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## The Bates Model: Fourier Transform For Option Pricing Under Affine Jump-Diffusions In The South African Market

by

Blessing Munhumwe

(03/1)

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of

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 $\mathbf{at}$ 

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### Supervisor: Professor Ronald Becker

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## Abstract

The purpose of this study is to price options under jump diffusions using Fourier Transforms and obtain the implied volatility surface from these option prices. Modeling the dynamics of the underlying asset combined with jumps in asset returns was developed by Bates [10](1996), resulting in a model known as the Bates model. The approach that we use for solving our pricing problem is similar to the one used by Sepp [64](2003). It is the method of using Fourier Transforms under the assumption that the price process is given by a general model. Two methods for evaluating options using Fourier Transforms under this price process are discussed. Having obtained the option prices we can use a numerical procedure to invert Black's (1976) formula to obtain the implied volatilities. Using the model implied volatility and the market volatility we discuss how this model can be calibrated. For our empirical analysis we use data obtained from the South African market. Exchange traded data for the ALSI call option at a given date is obtained from the SAFEX website which we use to calibrate our model. The volatilities obtained using the Bates model are compared to the market volatilities and this helps us to determine how good our model is. The results suggest that the model is good for options with short maturities. Since the methods for modeling volatility in the South African market discussed so far in literature do not model appropriately shorter maturities we recommend modeling volatilities using this method.

# Contents

| 1        | Inti        | roducti          | ion  | 1         |
|----------|-------------|------------------|--|-----------|
| <b>2</b> | The         | e Bates          | Model And Option Pricing Problem Formulation         | <b>4</b>  |
|          | 2.1         | The B            | lack-Scholes-Merton Model                            | 4         |
|          | 2.2         | Stocha           | astic Volatility Models                              | 5         |
|          | 2.3         | Jump             | Diffusion Processes                                  | 6         |
|          | 2.4         | Affine           | Jump Diffusions With Stochastic Volatility           | 7         |
|          | 2.5         | Option           | n Pricing Problem Formulation                        | 9         |
| 3        | Solu<br>3 1 | ution (<br>The P | Of The Pricing Problem Using Fourier Transforms      | 10        |
|          | 0.1         | Option           | ns In The Bates Model                                | 10        |
|          | 3.2         | Pricin           | g European Options Using Fourier Transforms          | 11        |
|          | 0.1         | 3.2.1            | Method 1: The Characteristic Formula                 | 13        |
|          |             | 3.2.2            | Method 2: The Black-Scholes-style Formula            | 15        |
|          |             |                  | G  |           |
| 4        | Mo          | del Ca           | llibration And Finding The Option Price Together     | •         |
|          | Wit         | h The            | Volatility   | 17        |
|          | 4.1         | Model            | Calibration  | 17        |
|          | 4.2         | Solvin           | g The Optimisation Problem Using R GUI Language      | 18        |
|          | 4.3         | Findir           | ng The Price Of The Option And The Volatility        | 19        |
|          | 4.4         | Uses (           | Of Parameters And The Volatility                     | 20        |
| <b>5</b> | Em          | pirical          | Study  | <b>21</b> |
|          | 5.1         | The S            | outh African Equity Futures Derivatives Market       | 21        |
|          | 5.2         | Proble           | em Formulation                                       | 22        |
|          | 5.3         | Volati           | lity Models On SAFEX And The Motivation In Using The |           |
|          |             | Bates            | Model  | 24        |
|          |             | 5.3.1            | SABR Model   | 24        |
|          |             | 5.3.2            | Polling Method                                       | 24        |
|          |             | 5.3.3            | Deterministic Model                                  | 24        |
|          |             | 5.3.4            | The Motivation In Using The Bates Model Approach     | 25        |
|          | 5.4         | The M            | Iarket Data  | 25        |
|          |             | 5.4.1            | Data Screening                                       | 26        |

|   | $5.5 \\ 5.6 \\ 5.7$      | Model Calibration Using ALSI Futures Derivatives DataEmpirical Results                  | 26<br>26<br>30        |
|---|--------------------------|---|-----------------------|
| 6 | <b>Sum</b><br>6.1<br>6.2 | Immary And Conclusions         Summary  | <b>41</b><br>41<br>41 |
| Α | Pric<br>A.1<br>A.2       | Cing Formulas         The Coefficients For (3.27).         The Coefficients For (3.31). | <b>43</b><br>43<br>44 |
| в | RG                       | UI Code For Option Pricing Using The Bates Model  | 45                    |

GUI Code For Option Pricing Using The Bates Model

# List of Figures

| 5.1 | ALSI Volatility Surface                           | 28 |
|-----|---|----|
| 5.2 | Bates Model Volatility Surface                    | 36 |
| 5.3 | ALSI Volatility Surface For Certain Strike Range  | 37 |
| 5.4 | Bates Volatility Surface For Certain Strike Range | 38 |
| 5.5 | Bates Model Error Histogram For Errors            | 39 |
|     | iversity cape                                     |    |

# List of Tables

| 5.1<br>5.2<br>5.3<br>5.4<br>5.5<br>5.6<br>5.7 | ALSI Data | 27<br>31<br>32<br>33<br>34<br>35<br>40 |
|---|-----------|--|
|   | Gov Gov   |  |

## Chapter 1

# Introduction

Modeling the dynamics of volatility in different market environments has been an area of interest to most researchers as evidenced by the amount of literature on the subject. One use of volatility models is to price options. Options can be used as protection against unfavourable market movements. An option is a derivative that gives the right but not the obligation to buy or sell some underlying asset at some future time. A call option gives the right but not the obligation to buy the underlying asset whilst a put option gives the right but not the obligation to sell the underlying asset. There are several option styles that are discussed in literature. European options may only be exercised on the maturity date whilst American options may only be exercised on or before the maturity date.

It was in the early 1970s when Fisher Black, Myron Scholes and Robert Merton achieved a major breakthrough in the pricing of options. This involved what became known as the Black-Scholes (or Black-Scholes-Merton) model. One of the assumptions made about this model is that volatility is constant. They used historical volatility as the proxy for determining volatility in their model (Black and Scholes [12](1972)). In contrast to historical volatility some research has been done on implied volatility, for example (Fleming [38](1998) and Dumas, Fleming and Whaley [34](1998)). This is the volatility implied by the current market option prices.

The notion of constant volatility implies that options with different strikes and maturities for the same underlying asset have the same implied volatility. However, if we observe a plot of market implied volatilities against strikes for all option markets, a pattern known as a smile is seen. This might mean that values obtained using the Black-Scholes model do not reflect the true observed market values. Another concept of interest is the term structure of volatility. This refers to how implied volatility differs for related options with different maturities. A three-dimensional plot which combines the volatility smile and the term structure of volatility is known as an implied volatility surface.

Asset price dynamics are determined by the factors which affect the asset's price movements. Jumps, deterministic volatility, stochastic volatility, etc have an effect on the movement of asset prices. Since the Black-Scholes model does not cater for this, models that reflect these factors have been developed. Merton [54](1976) added a Poisson jump process to the price process. Heston [42](1993)developed a stochastic volatility model which became known as the Heston model. Dupire [35](1994) looked at how to price with a smile. He assumed that volatility is a deterministic function of time and the asset price. Bates [10](1996)further developed Heston's model by adding jumps to the model and as result we have what is known as the Bates model. Scott [62](1997) in his paper researched on pricing the stock options in a jump diffusion model with stochastic volatility and interest rates. Fang [37](2000) considered the Bates' model and further developed it by assuming that the jump intensity is stochastic. That same year Duffie. Pan and Singleton also studied the Bates model and proposed a jump diffusion process that has both price and volatility jumps (Duffie-Pan-Singleton [33](2000)). These models were developed with the aim of modeling volatility in such a way that the volatility obtained is close to market volatilities.

Different market environments have their own approaches in modeling volatility dynamics. For our empirical analysis we look at the South African market. Several approaches have been used to model the volatility surface. West [72](2005) looked at calibration of the SABR model in illiquid markets and considered the South African market for his analysis. Araujo and Mare [4](2006) examined the volatility skew in the South African equity market using risk-neutral historical distributions. Davies [26](2006) in his thesis focused on option models with jumps in the context of the JSE's Top40 index. Bonney, Shannon and Uys [14](2008) describe the method of modeling the Top40 volatility skew using Principal Component Analysis approach. Kotze and Joseph [49](2009) constructed a South African index volatility surface using stock exchange traded data. They assumed that volatility is deterministic.

In this project we discuss option pricing in the South African market using Fourier transforms under the assumption that the underlying asset's dynamics are given by the Bates model. From these option prices we obtain the volatilities for different strikes and maturities by using a numerical procedure to invert the Black's (1976) formula. Using these volatilities we plot the volatility surface to get a visual idea. The methods of modeling volatility in the South African market discussed so far in literature do not model appropriately shorter maturities. We also seek to investigate and see if the method models well volatilities with shorter maturities.

This is the structure of our project. In Chapter 2 we examine the Bates model and how it came to be. The last section of this chapter describes the formulation of the option pricing problem, based on the assumption that the asset price dynamics are given by the Bates model. In Chapter 3 we study the solution of the option pricing problem under the considered price process using Fourier transforms. In Chapter 4 we go into detail on how to calibrate the model, find the option price and obtain volatility from the option price. In Chapter 5 we focus on our empirical study. We summarize and conclude in Chapter 6.

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## Chapter 2

# The Bates Model And Option Pricing Problem Formulation

In this chapter we examine the Bates model and how it came to be. Prior to the use of the Bates model, the Black-Scholes model and stochastic volatility models were mostly in use. These models are briefly discussed before we study the Bates model. At the end of the chapter we formulate our option pricing problem under the assumption that the asset dynamics are given by the Bates model.

## 2.1 The Black-Scholes-Merton Model

This is an option pricing model for pricing standard European call and put options. It was developed by Black and Scholes in 1973 and they gave their name to this formula (Black and Scholes [13](1973)). Fischer Black is quoted saying:

"As we worked on the paper, we had long discussions with Robert C. Merton, who was also working on the valuation of options (Merton [54](1973)). He suggested a method for deriving the formula that became the principal derivation in the paper". This Week's Citation Classic [70](1987)

As a result of their contributions to this option pricing model we have what is known as the Black-Scholes-Merton model or the Black-Scholes model. One of the assumptions made by the Black-Scholes model is that the price dynamics of the underlying asset follow a Geometric Brownian motion given by:

$$dS(t) = (r-d)S(t)dt + \sigma S(t)dW(t)$$
(2.1)

where S(t) is the underlying asset price, r is a risk-free interest rate, d is a dividend rate,  $\sigma$  is the volatility and W(t) is a standard Wiener process. Using the risk-neutral valuation principle, Black and Scholes [13](1973) showed that the value F(S, t) of a European option satisfies the partial differential equation:

$$F_t + \frac{1}{2}\sigma^2 S^2 F_{SS} + (r - d) SF_S - rF = 0$$
(2.2)

with the boundary condition

$$F(S,T) = \max\left\{\varphi\left[S-K\right],0\right\}$$
(2.3)

where K is the strike price and T is the maturity.  $\varphi$  denotes the option type  $(\varphi = 1 \text{ if the option is a call option and } \varphi = -1 \text{ if the option is a put option}).$ 

The solution to this partial differential equation gives the price of the option which is called the Black-Scholes formula and it is given by:

$$F(S,T) = \varphi[Se^{-d(T-t)}N(\varphi d_{+}) - Ke^{-r(T-t)}N(\varphi d_{-})]$$
(2.4)

where

$$d_{\pm} = \frac{\ln(S/K) + \left(r - d \pm \frac{1}{2}\sigma^2\right)(T - t)}{\sigma\sqrt{T - t}}$$
(2.5)

 $N(\cdot)$  is the cumulative distribution function and is given by

$$N(x) = \int_{-\infty}^{x} n(y) dy = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^{2}} dy$$
(2.6)

One of the assumptions made by the Black-Scholes model is that volatility is constant. Other models were introduced which seek to address this because in reality, volatility is not constant. We look at stochastic volatility models in the section that follows.

