675,70	14,63	0,67	42,55	45,96	10,95
wheat	223,96	8,15	0,67	17,07	19,91	10,95
maize	230,30	8,29	0,67	22,90	27,67	10,95

Table 3.2 Performance and Volatility of 25 securities for South African rand investors, February 1965 to January 1980

Finally, the exchange gain, defined as

$$\text{exchange gain} = \frac{\text{exchange rate}_{\text{January 1980}} - \text{exchange rate}_{\text{February 1965}}}{\text{exchange rate}_{\text{February 1965}}} \times 100$$

is displayed for each of the three currencies under study, that is, South African rands, US dollars and UK pounds. The exchange gain indicates how these currencies have moved relative to the South African rand over the entire 180 month period and is measured in percent per annum. Note that the exchange gain is positive in the case of UK pounds-quoted securities, negative in the case of US dollars-quoted securities and zero in the case of the South African securities.

Table 3.2 shows that the best returns from a South African investor's point of view have been achieved by investing in gold, silver and sugar. Sugar carries the greatest total risk, while antimony and South African financial shares are also extremely volatile. Of interest is that both gold and silver are only moderately risky to the South African investor. This is because the correlation between these securities and the exchange rate was large and negative, largely "smoothing" the price series out. When the standard deviation of returns of these series are calculated they are much smaller than those of the original series. Hence the following comment by Solnik and Noetzlin (1983):
"*It is often said that the currency factor has been a major element in performance and risk* [in recent years]. *This*

assertion may depend on what base currency is referred to. Furthermore, what may be true for specific investments is not necessarily so for international portfolios where exchange risk is diversified, due to the coexistence of holdings in several currencies."

For the same reason as above, the differential in Table 3.2 between volatility in rands (total risk) of each security and volatility in local currency (domestic risk) is generally less than the exchange risk.

The foreign stock exchanges both had very small annual returns of between about 3% and 5½% with moderate risks. South African shares had annual returns of between 6% and 12½%, while the range of the metals was between about 7% and 26%. The soft commodities attained returns of between 6½% and 14%. The average risk for a South African investor based in rands was about 27½% annually on the South African equity market, about 21% on foreign equity markets, about 28½% on metals and about 29% on soft commodities.

3.3 A multidimensional scaling of the data

The technique of multidimensional scaling has the effect of displaying points in very high dimensional space as points in much lower dimensionality (Kruskal 1964a,b).

Greenacre and Underhill (1982) expound the usefulness of this method when they note that *"scaling techniques provide a natural first stage in the exploration of a data matrix.*

From a point of view of communication, a graphical description is very easily and quickly assimilated by the researcher. In particular, scaled data has the familiar appearance of a map of points and the proximities, distances and groupings of the points are readily picked up by the eye."

The original data vector can be considered as a point in high-dimensional space and the scaling process maps this point to a point in a low-dimensional subspace. In general the subspace will be of two dimensions so the original vectors are "scaled" as points on a plane. To create the "scaling" a square symmetric data matrix of dissimilarities between points must be created.

In this study the raw data consisted of price series of 25 securities for 181 months, i.e. a 25 × 181 matrix, which had to be transformed into a 25 × 2 matrix which can then be displayed on a plane. A measure of the similarity between two securities α and β can be found by considering the correlation between them. The transformation

$$d_{\alpha\beta} = \frac{1}{2} - \frac{1}{2}\rho_{\alpha,\beta}$$

where $\rho_{\alpha,\beta}$ is the correlation between securities α and β

yields a measure of dissimilarity between securities α and β, with a value of 0 indicating high similarity and a value of 1 indicating high dissimilarity. The square symmetric matrix formed by all the $d_{\alpha\beta}$, $\alpha = 1,2,\ldots,25$;

$\beta = 1,2,\ldots,25$ is an adequate matrix of dissimilarities between the 25 securities.

Associated with each possible configuration of the points in two dimensions is a badness-of-fit measure, called the stress. The stress is thus a function of the configuration, and the scaling solution is one in which the stress is minimised. As there is no algebraic solution to the problem of minimising the stress an iterative procedure is employed. Such a procedure has been programmed by Greenacre and Underhill (1982).

Figure 3.1 is a display of the 25 securities in two dimensions obtained using Greenacre and Underhill's multi-dimensional scaling program. Using the stress function

$$S^2 = \sum_{i=1}^{25}(d_i-\hat{d}_i)^2/\sum_{i=1}^{25} d_i^2$$

where d_i and $\hat{d}_i$ are the true and fitted distances associated with the dissimilarity matrix respectively (Greenacre and Underhill (1982), equation 5.2.3), the stress for this configuration is 0.1116 which implies a good fit in two dimensions.

It will be noticed that the axis AA' indicates, in general, an increasing likeness to South African securities.

Thus all but two of the South African securities lie in the same quadrant of the display. The exceptions are South African gold shares, which are probably more closely related to the price of gold and other precious metals than

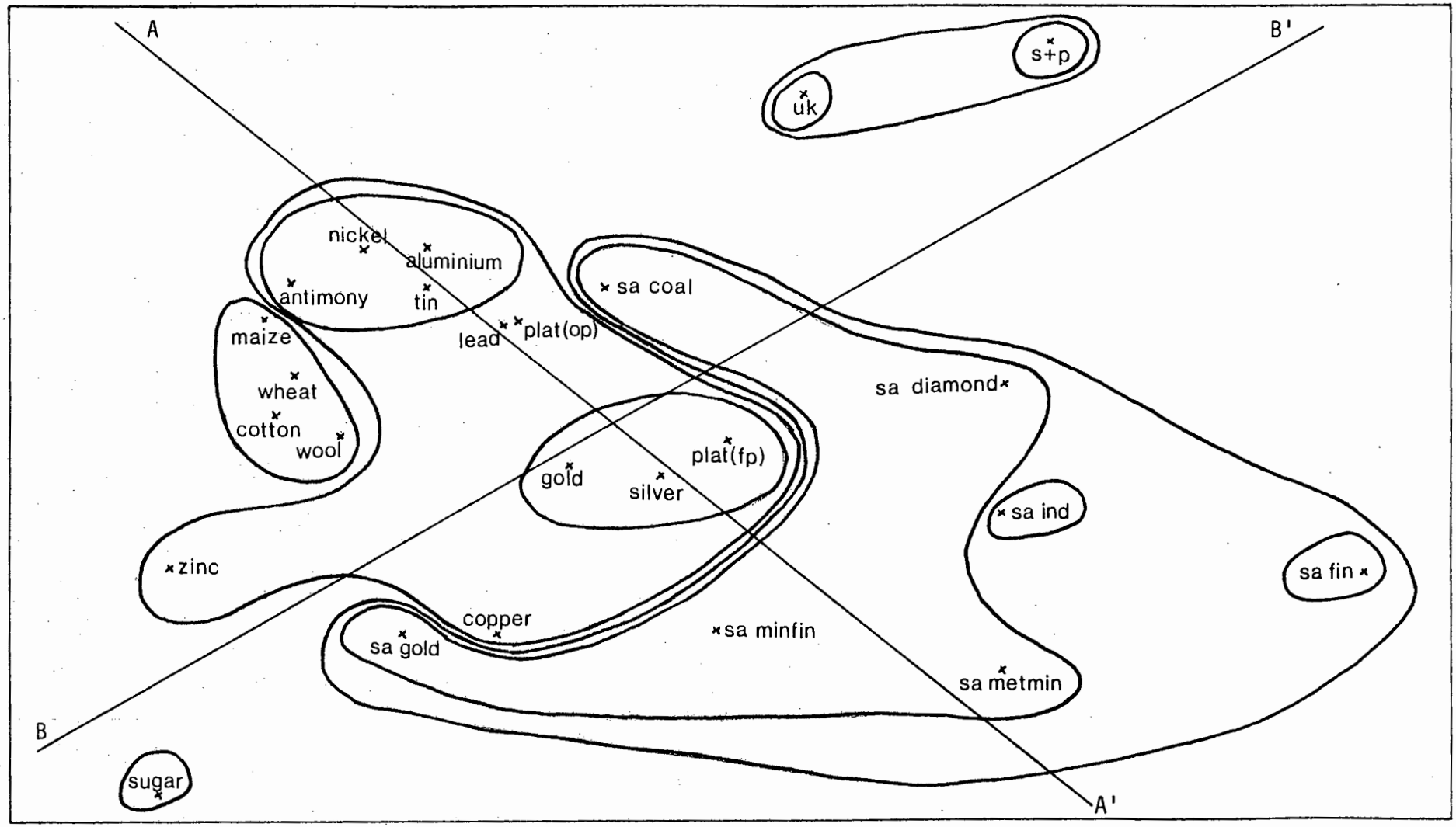


Figure 3.1 A multidimensional scaling of 25 securities

Key for interpreting Figure 3.1

code	Security
sa coal	JSE Coal Index (SA rands)
sa diamond	JSE Diamond Index (SA rands)
sa gold	JSE All Gold Index (SA rands)
sa metmin	JSE Metals and Minerals Index (SA rands)
sa minfin	JSE Mining Financial Index (SA rands)
sa fin	JSE Financial Index (SA rands)
sa ind	JSE Industrial Index (SA rands)
s+p	Standard and Poor's '500' Index (US dollars)
uk	UK Actuaries Index (UK pounds)
lead	lead price - London fixing (£/ton)
tin	tin price - London fixing (£/ton)
zinc	zinc price - London fixing (£/ton)
silver	silver price - London fixing (p/troy oz)
aluminium	aluminium price - London fixing (£/ton)
antimony	antimony price - London fixing (£/ton)
copper	copper price - London fixing (£/ton)
nickel	nickel price - London fixing (£/ton)
plat(op)	platinum price (official price) - London fixing (£/troy oz)
plat(fp)	platinum price (free price) - London fixing (£/troy oz)
gold	gold price - New York fixing ($/troy oz)
wool	wool price - London fixing (p/kg)
cotton	cotton price - London fixing (£/ton)
sugar	sugar price - London fixing (£/ton)
wheat	wheat price - London fixing (£/ton)
maize	maize price - London fixing (£/ton)

to other South African shares, and South African coal shares which rely on the world mineral prices since a large proportion of South African coal is exported. Similarly almost all the foreign securities lie in the upper left quadrant of the display except for sugar which appears to be an outlier.

Along the axis BB' there is a tendency for both increasing liquidity and durability of the securities in question from B to B'. Thus the world's stock exchanges (the most liquid of the assets under study) are positioned mainly in the upper right quadrant, examples being the Standard and Poor's Index on the New York Stock Exchange and the UK Actuaries Index on the London Stock Exchange as well as most of the South African indices. An exception is the South African gold shares. This is because gold shares are very dependent on the gold price. Roughly in the centre of the display are the metals (with the exception of zinc), which are less readily convertible to cash than shares are. Closest to B are the soft commodities which are the least liquid investments of all the securities considered.

A share of stock of a company is considered as lasting forever, and will only cease to exist if the company ceases to exist. Thus shares are extremely durable investments, and are all in the upper right quadrant of the display. On the other hand the soft commodities are all of short durability. These securities are positioned in the lower left quadrant

of the display.

"Cluster loops" have been inserted to group various securities in Figure 3.1. The groupings are according to type of security (for example, South African shares, metals etc) and are not the product of cluster analysis. In the one quadrant of the display the South African mining and metal securities are grouped, and a larger cluster adds the remaining South African securities to the group. It will be noted that the South African shares form a group which is very widely spaced. The reason for this is that they are very diverse in nature. For example, the South African gold, coal and metal and mineral shares follow international metal prices, whereas the South African financial and industrial shares are more closely associated with foreign stock exchange prices. At the top of the display the two foreign stock exchanges are grouped together. In the centre of the display are the precious metals gold, silver and platinum (free price). Another group consists of all the industrial-related metals, notably nickel, antimony, aluminium and tin. All the metals are arranged in one cluster and all the soft commodities except sugar (an outlier) are tightly grouped together.

These groupings clearly indicate the similarities which exist between the 25 securities.

C H A P T E R 4

PORTFOLIO SELECTION USING THE MARKOWITZ APPROACH

4.1 Introduction

The overall aim of this thesis is to determine *ex post* whether or not the rational South African investor would have divested his funds out of South African securities in the event of a complete relaxation of the present exchange control regulations. In other words, had South African investors been allowed to invest outside of South Africa in the past say, twenty years, what securities would have been most profitable for them to invest in? In Chapter 2 it was argued that the possible alternatives to the South African share market were the major international stock markets and the commodity markets of the world.

The analysis of an investor's portfolio "..... *starts with information concerning individual securities. It ends with conclusions concerning portfolios as a whole. The purpose of the analysis is to find portfolios which best meet the objectives of the investor.*" (Markowitz (1959)).

Thus the first stage in portfolio selection is security analysis and concerns the collection of predictions about the future prospects of securities. There are two schools of

thought as to the best method of predicting future prospects. The fundamentalists believe that by studying the balance sheets, dividend records, management policies etc of the company it is possible to determine the intrinsic value of the security under observation. On the other hand the technical analysts believe that past patterns of price behaviour will recur in the future and thus past prices can be used as a prediction of future prices. More recently the Efficient Market Hypothesis has queried whether either of these two approaches can be used successfully in practice. However from the portfolio selection problem point of view all that is important is that the predictions from the security analysis phase must be used as predictions for the portfolio analysis phase. This is true regardless of whether the predictions are derived using the fundamental or technical approach, or any other approach for that matter.

In this chapter an overall view of the portfolio selection problem will be presented together with the formulation and solution proposed by Markowitz (1959). In section 2 the major basic definitions of the parameters which underlie modern portfolio theory are discussed. Section 3 contains a discussion of how and why the addition of more securities to the portfolio makes this portfolio more desirable, and in section 4 the original mathematical model proposed by Markowitz is developed. Section 5 considers an extension of this model to allow for borrowing and lending of capital.

4.2 Risk and Return

If there was no such thing as uncertainty, portfolio selection would not be necessary. An investor would simply buy that security which offered the greatest certain return. *"However, the real world is not one of certainty and so the individual is left with a choice and thus the need for portfolio theory arises."* (Affleck-Graves (1974)). In other words, *"risk in investment means that future returns are unpredictable."* (Brealey and Myers (1981)).

Thus all investors, whether they use a mathematical approach to selecting securities or not, will consider the possible risks associated with their expected return. It should be intuitively obvious that investors require two things from their investment:

(i) the return should be high
(ii) this return should be stable, dependable and not subject to any uncertainty.

Thus the investor should consider two factors when choosing a portfolio:

(i) the return he can expect to receive from the portfolio
(ii) the uncertainty associated with this return.

The rate of return is easily defined as

$$\text{Rate of return} = \frac{\text{receipt - expenditure}}{\text{expenditure}}$$

and it follows that the return associated with a portfolio

is the weighted sum of the return on each individual security.

When trying to quantify the terms "uncertainty" and "risk" Williamson (1970) argued that the individual who is faced with a risk problem does not know the final outcome, but does know the exact probability that any given outcome will occur. That is, the individual can determine the probability distribution associated with the problem. An example is the throwing of a die. However, the individual faced with an uncertainty problem has no knowledge of the probability distribution associated with the outcome. For example, an investor buying a share of stock is faced with a problem of uncertainty.

However, the borderline between the two terms is so close that it shall be assumed that they are interchangeable. Thus the risk associated with a portfolio can be thought of as a measure of the uncertainty of the expected return.

This uncertainty can be quantified in many ways, for example, the variance, standard deviation, mean absolute deviation, semi-variance or coefficient of variation. The variance is the most commonly used measure of risk in portfolio selection and is the one used in the Markowitz approach which will be discussed in the next section.

The expected return on a portfolio is simply a weighted average of the expected returns on the individual securities. A first inclination may be to assume that the standard

deviation of a portfolio is also a weighted average of the standard deviations on the individual securities. However, this is only true if the individual securities move together in perfect lock-step; that is, they are perfectly positively correlated.

Consider the case of a portfolio of just two securities.

Let X_i = proportion of funds invested in security i, i = 1,2

σ_i = standard deviation of security i, i = 1,2

σ_p = standard deviation of the portfolio

$\rho_{1,2}$ = correlation coefficient between security 1 and security 2.

Now

$$\begin{aligned}\sigma_p^2 &= \text{Var}(X_1.\text{return on security 1} + X_2.\text{return on security 2}) \\ &= \text{Var}(X_1.\text{return on security 1}) + \text{Var}(X_2.\text{return on security 2}) \\ &\quad + 2\,\text{Cov}(X_1.\text{return on security 1}, X_2.\text{return on security 2}) \\ &= X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + 2X_1X_2\sigma_1\sigma_2\rho_{1,2}\end{aligned}$$

But, if it is assumed that the standard deviation of a portfolio is a weighted average of the individual standard deviations, then

$$\begin{aligned}\sigma_p^{2*} &= (X_1\sigma_1+X_2\sigma_2)^2 \\ &= X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + 2X_1X_2\sigma_1\sigma_2 \\ &\geqslant X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + 2X_1X_2\sigma_1\sigma_2\rho_{1,2} \\ &\geqslant \sigma_p^2 \qquad \text{since } -1 \leqslant \rho_{1,2} \leqslant 1\end{aligned}$$

Thus only if $\rho_{1,2} = 1$ will the risk involved in holding a portfolio of more than one security be the same as the

average of the risks of the individual securities involved. In all other cases the risk of holding a portfolio will be somewhat less than the average risk of the individual securities.

So if an investor is willing to choose a portfolio solely on the basis of the two variables, expected return and the uncertainty of that return, it may be assumed that the following rules would apply:

(i) If two portfolios have the same expected return the rational investor will choose the one with the lower risk.

(ii) If two portfolios have the same risk then the rational investor will choose the one with the greatest expected return.

(iii) If one portfolio has less risk and a greater return than another portfolio it will be preferred.

Thus a differentiation can be made between 'efficient' and 'inefficient' portfolios.

A portfolio is 'efficient' if it is impossible to achieve a greater expected return without bearing more risk and it is impossible to achieve a smaller risk without decreasing the expected return.

Likewise, a portfolio is 'inefficient' if it is possible to achieve a larger expected return without bearing greater risk, or to obtain a smaller risk without decreasing the expected return.

involved. However if the correlation between the two securities is decreased below 1 then the portfolio risk becomes somewhat less than the average risk of the individual securities, since

$$\sigma_p^2 = X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + 2X_1X_2\sigma_1\sigma_2\rho_{1,2}$$
$$< X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + 2X_1X_2\sigma_1\sigma_2 \quad \text{since} \quad \rho_{1,2} < 1$$

The greatest possible reduction in risk occurs when the two securities are perfectly negatively correlated. Then the portfolio risk is

$$\sigma_p^2 = X_1^2\sigma_1^2 + X_2^2\sigma_2^2 - 2X_1X_2\sigma_1\sigma_2$$

When there is perfect negative correlation there is always a portfolio strategy (represented by a particular set of portfolio weights) which will completely eliminate risk. Say, for example, that the standard deviation of security 1 is α times that of security 2. Then to eliminate risk the best strategy is to invest α times as much in security 2. Then

$$\sigma_1 = \alpha\sigma_2$$

and if X_1 is invested in security 1 then invest αX_1 in security 2

$$\text{i.e.} \quad X_2 = \alpha X_1$$

$$\text{So} \quad \sigma_p^2 = X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + 2X_1X_2\sigma_1\sigma_2\rho_{1,2}$$
$$= X_1^2\sigma_1^2 + (\alpha X_1)^2\left(\frac{\sigma_1}{\alpha}\right)^2 - 2X_1(\alpha X_1)\sigma_1\left(\frac{\sigma_1}{\alpha}\right)$$
$$= X_1^2\sigma_1^2 + X_1^2\sigma_1^2 - 2X_1^2\sigma_1^2$$
$$= 0$$

Brealey and Myers (1981) comment wryly that "*its too bad perfect negative correlation doesn't really occur between common stocks.*"

The formula for the variance of a portfolio can be extended to a portfolio containing three or more securities. If the portfolio contains three securities then

$$\sigma_p^2 = X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + X_3^2\sigma_3^2 + 2X_1X_2\sigma_1\sigma_2\rho_{1,2} + 2X_1X_3\sigma_1\sigma_3\rho_{1,3} + 2X_2X_3\sigma_2\sigma_3\rho_{2,3}$$

In general the simplest formula for calculating the variance of a portfolio containing N securities is

$$\sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} X_iX_j\sigma_{ij}$$

where σ_{ij} = covariance of securities i and j

$= \sigma_i\sigma_j\rho_{i,j}$

When $i = j$ then $\sigma_{ij} = \sigma_i^2$, the variance of security i.

The expected return of a portfolio containing N securities is

$$E_p = \frac{1}{N}\sum_{i=1}^{N} X_i$$

Wagner and Lau (1971) conducted a study in which they formed portfolios of differing size from a sample of stocks and then calculated the standard deviation of returns for each of these portfolios. Figure 4.1 shows the generalised results of their study.

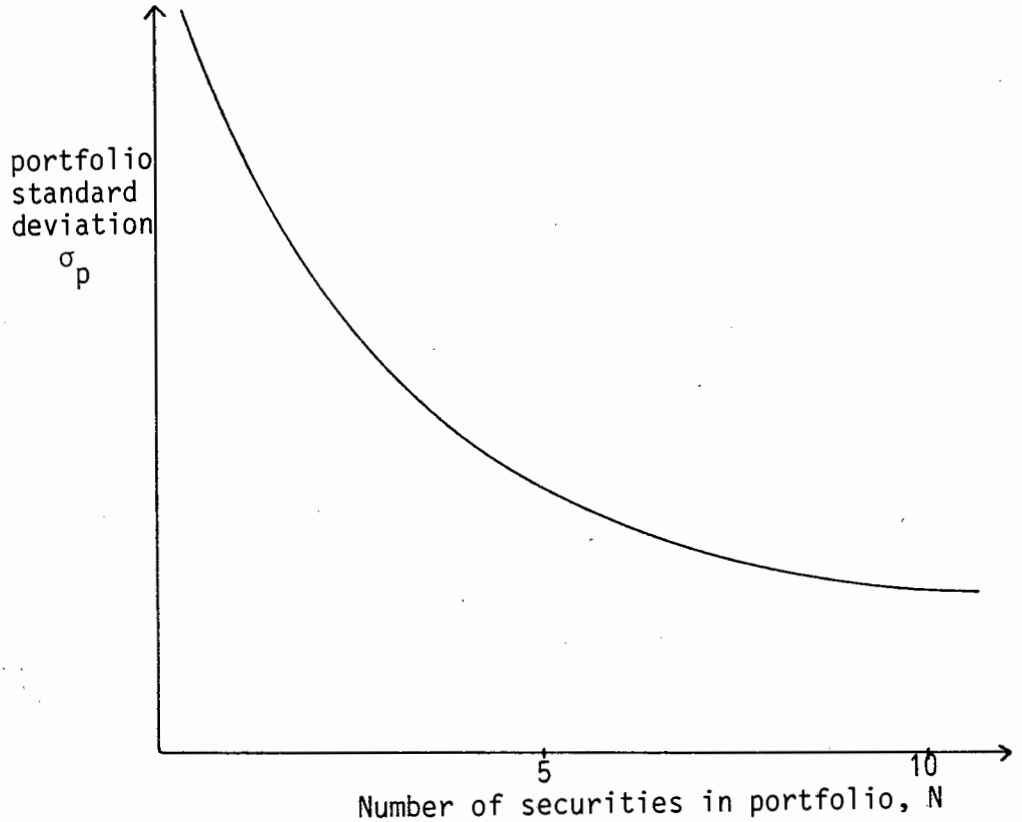


Figure 4.1 Increasing the number of securities in a portfolio generally decreases portfolio standard deviation

It can be seen that as more stocks are added to the portfolio there is a reduction in the portfolio risk. In their study Wagner and Lau found that "*on average, approximately 40% of the risk was reduced through the simple expedient of holding a randomly selected diversified portfolio.*" However, most of the benefit of diversification can be achieved with relatively few stocks : the improvement is slight when the number of stocks is increased beyond, say, 10.

Diversification reduces risk because prices of different securities do not move exactly together. That is, they are imperfectly correlated. This means that sometimes a decline in the price of one security is cancelled out by a rise in

the price of another. Thus the portfolio standard deviation is decreased and an opportunity exists to reduce risk by diversification.

Risk can be broken up into two parts:

(i) The risk that can potentially be eliminated by diversification is called 'unique risk' or 'unsystematic risk'. This is the variability not explained by general market movements and stems from the fact that *"many of the perils that surround an individual company are peculiar to that company and perhaps its immediate competitors."* (Brealey and Myers (1981)). Thus, for example, local strikes or bad management will constitute unique risk.

(ii) The risk that cannot be avoided however much diversification is employed is called 'market risk' or 'systematic risk'. This is caused by movements in the market as a whole and stems from the fact that *"there are economy-wide perils which threaten all business."* (Brealey and Myers (1981)). Thus investors are exposed to market uncertainties no matter how many stocks they hold.

The above two components of risk are combined in the following way:

Total risk = unique risk + market risk.

In Figure 4.2 below the portfolio risk (represented by the standard deviation of expected returns) has been divided into its two parts - unique risk and market risk.

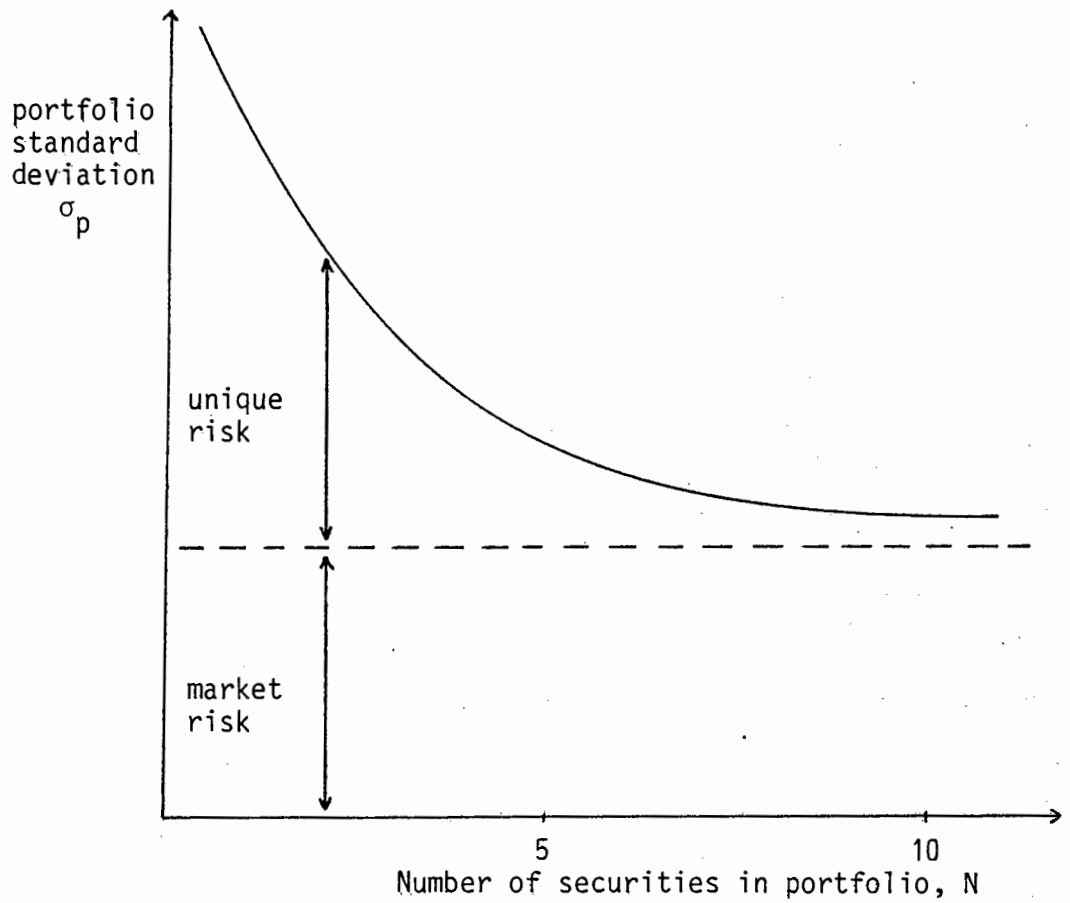


Figure 4.2 Portfolio risk expressed as a sum of unique risk and market risk

When only a small number of securities are included in the portfolio the unique risk is important, but when the portfolio consists of a large number of securities diversification no longer reduces risk and only market risk is important. As Brealey and Myers (1981) put it *"the predominant source of uncertainty for a diversified investor is that the market will rise or plummet, carrying the investor's portfolio with it."*

They continue by saying *"If we want to know the contribution of an individual security to the risk of a well-diversified portfolio, it is no good thinking about how risky that security is when held in isolation - we need to measure its market risk and that boils down to measuring how sensitive it is to market movements."*

Consider a portfolio in which equal investments are made in each of N stocks. The proportion invested in each stock is 1/N. Then the portfolio variance is

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j \sigma_{ij}$$

$$= N\left(\frac{1}{N}\right)\left(\frac{1}{N}\right) \times \text{average variance}$$

$$+ (N^2-N)\left(\frac{1}{N}\right)\left(\frac{1}{N}\right) \times \text{average covariance}$$

$$= \frac{1}{N} \times \text{average variance} + \left(1 - \frac{1}{N}\right) \times \text{average covariance}$$

As N increases, the portfolio variance σ_p^2 approaches the average covariance. Thus if the average covariance were zero it would be possible to eliminate all risk by holding sufficient securities. The securities considered do not move independently of one another, but are tied together in a web of positive covariances which set the limit to the benefits of diversification. Thus the market risk in Figure 4.2 is the average covariance which constitutes the risk remaining after diversification has done its work.

4.4 The Markowitz approach to portfolio selection

More than thirty years ago Harry Markowitz (1952) published a paper in which he proposed a model which has since become the cornerstone of portfolio selection. His basic approach makes use of the idea that the choice of a portfolio rests solely on two variables, that is the expected return an investor can achieve from the portfolio and the risk associated with that expected return. The most widely used measures of expected return and risk will be used here. They are the weighted average return of the securities comprising the portfolio, and the standard deviation of the returns on the portfolio respectively.

In generating portfolios the following notation will be used:

E_i = expected return on the ith security

σ_i = standard deviation of the return on the ith security

E_p = expected return on the portfolio

σ_p = standard deviation of the return on the portfolio

σ_{ij} = covariance between security i and security j

ρ_{ij} = correlation coefficient for the returns on securities i and j

X_i = proportion of funds invested in security i

N = total number of securities considered

Then $E_p = \sum_{i=1}^{N} X_i E_i$

$= X'E$

and $\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j \sigma_{ij}$

$= \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j \sigma_i \sigma_j \rho_{ij}$

$= X' \Sigma X$

where $X' = (X_1, X_2, \ldots, X_N)$

$E' = (E_1, E_2, \ldots, E_n)$

and $\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2N} \\ \vdots & \vdots & & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_N^2 \end{bmatrix}$

As has already been mentioned a rational investor will choose a portfolio on the basis of its expected return and the risk (standard deviation) associated with that return. Hence any portfolio may be represented as a single point in the E_p, σ_p plane. Figure 4.3 indicates the situation for a large number of securities.

Each cross in Figure 4.3 represents the combination of risk and return offered by a different individual security. By mixing these securities in different proportions you can reduce your risk and obtain an even wider selection of risk and expected return. Thus the range of attainable combinations may look something like the broken-egg-shaped area

in the E_p, σ_p plane as shown in Figure 4.3.

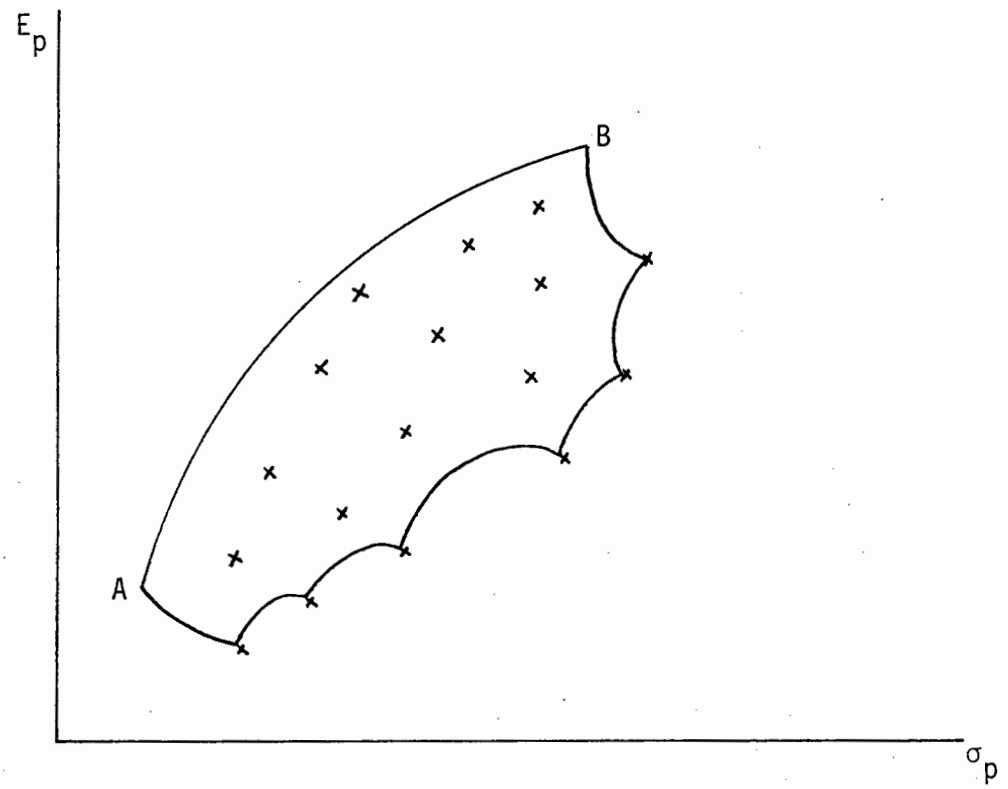


Figure 4.3 The set of feasible portfolios in the E_p, σ_p plane

Those portfolios lying along the convex upper boundary (curve AB) dominate all others in the sense that they are the portfolios for which no greater expected return is possible without incurring a greater risk, or equivalently, no smaller risk is attainable without sacrificing some expected return. These are commonly called the efficient set of portfolios, or the efficient frontier and a rational investor would only consider holding a portfolio from this efficient set. Brealey and Myers (1981) point out that

"since you wish to increase expected return and reduce standard deviation, you will only be interested in those portfolios that lie along the [efficient frontier]." Thus a rational investor will not hold a more risky portfolio without being offered a greater expected return. In the words of Lorie and Hamilton (1973) *"people do not like risk and if they expect to incur it, they expect to be paid,"* and Jensen (1969) *"in a world dominated by risk averse investors, a risky portfolio must be expected to yield higher returns than a less risky portfolio, or it would not be held."*

Thus given predictions about individual securities and their interrelationships, the efficient set is the same for all investors. But each investor's preference for return *vis-à-vis* risk is likely to differ. Thus once the efficient frontier has been created it is left to the individual investor to trade off the expected return and risk of each portfolio on that efficient frontier, and to choose the particular one which suits his risk profile. Brealey and Myers (1981) comment *"whether you want to choose the minimum risk portfolio (portfolio A) or the maximum expected return portfolio (portfolio B) or some other efficient portfolio depends on how much you dislike taking risk."*

So if a portfolio can be represented as

$$X' = (X_1, X_2, \ldots, X_N)$$

where X_i is the proportion of funds invested in security i,

the basic Markowitz portfolio selection problem reduces to the following mathematical programming problem:

$$\text{minimise} \quad -\lambda E_p + V_p \quad \text{for all possible} \quad \lambda \geq 0$$

$$\text{where} \quad E_p = X'E$$

$$\text{and} \quad V_p = \sigma_p^2 = X'\Sigma X$$

subject to the constraints

(i) $\sum_{i=1}^{N} X_i = 1$

that is, all the investor's funds must be invested, and

(ii) $X_i \geq 0$ for all $i = 1,2,\ldots,N$

that is, no security may be held in negative quantities (i.e. no short position is allowed).

It will be seen that the objective function is linear in the E_p, σ_p^2 plane with slope λ and will be minimised at the point where the line $-\lambda E_p + V_p$ is tangent to the efficient frontier. Thus by varying λ from 0 to ∞ each point on the efficient frontier can be created.

A quadratic programming method must be employed to solve the above problem since V_p contains terms in X_i^2.

Additional linear equality constraints could be included to ensure that a certain proportion of funds are invested in a particular section of the universe of securities available. For example

$$\sum_{j=1}^{N} d_{ij} X_j = R_i \,, \qquad 0 \leq R_i \leq 1$$

would ensure that a proportion R_i would be invested in section i of the universe of securities.

Furthermore, upper and lower bounds may be placed on the amount to be invested in each security by constraints of the form

$$L_i \leqslant X_i \leqslant U_i \qquad 0 \leqslant L_i, U_i \leqslant 1 \quad \text{and} \quad i = 1,2,\ldots,N$$

where L_i and U_i are the lower and upper bounds respectively for the proportion of funds invested in security i.

Thus a more general form of the problem, termed the 'standard problem' can be written as

$$\text{minimise} \quad -\lambda E_p + V_p$$

$$\text{for all possible} \quad \lambda \geqslant 0$$

$$\text{subject to} \quad \sum_{i=1}^{N} X_i = 1$$

plus any other linear equality constraints

$$\text{plus} \quad L_1 \leqslant X_1 \leqslant U_1$$

$$L_2 \leqslant X_2 \leqslant U_2$$

$$\vdots$$

$$L_N \leqslant X_N \leqslant U_N$$

$$\text{plus all} \quad X_i \geqslant 0 \qquad i = 1,2,\ldots,N$$

Various algorithms have been devised to solve the 'standard problem' and an adaption of the method proposed by Sharpe (1970) by Affleck-Graves (1974) has been used for computational purposes.

4.5 In which borrowing and lending are introduced

When considering a portfolio selection problem it should

not be forgotten that not all funds need be invested in risky securities. There exist certain riskless assets, for example investments in banks, building societies or government securities. Lorie and Hamilton (1973) state that "*there are assets whose rates of return can be predicted with virtual certainty ..... most investors have an extraordinarily great confidence that they can predict accurately the rate of return on securities of the federal government for any period which is equal to their maturity. For example, Treasury bills maturing in one year have a precisely predictable rate of return for one year.*" Furthermore portfolios can be purchased in part with borrowed funds.

Thus allowance must be made in the portfolio selection problem for lending (investment in a riskless security like Treasury bills or a bank) and borrowing (issuing a riskless security to the lender which will be repaid with interest at some future date). So if security r is riskless

$X_r > 0$ implies the investor lends

$X_r < 0$ implies the investor borrows

$X_r = 0$ implies he neither borrows nor lends

i.e. he invests all his funds in risky securities.

Initially assume equal risk-free borrowing and lending rates.

Then $E_r = r_f$

where r_f is the risk-free rate of return, and

$\sigma_r = 0$

since the security is riskless.

Thus $\sigma_{ri} = 0$ for $i = 1,2,\ldots,N$

since $\sigma_{ri} = \sigma_r \sigma_i \rho_{r,i}$

$= 0.\sigma_i \rho_{r,i}$

$= 0$

So if any two securities are combined into a portfolio, one of which is risk-free (security r) and the other being risky (security k) then

$$E_p = X_r E_r + X_k E_k$$

and $\sigma_p^2 = X_r^2 \sigma_r^2 + X_k^2 \sigma_k^2 + 2X_r X_k \sigma_r \sigma_k \rho_{r,k}$

$= 0 + X_k^2 \sigma_k^2 + 0$

$= X_k^2 \sigma_k^2$ since $\sigma_r = 0$

Thus $\sigma_p = X_k \sigma_k$

Thus if two such securities are combined the result is a straight line through the points representing the two securities in the E_p, σ_p plane. This is shown in Figure 4.4 below.

The efficient frontier as described in section 4.2 is the curve AMB. An investor who wishes to invest in risky securities may invest in any portfolio along this curve. However, if it is assumed that he may lend to the market he may achieve a position anywhere along the line r_f M by lending a proportion of his funds to the market (at zero risk) and investing the rest in portfolio M at risk. If the investor wishes to bear no risk he may invest all his funds in r_f. His return would then be the risk-free rate of return

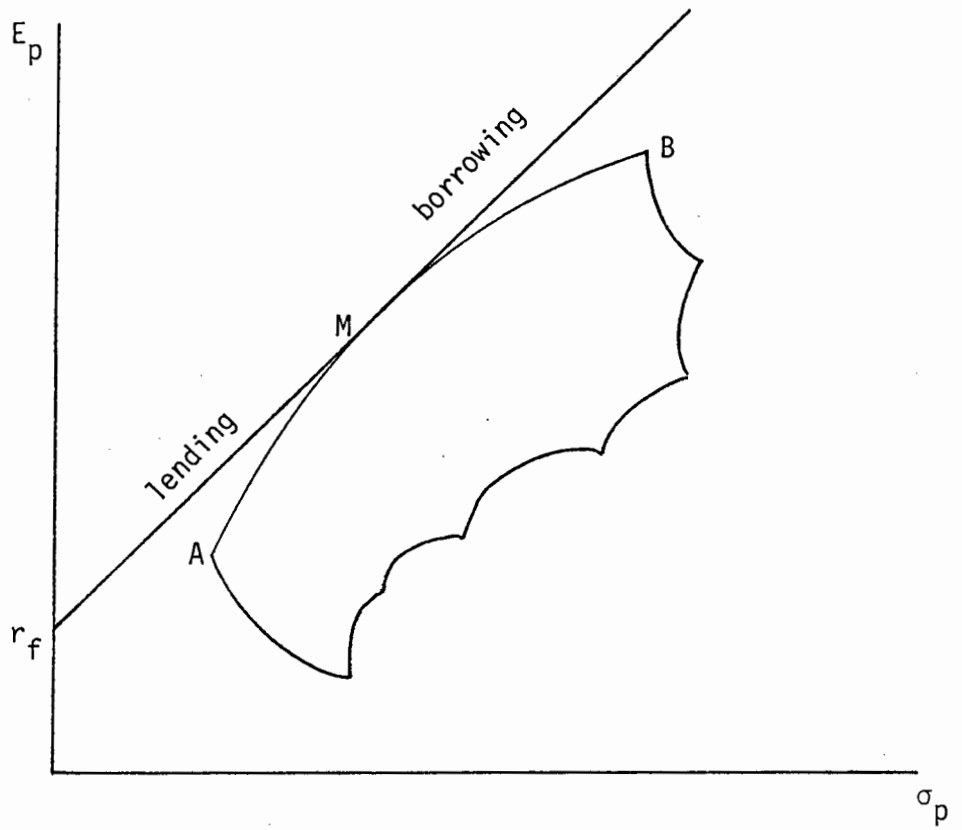


Figure 4.4 The effect of borrowing and lending

and his risk would be $r_f = 0$. If it is assumed that me may borrow at rate r_f equal to the lending rate then he may achieve a position on the straight line above portfolio M by borrowing and investing all his funds as well as his borrowed funds in portfolio M.

So regardless of what level of risk is chosen, the highest expected return can be achieved by a mixture of portfolio M and borrowing or lending. There is no reason ever to hold portfolio A, say. Brealey and Myers (1981) remark that *"we can separate the investor's job into two stages. First the 'best' portfolio of common stocks must be selected.*

Second, this portfolio must be blended with borrowing or lending to obtain an exposure to risk that suits the investor's particular tastes. Each investor, therefore, should put money into just two benchmark investments - a risky portfolio M and a risk-free loan (borrowing or lending)." This is known as the Separation Theorem.

Assume that all investors are faced with the same opportunities (that is, they can borrow and lend at the same risk-free rate), and they all agree on the expected returns for each security. Then clearly all investors will choose the same portfolio M of risky securities, which will contain all the securities in the market. The proportion of each security will be equal to its proportionate value in the market as a whole. It is thus commonly called the 'market portfolio'.

However, not all investors do invest in the market portfolio. This is because they are seldom faced with the same opportunities, nor do they all have the same expectations about each security's prospects. Hence Brealey (1969) argues *"The argument presented in this chapter does not imply that all investors should have the same mix of stocks, but only that they should have the same mix if they are faced with the same set of opprotunities and are agreed on the odds of realising various levels of return. In practice, of course, such agreement is rare. Even if they do share identical views of each stock's prospects, differences in the costs to which they are liable could result in differences in*

their expectations of return. For example, high-yielding stocks offer very low returns to investors with a high tax rate." Bradfield (1983) adds that "*brokerage costs are extensive for large portfolios such as the market portfolio. It is a well-known fact that diversification reduces risk, however researchers have shown that the improvement in risk reduction by diversifying portfolios by more than 10 securities is small."*

The initial assumption was equal borrowing and lending rates. It is unlikely that most investors will be able to borrow and lend at the same risk-free rate. Usually a higher borrowing rate will apply so, let

E_L = lending rate

E_B = borrowing rate

$E_L < E_B$

Figure 4.5 below shows the effect of a higher borrowing rate.

The investor is now faced with one of three decisions depending on his willingness to bear risk:

(i) if he requires a risk less than σ_R he may lend α% of his funds and invest (1-α)% of his funds in the risky security R to attain some position on the line $E_L R$.

(ii) if he wishes to bear a risk of between σ_R and σ_S he will choose one of the risky portfolios along the efficient frontier between R and S,

(iii) if he wishes to stake all and bear a risk of greater than σ_S, and if he can borrow at rate E_B, then the investor can achieve a position along the line SB by borrowing and investing that together with his own funds in portfolio S.

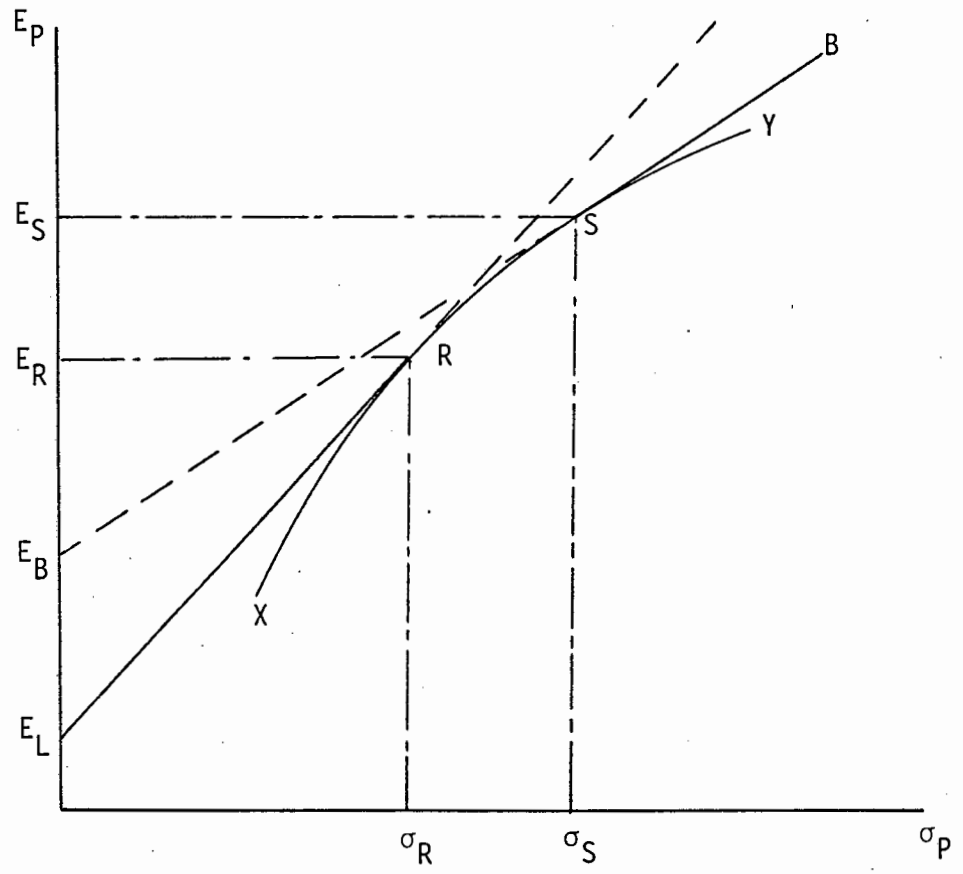


Figure 4.5 The effect of borrowing and lending when the borrowing rate is greater than the lending rate

C H A P T E R 5

RESULTS OF PORTFOLIO SELECTION FOR THE US, UK AND SOUTH AFRICAN INVESTOR

5.1 Introduction

In the previous chapter the theoretical formulation of the Markowitz portfolio selection model was presented. In this chapter various efficient portfolios are constructed, using this model and empirical data from several recent time periods. The constituents of these portfolios are closely examined.

Section 2 discusses the methodology used for the empirical study based on this portfolio selection model. The securities considered were those discussed in Chapter 2. The results of this initial look at portfolio selection from the South African investor's point of view assuming that exchange control did not exist are contained in section 3. The position as viewed by US and UK investors is discussed in section 4. Finally, in section 5 a summary and the main conclusions of the study appear.

It should be pointed out that in this and all subsequent empirical studies, all brokerage and tax payments have been ignored. Since the model is essentially a single period buy and hold model the effect of brokerage will be to reduce

the reported return by a maximum of 4% per annum and has no effect on the risk. The effects of tax are complicated by the fact that different rates are applied to companies and to individuals, and individuals are taxed at different rates. It was therefore decided to ignore all tax effects - the individual investor can make his own modifications depending on his personal tax position.

5.2 Methodology

In section 4.3 the linear objective function of the mathematical programming problem was formulated as

$$\text{minimise} \quad -\lambda E_p + V_p \quad \text{for all possible} \quad \lambda \geq 0$$

where E_p = expected return on the portfolio
= weighted average of expected returns of the securities in the portfolio

and $V_p = \sigma_p^2$ = variance of the returns on the portfolio

Thus the problem is one in two dimensions and depends on two variables, namely the expected return on the portfolio and the variance of the returns on the portfolio. Thus for any given λ the objective function will be minimised when a portfolio's expected return is large and this return is very stable (that is, the variance is small). Securities will only be included in the efficient portfolios if they have expected returns large enough to contribute towards this large portfolio return, and a risk small enough to maintain portfolio variance at a low level. Thus securities that did well

over a particular period, and where risks associated with holding such securities are small would tend to be included in the efficient portfolios.

