PHYSICAL GEODESY
with special reference to
DEFLECTIONS OF THE VERTICAL

by

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ABSTRACT

The general field of physical geodesy is described in outline and the application of the basic theories to the calculation of deflections of the vertical are discussed and analysed in detail. The results of the calculation of the deflections of the vertical at three points in South Africa are given and discussed. Some applications of physical geodesy are described and proposals are made for the continuation of work in this field in South Africa.

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REFERENCES

References are given by quoting the author's surname in capital letters , followed by the date of publication of the article or book (or the date of the delivery of the author's paper). For example , RICE 1952 , BOMFORD 1962 (or STOKES 1849). The references are listed in alphabetical order in chapter 10 page 79.

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CHAPTER 1

INTRODUCTION

1.00 Geodesy in general.

Geodesy, literally "dividing the earth", is the science of determining the size and shape of the earth and the provision of an accurate control point system on the earth. Geodetic science includes the observations, computations and adjustments for achieving the above. Geodesy overlaps into other earth sciences in the study of the gravity field and the internal structure of the earth.

We can say that the main problem in geodesy is to determine the space co-ordinates of any point on the physical surface of the earth by carrying out geodetic operations on this physical surface. These geodetic operations can be divided into three broad interdependent sections :-

A. Geometrical Geodesy.(Including triangulation, trilateration, traversing and electro-magnetic distance measurements)

B. Astronomical Geodesy.(Including the determination of latitude, longitude and azimuth, and satellite orbits.)

C. Physical Geodesy.(Including gravity measurements and spirit levelling)

Geometrical geodesy yeilds differences in the horizontal co-ordinates of points ; astronomical geodesy determines the direction of gravity and physical geodesy supplies the absolute and relative values of gravity, the differences of the potential of gravity and differences in height between points.

Gravity is the physical quantity which affects most geodetic operations and it is the chief characteristic of physical geodesy.

(BOMFORD 1962 , DE JONG 1963 , HEISKANEN 1958a and 1964b , HIRVONEN 1960)

1.01 Definition of terms in physical geodesy.

The following terms are frequently used in physical geodesy and the definitions currently accepted by geodesists are given. (BOMFORD 1962 , HEISKANEN 1958a and 1967 , MUELLER 1966)

Rather than give the terms in alphabetical order , they are given in a more or less logical sequence.

Gravitation.

The attraction of the earth's mass.

Gravity.

The resultant of the gravitation and the centrifugal force caused by the rotation of the earth. Gravity , g , is gravitation minus the effect of the centrifugal force and has the dimension of the acceleration. The value of g varies between 978 cm/sec^2 and 983 cm/sec^2 over the surface of the earth. In physical geodesy , the units used are the gal (named after Galileo) and the milligal (mgal).

$$1 \text{ gal} = 1 \text{ cm/sec}^2$$

$$1 \text{ mgal} = 1 \times 10^{-3} \text{cm/sec}^2$$

Potential.

A scalar function whose gradient is the force.

Geopotential.

The gravity potential of the earth. It is the sum of the gravitational potential and the potential of the centrifugal force.

Equipotential (or level) surface.

This is a surface in a field of force on which the potential is constant. That is , it is a surface about which an object can be moved without expenditure of work. The force is everywhere perpendicular to this surface.

Geop or geopotential surface.

An equipotential surface in the gravity field of the earth. Gravity is everywhere perpendicular to the geop.

Geoid.

The geoid is the geop which co-incides with mean sea level. The fact that mean sea level is not fixed in an absolute sense means that each country's mean sea level datum gives rise to a different geoid.

Geopotential number.

This is the geopotential difference between the geoid and the geop through an observation point. The number is given in geopotential units (g.p.u.).

$$1 \text{ g.p.u} = 1 \text{ kilogal metre } (10^5 \text{cm}^2 \text{sec}^{-2}).$$

Normal earth.

The normal earth is a mass such that its external bounding equipotential surface is the earth spherop , and its gravity (called normal gravity) is given by the gravity formula:-

$$\gamma = \gamma_E (1 + \beta \sin^2 \varphi + \epsilon \sin^2 2\varphi)$$

where γ_E = normal gravity at the equator obtained empirically.

φ = latitude of station.

β = gravitational flattening obtained empirically.

ϵ = a theoretically derived co-efficient.

For practical purposes we can regard the normal earth as being the same as the International Reference Spheroid.

Spherop.

An equipotential surface in the normal gravity field of the earth.

Spheropotential.

The potential of the normal gravity.

Disturbing potential.

The difference between the geopotential and the spheropotential at a given point.

Gravity anomaly.

The difference between the gravity on a geop and the normal gravity on the corresponding spherop.

Plumb line.

A continuous curve which is a line of force in the geopotential field. The direction of gravity is everywhere tangential to the plumb line.

Normal.

A normal is a straight line perpendicular to a particular surface.

Vertical.

The direction of gravity at a point.

Isostasy.

A hypothesis of equilibrium where the crustal elements at a certain depth below the geoid are under equal pressure or equal mass regardless of whether they are under mountains , lowlands or oceans. Isostasy has been dealt with in greater detail in LOON 1955.

Undulation of the geoid or geoidal heights.

The distance between the geoid and the earth-spherop (or reference spheroid).

1.02 Physical geodesy.

The mathematical basis of physical geodesy was laid down by STOKES 1849. He developed a formula for calculating the geoidal heights if gravity observations all over the earth's surface (land and sea) were available. In 1849 there did not seem to be any prospect of ever obtaining these measurements. Stokes commented "....These points of the theory are noted more for the sake of the ideas than on account of any application which is likely to be made of them....."

VENING MEINESZ 1929 developed a pendulum apparatus which could be used for gravity observations at sea. Recently (THOMPSON 1966) gravity observations have been made from aircraft and it is now becoming possible to partially fulfil Stokes' requirements. But it will still be many years before the gravity information is complete.

The basis of physical geodesy is the fact that gravity anomalies (or gravity disturbances) , the undulations of the geoid and the deflections of the vertical are all caused by surface and sub-crustal disturbing masses of high or low density. The chief characteristic of physical geodesy is the gravimetric method and the basic requirement (or tool) of the gravimetric method is the gravity anomaly. The undulations of the geoid and the deflections of the vertical can be computed from these anomalies. The great advantage of the gravimetric method lies in the fact that these computed values are independent of the local reference spheroid. They depend only on the gravity formula used for computing the theoretical value of gravity. This theoretical value is based on the International Spheroid which is the closest mathematical representation of the earth for pure geodetic purposes. (HIRVONEN 1960)

HEISKANEN 1958b says that "we are not very far from the truth when we say that the most important and actual problem of geodesy at the present time is the determination of the gravimetric undulations of the geoid and the deflection of the vertical components".

Included in the field of physical geodesy are heights above sea level. See chapter 5.

1.03 Reference surfaces.

The practice of determining the size and shape of the earth involves choosing a mathematical figure which best fits the figure of the earth and then determining the details of the lack of fit. In order to do this , the geodesist is concerned mainly with three reference surfaces. These are the actual surface of the earth , the geoid and the reference spheroid.

A. The actual surface of the earth is the physical surface on which the geodesist sets his instruments and makes his measurements. The shape of this surface is approximately an oblate spheroid (BOMFORD 1962) with local departures of up to 8 Km from this shape.

B. The geoid can be regarded as the fundamental surface of geodesy. The term "geoid" was first used by J.B.Listing in about 1872. (OXFORD 1933. But NAGY 1963 gives the date of Listing's article as 1873). The shape of the geoid is a smoother oblate spheroid than the actual surface of the earth with local departures of less than 100 metres. (HELMERT 1884 and HIRVONEN 1962 give a figure of about 50 metres.) The geoid can be regarded as a physical reality (BOMFORD 1962) when we consider that , at sea level , the vertical axis of a level theodolite is perpendicular to it. The process of spirit levelling measures heights above the geoid. See chapter 5.

C. The Reference Spheroid.

In order to compute co-ordinates for a geodetic control system , we need a point from which to compute , a direction in which to compute and a surface along which to compute. This surface is called the reference spheroid. The actual surface of the earth and the geoid cannot be defined mathematically and therefore cannot be used as surfaces along which to compute.

The reference spheroid is an arbitrarily defined geometrical figure on which the co-ordinates of points are computed. It is defined by seven constants :-

Two constants to define the shape and size of the spheroid. Either the major axis and the flattening or the major and minor axes are given;

Two constants to define the position of the spheroid axis. This is usually defined to be parallel to the earth's axis of rotation;

Three constants to define the position of the centre of the spheroid. This is usually done indirectly by choosing an initial (or datum) point on the actual surface of the earth and defining the geodetic latitude of this point , the geodetic azimuth from this point to another point and the height of the initial point above the spheroid. Instead of the azimuth the geodetic longitude of the initial point could be defined. Or the three constants could be the geoidal height and the two components of the deflection of the vertical.

The reference surface usually chosen is the oblate spheroid. In geodetic literature the words "ellipsoid" and "spheroid" are used but the concept meant is oblate spheroid. That is , a surface of revolution defined by two parameters.

Note on ellipsoid and spheroid.

Any surface represented by an algebraic equation of the second degree in three variables , is a quadric surface. Eg. $Ax^2+By^2+Cz^2+2Exy+2Fxz+2Gyz+2Px+2Qy+2Rz+D = 0$. If this surface is symmetric about the co-ordinate system it is called a central quadric and has the equation $Ax^2+By^2+Cz^2 = 1$. An ellipsoid is a special case of a central quadric surface and has the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

. An ellipsoid , therefore , is a tri-axial figure. There is a special case of an ellipsoid called a spheroid which is a surface obtained when two of the axes of the ellipsoid are equal. If the two equal axes are each longer than the third axis , then we have an oblate spheroid whose equation is

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{c^2} = 1$$

. (In the other case a prolate spheroid is obtained.) The oblate spheroid can also be regarded as the surface obtained by revolving an ellipse about its minor axis, i.e. an "ellipse of revolution" is obtained. Some mathematical text books use the term "ellipsoid of revolution" and some authors of geodetic literature have adopted this expression and the abbreviated term "ellipsoid". An ellipsoid is not a surface of revolution. The term "oblate spheroid" or its abbreviation "spheroid" is to be preferred. This note could be summarised by using biological expressions:-

the family is the quadric surface ;

the genus is the ellipsoid ;

the species is the oblate spheroid.

(FRAME 1960 , VAN NORSTRAND 1958)

In South Africa the reference spheroid is the "modified" Clarke 1880 Spheroid with

semi-major axis = 20 926 202 S.A.Geodetic feet

semi-minor axis = 20 854 895 S.A.Geodetic feet.

(See GILL 1896 where the above axes are given as English feet and HENDRIKZ 1956 where the S.A. Geodetic foot is explained).

The flattening given by Clarke is f = 1/293.465. As Hendrikz points out , using the above axes the flattening works out to be f = 1/293.466 307 656 which figure has been used in the geodetic survey of South Africa. (Correction tables have been given in HENDRIKZ 1943).

The datum point for the South African Geodetic System is the trigonometrical station No. 130 Buffelsfontein , height 926 feet above mean sea level , with co-ordinates

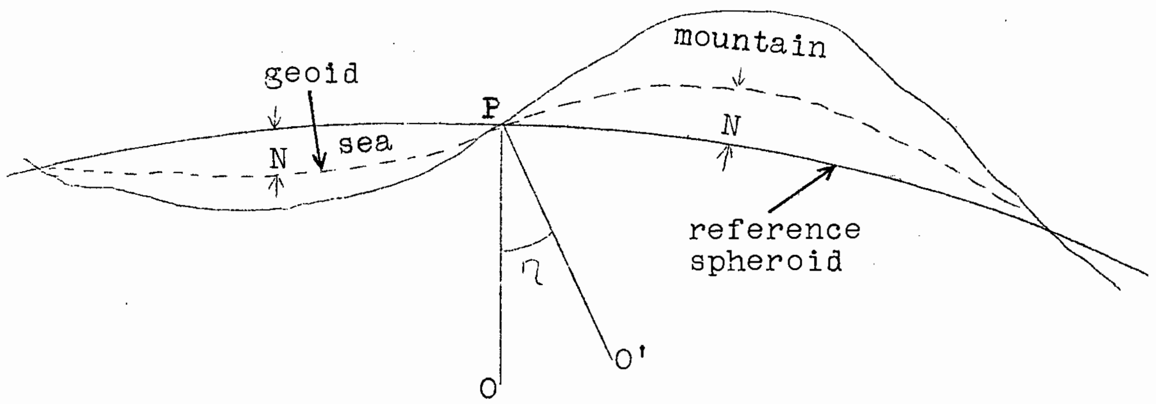
latitude = 33° 59' 32".00 South

longitude = 25° 30' 44".622 East

and the initial direction (azimuth) is Buffelsfontein to Zuurberg = 183° 58' 15".50 . (GILL 1896).

(In 1953/54 the latitude and longitude at Buffelsfontein were re-observed in connection with the 30th Arc of Meridian Project and values differing from the above were obtained. In view of the interesting remarks made by THOMAS 1965 about this discrepancy it appears that further investigation is needed).

The figure below shows an east-west section through the topography. η is the east-west component of the deflection of the vertical and N is the geoidal height. The relationship between the spheroid and the geoid as well as the effect



of topography on the geoid is shown. PO is perpendicular to the reference surface and PO' is perpendicular to the geoid. The same relationship between the geoid and the spheroid would be obtained if the mountain were replaced by a sub-crustal mass surplus and the sea by a sub-crustal mass deficiency, OR if the area to the right of PO has a gravity anomaly greater than zero and the area to the left of PO has a gravity anomaly of less than zero. It can be seen that the geoid is fixed and determined by the surface and sub-crustal masses and the reference spheroid is arbitrarily chosen.

Base lines are measured on the actual surface of the earth. The computation of the geodetic control system requires that these base line measurements should be reduced to the reference spheroid. But in the initial stages of a triangulation system of a country , the position of the reference spheroid in relation to the geoid (and therefore also to the actual surface of the earth) is only assumed at the initial point. As the height above mean sea level of the base line is usually known , the base line measurements are reduced to the geoid. But in the computations it is assumed that they have been reduced to the reference spheroid and this gives rise to distortions in the triangulation.

A knowledge of the deflections of the vertical can show whether the chosen reference spheroid is the best one for a particular country and a knowledge of the geoidal heights enable base lines to be reduced to the computation surface. The methods of physical geodesy can provide us with these geoidal heights and deflections of the vertical.

CHAPTER 2

GRAVITY MEASUREMENTS.

2.00 General.

The value of gravity , g , can be measured on the earth's surface , on the sea , underwater or in the air. The apparatus used is either the dynamic type (eg. the pendulum method or the falling body method) or the static type (eg. the spring balance principle). These gravity measurements are either absolute or relative.

2.01 Absolute gravity measurements.

In the absolute determination of g , measurements are made at some point without reference to any other point. Most absolute determinations have been made by the pendulum method using a multiple pendulum apparatus. Great care is required in these measurements and a number of effects which produce systematic errors must be studied. The accuracy of this method is 1 mgal (GARLAND 1965) or perhaps even 0.4 mgal (HEISKANEN 1960). Pendulum observations made at Potsdam at the turn of the century gave a value of $g = 981.274$ gal which today still serves as the basis of the world gravity network. Today it appears that this Potsdam value is between 10 and 14 mgal too high. But this does not affect work done in physical geodesy where the variation of gravity over the earth's surface is used and not so much the absolute values.