### 2.2 Stochastic Volatility Models

If the assumption of constant volatility is valid, the implied volatility obtained from observed market prices would be the same for all options with different strikes and maturities. This means that the volatility surface would be flat across all strikes and maturities. However, traded option data shows that this is not so because the volatility surface is dynamic and is changing over time.

In stochastic volatility models the underlying asset price and its volatility are modeled by two stochastic processes with different parameters. A huge number of stochastic volatility models have been developed in the academic literature. Of interest to us is the square root model that was developed by Heston [42](1993) and is also known as the Heston model:

$$dS(t) = (r-d)S(t)dt + \sqrt{V(t)}S(t)dW^{s}(t), \quad S(0) = S;$$
(2.7)

$$dV(t) = \kappa \left(\theta - V(t)\right) dt + \epsilon \sqrt{V(t)} dW^{v}(t), \quad V(0) = V$$
(2.8)

where  $\kappa$  is a mean-reverting rate,  $\theta$  is a long-term variance,  $\epsilon$  is a volatility of volatility, V(t) is the variance,  $W^s(t)$  and  $W^v(t)$  are correlated Wiener processes with constant correlation  $\rho$ . The other parameters are defined as given previously.

Derman [29](2003) explains the advantages and disadvantages of using these models.

"Stochastic volatility models have attractive features. Their smile is stable, unchanging over time, and in that sense more like realworld smiles. They therefore produce more realistic smiles. But there are disadvantages too. If volatility is stochastic you have to hedge it to replicate and price the option. Unlike a stock or a currency, volatility is not a traded variable with an observable price. In practice you must hedge one option with another, and calibrate the evolution of future volatility in the model to fit current option prices in order to get going." Derman [29](2003)

### 2.3 Jump Diffusion Processes

Prices of individual stocks often jump due to certain events happening. Hence to obtain a more realistic model of stock prices, researchers have added jumps to stochastic volatility models (e.g. Bates [10](1996); Bakshi, Cao and Chen [7](1997)). These types of models are important for explaining shorter maturity smiles because the Heston model matches the smile well except at very short maturities.

Merton [54](1976) added a Poisson jump process to the price process that is uncorrelated with the Brownian motion driving the price process and a generalised jump diffusion process is given as:

$$dS(t) = \mu(S, t)dt + \sigma(S, t)dW(t) + \gamma(S, t)JdN(t)$$
(2.9)

where  $\mu(S,t)$  is the process' drift and  $\sigma(S,t)$  is the process' volatility.  $\mu(S,t)$ and  $\sigma(S,t)$  are deterministic functions. N(t) is a Poisson process, J is the jump magnitude which is a random variable with the probability density function  $\varpi(J)$ ,  $\gamma(S,t)$  is a function of S and t which may be found in the process' jump component e.g.  $\gamma(S,t) = S(t)$  in some cases. The Poisson process N(t) counts the number of jumps that occur at or before time t. These jumps arrive at an average rate of  $\lambda$  per unit time, where  $\lambda$  is a constant. Hence we say that the Poisson Process N(t) has intensity  $\lambda$ . Using the risk-neutral valuation principle the dynamics of S(t) are such that  $e^{-rt}S(t)$  is a Martingale. In the Poisson process we know that the arrival of one jump does not depend on the arrival of previous jumps and the probability of having two simultaneous jumps is zero. We use these properties in computing the expected value of the increment dN(t). Within the next interval dt one jump will arrive with probability  $\lambda dt$  and the probability of no jumps arriving within the same interval is  $1 - \lambda dt$ . Hence the expected value of the increment is computed as:

$$E[dN(t)] = 1 \cdot \lambda dt + 0 \cdot (1 - \lambda dt) = \lambda dt$$

Let the increments of the process M(t) be given as

$$dM(t) = dN(t) - \lambda dt$$

Taking expectations,

$$E[dM(t)] = E[dN(t) - \lambda dt] = E[dN(t)] - \lambda dt = 0$$

From the Martingale property (Shreve [66](2004)) the process M(t) is a Martingale.

Assuming that the jump-diffusion model is a Geometric Brownian Motion with jumps, the risk neutral dynamics for S(t) under the Martingale measure Q is given by:

$$dS(t) = (r - d - \lambda m) S(t)dt + \sqrt{V(t)}S(t)dW^{s}(t) + (e^{J} - 1) S(t)dN(t)$$

where m is the average jump amplitude given by

$$m = \int [e^J - 1] \varpi(J) dJ$$

A number of authors have studied this approach. One of the most common models developed was a model by Bates [10](1996) and this became known as the Bates model. The following section expounds on this model.

### 2.4 Affine Jump Diffusions With Stochastic Volatility

Bates [10](1996) developed a method for pricing American options on combined stochastic volatility and jump-diffusion processes in the presence of systematic jump and volatility risk. The Bates model is based on the approach used by Stein and Stein [68](1991) and Heston [42](1993). The risk-neutral version of this model is given mathematically as:

$$dS(t) = (r - d - \lambda m) S(t)dt + \sqrt{V(t)S(t)}dW^{s}(t) + (e^{J} - 1) S(t)dN(t) \quad (2.10)$$

$$dV(t) = \kappa \left(\theta - V(t)\right) dt + \epsilon \sqrt{V(t)} dW^{v}(t)$$
(2.11)

This type of model gives us one of the most realistic dynamics for the smile. The advantages and disadvantages of jumps and stochastic volatility presented in the previous sections still apply to this model. Bates [10](1996) mentions two major advantages in using this model. First, the model's uniqueness lies in that it can allow for systematic volatility risk unlike other models. The second advantage is that the method is quite easy to use for pricing options and at the same time maintaining precise values for the parameters in the model.

Possible probability density functions  $(\varpi(J))$  for jumps sizes are given below. These were also studied by Sepp [64](2003).

#### (a) Log-normal jump diffusions

The probability density function in which the logarithm of jump size is normally distributed is given as:

$$\varpi(J) = \frac{1}{\sqrt{2\pi\delta^2}} e^{-(J-v)^2/2\delta^2}$$
(2.12)

This was proposed by Merton [54](1976).  $\nu$  and  $\delta$  are mean and volatility for the Merton jump-diffusion. Hence  $e^J$  is lognormally distributed.

#### (b) Double-Exponential Jump Diffusions

Kou [48](2002) studied the double-exponential jump-diffusions and it is given as:

$$\varpi(J) = p \frac{1}{\eta_u} e^{-\frac{1}{\eta_u} J} \mathbf{1}_{\{J \ge 0\}} + q \frac{1}{\eta_d} e^{\frac{1}{\eta_d} J} \mathbf{1}_{\{J < 0\}}$$
(2.13)

where  $1 > \eta_u > 0, \eta_d > 0$  are the averages of positive and negative jumps, respectively;  $p, q \ge 0, p + q = 1$ . p and q are probabilities of positive and negative jumps.  $\eta_u$  should be less than one so that  $E[J] < \infty$  and  $E[S] < \infty$ .

#### (c) Jump Diffusions with a Mixture of Independent Jumps

The jump-diffusion with a mixture of independent jumps has a probability density function given by:

$$\varpi(J) = \sum_{j=1}^{n} \Omega_j \varpi_j(J) \tag{2.14}$$

where  $\Omega_j$  is a weighting function,  $\sum_{j=1}^{n} \Omega_j = 1$ , and  $\varpi_j(J)$  is a probability density function corresponding to an individual jump size.

For the Bates model jumps can be drawn from either normal or double-exponential distribution (Becker [11](2009)).

#### **Option Pricing Problem Formulation** 2.5

The problem at hand is an option pricing problem. We need to price European style options under the assumption that the underlying asset dynamics are given by the Bates model ((2.10) - (2.11)). In the chapter which follows we explain how we can price the options using Fourier Transforms.



## Chapter 3

# Solution Of The Pricing Problem Using Fourier Transforms

This chapter looks at the solution to the pricing problem using the method of Fourier Transforms. We use different formulas under this method. They were discussed by Sepp [64](2003) and Becker [11](2009). In obtaining these formulas we begin by finding the price of European options using risk-neutral valuation. In order to evaluate some expressions obtained under this approach we use Complex Fourier Transforms. Before studying these formulas we state the partial integro-differential equation (PIDE) which is satisfied by European options in the Bates model.

## 3.1 The Partial Integro-Differential Equation Satisfied By European Options In The Bates Model

The asset dynamics for the Bates model are given as:

$$dS(t)/S(t) = (r - d - \lambda m) dt + \sqrt{V(t)} dW^{s}(t) + (e^{J} - 1) dN(t); \qquad (3.1)$$

$$dV(t) = \kappa \left(\theta - V(t)\right) dt + \epsilon \sqrt{V(t)} dW^{v}(t).$$
(3.2)

If we change the variable S to  $x = \ln S$  and t to  $\tau = T - t$  and use the Feynman-Kac theorem for the dynamics (3.1) to (3.2) the value of a European-style claim denoted by  $f(x, V, \lambda, \tau)$  satisfies the partial integro-differential equation:

$$-f_{\tau} + \left(r - d - \frac{1}{2}V - \lambda m\right)f_x + \frac{1}{2}Vf_{xx} + \kappa\left(\theta - V\right)f_V + \frac{1}{2}\epsilon^2 Vf_{VV} + \rho\epsilon Vf_{xV}$$

$$+\lambda \int_{-\infty}^{\infty} \left[ f(x+J) - f(x) \right] \varpi(J) dJ = rf; \quad f(x, V, \lambda, 0) = g(e^x, K) \tag{3.3}$$

where g is the payoff function assumed to be independent of the variance (Sepp [64](2003)).