The data discussed in Chapter 2 showed that the New York Stock Exchange (NYSE) and the London Stock Exchange (LSE) were represented in this study by only one index each, namely the Standard and Poor's '500' Index and the UK Actuaries Industrial Index respectively. However, the Johannesburg Stock Exchange (JSE) was represented by seven composite indices and so clearly those sections of the JSE that did very well in a given period with low associated risks would be selected for inclusion in the efficient frontier. Similarly those sections that did not do well or had high risks would be left out. This may lead to overall results that would indicate that South African shares were very good investments over the period under study, whereas in fact only one or two sections of the market performed particularly well, the others doing at most only averagely well. To eliminate this possible source of portfolio selection bias it was decided to employ a single overall index which would represent the JSE as a whole. For this purpose the JSE All Share Index was used to represent all South African shares. Figure 2.1 shows the percentage contribution of each of the seven composite indices already discussed to the JSE All Share Index. Results achieved would then be comparable to those on the NYSE and the LSE.

For the same reason as described above the eleven metals and five soft commodities selected in Chapter 2 should also be combined into two separate indices for comparison purposes. Unfortunately there was no easily-obtainable index of metal or soft commodity prices. Thus indices for these two groups of securities had to be constructed. There are several different methods of constructing a stock market index, for example arithmetic averages, weighted averages, geometric averages etc, and a discussion of these methods can be found in Affleck-Graves (1977).

It has been argued by Cohen and Fitch (1966) that since investors are generally interested in return and not usually in price *per se*, stock market indices should be based on return and not price. Thus an arithmetic average of returns was decided on for the metal and soft commodity indices. Most of the empirical work pertaining to stock market indices based on return have not used return in the traditional sense of the word (difference in price over some period divided by price at the beginning of the period) but have used a related measure, the price relative

$$PR = \frac{P_{i;t}}{P_{i;t-1}}$$ where $P_{i;t}$ is the price of security i in period t.

Thus, this type of index is usually constructed as follows:

$$I_t = [1/N \sum_{i=1}^{N} (P_{i;t}/P_{i;t-1})] \times I_{t-1}$$

where I_t is the level of the index at period t

and $P_{i;t}$ is defined as above.

This type of index, which is essentially an index of rates of return, is often called an investment performance index (IPI). This index is equivalent to the performance of an investor who invests equal monetary amounts in each security and re-allocates back to equal amounts at the start of each new period (whether a day, a week, a month or a year). The United Press International Market Indicator on the New York Stock Exchange is an example of a stock market index based on this methodology.

Thus an universe of 5 securities existed, namely

(i) South African shares (represented by the JSE All Share Index),

(ii) US shares (represented by the Standard and Poor's '500' Index),

(iii) UK shares (represented by the UK Actuaries Industrial Index),

(iv) Metals (represented by an IPI-type index calculated from the eleven metal price series),

(v) Soft commodities (represented by an IPI-type index calculated from the five soft commodity price series).

Portfolio selection on the five above-mentioned securities by means of the Markowitz approach as discussed in section 4.4 was desired.

The study was conducted over a period of 180 successive months, from February 1965 to January 1980. This time period was subdivided into three equal non-overlapping sub-periods, namely

1) February 1965 to January 1970

2) February 1970 to January 1975

3) February 1975 to January 1980.

These three periods were not chosen to coincide with or represent any market cycle, but provided three convenient periods for comparison of the Markowitz selection.

As this study was aimed at observing the effect of various forms of investment on the South African investor, all prices were initially expressed in South African rands by multiplying by the dollar/rand or pound/rand exchange rates where necessary:

$$P_R = (P_\$)(ER_{\$,R})$$

or $$(P_£)(ER_{£,R})$$

where P_R = price of the security in SA rands

$P_\$$ = price of the security in US dollars

$P_£$ = price of the security in UK pounds

$ER_{\$,R}$ = exchange rate from US dollars to SA rand

$ER_{£,R}$ = exchange rate from UK pounds to SA rand

The commodities expressed in pence were first converted to UK pounds. The indices on the JSE, originally created from series of prices quoted in SA rands, were not converted.

This study is done *ex post* on prices collected over a past 15 year period. The results of the portfolio selection model will indicate the proportion of funds that a South

$$R_{i,t} = \log_e P_{i,t} - \log_e P_{i,t-1}$$

This is the monthly rate of return with continuous compounding.

Thus for each of the sub-periods sixty monthly returns (spanning five years of data) were calculated. Mean monthly returns for each security in each sub-period were calculated, as was the 5×5 covariance matrix for each sub-period. This data was used as input to the Markowitz portfolio selection program coded by Affleck-Graves (1974), and the efficient frontiers were calculated.

5.3 Empirical results of unconstrained efficient frontiers

The unconstrained efficient frontiers created using the universe of five securities are displayed in Figure 5.1. These frontiers are unconstrained in the sense that no limitation was placed on the proportion of the total funds which could be invested in any one security. The minimum proportion of zero was applied throughout which means that short positions were not allowed.

As discussed in section 5.1, tax and brokerage effects were not considered.

A close examination of these three frontiers reveals several important points:

(a) The range of risk/return combinations available in period 3 was slightly larger than that of period 2. The range of risk/return combinations available in

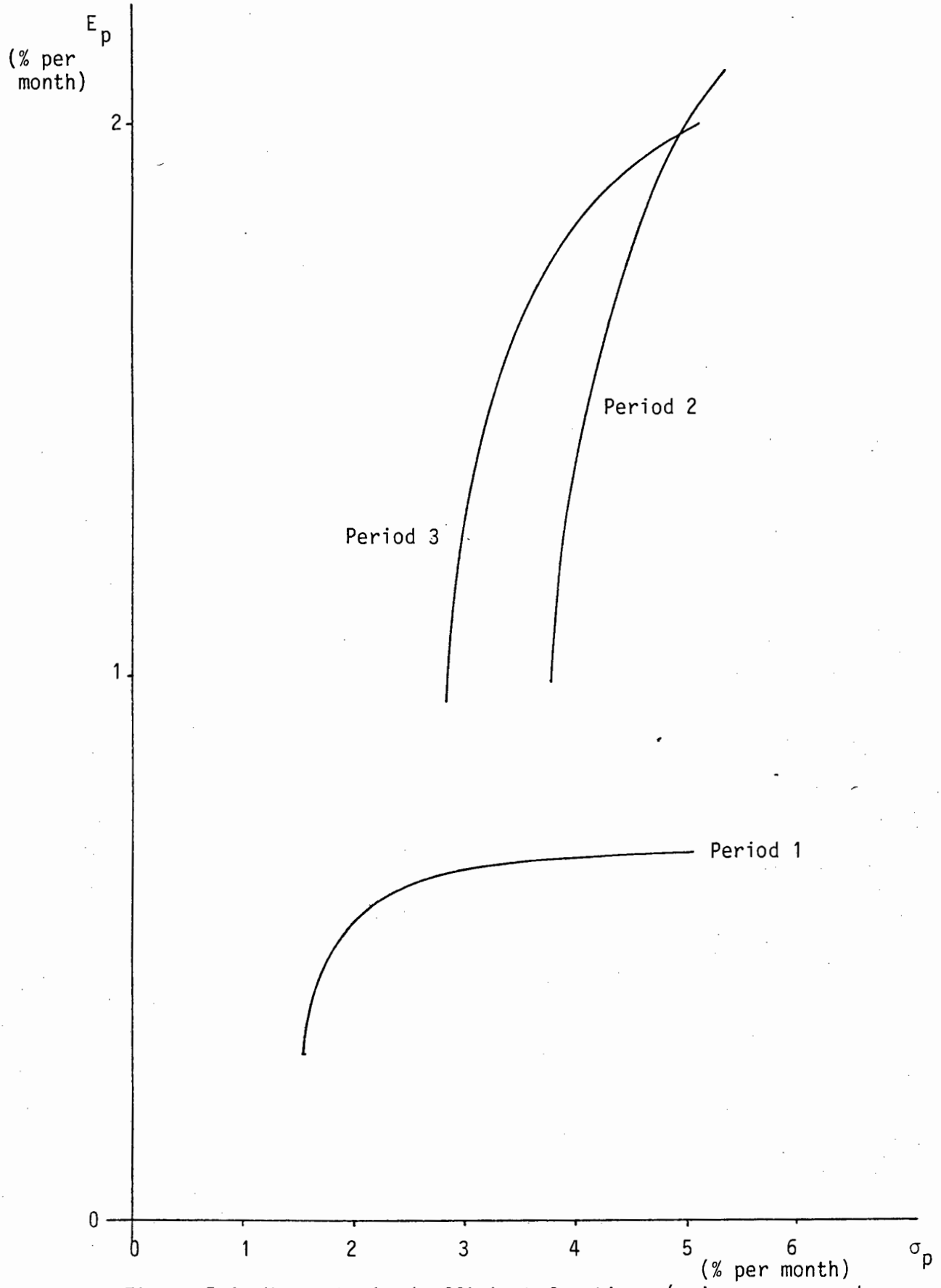


Figure 5.1 Unconstrained efficient frontiers (prices expressed in SA rands)

period 1 was very restricted, although a considerably smaller risk (with associated smaller return) than in periods 2 or 3 was achievable.

(b) For a given risk level dramatically different returns were achieved depending on the period under consideration. For example, for $\sigma_p = 4,0\%$ the following range of average monthly returns were achieved:

period	σ_p	E_p
1	4,0%	0,66%
2	4,0%	1,40%
3	4,0%	1,81%

This would be important to an investor who had a portfolio which included investment in securities other than those considered here, and who wished to bear a risk which was uniform over time on the portion invested in the securities considered in this study.

Similarly, for a given return a varying amount of risk was incurred, depending on the period under consideration. For example, for $E_p = 1,10\%$ the following range of risks were borne:

period	σ_p	E_p
1	This return was unobtainable in this period at any risk level	
2	3,82%	1,10%
3	2,91%	1,10%

This could be important to an investor who required a stable return (for example, a widow or pensioner who must live off the income created by the return on his/her investment) but was prepared to accept varying risks to achieve this stable return.

(c) The slope of the efficient frontier over particular ranges was sometimes very different from period to period. The table below indicates, for example, the percentage increase in expected return an investor could have achieved in each time period when the risk level was increased by 30% from 3,75% per month to 4,88% per month.

period	E_p when σ_p = 3,75%	E_p when σ_p = 4,88%	% increase in E_p
1	0,66	0,68	3,03%
2	0,99	1,95	96,97%
3	1,75	1,97	12,56%

This confirms that the slope at different risk levels is of great importance in deciding whether bearing additional risk is worthwhile in terms of an investor's risk/return expectations.

Figures 5.2, 5.3 and 5.4 again represent the efficient frontiers respectively for the three periods under consideration. In these figures various individual portfolios along the efficient frontier have been numbered. An examination of the composition of these portfolios (each constituting a widely different risk level) is instructive to observe

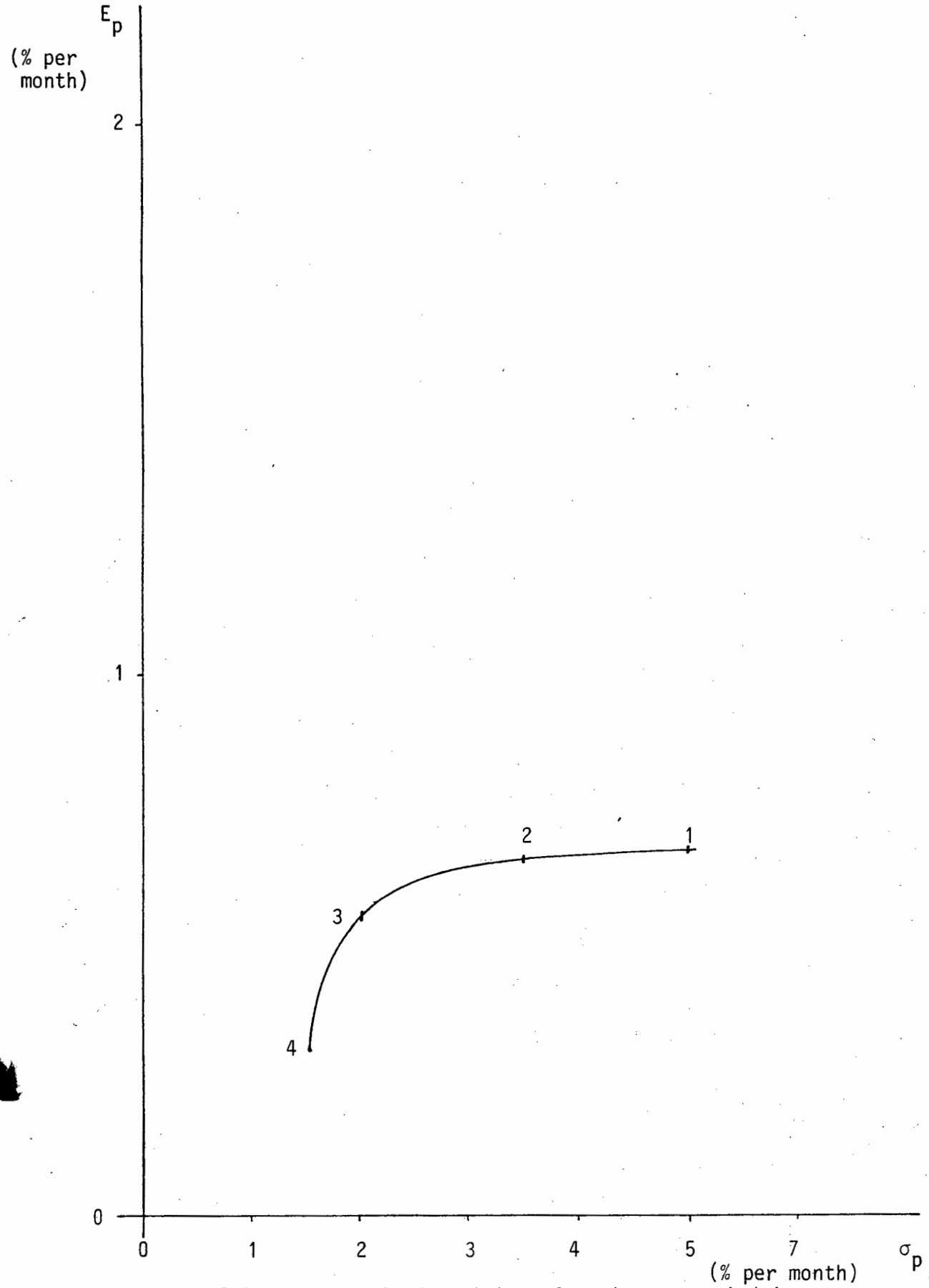


Figure 5.2 Unconstrained efficient frontiers - period 1
(Proces expressed in SA rands)

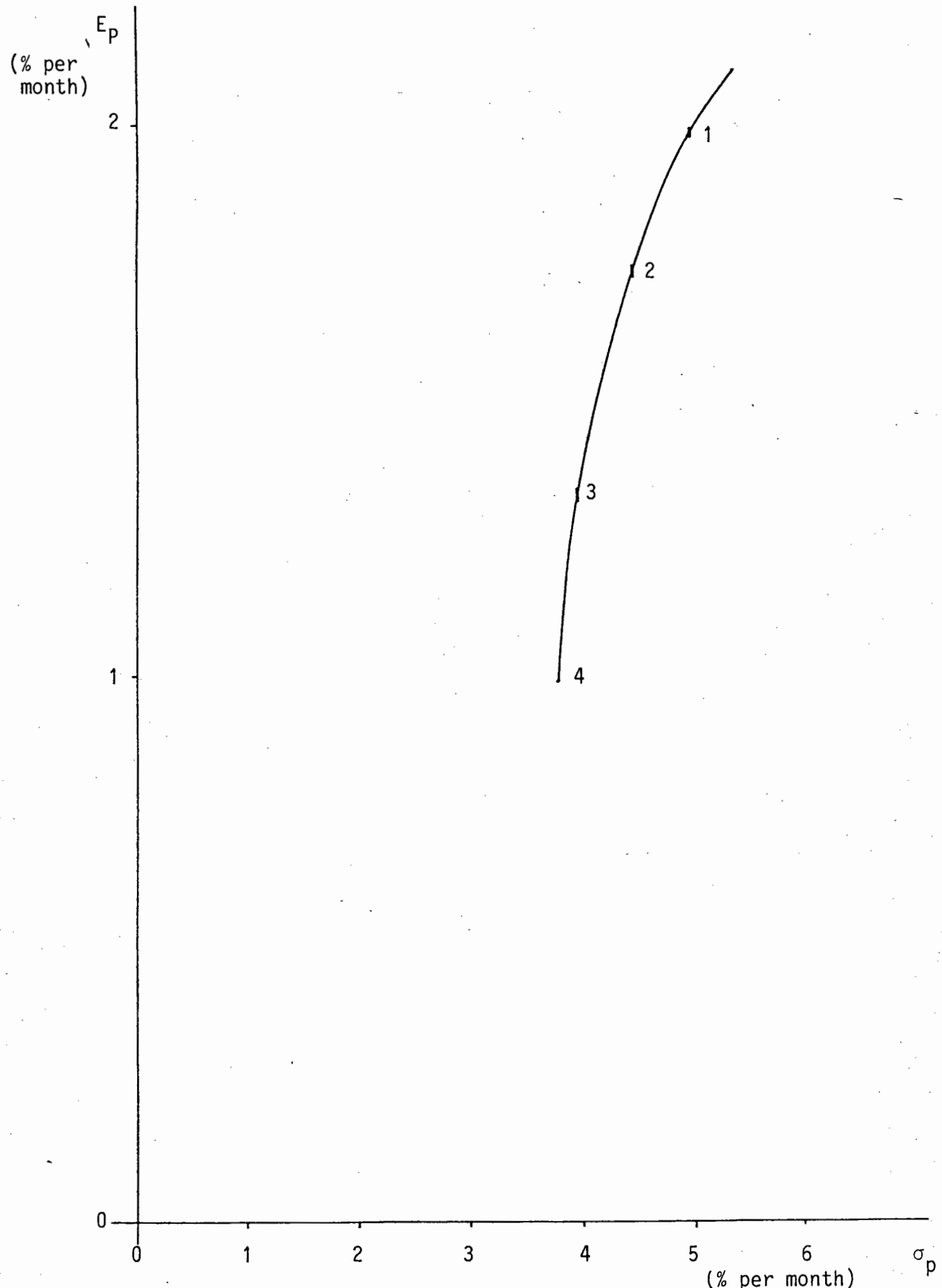


Figure 5.3 Unconstrained efficient frontiers - period 2
(Prices expressed in SA rands)

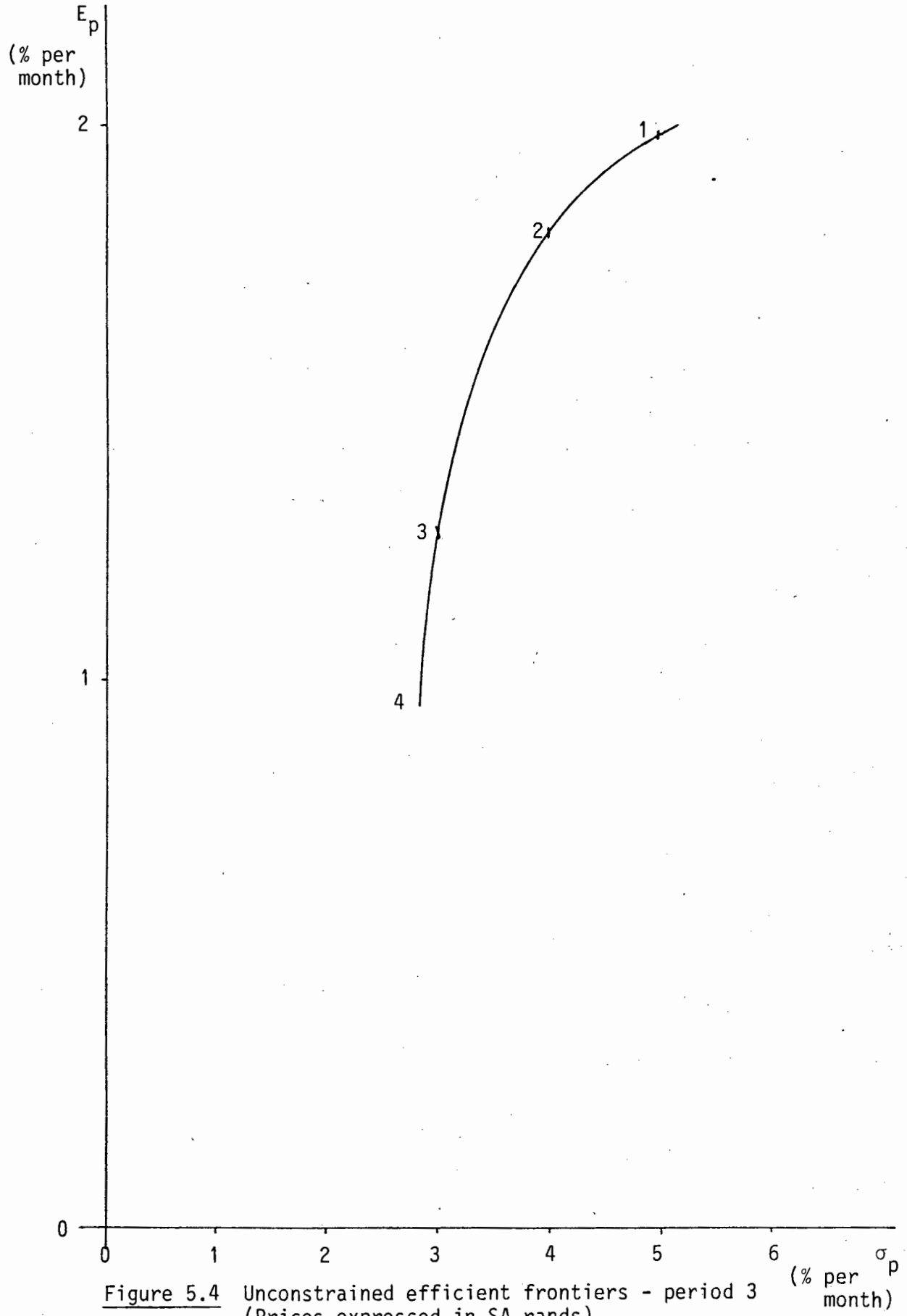


Figure 5.4 Unconstrained efficient frontiers - period 3
(Prices expressed in SA rands)

the level of diversification and magnitude of weighting in each security. Tables 5.1, 5.2 and 5.3 refer to Figures 5.2, 5.3 and 5.4 respectively and represent periods 1, 2 and 3 as defined in section 5.2.

It can be seen that at very high risks, very limited diversification occured. This was due to the fact that only a limited number of securities in each period had sufficiently high return to compensate the investor for the high risk.

As the risk of the efficient portfolios decreased, diversification increased with concommitant smaller weightings in each security. This is due to the fact that these portfolios are approaching the area in which the 'market portfolio' lies, and since by definition the market portfolio is fully diversified it is clear that these portfolios show more diversification. Lower variance in portfolio returns is also to be expected as diversification increased. In fact the least risky efficient portfolio in periods 1, 2 and 3 contained 5, 5 and 3 securities respectively. In other words, except for period 3, all five securities considered were held in some proportion in the lowest risk portfolio.

Certain of the securities appear to persist in the efficient portfolios over large risk ranges. For example, metals appeared in all portfolios considered in periods 1 and 2, as do SA shares in period 1 and soft commodities in period 2. The only way this can occur is if the security not only

	high risk	medium risk	low risk	lowest risk
SA shares	0,9957	0,3529	0,1577	0,0775
US shares	-	-	-	0,0968
UK shares	-	-	0,0771	0,0302
metals	0,0043	0,6471	0,7498	0,4617
soft commods.	-	-	0,0154	0,3338
σ_p	5,000	3,500	2,000	1,561
E_p	0,676	0,636	0,551	0,307

Table 5.1 Proportion of funds invested in securities at various levels of risk (SA rands - period 1)
SA shares ≡ JSE All Share Index
metals, soft commodities ≡ 1 IPI-type index each

	high risk	medium risk	low risk	lowest risk
SA shares	-	-	0,0172	0,0439
US shares	-	-	0,0730	0,1871
UK shares	-	-	0,0377	0,0647
metals	0,1143	0,3250	0,3669	0,3956
soft commods.	0,8857	0,6750	0,5052	0,3087
σ_p	5,000	4,500	4,000	3,767
E_p	1,985	1,774	1,416	0,989

Table 5.2 Proportion of funds invested in securities at various levels of risk (SA rands - period 2)
SA shares ≡ JSE All Share Index
metals, soft commodities ≡ 1 IPI-type index each

	high risk	medium risk	low risk	lowest risk
SA shares	-	0,1158	0,2075	0,2550
US shares	-	0,1331	0,2824	0,2928
UK shares	0,0489	0,1175	0,0188	-
metals	0,9511	0,6336	0,2278	-
soft commod.	-	-	0,2635	0,4522
σ_p	5,000	4,000	3,000	2,841
E_p	1,987	1,741	1,220	0,951

Table 5.3 Proportion of funds invested in securities at various levels of risk (SA rands - period 3)
SA shares ≡ JSE All Share Index
metals, soft commodities ≡ 1 IPI-type index each

had a high return during that period, but also showed very little covariance with any of the other currently efficient securities.

It may be noticed that some securities appearing in adjacent portfolios came in at low proportions, rose to a peak and then fell again. Examples are metals in period 1 and UK shares in period 3. These securities appeared to be efficient over a wide risk range. Other securities came in at a high proportion of the portfolio and steadily diminished. Examples of securities displaying this behaviour are SA shares in period 1, soft commodities in period 2 and metals in period 3. Still further securities started in low proportions and grew in importance as the risk level dropped, for example metals in period 2, soft commodities in periods

1 and 3, US shares in periods 2 and 3, and SA shares in periods 2 and 3.

It is of interest that every security appeared in at least one of the efficient portfolios in each period.

The highest proportion for a security appearing in only one portfolio in any given period was US shares (0,0968) in period 1. This security occurred in the lowest risk portfolio possible. No securities appeared in a non-peripheral portfolio only. This tends to substantiate the claim that "*there is a definite hierachy of efficiency dominance in each period which implies that when a* [security] *is efficient enough to be included in efficient portfolios on the efficient frontier it tends to persist in these portfolios for quite a range in risk. Seldom does a* [security], *once having achieved efficiency dominance, only appear in a very localised area of the efficient frontier.*" (Carter, Affleck-Graves and Money (1982)).

5.4 US and UK Investors

Although the aim of this study is to determine *ex post* what securities, and in what quantities, a South African investor should have invested in if the present exchange control regulations had not existed over the period February 1965 to January 1980, it is of interest to observe the situation from the US and UK investor's point of view.

To achieve these situations it is necessary to convert all the security prices into US dollars or UK pounds by multiplying the originally quoted price series by the appropriate exchange rates. For example, to convert to US dollars

$$P_{\$} = (P_R)(ER_{R,\$})$$

or $$(P_{£})(ER_{£,\$})$$

where the symbols are as previously defined in section 5.2. Again, commodity prices expressed in pence were first converted into UK pounds, and the two price series quoted in US dollars (the Standard and Poor's '500' Index and the price of gold) were left as they were.

The monthly returns (calculated as the change in the natural logarithm of the price) were calculated for each security, mean monthly returns for each security for each of the same three sub-periods were found, as were the 5×5 covariance matrix for each sub-period. This data was then used as input for the portfolio selection program of Affleck-Graves (1974).

Tables 5.4, 5.5 and 5.6 show the efficient portfolios at various widely differing risk levels as viewed by a US investor in periods 1, 2 and 3 respectively. Tables 5.7, 5.8 and 5.9 show the efficient portfolios for a UK investor in each of the periods 1, 2 and 3. Each of the given portfolios constitutes a widely differing risk level.

	high risk	medium risk	low risk	lowest risk
SA shares	0,9970	0,6648	0,1579	0,0801
US shares	-	-	-	0,0964
UK shares	-	-	0,0857	0,0393
metals	0,0030	0,3352	0,7288	0,4496
soft commods.	-	-	0,0276	0,3345
σ_p	5,000	3,5000	2,000	1,587
E_p	0,682	0,640	0,548	0,311

Table 5.4 Proportion of funds invested in securities at various levels of risk (US $ - period 1)
SA shares ≡ JSE All Share Index
metals, soft commodities ≡ 1 IPI-type index each

	high risk	medium risk	low risk	lowest risk
SA shares	-	-	-	-
US shares	-	0,0496	0,1540	0,2568
UK shares	-	-	0,0222	0,0477
metals	0,1252	0,2505	0,3404	0,3105
soft commods.	0,8748	0,6999	0,4834	0,3850
σ_p	4,500	4,000	3,500	3,322
E_p	2,056	1,812	1,464	1,112

Table 5.5 Proportion of funds invested in securities at various levels of risk (US $ - period 2)
SA shares ≡ JSE All Share Index
metals, soft commodities ≡ 1 IPI-type index each

	high risk	medium risk	low risk	lowest risk
SA shares	-	-	-	0,0672
US shares	-	0,1233	0,4015	0,4567
UK shares	0,0153	0,1702	0,0721	-
metals	0,9847	0,7065	0,4819	0,0679
soft commods.	-	-	0,7303	0,4083
σ_p	5,000	4,000	3,000	2,392
E_p	1,685	1,516	1,186	0,659

Table 5.6 Proportion of funds invested in securities at various levels of risk (US $ - period 3)
SA shares ≡ JSE All Share Index
metals, soft commodities ≡ 1 IPI-type index each

	high risk	medium risk	low risk	lowest risk
SA shares	0,8644	0,5998	0,2531	0,0479
US shares	-	-	-	0,0311
UK shares	-	-	0,0935	0,2316
metals	0,1356	0,4002	0,6534	0,2955
soft commods.	-	-	-	0,3940
σ_p	5,000	4,000	3,000	2,323
E_p	0,918	0,885	0,827	0,526

Table 5.7 Proportion of funds invested in securities at various risk levels (UK £ - period 1)
SA shares ≡ JSE All Share Index
metals and soft commodities ≡ 1 IPI-type index each

	high risk	medium risk	low risk	lowest risk
SA shares	-	-	-	-
US shares	-	-	-	-
UK shares	-	-	0,0485	0,0927
metals		0,1541	0,2302	0,2619
soft commods.	1,0000	0,8459	0,7213	0,6454
σ_p	5,000	4,500	4,000	3,790
E_p	2,190	2,035	1,816	1,653

Table 5.8 Proportion of funds invested in securities at various risk levels (UK £ - period 2)
SA shares ≡ JSE All Share Index
metals and soft commodities ≡ 1 IPI-type index each

	high risk	medium risk	low risk	lowest risk
SA shares	-	-	0,0048	0,0885
US shares	-	0,0048	0,2399	0,1933
UK shares	0,0186	0,2684	0,1978	0,1197
metals	0,9814	0,7268	0,5059	0,1090
soft commds.	-	-	0,0516	0,4895
σ_p	4,500	3,750	3,000	2,475
E_p	1,777	1,713	1,400	0,893

Table 5.9 Proportion of funds invested in securities at various risk levels (UK £ - period 3)
SA shares ≡ JSE All Share Index
metals and soft commodities ≡ 1 IPI-type index each

To compare the efficient portfolios of the three investors (i.e. South African, US and UK investors) the lowest risk portfolio for each investor for each period will be considered. It is in this region that the most diversification occurs, and thus the most differences will be found.

From the tables it will be noticed that in general in all periods each investor should have held a greater proportion of his own domestic shares than any of the other investors should have held in that share. This is because the prices (in local currency) are not subject to variations in the exchange rate and thus we was not faced with any exchange rate risk. The only exception occurred when US investors held more South African shares that did the South African investors in period 1. However, in this case the difference was slight (8,01% as opposed to 7,75%).

The metals and soft commodities together always made up a large proportion of any investor's portfolio. This proportion ranged between 45,22% for South African investors in period 3 to 90,73% for UK investors in period 2.

In period 1 US and South African investors had very similar portfolios. UK investors should have held a smaller proportion of South African and US shares, but a much larger proportion of UK shares (23,16% as opposed to less than the 4% of the US investor). A far smaller proportion of metals were held by UK investors (29,55% as opposed to 44,96% by US investors and 46,17% by South African investors), whereas

the soft commodities formed a larger part of their portfolio (39,40% compared to about 33% for South African and US investors).

In period 2 the South African investor appeared to be more diversified at the lowest risk level, investing in all five security groups, whereas the US and UK investor invested in only four and three groups respectively. The foreign investors both held no South African shares. The UK investors also held no US shares although these formed major portions of US and South African investor's portfolios. UK shares were held in small quantities by all investors, although the UK investor favoured these more than his South African or US counterparts.

In period 3 the UK investor was much more diversified, investing in all five security groups. US investors and South African investors invested in only four and three of the groups respectively. The UK investor invested a fairly substantial portion of his portfolio in his domestic shares, whereas the US or South African investor did not. All chose US shares for their portfolios - the US investor invested as much as 45,67% of his funds on his domestic share market whereas the proportions for the South African and UK investors were substantially less - 29,28% and 19,33% respectively. South African shares were held predominantly by South African investors (25,50%) whereas the proportions for overseas investors were 8,85% (UK investors) and 6,72% (US investors).

Thus South African shares did not appear to be very attractive to overseas investors at all. In period 2 the US and UK portfolios did not include any South African shares and in period 3 very small amounts were included and then at the lowest risk levels only. One question which this study therefore raises is whether in fact there is any benefit at all to US or UK investors in holding South African shares.

This is beyond the scope of this thesis which aims only at addressing the problem from the South African investor's point of view. However, the South African authorities should note that although the percentages of South African shares included by UK and US investors are small, the size of those markets would still probably result in a significant inflow to South Africa even at these low percentages.

5.5 Summary and conclusions

In this chapter some empirical portfolio selection results based on recent data from the Johannesburg Stock Exchange and other international stock and commodity markets were presented.

The basic assumption on which the theory rests is that an investor chooses a portfolio solely on the basis of the return he expects to derive from holding that portfolio and the risk associated in holding it. In addition, it is assumed that any investor prefers more return to less, and

at the same time desires as low a risk as possible. For any investor who does not conform to the above assumption the model presented is useless.

From this basic assumption the Markowitz portfolio selection model was derived. This model was empirically tested over the same three non-overlapping periods for the situation where the investor is buying securities in South African rands, US dollars or UK pounds. These three situations represent the strategy that might be followed by a South African investor, a US investor and a UK investor respectively.

The main conclusion reached from this initial empirical study is that it would have been beneficial for a South African investor to have divested a large proportion of his funds in securities outside of the South African share market during the period February 1965 to January 1980 if exchange control regulations had not existed over that period. These proportions differed over the three non-overlapping sub-periods considered, and also from one risk level to another. A further conclusion is that the exchange rate between the three currencies (South African rands, US dollars and UK pounds) has a distinct bearing on which securities, and in what proportions, each investor will choose to invest his funds in.

In the next chapter a closer look is taken at the individual securities within the five groups defined in this chapter.

CHAPTER 6

EFFICIENT INTERNATIONAL PORTFOLIOS FOR THE SOUTH AFRICAN INVESTOR - AN *EX POST* ANNUAL STUDY

6.1 Introduction

Under the existing exchange control regulations the only equity investments that a local investor may purchase are shares quoted on the Johannesburg Stock Exchange. Kruger Rands may also be purchased, but they have been ignored as investment media in this study since gold shares provide very similar performance to that shown by Kruger Rands. In fact, according to Bradfield (1983), gold shares have proved superior investments to Kruger Rands over recent years.

In the absence of exchange control the South African would be faced with a much expanded universe of securities from which to choose, as he would then be able to include foreign securities or even commodities in his portfolio. He could then possibly earn higher returns from fast-growing economies and firms. In addition the investor could strive for exchange rate gains as well.

In addition to these potential gains Solnik (1974) has also claimed that substantial advantages in risk reduction can be attained through portfolio diversification in foreign securities. He explains that

".... movements in stock prices in different countries are almost unrelated : Changes in price on the Paris Bourse appear independent of stock price fluctuations on the London exchange, and so on. When securities of one country (say the US) are doing worse than expected, another market is likely to be doing better, hence offsetting the losses. Simply by investing in stocks of different countries, the risk is drastically reduced."

Solnick's results indicate that an internationally well-diversified portfolio would be one-tenth as risky as a typical security and half as risky as a well-diversified portfolio of US stocks with the same number of holdings.

Thus it is desirable to construct two efficient frontiers, the first representing purely South African securities, and the second an expanded set including both international securities and commodities as well as the South African securities. This has been done for several recent time periods.

In the following sections these frontiers are calculated and they are graphically displayed and compared. In addition, the composition of the efficient portfolios, and in particular the optimal combination of risky securities are examined from a South African investor's point of view.

6.2 The Potential Benefits of a Relaxation in Exchange Control - An Empirical Study

This study was conducted over 18 periods of one year each from February 1965 to January 1983. Each period thus ran from February of one year to January of the following year. These divisions were chosen to accommodate the data available, and provide equal, non-overlapping periods. For the remainder of this thesis each period will be referred to by the year in which the period began. For example, the first period (February 1965 to January 1966) will be referred to as 1965, and so on. Month-end data (in South African rands) for all 25 securities under consideration for the entire period (216 months) was available.

Returns for each security in each month, the mean monthly returns per annum for each security and the 25×25 covariance matrix for each year were calculated according to the methodology discussed in section 5.2.

The aim of this study was three-fold : firstly, to compare the range and domain of the efficient frontiers of a portfolio of purely South African securities with those of a portfolio of international securities; secondly, to determine the composition of portfolios on the efficient frontiers of both groups at varying levels of risk; and thirdly, to determine the composition of the optimal combination of risky securities as defined by the Separation Theorem.

6.2.1 Comparison of the Unconstrained Efficient Frontiers for the years 1965 to 1982

Initially the efficient frontiers selected entirely from the 7 South African securities were established for each of the 18 annual periods employing the portfolio selection algorithm of Sharpe (1970). Then the efficient frontiers resulting from the expanded set of international securities (that is, all 25 securities) were created for each year using the same portfolio selection algorithm. These frontiers are all unconstrained in the sense that no individual sector or security was assigned a maximum in terms of the proportion of the total funds which could be invested in that sector or security. The South African and international efficient frontiers for the years 1965 to 1982 appear in Figures 6.1 to 6.18 respectively. It should be noted that the portfolio returns and risks are expressed in units of average monthly percentages.

The most notable features which emerge from Figures 6.1 to 6.18 are:

(a) The international efficient portfolios always dominated the South African efficient portfolios. That is, for a given risk level the international efficient portfolios always offered a larger return, or for a given return they had a smaller risk than the purely South African portfolios. This, of course, is to be expected because the expanded set also includes all the South African securities. However, it is of interest to note

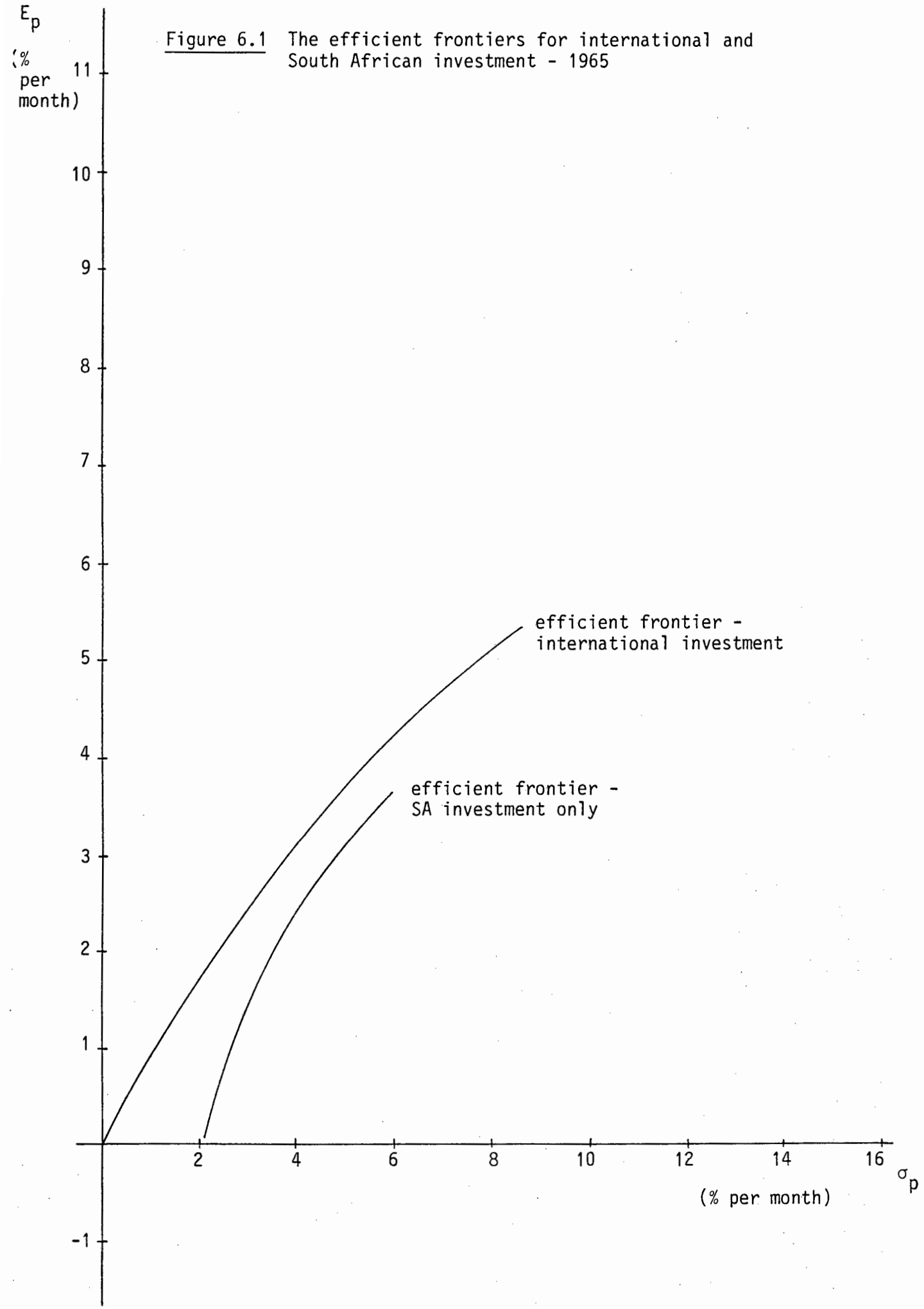
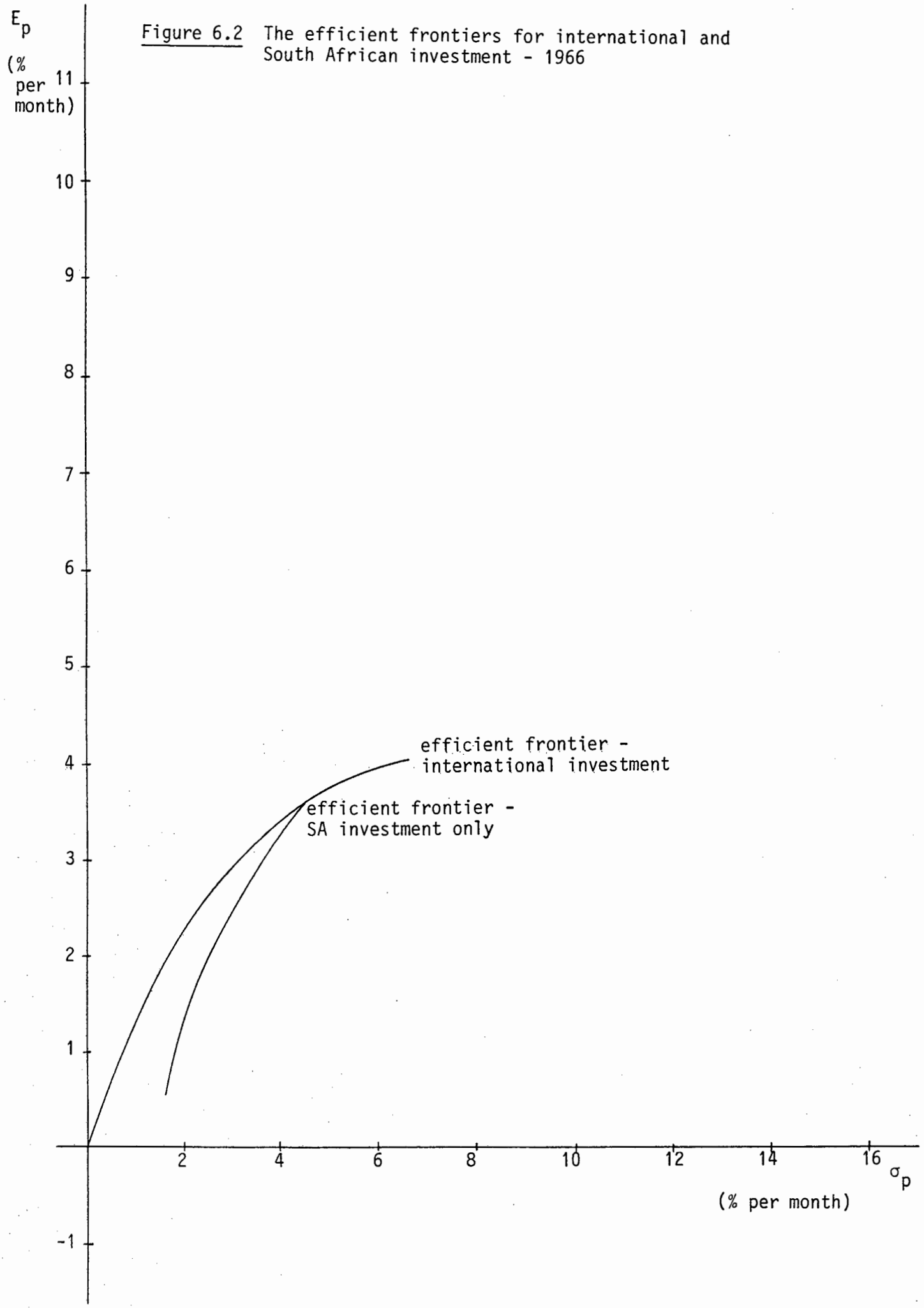


Figure 6.1 The efficient frontiers for international and South African investment - 1965

Figure 6.2 The efficient frontiers for international and South African investment - 1966



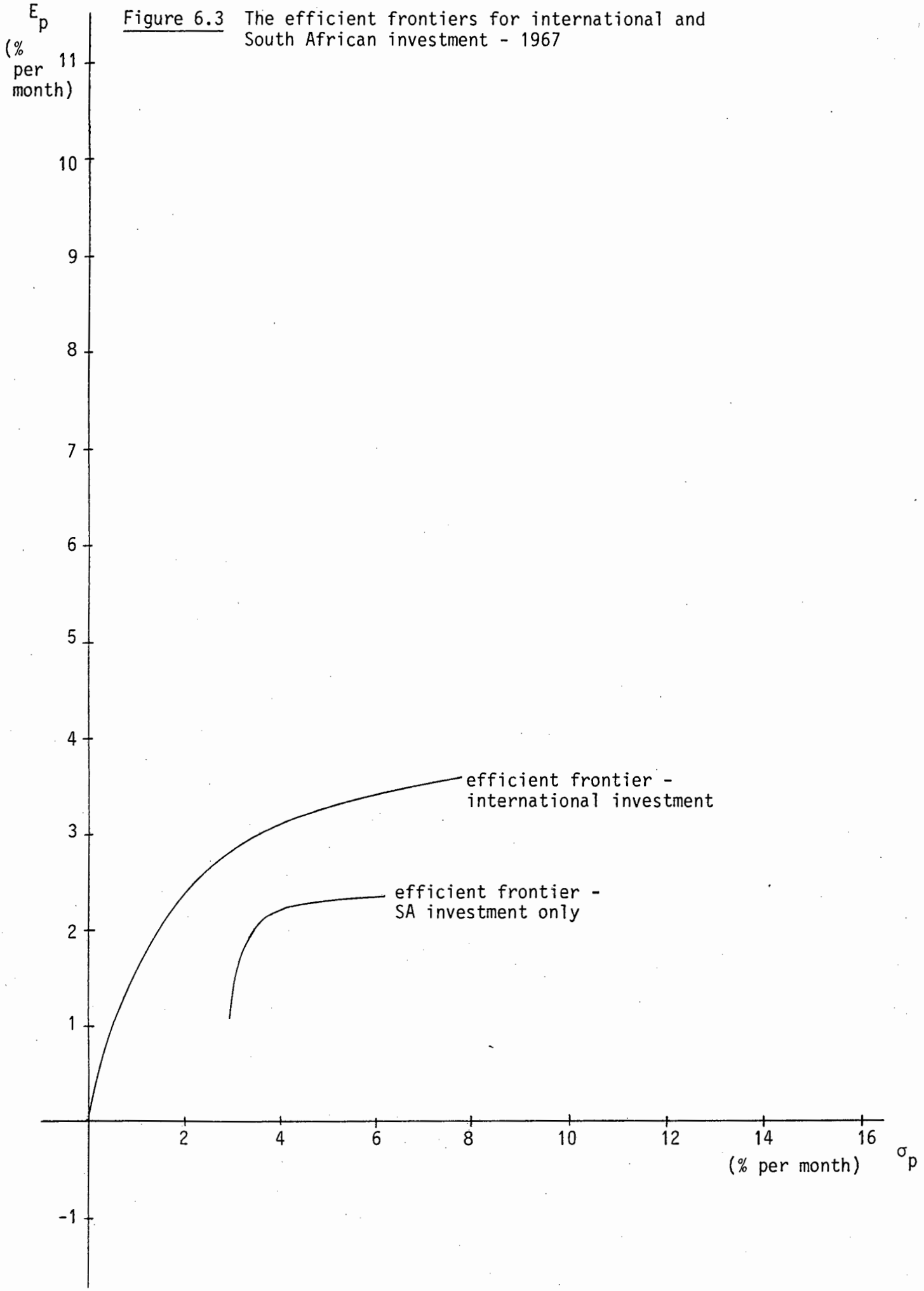


Figure 6.3 The efficient frontiers for international and South African investment - 1967