Falling body methods (first used by Galileo) for the determination of g have been used in recent years. Accuracies of better than 1 mgal are claimed for these methods. (HEISKANEN 1960)

2.02 Relative gravity measurements.

This type of measurement is most often used in physical geodesy. In this case the ratio between the gravity measured at a base station and the gravity measured at a field station is determined. Relative gravity obsevations can be made by the time-consuming pendulum method but mostly the spring balance system (and variations of this system) is used. In this system a weight hangs from a spring and the length of the spring depends on g at any point. The instrument using this system is very light and portable. It can detect very small differences in g and can be read very quickly.

For local surveys the range of the gravimeter need not be large but the geodetic gravimeter needs a range of about 5000 mgals to be able to be used anywhere on the earth. The fact that such gravimeters are available and have proved their ability of measuring differences in g of 0.01 mgal and better , has contributed to the tremendous progress in physical geodesy in the last two decades.

2.03 Measurements from a moving platform.

Physical geodesy requires gravity observations over the whole earth. But a large portion of this surface is sea and much of the remainder is not easily accesible by land. In recent years much progress has been made with seaborne and airborne gravimeters.

Vening Meinesz is regarded as the pioneer in the field of gravity observations at sea and since his initial work in the 1920's many reliable gravity observations have been made from submarines and surface ships. In one of the latest reports , BOWER 1967 gives a standard deviation between a series of observations of a ship's traverse run as 3.9 mgal for a LaCoste gravimeter and 2.7 mgal for a Graf-Askania gravimater. WORZEL 1965 says , in connection with gravimeters at sea , " differences between values at the same point made on different profiles may be as large as 20 mgals , although usually smaller." Part of this difference could be due to the uncertainty of the ship's position.

In South Africa we can look forward to increased activity in this field when the Decca navigation system is fully operational around the coast and when the Graf-Askania gravimeter is installed in the S.A.S. Natal.

Because of the speed of an aircraft , it seems that airborne measurements can only provide a generalised picture of the gravitational field. This type of gravity measurement is still in the experimental and testing stage.

2.04 Gravity measurements in South Africa.

Gravity measurements in South Africa have been mentioned briefly before in LOON 1967

Pendulum observations were made as far back as 1818. (MENZIES 1967). In 1948 and 1949 pendulum observations were carried out at 53 stations in Southern Africa. (HALES 1950). These formed the base stations for gravimeter connections done during 1949 to 1957. Some 6000 observations were made during this period.(SMIT 1962). A few pendulum observations aboard a submarine were made by Vening Meinesz in 1935.

Details of published gravity measurements in South Africa are to be found in HALES 1950 , SMIT 1962 and UOTILA 1960.

In addition to the above , local gravity surveys (results unpublished) have been made by oil prospecting companies , government departments and private geophysical consultants.

HALES 1950 used the value g(Cambridge) = 981.265 gals as the basis for his survey. This gave the values g(Mowbray) = 979.644 gals and g(Johannesburg) = 978.546 gals. The values used by Smit were g(Mowbray) = 979.6468 gals. (This is incorrectly printed as 978.6468 on page 9 of SMIT 1962). Smit's value for g(Johannesburg) = 978.5491 gals. Smit has corrected Hales' values by +3.3 mgal so as " to adjust them to the accepted value of g at the Museum pendulum station in Pretoria." The value g(Teddington) = 981.1963 gals was accepted by Smit who gives probable accuracies of the South African bases relative to this value. (These range from ± 0.1 mgal to ± 0.5 mgal.)

UOTILA 1960 gives a list of world national reference stations adjusted on to the Potsdam system. The differences between his and the above mentioned values are given in the following table. g(Potsdam) = 981.27400 gals.

	Hales gals	Smit gals	Uotila (Potsdam) gals
g(Teddington)		981.1963	981.1963
g(Mowbray)	979.644	979.6468	979.6475
g(Johannesburg)	978.546	978.5491	978.5514
g(Cambridge)	981.265		981.2688

From the above table it can be seen that Smit's values are closer to the Potsdam system and he was therefore justified in correcting Hales' values.

The geodesist must work on a World System. This has been stressed many times by HEISKANEN 1951 , 1952 , 1958a etc. When using the results of previous gravity surveys , the South African geodesist will have to make adjustments to bring everything onto the internationally accepted Potsdam System.

CHAPTER 3

THE REDUCTION OF GRAVITY MEASUREMENTS

3.00 General.

The gravity measurements obtained at different points on the earth's surface are not directly comparable one to the other. This is due mainly to the fact that these observations are made at different heights above sea level. All other factors being equal , the observations made at a **high** point (far from the centre of gravity) will be too small and the observations made at a low point (nearer the centre of gravity) will be larger. In the gravimetric method we have to compare measurements made at different points and these observations therefore have to be reduced to the same level , usually sea level (i.e. the geoid).

The reductions can be conveniently classified as follows:-

A. Non-isostatic reductions:

- Free air reductions
- Bouguer reductions
- Condensation and inversion reductions

B. Isostatic reductions :

Based on the hypotheses of

Pratt-Hayford or
Airy-Heiskanen or
Vening Meinesz.

(HEISKANEN 1958a)

3.01 The free air reduction.

This reduction is the one most commonly used in physical geodesy and it takes account only of the elevation of the station where the gravity observation has been made. No account is taken of the mass between the station and sea level. This is sometimes called Fayes' reduction after the man who first drew attention to it. (LAMBERT 1930 , NAGY 1963).

HEISKANEN 1958a and GARLAND 1965 , among others , give an approximate formula for the variation in gravity due to a change in distance from the earth's centre as

$$\frac{\partial g}{\partial r} = \frac{2g}{r} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \ (3\text{-}1)$$

where g = mean gravity on earth's surface = 981 gals

r = mean radius of earth = 6370 Km

This equation is known as the vertical gradient of gravity at sea level and using the numerical values

$$\frac{\partial g}{\partial r} = 0.3086 \text{ mgal/metre}$$

$$= 0.09406 \text{ mgal/foot}$$

Equation (3-1) can be used at most parts of the earth and the free air reduction is therefore

$$g_f = + 0.3086\ h \ \text{ mgal} \qquad \ldots\ldots\ldots\ldots \ (3\text{-}2)$$

where h is the height of the station in metres above mean sea level.

Note: A positive sign is used here because we are reducing from an observation point (above sea level) down to sea level. (For stations below sea level (3-2) would be negative).

More rigorous formulae are given by HEISKANEN 1958a as

$$g_f = \frac{2gh}{r}\left(1 - \frac{3h}{2r} + \ldots \right)$$

$$= +(0.3086\ h - 0.000\ 000\ 072\ h^2 + ..) \text{ mgal}$$

$$\ldots\ldots\ldots\ldots \ (3\text{-}3)$$

and by LAMBERT 1930 as

$$g_f = + (0.30857 + 0.00021 \cos 2\varphi)h - 0.000\ 000\ 072h^2 \text{ mgal}$$

$$\ldots\ldots\ldots\ldots \ (3\text{-}4)$$

(The actual formula given in LAMBERT 1930 is for g_f in gals).

GARLAND 1965 gives

$$g_f = -0.3085 - 0.00022 \cos 2\varphi + 0.000\ 144h$$

which is clearly incorrect as each term must be a function of the height. Note -Garland makes his comparisons at the observation station and not at sea level , therefore his g_f is negative.

3.02 The Bouguer reduction.

The formula for this reduction was derived by Bouguer in 1749. He used this reduction for comparing observed gravity values in South America.

This reduction takes into account the attraction of the material between the geoid and the observation station. The effect of the attraction of this mass must be subtracted from the observed gravity values.

The simplified Bouguer reduction is

$$g_b = - \frac{3\rho g h}{2\rho_m r} \text{ mgal} \qquad \ldots\ldots\ldots\ldots\ldots \text{(3-5)}$$

where ρ = density of regional mass

ρ_m = mean density of earth

g = mean gravity

h = height above sea level in metres

r = mean radius of the earth

From equation (3-1) it can be seen that

$$g_b = - g_f \frac{3\ \rho}{4\ \rho_m}$$

HEISKANEN 1958a gives the following corresponding values:-

ρ_m	ρ	g_b
5.576	2.67	- 0.1108 h
5.52	2.80	- 0.1912 h
5.53	2.67	- 0.1118 h

HALES 1962 used $g_b = -0.1118$ h for South Africa.

The drawback of the Bouguer reduction for geodetic purposes is that it changes the geop passing through the observation point and also changes the shape of the geoid.

3.03 Condensation and inversion reductions.

The condensation reduction was introduced to avoid the drawback of the Bouguer reduction for geodetic purposes. In this reduction the masses above the geoid are transferred inside the geoid or as near to it as possible. After applying the condensation reduction a so called " ideal geoid " is obtained. The differences between the actual and ideal geoids is less than 3 metres. (HEISKANEN 1958a)

The inversion reduction is applied in such a way (by manipulation of the masses) so as not to change the surface of the actual geoid.

3.04 Isostatic reductions

In gravimetric studies only three isostatic assumptions have been used. They are The Pratt-Hayford system
The Airy-Heiskanen system
The Vening Meinesz Regional system

In the Pratt-Hayford system it is assumed that the density of the earth's crust is smaller as the elevation increases. In the Airy-Heiskanen system the mountains are assumed to be floating in the heavier substratum. The higher the mountains the deeper they are sunk into the substratum. In the same way it is assumed that under the oceans there are " antiroots " of heavy material. Using this system HALES 1950 found a depth of compensation (T) of 30 Km in South Africa. The Vening Meinesz Regional system is a modification of the Airy-Heiskanen system where the compensating masses of the mountains and the oceans are assumed to be broadly distributed horizontally. In this system the load of the topographic mass causes the earth's crust to bend until equilibrium prevails. The Airy-Heiskanen system can be called a local floating system and the Vening Meinesz system a regional floating system. (HEISKANEN 1960)

The isostatic reductions therefore depend on the surface and sub-surface earth structures and various tables have been used for carrying out these reductions. (See HEISKANEN 1958a).

3.05 Summary and analysis.

The free air reduction.

The following table summarises some of the numerical values to be expected from equations

(3-3) viz. $g_f = + (0.3086\ h - 0.000\ 000\ 072\ h^2)$ and

(3-4) viz. $g_f = + (0.30857 + 0.00021\cos 2\varphi)\ h - 0.000\ 000\ 072h^2$

h	A	B	C
metres	mgal	mgal	mgal
1000	0.07	0.16	0.09
2000	0.3	0.32	0.15
3000	0.7	0.48	0.22
4000	1.2	0.64	0.56
5000	1.8	0.80	1.00

where A = the h^2 term of equations (3-3) and (3-4)

B = the maximum value of the cos 2φ term of (3-4) for South Africa.

C = the maximum value of the sum of the second and third terms of (3-4) in South Africa.

From the above table we see that

for h = 1000 metres

$g_f = 0.3086\ h - 0.07$ mgal using (3-3)

$g_f = 0.30857h - 0.09$ mgal using (3-4)

and for h = 2000 metres

$g_f = 0.3086\ h - 0.3$ mgal using (3-3)

$g_f = 0.30857h - 0.15$ mgal using (3-4)

Considering the accuracy of a geodetic gravimeter , like the Worden gravimeter , of 0.2 mgal (HEISKANEN 1958a) and the field procedure normally used (HEISKANEN 1956) where discrepancies between two measurements at the same point of less than 0.3 mgal are acceptable , we can say that the formula (3-2) $g_f = + 0.3086\ h$ mgal can be used for stations whose altitudes do not exceed 2000 metres (6562 feet).

HALES 1962 used $g_f = + 0.3086 (1 + 0.00071 \cos 2\varphi)h$ mgal for the South African gravity survey. Multiplying out we get $g_f = + 0.3086\ h + 0.000\ 22 \cos 2\varphi h$ which is almost identical with the first two terms of equation (3-4). In this survey only a few points were higher than 6000 feet (1830 metres) above M.S.L.

The free air reduction is the simplest and is often used in physical geodesy. For example , RICE 1952 , KAULA 1954 , the Columbus Group (HEISKANEN 1964a) and others have calculated geoidal undulations and deflections of the vertical making use of the free air reduction to obtain the free air anomalies.

The disadvantage of using the free air reduction is that free air anomalies are not sufficiently representative without any correction. (HEISKANEN 1958a).

The Bouguer reduction

From HALES 1960 we can deduce that for South Africa , Hales has used $g_b = - 0.36\ g_f$ i.e. the Bouguer reduction diminishes the effect of the free air reduction by about one-third.

The assumption made for the Bouguer reduction is that the topography around the station is level. In order to take account of topographic irregularities around the station , a " terrain correction " (Geländereduktion) is applied. This is always positive and can be as much as 123 mgals. (At Mont Blanc where h = 4807 metres.)

The Bouguer reduction is not used for geodetic purposes as it changes the shape of the geoid and the geop through the observation point. (BOMFORD 1962 and others). This change can be as much as 500 metres. (HEISKANEN 1958a)

The condensation and inversion reductions.

These reductions are not widely used as the isostatic reductions are preferred. (HEISKANEN 1958a)

Isostatic reductions

Although tables can be used to a certain extent , isostatic reductions are time consuming because they involve an examination of the regional topography around each station where a gravity observation has been taken.

HEISKANEN 1964a says that the use of isostatic reductions will give the most accurate values for geoidal undulations.

General

The reduction of gravity measurements has occupied the attention of geodesists for a long time. Many methods have been suggested and to date there is no agreement (in fact there is much argument) as to which method is best suited for any particular investigation. For example see BOMFORD 1962 , COOK 1962 , DE GRAAFF-HUNTER 1958 and HEISKANEN 1959.

In July 1961 the International Association of Geodesy held a symposium in England on the reduction of gravity data. RICE 1962 reports on this symposium and sums up the proceedings as follows:-

" When the discussion session was concluded , it was apparent that there remained fundamental differences of opinion as to the best methods of reducing surface gravity to calculate the form of level surfaces for geodetic purposes. "

BOMFORD 1962 and others state that the particular reduction system used must depend on the objects of the investigation , but still there is no agreement. For example , in connection with deflections of the vertical , Browne (see RICE 1962) advocates the use of isostatic reductions and COOK 1962 states that free air anomalies should be used.

HEISKANEN 1958a says that for geodetic applications , the following can be applied :-

(a) the free air reduction with elevation correction or condensation correction ;

(b) the inversion reduction ;

(c) the isostatic reduction.

In South Africa , the Geological Survey has used the Bouguer and isostatic reductions and has published maps showing Bouguer and isostatic anomalies. HALES 1950 and SMIT 1962 have also worked out and tabulated the free air anomalies.

In chapter 8, for the South African calculations , the isostatic reductions based on the Airy-Heiskanen hypothesis (depth of compensation = 30 Km) have been used because gravity anomaly maps based on these reductions were readily available.

CHAPTER 4

GRAVITY ANOMALIES.

4.00 General.

The gravity anomaly , Δg , is the difference between the reduced value of g (from observations) and the theoretical value of gravity , G.

$$\Delta g = g + corr - G \quad \text{usually expressed in mgals}$$

where g is the observed gravity and corr is the correction as obtained from one of the methods described in chapter 3.

The formula for the value of the theoretical (normal) gravity is

$$G = 978.049 \; (1 + 0.0052884 \sin^2\varphi - 0.0000059 \sin^2 2\varphi) \text{ gal} \qquad \ldots\ldots\ldots\ldots\ (4\text{-}1)$$

where φ is the latitude.

This equation was adopted by the International Union of Geodesy and Geophysics in 1930 (Stockholm) and is based on the absolute value of g of 981.274 gals measured at Potsdam by Kühnen and Furtwängler in 1906 . (HEISKANEN 1958a; JORDAN 1958 ; PARASNIS 1962. Note : Parasnis has omitted the seventh decimal figure in the second term of (4-1) above). The co-efficients of (4-1) were computed by Heiskanen (in 1928) and Cassinis (in 1930).