## 3.2 Pricing European Options Using Fourier Transforms

In using the Fourier Transform method to price European options we start from the risk-neutral valuation formula. The payoff of a European call or put option at exercise time T with strike K is given by:

$$g(e^{x(T)}, K) = max \left\{ \varphi \left[ e^{x(T)} - K \right], 0 \right\}$$
(3.4)

(3.4) can also be written as:

$$\max\left\{\varphi\left[e^{x(T)}-K\right],0\right\} = \frac{1+\varphi}{2}e^{x(T)} + \frac{1-\varphi}{2}K - \min\left\{e^{x(T)},K\right\}$$
(3.5)

The last term of (3.5) is bounded in the interval  $0 \leq \min\{e^{x(T)}, K\} \leq K$ . It is more convenient to consider options with the bounded payoff function since they are easier to deal with under integration (Becker [11](2009)). Hence taking risk neutral expectations and discounting the expected payoff at the risk-free interest rate, we can represent the value of a call or put F(x, t) as follows, where  $\tau = T - t$ :

$$F(x(t),t) = \mathbf{E}^{\mathbf{Q}} \left[ e^{-(T-t)r} \max \left\{ \varphi \left[ e^{x(T)} - K \right], 0 \right\} \right]$$
  
=  $e^{-(T-t)r} \left[ \frac{1+\varphi}{2} \mathbf{E}^{\mathbf{Q}} [e^{x(T)}] + \frac{1-\varphi}{2} \mathbf{E}^{\mathbf{Q}} [K] - \mathbf{E}^{\mathbf{Q}} \left[ \min \left\{ e^{x\{T\}}, K \right\} \right] \right]$   
=  $\frac{1+\varphi}{2} e^{x(t)-\tau d} + \frac{1-\varphi}{2} e^{-\tau r} K - \mathbf{E}^{\mathbf{Q}} \left[ e^{-\tau r} \min \left\{ e^{x\{T\}}, K \right\} \right].$  (3.6)

$$= \frac{1+\varphi}{2}e^{x(t)-\tau d} + \frac{1-\varphi}{2}e^{-\tau r}K - f(x, V, \lambda, \tau).$$
(3.7)

Therefore we need to use the more convenient bounded form of the payoff function and calculate:

$$f(x, V, \lambda, \tau) = \mathbf{E}^{\mathbf{Q}} \left[ e^{-\tau r} \min \left\{ e^{x(T)}, K \right\} \right]$$
(3.8)

with the initial condition

$$f(x, V, \lambda, 0) = \min\left\{e^x, K\right\}$$

Using Feynman-Kac theorem this value of the option function  $f(x, V, \lambda, \tau)$  is the solution of the partial integro-differential equation (3.3) with the initial condition:

$$g(e^x, K) = f(x, V, \lambda, 0) = \min\{e^x, K\}$$
(3.9)

For a function f(x) its forward Fourier Transform is given by:

$$\boldsymbol{F}[f(x)](z) = \hat{f}(z) = v.p. \int_{-\infty}^{\infty} e^{izx} f(x) \, dx \tag{3.10}$$

The inverse Fourier Transform is given by:

$$f(x) = \boldsymbol{F}^{-1}[\hat{f}(z)] = \frac{1}{2\pi} v.p. \int_{iv-\infty}^{iv+\infty} e^{-izx} \hat{f}(z) dz$$
(3.11)

where  $\mathbf{i} = \sqrt{-1}$  and  $z \in \mathbf{C}, z = k + vi$  where the real part  $k = \Re z \in \mathbf{R}$ , the imaginary part  $v = \Im z \in \mathbf{R}$  is the transform variable. (3.10) may exist if  $\Re z$  is restricted to the strip of irregularity  $\alpha < \Re z < \beta$  for some option payoffs. Hence in using Fourier Transforms for option pricing we do the following (Sepp [64](2003)):

- find an explicit expression for the Fourier Transform.
- invert the result obtained with the z-plain integration whilst keeping  $\Im z$  within the strip of regularity.

This principle is used in finding option pricing formulas that are given in the subsections below. The complex Fourier Transform for the initial condition (3.9) is obtained by using integration as shown below. This result will be used in the subsections which follow.

$$\widehat{f}(x, v, \lambda, 0) = \int_{-\infty}^{\infty} e^{izx} \min\{e^{x}, K\} dx$$

$$= \lim_{U \to \infty} \int_{-U}^{\ln K} e^{izx} e^{x} dx + K \lim_{U \to \infty} \int_{lnK}^{U} e^{izx} dx$$

$$= \lim_{U \to \infty} \frac{e^{(iz+1)x}}{iz+1} \Big|_{x=-U}^{x=\ln K} + K \lim_{U \to \infty} \frac{e^{izx}}{iz} \Big|_{x=\ln K}^{x=U}$$

$$= \frac{e^{(iz+1)\ln K}}{iz+1} - 0 + K \left(0 - \frac{e^{iz\ln K}}{iz}\right)$$

$$= \frac{K^{iz+1}}{iz+1} - \frac{K^{iz+1}}{iz} = \frac{K^{iz+1}}{z^{2} - iz}$$
(3.12)

For the lower limit to exist the condition  $\Im z < 1$  should be satisfied and for the upper limit to exist  $\Im z > 0$  needs to be satisfied. Therefore the transform is defined for  $0 < \Im z < 1$  for the integrals to converge. We denote the strip of regularity as the payoff strip  $S_f$  (Sepp [64](2003)).

In the subsections which follow we present two formulas for pricing options.

#### 3.2.1 Method 1: The Characteristic Formula

If the characteristic function corresponding to the price dynamics is given in closed form, the characteristic function of  $x(t) = \ln S(t)$  is defined by

$$\phi_T(z) = \mathbf{E}^{\mathbf{Q}} \left[ e^{izx} \right] = \int_{-\infty}^{\infty} e^{izx} \varpi_T(x) dx \tag{3.13}$$

where  $\varpi_T(x)$  is the risk-neutral density of the logarithmic price x(t). A modified version of the formula proposed by Lewis [51](2001) is stated below for option pricing under general stochastic processes.

#### Theorem 1 (The Characteristic Formula)

We assume that x(t) has the analytic characteristic function  $\phi_T(z)$  with the strip of regularity  $S_z = \{z : \alpha < \Im z < \beta\}$ . Next we assume that  $e^{-vx}f(x) \in L^1(\mathbf{R})$ where v is located in the payoff strip  $S_f$  with transform  $\widehat{f}(z), \Im z \in S_f$ . Then, if  $S_F = S_f \cap S_z$  is not empty, the option value is given by:

$$f(x(t)) = \frac{e^{-r(T-t)}}{2\pi} \int_{iv-\infty}^{iv+\infty} \phi_T(-z) \,\widehat{f}(z) dz$$
(3.14)

where  $v = \Im z, z \in S_F = S_f \cap S_z$ .

#### Analysis of the Characteristic Formula

Let  $G(\Phi, x, V, \lambda, \tau)$  be the moment generating function. The relationship between the moment generating function and the complex valued characteristic function is given by:

$$\phi_T(z) = G(iz) \quad \Rightarrow \quad \phi_T(-z) = G(-iz)$$
(3.15)

Let the payoff function for the option whose value is given by (3.14) be represented by (3.9). Then we can re-write (3.14) as:

$$f(x,V,\lambda,\tau) = \frac{Ke^{-r\tau}}{2\pi} \int_{iv-\infty}^{iv+\infty} e^{iz\ln K} \frac{G(-iz,x,V,\lambda,\tau)}{z^2 - iz} dz$$
(3.16)

We evaluate (3.16) along a straight line v = 1/2 in the complex z-plain parallel to the real axis. By substituting  $z = k + i/2, k \in \mathbf{R}$ , into (3.16) we get:

$$f(x,V,\lambda,\tau) = \frac{Ke^{-r\tau}}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-(-ik+1/2)\ln K}G\left(-ik+\frac{1}{2},x,V,\lambda,\tau\right)}{k^2+1/4} dk.$$
 (3.17)

Let

$$Q(k, x, V, \lambda, \tau) = e^{-\tau r - (-ik + 1/2) \ln K} G(-ik + \frac{1}{2}, x, V, \lambda, \tau)$$
(3.18)

The integrand in (3.17) is a symmetric function and hence for option pricing we need to evaluate the expression:

$$f(x, V, \lambda, \tau) = \frac{K}{\pi} \int_0^\infty \Re\left[\frac{Q(k, x, V, \lambda, \tau)}{k^2 + 1/4}\right] dk$$
(3.19)

Having obtained the value of  $f(x, V, \lambda, \tau)$  it can be substituted in (3.7) to get the option prices. To find the expression for the moment generating function  $G(\Phi, x, V, \lambda, \tau)$  we make use of standard considerations used by Heston [42](1993). We let the moment generating function associated with the log of the terminal asset price  $x(\tau) = \ln S(\tau)$  under the measure **Q** be given by:

$$G(\Phi, x, V, \lambda, \tau) = \mathbf{E}^{\mathbf{Q}} \left[ e^{\Phi_{x(\tau)}} \right] = e^{-r\tau} \mathbf{E}^{\mathbf{Q}} \left[ e^{r\tau} e^{\Phi_{x(\tau)}} \right]$$
(3.20)

If we apply the Feynman-Kac theorem on the price dynamics (3.1) - (3.2),  $G(\Phi, x, V, \lambda, \tau)$  is the solution to:

$$-G_{\tau} + \left(r - d - \frac{1}{2}V - \lambda m\right)G_{x} + \frac{1}{2}VG_{xx} + \kappa\left(\theta - V\right)G_{V} + \frac{1}{2}\epsilon^{2}VG_{VV}$$
$$+\rho\epsilon VG_{xV} + \lambda\int_{-\infty}^{\infty}\left[G\left(x + J\right) - G\right]\varpi(J)dJ = 0$$
$$G(\Phi, x, V, \lambda, 0) = e^{\Phi x}.$$
(3.21)

To solve (3.21) we use the method of undetermined coefficients. The initial guess for our solution takes the form

$$G = e^{A(\tau) + B(\tau)V + C(\tau)\lambda}$$
(3.22)

Since this solution is an affine function these models are called affine jump diffusion. The complete solution is given by the proposition below (Sepp [64](2003)).

**Proposition 1** The solution to the partial integro-differential equation (3.22) is given by

$$G(\Phi, x, V, \lambda, \tau) = e^{x\Phi + (r-d)\tau\Phi + A(\Phi, \tau) + B(\Phi, \tau)V + C(\Phi, \tau) + D(\Phi, \tau)\lambda}$$
(3.23)

where

$$\begin{aligned} A(\Phi,\tau) &= -\frac{\kappa\theta}{\epsilon^2} \left[ \psi_+ \tau + 2\ln\left(\frac{\psi_- + \psi_+ e^{-\zeta\tau}}{2\zeta}\right) \right] \\ B(\Phi,\tau) &= -\left(\Phi - \Phi^2\right) \frac{1 - e^{-\zeta\tau}}{\psi_- + \psi_+ e^{-\zeta\tau}}, C = 0, D = 0 \\ \psi_+ &= -\left(\kappa - \rho\epsilon\Phi\right) + \zeta, \ \psi_- &= \left(\kappa - \rho\epsilon\Phi\right) + \zeta, \\ \zeta &= \sqrt{\left(\kappa - \rho\epsilon\Phi\right)^2 + \epsilon^2 \left(\Phi - \Phi^2\right)}, \\ \Lambda(\Phi) &= \int_{-\infty}^{\infty} e^{J\Phi} \varpi(J) dJ - 1 - m\Phi, \ m = \int_{-\infty}^{\infty} e^{J} \varpi(J) dJ - 1, \end{aligned}$$

In the previous sections we defined  $\Lambda(\Phi)$  to be the jump transform. Possible jump transforms for different jump types are given as (Sepp [64](2003)):

Log-normal price-jumps

$$\Lambda(\Phi) = e^{\nu \Phi + \delta^2 \Phi^2/2} - 1 - \Phi \left( e^{\nu + \delta^2/2} - 1 \right)$$
(3.24)

**Double-exponential jumps** 

$$\Lambda(\Phi) = \frac{p}{1 - \Phi\eta_u} + \frac{q}{1 + \Phi\eta_d} - 1 - \Phi\left(\frac{p}{1 - \eta_u} + \frac{q}{1 + \eta_d} - 1\right)$$
(3.25)

provided  $-1/\eta_d < \Im z < 1/\eta_u$ 

#### Mixture of independent jumps

$$\Lambda(\Phi) = \sum_{j=1}^{n} \Omega_j \Lambda_j(\Phi)$$
(3.26)

Now that we have an explicit expression for the moment generating function we use it in the pricing formulas. The general form of  $Q(k, x, V, \lambda, \tau)$  is found by substituting (3.23) in (3.18). It is given by (Sepp [64](2003)) as:

$$Q(k, x, V, \lambda, \tau) = e^{(-ik+1/2)X + A(k,\tau) + B(k,\tau)V + C(k,\tau) + D(k,\tau)\tau}$$
(3.27)

where  $X = \ln(S/K) + (r - d)\tau$  and the other coefficients are defined in Appendix A.1.