Figure 6.4 The efficient frontiers for international and South African investment - 1968

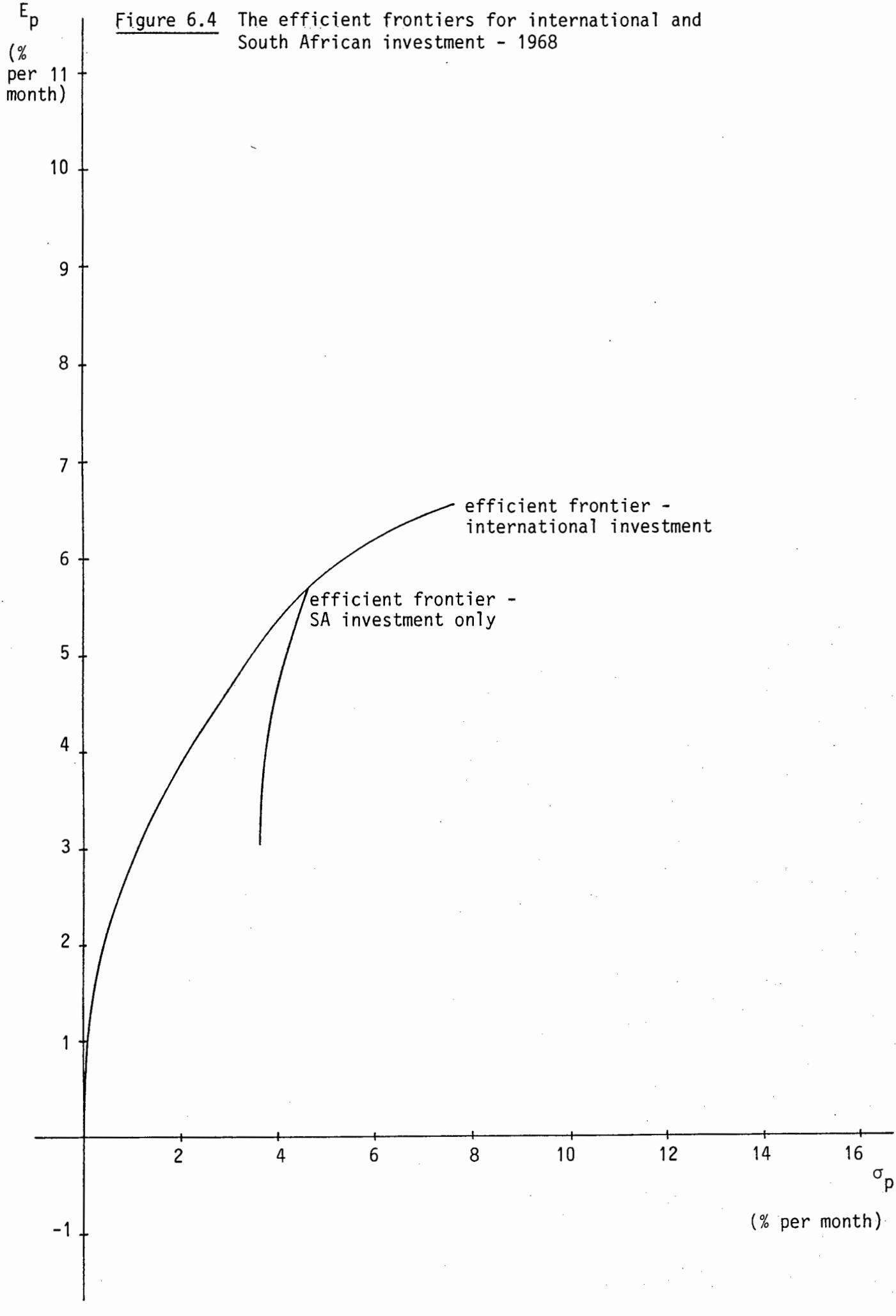


Figure 6.5 The efficient frontiers for international and South African investment - 1969

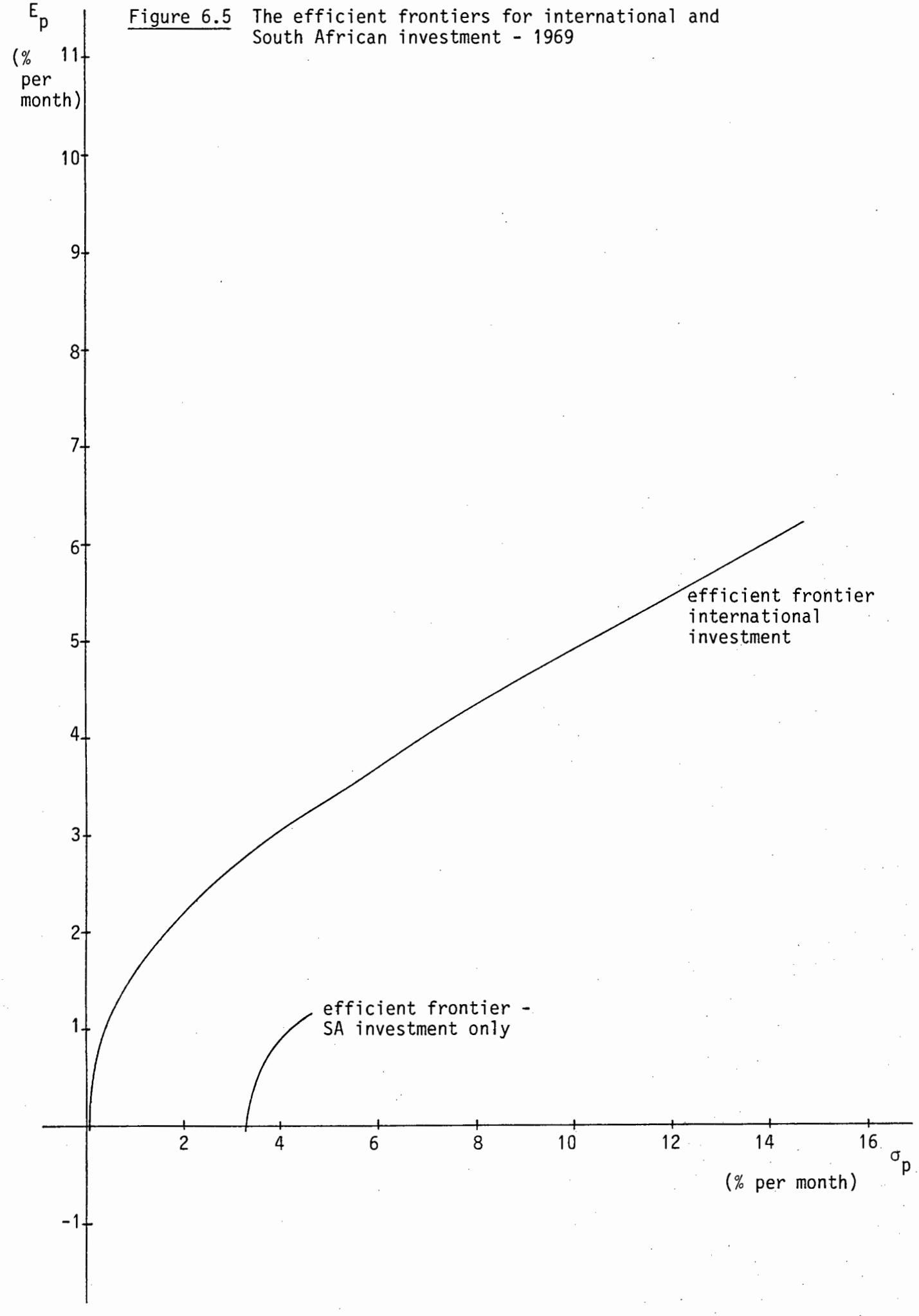


Figure 6.6 The efficient frontiers for international and South African investment - 1970

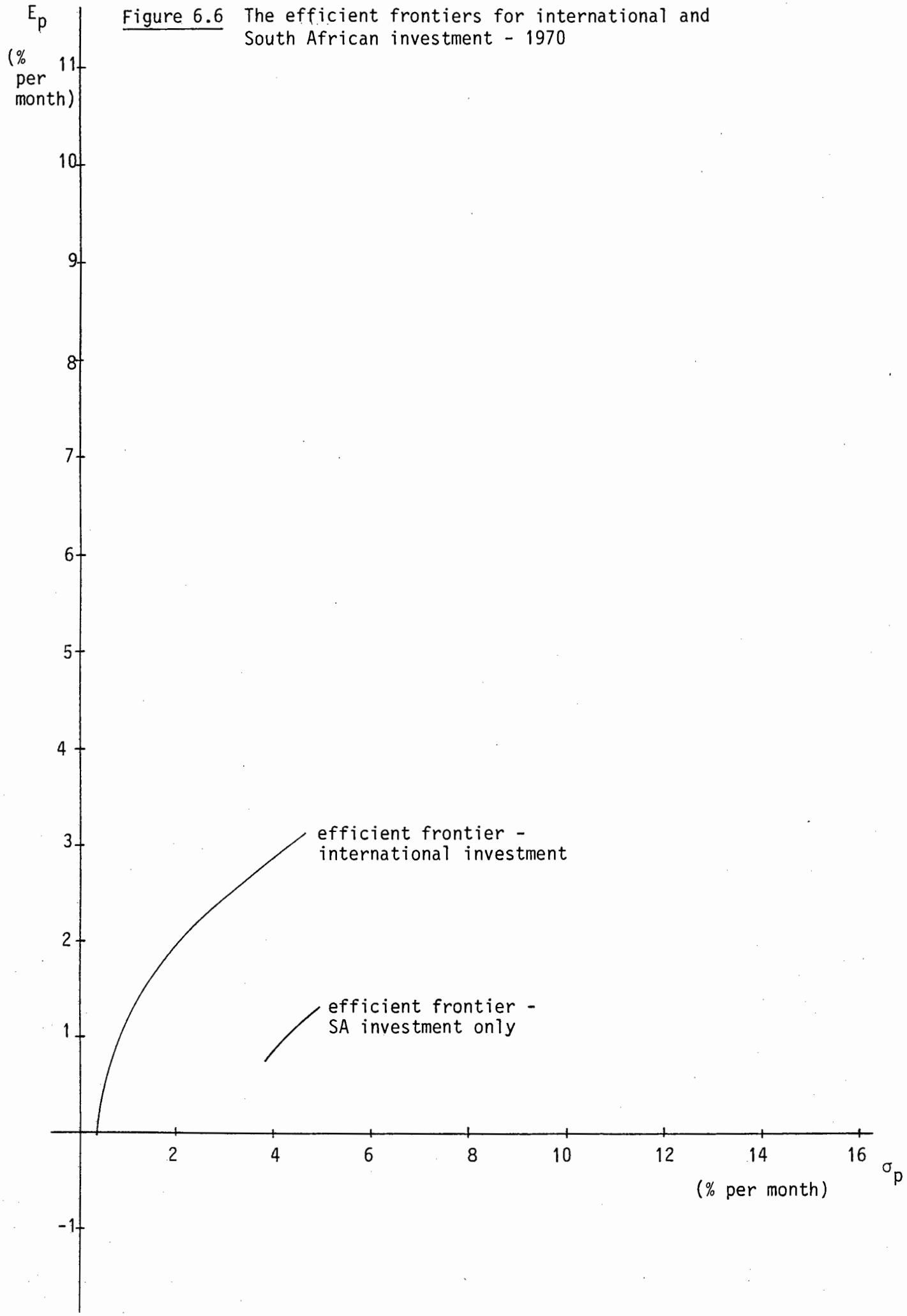


Figure 6.7 The efficient frontiers for international and South African investment - 1971

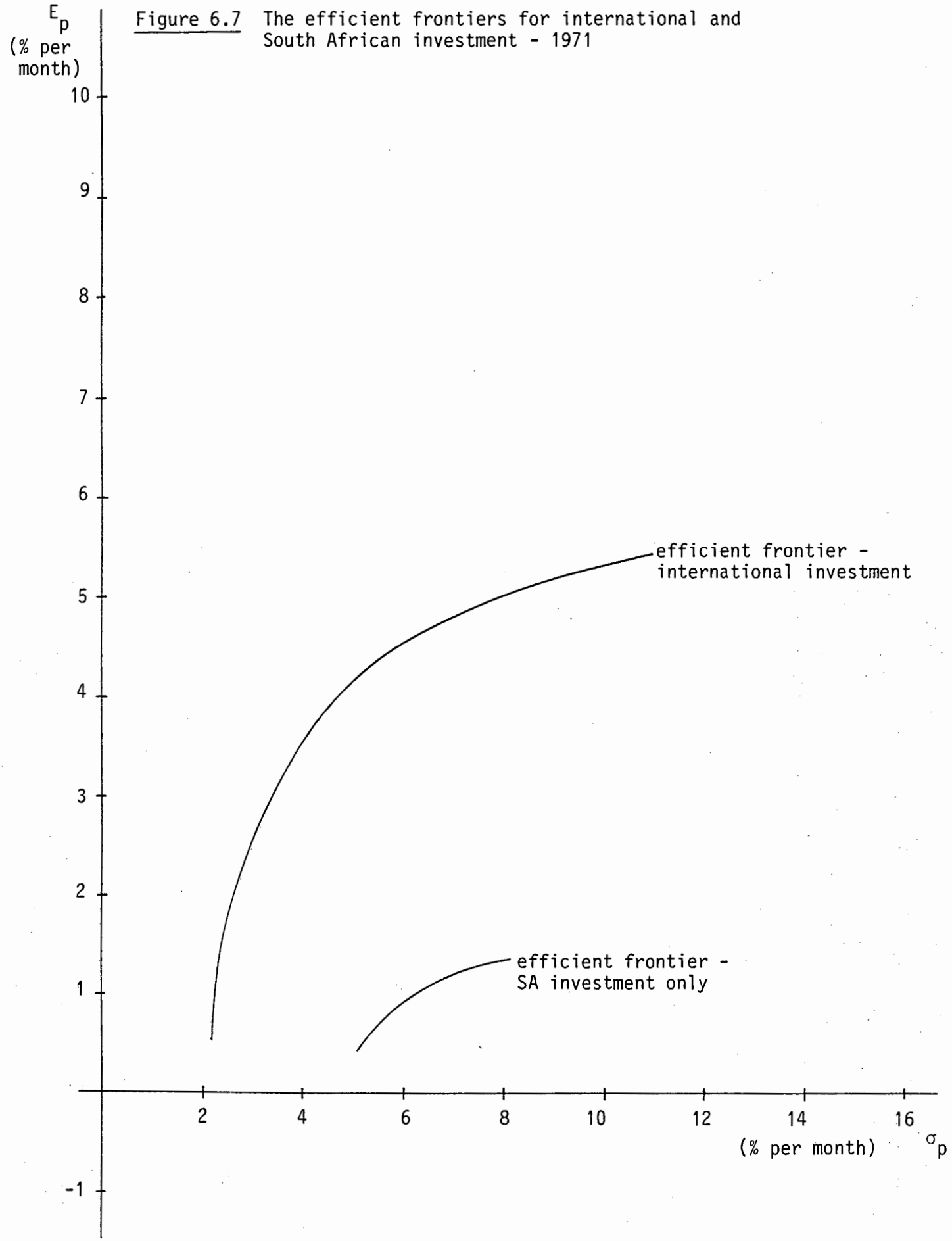
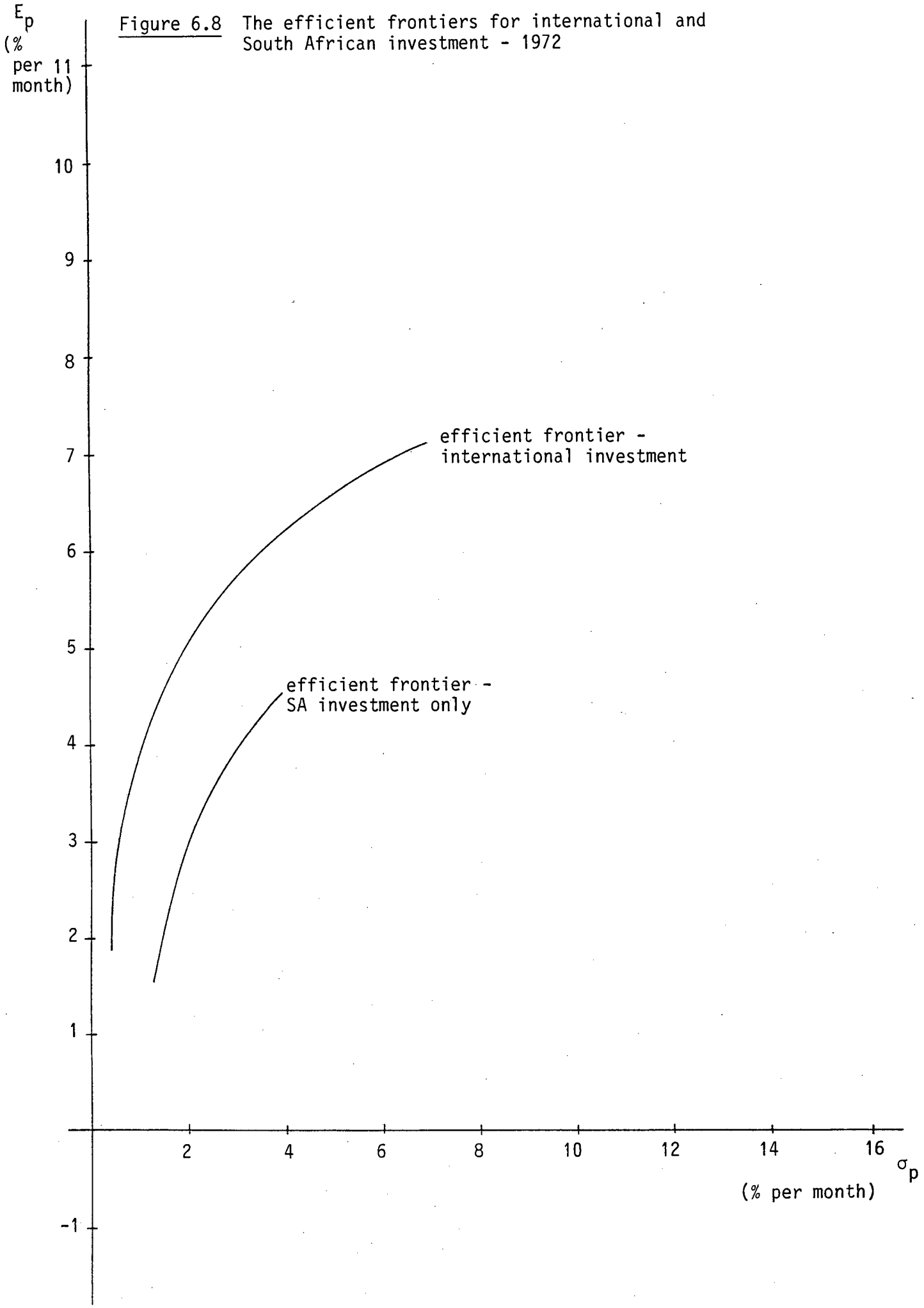


Figure 6.8 The efficient frontiers for international and South African investment - 1972



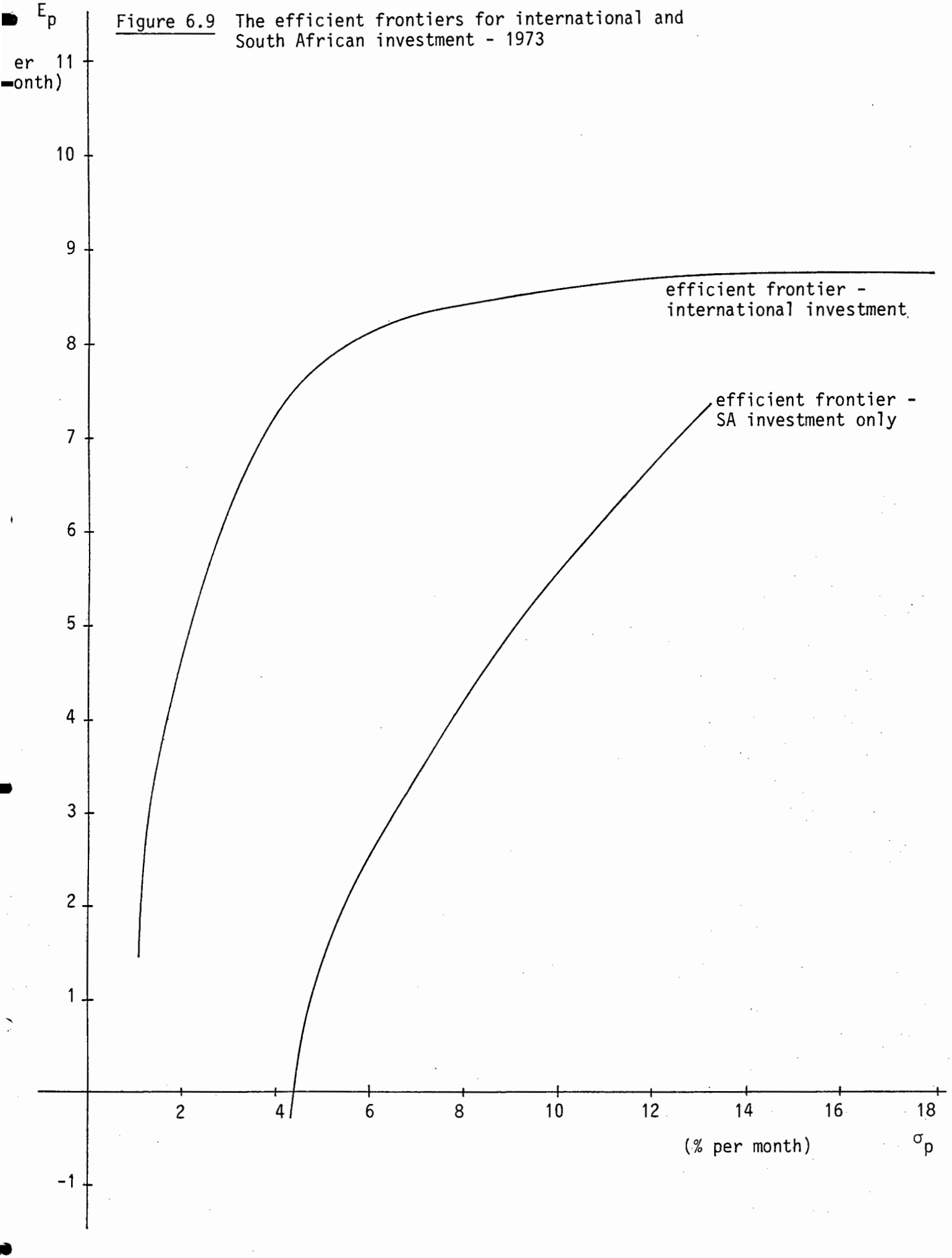
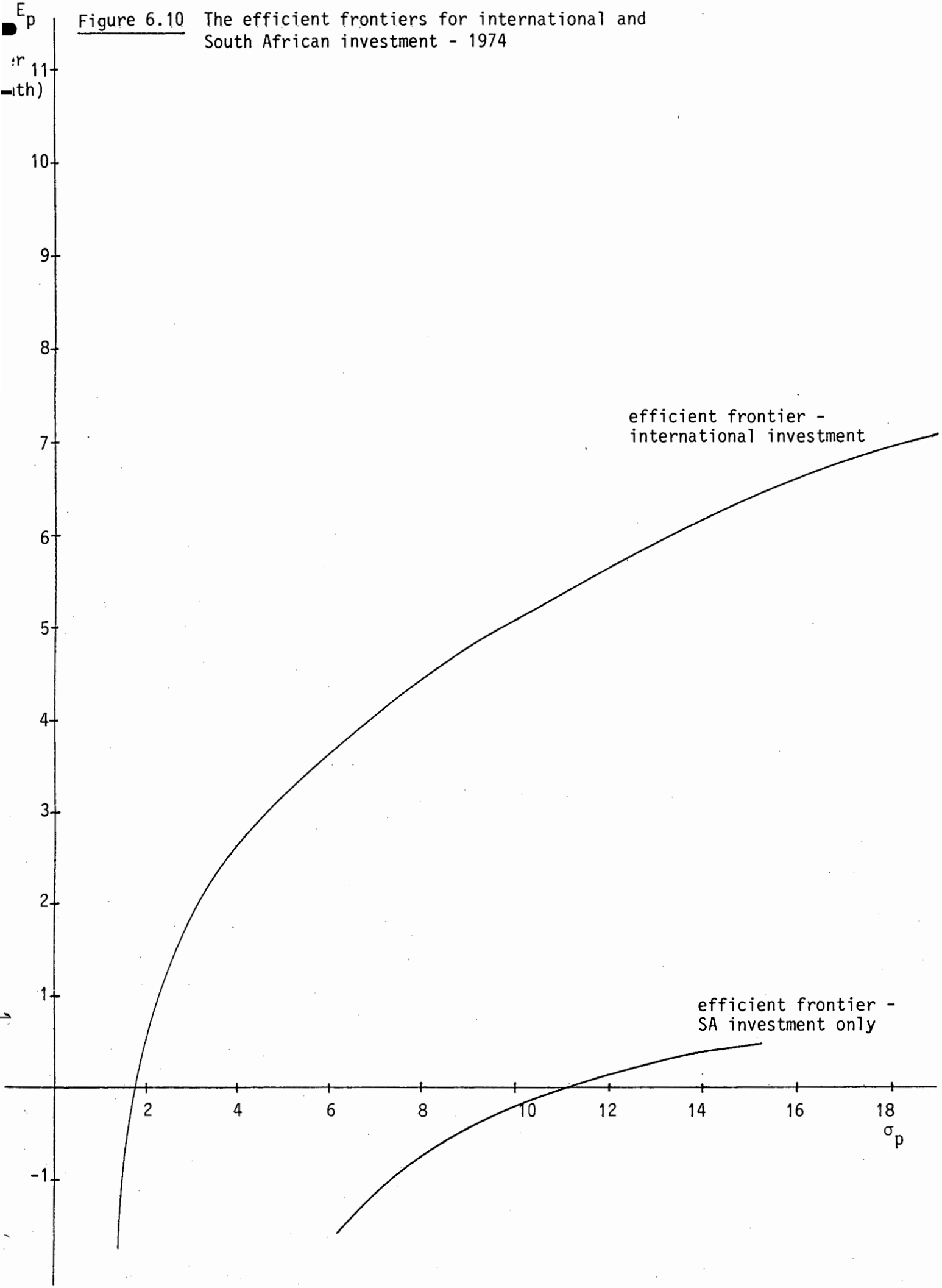


Figure 6.9 The efficient frontiers for international and South African investment - 1973

Figure 6.10 The efficient frontiers for international and South African investment - 1974



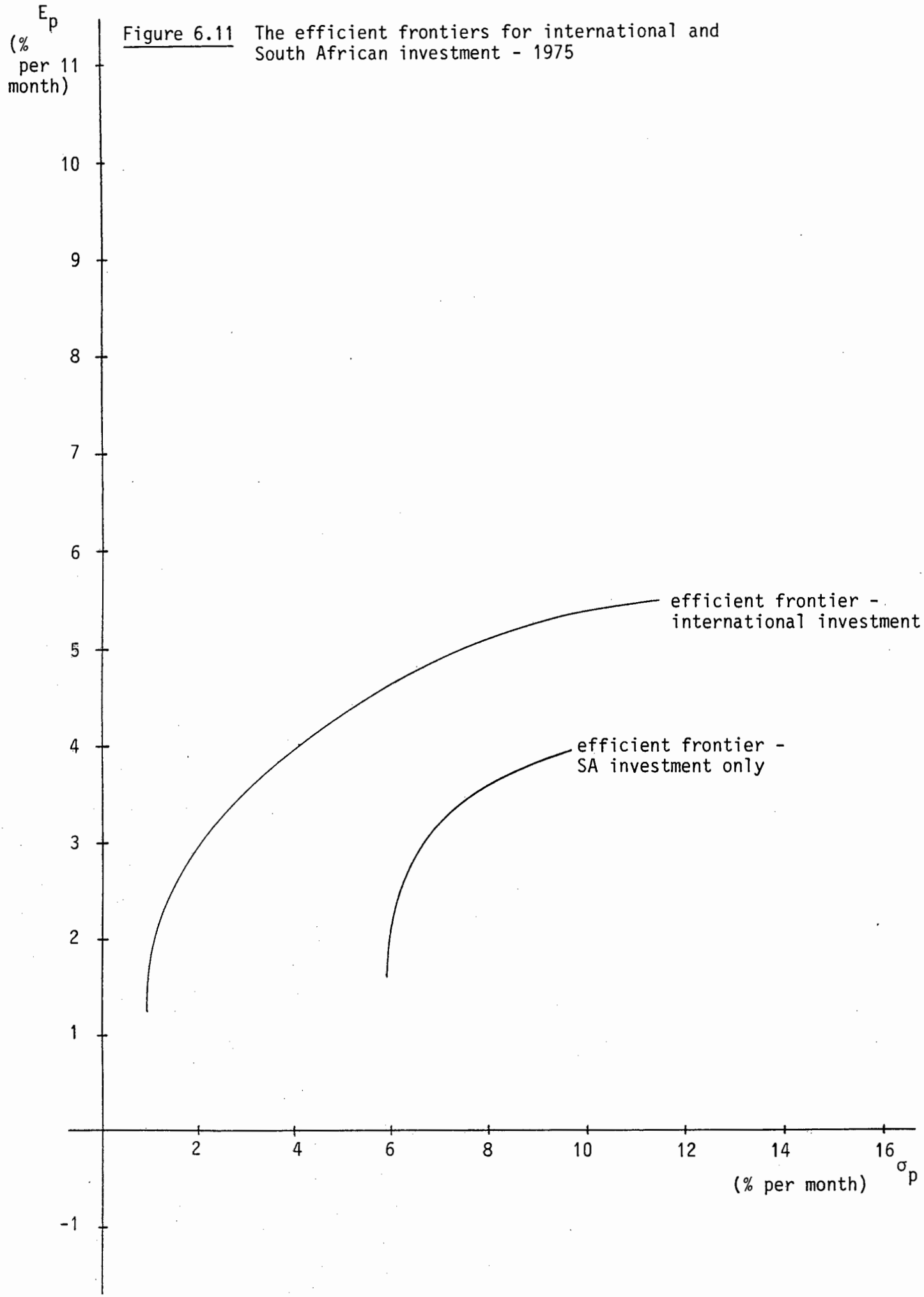


Figure 6.11 The efficient frontiers for international and South African investment - 1975

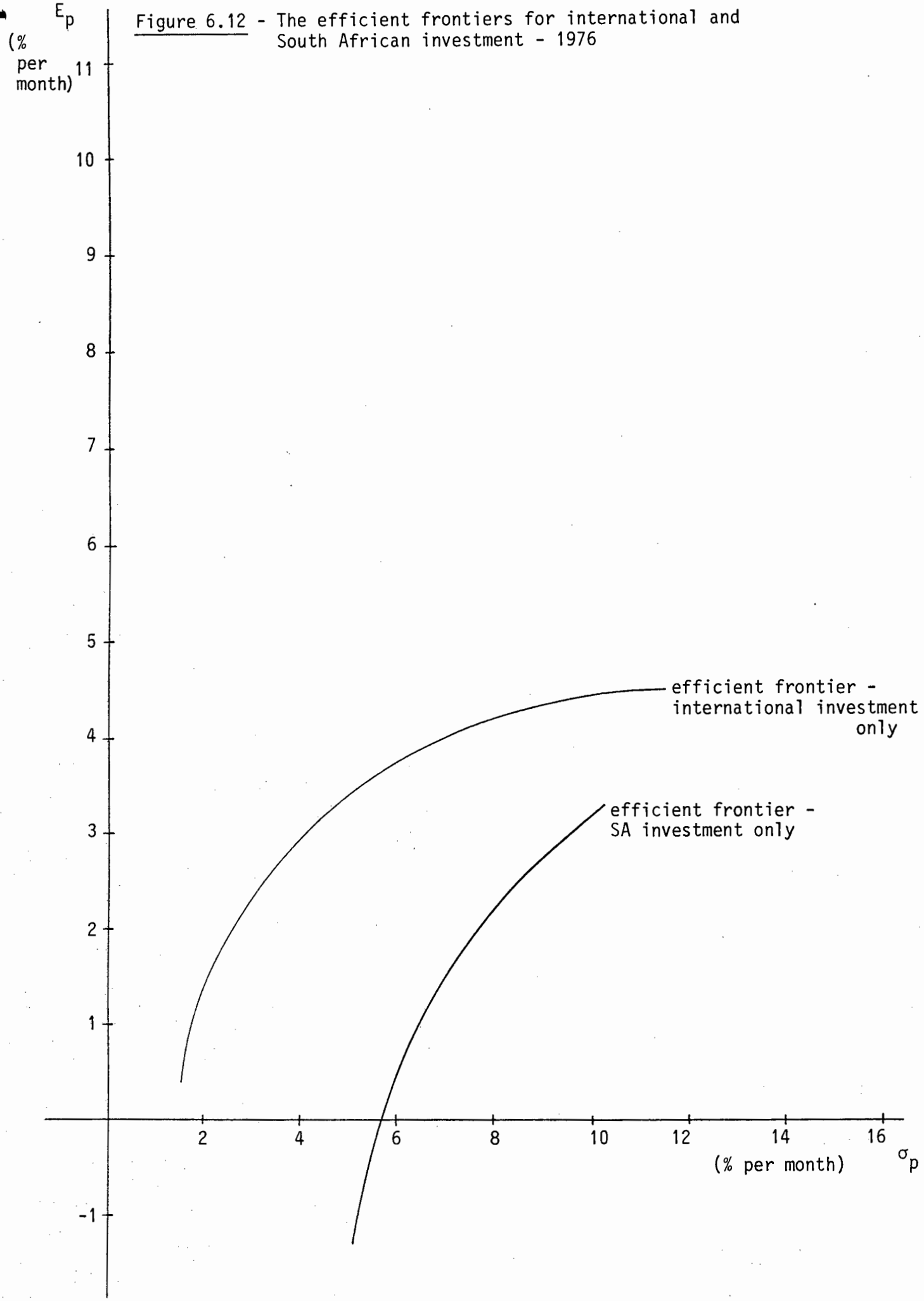


Figure 6.12 - The efficient frontiers for international and South African investment - 1976

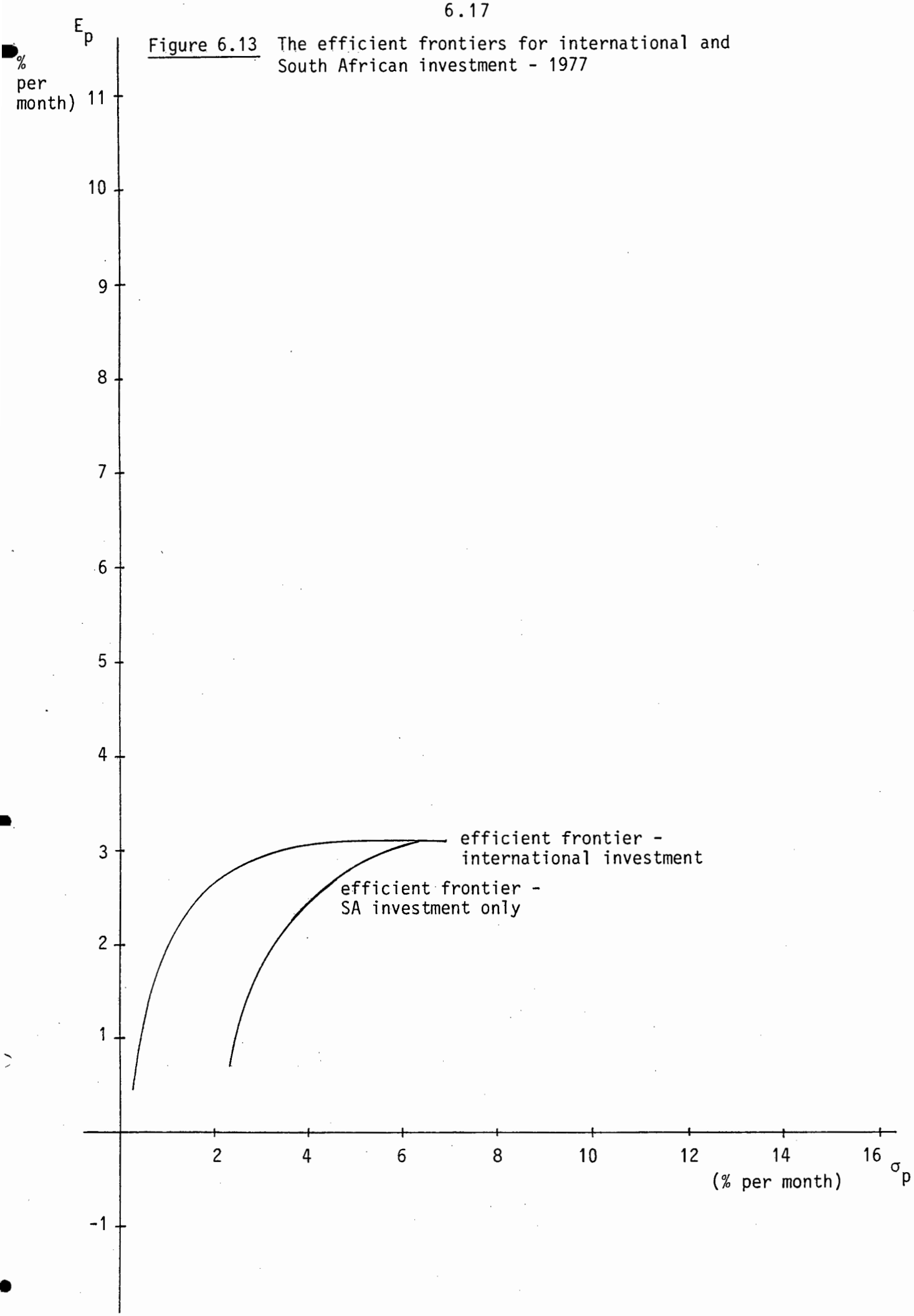


Figure 6.13 The efficient frontiers for international and South African investment - 1977

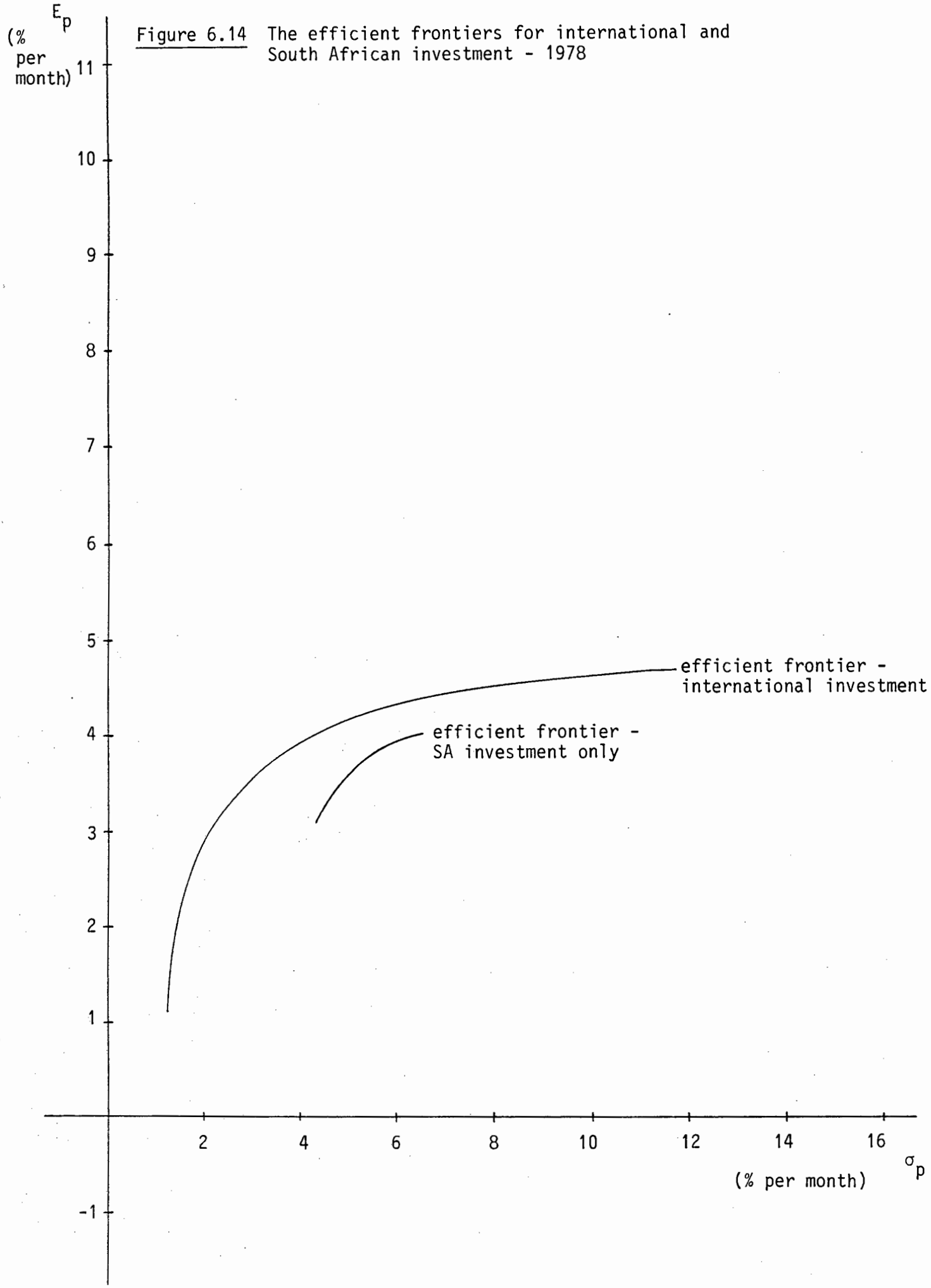


Figure 6.14 The efficient frontiers for international and South African investment - 1978

Figure 6.15 The efficient frontiers for international and South African investment - 1979

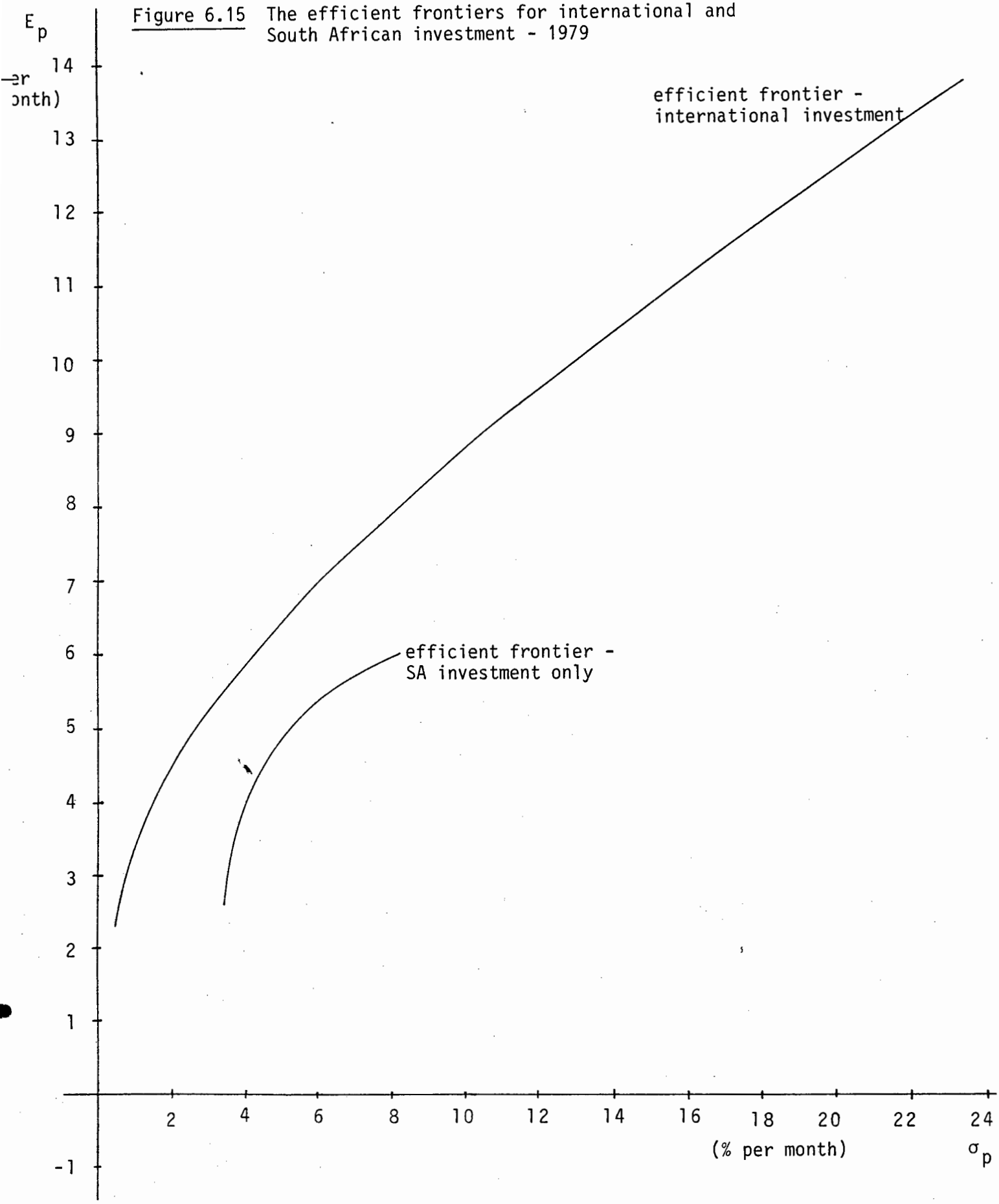


Figure 6.16 The efficient frontiers for international and South African investment - 1980

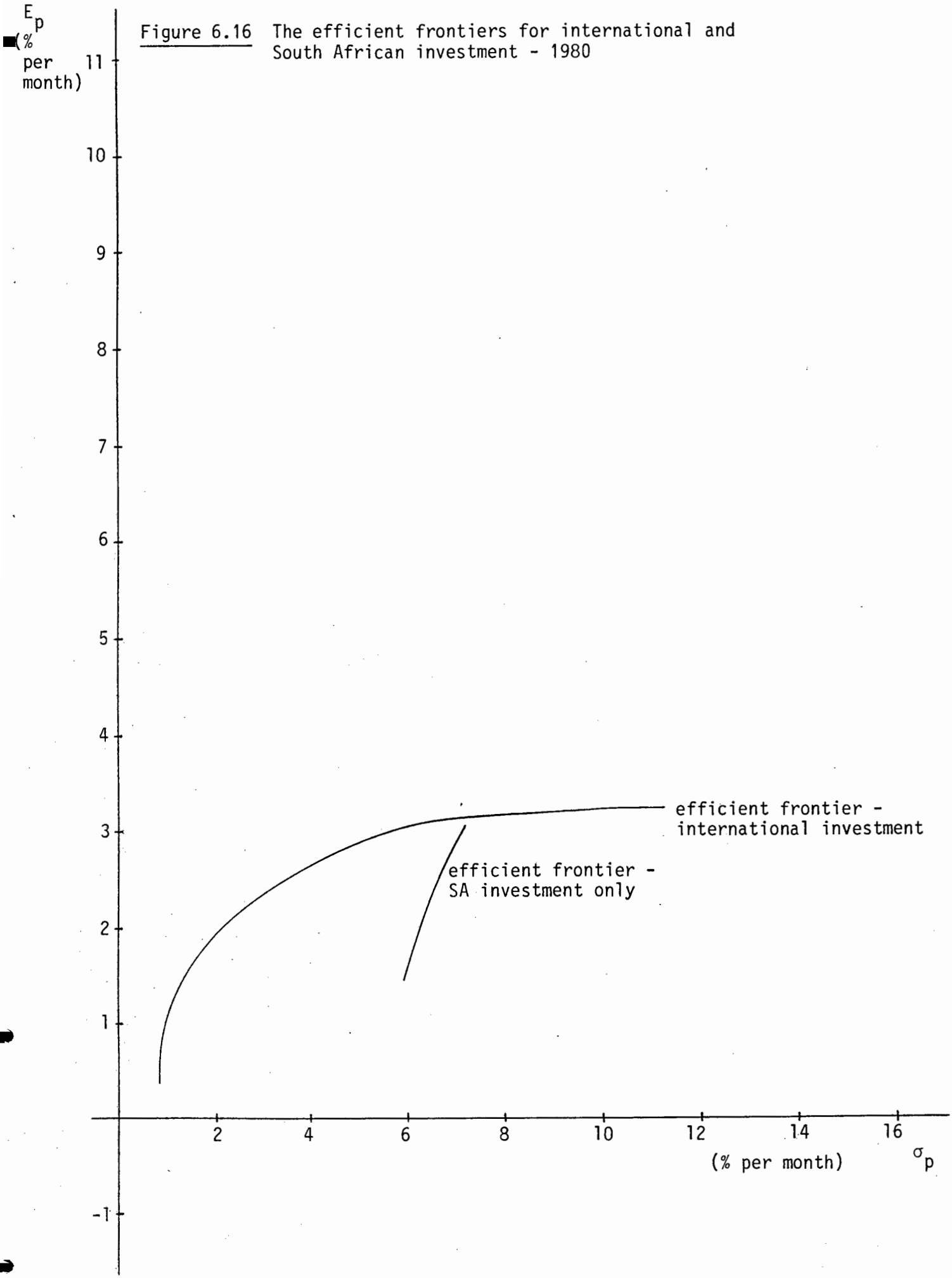


Figure 6.17 The efficient frontiers for international and South African investment - 1981