HEISKANEN 1958a suggests that the Potsdam value must be corrected by between -10 to -15 mgals. Since the exact correction is unknown , successive General Assemblies of theI.U.G.G (1954 and 1957) decided to await more information before making any change. Heiskanen's Columbus Group calculated a gravity formula using up to date (1957) gravity anomalies. The difference between the two formulae is small and Heiskanen recommends that formula (4-1) be used until new gravity material and new methods change it considerably. (HEISKANEN 1958b).

The international gravity formula (4-1)is a function of the latitude and as such is a function of the International Spheroid. The main parameters of the International Spheroid are

semi-major axis = 6 378 388 metres

flattening = 1/297.00 (JORDAN 1958)

The gravity anomaly , Δg , is required all over the earth's surface to evaluate the formulae of Stokes and Vening Meinesz. The practical application of these formulae present difficulties due to the scarcity of gravity information in large portions of the world and the fact that very seldom are gravity observations equally numerous on high and low ground. (COOK 1950)

Recent advances in gravity surveying have included gravity measurements at sea and in the air as described in chapter 2. At the present time the gravimetrically unsurveyed areas of the earth are large and the main problem is to decide what gravity anomalies to assign to these areas.

4.01 Anomalies in unsurveyed areas.

All the problems of physical geodesy can be solved if we know the gravity anomalies all over the world. But a large part of the earth's surface consists of oceans and land areas which are gravimetrically unsurveyed. The question of how to fill these gaps in the gravity anomaly field arises. The making of additional measurements in these areas will take time and , especially in the oceans , certain technical difficulties will have to be overcome. But research in the field of physical geodesy cannot wait for these additional measurements , so some way must be found to fill these gaps.

The method used by researchers in this field is to fill these gaps by extrapolation. This extrapolation can be done by either statistical methods or by geophysical methods. The results required from these methods are mostly in the form of mean area anomalies i.e. the area means of squares 1° or 5° or 10° and so on. For calculations for deflections of the vertical, point anomaly predictions are required.

Researchers working with statistical methods have made much progress in this field. The error of representation has been evaluated , predictions have been made using estimation techniques , least squares techniques and using Fouriers series and spherical harmonics. (RAPP 1966 , MATHER 1967).

The statistical method , however , cannot be used over long distances or for very large areas and in these areas the geophysical method is used. In this method use is made of the fact that the earth's crust is in isostatic equilibrium. This means that the topogrphic and bathymetric masses are more or less balanced by compensating masses. We therefore expect the gravity anomalies and the geoidal undulations to be relatively small. Knowledge of these topographic and bathymetric masses enable the effect of the isostatic compensating masses to be estimated and the gravity anomalies caused by these compensating masses are obtained. This method has been widely used especially by the Columbus group.(HEISKANEN 1965 and 1966).

For details of some of the above methods see ORLIN 1966.

When considering these extrapolation methods , one must remember that a prediction remains a prediction and that the ultimate test of whatever theory is being used is to compare the predicted anomaly with the observed anomaly.

4.02 Summary and analysis.

Gravity anomalies can be of different kinds depending on which particular reduction method is used to obtain the correction to the observed gravity. In general

gravity anomaly = observed gravity + correction obtained from particular reduction method + theoretical gravity as from (4-1)

For example , we have

Free air anomalies = observed gravity + free air correction
- theoretical gravity

Bouguer anomaly = observed gravity + Bouguer correction
+ free air correction + terrain
correction - theoretical gravity

(The numerical value of the bouguer correction
is negative)

Isostatic anomaly = observed gravity + free air correction
+ Bouguer correction + terrain
correction - theoretical gravity

For South Africa , HALES 1962 used

$$\text{Free air anomaly} = g + 0.3086\ (1 + 0.00071 \cos 2\varphi)\ h - G \ \text{mgal}$$

$$\text{Bouguer anomaly} = g - (0.1118\ h + B) + 0.3086\ (1 + 0.00071 \cos 2\varphi)\ h - G \ \text{mgal}$$

$$\text{Isostatic anomaly} = g + 0.3086\ (1 + 0.00071 \cos 2\varphi)\ h - (0.1118\ h + B) + I_n - G$$

where B = a curvature correction

I_n = isostatic correction

As mentioned in chapter 3 , Bouguer anomalies cannot be used in geodesy.

Figures 4/1 and 4/2 show the gravity anomaly profiles along 29° South Latitude and 28° East Longitude resp. (i.e. they are two South African profiles at right angles to each other). They were compiled from the data published in SMIT 1962 and HALES 1962. The isostatic anomalies in these figures are based on the Airy-Heiskanen system with a crustal thickness of 30 Km.

In the study of these figures we note the following interesting points :-

GRAVITY ANOMALY PROFILE ALONG 29°SOUTH LATITUDE

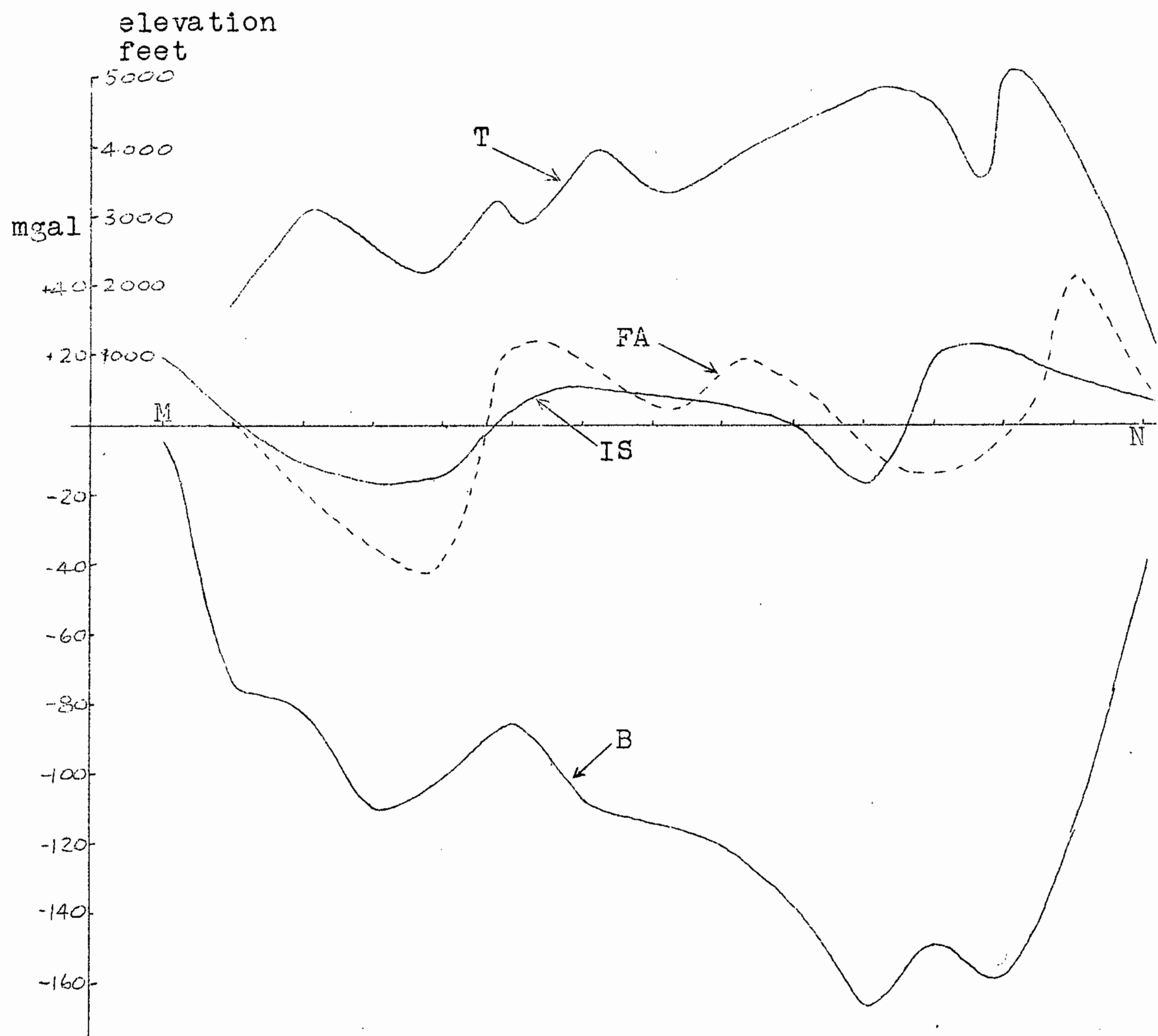


FIGURE 4/1

T = profile of topography

FA = profile of free air anomalies

IS = profile of isostatic anomalies (AH30)

B = profile of Bouguer anomalies

M = longitude 17°East

N = longitude 31°East

GRAVITY ANOMALY PROFILE ALONG 28°EAST LONGITUDE

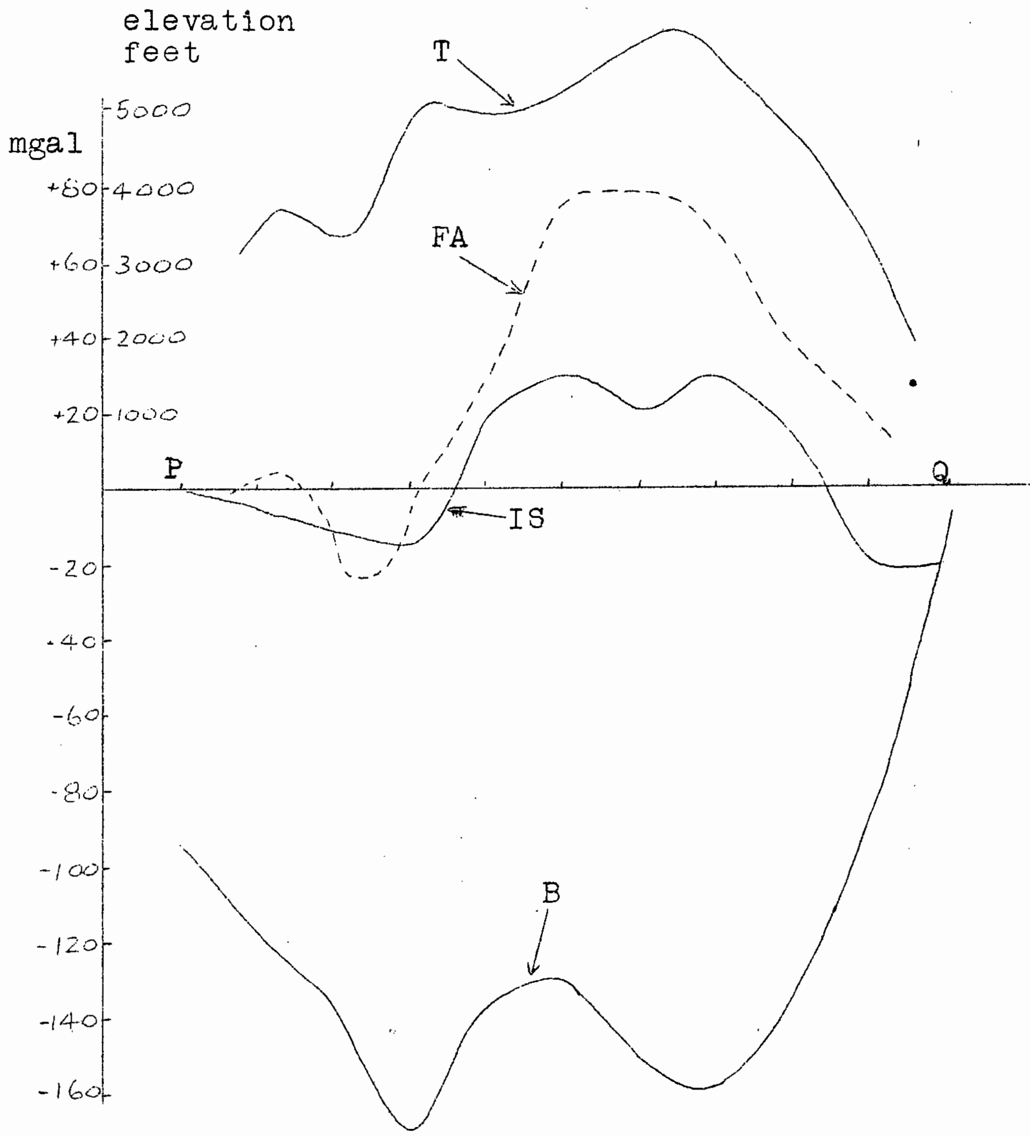


FIGURE 4/2

T = profile of topography

FA = profile of free air anomalies

IS = profile of isostatic anomalies (AH30)

B = profile of Bouguer anomalies

P = latitude 23°South

Q = latitude 33°South

(a) The Bouguer anomaly profile takes the form of an exaggerated mirror image of the topographic profile. High mountains give large negative Bouguer values.

(b) The free air anomaly profile follows the topography more closely than the isostatic anomaly profile. The free air anomalies are therefore more representative of the topography than the isostatic anomalies , while the Bouguer anomalies are more representative of the topography (in a negative sense) than the free air anomalies.

(c) The isostatic anomalies are , in general , smaller than the free air and Bouguer anomalies. This is an important fact , because if we use isostatic anomalies for calculations and give the value zero to the unsurveyed areas , then we are nearer the truth than with free air and Bouguer anomalies. For this reason the isostatic anomalies will give a smoother figure of the earth. (i.e. geoidal heights). SZABO 1962 says that in most parts of the world the isostatic anomalies are smaller because the isostatic compensation has a smoothing out effect upon the geoid

The points noted from figures 4/1 and 4/2 agree with the data published by BOMFORD 1962 and HEISKANEN 1958a.

From a practical point of view , the computation and drawing of isostatic anomalies involves much more work than for free air anomalies.

KAULA 1953 says that gravity anomalies must be reduced to sea level for application of Stokes' and Venning Meinesz formulae and therefore recommends that free air anomalies should be used. He also makes the point that further than $12^{\circ}.65$ from the computation station either free air or isostatic anomalies could be used.

As far as can be ascertained , SZABO 1962 is the only person who has used both free air and isostatic anomalies for the computation of deflections of the vertical at a number of points. In his comparative studies he shows that the deflections computed with isostatic anomalies are in reasonably good agreement with those computed by using free air anomalies.

CHAPTER 5

HEIGHTS ABOVE SEA LEVEL

5.01 General.

The general fomula for the height systems at present being used in the field of physical geodesy can be expressed as follows

$$\text{Height} = \frac{C}{G}$$

where C = the geopotential number

G = the value of gravity

For the dynamic height system , G = the normal gravity at an arbitrary latitude.

For the orthometric height system , G = the mean actual gravity value along the plumb line to the geoid.

For the normal height system , G = the mean normal gravity value along the plumb line to the reference spheroid. (HEISKANEN 1967)

5.02 Geopotential numbers.

The geopotential number , C , is not a height in a geometrical sense but it is important , because it is the most direct result of spirit levelling. It is a natural measure even if it does not have the dimension of length.

The geopotential number of a point is the difference between the potential at the geoid and the potential at the point. It is expressed in geopotential units , g.p.u.
1 g.p.u. = 1 Kgal.metre = 1000 gal.metre and
C = 0.98 H approx. where H is the height above mean sea level in metres.