#### 3.2.2 Method 2: The Black-Scholes-style Formula

The Black-Scholes-style formula is the probabilistic version of the pricing formula. Heston [42](1993), Bates [10](1996), Sepp [64](2003), etc. studied this type of formula before. An analysis of this formula is given just below the theorem (Sepp [64](2003)).

#### Theorem 2 The Black-Scholes-style formula

We assume that the characteristic function  $\phi_T(z) = \mathbf{E}^q \left[e^{izx(T)}\right]$  corresponding to the market model is analytic and bounded in the strip  $0 \leq \Im z \leq 1$ . Two characteristics,  $\phi_j(k)$  (j = 1, 2),  $k \in \mathbf{R}$ , are given by  $\phi_1(k) = e^{-\ln S(t) - (r-d)(T-t)}\phi_T(k-i)$ and  $\phi_2(k) = \phi_T(k)$ . The CDF-s,  $\Pi_j$ , in the variable  $y = \ln K$  of the log-spot price  $x(t) = \ln S(t)$  are given by

$$\Pi_j = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \Re\left[\frac{\phi_j(k)e^{-iky}}{ik}\right] dk$$
(3.28)

and variables  $P_{j}(\varphi)$  are defined by

$$P_{j}\left(\varphi\right) = \frac{1-\varphi}{2} + \varphi\Pi_{j} \tag{3.29}$$

Then the current value of a European-style contingent claim, F(S,T), that pays off max { $\varphi[S_T - K], 0$ }, where the binary variable  $\varphi = +1$  for a call and  $\varphi = -1$  for a put, at time of the expiration date T has the form

$$F(S,T) = \varphi \left[ e^{-d(T-t)} SP_1(\varphi) - e^{-r(T-t)} KP_2(\varphi) \right]$$
(3.30)

#### Analysis of the Black-Scholes-style Formula

(3.28) already has its integral expressed in terms of the real valued transform parameter k. Hence in the analysis of this formula we just need to obtain the expression for the characteristics  $\phi_j(k)$ . The expression for the characteristic functions  $\phi_j(k)$ , (j = 1, 2) is obtained using the moment generating function given by (3.23) and this is given as follows:

$$\phi_j(k) = e^{ikX + A(k,\tau) + B(k,\tau)V + C(k,\tau) + D(k,\tau)\lambda}$$
(3.31)

The coefficients for (3.31) are given in Appendix A.2.

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Using Fourier Transforms to price options has quite a number of advantages. The use of the technique itself means that computations can be made quicker because the method is fast. The other advantage again is the fact that prices of options with a range of strikes and time periods can be computed all at the same time. These advantages contribute to its wide use nowadays since efficient use of time has become an important factor in most working environments.

## Chapter 4

# Model Calibration And Finding The Option Price Together With The Volatility

In this short chapter we look at model calibration using one of the option pricing formulas that were presented in the Chapter 3. Since in model calibration we seek to solve an optimisation problem which yields parameter values, we explain how to solve this optimisation problem using the R GUI language. We also describe how to obtain option prices using the parameters obtained after calibration. Using these option prices we then show how to get volatilities. In the last section we explore possible uses of the volatilities and the parameters that have been obtained.

### 4.1 Model Calibration

Model calibration consists of modifying the input parameters of the Bates model ((2.10)-(2.11)) in the option pricing formulas until the output from this model matches the observed set of data, that is, market data. Since options are traded using volatilities, we will use these volatilities as our data to a good approximation to calibrate the model. For model calibration, we need to solve the following minimisation problem in which we are minimising the sum of errors:

$$\min_{\Theta} \sum_{j=1}^{N} \left( \sigma_j^{market}(K,T) - \sigma_j^{model}(K,T;\Theta) \right)^2$$
(4.1)

where  $\sigma_j^{market}(K,T)$  is the market volatility that has strike K and maturity time T,  $\Theta$  is a vector of parameters that we are fitting and is given by:

$$\Theta = (V_0, \kappa, \theta, \epsilon, \rho, \lambda, \nu, \delta)$$
(4.2)

and  $\sigma_j^{model}(K, T; \Theta)$  is the model volatility with strike K, maturity time T and a vector of parameters defined by (4.2).

We begin by inputing starting values for the vector of parameters given by (4.2). Selection of these values will depend on the user's estimating capabilities after observing how the options market is performing. These estimated parameters, together with the underlying asset's price S, strike price K, risk-free interest rates r, the dividend rates d and the time durations T - t are substituted in the following formulas depending on the terms that make up the formula. We first substitute in (3.31) to obtain the two values for the characteristic function  $\phi_1$  and  $\phi_2$ . These two values obtained are each substituted in turn in (3.28) to get  $\pi_1$  and  $\pi_2$  respectively. Having obtained  $\pi_1$  and  $\pi_2$  we substitute them in (3.29) in turn to obtain  $P_1$  and  $P_2$  respectively.  $P_1$  and  $P_2$  are then substituted in (3.30) to obtain the model option price.

Having found the model option prices using the estimated vector of parameters, we find the model volatilities. To accomplish this we use a numerical procedure to invert Black's (1976) formula to find a formula for obtaining volatilities given the option price and the other parameters. Black's (1976) formula is given below as:

$$Price_{Call \ Option} = FN(d_1) - KN(d_2)$$

$$(4.3)$$

$$Price_{Put \ Option} = KN(-d_2) - FN(-d_1)$$

$$(4.4)$$

$$d_1 = \frac{ln\frac{F}{K} + \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}} \tag{4.5}$$

$$d_2 = \frac{ln\frac{F}{K} - \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}} \tag{4.6}$$

where F is the futures price, K is the strike price,  $\sigma$  is the volatility and  $\tau$  is the time to expiration. This formula applies in the case where the underlying asset is a future. These model volatilities obtained are used together with the market volatilities in the minimisation formula (4.1) above. On solving the minimisation problem above we obtain the parameters that we require.

## 4.2 Solving The Optimisation Problem Using R GUI Language

To solve the optimisation problem we use the R GUI (Graphical User Interface) language. This is one of the best languages for solving optimisation problems.

R has a combination of software facilities for manipulating data, calculation, graphical display, etc. Using this language one can also manage and analyse data in an efficient way. R has a command line interface which allows the user to have direct control on the calculations and hence it is flexible. It also has the ability to produce complex graphs and it has methods of displaying the data/results. (http://www.texniccenter.org/). All these factors make it a very convenient language for us to use in solving the problem at hand.

In minimising the function given in (4.1) several programming languages (e.g. Matlab, C++, Excel, etc) could have been used. One of the advantages with using the R GUI language is that it can perform integration directly. Since we have chosen Fourier Transforms for pricing our options we need to compute the Fourier integral. In using other languages one might need numerical recipes to perform the integration.

We optimise using PORT routines through the use of the *nlminb* function. This function requires us to make some specifications concerning some functions and some values. It takes the form:

nlminb(start, objective, gradient=NULL, hessian=NULL, ..., scale = 1, control = list(), lower = -Inf, upper = Inf)

The values that we will need to input and that are of interest to us are the starting parameter values denoted by *start*, the objective function to be minimised which is denoted by *objective* and the vectors of lower and upper bounds denoted by *lower* and *upper* which are supposed to be of the same length as the vector of input parameters.

If we input these values and run the program we obtain the vector of parameters  $V_0, \kappa, \theta, \epsilon, \rho, \lambda, \nu$  and  $\delta$ . These parameters are the parameters that minimise the square error function between the market volatilities and the model volatilities.

## 4.3 Finding The Price Of The Option And The Volatility

The parameters obtained from Section (4.1), together with the underlying asset's price S, strike price K, risk-free interest rates r, the dividend rates d and the time durations T-t are substituted back into the pricing formulas depending on the terms that make up the formula. We first substitute in (3.31) to obtain the two values for the characteristic function  $\phi_1$  and  $\phi_2$ . These two values obtained are each substituted in turn in (3.28) to get  $\pi_1$  and  $\pi_2$  respectively. Having obtained  $\pi_1$  and  $\pi_2$  we substitute them in (3.29) in turn to obtain  $P_1$ and  $P_2$  respectively.  $P_1$  and  $P_2$  are then substituted in (3.30) to obtain the model option price. Using these model option prices we find the model volatilities. We use a numerical procedure again to invert Black's (1976) formula to obtain volatilities given the option price and the other parameters. These implied volatilities can be compared to the market volatilities to allow us to observe how good our model is. The smaller the errors between the market volatilities and the model implied volatilities, the better the model is. The shapes of the market volatility surface and the model implied volatility surface can also be compared. A histogram of errors can also be obtained to help us get a visual idea about the errors between the market volatilities and the model implied volatilities.

### 4.4 Uses Of Parameters And The Volatility

Now that we have obtained parameter values through minimisation based on market data they can be used as starting parameter values for obtaining option prices for the future dates. If there are minor changes in the market movements these parameters can be modified accordingly.

The volatility surface obtained can be used for a number of purposes. It can be used to price European put and call options within a specified range of strikes and maturities (Badshah [6](2008)). We can also price and hedge exotic options (Badshah [6](2008), Detlefsen [31](2005)). In some market environments margin requirements are calculated using volatility surfaces (West [72](2005), Kotze and Joseph [49](2009)).

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## Chapter 5

# **Empirical Study**

For this project our empirical study focuses on the South African market. In this chapter we will apply all that we have deliberated upon in the previous chapters to this market. As we begin we have an overview of the market first. We then present the problem formulation in the section which follows. In solving this problem we make use of traded options data collected from SAFEX website (www.safex.co.za). We also examine this data and we make graphical representation on it to help us visualise certain concepts. The data is used to calibrate our model using the optimisation method in Chapter 4. Having obtained the results (parameter values) we use them to price options. From these option prices we get implied volatilities by inverting the Black's (1976) formula. These model volatilities are compared to the market volatilities to determine the model errors. From these differences we are able to assess how good the model is.

## 5.1 The South African Equity Futures Derivatives Market

The South African Futures Exchange (SAFEX) equity derivatives market is the Johannesburg Stock Exchange (JSE)'s financial futures and options market. Two option types that are traded on the JSE's Equity Derivatives Market are put option and call option. This derivatives market only supports the American style options.