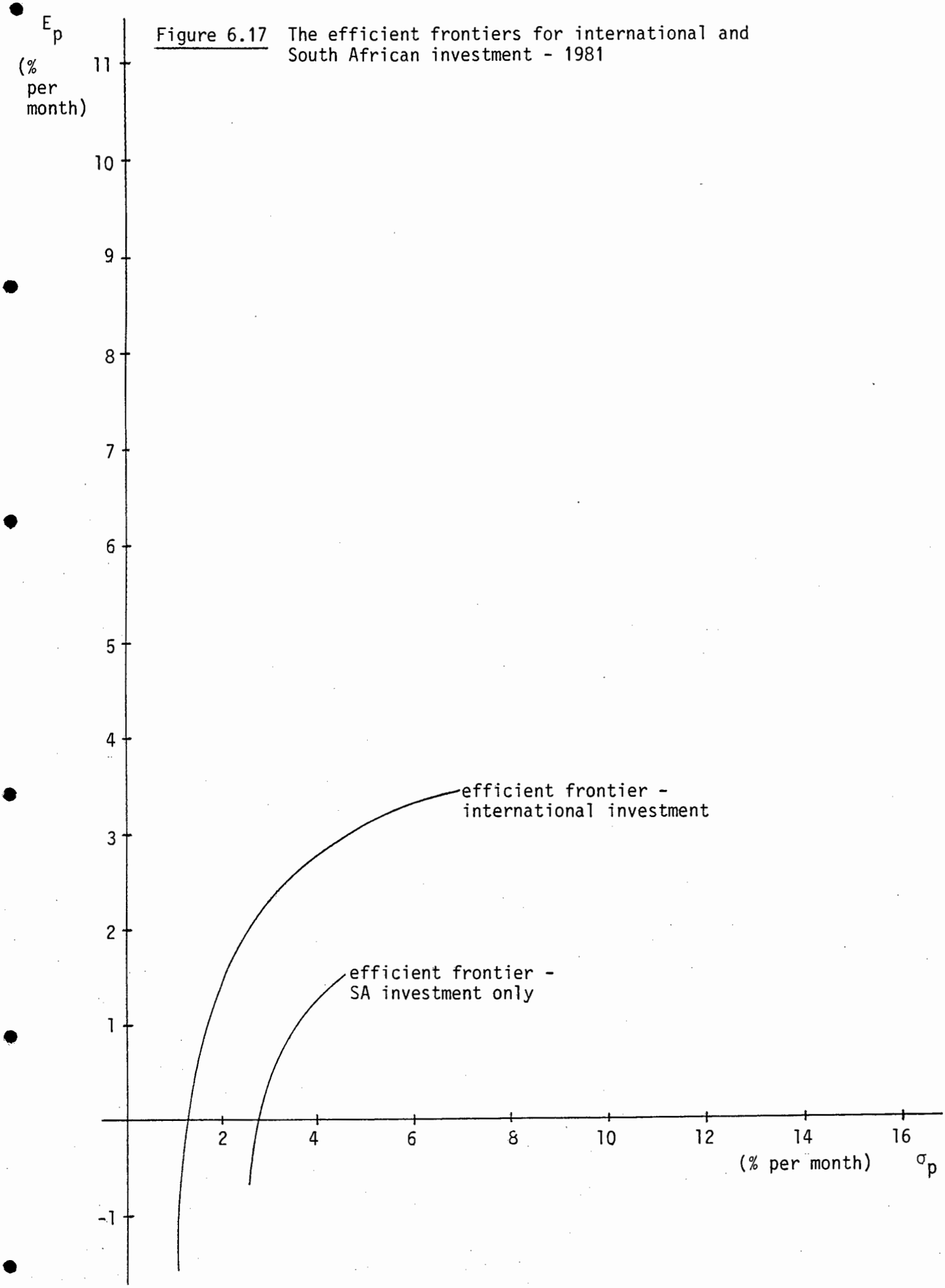
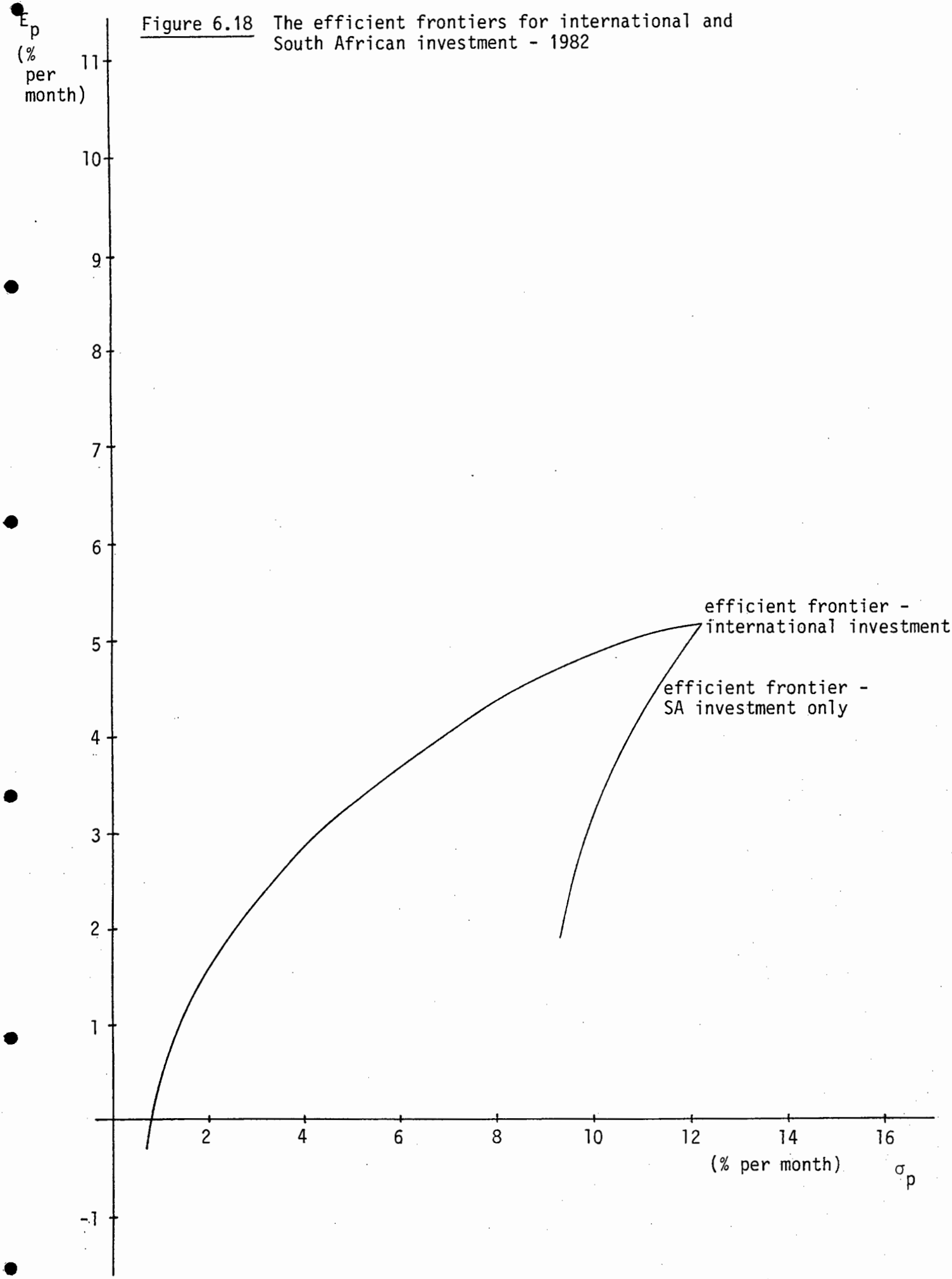


Figure 6.18 The efficient frontiers for international and South African investment - 1982



that in years 1966, 1968, 1977 and 1982 one of the South African securities offered the greatest return and hence in these 4 years the South African and international efficient frontiers coincided at the highest risk/return portion of the frontiers since this security would have been included in both efficient frontiers. The two efficient frontiers diverged at lower risk/return levels in these cases, however. This result indicates that a South African investor could have increased or, at least equalled his return for any given risk level if he were permitted to invest his funds at risk in foreign securities or commodities.

(b) Large differences in the range of returns of the efficient portfolios existed in the individual years. For the purely South African efficient portfolios the range of average monthly returns amounted to 0,6% in 1970 while the range of average monthly returns was as large as 7,7% in 1973. For the international efficient portfolios the range of average monthly returns was as low as 2,7% in 1977 and was largest in 1979 at 11,6%. This was due to the respective bear and bull phases during these periods on the South African and international markets.

(c) Large differences in the magnitude of the average monthly returns existed during the various periods. For a portfolio of South African securities average monthly returns of 7,4% were attainable during 1973

whereas the highest average monthly return attainable in 1974 was only 0,45%. For a portfolio of international securities average monthly returns of 13,9% were attainable during 1979 whereas the highest average monthly return available during 1977 was only 3,12%.

(d) There existed large differences in both the range and the magnitude of the average monthly risks in various periods. The following table indicates the maximum and minimum range of the average monthly risks, and the year in which they occurred.

	minimum range of average monthly risks	maximum range of average monthly risks
South African portfolio	1,09% (1970)	9,09% (1974)
International portfolio	4,33% (1970)	22,85% (1979)

The following table shows the maximum and minimum values of the greatest average monthly risk attainable in any one year, and the year in which it occurred.

	minimum value of greatest average monthly risk attainable	maximum value of greatest average monthly risk attainable
South African portfolio	4,07% (1972)	15,27% (1974)
International portfolio	4,65% (1970)	23,38% (1979)

(e) The slope of both the South African and international efficient frontiers over particular risk ranges was sometimes very different from one year to another. For example, in 1967, a 50% increase in risk borne from a portfolio of only South African securities from 4% per month to 6% per month achieved a 3,08% increase in return (from 2,27% per month to 2,34% per month) while the same 50% increase in risk borne in 1968 produced a 38,44% increase in return (from 4,50% per month to 6,23% per month). This was much the same in the case of the international portfolios, where the range in increases of monthly returns (when the risk borne was increased 50% from 4% to 6% per month) was from 3,7% per month in 1977 (monthly return increased from 2,96% per month to 3,07% per month) to 36,04% per month in 1965 (monthly return increased from 3,08% per month to 4,19% per month). This illustrates the importance of the slope at different risk levels which is the crucial factor in deciding whether bearing additional risk is worthwhile in terms of an investor's risk/return expectations.

6.2.2 The Composition of the Unconstrained Efficient Frontiers

In section 4.4 it was shown that the objective function of the Markowitz portfolio selection problem was

minimise $-\lambda E_p + V_p$ for all possible $\lambda \geq 0$

where E_p = expected return on the portfolio

and V_p = variance of the return on the portfolio

This objective function is linear in the E_p, σ_p^2 plane with slope λ and the minimum point occurs where the line $-\lambda E_p + V_p$ is tangent to the efficient frontier. The entire efficient frontier can thus be created by varying λ from 0 to ∞. Thus the parameter λ indicates the level of risk associated with a particular portfolio.

If $\lambda = 0$, the linear objective function reduces to

minimise V_p

and this gives rise to the least risky portfolio.
If $\lambda = 1$ the linear objective function is

minimise $-E_p + V_p$

and this situation corresponds to one in which the investor weighs expected return and risk equally. If $\lambda = \infty$ the linear objective function becomes

minimise $-\infty(E_p) + V_p$

and it is clear that this is minimised when the expected return offered is greatest.

The parameter λ is commonly known as the 'coefficient of risk aversion'.

Tables 6.1 to 6.7 show the percentage composition of the unconstrained efficient portfolios in each of the years

YEAR	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	MEAN	STANDARD DEVIATION	RANGE	NO OF YEARS PRESENT
E_p	0,00	0,00	0,00	0,00	-0,03	-0,01	0,54	1,90	1,46	-1,76	1,26	0,40	0,46	1,11	2,33	0,34	-1,56	-0,30				
σ_p	0,00	0,00	0,00	0,00	0,07	0,32	2,16	0,44	1,05	1,36	0,91	1,50	0,30	1,23	0,53	0,89	1,03	0,70				
Security																						
JSE Coal	-	-	4,6	9,2	-	-	25,7	-	-	9,2	-	12,4	-	-	-	-	-	-	3,39	6,85	25,7	5
JSE Diamonds	-	-	-	10,1	-	-	-	-	-	-	-	-	-	-	10,2	-	6,5	11,7	2,14	4,22	11,7	4
JSE All Gold	-	2,3	-	-	4,9	-	13,5	14,2	2,7	-	-	2,7	-	-	-	-	-	-	2,24	4,46	14,2	6
JSE Mets & Mins	-	2,6	-	-	2,5	-	-	2,5	2,6	-	-	-	6,7	-	-	-	-	-	0,94	1,80	6,7	5
JSE Min Fin	-	2,1	-	-	-	-	-	-	-	-	-	13,1	-	-	-	-	-	-	0,84	3,10	13,1	2
JSE Financial	-	0,7	0,4	-	-	-	-	10,8	-	4,0	-	0,8	-	-	19,8	19,2	1,4	-	3,17	6,49	19,8	8
JSE Industrial	-	-	-	11,4	-	-	-	-	-	-	38,7	-	-	-	17,7	-	-	-	3,77	9,96	38,7	3
S & P	-	2,8	2,5	7,7	-	-	-	11,3	-	8,0	-	40,8	27,0	36,5	-	5,1	-	11,5	8,51	12,94	40,8	10
UK Act Index	-	-	11,6	-	-	-	-	-	-	-	2,5	-	16,8	-	-	-	1,6	4,1	2,03	4,65	16,8	5
Lead	-	-	17,4	-	-	7,8	-	-	-	-	-	-	-	-	7,4	5,2	-	-	2,10	4,64	17,4	4
Tin	-	-	-	3,3	-	-	-	-	10,4	-	-	-	-	-	-	-	-	-	0,76	2,53	10,4	2
Zinc	-	-	-	-	-	4,2	-	-	7,0	-	0,7	-	-	-	-	-	-	-	0,66	1,87	7,0	3
Silver	-	7,4	-	1,2	5,7	-	-	1,2	-	-	-	-	-	6,8	-	-	10,0	0,9	1,84	3,22	10,0	7
Aluminium	-	-	-	-	20,7	63,6	-	13,0	-	-	-	12,5	-	-	20,9	-	-	-	7,26	15,88	63,6	5
Antimony	-	-	-	-	2,7	0,2	-	-	12,0	-	-	14,7	-	-	-	17,7	38,6	-	4,77	10,18	38,6	6
Copper	5,7	-	-	-	-	-	0,7	-	-	-	18,6	-	-	-	0,4	9,2	-	-	1,92	4,83	18,6	5
Nickel	-	5,7	3,7	-	-	11,9	-	-	-	-	-	-	22,7	-	12,1	3,7	-	-	3,32	6,25	22,7	6
Platinum (OP)	-	-	-	28,6	-	2,0	32,9	-	49,4	20,4	-	-	3,9	9,4	-	-	-	-	8,15	14,69	49,4	7
Platinum (FP)	3,9	-	-	-	-	-	10,5	2,3	-	-	-	-	-	-	-	-	-	-	0,93	2,60	10,5	3
Gold	61,7	73,8	29,0	4,5	10,8	-	26,2	-	-	-	-	-	-	-	-	0,2	18,7	-	12,49	22,30	73,8	8
Wool	17,4	-	-	20,8	15,0	3,3	-	12,0	6,6	55,7	-	13,9	-	30,6	-	-	5,8	39,3	12,24	15,85	55,7	11
Cotton	-	-	1,6	-	7,4	-	-	7,0	-	1,1	3,9	1,2	-	5,0	-	-	16,8	-	2,44	4,36	16,8	8
Sugar	-	-	-	-	-	-	-	-	-	0,9	2,7	0,3	-	-	-	-	0,6	10,8	0,85	2,57	10,8	5
Wheat	9,3	2,6	24,0	-	21,1	1,8	5,7	-	9,3	3,6	9,8	-	3,9	11,7	2,6	39,7	-	13,2	8,79	10,47	39,7	14
Maize	2,0	-	5,2	12,4	-	5,2	10,5	-	-	6,3	13,9	-	6,6	-	8,9	-	-	8,5	4,42	4,84	13,9	10
No of securities in portfolio	6	9	10	9	10	9	7	10	8	8	9	9	8	6	9	8	9	8				

Table 6.1 The percentage composition of the unconstrained efficient portfolios for 1965 to 1982 (λ = 0,00)

YEAR	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	MEAN	STANDARD DEVIATION	RANGE	NO OF YEARS PRESENT
E_p	1,62	2,18	2,25	3,89	1,89	1,79	2,67	5,25	6,25	1,74	2,81	1,89	2,44	2,94	4,41	1,69	1,70	1,68				
σ_p	1,84	1,93	1,71	2,11	1,46	1,85	3,12	2,22	3,01	2,79	1,67	2,49	1,55	2,09	1,96	1,60	2,18	2,09				
Security																						
JSE Coal	-	-	-	-	15,2	-	-	-	-	-	15,9	15,9	-	-	-	16,0	-	-	3,50	6,74	16,0	4
JSE Diamonds	10,1	26,4	-	36,4	-	-	-	5,6	-	-	-	-	17,7	13,4	-	-	-	-	6,09	10,79	36,4	6
JSE All Gold	-	-	-	-	-	1,9	6,6	-	35,6	-	-	-	-	-	-	-	-	12,9	3,17	8,75	35,6	4
JSE Mets & Mins	6,0	24,4	-	-	-	-	-	29,9	-	-	-	-	-	-	-	-	-	-	3,35	8,82	29,9	3
JSE Min Fin	-	-	-	-	-	-	-	-	-	-	-	-	-	-	13,6	0,3	-	-	0,77	3,20	13,6	2
JSE Financial	-	1,6	47,9	22,1	-	-	-	19,0	-	-	-	-	-	-	-	-	24,6	9,3	6,92	13,23	47,9	6
JSE Industrial	-	-	-	-	-	-	-	-	-	-	26,6	-	-	-	26,1	-	-	-	2,93	8,52	26,6	2
S & P	-	-	10,3	-	-	-	-	-	-	14,1	-	3,3	-	27,9	-	18,2	-	29,8	5,76	10,05	29,8	6
UK Act Index	-	-	12,5	-	-	-	13,6	-	-	-	11,4	-	16,0	-	-	11,5	-	-	3,61	6,06	16,0	5
Lead	-	-	-	10,4	25,8	-	-	-	-	-	-	-	-	-	-	-	-	-	2,01	6,42	25,8	2
Tin	0,3	-	-	-	-	-	-	-	-	-	-	14,9	3,6	-	-	-	-	-	1,04	3,56	14,9	3
Zinc	-	-	-	-	1,9	-	21,4	-	25,7	-	-	-	-	6,0	-	-	0,7	-	3,09	7,62	25,7	5
Silver	-	-	22,0	-	2,2	-	-	-	-	-	-	-	-	-	-	-	-	0,2	1,36	5,18	22,0	3
Aluminium	-	-	-	-	-	-	-	-	-	-	-	2,0	7,6	-	-	-	-	-	0,53	1,83	7,6	2
Antimony	-	-	-	-	10,3	-	-	-	17,6	-	-	23,7	-	-	-	21,0	-	-	4,04	8,14	23,7	4
Copper	17,2	-	-	-	19,7	-	0,2	-	-	-	2,4	-	-	8,0	0,9	-	-	-	2,69	6,06	19,7	6
Nickel	-	47,6	-	-	-	-	-	-	-	40,7	-	-	-	-	21,3	-	-	-	6,09	14,77	47,6	3
Platinum (OP)	-	-	-	18,9	-	-	-	-	-	36,9	-	-	-	44,7	-	-	32,9	-	7,41	14,97	44,7	4
Platinum (FP)	20,0	-	7,2	2,9	-	-	-	5,2	-	-	-	-	-	-	-	-	-	-	1,96	14,91	20,0	4
Gold	-	-	-	-	-	9,6	52,2	-	-	-	-	23,0	36,2	-	1,9	-	4,5	-	7,08	14,94	52,2	6
Wool	30,1	-	-	-	-	-	-	40,3	-	-	-	17,2	-	-	-	-	37,3	16,2	7,84	6,94	40,3	5
Cotton	-	-	-	-	-	-	6,0	-	20,8	-	19,2	-	-	-	-	-	-	13,3	3,29	10,31	20,8	4
Sugar	-	-	-	9,3	13,3	39,8	-	-	-	1,2	-	-	-	-	18,2	-	-	-	4,54	9,33	39,8	5
Wheat	16,3	-	-	-	-	20,0	-	-	0,3	7,1	-	-	-	-	-	33,0	-	-	4,26	9,33	33,0	5
Maize	-	-	-	-	11,6	28,7	-	-	-	-	24,5	-	18,9	-	18,0	-	-	18,2	6,67	10,21	28,7	6
No of securities in portfolio	7	4	5	6	8	5	6	5	5	5	6	7	6	5	7	6	5	7				

Table 6.2 The percentage composition of the unconstrained efficient portfolios for 1965 to 1982 (λ = 0,05)

YEAR	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	MEAN	STANDARD DEVIATION	RANGE	NO OF YEARS PRESENT
E_p	2,63	2,74	2,58	5,38	2,26	2,42	3,39	5,64	7,40	2,34	3,32	2,49	2,72	3,56	5,42	1,91	2,25	2,35				
σ_p	3,32	2,81	2,29	3,95	2,20	2,84	3,89	2,79	4,14	3,51	2,56	3,24	2,03	3,01	3,38	2,05	2,96	3,04				
Security																						
JSE Coal	-	-	-	-	15,5	-	-	-	-	-	4,2	17,3	-	16,9	-	21,3	-	-	5,29	8,95	24,2	5
JSE Diamonds	8,6	33,4	-	54,8	-	-	-	-	-	-	-	-	28,0	12,8	-	-	-	-	7,64	15,51	54,8	5
JSE All Gold	-	-	-	-	-	-	3,1	-	46,3	-	-	-	-	-	-	-	-	18,9	3,79	11,51	46,3	3
JSE Mets & Mins	19,9	36,8	-	-	-	-	-	33,5	-	-	-	-	-	-	-	-	-	-	5,01	11,93	36,8	3
JSE Min Fin	-	-	-	-	-	-	-	-	-	-	-	-	-	-	22,2	-	-	-	1,23	5,23	22,2	1
JSE Financial	-	-	58,8	34,4	-	-	-	18,2	-	-	-	-	-	-	-	-	21,3	3,2	7,55	16,11	58,8	5
JSE Industrial	-	-	-	-	-	-	-	-	-	-	15,3	-	-	-	3,9	-	-	-	1,07	3,67	15,3	2
S & P	-	-	-	-	-	-	-	-	-	-	-	-	-	6,9	-	17,2	-	44,4	3,82	10,99	44,4	3
UK Act Index	-	-	8,9	-	-	-	31,9	-	-	1,0	17,1	-	29,0	-	-	13,3	-	-	5,62	10,40	31,9	6
Lead	-	-	-	-	31,9	-	-	-	-	-	-	-	-	-	-	-	-	-	1,77	7,52	31,9	1
Tin	-	-	-	-	-	-	-	-	-	-	-	34,3	2,2	-	-	-	15,9	-	2,91	8,68	34,3	3
Zinc	-	-	-	-	-	-	26,6	-	35,2	-	-	-	-	-	-	-	13,1	-	4,17	10,31	35,2	3
Silver	-	-	32,1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1,7	1,88	7,55	32,1	2
Aluminium	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,00	0
Antimony	-	-	-	-	14,9	-	-	-	-	-	-	25,2	-	-	-	23,5	-	-	3,53	8,35	25,2	3
Copper	25,8	-	-	-	21,9	-	-	-	-	-	-	-	-	10,7	0,8	-	-	-	3,29	7,92	25,8	4
Nickel	-	29,8	-	-	-	-	-	-	-	43,8	-	-	-	-	26,4	-	-	-	5,56	13,17	43,8	3
Platinum (OP)	-	-	-	1,0	-	-	-	-	-	44,4	-	-	-	52,7	-	-	43,1	-	7,84	17,99	52,7	4
Platinum (FP)	30,2	-	-	-	-	-	-	-	-	-	-	-	24,8	-	-	-	-	-	3,06	8,94	30,2	2
Gold	-	-	-	-	-	-	24,8	-	-	-	-	13,0	16,0	-	6,5	-	-	-	3,35	7,20	24,8	4
Wool	13,2	-	-	-	-	-	-	48,3	-	-	-	10,2	-	-	-	-	6,6	-	4,35	11,67	48,3	4
Cotton	-	-	-	-	-	-	13,6	-	18,5	-	19,9	-	-	-	-	-	-	5,3	3,18	6,73	19,9	4
Sugar	-	-	0,1	9,8	14,3	61,9	-	-	-	3,3	-	-	-	-	32,8	0,1	-	-	6,79	16,07	61,9	7
Wheat	2,3	-	-	-	-	-	-	-	-	7,5	-	-	-	-	-	24,6	-	-	1,91	5,95	24,6	3
Maize	-	-	-	-	1,5	38,1	-	-	-	-	23,5	-	-	-	7,4	-	-	26,5	5,39	11,48	38,1	5
No of securities in portfolio	6	3	4	4	6	2	5	3	3	5	5	5	5	5	7	6	5	6				

Table 6.3 The percentage composition of the unconstrained efficient portfolios for 1965 to 1982 (λ = 0,10)

YEAR	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	MEAN	STANDARD DEVIATION	RANGE	NO OF YEARS PRESENT
E_p	4,16	3,65	2,83	5,93	3,08	3,01	4,32	6,29	7,91	3,19	4,23	3,40	2,81	3,92	7,04	2,63	2,98	3,34				
σ_p	5,94	4,58	3,02	4,92	4,38	4,27	5,35	4,37	5,10	5,19	4,55	4,93	2,34	3,72	6,27	4,10	4,58	5,10				
Security																						
JSE Coal	-	-	-	-	-	-	-	-	-	-	37,9	10,9	-	38,7	-	38,4	-	-	6,99	14,65	38,7	4
JSE Diamonds	-	40,3	-	51,0	-	-	-	-	-	-	-	-	34,5	1,2	-	-	-	-	7,06	16,31	51,0	4
JSE All Gold	-	-	-	-	-	-	-	-	55,3	-	-	-	-	-	-	-	-	19,0	4,13	13,53	55,3	2
JSE Mets & Mins	23,0	59,7	-	-	-	-	-	19,8	-	-	-	-	-	-	-	-	-	-	5,69	15,15	59,7	3
JSE Min Fin	-	-	-	-	-	-	-	-	-	-	-	-	-	-	8,9	-	-	-	0,49	2,10	8,9	1
JSE Financial	-	-	56,5	49,0	-	-	-	9,7	-	-	-	-	-	-	-	-	-	6,7	6,77	16,99	56,5	4
JSE Industrial	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,0	0
S & P	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	13,8	-	63,5	4,29	15,13	63,5	2
UK Act Index	-	-	-	-	-	-	55,2	-	-	-	34,6	-	31,5	-	-	11,4	-	-	7,38	16,07	55,2	4
Lead	-	-	-	-	34,3	-	-	-	-	-	-	7,0	-	-	-	-	-	-	2,29	8,16	34,3	2
Tin	-	-	-	-	-	-	-	-	-	-	-	66,3	-	-	-	-	42,6	-	6,06	18,07	66,3	2
Zinc	-	-	-	-	-	-	16,4	-	42,6	-	-	-	-	-	-	-	34,5	-	5,19	12,81	42,6	3
Silver	-	-	43,5	-	-	-	-	-	-	-	-	-	-	-	13,9	-	-	10,8	3,79	10,69	43,5	3
Aluminium	-	-	-	-	-	-	-	-	-	13,5	-	-	-	-	-	-	-	-	0,75	3,18	13,5	1
Antimony	-	-	-	-	30,2	-	-	-	-	-	-	15,8	-	-	-	31,2	-	-	4,29	10,30	31,2	3
Copper	57,8	-	-	-	25,2	-	-	-	-	-	-	-	-	-	-	-	-	-	4,61	14,54	57,8	2
Nickel	-	-	-	-	-	-	-	-	-	30,3	-	-	-	-	22,1	-	-	-	2,91	8,59	30,3	2
Platinum (OP)	-	-	-	-	-	-	-	-	-	37,8	-	-	-	57,1	-	-	22,9	-	6,54	16,17	57,1	3
Platinum (FP)	19,2	-	-	-	-	-	-	-	-	-	-	-	34,0	3,0	-	-	-	-	3,13	8,94	34,0	3
Gold	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1,0	-	-	-	0,06	0,24	1,0	1
Wool	-	-	-	-	-	-	-	70,5	-	-	-	-	-	-	-	-	-	-	3,92	16,62	70,5	1
Cotton	-	-	-	-	-	-	28,4	-	2,1	-	12,1	-	-	-	-	-	-	-	2,37	7,10	28,4	3
Sugar	-	-	-	-	10,3	93,2	-	-	-	15,8	-	-	-	-	54,1	5,2	-	-	9,92	24,49	93,2	5
Wheat	-	-	-	-	-	-	-	-	-	2,6	-	-	-	-	-	-	-	-	0,14	0,61	2,6	1
Maize	-	-	-	-	-	6,8	-	-	-	-	15,4	-	-	-	-	-	-	-	1,23	3,88	15,4	2
No of securities in portfolio	3	2	2	2	4	2	3	3	3	5	4	4	3	4	5	5	3	4				

Table 6.4 The percentage composition of the unconstrained efficient portfolios for 1965 to 1982 (λ = 0,25)

YEAR	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	MEAN	STANDARD DEVIATION	RANGE	NO OF YEARS PRESENT
E_p	5,35	3,78	3,07	6,20	4,22	3,13	4,72	7,14	8,00	4,48	4,89	3,84	2,85	4,02	9,14	3,07	3,26	3,99				
σ_p	8,68	5,08	4,24	5,86	7,84	4,65	6,48	6,99	5,36	8,66	6,62	6,26	2,63	4,20	10,80	5,55	5,43	7,08				
Security																						
JSE Coal	-	-	-	-	-	-	-	-	-	-	41,8	-	-	41,7	-	57,1	-	-	7,81	18,23	57,1	3
JSE Diamonds	-	26,3	-	29,2	-	-	-	-	-	-	-	-	42,6	-	-	-	-	-	5,45	12,89	42,6	3
JSE All Gold	-	-	-	-	-	-	-	-	54,6	-	-	-	-	-	-	-	-	38,9	5,19	15,36	54,6	2
JSE Mets & Mins	-	73,7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4,09	17,37	73,7	1
JSE Min Fin	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,0	0
JSE Financial	-	-	39,4	70,8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	6,12	18,61	70,8	2
JSE Industrial	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,0	0
S & P	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	42,4	2,36	9,99	42,4	1
UK Act Index	-	-	-	-	-	-	56,4	-	-	-	58,2	-	25,5	-	-	-	-	-	7,78	18,99	58,2	3
Lead	-	-	-	-	27,0	-	-	-	-	-	-	20,8	-	-	-	-	-	-	2,66	7,80	27,0	2
Tin	-	-	-	-	-	-	-	-	-	-	-	79,2	-	-	-	-	60,6	-	7,77	22,83	79,2	2
Zinc	-	-	-	-	-	-	-	-	45,4	-	-	-	-	-	-	-	39,4	-	4,71	13,75	45,4	2
Silver	-	-	60,6	-	-	-	-	-	-	-	-	-	-	-	34,3	-	-	18,7	6,31	16,24	60,6	3
Aluminium	-	-	-	-	-	-	-	-	-	31,7	-	-	-	-	-	-	-	-	1,76	7,47	31,7	1
Antimony	-	-	-	-	54,2	-	-	-	-	-	-	-	-	-	-	33,5	-	-	4,87	14,62	54,2	2
Copper	100,0	-	-	-	18,8	-	-	-	-	-	-	-	-	-	-	-	-	-	6,60	23,73	100,0	2
Nickel	-	-	-	-	-	-	-	-	-	11,4	-	-	-	-	2,8	-	-	-	0,79	2,73	11,4	2
Platinum (OP)	-	-	-	-	-	-	-	-	-	20,2	-	-	-	44,7	-	-	-	-	3,61	11,30	44,7	2
Platinum (FP)	-	-	-	-	-	-	-	-	-	-	-	-	31,9	13,6	-	-	-	-	2,53	8,00	31,9	2
Gold	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,0	0
Wool	-	-	-	-	-	-	-	100,0	-	-	-	-	-	-	-	-	-	-	5,56	23,57	100,0	1
Cotton	-	-	-	-	-	-	43,6	-	-	-	-	-	-	-	-	-	-	-	2,42	10,28	43,6	1
Sugar	-	-	-	-	-	100,0	-	-	-	36,7	-	-	-	-	62,9	9,4	-	-	11,61	27,62	100,0	4
Wheat	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,0	0
Maize	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,0	0
No of securities in portfolio	1	2	2	2	3	1	2	1	2	4	2	2	3	3	3	3	2	3				

Table 6.5 The percentage composition of the unconstrained efficient portfolios for 1965 to 1982 (λ = 0,50)

YEAR	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	MEAN	STANDARD DEVIATION	RANGE	NO OF YEARS PRESENT
E_p	5,35	4,02	3,54	6,56	6,09	3,13	5,03	7,14	8,06	6,17	5,12	4,03	2,93	4,22	12,08	3,09	3,34	5,10				
σ_p	8,68	6,60	7,29	7,66	14,15	4,65	8,07	6,99	5,73	13,91	7,82	7,31	3,56	5,73	18,31	5,67	5,92	11,43				
Security																						
JSE Coal	-	-	-	-	-	-	-	-	-	-	27,7	-	-	45,9	-	55,8	-	-	7,19	17,25	55,8	3
JSE Diamonds	-	-	-	-	-	-	-	-	-	-	-	-	58,9	-	-	-	-	-	3,27	13,88	58,9	1
JSE All Gold	-	-	-	-	-	-	-	-	50,7	-	-	-	-	-	-	-	-	68,5	6,62	19,51	68,5	2
JSE Mets & Mins	-	100,0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5,56	23,57	100,0	1
JSE Min Fin	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,0	0
JSE Financial	-	-	5,1	100,0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5,84	23,53	100,0	2
JSE Industrial	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,0	0
S & P	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,0	0
UK Act Index	-	-	-	-	-	-	32,0	-	-	-	72,3	-	13,6	-	-	-	-	-	6,55	18,25	72,3	3
Lead	-	-	-	-	-	-	-	-	-	-	-	43,2	-	-	-	-	-	-	2,40	10,18	43,2	1
Tin	-	-	-	-	-	-	-	-	-	-	-	56,8	-	-	-	-	77,8	-	7,48	22,06	77,8	2
Zinc	-	-	-	-	-	-	-	-	49,3	-	-	-	-	-	-	-	22,2	-	3,97	12,46	49,3	2
Silver	-	-	94,9	-	-	-	-	-	-	-	-	-	-	-	74,4	-	-	31,5	11,16	27,96	94,9	3
Aluminium	-	-	-	-	-	-	-	-	-	27,3	-	-	-	-	-	-	-	-	1,52	6,43	27,3	1
Antimony	-	-	-	-	96,8	-	-	-	-	7,6	-	-	-	-	-	37,0	-	-	7,85	23,87	96,8	3
Copper	100,0	-	-	-	3,2	-	-	-	-	-	-	-	-	-	-	-	-	-	5,73	23,54	100,0	2
Nickel	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,0	0
Platinum (OP)	-	-	-	-	-	-	-	-	-	-	-	-	-	19,3	-	-	-	-	1,07	4,55	19,3	1
Platinum (FP)	-	-	-	-	-	-	-	-	-	-	-	-	27,5	34,8	-	-	-	-	3,46	10,15	34,8	2
Gold	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,0	0
Wool	-	-	-	-	-	-	-	100,0	-	-	-	-	-	-	-	-	-	-	5,56	23,57	100,0	1
Cotton	-	-	-	-	-	68,0	-	-	-	-	-	-	-	-	-	-	-	-	3,78	16,03	68,0	1
Sugar	-	-	-	-	-	100,0	-	-	-	65,1	-	-	-	-	25,6	7,2	-	-	10,99	27,41	100,0	4
Wheat	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,0	0
Maize	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,0	0
No of securities in portfolio	1	1	2	1	2	1	2	1	2	3	2	2	3	3	2	3	2	2				

Table 6.6 The percentage composition of the unconstrained efficient portfolios for 1965 to 1982 (λ = 1,00)

YEAR	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	MEAN	STANDARD DEVIATION	RANGE	NO OF YEARS PRESENT
E_p	5,35	4,02	3,61	6,56	6,23	3,13	5,44	7,14	8,74	7,20	5,57	4,51	3,12	4,72	13,90	3,28	3,44	5,19				
σ_p	8,68	6,60	7,78	7,66	14,65	4,65	10,93	6,99	17,74	19,00	11,34	11,50	6,90	11,73	23,38	11,27	6,91	12,28				
Security																						
JSE Coal	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,0	0
JSE Diamonds	-	-	-	-	-	-	-	-	-	-	-	-	100,0	-	-	-	-	-	5,56	23,57	100,0	1
JSE All Gold	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	100,0	5,56	23,57	100,0	1
JSE Mets & Mins	-	100,0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5,56	23,57	100,0	1
JSE Min Fin	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,0	0
JSE Financial	-	-	-	100,0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5,56	23,57	100,0	1
JSE Industrial	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,0	0
S & P	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,0	0
UK Act Index	-	-	-	-	-	-	-	-	-	-	100,0	-	-	-	-	-	-	-	5,56	23,57	100,0	1
Lead	-	-	-	-	-	-	-	-	-	-	-	100,0	-	-	-	-	-	-	5,56	23,57	100,0	1
Tin	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	100,0	-	5,56	23,57	100,0	1
Zinc	-	-	-	-	-	-	-	-	100,0	-	-	-	-	-	-	-	-	-	5,56	23,57	100,0	1
Silver	-	-	100,0	-	-	-	-	-	-	-	-	-	-	-	100,0	-	-	-	11,11	32,34	100,0	2
Aluminium	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,0	0
Antimony	-	-	-	-	100,0	-	-	-	-	-	-	-	-	-	-	100,0	-	-	11,11	32,34	100,0	2
Copper	100,0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5,56	23,57	100,0	1
Nickel	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,0	0
Platinum (OP)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,0	0
Platinum (FP)	-	-	-	-	-	-	-	-	-	-	-	-	-	100,0	-	-	-	-	5,56	23,57	100,0	1
Gold	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,0	0
Wool	-	-	-	-	-	-	-	100,0	-	-	-	-	-	-	-	-	-	-	5,56	23,57	100,0	1
Cotton	-	-	-	-	-	-	100,0	-	-	-	-	-	-	-	-	-	-	-	5,56	23,57	100,0	1
Sugar	-	-	-	-	-	100,0	-	-	-	100,0	-	-	-	-	-	-	-	-	11,11	32,34	100,0	2
Wheat	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,0	0
Maize	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0,00	0,00	0,0	0
No of securities in portfolio	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1				

Table 6.7 The percentage composition of the unconstrained efficient portfolios for 1965 to 1982 ($\lambda = \infty$)

1965 to 1982. Each table displays the position at a different risk level, indicated by a different value of λ. The important features of these tables are laid out below.

1. There appeared to be no dominance by any one security over any risk level. This is evident from the number of years (out of 18) that each security appeared in the efficient portfolios. At low risk levels ($\lambda < 0,05$) all securities appeared in the efficient portfolios in at least 2 years out of the 18, with a maximum of 14 appearances. As the risk increased (that is, as $\lambda \to \infty$) securities appear less and less frequently during the 18 year period, until when $\lambda = \infty$ each security appeared in a maximum 2 years out of 18 years. In fact, 10 out of the 25 securities did not appear at all. The table below shows the average number of years (out of 18) that each security is present for various values of λ.

λ	average number of years present for each security
0,00	6,08
0,05	4,20
0,10	3,40
0,25	2,44
0,50	1,64
1,00	1,40
∞	0,72

From an overall point of view each of the 25 securities appeared at least once in the efficient portfolios during

the 18 years at some risk level indicating that there was no dominance by any one security over the rest.

2. The number of securities included in an efficient portfolio depends to a large extent on the value of λ, the risk. At low levels of λ the number of securities present in the efficient portfolios each year is high, and as λ increases diversification decreases. Table 6.8 below shows the minimum, the maximum and the average number of securities contained in the efficient portfolios over the 18-year period.

	Number of securities in efficient portfolios		
λ	Minimum	Maximum	Average
0,00	6	10	8,44
0,05	4	7	5,83
0,10	2	7	4,72
0,25	2	5	3,39
0,50	1	4	2,28
1,00	1	3	1,94
∞	1	1	1,00

Table 6.8 The number of securities contained in the efficient portfolios

3. On average over the period 1965 to 1982 a South African investor would have invested no more than 12,49% of his funds in any one security (gold; at the lowest risk level, $\lambda = 0$). However the average amount invested in each security over the 18-year period varies significantly from one security to another and from one risk level to

another. This is particularly noticeable at high risk levels where few securities can offer a return large enough to compensate for the risk borne, leaving each year's portfolio consisting of a few securities each held in fairly large proportions. The table below indicates the average over all 18 years of the maximum proportion held in any one security in that year for each value of λ.

λ	average of the maximum proportion held in any one security	average number of securities in efficient portfolios
0,00	40,17	8,44
0,05	36,32	5,83
0,10	40,52	4,72
0,25	53,64	3,39
0,50	64,15	2,28
1,00	76,99	1,94
∞	100,00	1,00

This shows that at low levels of λ there was much diversification, with small proportions held in each security. As λ increased, diversification decreased with concommitant larger proportion held in each security included in the efficient portfolios.

Tables 6.9, 6.10, 6.11 and 6.12 show the proportion that should have been held by a South African investor at various risk levels in each of the four main security groups; South African securities, foreign stocks, metals and soft

λ	0,00	0,05	0,10	0,25	0,50	1,00	∞
YEAR							
1965		16,1	28,5	23,0			
1966	7,7	52,4	70,2	100,0	100,0	100,0	100,0
1967	5,0	47,9	58,8	56,6	39,4	5,1	
1968	21,5	58,5	89,2	100,0	100,0	100,0	100,0
1969	16,6	15,2	15,5				
1970		1,9					
1971	13,5	6,6	3,1				
1972	53,2	54,5	51,7	29,5			
1973	5,3	35,6	46,3	55,3	54,6	50,7	
1974	4,0						
1975	37,9	42,5	39,5	37,9	41,8	27,7	
1976	16,6	15,9	17,3	10,9			
1977	19,1	17,7	28,0	34,5	42,6	58,9	100,0
1978		13,4	29,7	39,9	41,7	45,9	
1979	47,7	39,7	26,1	8,9			
1980	19,2	16,3	21,3	38,4	57,1	55,8	
1981	7,9	24,6	21,3				
1982	11,7	22,2	22,1	25,7	38,9	68,5	100,0
AVERAGE	15,9	26,7	31,6	31,1	28,7	28,5	22,2

Table 6.9 Proportions of a South African investor's funds invested in South African securities at various risk levels

λ	0,00	0,05	0,10	0,25	0,50	1,00	∞
YEAR							
1965							
1966	2,8						
1967	14,1	22,8	8,9				
1968	7,7						
1969							
1970							
1971		13,6	31,9	55,2	56,4	32,0	
1972	11,3						
1973							
1974	8,0	14,1	1,0				
1975	2,5	11,4	17,1	34,6	58,2	72,3	100,0
1976	40,8	3,3					
1977	43,8	16,0	29,0	31,5	25,5	13,6	
1978	36,5	27,9	6,9				
1979							
1980	5,1	29,7	30,5	25,2			
1981	1,6						
1982	15,6	29,8	44,4	63,5	42,4		
AVERAGE	10,5	9,4	9,4	11,7	10,1	6,6	5,6

Table 6.10 Proportions of a South African investor's funds invested in foreign stocks at various risk levels

λ	0,00	0,05	0,10	0,25	0,50	1,00	∞
YEAR							
1965	71,3	37,5	56,0	77,0	100,0	100,0	100,0
1966	86,9	47,6	29,8				
1967	50,1	29,2	32,1	43,5	60,6	94,9	100,0
1968	37,6	32,2	1,0				
1969	39,9	59,9	68,7	89,7	100,0	100,0	100,0
1970	89,7	9,6					
1971	70,3	73,8	51,4	16,4			
1972	16,5	5,2					
1973	78,8	43,3	35,2	42,6	45,4	49,3	100,0
1974	20,4	77,6	88,2	81,6	63,3	34,9	
1975	19,3	2,4					
1976	27,2	63,6	72,5	89,1	100,0	100,0	100,0
1977	26,6	47,4	43,0	34,0	31,9	27,5	
1978	16,2	58,7	63,4	60,1	58,3	54,1	100,0
1979	40,8	24,1	42,7	37,0	37,1	74,4	100,0
1980	36,0	21,0	23,5	31,2	33,5	37,0	100,0
1981	67,3	38,1	72,1	100,0	100,0	100,0	100,0
1982	0,9	0,2	1,7	10,8	18,7	31,5	
AVERAGE	44,2	37,3	37,9	39,6	41,6	44,6	50,0

Table 6.11 Proportions of a South African investor's funds invested in metals at various risk levels

λ	0,00	0,05	0,10	0,25	0,50	1,00	∞
YEAR							
1965	28,7	46,6	15,5				
1966	2,6						
1967	30,8		0,1				
1968	33,2	9,3	9,8				
1969	43,5	24,9	15,8	10,3			
1970	10,3	88,5	100,0	100,0	100,0	100,0	100,0
1971	16,2	6,0	13,6	28,4	43,6	68,0	100,0
1972	19,0	40,3	48,4	70,5	100,0	100,0	100,0
1973	15,9	21,1	18,5	2,1			
1974	67,6	8,3	10,8	18,4	36,7	65,1	100,0
1975	30,3	43,7	43,4	27,5			
1976	15,4	17,2	10,2				
1977	10,5	18,9					
1978	47,3						
1979	11,5	36,2	40,2	54,1	62,9	25,6	
1980	39,7	33,0	24,7	5,2	9,4	7,2	
1981	23,2	37,3	6,6				
1982	71,8	47,7	31,8				
AVERAGE	26,5	26,6	20,5	17,6	19,6	20,3	22,2

Table 6.12 Proportions of a South African investor's funds invested in soft commodities at various risk levels

commodities respectively. In Table 6.9 it should be noticed that, apart from the highest and lowest risk portfolios, the South African investor should have invested on average between 26,7% and 31,6% of his funds in South African securities. At the highest and lowest risks possible these proportions are somewhat less. These figures serve to indicate that if exchange control regulations were abolished the South African investor should have divested a large proportion of his funds at risk outside of this country. In some years (e.g. 1966, 1968, 1977 and 1978) the proportion invested in South African securities started off low and increased monotonically as the risk increased. In other years (e.g. 1971 and 1979) the proportion started off high and decreased monotonically as λ increased, and in some years (e.g. 1965, 1967, 1972, 1973 and 1981) the proportion invested started off low, rose to a peak and then decreased again as risk increased.

From Tables 6.10 to 6.12 it will be noticed that on average between 5,6% and 11,7% should have been invested in foreign stocks; between 37,3% and 50,0% in metals and between 17,6% and 26,6% in soft commodities.

A point of interest is that the ranges mentioned above contained the actual fraction of the number of securities that each security group contributed to the entire 25-security universe. For example, there are 7 South African securities in the 25-security universe. This fraction is 28%, which lies within the range 26,7% to 31,6% that should

have been invested by the South African investor in his local securities. Table 6.13 below displays this fully:

security group	no. of securities in group	percentage of total no. of securities	range of investment proportion calculated by Markowitz portfolio selection
SA securities	7	28%	26,7% - 31,6%
foreign stocks	2	8%	5,6% - 11,7%
metals	11	44%	37,3% - 50,0%
soft commodities	5	20%	17,6% - 26,6%

Table 6.13 Ranges of investment in each security group as calculated by the Markowitz portfolio selection model and actual fractions of the number of securities contributed by each group to the 25-security universe.

Thus on average over the 18-year period the South African investor should have chosen to invest in each security group in roughly the same proportion as each group contributed to the total universe of securities.

6.2.3 The Capital Market Line Approach

The Separation Theorem introduced in section 4.5 argues that the rational investor would divide his funds amongst two benchmark investments:

(i) a risky portfolio (the 'market portfolio')
and (ii) borrowing or lending at the risk-free rate.

The market portfolio is the optimal combination of risky securities and can be determined in any period by

finding the point on the efficient frontier which is tangent to the line with the risk-free rate as y-intercept. This line is commonly known as the Capital Market Line.

Figures 6.19 to 6.36 show the range of risk and return along the Capital Market Lines R_f to A with the introduction of borrowing and lending at the risk-free rate for the years 1965 to 1982 respectively. It should be noted that the risk-free rate was taken to be the average of the prevailing Treasury Bill rates over the respective years, and that these rates were divided by 12 in order to be comparable to the monthly returns used in the study. The risk-free rates can be found in Appendix A. The percentage composition of the optimal combination of securities at risk, marked P in Figures 6.19 to 6.36, are shown in Table 6.14. The means of the proportion of each security taken over all 18 years, as well as the standard deviations are also included. The important features of the optimal combination of risky securities are:

(a) The composition of the optimal combination of risky se-
securities differed significantly during each period.

(b) In each year's optimal combination of risky securities relatively few securities are found to be present. This ranges between 4 different securities present in 1981 and 10 securities present in 1968, 1969 and 1972.

(c) No individual security was found to be dominant in the optimal combination of risky securities during the 18 years of the study. Every security appeared in at least two years, but never more than seven years.