5.03 Dynamic heights.

$$\text{Dynamic height} = DH = \frac{C}{\gamma_o}$$

where γ_o is the normal gravity at an arbitrary latitude on the international spheroid. This latitude is usually 45° and $\gamma_{45^\circ} = 980.6294$ gals.

If N is the sum of the levelling increments (i.e. the height differences as measured by a spirit level) then

$$DH = N + DC$$

where DC is the dynamic correction.

$$DC = \frac{g - \gamma_o}{\gamma_o} N$$ where g = mean gravity between the points whose dynamic height is being determined.

(Details and derivation of DC are given in BOMFORD 1962 and HEISKANEN 1967)

Dynamic corrections are large. For example , g between Bloemfontein and Kimberley is about 978.9 gals. (SMIT 1962) If $\gamma_o = \gamma_{45^\circ} = 980.6$ gals then the dynamic correction between Bloemfontein and Kimberley(height difference = 666 feet , see TRIGSURVEY 1966) is

$$DC = \frac{978.6 - 980.6}{980.6} 666 = -1.16 \text{ feet.}$$

Because of the large corrections to the measured height differences , dynamic heights are not much used for practical purposes.

The dynamic height is the distance between the geop through a point and the geoid measured along a plumb line at some chosen latitude , usually 45°. Dynamic heights therefore have no geometrical meaning but are significant in that points on the same geops have the same dynamic height.

5.04 Orthometric height.

$$\text{Orthometric height} = OH = \frac{C}{g_m}$$

where g_m is the mean gravity value (along the plumb line through the point whose OH is being determined) between the geoid and the point.

$$OH = N + OC$$

where OC is the orthometric correction.

HEISKANEN 1967 gives the following Ledersteger relation between dynamic and orthometric heights

$$OC_{AB} = DC_{AB} + DC_{A'A} + DC_{B'B}$$

where A and B are points on the earth's surface and A' and B' are the corresponding points on the geoid and the dynamic corrections are as given in 5.03

TRIGSURVEY 1966 use the following relationship

$$OC = -\ 2B.N.\sin 2\varphi . \Delta\varphi$$

where $B = 0.002644$

φ = mean latitude

$\Delta\varphi$ = difference in latitude.

Orthometric corrections are generally small. For example , the orthometric correction for the levelling route from Bloemfontein to Kimberley (height difference = 666 ft) is + 0.1168 feet i.e. about 0.018 ft per 100 feet of measured height distance. (TRIGSURVEY 1966) Because orthometric corrections are small , orthometric heights can be obtained with great accuracy.

The orthometric height is the geometric distance between the geop through a point and the geoid measured along the plumb line through the point.

5.05 Normal heights.

MOLODENSKY 1958 introduced the concept of normal heights in his study of the figure of the earth.

$$\text{Normal height} = NH = \frac{C}{g_r}$$

where g_r is the mean normal gravity along the plumb line between the point and the reference spheroid.

The normal height is a geometric height above the reference spheroid. The surface which is always a distance NH above the reference spheroid is called the " telluroid " by HIRVONEN 1960. The locus of points whose distances below the actual surface of the earth are equal to the normal heights , is called the quasi-geoid by Molodensky. The quasi-geoid has no physical meaning and is not an equipotential surface.

Normal heights , telluroid and quasi-geoid are terms used in the modern methods of determining the figure of the earth. See HEISKANEN 1967.

5.06 Trigonometric heights.

As trigonometric heights are obtained by observing vertical angles and as these angles are with reference to the direction of gravity at a point , these heights fall within the scope of physical geodesy.

If a single vertical angle is taken , then the computed height is the orthometric difference of height above a spheroid whose axes are the same as those of the reference spheroid. But in this case the spheroid is tangent to the geops at the point of observation. If reciprocal vertical angle observations are made , if the deflection of the vertical is the same at both ends of the line and if the geoidal section can be represented by a circle , then the mean computed height differences give the difference of orthometric height above the geoid. (BOMFORD 1962).

A large factor in trigonometric heights is the effect of atmospheric refraction , which to a certain extent is eliminated by taking reciprocal observations.

5.07 Barometric heights.

Barometric heights fall into the field of physical geodesy in that they depend on the variation of gravity with latitude and with altitude. But as the accuracy obtainable is nowhere near geodetic standards , barometric heights will not be discussed here.

5.08 Accuracy.

Dynamic and normal heights are as accurate as the geopotential numbers. If , for the geopotential number , we assume a standard error of $\pm$ 0.1 mm per Km of distance for geodetic levelling , then the geopotential number can be determined with an accuracy of $\pm$ 0.1 gal.metre per Km of distance. (HEISKANEN 1967). i.e. 10^{-4} g.p.u. per Km of distance.

As orthometric heights depend on the mean gravity along the plumb line between the point and the geoid , they also depend on factors such as the density below the point. This information is not known exactly. BOMFORD 1962 states that the mean gravity , g_m , can be estimated to about 1 in 10 000.

In trigonometric levelling , the uncertainty and variations of the atmospheric refraction make an accuracy of greater than 1 second of arc in the vertical angle difficult to attain. Over a distance of 10 Km , the standard error of the elevation difference for reciprocal observations is $\pm$ 10 cm. (HEISKANEN 1967). If short distances with reciprocal angles are observed , fairly accurate orthometric heights above the geoid are obtained.

CHAPTER 6

DEFLECTIONS OF THE VERTICAL

6.00 General.

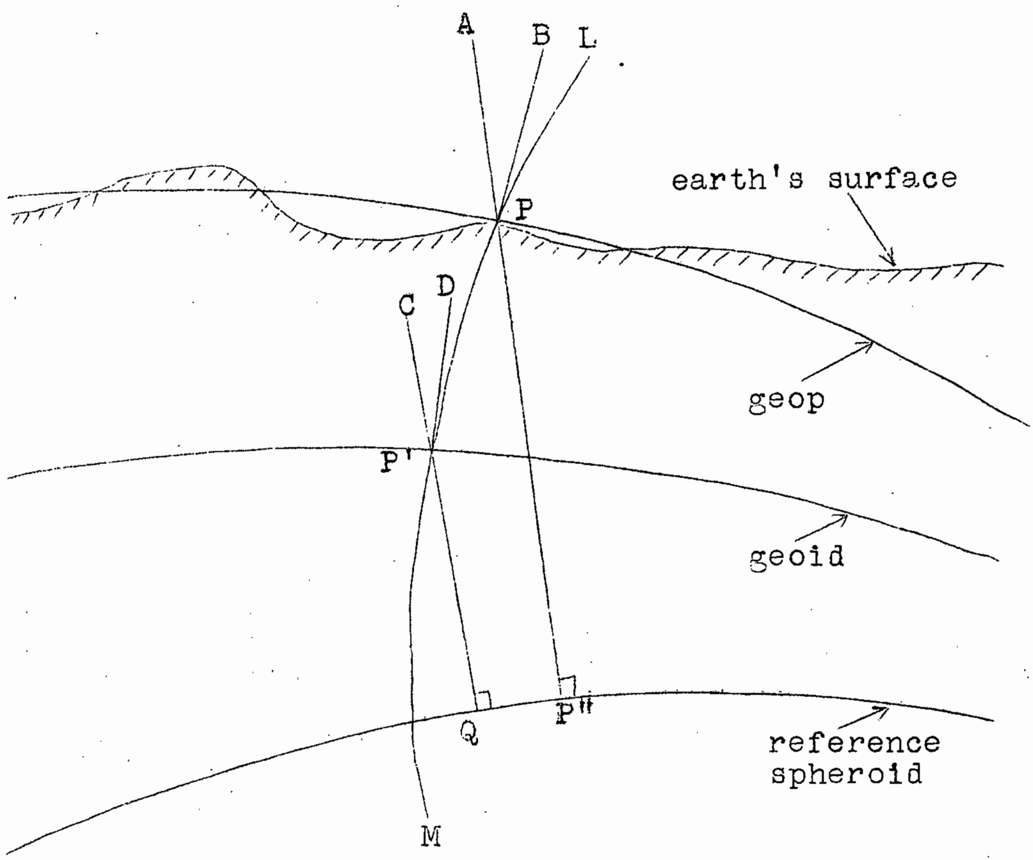


FIGURE 6/1

In the above figure , P is a point on the earth's surface. The geop passing through P , the geoid and the reference spheroid are shown. The curved line LM is the plumb line passing through P. This plumb line meets the geoid at P'. PA and P'C are the normals to the reference spheroid. PB is the vertical at P and P'D is the vertical at P'. The position of P is given by the astronomical latitude and longitude (as observed) and the position P" is given by the geodetic co-ordinates of P as computed.

Consider PB and P'D as gravity vectors and PA and P'C as the normal gravity vectors (I.E. perpendicular to the normal earth or reference spheroid. See section 1.01.) Then the difference in direction between PA and PB is the astronomical deflection of the vertical and the difference in direction between P'C and P'D is the gravimetric deflection of the vertical. Both these deflections are deflections as refered to P , the point on the earth's surface.

As LEDERSTEGER 1956 points out , " deflection of the vertical " is really a misnomer. The vertical is defined by the plumb line , we cannot therefore speak of a deflection. A better expression would be " deflection of the normal."

Note:- The difference in magnitude between P'C and P'D is the gravity anomaly at P. (Not at P'.)

The definition of the astronomical deflection of the vertical given above is known as Pizetti's definition and that for the gravimetric deflection is known as Helmert's definition.

Although the gravimetric deflection is regarded as the " absolute " deflection , it is dependent on the parameters of the International Spheroid. Strictly speaking there is no such thing as an " absolute deflection " because there is no absolute reference surface.

If figure 6/1 is considered as a north-south section , then the angles APB and CP'D are the north-south components of the respective deflections and if the figure is considered as an east-west section then these angles are the east-west components.

For astronomical deflections of the vertical

the north-south component = ξ_a

the east-west component = η_a

For gravimetric deflections of the vertical

the north-south component = ξ_g

the east-west component = η_g

And in general, the deflection = $\sqrt{\xi^2 + \eta^2}$

6.01 The astronomical deflection of the vertical.

We have the following relationships

$$\xi_a = \varphi_a - \varphi$$

$$\eta_a = (\lambda_a - \lambda)\cos\varphi$$

also $\eta_a = (\alpha_a - \alpha)\cot\varphi$

where $\varphi_a, \lambda_a, \alpha_a$ = astronomical latitude , longitude and azimuth.

φ, λ, α = geodetic latitude , longitude and azimuth.

The astronomical deflections are relative because they depend on the geodetic co-ordinates φ, λ, α These geodetic co-ordinates , in turn , depend on

(a) the particular reference spheroid used

(b) the orientation of the reference spheroid i.e. the deflection components and azimuth adopted at the initial point.

(c) the accuracy of the triangulation net , which involves many factors.

6.02 Corrections to the astronomical deflections.

In order to compare astronomical deflections with gravimetric deflections , certain corrections have to be applied to the former. These corrections arise because of

(a) the different star catalogues which may have been used in the computations ;

(b) the variation of the pole ;

(c) the fact that astronomical deflections are usually referred to the earth's surface while gravimetric deflections are referred to the geoid;

(d) the fact that the spheroid used when calculating the astronomical deflections might not be the same as the spheroid used in the International Gravity Formula.

6.03 Reduction to the same star catalogue.

It is possible that the astronomical latitudes and longitudes have been observed and computed over a large number of years and that different star catalogues have been used in the computations. These computed values, for the purposes of comparison, have to be reduced to the same star catalogue system (usually the FK4 system). These reductions are done by using specially prepared tables. (RICE 1952)

6.04 Variation of the pole

The pole of the earth wanders around its mean position along an irregular path which lies inside a circle of about 10 metres (or 0.3 seconds of arc). All astronomical observations have therefore to be reduced to the same epoch.

The corrections to the latitude and longitude as computed are

$$d\varphi = - Y \sin\lambda + X \cos\lambda$$

$$d(\lambda_2 - \lambda_1) = - (Y \cos\lambda_2 + X \sin\lambda_2)\tan\varphi_2 + (Y \cos\lambda_1 + X \sin\lambda_1)\tan\varphi_1$$

where λ = the longitude of the point, positive eastwards from Greenwich.

X, Y = co-ordinates of the pole in seconds of arc referred to its mean position. (Published by the International Latitude Service.)

The subscript " 1 " refers to the observatory sending the time signals and the subscript " 2 " refers to the computation point (i.e. the field station).

6.05 Reduction to sea level.

As explained in 6.00 the astronomical deflection refers to a point on the actual surface of the earth and the gravimetric deflection refers to a point on the geoid. In order, therefore, to compare these deflections a sea level correction must be applied. This is done indirectly by applying the corrections to the astronomical latitude and longitude.

As an approximation , the correction to the observed latitude = $d\varphi'' = -0.000171\ h \sin 2\varphi$

where φ = latitude of computation point

h = elevation of point in metres

(DERENYI 1963 incorrectly gives the co-efficient as 0.00171).

The above correction is consistent with the International Gravity Formula (4-1). As this formula has no longitude term , no correction is needed for the astronomical longitude.

GILL 1896 used the reduction $-0.052\ h \sin 2\varphi$ where h = height of station in Kilo-feet for the South African Geodetic Survey. This is the same as the previously mentioned reduction for h in metres.

In reducing the astronomical point to sea level , we must also take into account the curved line of the plumb line (or the irregular gravity anomalies). The further corrections which are needed are

$$d\varphi = 0''.21 \frac{\delta \Delta g}{\delta X} h$$

$$d\lambda = 0''.21 \frac{\delta \Delta g}{\delta Y} h$$

where $\frac{\delta \Delta g}{\delta X}$ and $\frac{\delta \Delta g}{\delta Y}$ are the north-south and east-west gravity gradients in milligals, and h is in the same linear units as X and Y.

In practice , the above mentioned three corrections are the only ones applied for reduction to sea level. (RICE 1952)

6.06 Change of spheroid.

We use approximations of the VENING MEINESZ 1950 formulae by leaving out squares and products of small quantities.

$$\begin{aligned} d\xi = & \left[\sin(\varphi-\varphi_o) - 2\cos\varphi_o \sin\varphi \sin^2\tfrac{1}{2}(\lambda-\lambda_o)\right]\Delta\beta \\ & - \left[4\cos\varphi\cos\tfrac{1}{2}(\varphi+\varphi_o)\sin\tfrac{1}{2}(\varphi-\varphi_o)\right]\Delta\alpha \\ & - (2+\tfrac{3}{4}\tan\varphi_o \sin 4\varphi_o)\sin(\varphi-\varphi_o)\alpha\Delta\alpha \end{aligned} \qquad (6\text{-}1)$$

$$\begin{aligned} d\eta = & -\cos\varphi_o \sin(\lambda-\lambda_o)\Delta\beta \\ & + \tfrac{1}{4}\sin\varphi_o . \sin 4\varphi_o \sin(\lambda-\lambda_o)\alpha\,\Delta\alpha \end{aligned} \qquad (6\text{-}2)$$

where $d\xi$ = correction to ξ_a

$d\eta$ = correction to η_a

φ_o, λ_o = geodetic co-ordinates of initial point

φ, λ = geodetic co-ordinates of computation point

Δa = (major axis International Spheroid) − (major axis local spheroid)

$\Delta\alpha$ = (flattening International Spheroid) − (flattening local spheroid)

$\Delta\beta = \frac{\Delta a}{a} + \sin^2\varphi_o \cdot \Delta\alpha$

Notes: (1) $-d\xi$ is the change in the geographical latitude and $-d\eta \sec\varphi$ is the change in the geographical longitude at the computation point due to changes in the elements a and α of the spheroid.