The underlying asset on which options trading is taking place is the equity futures contract. Hence the options are also called Future Style Options. A futures contract is an agreement between two parties to buy an underlying asset at a fixed date in the future at a price agreed now. Three main types of futures traded on the JSE on which options are based are:

i) Single stock futures options

These are future's contracts on individual stocks. The contracts are standardised, that is, they have certain specifications for the size, expiry date and tick movements. The value of a single stock futures contract is equal to 100 times the particular share's future price. Hence they are usually based on 100 underlying shares.

ii) Index future options

These are based on a basket of shares.

iii) Can do options

These are customised baskets or non-standardised options.

Few trades take place in this market compared to other markets hence it is a very illiquid market. Options on ALSI index futures contracts trade in large volumes compared to other futures options on the SAFEX market, hence we will focus on the ALSI index futures contracts and its options. SAFEX options are quoted using implied volatility. In order to obtain the price of these options Black's (1976) formula is used.

### 5.2 Problem Formulation

Researchers take time to find good models for volatility surfaces because of their usefulness. Below we mention some of the uses of volatility surfaces:

- They are used to price European put and call options within a specified range of strikes and maturities. Both liquid and illiquid options can be priced using implied volatility surface (West [72](2005), Badshah [6](2008)).
- They can be used to price and hedge exotic options (Badshah [6](2008), Detlefsen [31](2005)). Exotic options have payoffs which depend on the path of the underlying asset and as a result, the rules used to determine the payoffs are more complex in comparison to the ones used for standard options (Shreve [66](2004), Hull [44](2009)).
- In some markets margin requirements are calculated based on the volatility surface (West [72](2005), Kotze and Joseph [49](2009)).
- These volatility models also give the opportunity to compute Value at Risk (Var) for portfolios whose returns depend on particular option/index whose volatility surface has been obtained (Cassese and Guidolin [19](2006)).
- The availability of the volatility surface helps to decide on the choice of portfolios (Cassese and Guidolin [19](2006)). Barberis [8](2000) and Campbell & Viceira [16](2003) investigated on choosing optimal portfolios when excess stock returns dynamics are given by a stochastic process.

Of interest to us is the use of the volatility surface at the JSE. The JSE has provided useful information for our study but even more information can be obtained form their website www.safex.co.za. Kotze and Joseph [49](2009) and West [72](2005) provide clearer insights on this area too. At the JSE the option price is not paid at once but is paid in installments each day. This is done as a way of managing risk associated with trading futures options in the exchange and this is called margining. This implies that derivative traders must pay the loss amount or receive the amount gained on each day during the term of the contract.

To further reduce risk the JSE uses the services offered by clearing houses. When two parties would have made a trade on the exchange the clearing house then takes over the trade and comes in between the two parties' clearing firms. By doing so it takes the full legal responsibility of any risks that might be involved in these trades. The process of passing the trade title to the clearing house is called Novation. JSE's futures clearing house is called SAFCOM.

Two main types of margin in option trading at the JSE are:

1) Initial Margin

Initial margin is the amount of money that is required to open a buy or sell position on a futures options contract. Participants will earn a competitive interest rate on it and it will be returned upon the expiry or closure of the contract (www.safex.co.za).

#### 2) Premium Variation Margin •

These are the cash flow premiums paid or received by the counterparties to the option transaction each day. Each night the JSE calculates the value of the position using the Black's (1976) pricing formula. This is an estimate of what the position is worth every day. The counter parties will therefore either pay or receive the premium variation margin (www.safex.co.za).

Two stages in the estimation of possible future losses and initial margin requirements (Kotze and Joseph [49](2009)) are:

- In the first stage the JSE performs a statistical analysis on the way the market has been performing in the past and subjective assessment of the state of the market. They express the maximum anticipated price and volatility moves between the present and the next mark-to-market day.
- In the second stage the exchange re-values each position at this maximum anticipated price and volatility at the next mark-to-market day. The margin covers this maximum conceivable mark-to-market loss that the position could suffer.

Hence the initial margin requirements for options are directly linked to the volatility surface. This is where our problem formulation stems from. We use a

mathematical model to come up with a market volatility surface that will lead to initial margin that indicates the current risks of the market. This volatility surface can also be used for many other purposes like the ones we briefly mentioned when we began this chapter.

## 5.3 Volatility Models On SAFEX And The Motivation In Using The Bates Model

In this section we begin by describing the different methods that have been used so far by SAFEX to model volatility, based on literature that we found. The first model that we reflect on is the SABR model. We also study briefly the polling method that was used to determine the volatility surface. Currently the deterministic model is being used and it is also studied in another subsection. We then discuss the motivation for modeling the South African ALSI volatility surface using the Bates model approach.

#### 5.3.1 SABR Model

West [72](2005) looked at the SABR model and considered the South African Market in his analysis. Prior to that the JSE was using flat volatility up until April 2001. It was then that a skew was introduced into the mark to market and margining of exchange positions. The process of constructing the skew was supposed to be done through auctioning but it ended up being just a poll.

#### 5.3.2 Polling Method

On using the SABR model in the JSE they found that the SABR model gave too flat a skew compared to the market skew. Hence they used the method of polling for the volatility surface. This method involved contacting market participants and requesting their volatility surfaces. A weighted average of the contributed volatility surfaces was then obtained. The polled volatility surface was used to inform the shape of the skew. A four factor model was used for the skew and they would then back out the SABR parameters required to construct the surface, for the benefit of those who already used the SABR model (www.safex.ac.za).

#### 5.3.3 Deterministic Model

From the 8th of October 2009 SAFEX started using an ALSI volatility surface obtained using a deterministic model. In modeling the volatilities, traded data was used and the calibration was done by the exchange. This was done because the modeled surface would truly indicate the state of the market and since this is a mathematical model, the surface could be updated on a frequent basis. Kotze and Joseph [49](2009) show how to generate the implied volatility surface by fitting a quadratic deterministic function to implied volatility data from ALSI index options.

#### 5.3.4 The Motivation In Using The Bates Model Approach

We make an investigation into modeling the South African ALSI volatility surface using the Bates model. The use of the Bates model is motivated by the stochastic nature of volatility in the South African market and the fact that in any market, including the South African market jumps do occur. We investigate and see how close the model volatilities (using the Bates model) are to the market volatilities.

### 5.4 The Market Data

Data on daily trades for futures and their derivatives is kept on the SAFEX website. On observing the derivatives data on the 25th of November 2009 one would note the following. Most data sets for the other dates seem to take a similar format.

- The market is illiquid compared to other markets, that is, few trades occur for most options traded.
- Trades could be done on index futures and its options, can do futures and its options, single stock futures and its options, dividend futures options and its options, international derivatives futures and its options, international derivatives dividend futures and its options.
- The ALSI index futures options are the most liquid followed by DTOP. Hence we will focus on the ALSI derivative securitiues.
- Trades occur for both puts and call options and the options are American options. However, we will only use call options data.
- Expiries are for December 2009, March 2010, June 2010, September 2010, December 2010, March 2011 and December 2011. For the later expiries fewer trades occur. The December 2009, March 2010 and June 2010 are the most liquid.
- For each option type we have data on the futures price, the initial date, the options and futures expiration month, the style of the option (whether it is a put or a call option), the M-t-M, the volatility at which the option is trading, the number of contracts traded, the bid and the offer prices for both the futures and the options, the first, last, high and low option's future and option's price, etc. For the purposes of this analysis we make use of data on the future's price, type of option, strike price, expiry date, option strike price and volatility.
- Since these types of options have the futures as the underlying the risk-free rate of interest and the dividend rate are already included in the futures price. Hence these will equate to zero for our analysis.
- Due to some constraints, not all data will be used in our analysis. Thus in screening our data an analysis was made and is described as below.

#### 5.4.1 Data Screening

The JSE does not set strike prices and hence making allowance for more flexibility and more efficient combination positions. However strike prices for index options are set to a 50 point strike interval. As a result one would realise that the data does not have the same strikes for all the expirations. Because of this an analysis was made so as to determine how the inclusion of more strikes will affect the model results under similar conditions.

We started by the least number of strikes. The few strikes meant almost all different expirations could be included in the data set. We carried on the exercise until we had most strikes but only three expirations, that is, December 2009, March 2010 and June 2010. In the few cases where a certain expiration did not have a particular strike we used linear interpolation to find the required volatility for that strike. At the end of this analysis we realised that under the same conditions, the model which made use of the data set with most strikes mimics the market model volatilities more than in using any other dataset. Hence for our analysis we will make use of the data that contains most strikes. This data is given in Table 5.1. It shows the different volatilities given the strike price, expiry date, risk-free interest rates and the dividend rate. The volatility surface obtained from using this data is given in Figure 5.1 below and this is the ALSI volatility surface on the 25th of November 2009.

## 5.5 Model Calibration Using ALSI Futures Derivatives Data

In calibrating our model we solve the minimisation problem given in (4.1). Our market volatilities are given in Table 5.1. The model volatilities are obtained from the option prices obtained by using the Bates model. We solve for the parameters  $V_0, \kappa, \theta, \epsilon, \rho, \lambda, \nu, \delta$  through minimising the sum of squared errors.

For minimisation we use the *nmlib* function found in R GUI. We have to specify the initial values for the parameters we need to solve for, together with the upper and the lower bounds. We also need to specify the minimisation function and this is given by (4.1). Part of the R GUI program used was a modification of the program given by J. Gatheral, the author of Gatheral [39](2004) and Gatheral [40](2006). The code for option pricing and minimisation problem using R GUI is given in the appendix. The results are discussed in the section that follows.

### 5.6 Empirical Results

The initial parameters that were used together with the lower bound and upper bound values are given in Table 5.2. These are estimated values. The results that were obtained as a result of minimisation in R GUI are given in Table 5.3.

| nitial Date | 555       | 25-Nov-09    |           |
|-------------|-----------|--------------|-----------|
| Expiry      | 17-Dec-09 | 18-Mar-10    | 17-Jun-10 |
| T           | 0.06027   | 0.30959      | 0.55890   |
| r           | 0.00000   | 0.00000      | 0.00000   |
| d           | 0.00000   | 0.00000      | 0.00000   |
| Strike      |           | Volatilities |           |
| 16000       | 0.3437    | 0.3497       | 0.3516    |
| 16150       | 0.3413    | 0.3474       | 0.3493    |
| 19200       | 0.2920    | 0.3004       | 0.3040    |
| 19950       | 0.2798    | 0.2888       | 0.2928    |
| 20000       | 0.2790    | 0.2880       | 0.2920    |
| 21000       | 0.2660    | 0.2741       | 0.2783    |
| 22000       | 0.2532    | 0.2609       | 0.2653    |
| 23000       | 0.2414    | 0.2479       | 0.2524    |
| 24000       | 0.2313    | 0.2363       | 0.2408    |
| 25000       | 0.2225    | 0.2257       | 0.2298    |
| 25900       | 0.2161    | 0.2168       | 0.2206    |
| 26000       | 0.2154    | 0.2158       | 0.2196    |
| 26400       | 0.2127    | 0.2120       | 0.2157    |
| 26500       | 0.2122    | 0.2111       | 0.2147    |
| 27000       | 0.2094    | 0.2068       | 0.2100    |
| 28000       | 0.2050    | 0.1986       | 0.2011    |
| 28350       | 0.2040    | 0.1962       | 0.1984    |

Table 5.1: Implied volatility matrix of ALSI futures call options of 25 November 2009 used in calibrating our model. The futures price is 24723.