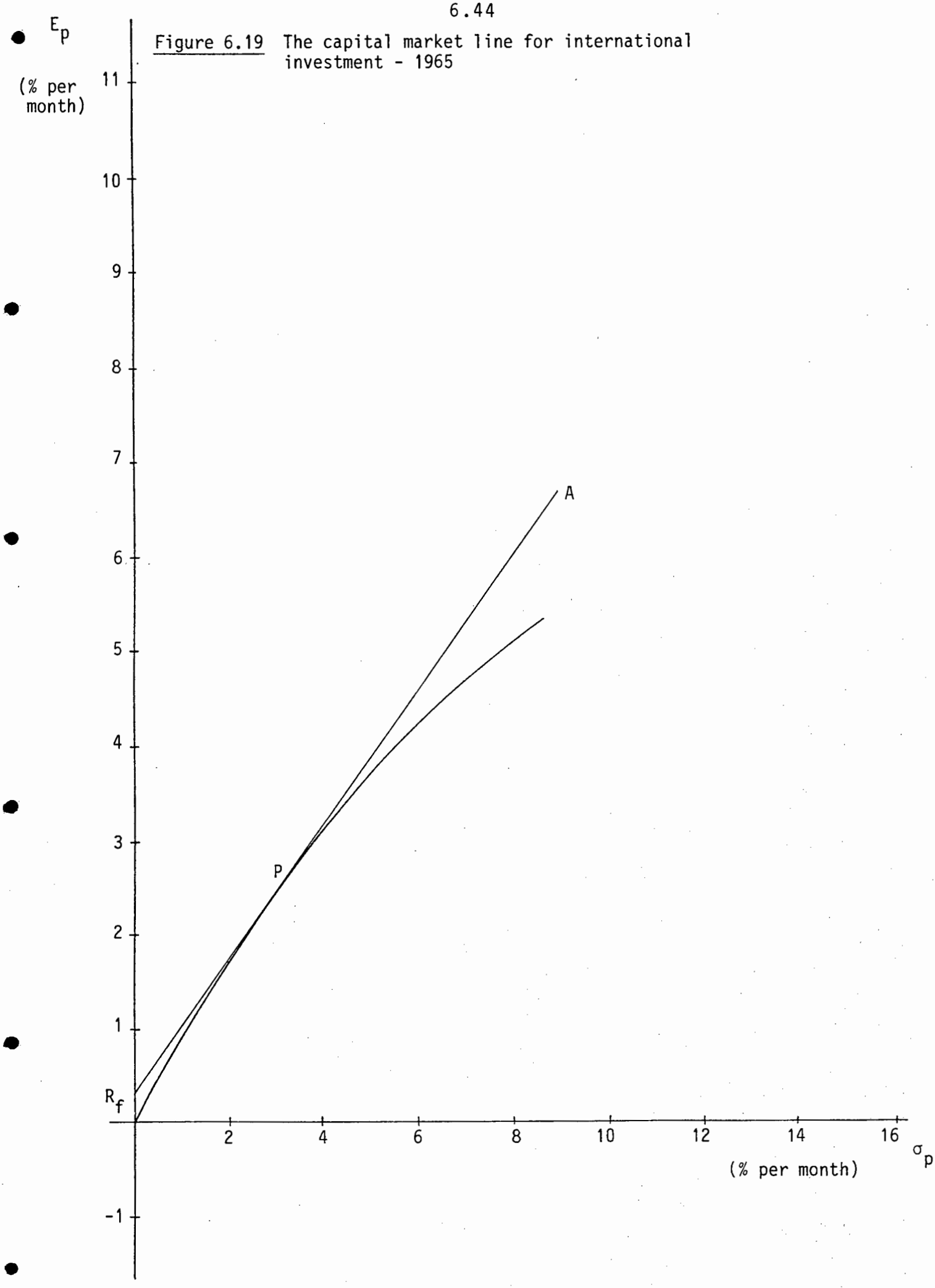
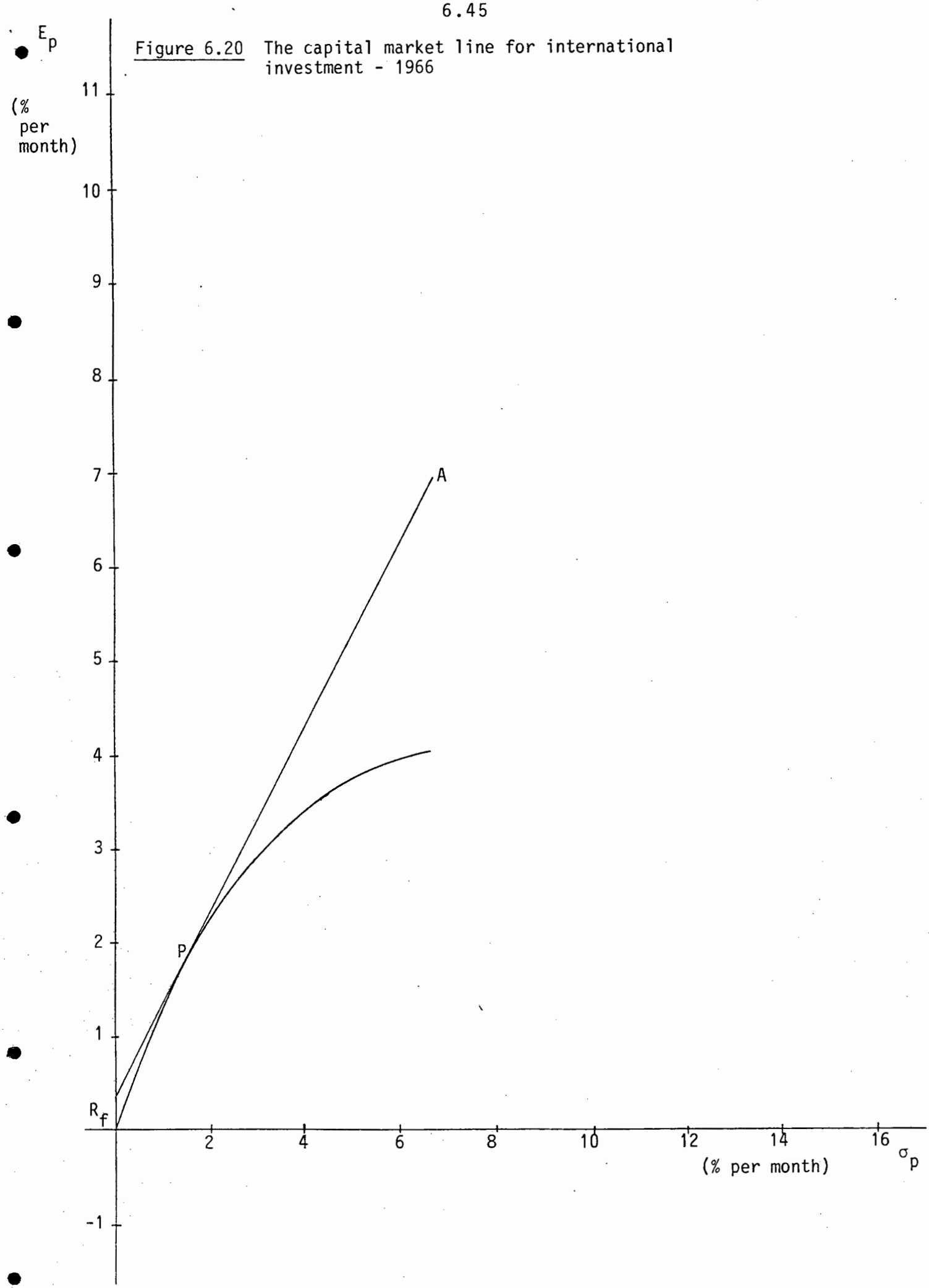


Figure 6.19 The capital market line for international investment - 1965

Figure 6.20 The capital market line for international investment - 1966



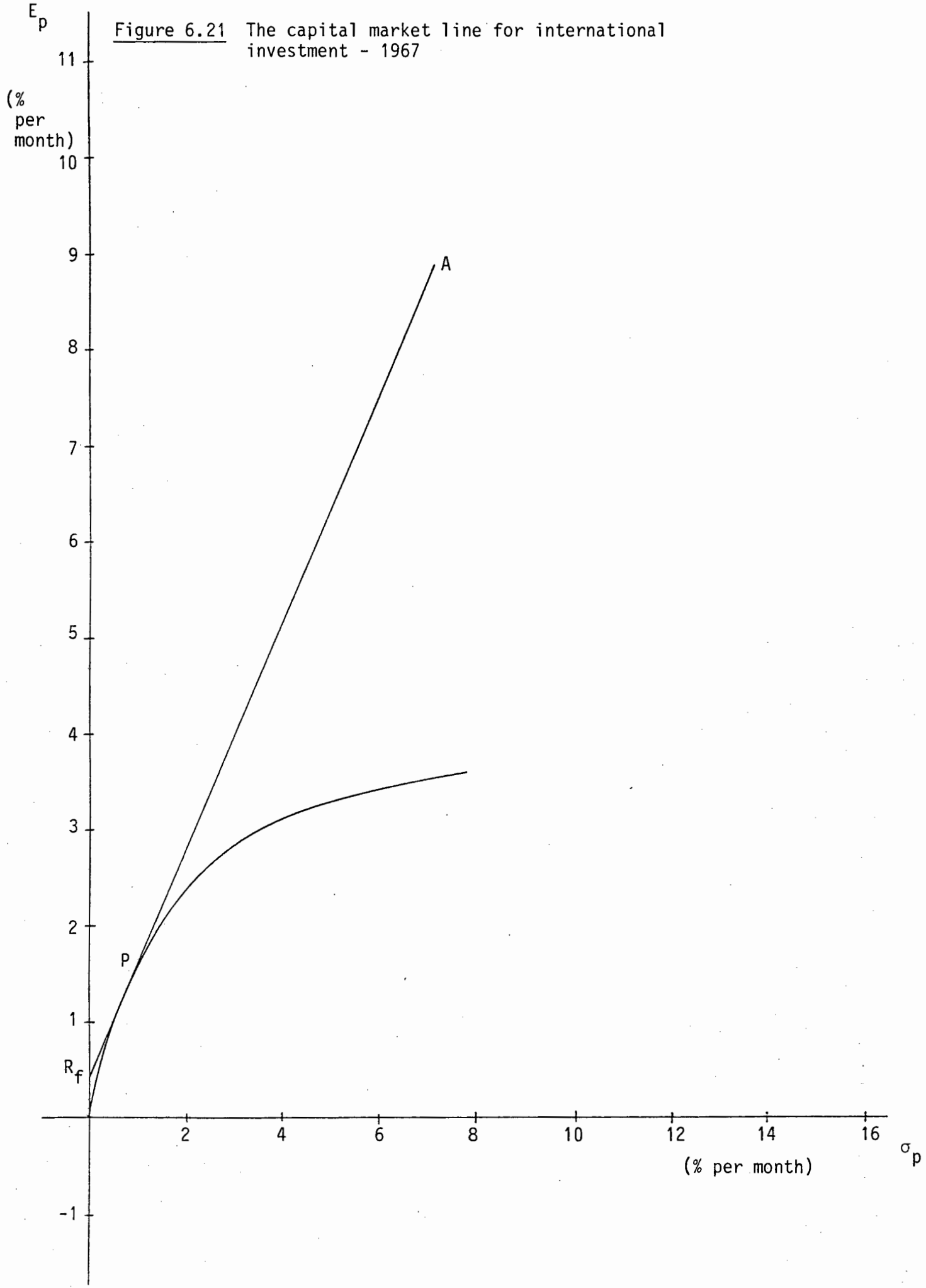
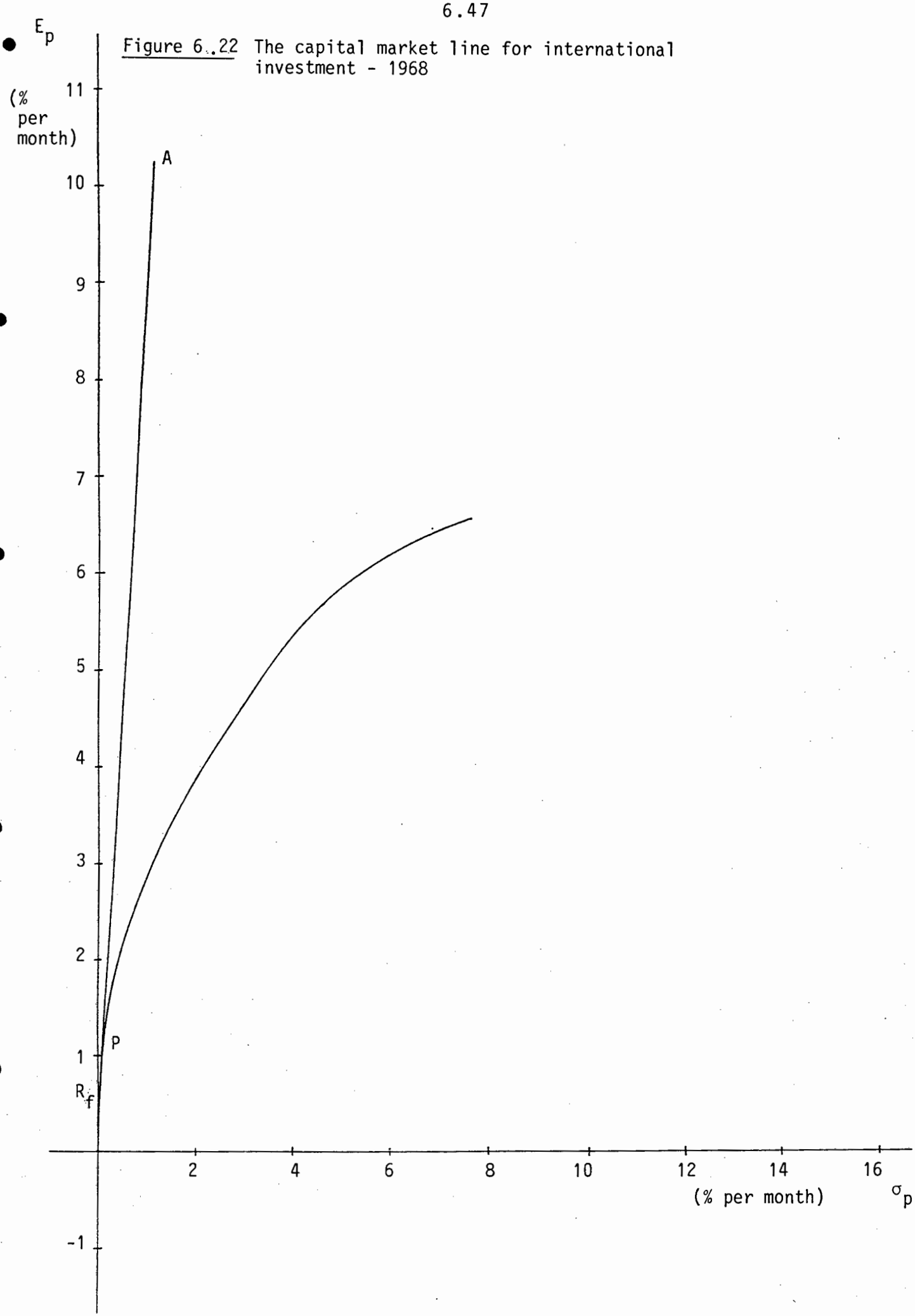


Figure 6.21 The capital market line for international investment - 1967

Figure 6.22 The capital market line for international investment - 1968



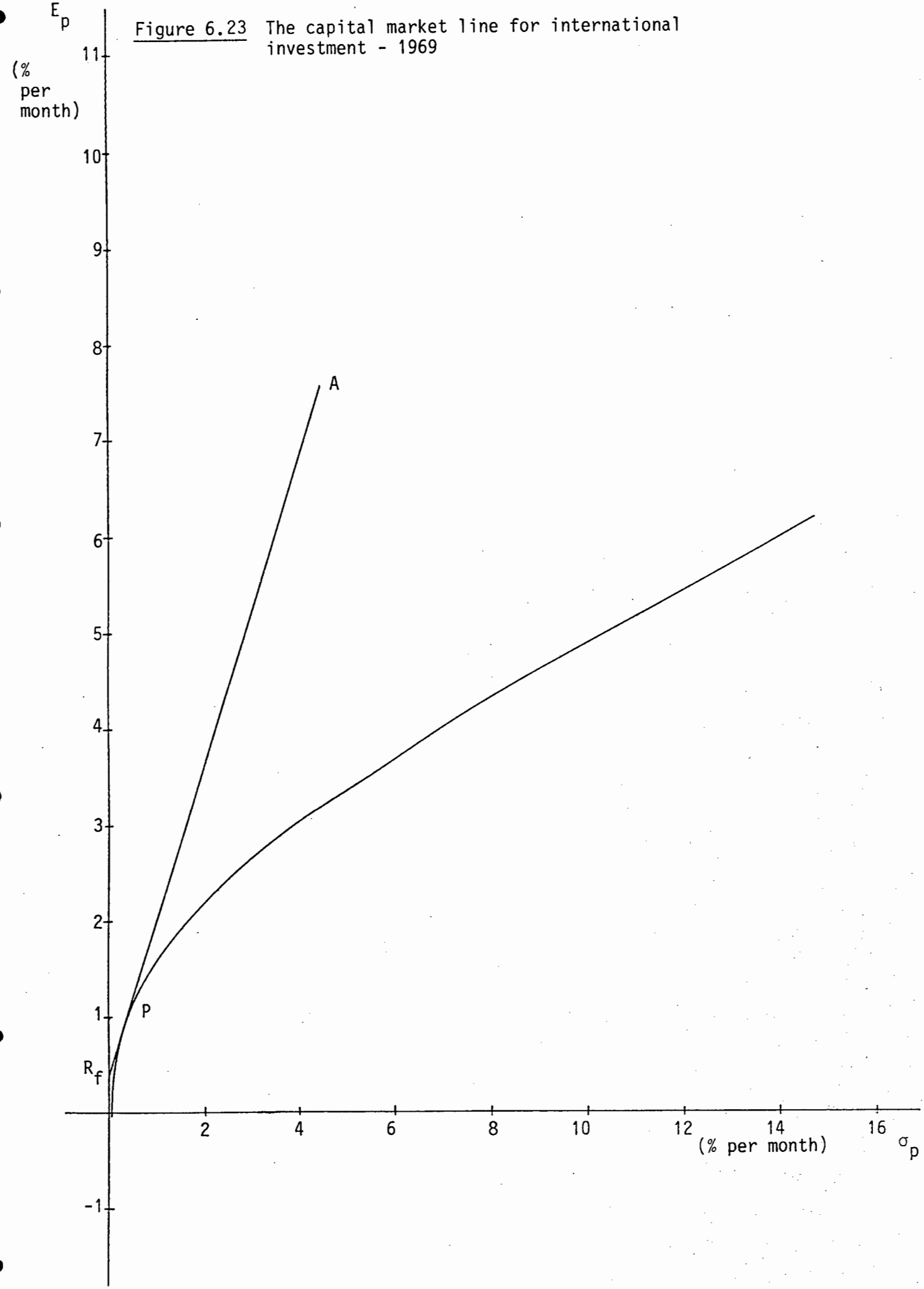


Figure 6.23 The capital market line for international investment - 1969

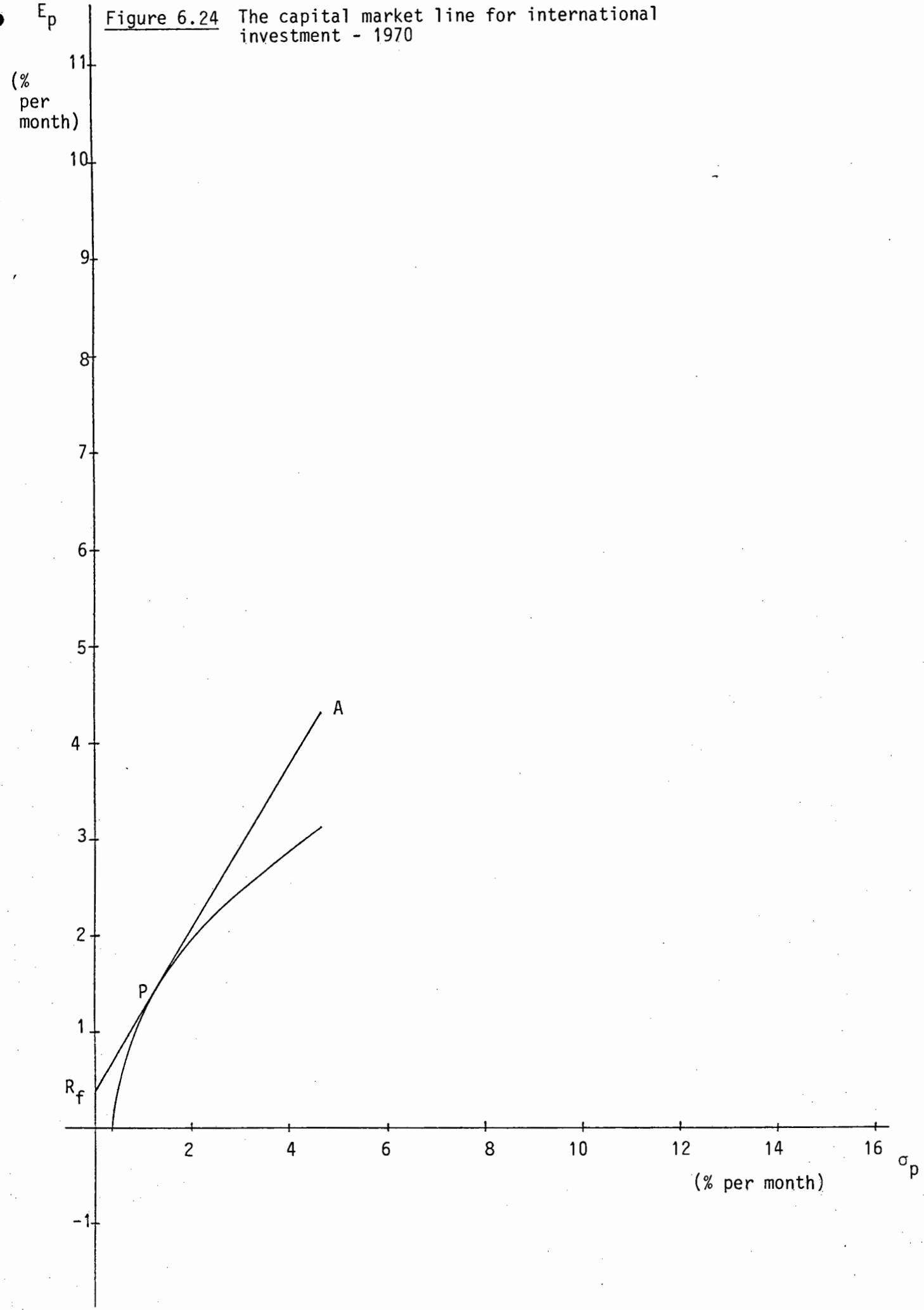
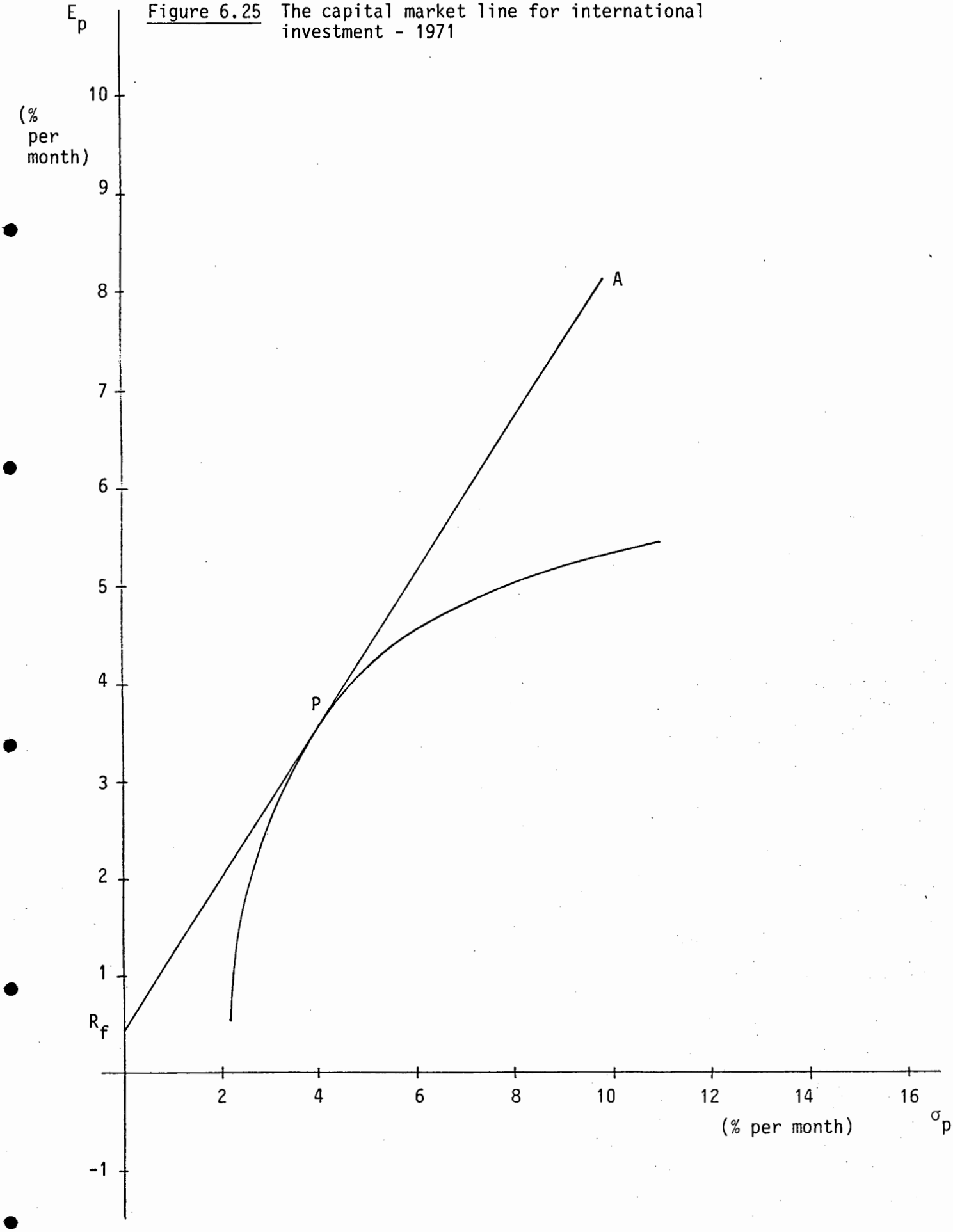


Figure 6.24 The capital market line for international investment - 1970

Figure 6.25 The capital market line for international investment - 1971



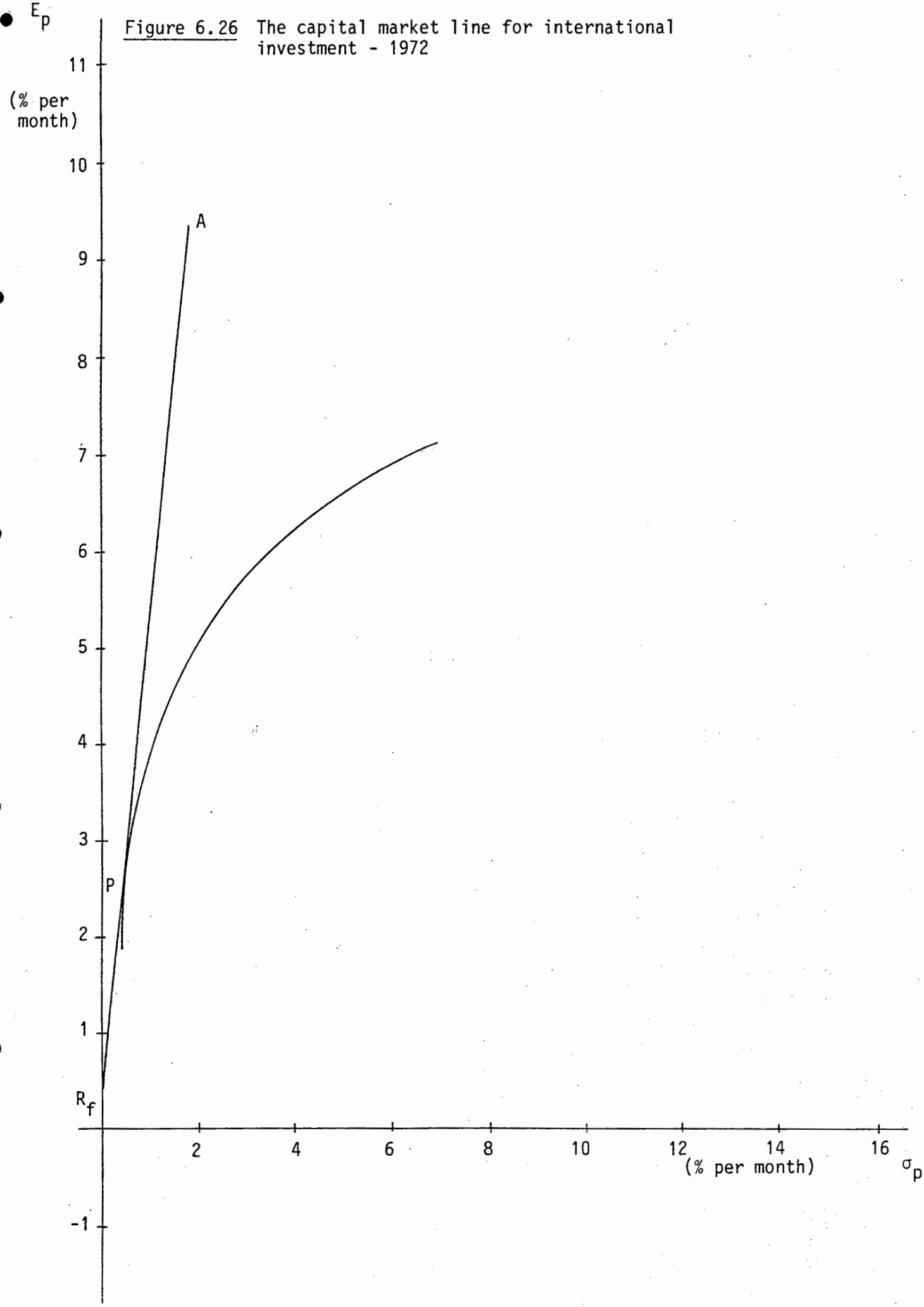


Figure 6.26 The capital market line for international investment - 1972

Figure 6.27 The capital market line for international investment - 1973

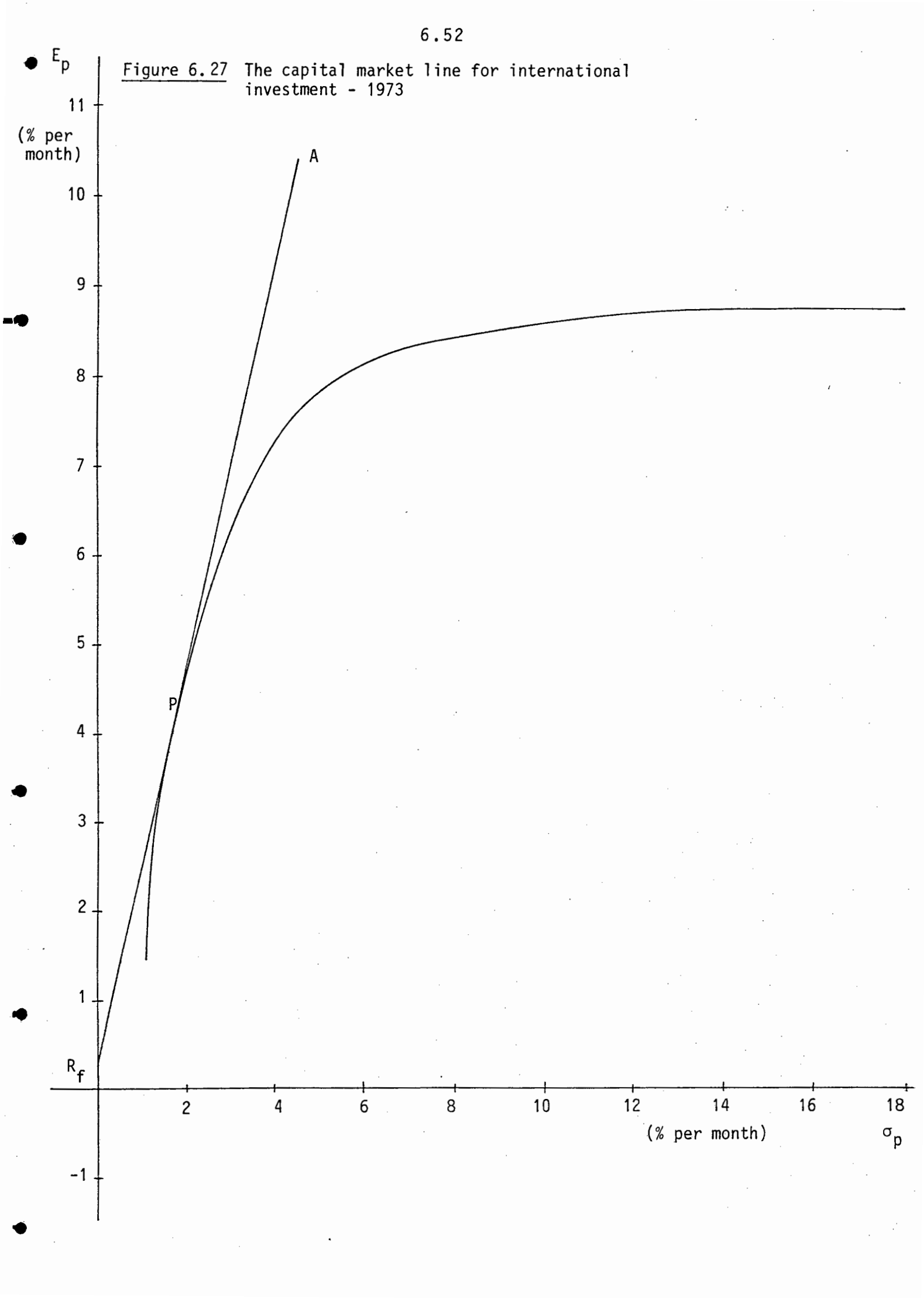


Figure 6.28 The capital market line for international investment - 1974

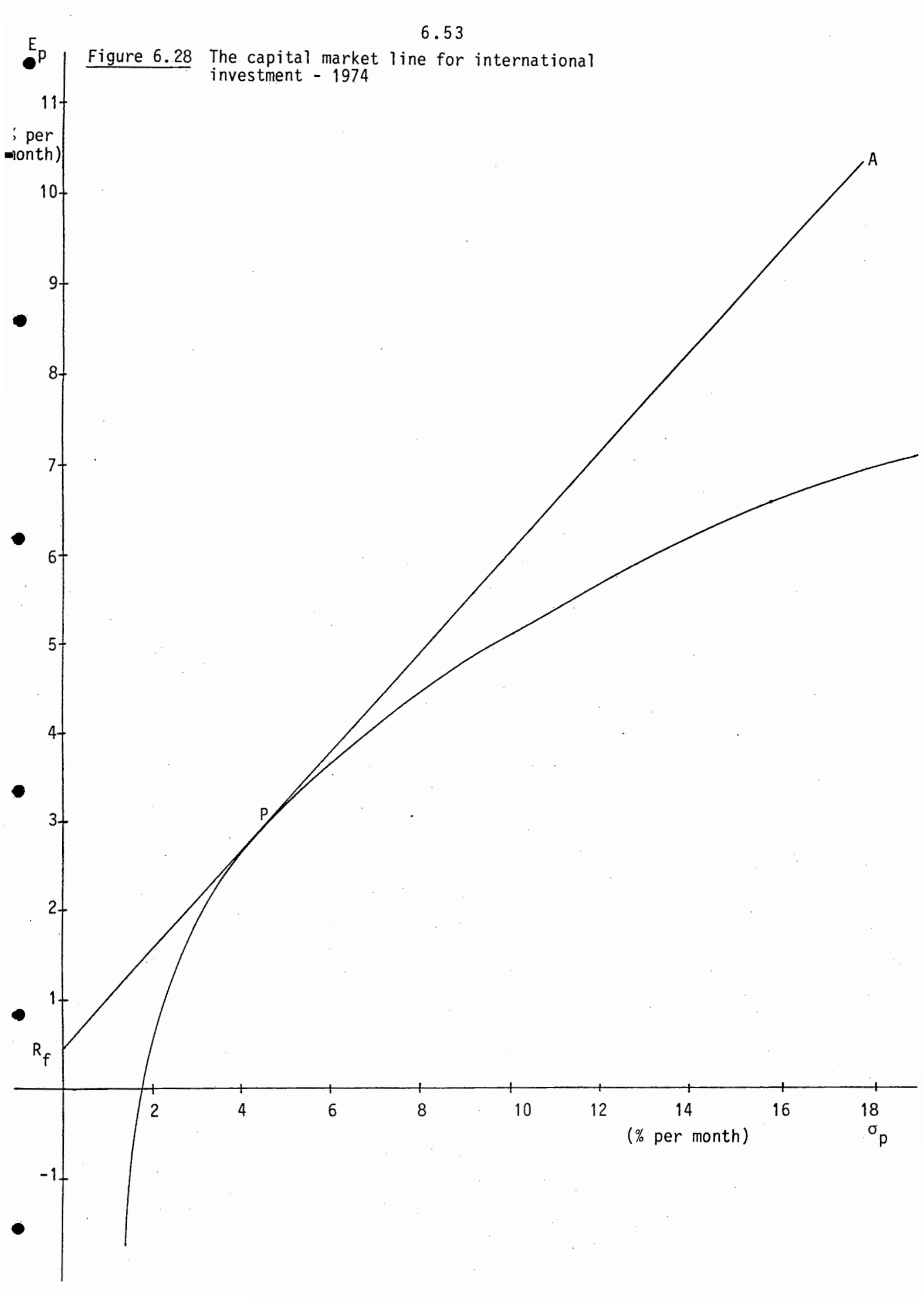


Figure 6.29 The capital market line for international investment - 1975

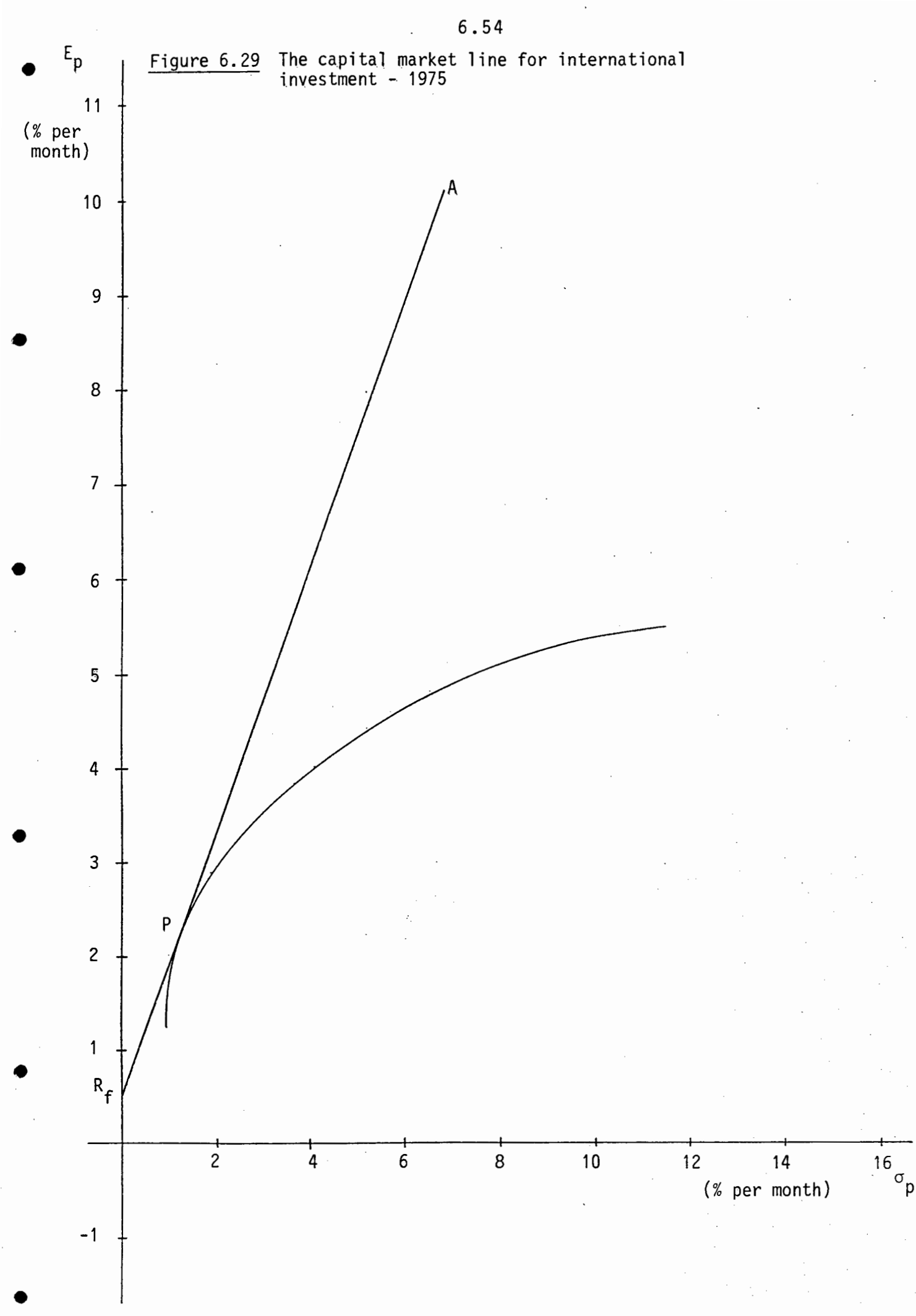


Figure 6.30 The capital market line for international investment - 1976

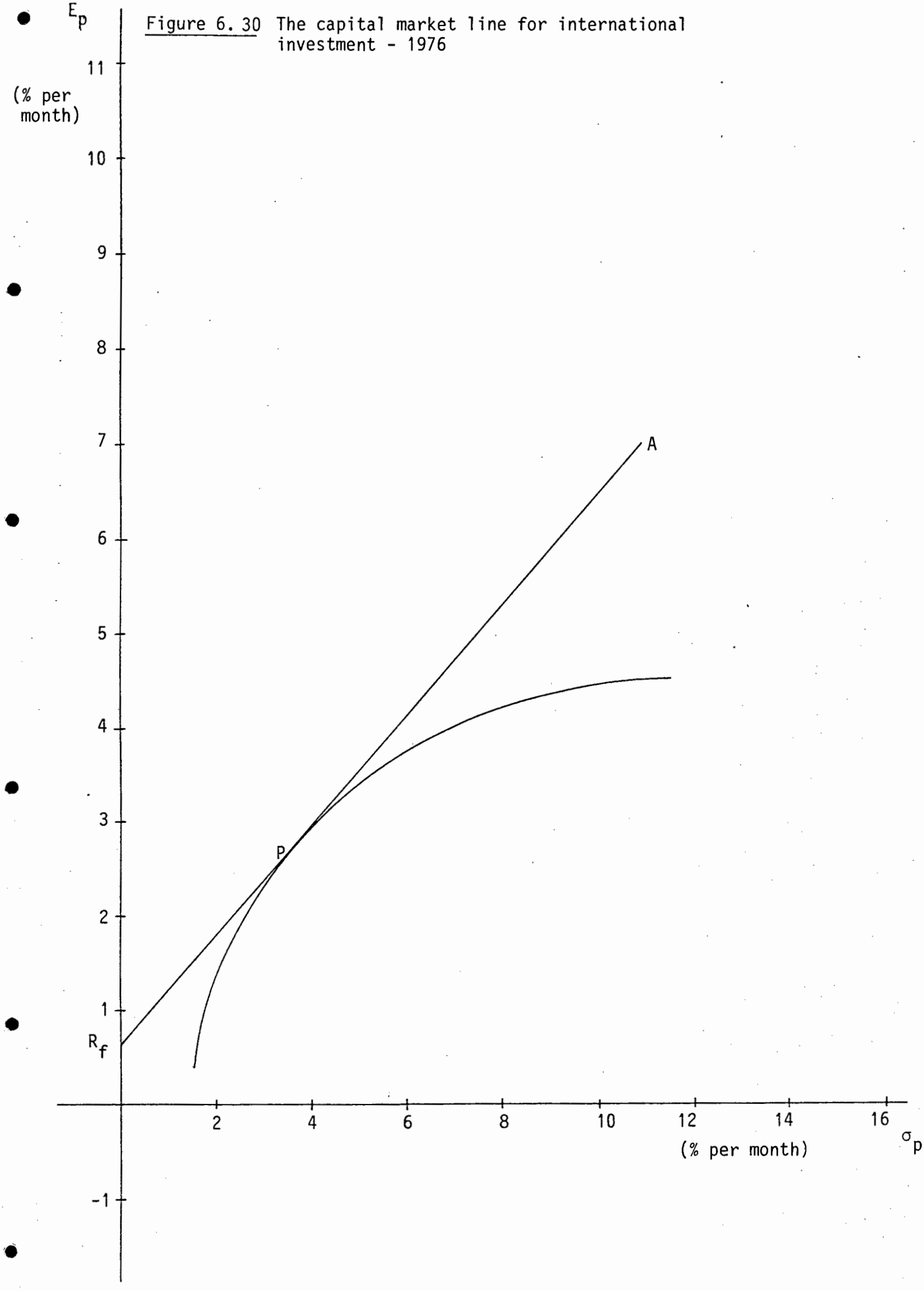
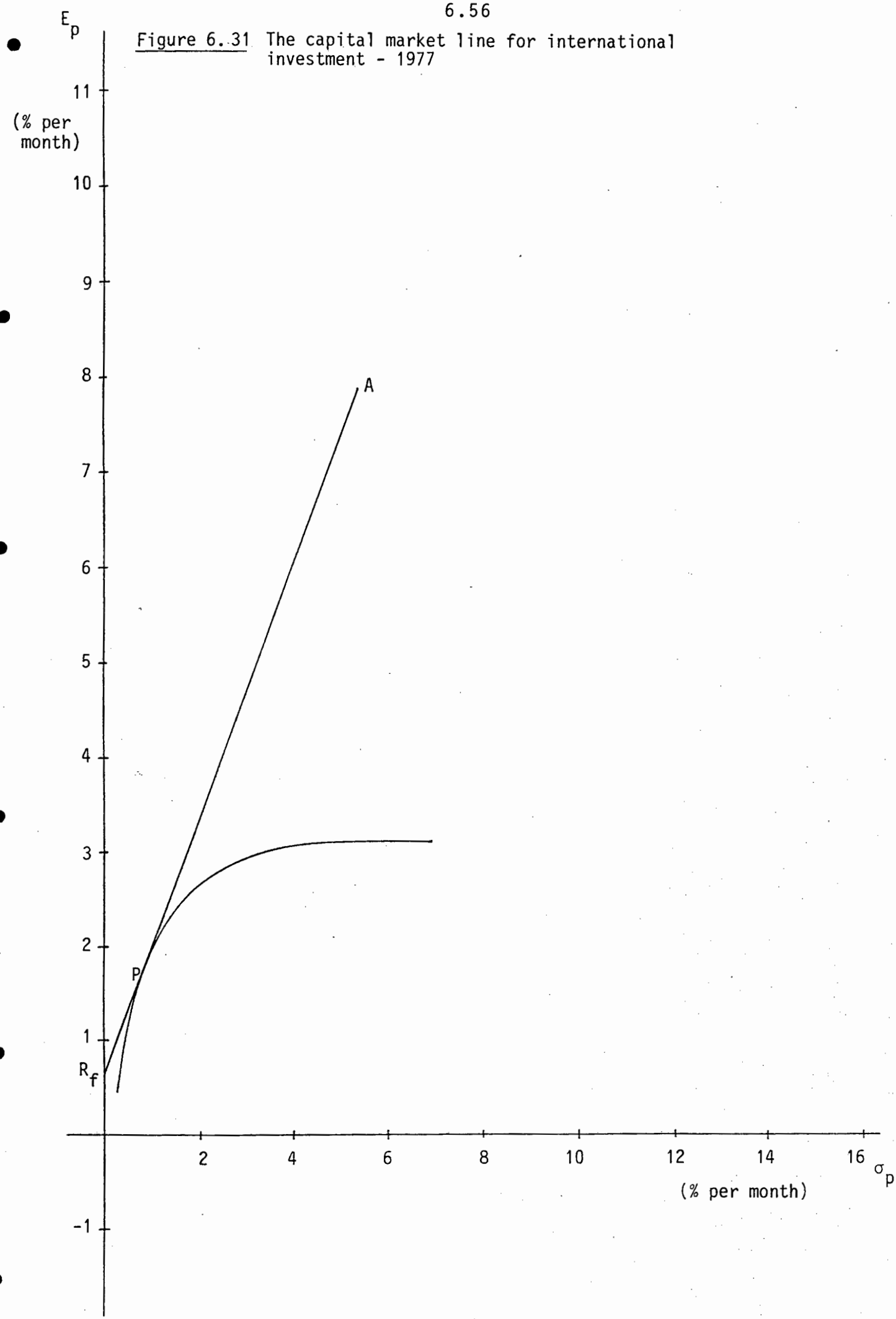