(2) These formulae are approximations which assume

(a) changes of the semi axis of the spheroid are not greater than 1/20 000 th part of these axes. Therefore squares of $\frac{\Delta a}{a}$ and $\frac{\Delta b}{b}$ have been neglected. Also $\alpha^2\frac{\Delta a}{a}$ and $\alpha^2\frac{\Delta b}{b}$ have been neglected

(b) $\alpha\Delta\alpha$ and $\alpha\Delta\beta$ have an order of magnitude of less than 1 in 6 000 000 and therefore when they are multiplied by terms containing squares and products of $(\varphi-\varphi_o)$ and $(\lambda-\lambda_o)$ we can neglect these terms.

If the triangulation system being investigated is of world wide extent , then the above assumptions should not be made.

For South Africa :-

$$\left.\begin{aligned} \varphi_o &= 33^\circ\ 59'\ 32''.00 \text{ South} \\ \lambda_o &= 25^\circ\ 30'\ 44''.622 \text{ East} \end{aligned}\right\} \text{Buffelsfontein}$$

Δa = (major axis International spheroid)

– (major axis Modified Clarke 1880 spheroid)

= (6 378 388 – 6 378 249.145 326) metres

(JORDAN 1958 , HENDRIKZ 1956)

= 138.854 674 metres

$\Delta\alpha$ = 0.00336 70033 67003 – 0.00340 75461 94953

(JORDAN 1958 , and evaluating f given by HENDRIKZ 1956)

= – 0.00004 05428 27950

$\Delta\beta$ = + 0.00000 90982

6.07 Re-orientation of spheroid.

$$d\xi = \frac{M_o}{M}\left[\cos(\varphi-\varphi_o) - 2\sin\varphi\sin\varphi_o\sin^2\tfrac{1}{2}(\lambda-\lambda_o)\right]d\xi_o + \frac{N_o}{M}\sin\varphi\sin(\lambda-\lambda_o)\,d\eta_o + \frac{1}{M}\left[\sin(\varphi-\varphi_o) - 2\sin\varphi\cos\varphi_o\sin^2\tfrac{1}{2}(\lambda-\lambda_o)\right]dN_o$$

$$d\eta = -\frac{M_o}{N}\sin\varphi_o\sin(\lambda-\lambda_o)\,d\xi_o + \frac{N_o}{N}\cos(\lambda-\lambda_o)\,d\eta_o - \frac{1}{N}\cos\varphi_o\sin(\lambda-\lambda_o)\,dN_o$$

where

$d\xi$ = correction to ξ_a

$d\eta$ = correction to η_a

M_o = radius of curvature of meridian at initial point

M = radius of curvature of meridian at computation point

N_o = radius of curvature of prime vertical at initial point

N = radius of curvature of prime vertical at computation pt

$d\xi_o, d\eta_o$ = deflection components at initial point

dN_o = geoidal height at the initial point

For South Africa :-

M_o = 20 850 330 Eng. feet. (UCT 1965)

N_o = 20 948 500 Eng. feet. (UCT 1965)

$d\xi_o$ = 0 , assumed (or 3"46 to North using TSO 1954)

$d\eta_o$ = 0 , assumed (or 0"88 to West using TSO 1954)

dN_o = 0 , assumed.

CHAPTER 7

GRAVIMETRIC DEFLECTIONS OF THE VERTICAL

7.01 General.

On 23rd April , 1849 G.G.Stokes read a paper " On the variation of gravity at the surface of the earth " to the Cambridge Philosophical Society. (STOKES 1849). Since then the calculation of geoid undulations using gravity data has been one of the clssical problems in geodesy. In 1928 Vening Meinesz , using Stokes' results , derived formulae for calculating deflections of the vertical using gravity anomalies. (VENING MEINESZ 1928). From the theoretical point of view , not much need be added to the formulae obtained by Stokes and Vening Meinesz.

These formulae presuppose complete and comprehensive gravity data over the whole earth. The fact is that this complete gravity data does not exist and it does not seem likely that such coverage will be possible in the near future. In view of this , and other considerations , the formulae derived by Stokes and Vening Meinesz must be used in a modified form.

7.02 Basic Theory.

In the following review of the basic theory , we will use the nomenclature in general use today and not the symbols used in the original papers quoted.

In article 31 of his paper , STOKES 1849 arrived at a formula for determining the geoid from gravity observations , namely

$$N = \frac{R}{2\pi G} \int_0^{2\pi} da \int_0^{\pi} \Delta g . f(\psi) \sin\psi . d\psi \qquad \ldots\ldots\ldots\ldots (7\text{-}1)$$

where N = geoidal height

R = mean radius of the earth

G = mean gravity

Δg = the gravity anomaly

$$2f(\psi) = \operatorname{cosec} \psi/2 + 1 - 6 \sin \psi/2 - 5 \cos \psi - 3 \cos \psi . \log_e [\sin \psi/2 (1 + \sin \psi/2)] \quad \ldots\ldots (7\text{-}2)$$

ψ, a = polar co-ordinates of the point where Δg applies. (the centre of the co-ordinate system is where N applies)

Note : In the original paper , STOKES 1849 , and in some subsequent literature , the constant in equation (7-1) is given as $\frac{R}{4\pi G}$. In these cases the function of ψ used is equal to twice the value of $f(\psi)$ above.

Equation (7-1) is called Stokes' Theorem and it enables us to find the geoid (or the geoidal heights) over the whole earth , provided the gravity anomalies over the whole earth are known. But as $f(\psi)$ is small for great distances from the point where N is being computed , distant anomalies have a smaller effect.

Because of the manner in which (7-1) has been derived , the theorem is valid only if no masses are present outside the geoid. Hence we have the various reduction methods as described in chapter 3. HIRVONEN 1962 says , " The conclusion can be made , however , that the errors caused by the theoretical defects of Stokes' formula are essentially smaller than the errors which still at present and long in the future will be caused by the scantiness of the gravity measurements actually carried out. "

The gravimetric deflection of the vertical is the slope of N. This deflection can be expressed in terms of the two components, ξ_g in the north-south direction and η_g in the east-west direction.

Note : The suffix g to ξ and η will be omitted hereafter and gravimetric components will be assumed unless otherwise stated.

$$\xi = -\frac{\partial N}{\partial X} \quad \text{and} \quad \eta = -\frac{\partial N}{\partial Y} \qquad \ldots\ldots\ldots (7\text{-}3)$$

The minus sign in (7-3) is a convention used to correspond with the definitions

ξ = (astronomical latitude) - (geodetic latitude)

η = (astronomical longitude - geodetic longitude) x (cosine geodetic latitude)

.... (7-3a)

(HEISKANEN 1967)

In (7-3) X is taken in a north-south direction and Y is taken in an east-west direction being positive towards the east.

In 1928 Vening Meinesz differentiated Stokes' theorem to obtain the deflection of the vertical, which can be expressed as follows :-

$$\left.\begin{aligned} \xi'' &= -\frac{\rho''}{2\pi G}\int_0^{2\pi}\cos a\,da\int_0^{\pi}\frac{df(\psi)}{d\psi}\sin\psi.\Delta g.d\psi \\ \eta'' &= -\frac{\rho''}{2\pi G}\int_0^{2\pi}\sin a\,da\int_0^{\pi}\frac{df(\psi)}{d\psi}\sin\psi.\Delta g.d\psi \end{aligned}\right\} \ldots (7\text{-}4)$$

SOLLINS 1947 has computed tables for $\frac{df(\psi)}{d\psi}\sin\psi$; $\int\frac{df(\psi)}{d\psi}\sin\psi.d\psi$ and $\frac{df(\psi)}{d\psi}$

Notes : (A) The gravimetric deflections calculated from (7-4) under ideal conditions (a knowledge of gravity over the entire earth's surface) represents absolute values in contrast to astronomical deflections which represent relative values. (See 6.00 with regard to " absolute " values).

(B) For the calculations of the gravimetric deflections , the following conditions must apply :-

(1) The gravity field observations must be converted to the same world gravity system;

(2) The gravity formula used must give the value zero for the mean gravity anomaly of the earth;

(3) The flattening of the reference spheroid and that of the spheroid used in the theoretical gravity formula must be the same.

7.03 A practical computation procedure.

The literature in the field of physical geodesy shows that the theoretical developments of the concepts are usually more advanced than the practical applications of these concepts. This is mainly due to insufficient gravity material being available.

The problem , then , in the application of the theory is to work out a practical method for performing the computations. The gravimetric methods used to compute the vertical deflection components will depend on the behaviour of equation (7-4) when ψ is small.

The principles used in working out a practical computation procedure are

(a) The gravity field immediately surrounding the computation point plays a decisive part in the deflection of the vertical ;

(b) The effect of the distant areas , while not being neglected , play a smaller part in the deflections ;

(c) At the computation point , the Vening Meinesz formulae are indeterminate.

7.04 The square method.

If we consider a surface element to be represented by an infinitely small square dq , we could replace $\sin\psi . d\psi . da$ by dq in equation (7-4) and get

$$\left.\begin{aligned} \xi'' &= -\frac{\rho''}{2\pi G}\int_q \frac{df(\psi)}{d\psi}\cos a\,\Delta g\,.\,dq \\ \eta'' &= -\frac{\rho''}{2\pi G}\int_q \frac{df(\psi)}{d\psi}\sin a\,.\,\Delta g\,.\,dq \end{aligned}\right\} \quad \ldots\ldots \text{(7-5)}$$

Note : a , the azimuth of dq , is measured from South through West.

If we now replace dq by a small square of finite dimension q and evaluate the constant taking ρ = 206265" , G = 979800 mgals , we get

$$\left.\begin{aligned} \xi'' &= -0.''0335 \sum \frac{df(\psi)}{d\psi}\cos a\,.\,q\,.\,\Delta g \\ \eta'' &= -0.''0335 \sum \frac{df(\psi)}{d\psi}\sin a\,.\,q\,.\,\Delta g \end{aligned}\right\} \quad \ldots\ldots \text{(7-6)}$$

This summation must be extended over half the earth around the computation point.

The use of equation (7-6) is known as the "square" method. (The figures are actually spherical trapezoids) .

7.05 To calculate the area of the square , q , in (7-6) we use the following formulae (JORDAN 1958) :-

$$\begin{aligned} q\ (\text{sq. Km}) = \frac{b^2\pi\Delta\lambda}{90}\Big(& A\cos\varphi\sin\frac{\Delta\varphi}{2} - B\cos 3\varphi\sin 3\frac{\Delta\varphi}{2} \\ & + C\cos 5\varphi\sin 5\frac{\Delta\varphi}{2} - D\cos 7\varphi\sin 7\frac{\Delta\varphi}{2} \\ & + E\cos 9\varphi\sin 9\frac{\Delta\varphi}{2} \quad \ldots\ldots\ldots\Big) \end{aligned}$$

$$\ldots\ldots\ldots\ldots \text{(7-7)}$$

where

$$A = 1 + \frac{1}{2}e^2 + \frac{3}{8}e^4 + \frac{5}{16}e^6 + \frac{35}{128}e^8 + \frac{63}{256}e^{10}$$

$$B = \frac{1}{6}e^2 + \frac{3}{16}e^4 + \frac{3}{16}e^6 + \frac{35}{192}e^8 + \frac{45}{256}e^{10}$$

$$C = \frac{3}{80}e^4 + \frac{1}{16}e^6 + \frac{5}{64}e^8 + \frac{45}{512}e^{10}$$

$$D = \frac{1}{112}e^6 + \frac{5}{256}e^8 + \frac{15}{512}e^{10}$$

$$E = \frac{5}{2304}e^8 + \frac{3}{512}e^{10}$$

$$F = \frac{3}{5632}e^{10}$$

e = eccentricity of the spheroid. (For the international spheroid $e^2 = 0.00676\ 81701\ 97224$).

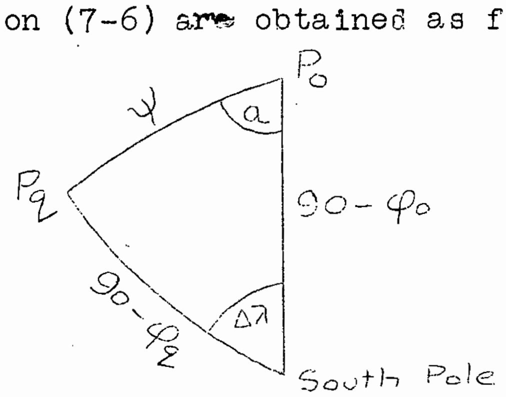
b = semi-minor axis of the spheroid. (For the international spheroid $b = 6\ 356\ 911.946\ 13$ metres).

φ = mean latitude of square

$\Delta\varphi$ = difference (in degrees) between northern and southern limits of square.

$\Delta\lambda$ = difference in degrees between eastern and western limits of square.

7.06 The azimuth , a , and the angular distance , ψ , required in equation (7-6) are obtained as follows :-



In the above figure

P_o = computation point

φ_o = latitude of computation point

P_q = centre of square q

φ_q = Latitude of P_q

$\Delta\lambda$ = difference in longitude between P_o and P_q

$$\left.\begin{aligned} \cos\psi &= \sin\varphi_o \sin\varphi_q + \cos\varphi_o \cos\varphi_q \cos\Delta\lambda \\ \sin a &= \frac{\cos\varphi_q \sin\Delta\lambda}{\sin\psi} \end{aligned}\right\} \quad \text{..(7-8)}$$

7.07 The last quantity needed to compute equation (7-6) is $\frac{d f(\psi)}{d \psi}$

From equation (7-2) we get

$$\frac{df(\psi)}{d\psi} = -0.25 \frac{\cos \psi/2}{\sin^2 \psi/2} - 1.5 \cos \psi/2 + 2.5 \sin \psi$$
$$+ 1.5 \sin \psi \log_e(\sin \psi/2 + \sin^2 \psi/2)$$
$$- 0.75 \left(\frac{1 + 2 \sin \psi/2}{1 + \sin \psi/2} \right) \cot \psi/2 \cos \psi \quad \ldots\ldots. \quad (7\text{-}9)$$

and SOLLINS 1947 has given

$$\frac{df(\psi)}{d\psi}\sin \psi = \tfrac{1}{2} \Big[- \operatorname{cosec} \psi/2 - 3 - 8 \sin \psi/2 + 32 \sin^2 \psi/2$$
$$+ 12 \sin^3 \psi/2 - 32 \sin^4 \psi/2$$
$$+ 3 \sin^2 \psi \log_e(\sin \psi/2 + \sin^2 \psi/2) \Big] \ldots. \quad (7\text{-}9a)$$

7.08

When using equation (7-6) for computing ξ and η for any point , we divide the earth into $1^\circ \times 1^\circ$ or $5^\circ \times 5^\circ$ spherical trapezoids ("squares").

HEISKANEN 1959 says that experience has shown that for radial distances greater than 20° from the computation point , $5^\circ \times 5^\circ$ squares should be used and that for radial distances from 3° to 20° from the computation point , squares of $1^\circ \times 1^\circ$ should be used.

For radial distances of less than 3° from the computation point , the squares have too large an effect, as $\frac{d f(\psi)}{d \psi}$ approaches infinity as ψ approaches zero and therefore a different method of computation will have to be used. In this case a further sub-division is made and we use the " circle-ring " method.