Figure 5.1: The ALSI volatility surface on the 25th of November 2009.

However the values that we are going to use for further computations are the parameter values. These were used in the option pricing formulas to find the Bates model option prices. These option prices are given by Table 5.4. Having found the option prices we used a numerical procedure to invert Black's (1976) formula to find the Bates model implied volatilities. These volatilities obtained are shown in Table 5.5. The Bates model implied volatility surface is given in Figure 5.2. By observing the table of values we note that the volatilities on the extremes are zeros and this explains the shape of the volatility surface in Figure 5.2. The inside part of the volatility surface seems to be similar in shape to Figure 5.1.

In determining how good our model is, we need to compare the volatilities obtained using the Bates model in Table 5.5 with the ALSI market volatilities in Table 5.1. This comparison can be made by finding percentage differences between the market volatilities and the Bates model volatilities. The differences are shown in Table 5.6. Most of the errors were reasonably small as evidenced by the fact that 75% of the percentage differences were less than 40%, the least percentage difference being 0.01% and the greatest percentage difference being being 35.68%.

Having obtained this range of errors, we selected the volatilities with percentage errors that ranged from 5.366% to 25.987%. For these percentage errors the strike range is between 25000 and 27000. In selecting the values we included all the dates that were used for the analysis and tried to select an acceptable range of errors. Selection of the values also depends on the degree of accuracy that one is willing to accept. The selected volatilities were used to construct the Bates model volatility surface. Figure 5.3 shows the ALSI market volatility surface within the selected range. Figure 5.4 shows the Bates model volatilities within the same range.

On looking at the two volatility surfaces they seem to be similar, with few differences. One will note that most of the errors obtained for the Bates model volatilities for the data on the 17th of December 2009 are negative as shown in Table 5.5. Although the percentage errors are small in magnitude, the fact that the values are negative helps to explain the difference in the shape of the graphs in Figure 5.3 and Figure 5.4. The errors obtained for the Bates model volatilities for the data on the 18th of March and on the 17th of June exhibit very small differences as shown again in Table 5.6. The errors obtained for the data. A quantitative comparison is also given in the summary of the results for the volatilities in Table 5.7.

On observing Figure 5.1 and Figure 5.3 we note that the volatilities seem to almost follow a linear trend. This suggest that we can use linear interpolation to reproduce some values on the ALSI market data. This was done on the original data and we realised that most of the volatilities could be reproduced using linear interpolation, some of them with zero error. Hence similarly with our results, we can use linear interpolation to reproduce the rest of the volatilities that we require within the range of the original strikes data. The histogram for all the errors is shown in Figure 5.5. This helps us to visualise the errors obtained as a result of using the Bates model for option pricing. Some differences are negative and some are positive. One will also note that the largest differences are on the extreme ends. Most of the errors are observed to be quite small.

### 5.7 Conclusion

Based on this empirical study we deduce that the Bates model is a good model for modeling the South African ALSI volatility surface. We also note that this method models well volatilities with short maturities as indicated by the results obtained. In the problem formulation we stated that we are using a mathematical model to come up with a market volatility surface that will lead to initial margin that indicates the current risks of the market. We conclude that this volatility surface can be used by the clearing houses as its errors are small compared to the market volatility and hence it indicates the current market risks.

Since we have been able to find parameter values through minimisation, they can also be used as future starting parameter values for obtaining option prices. If there are minor changes in the market movements these parameter values can be modified accordingly. From these option prices, volatility surfaces can be obtained.

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| Parameter Symbol | Initial Parameter Values | Lower Bound | Upper Bound |
|------------------|--------------------------|-------------|-------------|
| V <sub>0</sub>   | 0.2                      | 0.1         | 0.3         |
| к                | 0.05                     | 1           | 10          |
| θ                | 0.09                     | 0.015       | 0.1         |
| ε                | 1.5                      | 0.5         | 2           |
| ρ                | -0.9                     | -0.9        | -0.5        |
| λ                | 1                        | 0.01        | 2           |
| v                | -0.2                     | -0.3        | -0.1        |
| δ                | 0.3                      | 0.1         | 0.5         |
|                  |                          | ·           |             |

Table 5.2: The initial parameters used together with the upper and the lower bounds.



| \$par                      |           |           |           |            |           |            |           |
|----------------------------|-----------|-----------|-----------|------------|-----------|------------|-----------|
| vzero                      | kappa     | theta     | epsilon   | rho        | lambda    | nu         | delta     |
| 0.1000000                  | 9.7836472 | 0.0150000 | 1.5678556 | -0.5000497 | 1.5666190 | -0.1000000 | 0.1894760 |
| \$objective                |           |           |           |            |           |            |           |
| [1] 2.645315               | 5         |           |           |            |           |            |           |
| \$iteration                | IS        |           |           |            |           |            |           |
| [1] 85                     |           |           |           |            |           |            |           |
| \$ <mark>evaluat</mark> io | ons       |           |           |            |           |            |           |
| function g                 | radient   |           |           |            |           |            |           |
| 200 9                      | 60        |           |           |            |           |            |           |
|                            |           |           |           |            |           |            |           |
|                            |           |           |           |            |           |            |           |
|                            |           |           |           |            |           |            |           |
|                            |           |           |           |            |           |            |           |

Table 5.3: The minimisation results for the parameters.

| Initial Date |            | 25-Nov-09           |                 |
|--------------|------------|---------------------|-----------------|
| Expiry       | 17-Dec-09  | 18-Mar-10           | 17-Jun-10       |
| Т            | 0.06027    | 0.30959             | 0.55890         |
| r            | 0.00000    | 0.00000             | 0.00000         |
| d            | 0.00000    | 0.00000             | 0.00000         |
| Strikes      | Option Pr  | ices Obtained Using | the Bates Model |
| 1600         | 8 715.8535 | 8 717.2823          | 8 719.2276      |
| 1615         | 8 565.4519 | 8 567.7892          | 8 570.7525      |
| 1920         | 5 523.0000 | 5 597.3198          | 5 649.0284      |
| 1995         | 4 786.3966 | 4 902.4633          | 4 975.4214      |
| 2000         | 4 737.5892 | 4 856.8720          | 4 931.3838      |
| 2100         | 3 772.1149 | 3 967.2236          | 4 076.3418      |
| 2200         | 2 836.2109 | 3 126.4781          | 3 276.2934      |
| 2300         | 1 952.3976 | 2 346.2872          | 2 540.0539      |
| 2400         | 1 164.9610 | 1 643.9618          | 1 877.3105      |
| 2500         | 552.8509   | 1 044.6949          | 1 299.2537      |
| 2590         | 217.1237   | 616.3963            | 860.9746        |
| 2600         | 192.6079   | 576.0058            | 817.4017        |
| 2640         | 115.8410   | 429.2314            | 653.6728        |
| 2650         | 101.2873   | 396.2225            | 615.4002        |
| 2700         | 49.1976    | 252.7346            | 440.0528        |
| 2800         | 3.4920     | 63.2291             | 167.7812        |
| 2835         | -3.9850    | 22.2387             | 95.8089         |

Table 5.4: The Bates model option prices on the 25th of November 2009.

| Initial Date |           | 25-Nov-09           |           |
|--------------|-----------|---------------------|-----------|
| Expiry       | 17-Dec-09 | 18-Mar-10           | 17-Jun-10 |
| Т            | 0.06027   | 0.30959             | 0.55890   |
| r            | 0.00000   | 0.00000             | 0.00000   |
| d            | 0.00000   | 0.00000             | 0.00000   |
| Strikes      | Bat       | es Model Volatiliti | 25        |
| 16000        | 0.00000   | 0.00000             | 0.0000    |
| 16150        | 0.00000   | 0.00000             | 0.00000   |
| 19200        | 0.00000   | 0.27980             | 0.23398   |
| 19950        | 0.41065   | 0.27803             | 0.23285   |
| 20000        | 0.41156   | 0.27781             | 0.23272   |
| 21000        | 0.40748   | 0.27121             | 0.22882   |
| 22000        | 0.38326   | 0.26087             | 0.22278   |
| 23000        | 0.35026   | 0.24730             | 0.21480   |
| 24000        | 0.31384   | 0.23120             | 0.20500   |
| 25000        | 0.28032   | 0.21359             | 0.19350   |
| 25900        | 0.25863   | 0.19712             | 0.18162   |
| 26000        | 0.25687   | 0.19526             | 0.18020   |
| 26400        | 0.25113   | 0.18769             | 0.17425   |
| 26500        | 0.24996   | 0.18575             | 0.17269   |
| 27000        | 0.24487   | 0.17565             | 0.16434   |
| 28000        | 0.20971   | 0.14876             | 0.14255   |
| 28350        | 0.00000   | 0.13067             | 0.13112   |

Table 5.5: The Bates model volatilities on the 25th of November 2009.

| Initial Date | 25-Nov-09                                      |              |            |              |                      |              |  |  |  |  |
|--------------|--|--------------|------------|--------------|----------------------|--------------|--|--|--|--|
| Expiry       | 17-0   | )ec-09       | 18-N       | /lar-10      | 17-Jun-10<br>0.55890 |              |  |  |  |  |
| т            | 0.0  | 6027         | 0.3        | 0959         |                      |              |  |  |  |  |
| r            | 0.0  | 0000         | 0.0        | 0000         | 0.0                  | 0000         |  |  |  |  |
| d            | 0.0  | 0000         | 0.0        | 0000         | 0.0                  | 0000         |  |  |  |  |
| Strikes      | Market Volatilities - Bates Model Volatilities |              |            |              |                      |              |  |  |  |  |
|              | Difference                                     | % Difference | Difference | % Difference | Difference           | % Difference |  |  |  |  |
| 16000        | 0.343700                                       | 100.000%     | 0.349700   | 100.000%     | 0.351600             | 100.000%     |  |  |  |  |
| 16150        | 0.341280                                       | 100.000%     | 0.347390   | 100.000%     | 0.349300             | 100.000%     |  |  |  |  |
| 19200        | 0.291970                                       | 100.000%     | 0.020550   | 6.842%       | 0.070015             | 23.031%      |  |  |  |  |
| 19950        | -0.130849                                      | -46.765%     | 0.010773   | 3.730%       | 0.059948             | 20.474%      |  |  |  |  |
| 20000        | -0.132563                                      | -47.514%     | 0.010192   | 3.539%       | 0.059281             | 20.302%      |  |  |  |  |
| 21000        | -0.141476                                      | -53.186%     | 0.002888   | 1.054%       | 0.049479             | 17.779%      |  |  |  |  |
| 22000        | -0.130062                                      | -51.367%     | 0.000030   | 0.011%       | 0.042519             | 16.0279      |  |  |  |  |
| 23000        | -0.108856                                      | -45.093%     | 0.000602   | 0.243%       | 0.037601             | 14.8989      |  |  |  |  |
| 24000        | -0.082539                                      | -35.685%     | 0.005096   | 2.157%       | 0.035796             | 14.8669      |  |  |  |  |
| 25000        | -0.057821                                      | -25.987%     | 0.012110   | 5.366%       | 0.036303             | 15.797%      |  |  |  |  |
| 25900        | -0.042518                                      | -19.674%     | 0.019642   | 9.062%       | 0.038979             | 17.670%      |  |  |  |  |
| 26000        | -0.041475                                      | -19.255%     | 0.020545   | 9.520%       | 0.039402             | 17.943%      |  |  |  |  |
| 26400        | -0.038387                                      | -18.044%     | 0.024313   | 11.468%      | 0.041450             | 19.2179      |  |  |  |  |
| 26500        | -0.037793                                      | -17.813%     | 0.025376   | 12.019%      | 0.042009             | 19.566%      |  |  |  |  |
| 27000        | -0.035470                                      | -16.939%     | 0.031149   | 15.062%      | 0.045664             | 21.7459      |  |  |  |  |
| 28000        | -0.004706                                      | -2.296%      | 0.049843   | 25.097%      | 0.058546             | 29.113%      |  |  |  |  |
| 28350        | 0.203990                                       | 100.000%     | 0.065536   | 33.401%      | 0.067281             | 33.912%      |  |  |  |  |

Table 5.6: Market Volatilities and Bates Model Volatilities differences on the 25th of November 2009.