Figure 6.31 The capital market line for international investment - 1977



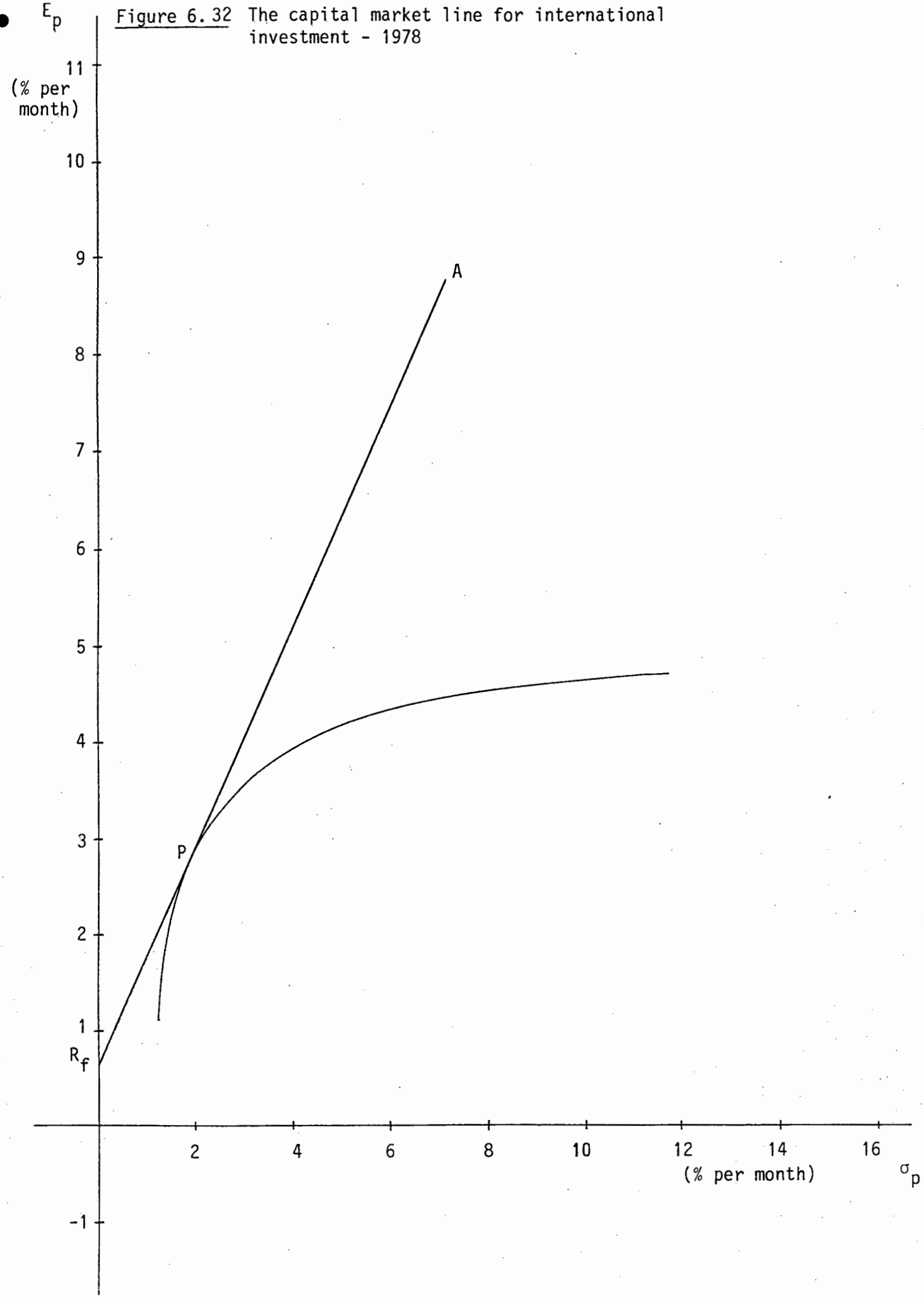


Figure 6.32 The capital market line for international investment - 1978

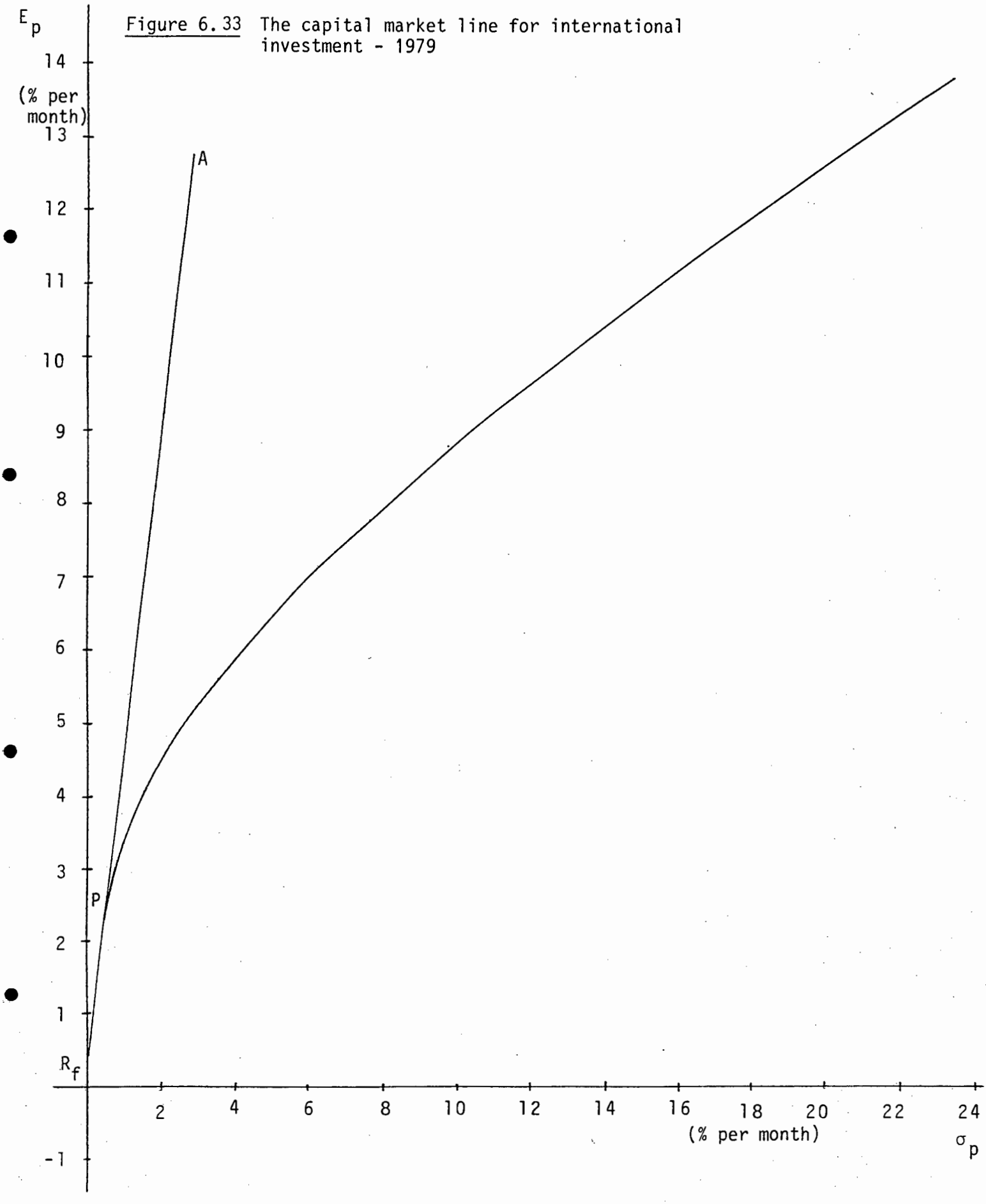


Figure 6.33 The capital market line for international investment - 1979

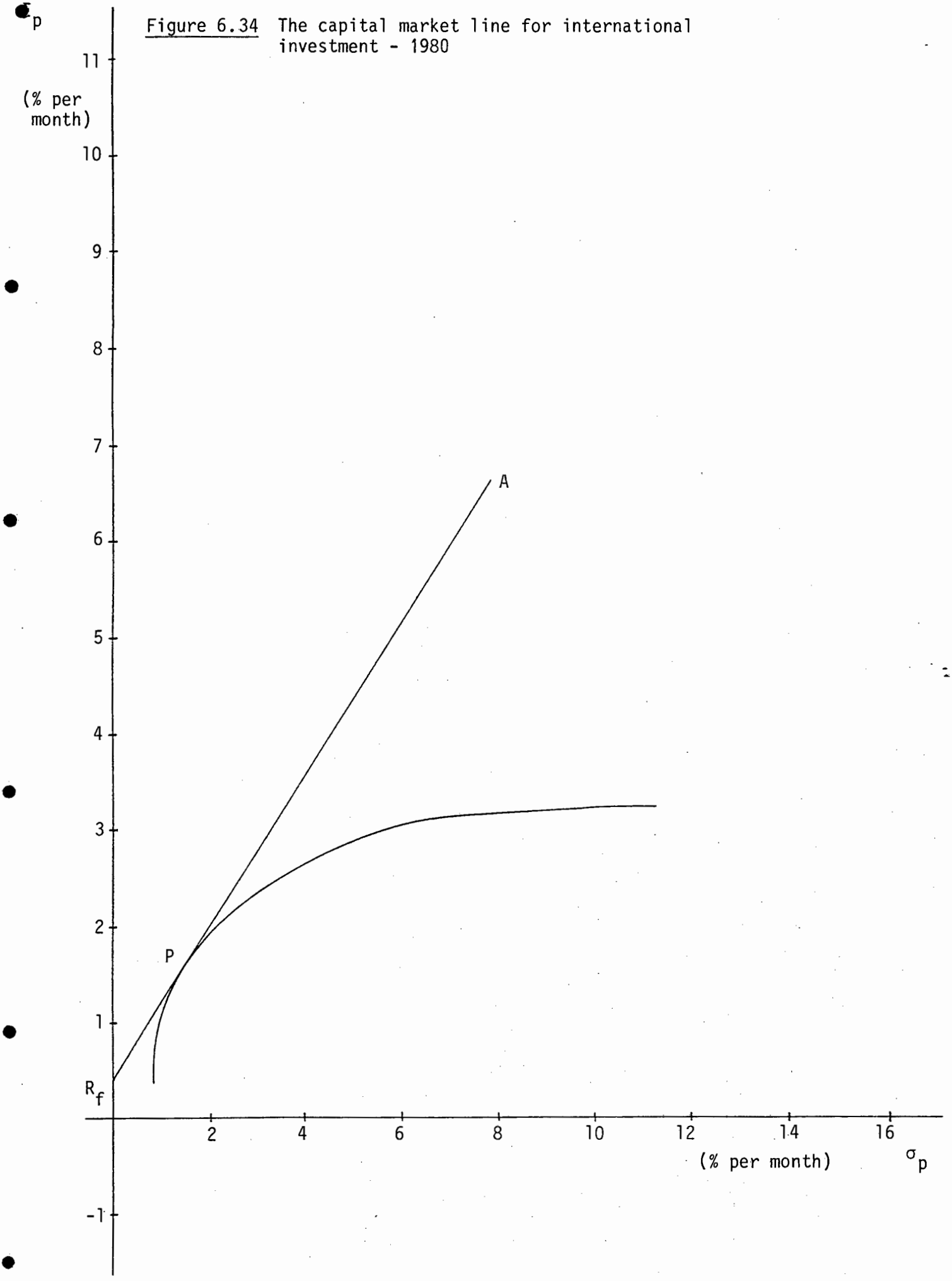


Figure 6.34 The capital market line for international investment - 1980

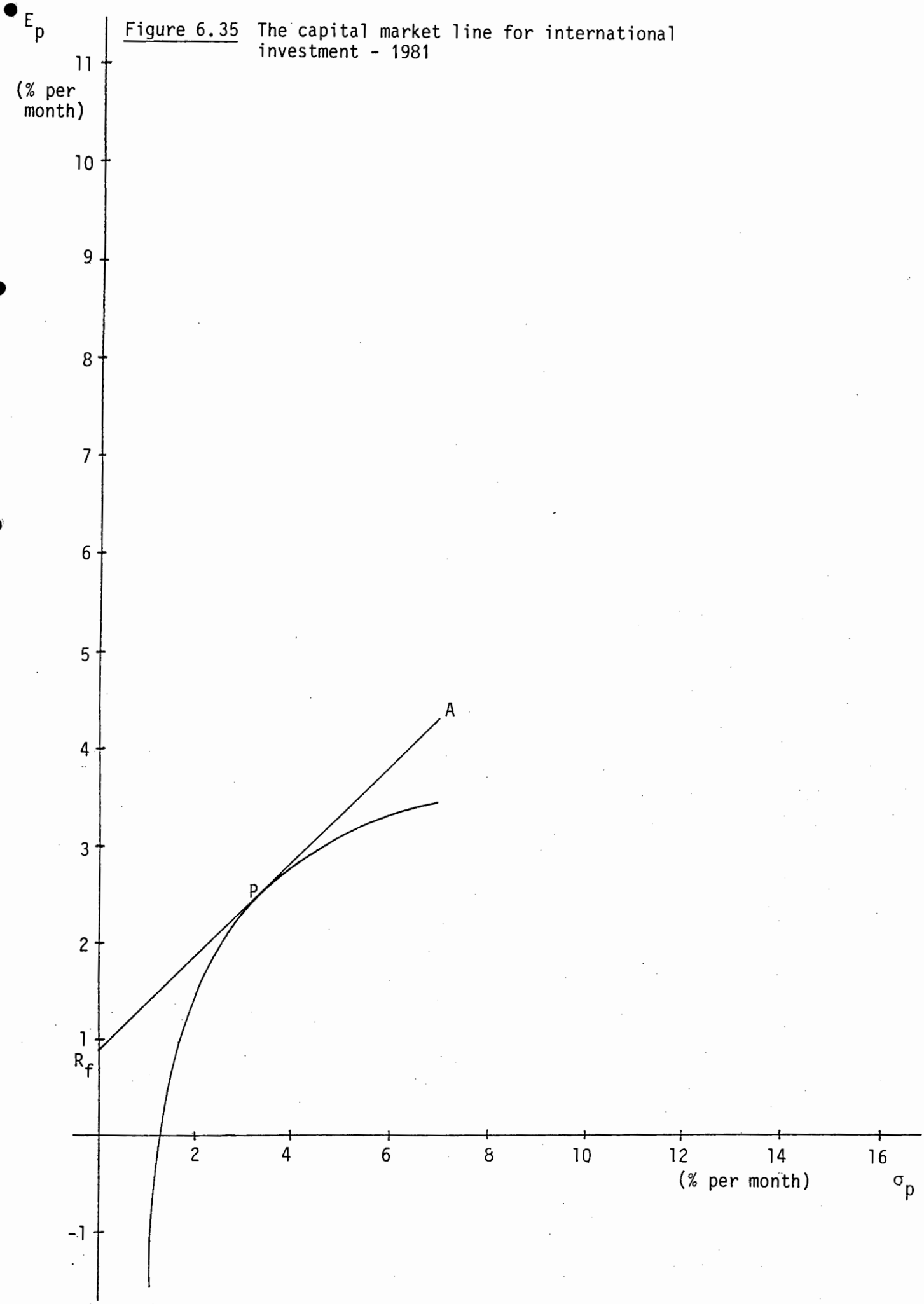
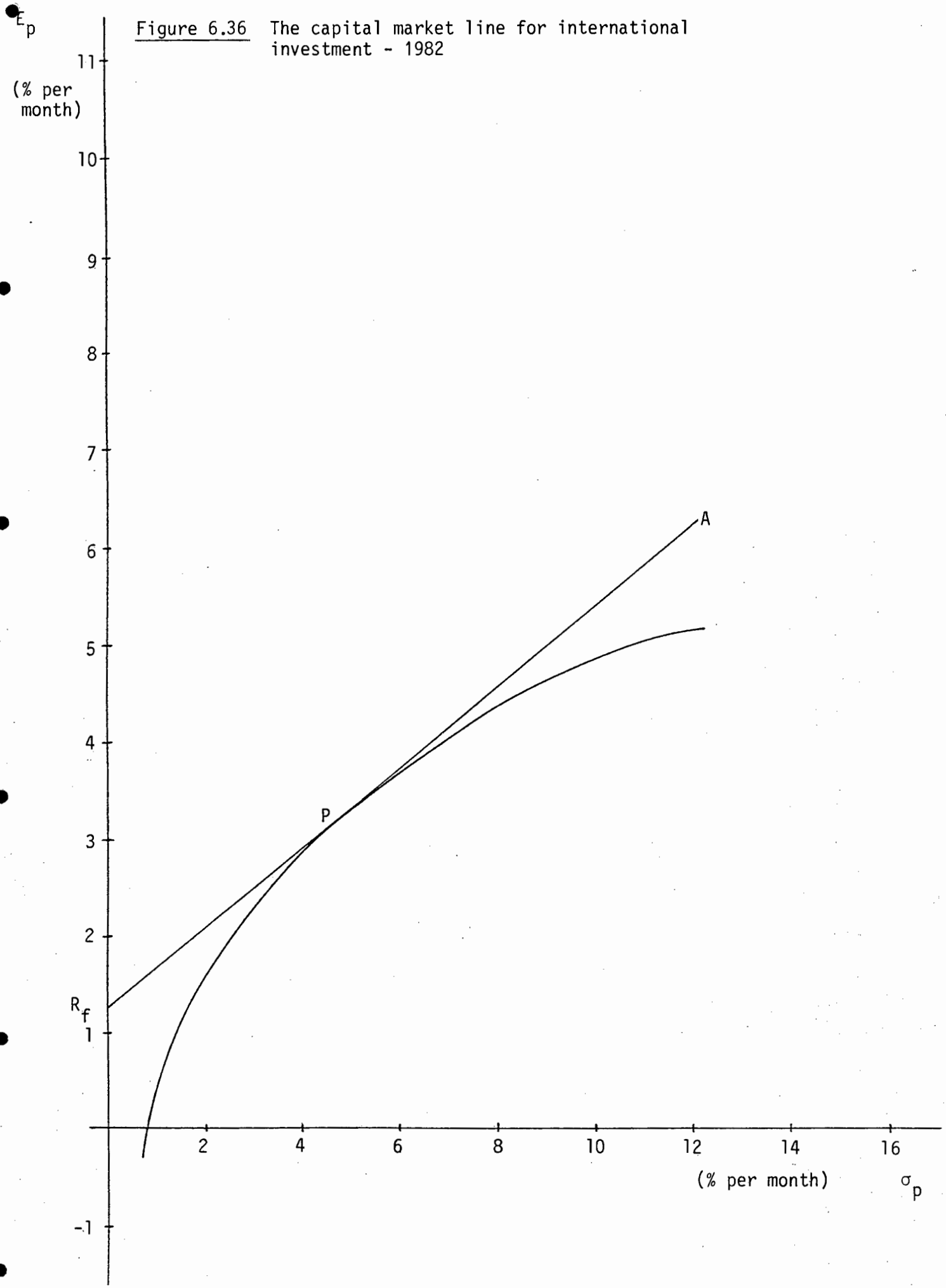


Figure 6.35 The capital market line for international investment - 1981

Figure 6.36 The capital market line for international investment - 1982



YEAR	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	MEAN	STANDARD DEVIATION
E_p	2,27	1,73	1,21	0,81	0,90	1,24	3,41	2,49	3,82	2,77	2,46	2,71	1,65	2,72	2,33	1,63	2,44	2,94		
σ_p	2,78	1,42	0,68	0,10	0,25	1,22	3,91	0,49	1,62	4,25	1,21	3,60	0,83	1,87	0,53	1,53	3,30	4,16		
Security																				
JSE Coal	-	1,35	-	-	15,22	-	-	21,07	-	-	11,52	17,27	16,75	-	-	14,31	-	-	5,42	7,91
JSE Diamonds	9,18	19,02	-	6,21	-	-	-	9,12	-	-	-	-	1,98	7,61	10,24	-	-	-	3,52	5,48
JSE All Gold	-	-	-	-	-	1,13	3,03	8,12	18,06	-	-	-	-	-	-	-	-	19,14	2,75	6,11
JSE Mets & Mins	14,94	18,19	-	-	-	-	-	3,35	-	-	-	-	-	-	-	-	-	-	2,03	5,37
JSE Min Fin	-	-	-	-	-	-	-	-	-	-	-	-	1,61	-	-	1,42	-	-	0,17	0,49
JSE Financial	-	5,99	19,23	-	-	-	-	-	4,74	-	-	-	-	-	19,79	-	16,75	4,14	3,92	7,03
JSE Industrial	-	-	-	7,25	-	-	-	7,93	-	-	31,90	-	-	-	17,72	-	-	-	3,60	8,45
S & P	-	-	12,31	4,78	-	-	-	17,03	-	-	-	-	-	36,32	-	18,25	-	56,14	8,04	15,54
UK Act Index	-	-	14,68	-	-	-	32,17	-	-	-	7,76	-	14,45	-	-	10,49	-	-	4,42	8,67
Lead	-	-	8,19	-	13,31	-	-	-	-	-	-	-	-	-	7,26	-	-	-	1,60	3,84
Tin	-	-	-	2,12	-	-	-	-	14,27	-	-	42,03	-	-	-	-	22,96	-	4,52	11,20
Zinc	-	-	-	-	-	-	26,68	-	12,67	-	-	-	-	7,16	-	-	17,86	-	3,58	7,72
Silver	-	-	7,86	0,79	5,60	-	-	-	-	-	-	-	-	-	-	-	-	7,12	1,19	2,65
Aluminium	-	-	-	-	46,75	-	-	3,49	-	4,93	-	-	21,66	-	20,90	-	-	-	5,42	12,37
Antimony	-	-	-	36,43	3,26	-	-	-	25,78	-	-	24,95	-	-	-	20,63	-	-	6,17	11,80
Copper	22,71	-	-	-	0,52	-	-	-	-	-	9,56	-	-	6,10	0,43	-	-	-	2,18	5,74
Nickel	-	43,77	-	-	-	-	-	-	-	39,19	-	-	17,95	-	12,14	-	-	-	6,28	13,74
Platinum (OP)	-	-	-	18,13	-	19,81	-	-	-	40,79	-	-	-	42,81	-	-	42,43	-	9,11	16,31
Platinum (FP)	26,48	-	2,36	-	-	-	-	5,17	-	-	-	-	-	-	-	-	-	-	1,89	6,27
Gold	-	-	20,97	2,93	5,22	9,31	24,34	-	-	-	-	8,52	6,92	-	-	-	-	-	4,35	7,41
Wool	19,26	11,68	-	13,54	-	-	-	16,75	4,90	-	-	7,23	-	-	-	-	-	-	4,08	6,63
Cotton	-	-	-	-	1,14	-	13,78	7,97	11,51	-	17,00	-	-	-	-	-	-	-	2,86	5,58
Sugar	-	-	-	-	1,09	28,06	-	-	-	9,13	-	-	-	-	-	-	-	-	2,13	6,82
Wheat	7,43	-	14,40	-	-	21,58	-	-	8,06	5,96	-	-	-	-	2,59	34,90	-	-	5,27	9,56
Maize	-	-	-	7,82	7,89	20,11	-	-	-	-	22,26	-	18,69	-	8,94	-	-	13,46	5,50	8,00
TOTAL	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	

Table 6.14 The percentage composition of the optimal combination of securities at risk, 1965 to 1982, as well as mean proportions and standard deviations of each security

Table 6.15 shows the percentage composition of the optimal combination of securities at risk amongst the four main groups, South African securities, foreign stocks, metals and soft commodities. Of paramount importance to the local investor is the proportion that should have been invested in South African securities. This differs significantly from year to year, ranging from a minimum of 0% in 1974 to a maximum of 49,56% in 1972. On average over the 18-year period 1965 to 1982 the proportion that should have been invested in South African securities was 23,42%. There is also a large range in the proportion that should have been invested in foreign stocks, metals and soft commodities in each year at the risk level constituting the optimal combination of risky securities. On average, however, these proportions are 12,47% for foreign stocks, 44,27% for metals and 19,84% for soft commodities.

	proportion invested in South African securities	proportion invested in foreign stocks	proportion invested in metals	proportion invested in soft commodities
1965	24,12	0,00	49,19	26,69
1966	44,55	0,00	43,77	11,68
1967	19,23	26,99	66,37	14,40
1968	13,46	4,78	60,40	21,36
1969	15,22	0,00	74,66	10,12
1970	1,13	0,00	29,12	69,75
1971	3,03	32,17	51,02	13,78
1972	49,59	17,03	8,66	24,72
1973	22,80	0,00	52,73	24,47
1974	0,00	0,00	84,91	15,09
1975	43,42	7,76	9,56	39,26
1976	17,27	0,00	75,50	7,23
1977	20,34	14,45	46,52	18,69
1978	43,93	36,32	19,75	0,00
1979	47,75	0,00	40,72	11,53
1980	15,73	28,74	20,63	34,90
1981	16,75	0,00	83,25	0,00
1982	23,28	56,14	7,12	13,46
AVERAGE	23,42	12,47	44,27	19,84

Table 6.15 Composition of the optimal combination of risky securities amongst the four groups South African securities, foreign stocks, metals and soft commodities.

6.3 Summary and Conclusions

In this chapter an examination was made of the annual Markowitz efficient portfolios for the years 1965 to 1982.

The efficient frontiers for both South African and international investments were plotted and the composition of the international efficient portfolios examined with special reference to the optimal combination of risky securities. It was seen that the composition of the efficient portfolios differed widely from year to year and from one risk level to another. Furthermore, on average over all 18 years in the study, the proportion invested in South African securities was always less than 32%, irrespective of the risk level. This indicates that the local investor would be better off if he could invest a large proportion of his funds at risk outside of South Africa. Over the period 1965 to 1982 it appears that on average he should have invested between two-thirds and three-quarters of his funds at risk in foreign stocks, metals and soft commodities although this amount differed widely from year to year and from one risk level to another.

In the next chapter an attempt will be made at quantifying the improvement in portfolio performance that a local investor could achieve given an abolition of exchange control regulations.

However, before concluding this chapter it must be pointed out that this study is an *ex post* study. In practice of course investors have to act *ex ante* and one would not expect the *ex ante* efficient frontier perceived by investors to in fact be identical to the subsequent *ex post* efficient frontier. Thus it is not claimed that a South African investor could actually have attained the risk-return combinations given above. Nevertheless the results do provide some indication based on past experience of what proportion of funds the South African investor should invest outside of South Africa should exchange control be abolished.

CHAPTER 7

QUANTIFYING THE BENEFITS OF INTERNATIONAL DIVERSIFICATION

7.1 Introduction

In Chapter 6 it was shown that the efficient portfolios made up from investments chosen from a universe of international securities always dominated the efficient portfolios made up from investments chosen from a universe of South African securities. That is, for a given risk level the international efficient portfolios always offered at least the same, and most often a greater return than the South African efficient portfolios. This would lead to the local investor divesting a certain proportion of his funds outside of South Africa in the event of a relaxation or abolishment of exchange control. An obvious extension to this result is the question: what is the cost to the South African investor of the current exchange control regulations? Or equivalently: how much would local investors gain if these exchange control restrictions were removed? In this chapter a number of practical ways of comparing the two efficient frontiers in each year will be investigated, and an attempt will be made to quantify the gains that are to be had from international investment.

Since there are two relevant criteria involved in evaluating portfolio performance, namely return and risk, a problem exists in the combination of these two criteria into a single meaningful measure of portfolio performance for comparison purposes. A number of techniques have been suggested in the literature, c.f., for example, Lorie and Hamilton (1973), and several of these will be discussed in this chapter. In addition some new procedures will be proposed.

When comparing the two non-linear efficient frontiers in any period what is required is some measure of the distance separating the two curves. For a given risk level this measure would obviously be the difference in returns (measured in percent per annum) between the two efficient frontiers. However it is not obvious at which risk level this difference should be measured. Section 2 contains a discussion leading to the possible choice of a particular risk level at which to measure this difference, and some empirical results from the 18-year period 1965 to 1982.

In section 3 this method is generalised to include all possible risk levels. In section 4 a risk-free asset is introduced, giving rise to the capital market approach. Efficient frontiers are compared on the basis of the capital market lines thus produced. Sharpe (1966) computed the so-called reward-to-variability ratio to compare portfolios, and this method is discussed and empirical results presented in section 5.

The final method of comparison follows a completely different approach and assumes unknown future security performance. Portfolios for the South African investor with and without access to the international markets are selected on this basis and compared in section 6. Finally, conclusions and implications are discussed in section 7.

7.2 Comparison of International and South African efficient frontiers at multiples of the market risk

In an attempt to quantify the gains that are to be made from investing in the enlarged universe consisting of foreign as well as local securities (the case if exchange control restrictions were removed), an initial approach might be to choose some risk level and observe the gain or percentage improvement from such an investment over an investment in a portfolio of purely South African securities.

As mentioned in section 6.2.1 the parameter λ in the Markowitz portfolio selection problem is the "coefficient of risk aversion". Each different value of λ from 0 to ∞ will plot a different point on the efficient frontier from the lowest risk/return point to the highest risk/return point. An initial approach might be to select some value (or values) of $\lambda > 0$ and observe the increase in returns at these values of λ when there is a switch from the purely South African efficient portfolios to the international efficient portfolios. However the value of λ is merely the slope of the efficient frontier at a particular risk level,

and does not consider the actual risk of the portfolio, σ_p. In Figure 7.1 below the international and South African efficient portfolios for the year 1978 are plotted. When

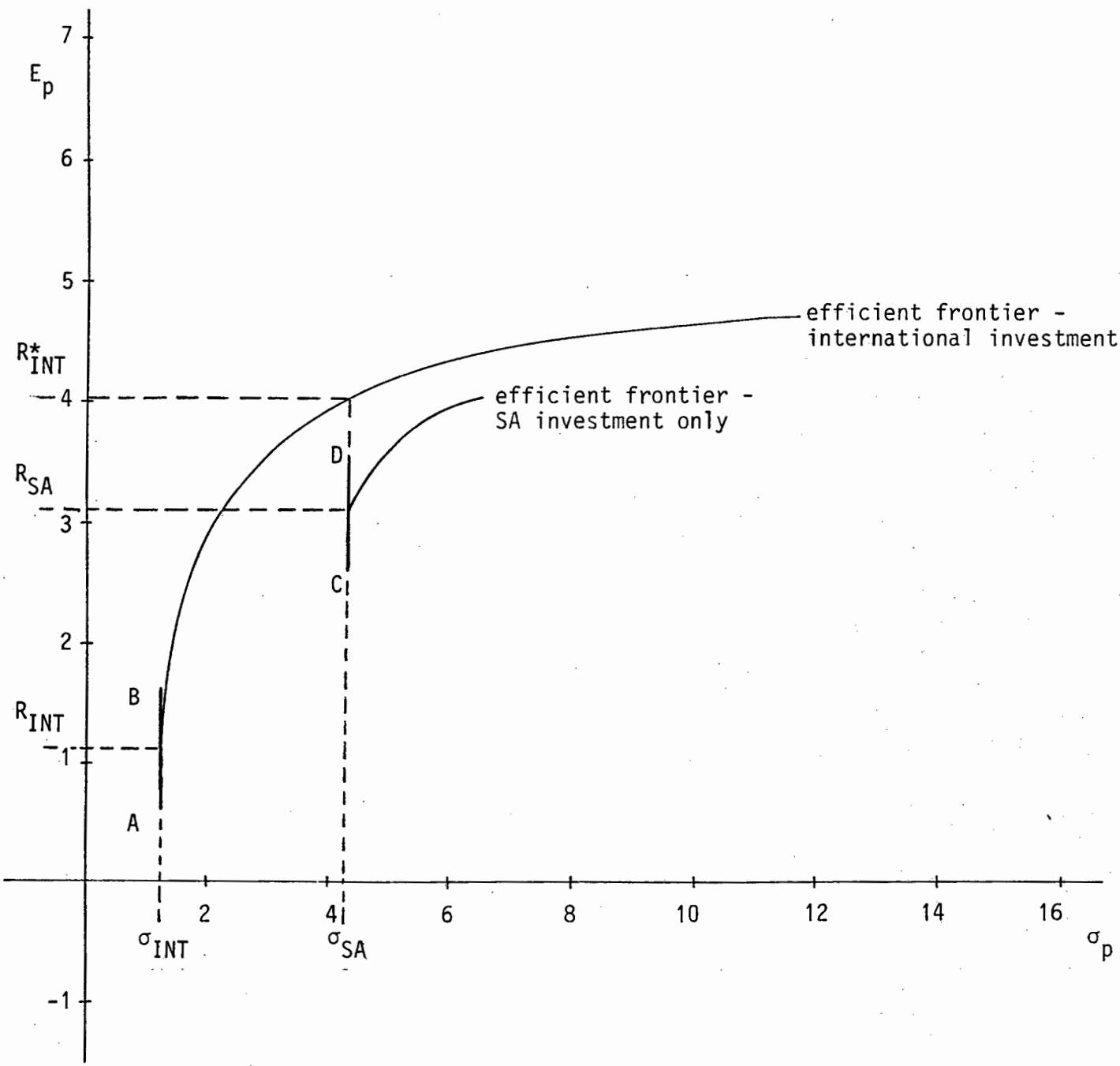


Figure 7.1 The importance of risk and return in dominance relationships of efficient frontiers. (The South African and International efficient frontiers of 1978 are used to illustrate this concept).

$\lambda = 0$ (that is, the lowest possible risk level is achieved) both efficient frontiers are vertical (lines AB and CD in Figure 7.1 indicate the slope of the international and South African frontiers respectively). If return is the only criterion on which the two curves are compared it will be seen that the South African efficient portfolio outperforms the international efficient portfolio at this level of λ (in Figure 7.1 $R_{SA} > R_{INT}$ by approximately 2% where R_{SA} = return on the South African efficient portfolio and R_{INT} = return on the international efficient portfolio). The actual level of portfolio risk, σ_p, is much larger in the case of the South African efficient portfolio, however ($\sigma_{SA} > \sigma_{INT}$ by approximately 3,1%, where σ_{SA} = risk of South African efficient portfolio and σ_{INT} = risk of international efficient frontier). Indeed, given a risk level of σ_{SA} it is clear that in fact the international efficient frontier offers much more return than does the South African frontier at this level of risk ($R_{SA} < R^*_{INT}$). The apparent dominance of the South African efficient portfolios at some levels of λ is thus an illusion and is caused by the non-inclusion of the portfolio risk.

Since the use of λ as a selection criteria is inappropriate other alternatives must be examined. One approach might be to select a specific value of σ_p and observe the increase in return from one curve to the other at this level. However the ranges in risk of the efficient portfolios, both international and South African, vary dramatically from year to year, causing difficulty in choosing suitable risks.

Because of the above-mentioned problems a particular market-related risk was chosen each year as a basis for the comparison of the efficient portfolios. This was the standard deviation of the monthly returns of the JSE All Share Index for each year and gave an indication of the risk of all securities traded on the JSE in that year. This will be referred to as the "market risk" and indicated by σ_{JSE}. For each of the 18 years in the study (1965 to 1982) the monthly returns for both the South African and international efficient portfolios at various multiples of the market risk were computed. The average values of the monthly returns for the entire 18-year period for both efficient frontiers under consideration, as well as average monthly and annual gains can be found in Table 7.1. The average relative performance of the international portfolio to the South African portfolio at each of the risk levels are also displayed.

Table 7.1 clearly supports the assertion that the international efficient portfolios dominate the South African efficient portfolios. This dominance ranges between 1,57% per month (18,84% per year) and 2,28% per month (27,36% per year) on average and depends on the level of risk chosen. However, the larger the risk the smaller the average gain to be made from international investment. A risk-averse investor who is only prepared to accept a risk equal to three-quarters of that of the overall South African market in each year could have achieved average annual gains of 27,36% (from 21,00% per annum to 48,36% per annum) from inter-

	$0,75\ \sigma_{JSE}$	$0,90\ \sigma_{JSE}$	$1,00\ \sigma_{JSE}$	$1,10\ \sigma_{JSE}$	$1,25\ \sigma_{JSE}$
Average monthly return on SA securities (% per month)	1,75	2,40	2,73	2,95	3,18
Average monthly return on international securities (% per month)	4,03	4,30	4,45	4,58	4,75
Average monthly gain from international investment (% per month)	2,28	1,90	1,72	1,63	1,57
Average annual gain from international investment (% per annum)	27,36	22,80	20,64	19,56	18,84
Average relative performance of the international portfolio to the South African portfolio	2,3029	1,7917	1,6300	1,5525	1,4937

Table 7.1 Average returns, gains and relative performances for the period 1965 to 1982 at various multiples of the market risk. (Average σ_{JSE} = 6,56%)

national investment. This represents an average relative increase of 2,3029 . The risky investor who is prepared to accept a risk of as much as 1,25 times that of the overall South African market in each year could have benefitted by 18,84% on average per year (from 31,16% per annum to 57,00% per annum) from international investment. This is an average relative increase of 1,4937 .

A problem exists with this approach in that in certain years the market risk is so great that some of the multiples of this market risk under consideration fall outside of the risk range of the South African and/or international efficient portfolios. This is because the market risk is calculated *ex post*, and thus will not necessarily plot on the South African efficient frontier. In these years the returns at the largest risk attainable is reported. This is equivalent to the case in which an investor cannot achieve a risk level as high as he desires, and instead settles for the largest possible risk level in that period. There are also certain years in which the market risk is small and some of the multiples under consideration of this market risk fall outside of the risk range of one or both of the efficient frontiers. In these cases the returns at the smallest risk attainable is reported. This is the case of an investor desiring a lower risk than is attainable, and thus settling for the smallest possible risk in that period. The above-mentioned two situations occurred in 1969,

1970, 1980 and 1981. Table 7.2 repeats Table 7.1, but ignores the four years just mentioned. It will be seen that the results are substantially the same as before. Thus while the problems mentioned in the previous paragraph exist, they do not appear to seriously invalidate the results.

The main implication of these tables is that at all levels of risk, investors would benefit from a relaxation or removal of exchange control regulations. An interesting facet of the results is that the investors who prefer lower-risk investment (and this includes the Unit Trusts) would benefit by a larger amount in the event of a removal of exchange control regulations than those speculators who are willing to accept a very high risk.

	$0,75\ \sigma_{JSE}$	$0,90\ \sigma_{JSE}$	$1,00\ \sigma_{JSE}$	$1,10\ \sigma_{JSE}$	$1,25\ \sigma_{JSE}$
Average monthly return on SA securities (% per month)	1,82	2,58	3,01	3,29	3,59
Average monthly return on international securities (% per month)	4,25	4,54	4,71	4,85	5,04
Average monthly return on international investment (% per month)	2,43	1,96	1,70	1,56	1,45
Average annual gain from international investment (% per month)	29,16	23,52	20,40	18,72	17,40
Relative performance of the international portfolio to the South African portfolio	2,3352	1,7597	1,5648	1,4742	1,4039

Table 7.2 Average returns, gains and relative performances for the period 1965 to 1982 (except 1969, 1970 and 1981) at various multiples of the market risk. (Average σ_{JSE} = 6,19%)

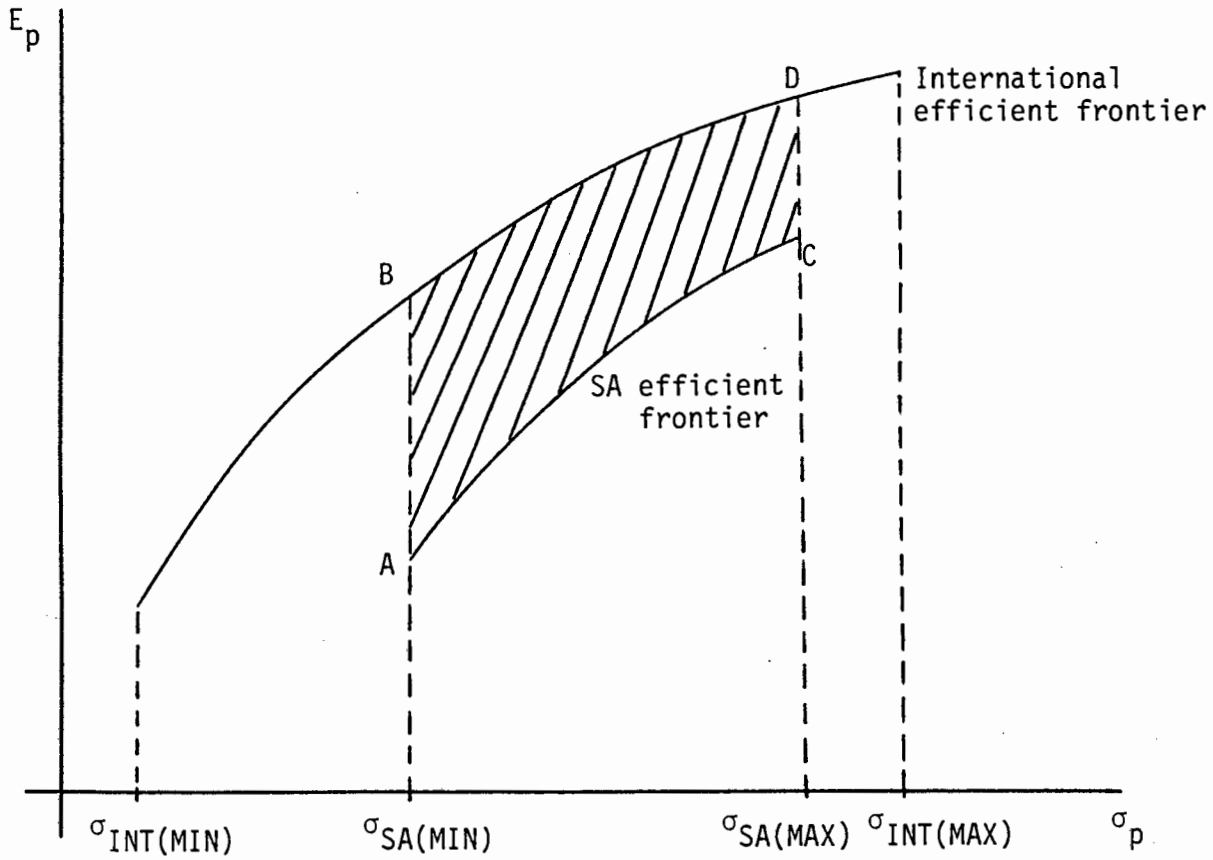
7.3 Approximate areas between efficient frontiers

In the previous section the difference between the international and the South African efficient portfolios was measured at various values of portfolio risk. Ideally it would be desired that the difference be measured at all possible values of risk over which the efficient portfolios span. This could be measured by the area between the two curves.

In general, in any particular year the range of risks attainable were different for the international efficient portfolios and the South African efficient portfolios. For this reason it was decided to measure the area between the two curves over the range of risk common to both curves.

In general the area measured was that between $\max(\sigma_{SA(MIN)}; \sigma_{INT(MIN)})$ and $\min(\sigma_{SA(MAX)}; \sigma_{INT(MAX)})$. This area is illustrated in Figure 7.2 by the shaded area and represents a case where $\sigma_{INT(MIN)} < \sigma_{SA(MIN)}$ and $\sigma_{INT(MAX)} > \sigma_{SA(MAX)}$.

An approximation of this area is achieved by calculating the average of the distances between the two efficient frontiers at the extremes of the risk range common to both curves (the average length of lines AB and CD in Figure 7.2), since the curves diverge as risk decreases. Table 7.3 below shows the monthly and annual averages of these two measurements for each year 1965 to 1982, as well as average values over all 18 years.



$\sigma_{SA(MIN)}$ = minimum value of risk attainable for South African efficient portfolios

$\sigma_{SA(MAX)}$ = maximum value of risk attainable for South African efficient portfolios

$\sigma_{INT(MIN)}$ = minimum value of risk attainable for international efficient portfolios

$\sigma_{INT(MAX)}$ = maximum value of risk attainable for international efficient portfolios

Figure 7.2 The approximate area between two efficient frontiers

Year	Average monthly difference (%)	Average annual difference (%)
1965	1,20	14,40
1966	0,66	7,92
1967	2,43	29,16
1968	1,07	12,84
1969	2,38	28,56
1970	2,02	24,24
1971	3,73	44,76
1972	2,20	26,40
1973	4,45	53,40
1974	5,65	67,80
1975	2,26	27,12
1976	2,70	32,40
1977	1,05	12,60
1978	0,64	7,68
1979	2,42	29,04
1980	2,00	24,00
1981	2,01	24,12
1982	1,34	16,08
Average	2,23	26,81

Table 7.3 Monthly and annual averages of the distance between South African and international efficient frontiers at the extremes of the risk ranges common to both frontiers

This average additional return of 26,81% per year is close to the average increase attained when the two frontiers were compared at different multiples of the market risk (Tables 7.1 and 7.2).

To calculate the relative performance of the two frontiers, the returns of the international portfolios at two extremes of the risk range common to both frontiers were averaged, as were the returns of the South African portfolios at the same risk levels. The ratio of these returns indicates the relative performance of the two frontiers. This average turns out to be 2,0769, or an average percentage increase of 107,69%. This compares with the relative performance of between 1,4937 and 2,3029 (dependent on risk) as calculated in section 7.2.

Thus it appears that on average over the period 1965 to 1982 investors who included international securities in the portfolios could have achieved average returns which were in the range of 20% to 30% per annum above the returns of investors who relied purely on South African securities.

7.4 Introduction of a risk-free asset and an extension to the capital market approach

In section 6.2.3 the capital market line was defined as the line tangent to the efficient frontier with the risk-free rate of return as the y-intercept. The point of tangency between the capital market line and the efficient frontier is known as the optimal combination of risky securities. The Separation Theorem pointed out that the rational investor would divide his funds between this optimal combination of securities at risk and either borrowing or lending at the risk-free rate, the proportion of each being determined by the risk he required. Thus he could attain any position on the capital market line.

For each of the 18 years 1965 to 1982 the capital market lines were drawn from the South African risk-free rate (indicated by R_f) tangent to both the international and South African efficient frontiers. The risk-free rate used for each year was the average of the twelve month-end Treasury Bill rates. The South African risk-free rate was used in both cases as it was assumed that the South African investor would be more likely to invest in a local risk-free asset since in this way he would not incur any exchange rate risk. Indeed, because of exchange rate risk, one could not argue that a USA Treasury Bill is risk-free from a South African investor's point of view. An example of the capital market lines thus produced, using the efficient portfolios for 1981, can be found in Figure 7.3. The optimal combina-

tion of risky securities for the international and South African efficient frontiers are labelled P and Q respectively.

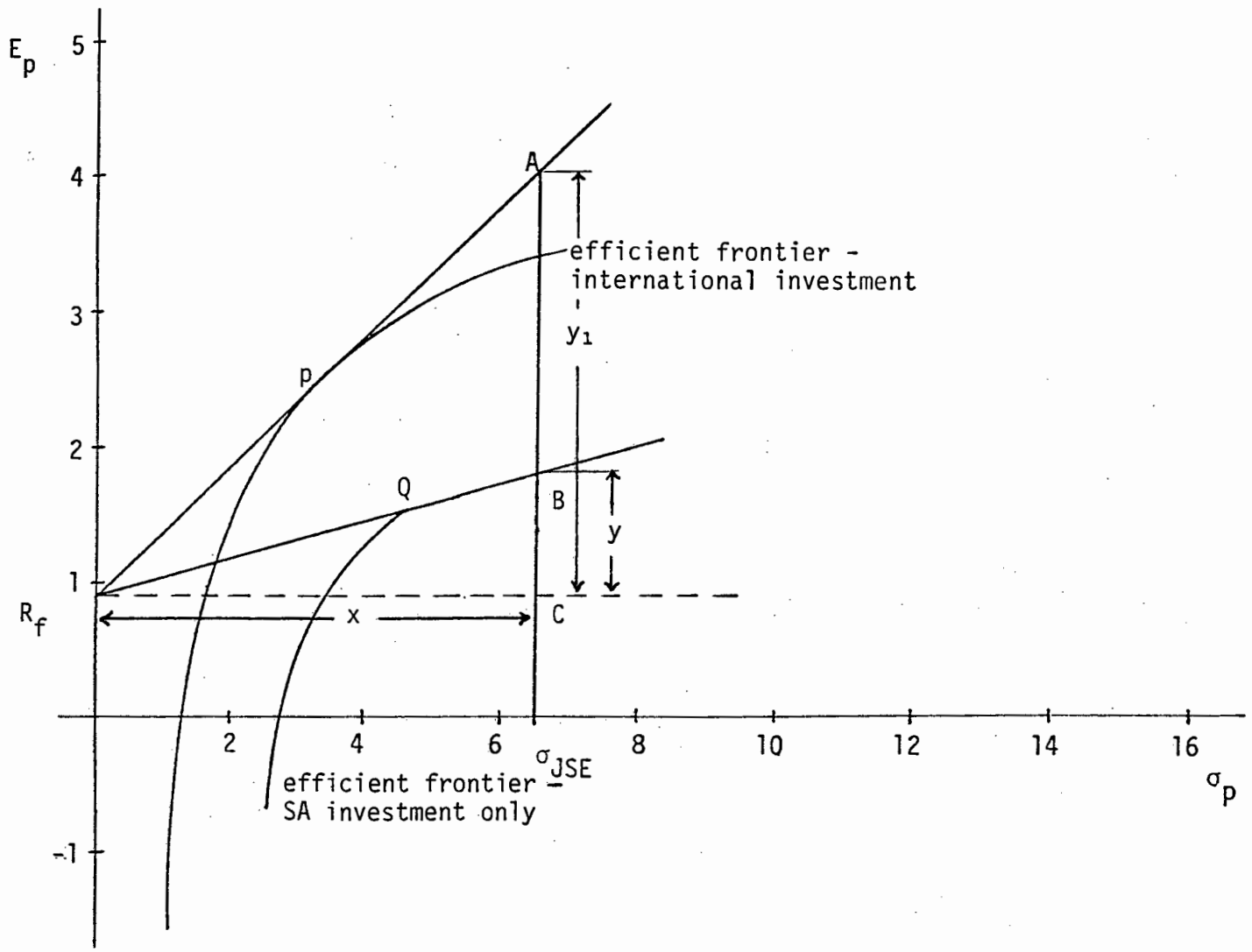


Figure 7.3 Capital market lines for international and South African investment (efficient frontiers for 1981 have been used to illustrate this concept)

Since the rational investor would only take up some position on the capital market line, and not any position other than the optimal combination of securities at risk on the efficient frontier, it is only necessary to compute the distance between the two capital market lines for each year.