7.09 The circle-ring method

Equation (7-4) was

$$\xi'' = -\frac{\rho''}{2\pi G}\int_0^{2\pi}\cos a\, da \int_0^{\pi}\frac{df(\psi)}{d\psi}\sin\psi.\Delta g.d\psi$$

From this equation we get

$$d\xi'' = -\frac{\rho''}{2\pi G}\int_{a_1}^{a_2}\cos a\, da\int_{\psi}^{\psi+d\psi} f'(\psi)\sin\psi.\Delta g_{\psi}\, d\psi$$

or

$$\left.\begin{aligned} d\xi'' &= -\frac{\rho''}{2\pi G}(\sin a_2 - \sin a_1)\int_{\psi}^{\psi+d\psi} f'(\psi)\sin\psi\,\Delta g_{\psi}.d\psi \\ d\eta'' &= -\frac{\rho''}{2\pi G}(\cos a_1 - \cos a_2)\int_{\psi}^{\psi+d\psi} f'(\psi)\sin\psi.\Delta g_{\psi}.d\psi \end{aligned}\right\} \text{..(7-10)}$$

where $d\xi''$ and $d\eta''$ are the effects on the components ξ'' and η'' of a circular ring compartment (with radial boundaries ψ and $\psi + d\psi$ and azimuth boundaries a_1 and a_2) and the gravity anomaly Δg_{ψ}

The best practical procedure is that suggested and used by RICE 1952. This is a modification and improvement on Kasansky's method. (SAKATOW 1957 , HEISKANEN 1958a) Rice used circular templates having a uniform angular aperture of 10 degrees . The tables published in SOLLINS 1947 were used and radii computed so that the effect of each compartment has a radial deflection effect of 0.001 seconds of arc for a mean anomaly of one milligal. i.e. $\frac{\rho''}{2\pi G}\frac{df(\psi)}{d\psi}\sin\psi.d\psi\, da = 0''.001$ always. The radii in this case are in geometric progression with a common ratio of 1.1864. The mean gravity anomaly of each compartment is estimated and the effect of each compartment is computed. This effect is multiplied by cos a and the summation gives the total effect on ξ Multiplying the effects by sin a and summing gives the total effect on η

In using Stokes' function $f(\psi)$(and its variations) as outlined above for determining gravimetric deflections , finite summation cannot be applied to the area immediately surrounding the computation point. In the region within about 10 Km of the computation point , we must therefore use a different method.

7.10 The gradient method.

In order that the integration should lead to a finite result , the gravity anomaly of any point , in the small region surrounding the computation point , can be expressed as a function of position.

Regard a circular area with radius r_o around the computation point as a plane region. Take rectangular co-ordinates with the origin at the centre , the X-axis in a north-south direction and the Y-axis in an east-west direction. Assume that

$$\Delta g = \Delta g_o + X\frac{\delta \Delta g}{\delta X} + Y\frac{\delta \Delta g}{\delta Y}$$

$$= \Delta g_o + r\cos a\frac{\delta \Delta g}{\delta X} + r\sin a\frac{\delta \Delta g}{\delta Y} \quad \ldots\ldots \quad (7\text{-}11)$$

Consider the earth as a sphere with radius R

$\therefore \psi = \dfrac{r}{R}$ and $d\psi = \dfrac{dr}{R}$. Substitute these values in the first two terms of equation (7-9a) and we get

$$\frac{df(\psi)}{d\psi}\sin\psi . d\psi = -\left(\frac{1}{r} + \frac{3}{2R}\right)dr \quad \ldots\ldots\ldots \quad (7\text{-}12)$$

Substutute (7-11) and (7-12) in (7-4) to obtain

$$d\xi'' = \frac{\rho}{2\pi G}\int_0^{r_o}\int_0^{2\pi}\left[\Delta g_o + r\cos a\frac{\delta\Delta g}{\delta X} + r\sin a\frac{\delta\Delta g}{\delta Y}\right]\cos a\left[\frac{1}{r}+\frac{3}{2R}\right]da\,dr$$

$$d\eta'' = \frac{\rho}{2\pi G}\int_0^{r_o}\int_0^{2\pi}\left[\Delta g_o + r\cos a\frac{\delta\Delta g}{\delta X} + r\sin a\frac{\delta\Delta g}{\delta Y}\right]\sin a\left[\frac{1}{r}+\frac{3}{2R}\right]da . dr \quad (7\text{-}13)$$

and the integration of (7-13) gives

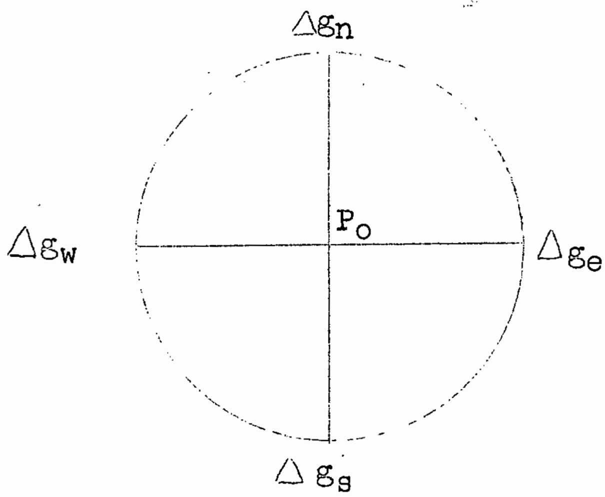
$$\left.\begin{aligned} d\xi'' &= \frac{\rho''}{2G}\left[r_o + \frac{3}{4R}r_o^2\right]\frac{\delta\Delta g}{\delta X} \\ d\eta'' &= \frac{\rho''}{2G}\left[r_o + \frac{3}{4R}r_o^2\right]\frac{\delta\Delta g}{\delta Y}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (7\text{-}14)$$

i.e. we now have expressions for the portions of ξ and η contributed by the inner zone of radius r_o .

Substitute the values 979800 for G and 6371 for R to get

$$\left.\begin{aligned} d\xi'' &= 0''.10526 \left[r_o + 0.00012\, r_o^2\right] \frac{\delta \Delta g}{X} \\ d\eta'' &= 0''.10526 \left[r_o + 0.00012\, r_o^2\right] \frac{\delta \Delta g}{Y} \end{aligned}\right\} \quad \ldots\ldots\ldots (7\text{-}15)$$

where $\frac{\delta \Delta g}{X}$ and $\frac{\delta \Delta g}{Y}$ are the gravity " gradients " in milligals in the north-south and east-west directions respectively and $r_o = \frac{\delta X}{2} = \frac{\delta Y}{2}$ in Kilometres



The figure above shows the situation near the computation point P_o . The radius of the circle is r_o and the gravity anomalies at the north , south , west and east points of the circle are Δg_n , Δg_s , Δg_w and Δg_e respectively.

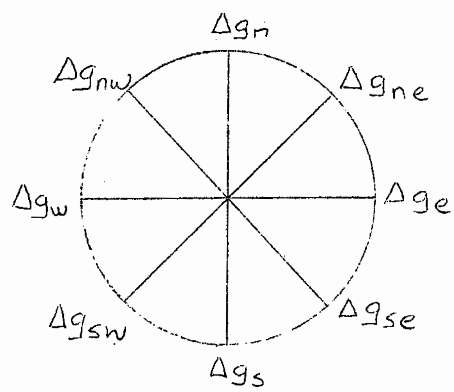
Formulae (7-15) now become

$$d\xi'' = 0''.05263\ (1 + 0.00012\, r_o)(\Delta g_s - \Delta g_n)$$
$$d\eta'' = 0''.05263\ (1 + 0.00012\, r_o)(\Delta g_e - \Delta g_w)$$

and neglecting the small term with r_o we have

$$\left.\begin{aligned} d\xi'' &= 0''.05263\ (\Delta g_s - \Delta g_n) \\ d\eta'' &= 0''.05263\ (\Delta g_e - \Delta g_w) \end{aligned}\right. \quad \ldots\ldots\ldots\ldots (7\text{-}16)$$

From the above reasoning , we note that equations (7-15) and (7-16) are strictly valid only if the gravity gradient is constant over the area within the circle i.e. we should have parallel gravity anomaly contours at a uniform spacing. RICE 1952 has shown that even if this condition does not apply , $d\xi''$ and $d\eta''$ could be evaluated to sufficient accuracy (if the local gravity survey is adequate) by increasing the accuracy of equations (7-16). This is done by taking four additional gradient lines as shown in the figure below.



i.e. the circle is divided into 45° sectors and Δg_{nw} , Δg_{ne} , Δg_{se} and Δg_{sw} are the additional anomalies at the north-west , north-east , south-east and south-west points on the circle.

The gravity gradients Δg_n to Δg_s and Δg_w to Δg_e are given weight 1 and the gravity gradients Δg_{nw} to Δg_{ne} , Δg_{sw} to Δg_{se} , Δg_{nw} to Δg_{sw} and Δg_{ne} to Δg_{se} are given weight $\frac{1}{2}$. When considering gradients of half weight it must be remembered that the distance over which the gradient applies is $(2)^{-\frac{1}{2}}$ of the radius. Taking the distances into account and using the above mentioned weights , we utilise all six gradient lines to get an improvement on equation (7-16) , namely

$$d\xi'' = 0''.026365\ (\Delta g_s - \Delta g_n) + 0''.01861\ \left[(\Delta g_{se}-\Delta g_{ne})+(\Delta g_{sw}-\Delta g_{nw})\right]$$
$$d\eta'' = 0''.026365\ (\Delta g_e - \Delta g_w) + 0''.01861\ \left[(\Delta g_{se}-\Delta g_{sw})+(\Delta g_{ne}-\Delta g_{nw})\right]$$

.... (7-17)

RICE 1952 gives the following equations (converted to our nomenclature) for the north-south and east-west gradients

$$\xi_1 = 0''.105\left(\frac{\delta \Delta g}{\delta Y}\right) r_o$$

$$= 0''.105 \; (\Delta g_s - \Delta g_n) \; \tfrac{1}{2}$$

$$= 0''.0525 \; (\Delta g_s - \Delta g_n)$$

and similarly $\eta_1 = 0''.0525 \; (\Delta g_e - \Delta g_w)$

From Rice's results and his published free air anomaly contour sketches , we can deduce that for the additional gradient lines he used

$$\xi_2 = 0''.105 \; (\Delta g_{se} - \Delta g_{ne})(2)^{-\frac{1}{2}}$$

$$= 0''.0742 \; (\Delta g_{se} - \Delta g_{ne})$$

$$\xi_3 = 0''.0742 \; (\Delta g_{sw} - \Delta g_{nw})$$

$$\eta_2 = 0''.0742 \; (\Delta g_{se} - \Delta g_{sw})$$

$$\eta_3 = 0''.0742 \; (\Delta g_{ne} - \Delta g_{nw})$$

and mean $\xi = \dfrac{2\xi_1 + \xi_2 + \xi_3}{4}$

mean $\eta = \dfrac{2\eta_1 + \eta_2 + \eta_3}{4}$.

The above will give results similar to those obtained from equations (7-17) but (7-17) are much quicker to use and the mean is obtained directly.

7.11 Summary and analysis.

Considering the basic theory and the limitations of some of the computation equations , we can set out a practical procedure for computing the gravimetric deflections of the vertical at a point , based on the methods and suggestions of HEISKANEN 1958a and 1958b and RICE 1952. A summary of part of the following has been published elsewhere in LOON 1967.

We divide our procedure into four stages. At the end of each stage a portion of the deflection component is obtained , so that

$$\xi = d_1\xi + d_2\xi + d_3\xi + d_4\xi$$
$$\eta = d_1\eta + d_2\eta + d_3\eta + d_4\eta \qquad \text{........... (7-18)}$$

Stage 1.

In this stage we use Rice's gradient method for computing the effect of the immediate neighbourhood of the station on ξ and η . i.e. we compute $d_1\xi$ and $d_1\eta$. A fundamental circle of radius between 0.5 Km and 10 Km is usually recommended. The nature of the gravity anomalies around the computation point will determine the radius chosen,i.e. that radius which will give a uniform gravity gradient. The circle is divided into eight sectors of 45° each to obtain the three north-south and the three east-west gravity gradients. $d_1\xi$ and $d_1\eta$ are found by using equations (7-17).

Note : For r_o ,Rice used various values eg. 0.279 Km , 0.554 Km and 4.320 Km. DE VOS VAN STEENWIJK 1947 used 30 Km. HEISKANEN 1958b suggests from 0.5 Km to 5.0 Km depending on the accuracy of the gravity anomalies.

Stage 2.

The area outside the fundamental circle is now divided into circular rings up to an angular distance of about 3^{o} from the computation point. The length of the radii of these rings are in geometric progression with a ratio of 1.1864 and are divided into 36 equal compartments of 10^{o} angular aperture each.

The following table has been adapted from RICE 1952

Angular distance degrees	Inner radius Km	Angular distance degrees	Inner radius Km
0.004	0.467	0.129	14.29
0.005	0.554	0.152	16.94
0.006	0.657	0.181	20.09
0.007	0.780	0.214	23.83
0.008	0.926	0.254	28.25
0.010	1.099	0.301	33.48
0.012	1.304	0.357	39.67
0.014	1.547	0.423	47.00
0.017	1.836	0.501	55.66
0.020	2.179	0.593	65.90
0.023	2.586	0.701	77.97
0.028	3.068	0.829	92.22
0.033	3.641	0.980	109.0
0.039	4.320	1.157	128.7
0.046	5.125	1.366	151.9
0.055	6.081	1.611	179.1
0.065	7.216	1.897	210.9
0.077	8.560	2.230	248.0
0.091	10.15	2.619	291.2
0.108	12.05	3.068	341.2

Table for radial deflection effect of 0".001 with mean compartment anomaly of 1 mgal and angular aperture = 10^{o}

The average gravity anomaly for each compartment is estimated. The effect for each compartment is computed i.e. average gravity anomaly in milligals multiplied by 0.001 = effect in seconds of arc. These effects are added up for each 10° sector. This sum is then multiplied first by cos a then by sin a . (Where a is the azimuth of the median of the sector i.e. 5° , 15° , 25° and so on). The sum of all the cos a terms gives $d_2\xi$ and the sum of all the sin a terms gives $d_2\eta$.

(RICE 1952 went up to about $4\overset{\circ}{.}8$ and HEISKANEN 1958a reports that the Columbus Group went up to $9\overset{\circ}{.}8$ for this method. As reported in chapter 8 , we have gone up to about 8° in this investigation.)

Stage 3.

From 3° to 20° away from the computation point , the mean gravity anomalies of $1^{\circ} \times 1^{\circ}$ squares are used. Equation (7-6) is used in this stage and can be written as follows - for the effects of any $1^{\circ} \times 1^{\circ}$ square :-

$$\begin{aligned} \Delta\xi'' &= -0.''0335 \frac{df(\psi)}{d\psi} \cos a . q . \Delta g = c'\xi . \Delta g_q \\ \Delta\eta'' &= \qquad\qquad\qquad\qquad\qquad\quad = c'\eta . \Delta g_q \end{aligned} \qquad \text{.............} \quad (7\text{-}19)$$

where $c'\xi = -0.''0335 \frac{df(\psi)}{d\psi} \cos a . q$

$c'\eta = -0.''0335 \frac{df(\psi)}{d\psi} \sin a . q$

a = azimuth from computation point to centre of square. Use equation (7-8).

q = area of square. Use equation (7-7).

Δg_q = mean gravity anomaly of the square.

$\frac{df(\psi)}{d\psi}$ can be found from tables, eg. SOLLINS 1947.

$c'\xi$ and $c'\eta$, called the Vening Meinesz co-efficients , can be computed beforehand.

$d_3\xi$ and $d_3\eta$ are the summations of all the equations (7-19)

Stage 4.

From 20° to the antipodes , repeat stage 3 above but use $5^{\circ} \times 5^{\circ}$ squares to obtain $d_4\xi$ and $d_4\eta$

Summing up the effects obtained in these four stages , we then get the components of the gravimetric deflections of the vertical at the computation point.