Figure 5.2: The Bates model volatility surface on the 25th of November 2009.



Figure 5.3: The ALSI volatility surface for certain strike range on the 25th of November 2009.

0.28 0.26 0.24 0.22 0.20 0.20 0.18 25000 25500 26000 26500 0.1 27000

Bates Model Volatility Surface Using ALSI Data For Selected Strikes

Figure 5.4: The ALSI volatility surface for certain strike range on the 25th of November 2009.



Figure 5.5: The Bates model histogram for errors on the 25th of November 2009.

| ŀ | 4 |
|---|---|
| ¢ | 0 |

| initial Date |                               |                  |           |         |                   | 25-Nov-09        |          |          |                   |                  |          |         |
|--------------|-------------------------------|------------------|-----------|---------|-------------------|------------------|----------|----------|-------------------|------------------|----------|---------|
| Expiry       | xpiry 17-Dec-09               |                  |           |         |                   | 18-Mar-10        |          |          |                   | 17-Jun-10        |          |         |
| Г            | 0.06027<br>0.00000<br>0.00000 |                  |           |         | 0.30959           |                  |          | 9)<br>19 | 0.55890           |                  |          |         |
| r            |                               |                  |           |         | 0.00000           |                  |          |          | 0.00000           |                  |          |         |
| d            |                               |                  |           | 0.00000 |                   |                  |          | 0.00000  |                   |                  |          |         |
| Strikes      | Market Volatility             | Model Volatility | Error     | % Error | Market Volatility | Model Volatility | Error    | % Error  | Market Volatility | Model Volatility | Error    | % Error |
| 16000        | 0.343700                      | 0.000000         | 0.343700  | 100.00% | 0.349700          | 0.000000         | 0.349700 | 100.00%  | 0.351600          | 0.000000         | 0.351600 | 100.00  |
| 16150        | 0.341280                      | 0.000000         | 0.341280  | 100.00% | 0.347390          | 0.000000         | 0.347390 | 100.00%  | 0.349300          | 0.000000         | 0.349300 | 100.00  |
| 19200        | 0.291970                      | 0.000000         | 0.291970  | 100.00% | 0.300350          | 0.279800         | 0.020550 | 6.84%    | 0.304000          | 0.233985         | 0.070015 | 23.03   |
| 19950        | 0.279800                      | 0.410649         | -0.130849 | -46.77% | 0.288800          | 0.278027         | 0.010773 | 3.73%    | 0.292800          | 0.232852         | 0.059949 | 20.47   |
| 20000        | 0.279000                      | 0.411564         | -0.132564 | -47.51% | 0.288000          | 0.277808         | 0.010192 | 3.54%    | 0.292000          | 0.232719         | 0.059281 | 20.30   |
| 21000        | 0.266000                      | 0.407476         | -0.141476 | -53.19% | 0.274100          | 0.271212         | 0.002888 | 1.05%    | 0.278300          | 0.228821         | 0.049479 | 17.78   |
| 22000        | 0.253200                      | 0.383262         | -0.130062 | -51.37% | 0.260900          | 0.260870         | 0.000030 | 0.01%    | 0.265300          | 0.222781         | 0.042519 | 16.03   |
| 23000        | 0.241400                      | 0.350256         | -0.108856 | -45.09% | 0.247900          | 0.247298         | 0.000602 | 0.24%    | 0.252400          | 0.214799         | 0.037601 | 14.90   |
| 24000        | 0.231300                      | 0.313839         | -0.082539 | -35.68% | 0.236300          | 0.231204         | 0.005096 | 2.16%    | 0.240800          | 0.205004         | 0.035796 | 14.87   |
| 25000        | 0.222500                      | 0.280321         | -0.057821 | -25.99% | 0.225700          | 0.213590         | 0.012110 | 5.37%    | 0.229800          | 0.193497         | 0.036303 | 15.80   |
| 25900        | 0.216110                      | 0.258628         | -0.042518 | -19.67% | 0.216760          | 0.197118         | 0.019642 | 9.06%    | 0.220600          | 0.181621         | 0.038979 | 17.67   |
| 26000        | 0.215400                      | 0.256875         | -0.041475 | -19.25% | 0.215800          | 0.195255         | 0.020545 | 9.52%    | 0.219600          | 0.180198         | 0.039402 | 17.94   |
| 26400        | 0.212740                      | 0.251127         | -0.038387 | -18.04% | 0.212000          | 0.187687         | 0.024313 | 11.47%   | 0.215700          | 0.174250         | 0.041450 | 19.22   |
| 26500        | 0.212170                      | 0.249963         | -0.037793 | -17.81% | 0.211130          | 0.185755         | 0.025376 | 12.02%   | 0.214700          | 0.172691         | 0.042009 | 19.57   |
| 27000        | 0.209400                      | 0.244870         | -0.035470 | -16.94% | 0.206800          | 0.175651         | 0.031149 | 15.06%   | 0.210000          | 0.164336         | 0.045664 | 21.74   |
| 28000        | 0.205000                      | 0.209706         | -0.004706 | -2.30%  | 0.198600          | 0.148757         | 0.049843 | 25.10%   | 0.201100          | 0.142554         | 0.058546 | 29.11   |
| 28350        | 0.203990                      | 0.000000         | 0.203990  | 100.00% | 0.196210          | 0.130675         | 0.065536 | 33.40%   | 0.198400          | 0.131119         | 0.067281 | 33.91   |

Table 5.7: Summary of the model and market volatilities together with the errors.

## Chapter 6

## Summary And Conclusions

### 6.1 Summary

We have been able to demonstrate how Fourier Transforms can be used to price options in the South African market, assuming that the price process follows the dynamics given by the Bates model. From these option prices we obtained the implied volatilities by using a numerical procedure to invert Black's (1976) formula. Most of the volatilities obtained seem to be similar to the ALSI volatilities. It is only the strikes on both extremes of the strike range that seem to have volatilities that differ slightly from the ALSI surface. As a result we decided to use volatilities with errors less than 26%. By using the method of linear interpolation/extrapolation we can obtain the volatilities that we require which are within the range of the strikes that were used as the original market data.

We used the R GUI language in obtaining our results and it has proved to be a very good program. We have been able to do our computations which consist of evaluating integrals without any intensive efforts required. The optimisation function however took some few minutes executing its commands before we could get the solution whilst the rest of the commands were giving solutions instantly like any other program. We have found R GUI to be such an invaluable tool that we recommend that it be used for similar projects.

### 6.2 Conclusion

We have been able to model well volatilities with short maturities. These have more trades done than in longer maturities. Since the methods for modeling volatility in the South African market discussed so far in literature do not model appropriately shorter maturities we recommend modeling the volatilities using this method. This volatility surface will lead to initial margin that indicates the current risks of the market as required by SAFEX. We conclude that this methodology is another good approach that could be used in the South African market to obtain the ALSI volatility surface. For future work, more research needs to be done using this model for the remaining maturities in this illiquid market.

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## Appendix A

# **Pricing Formulas**

## A.1 The Coefficients For (3.27)

1) Volatility

• Stochastic  $\begin{aligned} A\left(k,\tau\right) &= -\frac{\kappa\theta}{\epsilon^2} \left[ \psi_+\tau + 2\ln\left(\frac{\psi_- + \psi_+ e^{-\zeta\tau}}{2\zeta}\right) \right], B\left(k,\tau\right) = -\left(k^2 + 1/4\right) \frac{1 - e^{-\zeta\tau}}{\psi_- + \psi_+ e^{-\zeta\tau}}, \\ \psi_+ &= -\left(u + ik\rho\epsilon\right) + \zeta, \ \psi_- = \left(u + ik\rho\epsilon\right) + \zeta \\ \zeta &= \sqrt{k^2\epsilon^2 \left(1 - \rho^2\right) + 2ik\rho\epsilon u + u^2 + \epsilon^2/4}, u = \kappa - \rho\epsilon/2. \end{aligned}$ 

e c

### 2) Jump Rate Intensity

- Constant  $C(k,\tau) \equiv 0, D(k,\tau) = \tau \Lambda(k).$
- 3) Jump Size Distribution
  - Log-Normal  $\Lambda(k) = e^{-ik\left(\nu+\delta^2/2\right) - \left(k^2 - 1/4\right)\delta^2/2 + \nu/2} - 1 - \left(-ik + 1/2\right)\left(e^{\nu+\delta^2/2} - 1\right).$

#### The Coefficients For (3.31) A.2

Variables u, I, b are given as:

if 
$$j = 1$$
:  $u = 1$ ,  $I = 1$ ,  $b = \kappa - \rho \epsilon$ ; if  $j = 2$ :  $u = -1$ ,  $I = 0$ ,  $b = \kappa$ 

#### 1) Volatility

• Stochastic  

$$A(k,\tau) = -\frac{\kappa\theta}{\epsilon^2} \left[ \psi_+ \tau + 2\ln\left(\frac{\psi_- + \psi_+ e^{-\zeta\tau}}{2\zeta}\right) \right], \quad B(k,\tau) = -\left(k^2 - uik\right) \frac{1 - e^{-\zeta\tau}}{\psi_- + \psi_+ e^{-\zeta\tau}},$$

$$\psi_+ = -\left(b - \rho\epsilon ik\right) + \zeta, \quad \psi_- = \left(b - \rho\epsilon ik\right) + \zeta, \quad \zeta = \sqrt{\left(\kappa - \rho\epsilon ik\right)^2 + \epsilon^2 \left(k^2 - uik\right)}$$

#### 2) Jump Rate Intensity

 $\bullet$  Constant

#### 3) Jump Size Distribution

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 $C(k,\tau) \equiv 0, \ D(k,\tau) = \tau \Lambda(k)$  **Jump Size Distribution**• Log-Normal  $\Lambda(k) = e^{(\nu + I\delta^2/2)ik - \delta^2 k^2/2 + I(\nu + \delta^2/2)} - 1 - (ik + I) \left(e^{\nu + \delta^2/2} - 1\right)$ 