This could be measured by calculating the area between the two lines. If a maximum risk level is known or specified the required area becomes the area of some triangle of the form R_fAB. However, if it is assumed that the investor may borrow as much as he likes, the maximum risk attainable is theoretically infinite, giving rise to an infinite-sized triangle. This problem, as well as the case in which the investor can not borrow for investment purposes, can be overcome by measuring the relative increase of international investment over purely South African investment over and above the risk-free rate. This amounts to finding the ratio of the areas of two triangles at any risk level, for example the market risk. Referring to Figure 7.3 the required ratio is

$$\frac{\text{area of } \Delta R_fAC}{\text{area of } \Delta R_fBC} \quad .$$

The area of a triangle is $\frac{1}{2}$. base . height

$$= \tfrac{1}{2} \cdot x \cdot \text{height}$$

$$\therefore \text{ratio} = \frac{\frac{1}{2} \cdot x \cdot y_1}{\frac{1}{2} \cdot x \cdot y}$$

$$= \frac{y_1}{y} \quad .$$

Notice that

$$\frac{\text{slope of line } R_fA}{\text{slope of line } R_fB} = \frac{y_1/x}{y/x}$$

$$= \frac{y_1}{y}$$

So the ratio of the areas of the two triangles is given by y_1/y, the ratio of the slope of the capital market line for

international investments to the slope of the capital market line for South African investments. This is also the ratio of the height of the two triangles which is independent of the level of risk, x. So any convenient risk level will suffice and thus the market risk, σ_{JSE}, was chosen. Table 7.4 below shows the values of y_1 and y for each year from 1965 to 1982, as well as their ratio and the resulting relative increase for each year.

On average the area of the triangle formed by the international capital market line, the risk-free rate and the market risk is 4,45 times larger than the area of the triangle formed by the South African capital market line, the risk-free rate and the market risk. This is equivalent to a percentage increase of 345,15% over the risk-free rate on average over the 18 years if international investment is allowed.

It should be noted, however, that certain problems arise when this method is employed. Firstly, the South African capital market line might be only very slightly steeper than the risk-free rate of return. In this case the ratio of the two triangles and the percentage improvement would be extremely large, even if the slope of the international capital market line were not very steep. Figure 7.4 illustrates this position. The result would tend to inflate the average. Secondly, if the international efficient frontier offers a very low risk and a return fairly large in comparison to the risk-free rate (i.e. the international efficient frontier

Year	International capital market line y_1	South African capital market line y	relative increase = y_1/y
1965	4,24	3,36	1,2619
1966	5,28	3,87	1,3643
1967	8,58	3,47	2,4726
1968	48,29	9,30	5,1925
1969	27,20	2,81	9,6797
1970	14,11	3,11	4,5370
1971	11,43	1,66	6,8855
1972	36,54	9,38	3,8955
1973	43,68	10,31	4,2367
1974	10,90	0,60	18,1667
1975	19,55	5,39	3,6271
1976	8,76	3,89	2,2519
1977	11,50	3,90	2,9487
1978	13,47	7,10	1,8972
1979	40,29	8,39	4,8021
1980	13,03	6,10	2,1361
1981	6,30	1,81	3,4807
1982	9,75	7,55	1,2914
		Average	4,4515

Table 7.4 Ratios (relative increases) of heights of triangles (or alternatively, slopes of capital market lines) for 1965 to 1982.

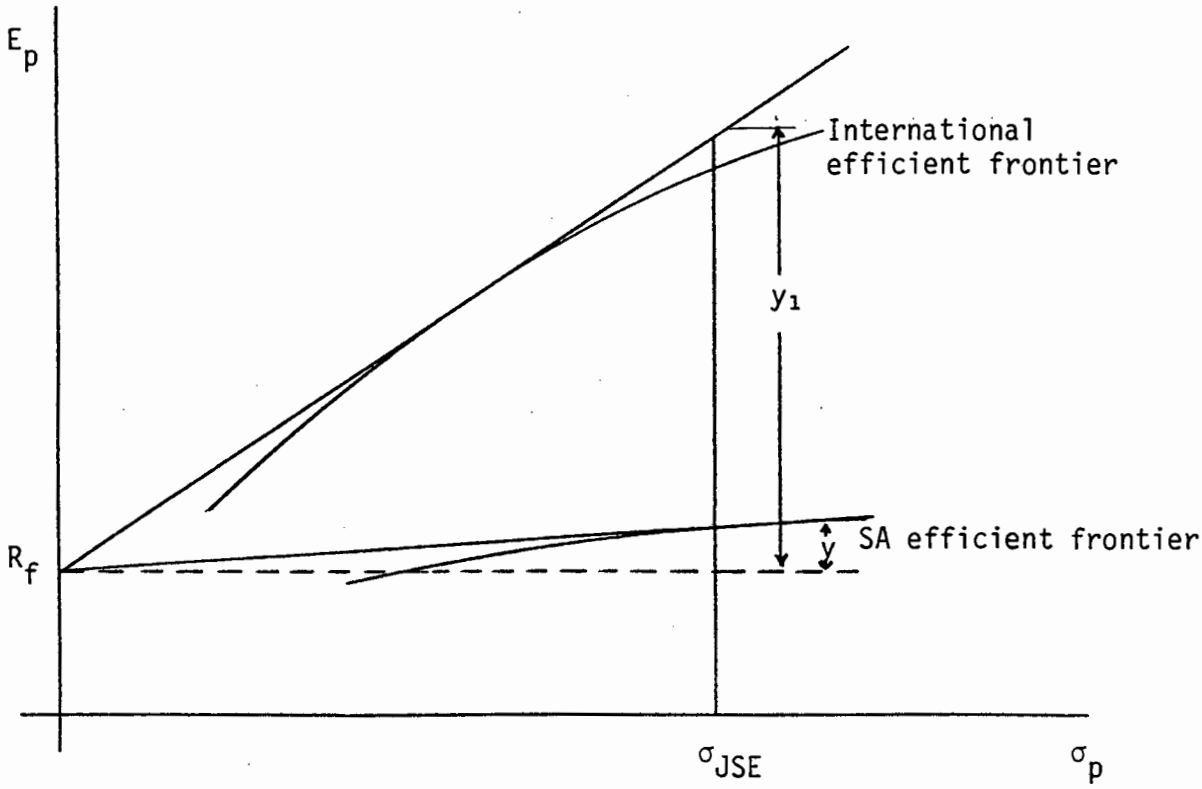


Figure 7.4 When the South African capital market line is only slightly steeper than the risk-free rate of return the ratio of the two triangles is large.

lies close to the vertical axis), the international capital market line will have a very steep slope, leading to a ratio and percentage improvement which is very large. This situation is depicted in Figure 7.5.

The first situation occurred in 1974, and the second situation in 1968, 1969 and 1979. When these four years are ignored in Table 7.4 the average relative increase from international divestment reduced to 3,02 and the average percentage increase over the risk-free rate each year from international investment was 202,05%.

Thus on average a local investor who invests in risky securities and a risk-free asset could have achieved a 200%

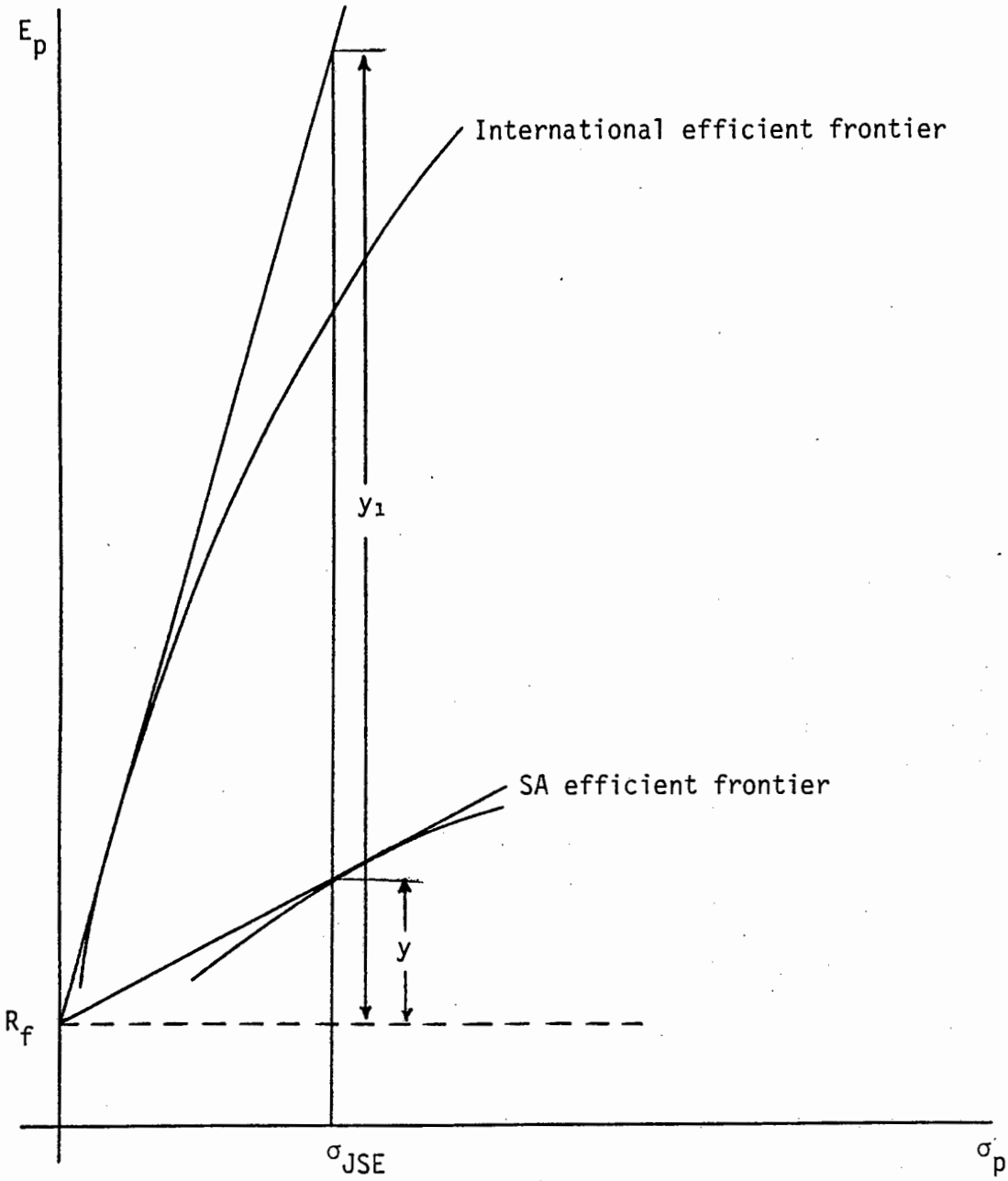


Figure 7.5 When the International efficient frontier offers low risk and a large return in comparison to the risk-free rate the ratio of two of the triangles is large

relative increase in his expected return by investing in the international rather than the local optimal combination of risky securities.

7.5 Sharpe's Reward-to-Variability Ratio

Sharpe (1966) compared several portfolios in a single period by means of the so-called reward-to-variability ratios of the portfolios concerned. For a particular portfolio this ratio is defined as the risk premium of the portfolio divided by the standard deviation of the portfolio. That is, for a portfolio A,

$$S_R = \frac{R_A - R_f}{\sigma_A}$$

where S_R = Sharpe's reward-to-variability ratio

R_A = return on portfolio A in the period

R_f = risk-free rate of return in the period

σ_A = standard deviation of returns of portfolio A in the period

This is indicated in Figure 7.6 below, and measures the rate of return above the risk-free rate per unit of risk borne.

It will be noticed that this ratio is merely the slope of the line from R_f to the portfolio plotted on the risk/return diagram.

The ratio was calculated for the optimal combination of securities at risk in each year from 1965 to 1982 as found in section 6.2.3 for both the investor with access to the international markets and for the investor who is restricted to South African securities. Table 7.5 contains these ratios, as well as the ratio of these ratios, labelled r*.

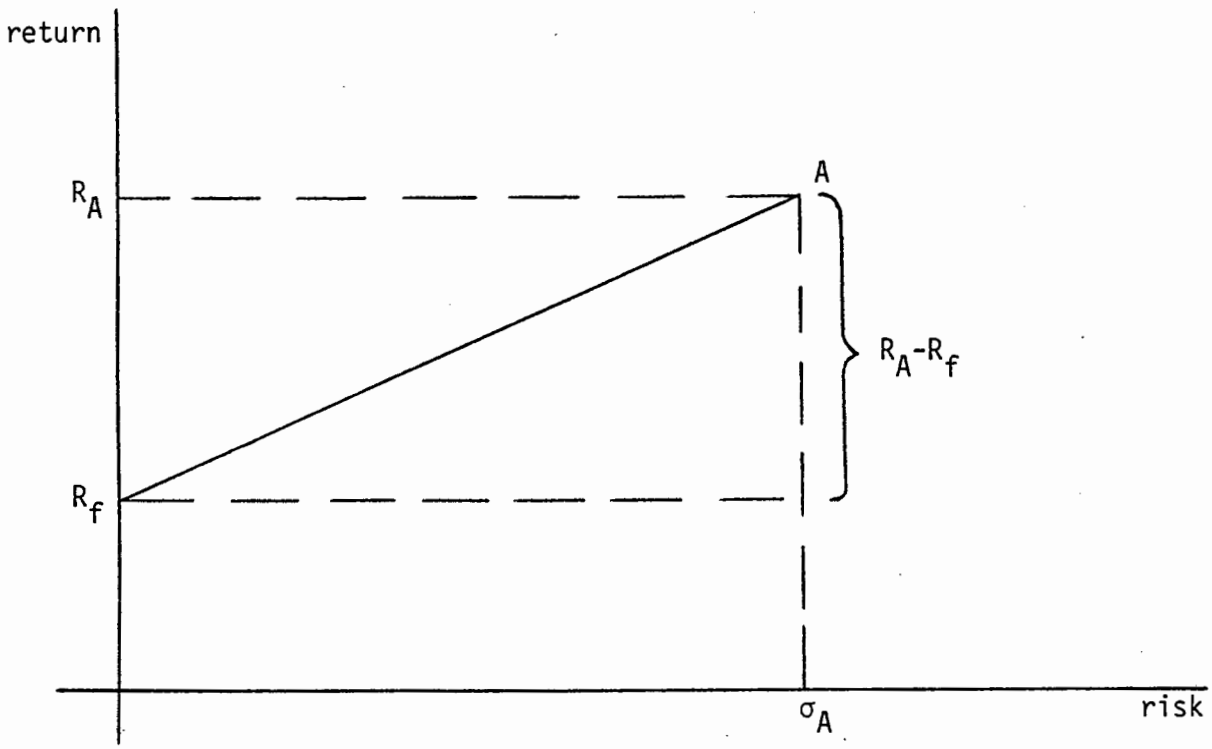


Figure 7.6 Sharpe's reward-to-variability ratio for an arbitrary portfolio A

It will be noted that r*, the ratio of the two reward-to-variability ratios in each year is always greater than 1, indicating that the international optimal combination of securities at risk offers a greater reward per unit of risk than does the South African optimal combination of securities at risk. This ratio varies from year to year, fluctuating between 1,27 in 1965 and 27,90 in 1974. The average value of r* over all 18 years is 4,76 which indicates a percentage increase of 376% from investing in the international portfolio. However, when the inordinately large values for 1969 and 1974 are removed the average value reduces to 2,99, indicating a percentage increase of

Sharpe's reward-to-variability ratio, S_{RTV}

YEAR	International portfolio	South African portfolio	r*
1965	0,6942	0,5486	1,27
1966	0,9718	0,7268	1,34
1967	1,1765	0,4708	2,50
1968	4,0000	1,1213	3,57
1969	2,0800	0,2091	9,95
1970	0,7131	0,1894	3,77
1971	0,7545	0,1123	6,72
1972	4,2041	1,3450	3,13
1973	2,1914	0,5394	4,06
1974	0,5412	0,0194	27,90
1975	1,6033	0,3840	4,18
1976	0,5778	0,2588	2,23
1977	1,1928	0,4506	2,65
1978	1,1123	0,5946	1,87
1979	3,6226	0,9512	3,81
1980	0,7974	0,3668	2,17
1981	0,4697	0,1422	3,30
1982	0,4014	0,3113	1,29
		Average	4,76

Table 7.5 Sharpe's reward-to variability ratios for international and South African investments, 1965 to 1982

199% from investing in the optimal combination of securities at risk from international investment. This compares with the average percentage increase in return over the risk-free rate of 202,05% as calculated in section 7.4.

7.6 The gains from international investment in the face of unknown future security performance

In section 7.4 the capital market lines for each year were constructed as rays from the South African risk-free rate tangent to both the international and South African efficient frontiers. These two lines indicated the best available positions that could have been achieved in practice during each year from the point of view of local investors who had access to (i) unlimited investment and (ii) no investment in foreign stocks or commodities, respectively. The optimal combination of risky securities thus arrived at was optimal *ex post* i.e. as viewed after the period had passed. This analysis may, quite clearly, not help in making decisions about the future.

If an investor is faced with a situation in which he has no knowledge of future security performance, it may be expected that he would "buy the market". By this it is meant that he would buy each risky security available in the market each year in proportion to their market capitalisation. In this way his portfolio would be fully diversified. This is clearly an *ex ante* investment strategy, since it refers to what was expected before the year actually began and does

not consider the effect of events that actually took place during the year.

The cases of two separate investors, each with their own market of securities at risk available to them, should be considered:

(i) the investor governed by current exchange control regulations and whose universe of risky securities is the seven South African securities chosen from the JSE as discussed in section 2.3; and

(ii) the investor with unlimited access to investment in foreign stocks and commodities, and whose universe of risky securities is all twenty-five securities as laid out in section 2.7. That is, seven South African securities, two foreign stocks, eleven metals and five soft commodities.

The twenty-five securities were divided into the following four groups:

(a) South African securities
(b) Foreign stocks
(c) Metals
(d) Soft commodities

It is assumed that each of the above two investors purchase equal rand amounts in each of the security groups available to them. In addition, all securities within each group are assumed to have been purchased in equal rand amounts.

For each year the actual return and actual risk for each of the above two portfolios was calculated using the relised prices of the securities concerned. Each of these two portfolios was plotted as a single point on a return/risk diagram in each year, and thus indicated the position of the market portfolios for South African investment and international investment respectively for that year. Market lines were drawn joining the South African risk-free rate to the above-mentioned two points in each year. The returns (in percent per month) for each of the two investment strategies in each year, as well as the difference between the two strategies were measured at both the South African market risk as well as the international market risk. These market risks were calculated as the standard deviation of the portfolio returns in each of the two markets under consideration in each year. These results are presented in Table 7.6.

Examination of Table 7.6 reveals that on average over the period 1965 to 1982 reasonably large positive returns could have been achieved by investors employing an *ex ante* selection procedure at either of the two risk levels considered. These amounted to 0,708% per month (8,493% per annum) and 0,850% per month (10,200% per annum) for investment in a portfolio of South African risky securities at the international market risk level and South African market risk level, respectively; and 0,715% per month (8,580% per annum) and 0,789% per month (9,467% per annum) for investment in a portfolio of international securities at the

Year	Risk = SA market risk			Risk = International market risk		
	Return (% per month)		Increase in return from international investment	Return (% per month)		Increase in return from international investment
	SA portfolio	International portfolio		SA portfolio	International portfolio	
1965	0,35	0,81	0,46	0,34	0,82	0,48
1966	1,68	-0,88	-2,56	1,27	-0,53	-1,80
1967	1,32	0,71	-0,61	0,80	0,55	-0,25
1968	4,19	3,50	-0,69	1,89	1,63	-0,26
1969	-2,65	-0,31	2,34	-0,74	0,11	0,85
1970	-1,79	-2,27	-0,58	-0,53	-0,73	-0,20
1971	0,67	1,63	0,96	0,59	1,08	0,49
1972	3,04	1,86	-1,18	1,46	0,98	-0,48
1973	-0,73	1,63	2,36	0,04	0,69	0,65
1974	-1,94	1,07	3,01	-1,97	1,00	2,97
1975	0,57	0,97	0,40	0,57	0,84	0,27
1976	-0,41	2,29	2,70	0,08	1,48	1,40
1977	2,21	1,79	-0,42	1,80	1,48	-0,32
1978	3,36	0,94	-2,42	3,16	0,92	-2,24
1979	3,03	3,90	0,87	1,74	2,17	0,43
1980	0,88	-1,07	-1,95	0,60	-0,36	-0,96
1981	0,23	1,13	0,90	0,35	1,09	0,74
1982	1,29	-3,50	-4,79	1,29	-0,35	-1,64
Average (% per month)	0,850	0,789	-0,067	0,708	0,715	0,007
Average (% per year)	10,200	9,476	-0,800	8,493	8,580	0,084

Table 7.6 Monthly returns for South African and international market portfolios at South African and international market risk levels for years 1965 to 1982

international and South African market risk levels respectively. It will be noticed that the returns fluctuate over a wide range, from as low as -3,50% per month (-42,00% per annum) for an international portfolio at the South African market risk in 1982, to as high as 3,90% per month (46,80% per annum) for an international portfolio at the South African market risk in 1979.

Furthermore, the investor with access to the international markets who spreads his funds over the entire universe of securities available to him in equal amounts amongst and within each of the four available security groups would, on average over the 18 years 1965 to 1982, end up with a deficit in return of 0,067% per month (or 0,800% per annum) when compared to the investor who is restricted to the local security market and who spreads his funds equally within that security group. These comparisons were at the level of risk available in the entire South African market in each year. When compared at the risk level attached to the international market (in all years it is smaller than the South African market risk) the investor who purchases foreign stocks and commodities as well as South African securities achieves a small gain of 0,007% per month (or 0,084% per annum) over his exchange control-restricted counterpart. These gains are significantly less than those achieved in the *ex post* studies.

However it should be borne in mind that this procedure measures the monthly returns from an international portfolio

and a purely South African portfolio at two different risk levels in each year, and does not consider the actual risk associated with each of the two portfolios. For this reason a standardised return (return adjusted for risk) was considered for each portfolio. The return and risk was calculated in each year for each of the two portfolios, and these averaged over all 18 years. The standardised return $(\frac{\text{return}}{\text{risk}})$ was calculated and is shown in Table 7.7 below:

	Average Return	Average Risk	standardised return
South African portfolio	0,849	6,436	0,1319
International portfolio	0,715	4,470	0,1600

Table 7.7 Standardised returns for South African and international portfolios chosen *ex ante*

It will be noted that the ratio of the international standardised return to the South African standardised return is 1,2127, i.e. the international portfolio has a relative performance which is 1,2127 times that of the South African portfolio when the actual risks associated with the two portfolios are taken into consideration.

7.7 Conclusions and Implications

In Chapter 6 it was clearly shown that large increases in returns are possible when the investor has access to an enlarged universe of securities - precisely the situation that would arise if investment in foreign securities was

allowed.

In this chapter an attempt has been made to quantify the gains that would have been achieved by a South African investor in the event of an abolishment of exchange control restrictions.

In section 7.2 the international and South African efficient frontiers were compared each year at various multiples of the South African market risk. It was shown that on average in the recent past the gains from international investment ranged from 18,84% to 27,36%, depending on the risk level chosen. In fact it would appear that the greater the risk level, the smaller the increase in returns achieved.

When the capital market lines were introduced (thus assuming that investment in a risk-free asset such as Treasury Bills was possible, as was borrowing at the same rate) it was found that the average percentage increase in return from international diversification over purely South African diversification was as high as 200%. This is, of course, the return that could be achieved over and above the risk-free rate. This method assumed that all investors would purchase just one portfolio of securities at risk as described by the Separation Theorem. If borrowing and lending are allowed the greatest absolute increases will be attained when the risk is large, since the capital market lines diverge. The percentage increase will, however, remain constant. This situation is not entirely realistic

since the borrowing and lending rates will not be the same, and infinite borrowing will not be allowed. These factors, will, however only mitigate slightly against the figures presented in this chapter.

Sharpe's reward-to-variability ratio measured the increase in risk premium for every unit increase in risk. It was shown that international diversification yielded returns that were as much as 199% greater than those achieved from investing in South African securities for every unit increase in risk.

When an *ex ante* portfolio selection approach was employed, and every security in the universe available was purchased it was found that the relative increase in average return from investing in foreign stocks or commodities was 1,2127 or over 20%.

The comparative figures above can be summarised in Table 7.8 below.

	method	average relative performance of international portfolios to SA portfolios
ex post	(1) Comparison at multiples of market risk (Section 7.2)	1,4937 - 2,3029 (dependent on risk level)
	(2) Approximate areas between efficient frontiers (Section 7.3)	2,0769
	(3) Capital market approach (Section 7.4)	3,0205 (over risk-free rate)
	(4) Sharpe's reward-to-variability ratio (Section 7.5)	2,99
ex ante	(5) equal funds amongst and within each security group available (Section 7.6)	1,2127

Table 7.8 Methods of comparing investment performance from an international portfolio and a South African portfolio

The implication of the figures displayed in this chapter is that *ex post* increases from international divestment are superior enough to warrant significant foreign investment by local investors in the event of an abolition of exchange control restrictions. It is not claimed that an investor will actually achieve a point on the *ex post* efficient frontier since they have to act *ex ante*. Nevertheless, even if the

investor's portfolio does not lie on the efficient frontier and providing the investor is equally inefficient in both the South African and international markets, the difference will still be the same, and the results in this chapter provide an indication of the costs to the South African investor of exchange control restrictions based on past experience.

CHAPTER 8

THE EFFECT OF CHANGES IN THE MAXIMUM PROPORTION OF FUNDS INVESTED EXTERNALLY

8.1 Introduction

The previous two chapters clearly indicated that when viewed *ex post* substantial increases in return may be achieved from international investment. These studies only considered an all-or-nothing approach, however. That is, either no investment in foreign securities was allowed, or an unlimited proportion of a South African investor's funds were allowed out of the country for investment in foreign securities.

Leading financiers envisage that initially a limit will be placed on the proportion of an investor's funds that will be allowed to be invested abroad. This is mainly because institutions have to meet their liabilities in rands and thus it would be inadvisable to invest a significant percentage of their assets abroad. Mr. Marinus Daling (1983), Senior General Manager of Sanlam, thus forsees that "a limit of five percent of total assets would therefore probably be a healthy maximum." Mr. Jui Lai (1983), Assistant General Manager (equity investments) of L & GV has been quoted as saying that that company, as well as many other local insti-

tutions would invest overseas on a limited scale in the event of the present restrictions on investment abroad being lifted.

Considering that liabilities are due in rands and finance for capital development must be retained, not all of an investor's funds should leave the country. With these limitations in mind it is desirable to determine what proportion of an investor's assets should be allowed to flow out of South Africa. From a different point of view, if the Reserve Bank were to allow a limited proportion of an investor's funds to be invested outside of South Africa it would be advantageous to determine this limit in such a way that investors can benefit as much as possible whilst at the same time a large proportion of funds are still retained in the country.

This problem can be analysed *ex post* by reconstructing the Markowitz efficient frontiers with certain restrictions placed on the maximum proportion allowed to be invested in foreign securities. These efficient frontiers are collectively compared and examined in sections 2 and 3 of this chapter by similar methods to those employed in Chapter 7. A conclusion as to the maximum percentage of an investor's funds that should have been allowed out of South Africa for investment purposes in recent years whilst still retaining a large proportion for local growth and payment of liabilities is discussed in section 4.

8.2 Changes in the Maximum Proportion of Funds Invested Externally

In order to compare the situations that would prevail under different proportions of an investor's funds being allowed out of South Africa for investment purposes, it is necessary to reproduce the Markowitz efficient frontiers after a further linear constraint has been included in the 'standard problem' formulation presented in section 4.4. This constraint is of the form

$$\Sigma X_{foreign} \leq \ell \quad , \quad 0 \leq \ell \leq 1.$$

Clearly, $\Sigma X_{foreign}$ is the sum of the proportions invested in non-South African securities, and ℓ is some value between 0 and 1. The value of ℓ is the limiting proportion for foreign securities, and was arbitrarily set at the following levels for the purposes of this study:

ℓ	Percentage of funds allowed for investment in foreign securities
0,00	0%
0,05	5%
0,10	10%
0,20	20%
0,25	25%
0,33	33$\frac{1}{3}$%
0,50	50%
1,00	100%

It will be noted that the case ℓ = 0,00 is merely the situation in which no foreign investment is allowed, and

ℓ = 1,00 implies an unlimited proportion of an investor's funds are allowed out of South Africa. These cases have already been examined in Chapter 6, section 6.2. They are presented here merely for the sake of completeness.

The efficient frontiers for each year were created for each value of ℓ, and an example of these frontiers (for the year 1975) can be found in Figure 8.1. A glance at Figure 8.1 reveals that as the level of ℓ increases the efficient frontiers so produced are positioned further and further away from the case ℓ = 0,00 (no foreign investment at all). That is, any increase in the limit placed on the percentage of funds allowed out of South Africa for investment purposes will produce a more desirable situation for the investor as he can earn a greater return for the same level of risk. This situation is maximised when ℓ = 1,00. That is, he cannot earn a higher return for a given level of risk than he can when the proportion he may invest in foreign securities is unlimited. What is of importance then is the increase, or alternatively the percentage gain, in return from increasing the value of ℓ above 0,00.

To achieve this end a similar procedure to the one employed in section 7.2 was initially followed. That is, the market risk in each year (defined as the standard deviation of the JSE All Share Index) was calculated and the returns at five arbitrarily-chosen multiples of this market risk were determined for each value of ℓ. Then the percentage gain in return from one value of ℓ to the next (larger)

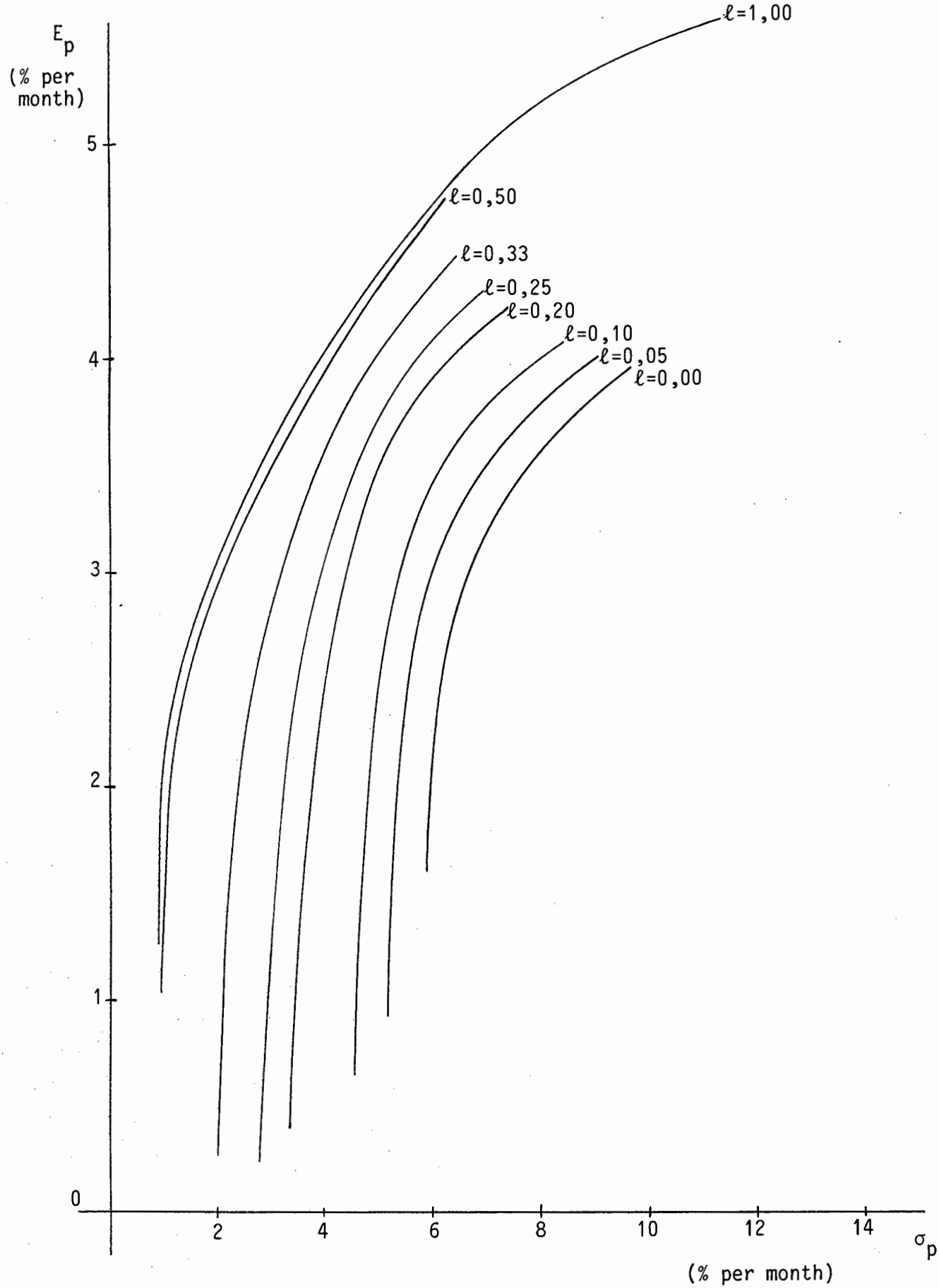


Figure 8.1 The effect of varying proportions of funds being allowed for investment in foreign securities and commodities in 1975 (ℓ = maximum proportion invested in foreign securities and commodities).

value of ℓ was calculated at each risk level, as was the cumulative percentage gain from the case when $\ell = 0,00$ (no foreign investment permitted). These returns, percentage gains and cumulative percentage gains for each year are contained in Appendix B whilst only the averages over the 18-year period 1965 to 1982 are presented in Table 8.1.

Several important points will be readily noted from Table 8.1. Firstly, for any given value of the maximum proportion of an investor's funds allowed out of the country for investment the average monthly return increased as the level of risk increased. Secondly, irrespective of the level of risk chosen, the average monthly return increased monotonically as the value of ℓ increased towards 1,00. This is exactly the position described in Figure 8.1. In other words, the larger the proportion of an investor's funds allowed out of South Africa, the greater his average monthly return was. Furthermore the average gain per annum from one value of ℓ to the next larger value (i.e. down the columns in Table 8.1) varies between 1,20% per annum to 6,36% per annum. These figures taken alone are, however, of little relevance since the values of ℓ considered are not equally spaced on the interval [0; 1].

As mentioned in Chapter 7, section 7.2, in certain years the *ex post* market risk was so great that even the smallest risk level considered (0,75 times the market risk) was not attainable. This situation occurred in 1969, 1970, 1980 and 1981. When these four years are removed from the study and

	0,75 σ_{JSE}			0,9 σ_{JSE}			1,0 σ_{JSE}			1,1 σ_{JSE}			1,25 σ_{JSE}		
ℓ	average monthly return	av. gain p.a.	average cumulative gain	average monthly return	av. gain p.a.	average cumulative gain	average monthly return	av. gain p.a.	average cumulative gain	average monthly return	av. gain p.a.	average cumulative gain	average monthly return	av- gain p.a.	average cumulative gain
0%	1,75	-	-	2,40	-	-	2,73	-	-	2,95	-	-	3,18	-	-
5%	2,10	4,20	4,20	2,74	4,08	4,08	2,99	3,12	3,12	3,16	2,52	2,52	3,36	2,16	2,16
10%	2,52	5,04	9,24	2,98	2,88	6,96	3,19	2,40	5,52	3,34	2,16	4,68	3,48	1,44	3,60
20%	3,02	6,00	15,24	3,34	4,32	11,28	3,49	3,60	9,12	3,58	2,88	7,56	3,68	2,40	6,00
25%	3,20	2,16	17,40	3,47	1,56	12,84	3,60	1,32	10,44	3,68	1,20	8,76	3,78	1,20	7,20
33⅓%	3,40	2,40	19,80	3,64	2,04	14,88	3,76	1,92	12,36	3,83	1,80	10,56	3,93	1,80	9,00
50%	3,67	3,24	23,04	3,91	3,24	18,12	4,03	3,24	15,60	4,12	3,48	14,04	4,22	3,48	12,48
100%	4,03	4,32	27,36	4,30	4,68	22,80	4,45	5,04	20,64	4,58	5,52	19,56	4,75	6,36	18,84

Table 8.1 Average returns (percent per month), gains (percent per annum) and cumulative gains (percent per annum) for the period 1965-1982 at various risk levels for various maximum proportions of investment allowed in foreign securities.

the monthly returns, gains per annum and cumulative gains per annum are averaged over the remaining 14 years, Table 8.2 results. The main points of Table 8.1 as just discussed are also true for Table 8.2.

It will be seen in Table 8.1 that the maximum average cumulative gain occurred when no limit was placed on the proportion of an investor's funds that may have been invested in foreign securities, irrespective of the risk level desired. Thus the average annual percentage gain in return at each level of ℓ can be expressed as a fraction of the maximum average annual percentage gain in return. This is shown in Table 8.3.

Clearly, when the risk level desired was low (0,75 σ_{JSE}) more than half of the average annual percentage gain in return could have been achieved by allowing a mere 20% of an investor's funds to be invested in foreign securities, and almost three-quarters of the average annual percentage gain could have been achieved if this limit was extended to $33\frac{1}{3}\%$. As the risk level increases the fractions decrease, until at the highest risk level considered (1,25 σ_{JSE}) almost half the average annual percentage gain in return could have been achieved by allowing $33\frac{1}{3}\%$ of an investor's funds to be invested abroad, and two-thirds of the average annual percentage gains could have been achieved if the limit was 50%.

Thus there is historical evidence from a recent time period that, irrespective of the risk level desired, the

	0,75 σ_{JSE}			0,9 σ_{JSE}			1,0 σ_{JSE}			1,1 σ_{JSE}			1,25 σ_{JSE}		
ℓ	average monthly return	av. gain p.a.	average cumulative gain	average monthly return	av. gain p.a.	average cumulative gain	average monthly return	av. gain p.a.	average cumulative gain	average monthly return	av. gain p.a.	average cumulative gain	average monthly return	av. gain p.a.	average cumulative gain
0%	1,82	-	-	2,58	-	-	3,01	-	-	3,29	-	-	3,59	-	-
5%	2,20	4,56	4,56	2,99	4,92	4,92	3,31	3,60	3,60	3,53	2,88	2,88	3,79	2,40	2,40
10%	2,69	5,88	10,44	3,26	3,24	8,16	3,53	2,64	6,24	3,73	2,40	5,28	3,91	1,44	3,84
20%	3,25	6,72	17,16	3,66	4,80	12,96	3,86	3,96	10,20	3,97	2,88	8,16	4,10	2,28	6,12
25%	3,45	2,40	19,56	3,80	1,68	14,64	3,96	1,20	11,40	4,07	1,20	9,36	4,20	1,20	7,32
33$\frac{1}{3}$%	3,65	2,40	21,96	3,96	1,92	16,56	4,12	1,92	13,32	4,21	1,68	11,04	4,33	1,56	8,88
50%	3,92	3,24	25,20	4,20	2,88	19,44	4,36	2,88	16,20	4,47	3,12	14,16	4,60	3,24	12,12
100%	4,25	3,96	29,16	4,54	4,08	23,52	4,71	4,20	20,40	4,85	4,56	18,72	5,04	5,28	17,40

Table 8.2 Average returns (percent per month), gains (percent per annum) and cumulative gains (percent per annum) for the period 1965-1982 (excluding years 1969, 1970, 1980 and 1981) at various risk levels for various maximum proportions of investment allowed in foreign securities.

greatest average annual percentage gains in return would have occurred if the proportion of an investor's funds that were allowed to be invested abroad was less than 50%.

	0,75 σ_{JSE}		0,90 σ_{JSE}		1,00 σ_{JSE}		1,1 σ_{JSE}		1,25 σ_{JSE}	
ℓ	A[1]	B[2]	A[1]	B[2]	A[1]	B[2]	A[1]	B[2]	A[1]	B[2]
0%	-	0,00	-	0,00	-	0,00	-	0,00	-	0,00
5%	4,20	0,15	4,08	0,18	3,12	0,15	2,52	0,13	2,16	0,11
10%	9,24	0,34	6,96	0,31	5,55	0,27	4,68	0,24	3,60	0,19
20%	15,24	0,56	11,28	0,49	9,12	0,44	7,56	0,39	6,00	0,32
25%	17,40	0,64	12,84	0,56	10,44	0,51	8,76	0,45	7,20	0,38
33$\frac{1}{3}$%	19,80	0,72	14,88	0,65	12,36	0,60	10,56	0,54	9,00	0,48
50%	23,04	0,84	18,12	0,79	15,60	0,76	14,04	0,72	12,48	0,66
100%	27,36	1,00	22,80	1,00	20,64	1,00	19,56	1,00	18,84	1,00

1. A is the average annual percentage gain in return.
2. B is the average annual percentage gain in return expressed as a fraction of the maximum average annual percentage gain in return.

Table 8.3 Average annual percentage gain in return expressed as a fraction of the maximum average annual percentage gain in return

8.3 Changes in the Maximum Proportion of Funds Invested Externally - An Alternative Approach

In Chapter 7, section 7.3 the gains to be made from investment in an international portfolio selected from South African and foreign stocks, metals and soft commodities rather than a portfolio selected from purely South African securities were quantified by measuring the distance between the two efficient frontiers over the entire range of risks common to both. This was approximated by calculating the average distance between the two frontiers at the two extremes of the risk range common to both frontiers.

If the proportion of an investor's funds allowed out of South Africa was allowed to vary, this average distance should be recalculated for each limiting value ℓ of the proportion of his funds which may be invested in foreign stocks and commodities. The average is the average of the distances XX_ℓ and YY_ℓ for each value of ℓ in Figure 8.2 below.

Table 8.4 below presents the average values over the 18 years 1965 to 1982 of the distances XX_ℓ and YY_ℓ (the distance between the frontier formed when no foreign investment was allowed, and the frontier formed when a maximum of 100 ℓ% of the portfolio consisted of foreign securities) for the same values of ℓ considered in section 8.2, as well as the average monthly and annual percentage increase in return at each of these values of ℓ.

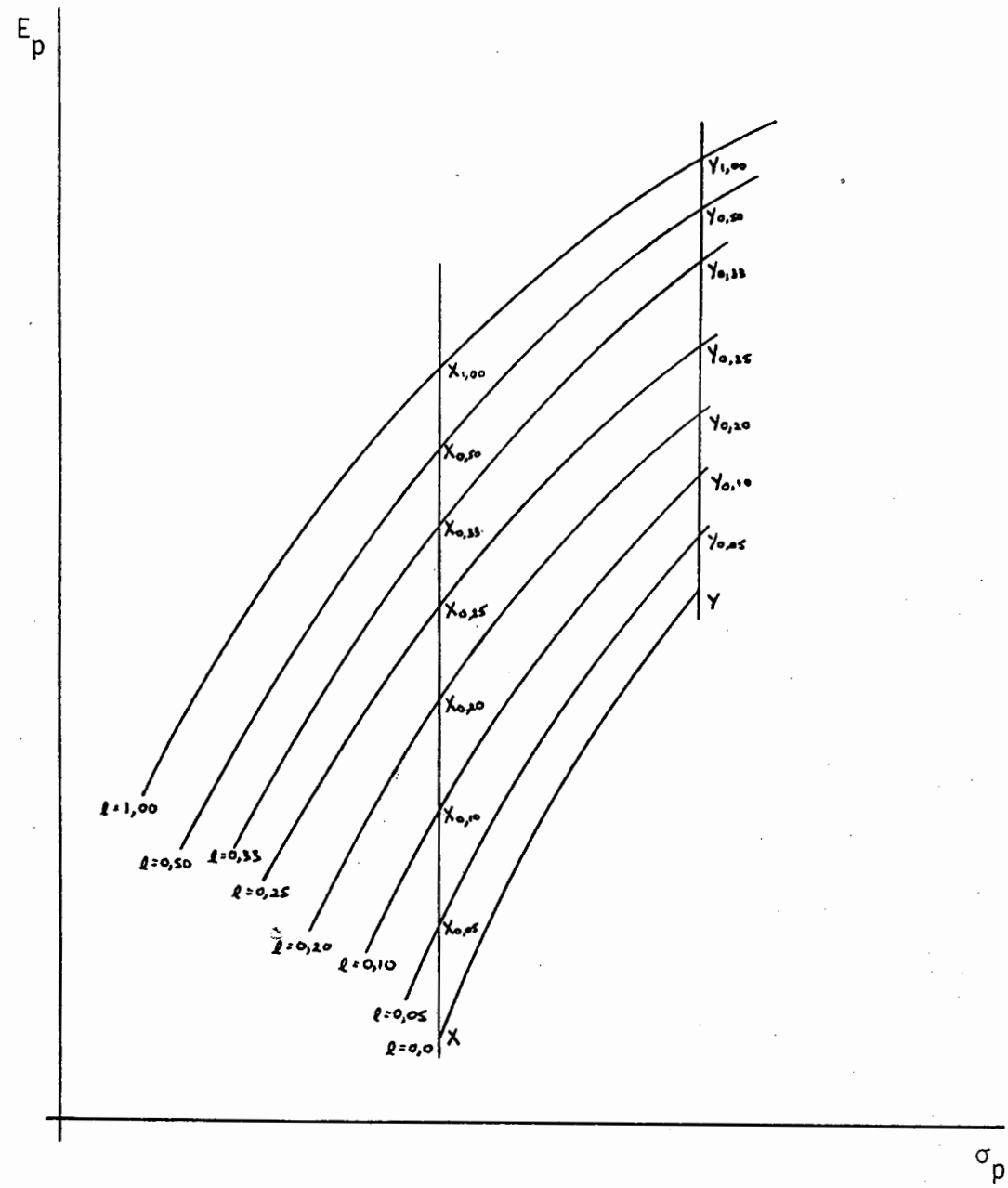


Figure 8.2 Increases in return over the risk range common to the South African efficient frontier and an efficient frontier where the proportion of funds allowed for foreign investment in 100 ℓ%

ℓ	XX_ℓ	YY_ℓ	$YY_\ell - XX_\ell$	average monthly increase in return (% per month)	average annual increase in return (% per annum)
0,05	0,91	0,21	-0,71	0,56	6,72
0,10	1,32	0,40	-0,92	0,86	10,32
0,20	1,87	0,75	-1,12	1,31	15,72
0,25	2,08	0,92	-1,16	1,50	18,00
0,33	2,34	1,16	-1,18	1,75	21,00
0,50	2,59	1,35	-1,24	1,97	23,64
1,00	2,98	1,48	-1,50	2,23	26,76

Table 8.4 Average values of XX_ℓ and YY_ℓ over the years 1965 to 1982 as well as average monthly and annual percentage increase in return

It will be noted from Table 8.4 that as ℓ increased both XX_ℓ and YY_ℓ also increased. That is, as the proportion of funds allowed for investment in foreign securities increased, so did the average increase in monthly returns from the case when no foreign investment was permitted, over two widely varying risk levels. These average increases in return amounted to 2,98% per month (35,76% per annum) at the lower risk level and 1,48% per month (17,76% per annum) at the higher risk level for the extreme case ℓ = 1,00. It will also be noted that for all levels of ℓ the average increase in return was greater at the lower risk level than at the higher risk level (indicated by the fact that all the entries in the column $YY_\ell - XX_\ell$ of Table 8.4 are negative). Furthermore, as ℓ increased this characteristic becomes more marked.

The average monthly increase in return is calculated by finding the mean of XX_ℓ and YY_ℓ for each ℓ considered. This is seen to increase monotonically as ℓ increased, to a maximum of 2,23% per month (26,76% per annum) when $\ell = 1{,}00$. Since the maximum increase in return occurred when no limit was placed on the proportion of an investor's funds that may have been invested in foreign securities, the increase in return at each level of ℓ can be expressed as a fraction of the maximum increase in return. This is shown in Table 8.5 below.

ℓ	average annual increase in return (% per annum)	increase in return as a percentage of the maximum increase in return
0,00	0,00	0,00
0,05	6,72	25,11
0,10	10,32	38,57
0,20	15,72	58,74
0,25	18,00	67,26
0,33	21,00	78,48
0,50	23,64	88,34
1,00	26,76	100,00

Table 8.5 Increase in return as a percentage of the maximum increase in return

It is clear that more than half of the increase in return could have been achieved by setting a limit of just 20% on the percentage of funds that an investor may have invested in foreign securities, and more than three-quarters of the

increase could have been achieved if this limit was extended to $33\frac{1}{3}\%$. Thus the greatest increases in return (taken over the entire risk range common to both the South African and international efficient frontiers) occurred when $\ell \leqslant 0{,}33$. The increases in return were small when ℓ was increased above 0,50. This is displayed graphically in Figure 8.3 below, where the average annual percentage increase was plotted against ℓ, the proportion of foreign investment allowed. The slope of the curve was initially steep, indicating large increases in return, but flattened off as ℓ increased above 0,50.