7.12. Other investigations.

Since Vening Meinesz's work on gravimetric deflections , many others have contributed to knowledge in this field. For example :-

TSUBOI 1954 has used a method of calculating deflections gravimetrically using the Bessel Fourier series and neglecting the curvature of the earth. His results agree in the main with those of RICE 1952.

COOK 1950 , using Jeffreys' spherical harmonic development obtained the following standard deviations of the different sources of error for the calculation of deflections of the vertical :-

A single deflection , neglecting gravity outside 20°: 1"
Difference of deflections " " " 5°: 0".5
Calculation of the effects of gravity from $0^{\circ}.05$ to 5°: 0".1
Calculation of the effects of gravity within $0^{\circ}.05$: between 0".1 and 0".5

WEISFELD 1967 worked from Stokes' formula and , by applying the theory of distributions , arrived at " the Vening Meinesz formulas with the singularities removed." Weisfeld and Schubert maintained that the method used by Vening Meinesz (differentiation of the integrand of Stokes' integral) is not valid , in that , at the computation point the integrals of the deflection formulae are divergent. As far as can be ascertained , no practical tests have been made using Weisfeld's formulae

DE GRAAFF-HUNTER 1951 has also re-derived the Vening Meinesz formulae.

7.13 Maps of deflections.

Tests were done using the results of RICE 1952 , in an attempt to establish whether contour deflection maps could be drawn and interpolations carried out. These tests showed that interpolation of ξ and η on deflection contour maps was a hazardous process and should not be attempted. Whereas geoidal undulations are area values , deflection components are point values. Attention is drawn to LUNDQUIST 1966 where contour maps for ξ and η are drawn at sea level , and at elevations of 1 000 Km , 10 000 Km and 100 000 Km. These maps are based on results obtained in the Satellite Geodesy Program of the Smithsonian Institution. Because of their scale , however , these maps can only be generalizations.

7.14 Anomaly maps.

The gravity anomaly maps for the computation of the deflection of the vertical should be drawn on the stereographic projection. The usefulness of this projection lies in the fact that great circles passing through the centre point are projected as straight lines and that all circles on the sphere remain circles on the projection. The formulae used are

$$x = 2r \frac{\cos\varphi \sin\varphi_0 \cos\lambda - \sin\varphi \cos\varphi_0}{1 + \cos\varphi \cos\varphi_0 \cos\lambda + \sin\varphi \sin\varphi_0}$$

$$y = 2r \frac{\cos\varphi \sin\lambda}{1 + \cos\varphi \cos\varphi_0 \cos\lambda + \sin\varphi \sin\varphi_0}$$

(JORDAN 1948) .... (7-20)

where r = radius of earth

φ = latitude of point to be calculated

λ = longitude difference

φ_0 = latitude of centre point

Using equations (7-20) , two stereographic maps were constructed. Annexure 1 is a 1 in 3,500,000 map of South Africa with centre at 30° South Latitude , 25° East Longitude. This map will be suitable for use with the circle ring method. Annexure 2 is a 1 in 400,000 map with grid intervals at 10 minutes of arc and can be used for the gradient method and for the circle ring method. Although the centre of this map used in the construction was 31° South Latidude , it could be used in South Africa in the latitude belt 29° to 32° South Latitude.

7.15 The effect on deflections of increasing gravity field radius.

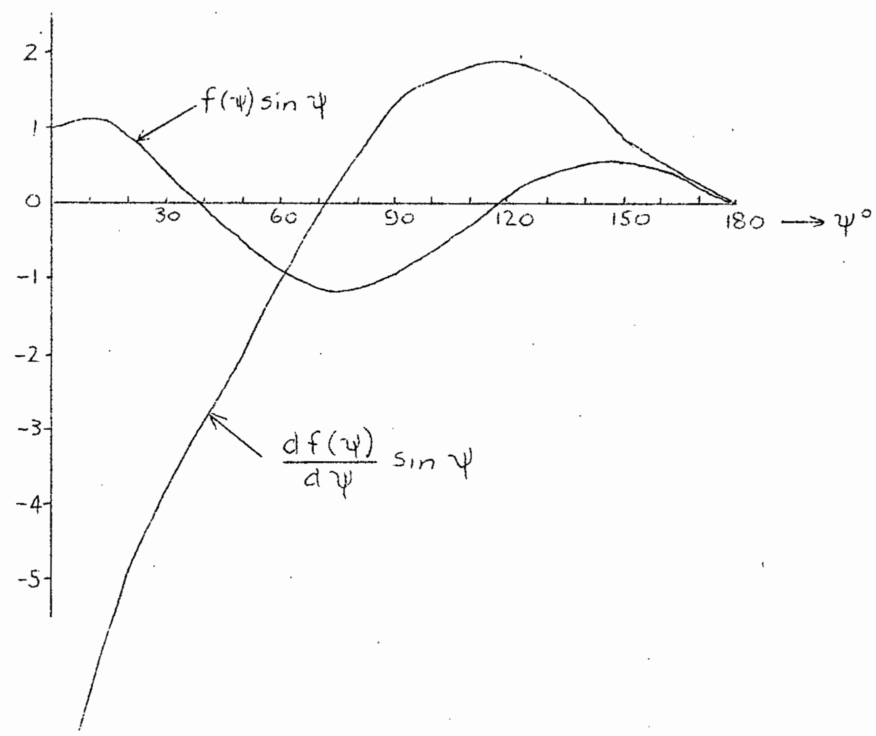


FIGURE 7/1

Figure 7/1 shows the graphs of $f(\psi)\sin\psi$ i.e. the Stokes' function as used for calculating geoidal heights and $\frac{d f(\psi)}{d\psi}\sin\psi$ i.e. the Vening Meinesz function used for calculating deflections of the vertical. Studies by the Columbus Group (HEISKANEN 1958b) have shown that the effect of the gravity field on the components of the deflection decreases with the <u>square</u> of the distance from the computation point , whereas the effect of the gravity field on the geoidal heights decreases with the <u>distance</u> from the computation point. An examination of figure 7/1 shows the correlation between these findings and the relevant functions.

The table on the next page has been compiled from results published by RICE 1952 and KAULA 1954.

The points A , B , C , D and E in column 1 are Kaula's points Conley , Stanforth , State , Columbus and Barr respectively.

The points E , F and G in column 1 are Rice's points Twin , P_W and P_Θ respectively.

The extension of the table (columns 9 to 14) has been calculated using only the ξ deflection components. In this extension we have

column 9 = column 7 divided by column 8
10 = differences using column 8
11 = differences using column 7
12 = differences using column 6
13 = differences using column 5
14 = differences using column 4

	ψ Km	14.29	179.1	600	836	1406	Whole field
	ψ°	$0^{\circ}.1$	$1^{\circ}.6$	$5^{\circ}.4$	$7^{\circ}.5$	$12^{\circ}.65$	180°
1	2	3	4	5	6	7	8
A	ξ	+0.19	+2.84		+2.56	+2.61	+3.16
	η	+1.35	+2.71		+2.10	+1.71	+2.92
B	ξ	-0.08	+1.37		+1.15	+1.19	+1.74
	η	-1.13	-2.90		-3.27	-3.70	-2.49
C	ξ	-0.58	-0.01		-0.11	-0.08	+0.49
	η	-2.11	-6.00		-6.24	-6.70	-5.49
D	ξ	-0.72	-0.60		-0.70	-0.68	-0.13
	η	-2.03	-6.20		-6.44	-6.90	-5.69
E	ξ	+1.16	+0.27		+0.57	+0.58	+1.13
	η	+0.90	-2.18		-2.35	-2.85	-1.65
F	ξ			-0.73		-0.58	-0.87
	η			-0.61		-0.60	-0.14
G	ξ			-0.53		-0.76	-1.58
	η			+0.68		-0.07	+0.93
H	ξ			+0.37		-0.70	-1.08
	η			+0.28		+0.05	+0.22

Table showing deflection components in seconds of arc as related to increasing gravity field radius

1	9	10	11	12	13	14
A	0.83					
		+1.42	+1.42	+1.41		+1.47
B	0.68					
		+1.25	+1.27	+1.26		+1.38
C	0.16					
		+0.62	+0.60	+0.59		+0.59
D	5.23					
		-1.26	-1.26	-1.27		-0.87
E	0.51					
F	0.67					
		+0.71	+0.18		+0.20	
G	0.48					
		+0.50	+0.06		-0.90	
H	0.65					

Table showing differences using ξ values only

From column 9 , it can be seen that the deflection component obtained using the gravity field radius up to $12^{\circ}.65$ is a large percentage of the deflection obtained using the whole field.

Columns 10 to 14 (especially for Kaula's points) show that differences between deflection components using a gravity field radius of from only $1^{\circ}.6$ compare well with the differences using the whole gravity field. The comparisons are not so good with Rice's points. This is probably due to the fact that Kaula had more and better gravity observations over the whole field available.

The above observation concerning the differences of the deflection components and Szabo's good agreement between gravimetric and astronomical deflections leads us to conclude that the following is a feasible precedure :-

Take astronomical deflections at a number of points , say 30 Km apart. Compute the gravimetric deflections at these points and at intermediate points at , say , 10 Km apart. The gravity field used in these computations need only extend for a limited radius. Using the differences in the gravimetric deflection components , we can easily arrive at the astronomical deflections of the intermediate points. We are thus "transporting" the astronomical deflections by means of easily computable gravimetric deflections. The reverse procedure could be adopted to compute the gravimetric deflection of , say , Buffelsfontein , where the gravity field consists mostly of the sea area. In this case , we would compute the gravimetric deflection at an inland station , say , Kimberley , using the gravity field of the whole earth. Now by means of astronomical deflections , we could transport the Kimberley gravimetric deflection to Buffelsfontein.

7.16 Gravimetric deflections using the electronic computer.

Many gravimetric deflections have been done using the electronic computer. Gravity anomalies over the whole earth are stored and used as required. For examples of these computations see UOTILA 1960 and NAGY 1963.

In this thesis , as reported in chapter 8 , the computations were of the "one-off" type and the electronic computer was therefore not used.

CHAPTER 8

COMPUTATIONS

8.01 Gradient effects for Rice's points.

With the information published in RICE 1952 , the gradient effects of Rice's points using equation (7-17) were calculated. The following table shows the good agreements obtained.

Station	Differences: Rice - Loon	
	ξ	η
	seconds of arc	
Bartley	-0.003	-0.002
Roby	+0.001	+0.010
Brooks	+0.004	+0.008
Lacassa	-0.017	-0.003
Sears	-0.022	-0.032
Bynum	0	-0.009
Little Rock	-0.069	-0.018
Legion	+0.012	-0.002
Polk	-0.007	-0.003
Burns	-0.003	0
Bogue	+0.006	-0.006
Ecore	-0.009	-0.004

Using Rice's published results as a control , the above table confirms the application of the method established in chapter 7 and the use of equation (7-17).

8.02 Circle ring method for Rice's points.

Again using information in RICE 1952 , the following results were obtained for the station Twin :-

	ξ	η
	seconds of arc	
Circle ring method (Loon)	-0.459	+1.180
Gradient effect (Rice)	+0.011	+0.008
Total deflection components	-0.448	+1.188

The computations for the circle ring method were from radius 0.657 Km. to radius 65.90 Km. Using Rice's graph to scale off the deflection components up to the last mentioned radius , we get ξ = -0".5 and η = +1".1

The above results confirm the application of the circle ring method , and , as mentioned in chapter 7 , the azimuth must be measured from South through West (i.e. clockwise) to agree with the usual convention of "astronomical - geodetic" for the deflections.

8.03 Deflections at Kimberley Hill and Hanover.

The gravimetric deflection of the vertical was calculated for the trigonometrical stations Kimberley Hill (longitude point) and Hanover (station No. 20). The gravity information used was that published by the Geological Survey of the Republic of South Africa in their Handbook 3 , 1962. (See SMIT 1962 and HALES 1962)

The gradient method was used with a fundamental circle of radius 4.32 Km in each case and the circle ring method used from 4.32 Km to 541.5 Km i.e. up to $4^{\circ}.8$. A template was drawn for the circle ring method and gravity anomalies estimated on the 1 in 2 000 000 Isostatic Anomaly (AH 30) map by HALES 1962.

NOTE : A stereographic projection map was not used for these and subsequent calculations because

(a) the error of estimating the mean anomaly plus the error in the drawing of the anomaly contours would be greater than any positional error of the grid lines ; and

(b) such a map is at present not available.

These gravimetric deflections were compared with the astronomical deflections as obtained from the South African Geodetic Reports with the following results :-

	Gravimetric	Astronomical
	seconds	of arc
Kimberley Hill	$\xi_g = +0.03$	$\xi_a = -1.56$
	$\eta_g = -2.37$	$\eta_a = -0.07$
Hanover	$\xi_g = +0.15$	$\xi_a = -0.01$
	$\eta_g = +0.87$	$\eta_a = -0.45$

8.04 Analysis of Kimberley Hill and Hanover results.

The poor comparisons obtained could be due to

(a) the fact that corrections for variation of the pole and to FK4 system have not been applied to the astronomical deflections. (The effect of change of spheroid for these points is negligible). These astronomical observations were made about 60 years ago.

(b) the poor gravity anomaly information available in the immediate vicinity of the computation points. (The gradient effects in each case were based on only two gravity anomaly values).

(c) observational errors

(d) the assumed astronomical deflections at Buffelsfontein may be wrong.

It is felt , however , that the greatest effect in this case is contributed by the poor gravity coverage in the vicinity of the points.

8.05 Deflections at Tunnel Shaft No. 3 .

The gravimetric deflection components were calculated for the ground position of Tunnel Shaft No. 3 of the proposed Orange - Fish River Tunnel. Tunnel Shaft 3 lies about 108 Km south of the Trompsberg Anomaly and about 25 Km south of Venterstad.

The following information was used :-

(a) the isostatic anomaly map (1 in 250 000) of the Tunnel area as published in KLEYWEGT 1964 ; and

(b) the isostatic anomaly map (1 in 2 000 000) of the Geological Survey. (HALES 1962).

Templates were drawn for use on the above maps.

The following results were obtained :-

TUNNEL SHAFT 3

Gradient method up to		ξ_g	η_g
Km	Degrees	seconds of arc	
6.08	0.055	-4.17	-1.62
12.05	0.108	-2.31	-1.25
23.83	0.214	-2.24	-0.63
47.00	0.423	-0.84	-0.05
92.22	0.829	-0.68	-0.65
In each case the circle ring method was used up to 835.9 Km (7°.5) and the total deflection components are given.			