## Appendix B

# R GUI Code For Option Pricing Using The Bates Model

#Calibration of the Bates model using ALSI data

```
getwd()
setwd("c:/")
#This is the directory I have been working on.
#It can be changed depending on where the user is
#working from.However I am working from the folder Bates,
#which I will make use of later on.
```

```
# Function to return implied vols for a range of strikes
setwd("Bates")
alsidata<-read.table("ALSIData3.txt",header=TRUE)</pre>
```

```
KVector<-rbind(16000,16150,19200,19950,20000,21000,22000,23000,
24000,25000,25900,26000,26400,26500,27000,28000,28350);
K<-rep(KVector,times=3);</pre>
```

```
TVector<-alsidata$T;
T<-rep(TVector,each=17);
rVector<-alsidata$r;
r<-rep(rVector,each=17);
dVector<-alsidata$d;
d<- rep(dVector,each=17);
S<-24723;
V1<-alsidata$V1;
V2<-alsidata$V2;</pre>
```

```
V3<-alsidata$V3;
V4<-alsidata$V4;
V5<-alsidata$V5;
V6<-alsidata$V6;
V7<-alsidata$V7;
V8<-alsidata$V8;
V9<-alsidata$V9;
V10<-alsidata$V10;
V11<-alsidata$V11;
V12<-alsidata$V12;
V13<-alsidata$V13;
V14<-alsidata$V14;
V15<-alsidata$V15;
V16<-alsidata$V16;
V17<-alsidata$V17;
MktvolsVectors<- rbind(V1,V2,V3,V4,V5,V6,V7,V8,V9,V10,V11,
V12,V13,V14,V15,V16,V17)
Mktvols<-c(MktvolsVectors)
Style<-1
#Finding P1
# Bates characteristic function
#The Bates parameters
Batesparams <- c(vzero=0.2,kappa=0.05,theta=0.09,
epsilon=1.5,rho=-0.9,lambda=1,nu=-0.2,delta=0.3);
pyone <- function(phi,K,T, params){</pre>
y < -log(K);
integrand <- function(k){Re(exp(-1i*k*y)*</pre>
phi (k,T)/(1i*k))};
res <- 1/2 + (1/pi)*integrate(integrand,</pre>
lower=0,upper=Inf)$value;
return(res);
}
phiBates1 <- function(params){</pre>
vzero <- params['vzero'];</pre>
kappa <- params['kappa'];</pre>
theta <- params['theta'];</pre>
epsilon <- params['epsilon'];</pre>
rho <- params['rho'];</pre>
nu<- params['nu'];</pre>
lambda <- params['lambda'];</pre>
delta <- params['delta'];</pre>
function(k,T,S){
```

```
S<-24723;
zeta<-sqrt(((kappa-(rho*epsilon*1i*k))^2)+</pre>
((epsilon<sup>2</sup>)*((k<sup>2</sup>)-(1i*k))));
psiminus<-(kappa-(rho*epsilon)-(rho*epsilon*1i*k)+zeta);</pre>
psiplus<-(-(kappa-(rho*epsilon)-(rho*epsilon*1i*k)))+zeta;</pre>
A<--((k*theta)/(epsilon^2))*((psiplus*T)+2*log((psiminus+</pre>
(psiplus*exp(-zeta*T)))/(2*zeta)));
B<-((1i*k)-(k^2))*(1-exp(-zeta*T))/(psiminus+
(psiplus*exp(-zeta* T)));
C<-0
Xi<-exp(((nu+((delta<sup>2</sup>)/2))*1i*k)-((delta<sup>2</sup>)*(k<sup>2</sup>)/2)+
(nu+((delta<sup>2</sup>)/2)))-1-((((1i*k)+1)*(exp(nu+((delta<sup>2</sup>)/2))-1)))
D<-T*Xi
return(exp(1i*k*(log(S))+A+(B*vzero)+C+(D*lambda ) ));
}
}
P1<-function(style,pyonefn)
ſ
((1-style)/2)+(style*pyonefn)
}
#Finding P2
pytwo <- function(phi,K,T, params){</pre>
y < -\log(K);
integrand <- function(k){Re(exp(-1i*k*y)* phi (k,T)/(1i*k))};</pre>
 res <- 1/2 + (1/pi)*integrate(integrand,</pre>
 lower=0,upper=Inf)$value;
return(res);
}
phiBates2 <- function(params){</pre>
vzero <- params['vzero'];</pre>
kappa <- params['kappa'];</pre>
theta <- params['theta'];</pre>
epsilon <- params['epsilon'];</pre>
rho <- params['rho'];</pre>
nu<- params['nu'];</pre>
lambda <- params['lambda'];</pre>
delta <- params['delta'];</pre>
function(k,T,S){
S<-24723:
zeta<-sqrt(((kappa-(rho*epsilon*1i*k))^2)+((epsilon^2)*</pre>
((k<sup>2</sup>)+(1i*k))));
psiminus<-(kappa-(rho*epsilon*1i*k)+zeta);</pre>
psiplus<-(-(kappa-(rho*epsilon*1i*k)))+zeta;</pre>
A<--((k*theta)/(epsilon^2))*((psiplus*T)+2*log((psiminus+
(psiplus*exp(-zeta*T)))/(2*zeta)));
```

```
B<--((1i*k)+(k^2))*(1-exp(-zeta*T))/(psiminus+
(psiplus*exp(-zeta*T)));
C<-0
Xi<-exp((nu*1i*k)-((delta<sup>2</sup>)*(k<sup>2</sup>)/2))-1-((1i*k)*
(exp(nu+((delta<sup>2</sup>)/2))-1 ) )
D<-T*Xi
return(exp((1i*k*(log(S)))+A+(B*vzero)+C+(D*lambda ) ));
}
}
P2<-function(style,pytwofnc)
ſ
((1-style)/2)+(style*pytwofnc)
}
#Using the functions above to find the option values
Optionvals<-function(S,K,d,r,P1fnc,P2fnc,style, params)</pre>
{
return((exp(-d*T)*S* P1fnc)-(exp(-r*T)*K* P2fnc));
 }
#The Black-Scholes formula code for option pricing
BSFormula <- function(S, K, T, r, sigma, params)
ſ
x \le \log(S/K) + r * T;
sig <- sigma*sqrt(T);</pre>
d1 <- x/sig+sig/2;
d2 <- d1 - sig;
pv <- exp(-r*T);
return( S*pnorm(d1) - pv*K*pnorm(d2));
}
#Code for finding implied volatilities using the Black-Scholes
BSImpliedVolCall <- function(S,K,r,d,T,C,params)</pre>
{
nK <- length(K);</pre>
sigmaL <- rep(1e-10,nK);</pre>
CL <- BSFormula(S, K, T, r, sigmaL,params);</pre>
sigmaH <- rep(10,nK);</pre>
CH <- BSFormula(S, K, T, r, sigmaH, params);
  while (mean(sigmaH - sigmaL) > 1e-10)
  {
    sigma <- (sigmaL + sigmaH)/2;</pre>
    CM <- BSFormula(S, K, T, r, sigma, params);</pre>
    CL <- CL + (CM < C)*(CM-CL);
    sigmaL <- sigmaL + (CM < C)*(sigma-sigmaL);</pre>
    CH <- CH + (CM >= C)*(CM-CH);
```

```
sigmaH <- sigmaH + (CM >= C)*(sigma-sigmaH);
  }
  return(sigma);
}
lowBates<-c(0.1,1,0.015,0.5,-0.9,0.01,-0.3,0.1)
upperBates<-c(0.3,10,0.1,2,-0.5,2,-0.1,0.5)
analyticBatesCalibr<-function(params)</pre>
{
Style<-1
S<-24723;
Nk<-length(K);
Callprice<-numeric(Nk);
Mktprice<-numeric(Nk);</pre>
BSV<-numeric(Nk);
Mktvols<-numeric(Nk);</pre>
BSVols<-numeric(Nk);
for (i in 1:Nk){
Callprice[i] <- Optionvals(S,K[i],d[i],r[i],P1(Style,
pyone(phiBates1(params),K[i],T[i], params)), P2(Style,
pytwo(phiBates2(params),K[i],T[i], params)),Style, params);
Mktprice[i] <- BSFormula(S,K[i],T[i],r[i],Mktvols[i], params);</pre>
BSVols[i] <- BSImpliedVolCall(S,K[i],r[i],d[i],T[i],</pre>
Callprice[i], params);
}
return(sum(((BSVols-Mktvols)^2)))
}
analyticBatesCalibr(Batesparams)
outBatesCalibr <-nlminb (Batesparams,</pre>
analyticBatesCalibr, lower=lowBates, upper=upperBates) #Takes some time
outBatesCalibr
Batesparams <- c(outBatesCalibr$par[1],outBatesCalibr$par[2],</pre>
outBatesCalibr$par[3],outBatesCalibr$par[4],outBatesCalibr$par[5],
outBatesCalibr$par[6],outBatesCalibr$par[7],outBatesCalibr$par[8]);
analyticBatesVolCalc<-function(params)</pre>
{
S<-24723;
Style<-1
Nk<-length(K)
Callprice<-numeric(Nk);
Mktprice<-numeric(Nk);</pre>
BSV<-numeric(Nk);
Mktvols<-numeric(Nk);</pre>
BSVols<-numeric(Nk);</pre>
for (i in 1:Nk){
```

```
Callprice[i] <- Optionvals(S,K[i],d[i],r[i],P1(Style,
pyone(phiBates1(params),K[i],T[i], params)), P2(Style,
pytwo(phiBates2(params),K[i],T[i], params)),Style, params);
Mktprice[i] <- BSFormula(S,K[i],T[i],r[i],Mktvols[i], params);</pre>
BSVols[i] <- BSImpliedVolCall(S,K[i],r[i],d[i],T[i],</pre>
Callprice[i], params);
}
return(BSVols)
}
analyticBatesVolCalc(Batesparams)
#Code for computing the difference between market
#and computed volatilities
analyticBatesVolDiff <- Mktvols-
analyticBatesVolCalc(Batesparams)
analyticBatesVolDiff
analyticBatesVolDiffMat<-matrix(analyticBatesVolDiff,
nrow=17,ncol=3)
analyticBatesVolDiffMat
#Code for plotting the market volatilities
x<-TVector
y<-KVector
zha<-matrix(Mktvols,nrow=17,ncol=3)</pre>
mktvolsurface<-persp(y,x, zha, theta = 30, phi = 30,</pre>
expand = 0.5, col = "lightblue", ltheta = 120,
shade = 0.75, ticktype = "detailed",xlab = "Time",
ylab = "Strike", zlab = "Volatility",
main="Market Model Volatility Surface")
mktvolsurface
#Code for plotting the computed volatilities
x<-TVector
y<-KVector
zbb<-matrix(analyticBatesVolCalc (Batesparams),</pre>
nrow=17,ncol=3)
Batesvolsurface<-persp(y,x, zbb, theta = 30,</pre>
phi = 30, expand = 0.5, col = "lightblue",
ltheta = 120, shade = 0.75, ticktype = "detailed",
xlab = "Time", ylab = "Strike", zlab = "Volatility",
main = "Bates Model Volatility Surface")
Batesvolsurface
```

#End of Bates model code

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