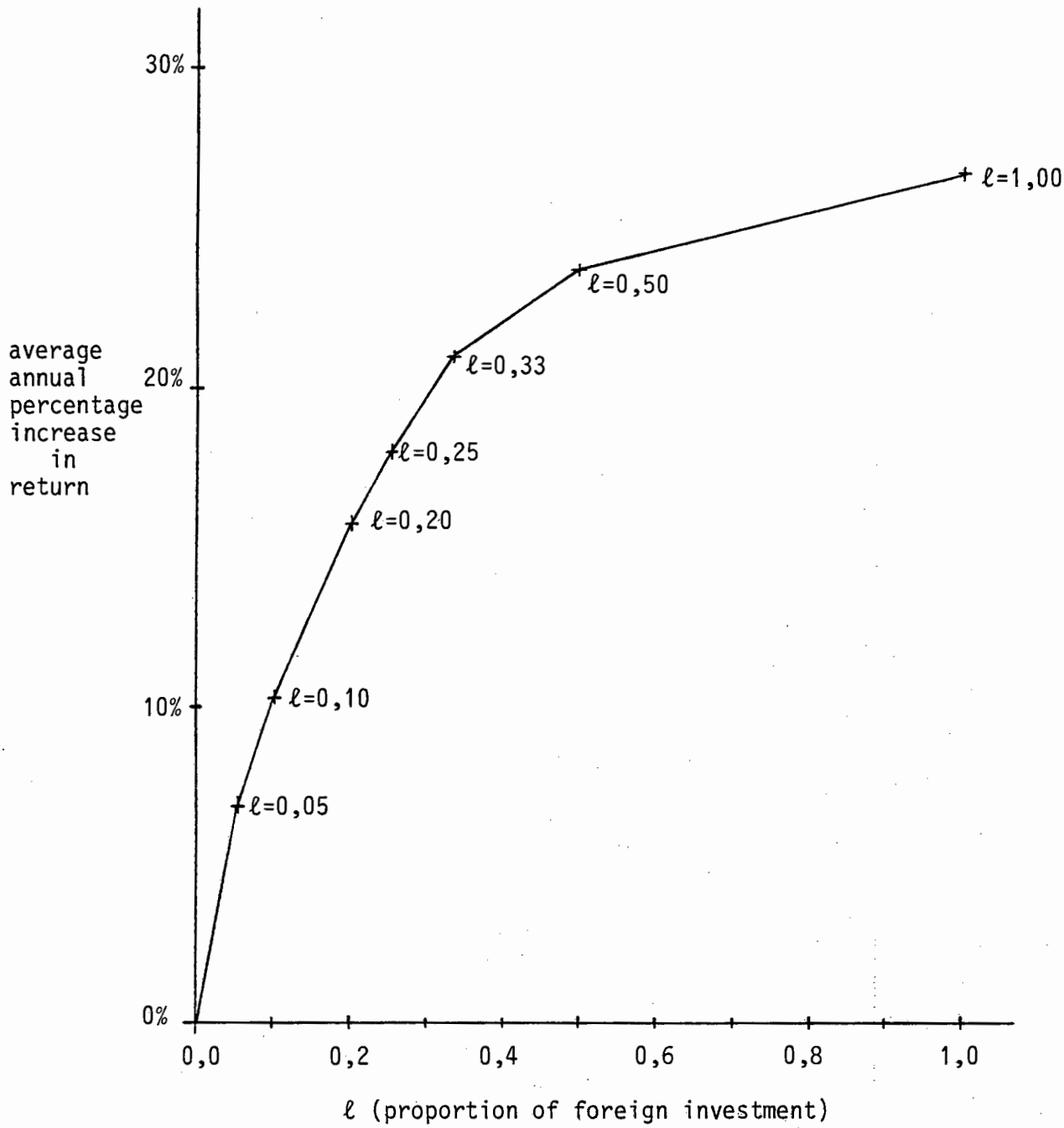


Figure 8.3 Plot of average annual percentage increase in return from foreign investment against ℓ (proportion of foreign investment)

8.4 Changes in the Maximum Proportion of Funds Invested Externally - An *Ex Ante* Approach

In Chapter 7, section 6 an *ex ante* investment strategy was investigated, which did not consider the effect of events that actually took place in the forthcoming year. That is, it was assumed that an investor was faced with a situation in which he has no knowledge of future security performance. Under these circumstances it was assumed that he would "buy the market", i.e. he would buy each risky security available in the market each year in proportion to its market capitalisation, yielding a fully diversified portfolio.

In this section an *ex ante* empirical study will be presented in which varying proportions of an investor's funds are distributed to the international markets.

Initially two indices were created:

(i) a South African index consisting of equal amounts in each of the seven South African securities chosen from the JSE (see Chapter 2, section 2.3);

(ii) a "foreign" index consisting of equal amounts in each of the three foreign security groups, namely foreign stocks, metals and soft commodities (see Chapter 2, section 2.7). All securities within the three groups were assumed to have equal weightings.

The mean return and variance of the returns for each year for each of the above two indices were calculated, as were the correlations between the two indices for each year.

The returns and the risks of the portfolios created by allowing varying proportions between 0% and 100% of an investor's funds to be invested in foreign securities were calculated for each year and appear in Appendix C. The average portfolio returns and risks over all 18 years from 1965 to 1982 are found in Table 8.6.

proportion of funds allowed for investment in foreign securities w_1	average portfolio return R_p (% per month)	average portfolio risk σ_p (% per month)
0,00	0,849	6,436
0,05	0,839	6,147
0,10	0,829	5,861
0,20	0,809	5,321
0,25	0,799	5,064
0,33	0,782	4,659
0,50	0,749	3,976
1,00	0,649	3,706

Table 8.6 Average portfolio returns and risks for 1965 to 1982 for varying proportions of investment allowed in foreign securities

It will be noted that both the average portfolio return, R_p, and the average portfolio risk, σ_p, decreased monotonically as the proportion of funds allowed out of the country, w_1, increased. However the average risk decreased at a much faster rate than did the average return. This

feature is readily seen in Table 8.7, which presents the average percentage decrease in return and risk from the case $w_1 = 0,00$ as the proportion of funds for investment in foreign securities increased.

proportion of funds allowed for investment in foreign securities w_1	average percentage decrease in return (%)	average percentage decrease in risk (%)
0,00	0,00	0,00
0,05	1,18	4,49
0,10	2,36	8,93
0,20	4,71	17,32
0,25	5,89	21,32
0,33	7,89	27,61
0,50	11,78	38,22
1,00	23,56	42,42

Table 8.7 Average percentage decrease in return and risk for 1965 to 1982 as the proportion of funds for investment in foreign securities increased

It will be noted that for any value of w_1 the average percentage reduction in risk is always far greater than the average percentage reduction in return. Thus, although an investor would have achieved smaller average returns by investing in an international portfolio of securities rather than a purely South African portfolio, he would have dramatically reduced his risk. This result is displayed graphically in Figure 8.4, where the average percentage decrease in risk is plotted against the average percentage decrease in return. The graph always lies above the line with slope 1,00.

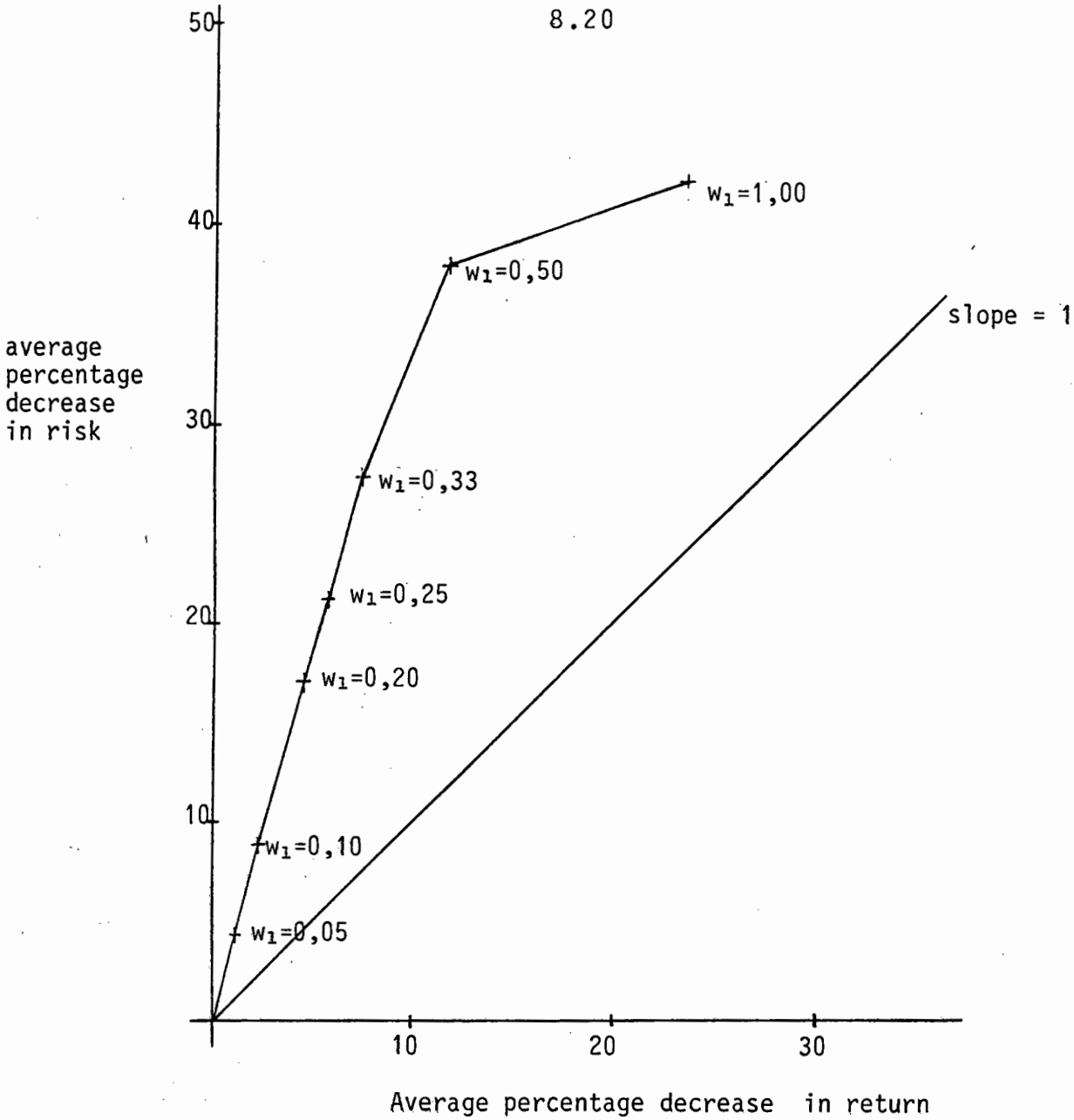


Figure 8.4 Plot of average percentage decrease in risk against average percentage decrease in return when foreign investment is permitted.

This feature is particularly marked for w_1 less than or equal to 0,50. For these values of w_1 the graph and the line with slope 1,00 diverge. When w_1 is greater than 0,50 the graph and the line with slope 1,00 converge towards one another.

8.5 Summary and Conclusions

In this chapter a closer examination was made of the benefits to the South African investor if investment in foreign securities was limited to some proportion less than 100% of an investor's total portfolio wealth.

It was initially shown that on average over a recent 18-year period the larger the proportion of an investor's funds allowed out of South Africa for investment purposes, the greater his average monthly return, irrespective of the level of risk chosen.

When the entire risk range common to both the South African and international efficient frontiers was considered it was shown that more than half of the increase in return could have been achieved by setting a limit of a mere 20% on the proportion of an investor's portfolio wealth that may be held in foreign securities, and more than three quarters of the increase could have been achieved if this limit was extended to $33\frac{1}{3}\%$. Thus the greatest increases in return occurred when the proportion of an investor's portfolio wealth which may be invested in foreign securities was less than one third. Thus even small relaxations in the current exchange control restrictions would have been very beneficial to the investor.

When an *ex ante* investment strategy was employed it was shown that on average over a recent eighteen year period the inclusion of foreign stocks and commodities into a portfolio

would have decreased the average return on that portfolio. However this reduction in average return was offset by the fact that the average risk associated with the portfolio decreased at an even greater rate, particularly when the proportion of an investor's funds allowed outside the country was less than 50%.

Thus there is evidence based on recent past experience that the South African Reserve Bank should aim at raising the proportion of an investor's funds that could be allowed out of South Africa for investment abroad. A reasonable proportion might be between 20% and $33\frac{1}{3}$% of an investor's funds. At these levels most of the benefits from foreign investment will have been gained whilst a majority of the investor's funds are retained in the country for payment of liabilities and for capital growth.

CHAPTER 9

CONCLUSIONS

This thesis has studied the benefits to the South African investor of a relaxation in the current exchange control regulations. Under these conditions an investor would be free to purchase securities from any of the world's stock markets or commodity and metal exchanges.

In an enlarged universe of securities an investor may be able to attain higher returns from some previously unattainable securities. Furthermore a repeatedly proven argument advanced in favour of diversification of an investor's risky assets is that the portfolio risk can be drastically reduced if the returns of the risky assets are not correlated. Many researchers have shown that the movements of security prices in different countries, and indeed various commodity prices, do not show high correlation with one another, and thus a strong case can be made for investment in the stocks of foreign countries and in metals and soft commodities. Garrone and Solnik (1983) comment on the advantage of diversification to alternative investment media:

"Asset diversification will reduce the amplitude of the portfolio price fluctuations. This simply

extends the well-known principle of diversification to all investment media. Indeed, the value of a portfolio diversified over all media is much less volatile (risky) than each individual investment. Media diversification is all the more important since it is so hard to make reliable long-term return forecasts on individual stock markets and media."

The efficient portfolios corresponding to international investment in a number of different media, such as local and foreign stocks, metals and soft commodities were shown to outperform the efficient portfolios corresponding to investment in purely South African securities in each of the eighteen years under study. On average the percentage gain from international investment was shown to be as much as 25%, and as much as 200% over and above the risk-free rate. From an *ex ante* point of view the relative increase in average return from foreign investment rather than a purely South African portfolio was over 20%.

Thus the opportunity exists for large increases in a South African investor's portfolio return if he were able to include the international markets in his universe of securities. However it is not claimed that he could actually attain the risk-return combinations described in this thesis. Solnik and Noetzlin (1983) put it this way:

"*There was of course no way of knowing in advance what these optimal asset allocations would be,*

and all that can be concluded is simply that opportunities are sizeable and that the gap between an optimal and an index fund approach is potentially very wide. Whether any money manager has sufficient ability to realize most or part of this performance differential is another story."

Furthermore, the study revealed that on average in the recent past the greatest benefits from diversification were attained if a mere 20% to 33$\frac{1}{3}$% of the investor's funds at risk were divested out of South Africa. This proportion would have allowed the local investor to obtain the diversification benefits to be had from investment in the international markets whilst still retaining a large proportion of his funds in the country for payment of liabilities and for capital growth.

A possible drawback of the results presented in this thesis is that the largest portion of this study was performed on an *ex post* basis. That is, conclusions were reached about the magnitude of the gains that could have been attained had exchange control not existed in the past, and decisions were made regarding the optimal proportion of an investor's funds at risk that should have been placed in foreign stocks and commodities in the recent past in the absence of exchange control restrictions. This analysis may, quite clearly, not necessarily help to make decisions about the future. Nevertheless, *ex post* studies do have value in that they demonstrate the best investment opportu-

nities that were actually available over any particular period. Furthermore if future price movements can be approximated by past price movements then these models could possibly be used for future predictions of portfolio returns and risks. However, Robichek, Cohn and Pringle (1972) warn against this:

> *"If mean returns and covariances tend to remain stable over time, then an analysis of ex post results will have value when it comes to making ex ante predictions. However, it is by no means safe to assume stability of these parameters over time, and considerable attention to this question is both justified and necessary."*

An attempt was made, however, to examine the *ex ante* question in addition to the *ex post* analysis. From an *ex ante* position it is assumed that the investor has no knowledge of future security prices and thus "buys the market", i.e. holds a market capitalisation index portfolio. Such a strategy was employed for investment in an international portfolio of stocks and commodities and a purely South African portfolio. On average the South African portfolio produced returns which equalled, and in some cases, bettered those offered by the international portfolio. However, the diversified international portfolio offered risks which were far smaller than those offered by the South African portfolio, thus providing empirical evidence supporting the diversification benefit arguments. Indeed, this decrease in risk is so large that it probably more than compensates for any loss

of return in the international portfolio.

It should be pointed out that the effect of brokerage and taxation have not been considered. Brokerage will be incurred whenever trading occurs on the foreign stock exchanges, the metal exchanges and the soft commodity exchanges as well as the JSE. However, brokerage rates of the various media examined in this thesis do not vary greatly and hence the introduction of brokerage charges is unlikely to change the results. All investors, whether trading on the local or international markets, will be liable to pay tax. The effects of tax are complicated by the fact that different rates are applied to companies and to individuals, and individuals are taxed at different rates. Each individual investor can make his own modifications depending on this personal tax situation.

It is important to note that all the results presented in this thesis are based on data which reflected the situation when no funds were allowed out of South Africa for investment purposes. Thus the results are conditional on the current exchange control regulations being enforced. In the event of a relaxation of the current exchange control regulations and a possible outflow of funds this situation might change, and the gains to be had from foreign investment might be affected. This situation is impossible to model, however, since reliable predictions of future prices are not available.

This thesis has by no means been an exhaustive study on the effect of an abolition or relaxation of exchange control regulations on the South African investor. Numerous non-security assets such as real estate, stamps, art and antiques have been ignored because their heterogeneous nature makes them extremely difficult to value. It was rather intended to provide some indication of the potential benefits that exist for the South African investor should foreign exchange restrictions be lifted, as well as an indication of what percentage of funds the local investor should invest on average outside the Republic of South Africa.

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APPENDIX A

Average South African Treasury Bill Rates

The Treasury Bill rate at the end of each month was collected for the entire period February 1965 to January 1983. These rates were averaged every 12 months.

Period	Treasury Bill rate	
	Annual (%)	monthly (%)
Feb 1965 - Jan 1966	4,12	0,3433
Feb 1966 - Jan 1967	4,25	0,3542
Feb 1967 - Jan 1968	4,90	0,4083
Feb 1968 - Jan 1969	4,86	0,4050
Feb 1969 - Jan 1970	4,59	0,3825
Feb 1970 - Jan 1971	4,42	0,3684
Feb 1971 - Jan 1972	5,51	0,4595
Feb 1972 - Jan 1973	5,14	0,4281
Feb 1973 - Jan 1974	3,19	0,2661
Feb 1974 - Jan 1975	5,60	0,4669
Feb 1975 - Jan 1976	6,25	0,5207
Feb 1976 - Jan 1977	7,54	0,6287
Feb 1977 - Jan 1978	7,89	0,6574
Feb 1978 - Jan 1979	7,73	0,6438
Feb 1979 - Jan 1980	4,91	0,4092
Feb 1980 - Jan 1981	4,93	0,4107
Feb 1981 - Jan 1982	10,72	0,8937
Feb 1982 - Jan 1983	15,23	1,2693

APPENDIX B

Returns, percentage gains and cumulative percentage gains (in percent per period) at various risk levels for various maximum proportions of investment allowed in foreign securities (1965-1982).

YEAR: 1965

MARKET RISK: 3,03%

	0,75 σ_{JSE}			0,9 σ_{JSE}			1,0 σ_{JSE}			1,1 σ_{JSE}			1,25 σ_{JSE}		
	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain
0%	0,34	-	-	0,97	-	-	1,35	-	-	1,71	-	-	2,23	-	-
5%	0,63	3,48	3,48	1,22	3,00	3,00	1,59	2,88	2,88	1,94	2,76	2,76	2,44	2,52	2,52
10%	0,91	3,36	6,84	1,47	3,00	6,00	1,82	2,76	5,64	2,16	2,64	5,40	2,57	1,56	4,08
20%	1,26	4,44	11,28	1,83	4,32	10,32	2,15	3,96	9,60	2,40	2,88	8,28	2,70	1,56	5,64
25%	1,40	1,68	12,96	1,94	1,32	11,64	2,24	1,08	10,68	2,46	0,72	9,00	2,75	0,60	6,24
33⅓%	1,62	2,64	15,60	2,07	1,56	13,20	2,31	0,84	11,52	2,52	0,72	9,72	2,82	0,84	7,08
50%	1,83	2,52	18,12	2,18	1,32	14,52	2,40	1,08	12,60	2,61	1,08	10,80	2,92	1,20	8,28
100%	1,92	1,08	19,20	2,23	0,60	15,12	2,44	0,48	13,08	2,64	0,36	11,16	2,94	0,24	8,52

* A risk level this low is not attainable. The given monthly return is the return at the lowest possible risk level of the portfolio

\+ A risk level this high is not attainable. The given monthly return is the return at the greatest possible risk level of the portfolio concerned.

[a] These returns were obtained from a portfolio in which the investment in foreign securities was less than the maximum allowable proportion.

YEAR: 1966

MARKET RISK: 2,67%

-	0,75 σ_{JSE}			0,9 σ_{JSE}			1,0 σ_{JSE}			1,1 σ_{JSE}			1,25 σ_{JSE}		
	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain
0%	1,33	-	-	1,82	-	-	2,11	-	-	2,38	-	-	2,75	-	-
5%	1,54	2,52	2,52	1,97	1,80	1,80	2,24	1,56	1,56	2,49	1,32	1,32	2,84	1,08	1,08
10%	1,69	1,80	4,32	2,10	1,56	3,36	2,35	1,32	2,88	2,59	1,20	2,52	2,93	1,08	2,16
20%	1,94	3,00	7,32	2,31	2,52	5,88	2,54	2,28	5,16	2,75	1,92	4,44	3,02	1,08	3,24
25%	2,04	1,20	8,52	2,39	0,96	6,84	2,61	0,84	6,00	2,79	0,48	4,92	3,02[a]	0,00	3,24
33⅓%	2,15	1,32	9,84	2,48	1,08	7,92	2,65	0,48	6,48	2,80	0,12	5,04	3,02[a]	0,00	3,24
50%	2,23	0,96	10,80	2,50	0,24	8,16	2,65	0,00	6,48	2,80[a]	0,00	5,04	3,02[a]	0,00	3,24
100%	2,24	0,12	10,92	2,50	0,00	8,16	2,66	0,12	6,60	2,81	0,12	5,16	3,03	0,12	3,36

YEAR: 1967

MARKET RISK: 3,76%

	0,75 σ_{JSE}			0,9 σ_{JSE}			1,0 σ_{JSE}			1,1 σ_{JSE}			1,25 σ_{JSE}		
	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain
0%	0,85	-	-	1,94	-	-	2,26	-	-	2,28	-	-	2,30	-	-
5%	1,53	8,16	8,16	2,32	4,56	4,56	2,34	0,96	0,96	2,36	0,96	0,96	2,37	0,84	0,84
10%	2,09	6,72	14,88	2,40	0,96	5,52	2,42	0,96	1,92	2,43	0,84	1,80	2,44	0,84	1,68
20%	2,52	5,16	20,04	2,54	1,68	7,20	2,55	1,56	3,48	2,56	1,56	3,36	2,58	1,68	3,36
25%	2,59	0,84	20,88	2,61	0,84	8,04	2,62	0,84	4,32	2,63	0,84	4,20	2,64	0,72	4,08
$33\frac{1}{3}$%	2,70	1,32	22,20	2,72	1,32	9,36	2,73	1,32	5,64	2,74	1,32	5,52	2,75	1,32	5,40
50%	2,75	0,60	22,80	2,91	2,28	11,64	2,93	2,40	8,04	2,94	2,40	7,92	2,96	2,52	7,92
100%	2,78	0,36	23,16	2,91	0,00	11,64	2,98	0,60	8,64	3,05	1,32	9,24	3,14	2,16	10,08

YEAR: 1968

MARKET RISK: 4,08%

	0,75 σ_{JSE}			0,9 σ_{JSE}			1,0 σ_{JSE}			1,1 σ_{JSE}			1,25 σ_{JSE}		
	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain
0%	3,03*	-	-	3,04	-	-	4,69	-	-	5,56	-	-	5,99	-	-
5%	3,03*	0,00	0,00	4,51	17,64	17,64	5,32	7,56	7,56	5,71	1,80	1,80	5,99[a]	0,00	0,00
10%	3,72	8,28	8,28	4,95	5,28	22,92	5,47	1,80	9,36	5,72	0,12	1,92	5,99[a]	0,00	0,00
20%	4,45	8,76	17,04	5,13	2,16	25,08	5,47[a]	0,00	9,36	5,72[a]	0,00	1,92	5,99[a]	0,00	0,00
25%	4,66	2,52	19,56	5,16	0,36	25,44	5,47[a]	0,00	9,36	5,72[a]	0,00	1,92	5,99[a]	0,00	0,00
33⅓%	4,67	0,12	19,68	5,16[a]	0,00	25,44	5,47[a]	0,00	9,36	5,72[a]	0,00	1,92	5,99[a]	0,00	0,00
50%	4,67[a]	0,00	19,68	5,16[a]	0,00	25,44	5,47[a]	0,00	9,36	5,72[a]	0,00	1,92	5,99[a]	0,00	0,00
100%	4,67	0,00	19,68	5,16	0,00	25,44	5,47	0,00	9,36	5,72	0,00	1,92	5,99	0,00	0,00

YEAR: 1969

MARKET RISK: 8,50%

	0,75 σ_{JSE}			0,9 σ_{JSE}			1,0 σ_{JSE}			1,1 σ_{JSE}			1,25 σ_{JSE}		
	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain
0%	1,17^{+}	-	-	1,17^{+}	-	-	1,17^{+}	-	-	1,17^{+}	-	-	1,17^{+}	-	-
5%	1,43^{+}	3,12	3,12	1,43^{+}	3,12	3,12	1,43^{+}	3,12	3,12	1,43^{+}	3,12	3,12	1,43^{+}	3,12	3,12
10%	1,68^{+}	3,00	6,12	1,68^{+}	3,00	6,12	1,68^{+}	3,00	6,12	1,68^{+}	3,00	6,12	1,68^{+}	3,00	6,12
20%	2,18^{+}	6,00	12,12	2,18^{+}	6,00	12,12	2,18^{+}	6,00	12,12	2,18^{+}	6,00	12,12	2,18^{+}	6,00	12,12
25%	2,44^{+}	3,12	15,24	2,44^{+}	3,12	15,24	2,44^{+}	3,12	15,24	2,44^{+}	3,12	15,24	2,44^{+}	3,12	15,24
33$\frac{1}{3}$%	2,86^{+}	5,04	20,28	2,86^{+}	5,04	20,28	2,86^{+}	5,04	20,28	2,86^{+}	5,04	20,28	2,86^{+}	5,04	20,28
50%	3,37	6,12	26,40	3,70^{+}	10,08	30,36	3,70^{+}	10,08	30,36	3,70^{+}	10,08	30,36	3,70^{+}	10,08	30,36
100%	3,75	4,56	30,96	4,16	5,52	35,88	4,42	8,64	39,00	4,68	11,76	42,12	5,06	16,32	46,68

YEAR: 1970

MARKET RISK: 8,21%

	0,75 σ_{JSE}			0,9 σ_{JSE}			1,0 σ_{JSE}			1,1 σ_{JSE}			1,25 σ_{JSE}		
	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain
0%	1,31+	-	-	1,31+	-	-	1,31+	-	-	1,31+	-	-	1,31+	-	-
5%	1,40+	1,08	1.08	1,40+	1,08	1,08	1,40+	1,08	1,08	1,40+	1,08	1,08	1,40+	1,08	1,08
10%	1,49+	1,08	2,16	1,49+	1,08	2,16	1,49+	1,08	2,16	1,49+	1,08	2,16	1,49+	1,08	2,16
20%	1,67+	2,16	4,32	1,67+	2,16	4,32	1,67+	2,16	4,32	1,67+	2,16	4,32	1,67+	2,16	4,32
25%	1,76+	1,08	5,40	1,76+	1,08	5,40	1,76+	1,08	5,40	1,76+	1,08	5,40	1,76+	1,08	5,40
33⅓%	1,92+	1,92	7,32	1,92+	1,92	7,32	1,92+	1,92	7,32	1,92+	1,92	7,32	1,92+	1,92	7,32
50%	2,22+	3,60	10,92	2,22+	3,60	10,92	2,22+	3,60	10,92	2,22+	3,60	10,92	2,22+	3,60	10,92
100%	3,13+	10,92	21,84	3,13+	10,92	21,84	3,13+	10,92	21,84	3,13+	10,92	21,84	3,13+	10,92	21,84

YEAR: 1971

MARKET RISK: 7,39%

	0,75 σ_{JSE}			0,9 σ_{JSE}			1,0 σ_{JSE}			1,1 σ_{JSE}			1,25 σ_{JSE}		
	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain
0%	0,75	-	-	1,15	-	-	1,30	-	-	1,36	-	-	1,37^{+}	-	-
5%	0,97	2,64	2,64	1,38	2,76	2,64	1,52	2,64	2,64	1,58^{+}	2,64	2,64	1,58^{+}	2,52	2,52
10%	1,19	2,64	5,28	1,61	2,76	5,28	1,74	2,64	5,28	1,78^{+}	2,40	5,04	1,78^{+}	2,40	4,92
20%	1,61	5,04	10,32	2,02	4,92	10,32	2,16	5,04	10,32	2,19^{+}	4,92	9,96	2,19^{+}	4,92	9,84
25%	1,82	2,52	12,84	2,22	2,40	12,72	2,36	2,40	12,72	2,39^{+}	2,40	12,36	2,39^{+}	2,40	12,24
33$\frac{1}{3}$%	2,16	4,08	16,92	2,54	3,84	16,80	2,70	4,08	16,80	2,73^{+}	4,08	16,44	2,73^{+}	4,08	16,32
50%	2,82	7,92	24,84	3,13	7,08	23,88	3,29	7,08	23,88	3,40^{+}	8,04	24,48	3,40^{+}	8,04	24,36
100%	4,40	18,96	43,80	4,76	19,56	43,32	4,91	19,44	43,32	5,04	19,68	44,16	5,21	21,72	46,08

YEAR: 1972

MARKET RISK: 3,67%

	0,75 σ_{JSE}			0,9 σ_{JSE}			1,0 σ_{JSE}			1,1 σ_{JSE}			1,25 σ_{JSE}		
	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain
0%	3,72	-	-	4,13	-	-	4,33	-	-	4,50	-	-	4,52^{+}	-	-
5%	4,12	4,80	4,80	4,44	3,72	3,72	4,62	3,48	3,48	4,65^{+}	1,80	1,80	4,65^{+}	1,56	1,56
10%	4,44	3,84	8,64	4,72	3,36	7,08	4,78^{+}	1,92	5,40	4,78^{+}	1,56	3,36	4,78^{+}	1,56	3,12
20%	4,95	6,12	14,76	5,04^{+}	3,84	10,92	5,04^{+}	3,12	8,52	5,04^{+}	3,12	6,48	5,04^{+}	3,12	6,24
25%	5,13	2,16	16,92	5,17^{+}	1,56	12,48	5,17^{+}	1,56	10,08	5,17^{+}	1,56	8,04	5,17^{+}	1,56	7,80
33⅓%	5,36	2,76	19,68	5,39^{+}	2,64	15,12	5,39^{+}	2,64	12,72	5,39^{+}	2,64	10,68	5,39^{+}	2,64	10,44
50%	5,61	3,00	22,68	5,81	5,04	20,16	5,83^{+}	5,28	18,00	5,83^{+}	5,28	15,96	5,83^{+}	5,28	15,72
100%	5,62	0,12	22,80	5,88	0,84	21,00	6,03	2,40	20,40	6,17	4,08	20,04	6,36	6,36	22,08

YEAR: 1973

MARKET RISK: 9,65%

	0,75 σ_{JSE}			0,9 σ_{JSE}			1,0 σ_{JSE}			1,1 σ_{JSE}			1,25 σ_{JSE}		
	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain
0%	3,61	-	-	4,66	-	-	5,29	-	-	5,88	-	-	6,73	-	-
5%	4,63	12,24	12,24	5,58	11,04	11,04	6,17	10,56	10,56	6,74	10,32	10,32	7,46^{+}	8,76	8,76
10%	5,54	10,92	23,16	6,43	10,20	21,24	6,99	9,84	20,40	7,52^{+}	9,36	19,68	7,52^{+}	0,72	9,48
20%	7,12	18,96	42,12	7,66^{+}	14,76	36,00	7,66^{+}	8,04	28,44	7,66^{+}	1,68	21,36	7,66^{+}	1,68	11,16
25%	7,73^{+}	7,32	49,44	7,73^{+}	0,84	36,84	7,73^{+}	0,84	29,28	7,73^{+}	0,84	22,20	7,73^{+}	0,84	12,00
33⅓%	7,84^{+}	1,32	50,76	7,84^{+}	1,32	38,16	7,84^{+}	1,32	30,60	7,84^{+}	1,32	23,52	7,84^{+}	1,32	13,32
50%	8,07^{+}	2,76	53,52	8,07^{+}	2,76	40,92	8,07^{+}	2,76	33,36	8,07^{+}	2,76	26,28	8,07^{+}	2,76	16,08
100%	8,18	1,32	54,84	8,27	2,40	43,32	8,32	3,00	36,36	8,38	3,72	30,00	8,45	4,56	20,64

YEAR: 1974

MARKET RISK: 9,73%

	0,75 σ_{JSE}			0,9 σ_{JSE}			1,0 σ_{JSE}			1,1 σ_{JSE}			1,25 σ_{JSE}		
	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain
0%	-1,01	-	-	-0,67	-	-	-0,48	-	-	-0,30	-	-	-0,05	-	-
5%	-0,57	5,28	5,28	-0,23	5,28	5,28	-0,04	5,28	5,28	0,14	5,28	5,28	0,38	5,16	5,16
10%	-0,16	4,92	10,20	0,19	5,04	10,32	0,38	5,04	10,32	0,56	5,04	10,32	0,81	5,16	10,32
20%	0,57	8,76	18,96	0,97	9,36	19,68	1,18	9,60	19,92	1,37	9,72	20,04	1,63	9,84	20,16
25%	0,86	3,48	22,44	1,33	4,32	24,00	1,55	4,44	24,36	1,75	4,56	24,60	2,02	4,68	24,84
$33\frac{1}{3}$%	1,29	5,16	27,60	1,87	6,48	30,48	2,14	7,08	31,44	2,36	7,32	31,92	2,65	7,56	32,40
50%	2,09	9,60	37,20	2,65	9,36	39,84	3,02	10,56	42,00	3,37	12,12	44,04	3,76	13,32	45,72
100%	4,00	22,92	60,12	4,52	22,44	62,28	4,85	21,96	63,96	5,18	21,72	65,76	5,66	22,80	68,52

YEAR: 1975

MARKET RISK: 6,97%

	0,75 σ_{JSE}			0,9 σ_{JSE}			1,0 σ_{JSE}			1,1 σ_{JSE}			1,25 σ_{JSE}		
	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain
0%	1,60*	-	-	2,63	-	-	3,20	-	-	3,44	-	-	3,72	-	-
5%	1,48	-1,44	-1,44	3,21	6,96	6,96	3,50	3,60	3.60	3,70	3,12	3,12	3,95	2,76	2,76
10%	2,77	15,48	14,04	3,54	3,96	10,92	3,75	3,00	6,60	3,93	2,76	5,88	4,10+	1,80	4,56
20%	3,60	9,96	24,00	4,00	5,52	16,44	4,17	5,04	11,64	4,26+	3,96	9,84	4,26+	1,92	6,48
25%	3,84	2,88	26,88	4,18	2,16	18,60	4,33	1,92	13,56	4,34+	0,96	10,80	4,34+	0,96	7,44
33⅓%	4,11	3,24	30,12	4,44	3,12	21,72	4,48+	1,80	15,36	4,48+	1,68	12,48	4,48+	1,68	9,12
50%	4,41	3,60	33,72	4,74	3,60	25,32	4,75+	3,24	18,60	4,75+	3,24	15,72	4,75+	3,24	12,36
100%	4,46	0,60	34,32	4,79	0,60	25,92	4,97	2,64	21,24	5,09	4,08	19,80	5,25	6,00	18,36

YEAR: 1976

MARKET RISK: 7,56%

	0,75 σ_{JSE}			0,9 σ_{JSE}			1,0 σ_{JSE}			1,1 σ_{JSE}			1,25 σ_{JSE}		
	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain
0%	-0,32	-	-	0,78	-	-	1,38	-	-	1,92	-	-	2,69	-	-
5%	0,31	7,56	7,56	1,27	5,88	5,88	1,82	5,28	5,28	2,34	5,04	5,04	3,08	4,68	4,68
10%	0,84	6,36	13,92	1,70	5,16	11,04	2,22	4,80	10,08	2,72	4,56	9,60	3,27	2,28	6,96
20%	1,69	10,20	24,12	2,46	9,12	20,16	2,94	8,64	18,72	3,22	6,00	15,60	3,47	2,40	9,36
25%	2,04	4,20	28,32	2,78	3,84	24,00	3,12	2,16	20,88	3,35	1,56	17,16	3,55	0,84	10,20
33⅓%	2,54	6,00	34,32	3,11	3,96	27,96	3,34	2,64	23,52	3,50	1,80	18,96	3,66+	1,32	11,52
50%	3,16	7,44	41,76	3,48	4,44	32,40	3,62	3,36	26,88	3,73	2,76	21,72	3,87+	2,52	14,04
100%	3,66	6,00	47,76	3,95	5,64	38,04	4,07	5,40	32,28	4,16	5,16	26,88	4,30	5,16	19,20

YEAR: 1977

MARKET RISK: 4,34%

	$0,75\ \sigma_{JSE}$			$0,9\ \sigma_{JSE}$			$1,0\ \sigma_{JSE}$			$1,1\ \sigma_{JSE}$			$1,25\ \sigma_{JSE}$		
	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain
0%	1,98	-	-	2,39	-	-	2,58	-	-	2,74	-	-	2,96	-	-
5%	2,15	2,04	2,04	2,51	1,44	1,44	2,69	1,32	1,32	2,84	1,20	1,20	3,01	0,60	0,60
10%	2,30	1,80	3,84	2,61	1,20	2,64	2,78	1,08	2,40	2,93	1,08	2,28	3,03	0,24	0,84
20%	2,53	2,76	6,60	2,80	2,28	4,92	2,95	2,04	4,44	3,00	0,84	3,12	3,03	0,00	0,84
25%	2,63	1,20	7,80	2,88	0,96	5,88	2,97	0,24	4,68	3,00	0,00	3,12	3,03[a]	0,00	0,84
$33\frac{1}{3}$%	2,77	1,68	9,48	2,94	0,72	6,60	2,97[a]	0,00	4,68	3,00[a]	0,00	3,12	3,03[a]	0,00	0,84
50%	2,88	1,32	10,80	2,94[a]	0,00	6,60	2,97[a]	0,00	4,68	3,00[a]	0,00	3,12	3,03[a]	0,00	0,84
100%	2,90	0,24	11,04	2,95	0,12	6,72	2,98	0,12	4,80	3,01	0,12	3,24	3,04	0,12	0,96

YEAR: 1978

MARKET RISK: 6,02%

	0,75 σ_{JSE}			0,9 σ_{JSE}			1,0 σ_{JSE}			1,1 σ_{JSE}			1,25 σ_{JSE}		
	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain
0%	3,28	-	-	3,81	-	-	3,97	-	-	4,02^{+}	-	-	4,02^{+}	-	-
5%	3,46	2,16	2,16	3,91	1,20	1,20	4,03	0,72	0,72	4,05^{+}	0,36	0,36	4,05^{+}	0,36	0,36
10%	3,60	1,68	3,84	3,99	0,96	2,16	4,07	0,48	1,20	4,09^{+}	0,48	0,84	4,09^{+}	0,48	0,84
20%	3,83	2,76	6,60	4,08	1,08	3,24	4,15	0,96	2,16	4,16^{+}	0,84	1,68	4,16^{+}	0,84	1,68
25%	3,92	1,08	7,68	4,11	0,36	3,60	4,18	0,36	2,52	4,19^{+}	0,36	2,04	4,19^{+}	0,36	2,04
33$\frac{1}{3}$%	4,00	0,96	8,64	4,15	0,48	4,08	4,22	0,48	3,00	4,25^{+}	0,72	2,76	4,25^{+}	0,72	2,76
50%	4,06	0,72	9,36	4,18	0,36	4,44	4,25	0,36	3,36	4,32	0,84	3,60	4,37^{+}	1,44	4,20
100%	4,07	0,12	9,48	4,19	0,12	4,56	4,26	0,12	3,48	4,32	0,00	3,60	4,41	0,48	4,68

YEAR: 1979

MARKET RISK: 6,07%

	0,75 σ_{JSE}			0,9 σ_{JSE}			1,0 σ_{JSE}			1,1 σ_{JSE}			1,25 σ_{JSE}		
	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain
0%	4,42	-	-	4,88	-	-	5,14	-	-	5,39	-	-	5,74	-	-
5%	4,66	2,88	2,88	5,10	2,64	2,64	5,38	2,88	2,88	5,64	3,00	3,00	6,01	3,24	3,24
10%	4,86	2,40	5,28	5,28	2,16	4,80	5,56	2,16	5,04	5,83	2,28	5,28	6,22	2,52	5,76
20%	5,18	3,84	9,12	5,58	3,60	8,40	5,85	3,48	8,52	6,12	3,48	8,76	6,51	3,48	9,24
25%	5,31	1,56	10,68	5,71	1,56	9,96	5,97	1,44	9,96	6,24	1,44	10,20	6,64	1,56	10,80
33⅓%	5,51	2,40	13,08	5,91	2,40	12,36	6,17	2,40	12,36	6,44	2,40	12,60	6,83	2,28	13,08
50%	5,84	3,96	17,04	6,26	4,20	16,56	6,53	4,32	16,68	6,79	4,20	16,80	7,19	4,32	17,40
100%	6,15	3,72	20,76	6,63	4,44	21,00	6,94	4,92	21,60	7,24	5,40	22,20	7,69	6,00	23,40

YEAR: 1980

MARKET RISK: 8,17%

	0,75 σ_{JSE}			0,9 σ_{JSE}			1,0 σ_{JSE}			1,1 σ_{JSE}			1,25 σ_{JSE}		
	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain
0%	2,05	-	-	3,01	-	-	3,02$^+$	-	-	3,02$^+$	-	-	3,02$^+$	-	-
5%	2,48	5,16	5,16	3,03$^+$	0,24	0,24	3,03$^+$	0,12	0,12	3,03$^+$	0,12	0,12	3,03$^+$	0,12	0,12
10%	2,74	3,12	8,28	3,05$^+$	0,24	0,48	3,05$^+$	0,24	0,36	3,05$^+$	0,24	0,36	3,05$^+$	0,24	0,36
20%	3,07	3,96	12,24	3,07$^+$	0,24	0,72	3,07$^+$	0,24	0,60	3,07$^+$	0,24	0,60	3,07$^+$	0,24	0,60
25%	3,09$^+$	0,24	12,48	3,09$^+$	0,24	0,96	3,09$^+$	0,24	0,84	3,09$^+$	0,24	0,84	3,09$^+$	0,24	0,84
33$\frac{1}{3}$%	3,11$^+$	0,24	12,72	3,11$^+$	0,24	1,20	3,11$^+$	0,24	1,08	3,11$^+$	0,24	1,08	3,11$^+$	0,24	1,08
50%	3,13	0,24	12,96	3,15$^+$	0,48	1,68	3,15$^+$	0,48	1,56	3,15$^+$	0,48	1,56	3,15$^+$	0,48	1,56
100%	3,13	0,00	12,96	3,18	0,36	2,04	3,21	0,72	2,28	3,23	0,96	2,52	3,26	1,32	2,88

YEAR: 1981

MARKET RISK: 6,48%

	0,75 σ_{JSE}			0,9 σ_{JSE}			1,0 σ_{JSE}			1,1 σ_{JSE}			1,25 σ_{JSE}		
	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain
0%	1,54$^+$	-	-	1,54$^+$	-	-	1,54$^+$	-	-	1,54$^+$	-	-	1,54$^+$	-	-
5%	1,63$^+$	1,08	1,08	1,63$^+$	1,08	1,08	1,63$^+$	1,08	1,08	1,63$^+$	1,08	1,08	1,63$^+$	1,08	1,08
10%	1,73$^+$	1,20	2,28	1,73$^+$	1,20	2,28	1,73$^+$	1,20	2,28	1,73$^+$	1,20	2,28	1,73$^+$	1,20	2,28
20%	1,92$^+$	2,28	4,56	1,92$^+$	2,28	4,56	1,92$^+$	2,28	4,56	1,92$^+$	2,28	4,56	1,92$^+$	2,28	4,56
25%	2,01$^+$	1,08	5,64	2,01$^+$	1,08	5,64	2,01$^+$	1,08	5,64	2,01$^+$	1,08	5,64	2,01$^+$	1,08	5,64
33$\frac{1}{3}$%	2,17$^+$	1,92	7,56	2,17$^+$	1,92	7,56	2,17$^+$	1,92	7,56	2,17$^+$	1,92	7,56	2,17$^+$	1,92	7,56
50%	2,49$^+$	3,84	11,40	2,49$^+$	3,84	11,40	2,49$^+$	3,84	11,40	2,49$^+$	3,84	11,40	2,49$^+$	3,84	11,40
100%	3,08	7,08	18,48	3,33	10,08	21,48	3,40	10,92	22,32	3,44$^+$	11,40	22,80	3,44$^+$	11,40	22,80

YEAR: 1982

MARKET RISK: 11,73%

	0,75 σ_{JSE}			0,9 σ_{JSE}			1,0 σ_{JSE}			1,1 σ_{JSE}			1,25 σ_{JSE}		
	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain	monthly return	gain p.a.	cumulative gain
0%	1,89*	-	-	4,55	-	-	5,02	-	-	5,19^{+}	-	-	5,19^{+}	-	-
5%	2,95	12,72	12,72	4,67	1,44	1,44	5.07	0,60	0,60	5.19^{a}	0,00	0,00	5,19^{a}	0,00	0,00
10%	3,86	10,92	23,64	4,75	0,96	2,40	5,11	0,48	1,08	5.19^{a}	0,00	0,00	5,19^{a}	0,00	0,00
20%	4,25	4,68	28,32	4,84	1,08	3,48	5,13^{+}	0,24	1,32	5,19^{a}	0,00	0,00	5,19^{a}	0,00	0,00
25%	4,32	0,84	29,16	4,87	0,36	3,84	5,13^{a}	0,00	1,32	5,19^{a}	0,00	0,00	5,19^{a}	0,00	0,00
33$\frac{1}{3}$%	4,40	0,96	30,12	4,89	0,24	4,08	5,13^{a}	0,00	1,32	5,19^{a}	0,00	0,00	5,19^{a}	0,00	0,00
50%	4,45	0,60	30,72	4,89^{a}	0,00	4,08	5,13^{a}	0,00	1,32	5,19^{a}	0,00	0,00	5,19^{a}	0,00	0,00
100%	4,45	0,00	30,72	4,89	0,00	4,08	5,14	0,12	1,44	5,19^{+}	0,00	0,00	5,19^{+}	0,00	0,00

APPENDIX C

Returns and risks (in percent per month) of *ex ante* portfolios for varying proportions allowed in foreign investment (1965-1982).

YEAR	1965		1966		1967		1968		1969		1970	
w_1 c.	R_p a.	σ_p b.	R_p	σ_p	R_p	σ_p	R_p	σ_p	R_p	σ_p	R_p	σ_p
0,00	0,352	2,726	1,684	2,985	1,319	4,074	4,189	4,173	-2,652	10,596	-1,792	8,892
0,05	0,378	2,670	1,550	2,848	1,275	3,856	4,026	3,969	-2,484	10,057	-1,724	8,525
0,10	0,404	2,620	1,416	2,718	1,232	3,642	3,863	3,766	-2,315	9,520	-1,655	8,161
0,20	0,457	2,540	1,148	2,482	1,145	3,228	3,538	3,364	-1,979	8,453	-1,518	7,447
0,25	0,483	2,510	1,014	2,378	1,102	3,030	3,375	3,166	-1,810	7,925	-1,450	7,098
0,33	0,526	2,476	0,791	2,231	1,029	2,716	3,104	2,839	-1,529	7,054	-1,336	6,531
0,50	0,614	2,469	0,345	2,057	0,885	2,187	2,562	2,215	-0,968	5,375	-1,107	5,472
1,00	0,875	2,913	-0,995	2,623	0,451	2,247	0,936	1,227	0,715	2,787	-0,422	3,711

a. $R_p = w_1 (R_{SA}) + (1-w_1)(R_{foreign})$

b. $\sigma_p = [w_1^2 \sigma_{SA}^2 + (1-w_1)^2 \sigma_{foreign}^2 + 2 w_1 (1-w_1) \sigma_{SA} \sigma_{foreign} \rho_{SA/foreign}]^{\frac{1}{2}}$

c. w_1 = porportion invested in international securities

YEAR	1971		1972		1973		1974		1975		1976	
w_1	R_p	σ_p	R_p	σ_p	R_p	σ_p	R_p	σ_p	R_p	σ_p	R_p	σ_p
0,00	0,668	7,587	3,042	4,198	-0,732	9,606	-1,943	8,619	0,566	6,460	-0,413	7,455
0,05	0,712	7,343	2,911	4,043	-0,677	9,095	-1,783	8,288	0,600	6,074	-0,317	7,090
0,10	0,756	7,101	2,780	3,889	-0,623	8,589	-1,622	7,978	0,634	5,702	-0,221	6,732
0,20	0,844	6,629	2,519	3,587	-0,513	7,597	-1,301	7,431	0,702	5,016	-0,029	6,044
0,25	0,889	6,399	2,388	3,440	-0,459	7,112	-1,141	7,201	0,736	4,709	0,067	5,717
0,33	0,962	6,026	2,170	3,200	-0,368	6,328	-0,874	6,889	0,792	4,271	0,227	5,205
0,50	1,109	5,329	1,734	2,749	-0,186	4,901	-0,339	6,583	0,906	3,774	0,546	4,355
1,00	1,551	3,942	0,426	1,885	0,360	4,034	1,265	8,249	1,246	5,763	1,506	4,363

YEAR	1977		1978		1979		1980		1981		1982	
w_1	R_p	σ_p	R_p	σ_p	R_p	σ_p	R_p	σ_p	R_p	σ_p	R_p	σ_p
0,00	2,207	4,208	3,360	5,410	3,029	5,464	0,877	7,687	0,227	4,941	1,286	10,765
0,05	2,157	4,045	3,219	5,287	2,971	5,119	0,808	7,385	0,280	4,735	1,191	10,215
0,10	2,106	3,888	3,077	5,172	2,913	4,781	0,740	7,086	0,332	4,490	1,097	9,669
0,20	2,004	3,592	2,794	4,967	2,797	4,127	0,603	6,498	0,437	4,188	0,908	8,593
0,25	1,954	3,455	2,653	4,879	2,739	3,815	0,535	6,210	0,489	4,035	0,814	8,064
0,33	1,869	3,246	2,417	4,755	2,642	3,328	0,420	5,742	0,576	3,819	0,657	7,202
0,50	1,700	2,923	1,946	4,601	2,449	2,562	0,192	4,863	0,751	3,568	0,343	5,590
1,00	1,192	2,982	0,532	4,935	1,868	3,536	-0,492	3,254	1,274	4,405	-0,600	3,853