TABLE 8/1

TUNNEL SHAFT 3

	ξ_g	η_g
	seconds of arc	
Gradient method up to 6.08 Km	-1.04	+0.04
PLUS circle ring method up to		
Km		
12.05	-1.95	-0.09
23.83	-3.10	-0.41
47.00	-3.20	-0.88
92.22	-3.95	-1.65
835.9	-4.17	-1.62

TABLE 8/2

8.06 Analysis of deflection at Tunnel Shaft 3 .

The final result for the deflection components is

$$\xi_g = -4''.17$$
$$\eta_g = -1''.62$$

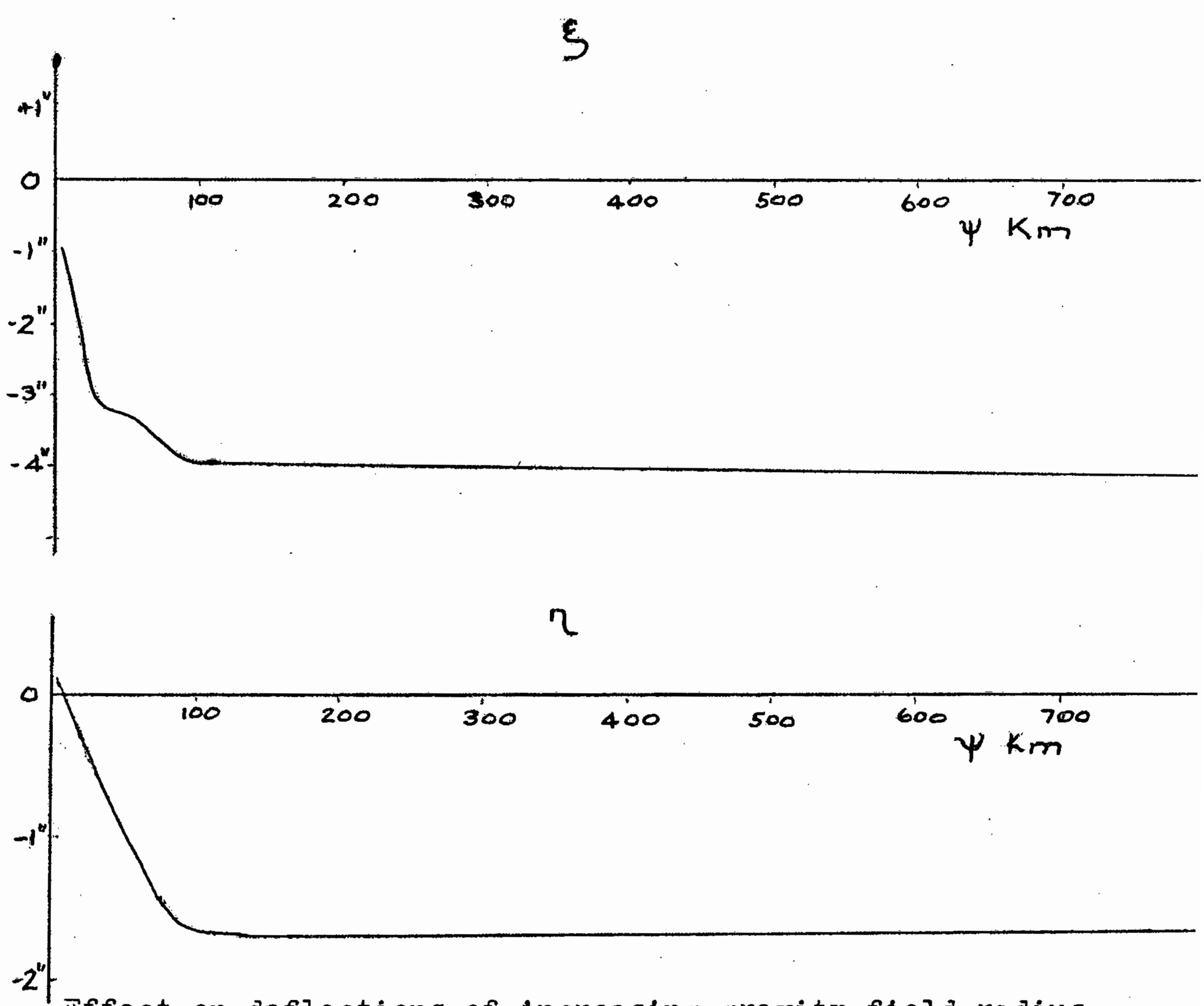
based on the gradient method with fundamental circle radius = 6.081 Km and the circle ring method up to 835.9 Km or 7.5 degrees.

If this radius for the fundamental circle is increased , then the results are less reliable because there is not a uniform gradient over the circle. (i.e. the gravity anomaly contours are not parallel). The effects of increasing this radius can be seen from Table 8/1. With a radius of 6.081 Km , the gravity anomaly contours are more or less parallel.

Although the accuracy depends on the size of the radius (the smaller the radius , the higher the accuracy) , a small radius can only be used if there is a dense gravity station network in the vicinity of the computation point. In this case there were 3 gravity stations in the 6.081 Km radius but many more in the surrounding area. HEISKANEN 1958a says that in many cases 4 to 8 gravity stations in a 5 Km circle are sufficient but this would depend on the nature of the gravity anomaly field. As a guide , Heiskanen gives a figure of 30 stations in a circle of 20 Km radius. In the case of Tunnel Shaft 3 , there were 19 stations in a radius of 20 Km.

Using results obtained by KAULA 1954 for 5 computation points (see 7.15) , we calculate that the deflections obtained using the field up to 7°.5 range from 60% to 88% of the deflections obtained using the whole field , for most of the points. But this observation can only be regarded as a generalisation to indicate that the final result given for Tunnel Shaft 3 will probably not change by more than 30-40% if the whole field is available.

With regard to the effect on deflections of increasing gravity field radius , the results shown in Table 8/2 can be shown graphically as in Figure 8/1 below.



Effect on deflections of increasing gravity field radius for Tunnel Shaft 3

FIGURE 8/1

The general characteristics of figure 8/1 compare well with the 30 graphs shown by RICE 1952. The general conclusion one can draw is that there is a "settling down" of the deflection component after about 350 Km from the computation point. This is significant when considering the method of transporting the deflection , as mentioned in chapter 7. It means that if we use a gravity field of only 350 Km , then the <u>differences</u> between deflection components of neighbouring points are fairly accurate.

HEISKANEN 1958a says that an accuracy of the deflection $\theta = \sqrt{\xi^2 + \eta^2}$ of 0.5 seconds of arc can be obtained with a dense gravity station net around the computation point plus a good regional gravity survey up to 2000 Km. Not every point with similar gravity coverage would , however , yield the same result. The greatest accuracy will be obtained where the area immediately surrounding the point has a smooth (or uniform) gravity anomaly field.

It should be mentioned here , that if the gravity survey in the Tunnel area was done while bearing in mind the possible computation of gravimetric deflections , then many more well placed gravity stations would have been recommended by a geodesist. This would have greatly increased the accuracy of the deflections.

CHAPTER 9

CONCLUSIONS AND RECOMMENDATIONS

9.01 General

Very little has been done in South Africa from the gravity-geodesy point of view. In order to make gravity information useful in geodetic work , the geodesist should have a hand in at least the planning stage of all gravity surveys. This would ensure that these surveys are properly connected to base stations , that adequate coverage is obtained around future computation points and that the surveys are carried out to the required accuracy.

9.02 Gravimetric deflections in South Africa.

As far as can be ascertained , the gravimetric deflections calculated in chapter 8 are the first done in South Africa. In order to facilitate the calculation of further gravimetric deflections , a manual for these calculations could be compiled. This manual could contain the following information :-

A. Requirements with regard to gravity field.

B. Details of the calculation procedures i.e. the gradient method , the circle ring method and the square method.

C. Tables for constructing maps on the stereographic projection in South Africa.

D. Tables for the calculation of G , theoretical gravity , in South Africa on the International Spheroid.

E. Areas of $5^{\circ} \times 5^{\circ}$ and $1^{\circ} \times 1^{\circ}$ squares for use in the square method.

F. Information for reducing astronomical deflections so that they can be compared with gravimetric deflections. This would include correction graphs for South Africa for change of spheroid from the

Modified Clarke 1880 to the International Spheroid , tables for reduction to the same star catalogue system and so on. This information will also be useful for "transporting" the deflections as outlined in 7.14 .

G. The computations for the square method could be done by the electronic computer and the mean gravity anomalies of the squares could be stored on punched cards and used when required. The layout of the programme and a suggested method of recoding on cards is described in UOTILA 1960. The proposed manual could include the latest available mean anomalies of the required squares.

H. An outline of the methods of predicting gravity anomalies in unsurveyed areas.

9.03 The applications of physical geodesy.

In this thesis , the calculation of gravimetric deflections of the vertical have been described in detail. A similar procedure can be worked out for the calculation of geoidal heights. With these quantities N , ξ and η the following applications are possible.

A. Determination of the best fitting reference spheroid.

The best fitting spheroid for a region will make $\sum(\xi_a^2+\eta_a^2)$ = minimum. This is the usual criterion applied but a better determination results from making

$$\sum\left[(\xi_a-\xi_g)^2+(\eta_a-\eta_g)^2\right] = \text{minimum},$$

because we now take unknown density anomalies and any lack of complete isostatic compensation into account.

B. Undulations of the geoid.

If N is known , especially in the regions where base lines are measured and at the initial point , then the geodetic co-ordinates can be accurately referred to the reference spheroid and not to the geoid as is usually done. For every 10 metres separation between the geoid and spheroid , the error in reducing the base line to the geoid and not to the spheroid , is of the order of 1 in 665 000. Various reports give N values for South Africa which range from +20 metres to -180 metres. (BOMFORD 1958 , HEISKANEN 1965 , UOTILA 1962). (These values , however , are not with reference to the Clarke 1880 spheroid). Reduction to the spheroid would eliminate certain distortions in a triangulation system.

C. Deflections at triangulation stations.

A deflection of the vertical at a triangulation station introduces an error in the observed direction similar to that given by the inclination of the horizontal axis of a theodolite. This error can be written as $\theta \tan h$ (HEISKANEN 1958a) where θ is the difference between the deflection components at both ends of a triangle side and h is the vertical angle. For $h = 10^\circ$ and $\theta = 15''$ the error is 2.9 seconds of arc. Even with $h = 5^\circ$ the error is 1.3 seconds of arc. A knowledge of these deflections , which can be computed gravimetrically , will eliminate this source of error.

D. Linking of existing geodetic systems.

In order to link existing geodetic systems onto one World System we need the absolute values of N , ξ and η at the initial points of all the local systems as well as astronomical observations at these and other points. Details of this type of computation are given in HEISKANEN 1958a .

E. Control points for small scale maps.

The formulae used in this case are

$$\varphi = \varphi' - \xi_g$$
$$\lambda = \lambda' - \eta_g \sec \varphi'$$
$$A = A' - \eta_g \tan \varphi'$$

where φ, λ, A are the geodetic latitude, longitude and azimuth ;

φ', λ', A' are the astronomical latitude, longitude and azimuth ;

ξ_g, η_g are the gravimetric deflection components.

The right hand sides of the above equations are obtained from astronomical and gravimetric observations and the geodetic co-ordinates are then calculated. Using gravimetric deflections in this way would overcome the effects of the deflection of the vertical on photogrammetric work in regions having no geodetic control as mentioned by KARARA 1960.

The above formulae can also be used for calculating distances over large areas

HEISKANEN 1952 gives a point accuracy of about 40 metres and a distance accuracy of about 60 metres irrespective of the distance. For maps of 1 in 100 000 and smaller scale this is more than adequate.

9.04 Physical geodesy in South Africa.

Some of the applications of physical geodesy have been given in 9.03. These can be used in South Africa but perhaps more important is to consider the future of geodesy in this country. At some future date the South African Geodetic Triangulation will be re-computed. (This will probably necessitate the reobserving of some of the triangles). It will then be a suitable time to change the Geodetic Datum point to some other point which will enable a strong gravimetric determination of ξ , η and N to be computed. This will give a better connection of the South African system to any other system. The changing of the datum point is not a complicated procedure and can be done "by the stroke of an administrative pen ! " (BOMFORD 1962).

Perhaps the most important task for physical geodesy in South Africa at present is the determination of the geoidal undulations. This information will be required for any re-computation of the Geodetic Triangulation as well as being needed for the launching and tracking of rockets and missiles. This is a project which will take some years and will have to be undertaken on a national level.

Much work has been done in the field of physical geodesy in the U.S.A. (the Columbus Group and others) , Canada (NAGY 1963 and others) , Australia (MATHER 1966 and 1967) any many other countries. It would be a pity if South Africa were to lag behind in this field.

CHAPTER 10

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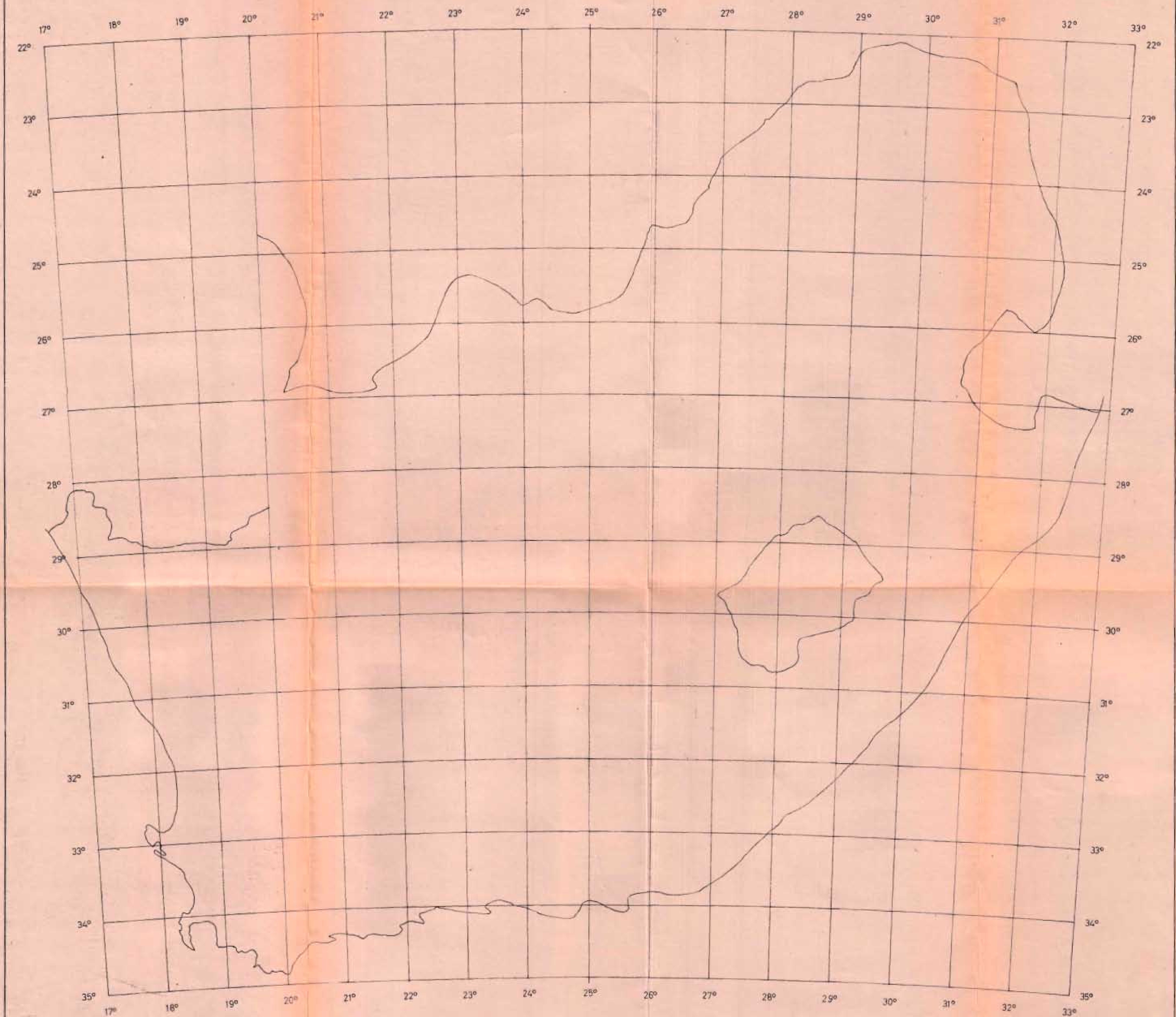

SOUTH AFRICA

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Stereographic Projection
Centre: 30° S Lat., 25°E Long

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Grid: 10' intervals

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