EVALUATING VALUE AT RISK MODELS: AN APPLICATION TO THE JOHANNESBURG STOCK EXCHANGE

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By

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Abstract

The management of market risk is an essential determinant of the stability of a financial institution, and by extension, of the overall financial system. There are various variables which impact on the accuracy of a market risk management system. For various reasons which are discussed in this study, Value at Risk (VaR) is used as a measure of market risk. VaR has certain key features which make it adaptable to several types of scenarios in order to provide a measure of market risk. In order to assess these features of VaR, this study evaluates VaR using a range of techniques. This study analyses the performance of some of the most popular VaR models using the JSE ALSI’s total daily returns. The VaR estimates were calculated for each model using varying parameters for confidence level, risk horizon, distributional assumptions and other variables. The study evaluates the relative accuracy of each model analysed, over specific subsets of the data set under consideration, and performs five different backtests to determine the accuracy of each model. The aim of this analysis is to identify the model most suited to predicting VaR in the South African environment. A key feature of this study is that it includes data during and after the financial crisis, and can, therefore, model the respective volatility characteristics of the data during this period. The results of the analysis indicate that the asymmetric GARCH models outperform the other models over both the full sample period and the crisis and post-crisis sub-periods, and that the t distribution assumption produces more accurate forecasts. This implies that such models are better suited to capturing the effects of volatility for data with these characteristics.

Keywords: Market Risk, Value at Risk, Backtesting
Declaration

I, Deepika Chotee, hereby declare that the work on which this thesis is based is my original work (except where acknowledgements indicate otherwise) and that neither the whole work nor any part of it has been, is being, or is to be submitted for another degree in this or any other University. I empower the University to reproduce for the purpose of research either the whole or any portion of the contents in any manner whatsoever.

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Chapter 1: Introduction

1.1 Background

Market Risk is defined as the potential risk of loss in an institution’s portfolio value as a result of fluctuations in market conditions. Banking institutions are exposed to market risk in instances where they hold financial instruments that are subject to changes in the market, and may, as a result of fluctuations in these instruments, incur significant losses. Inadequate market risk management measures can jeopardize an institution’s financial well-being and may, during highly volatile market conditions, drag an institution to its downfall.

Market Risk management is critical in ensuring an institution’s sustainability. The increasing diversity of drivers of market risk translates into the need for more adaptable and robust risk management techniques (Alexander, 2008). It is not necessarily possible to hedge against all potential sources of risk, which is most likely why there is not, to date, a model capable of mapping movements in drivers of market risk into one single structure. There is, however, continuous development in the area of risk management in order to develop models better suited to managing market risk.

There has been active innovation in the market for commercially available risk management tools and this has been, to a large extent, supported by regulatory authorities overseeing risk management practices in financial institutions (Dimson and Marsh, 1995).

The financial crisis which began in 2007 triggered the need for more accurate market risk management practices. There has, amongst other measures, been heightened emphasis on banking institutions around the world to develop and adjust internal risk models that are capable of more accurately managing risk exposure.

Owing to its high level of regulation, the banking sector has been governed by several changes to their market risk management procedures in recent years. The Basel
Committee on Banking Supervision (discussed in more detail in Section 2.1.1) has made revisions to its Basel II framework, with new requirements for the calculation of Value at Risk. The main aim of these revisions is to standardize risk management practices across banking institutions around the world.

Popularly used as a measure of market risk, the Value at Risk (VaR) of a portfolio is the maximum loss that will not be exceeded with a certain level of certainty over a certain period (Schroder, 1996, Hendricks, 1997; Berkowitz & O’Brien, 2001; Jorion, 2000). This indicates that the Value at Risk relies on the time horizon and the level of significance. Interpreted graphically, Value at Risk is the lower tail of a probability density function of profit and loss figures for a portfolio.

The above figure is a graphical representation of a hypothetical set of normally distributed portfolio returns. The figures on the left hand side of the mean (μ) represent losses on the portfolio over the period under consideration. Interpreted graphically, the Value at Risk of the portfolio is measured as the value that corresponds to the figure at the lower tail of the distribution. This would be the loss corresponding to r* in the figure above.
1.2 Problem Statement and Aim of Study

This study focuses on analysing the accuracy of Value at Risk models for the prediction of market risk on the Johannesburg Stock Exchange during periods of differing volatility, with specific reference to the periods before, during and after the financial crisis of 2007/8. Various well-known VaR models are tested on a range of pre-defined criteria which determine their accuracy using established backtesting methodologies.

The main aim of the study is to identify the VaR model which performs best given certain predefined parameters. The spectrum of parameters and backtesting procedures used in this study are as broad as possible to provide an in depth evaluation the level of accuracy of the models analysed within the study. The results of the analysis are then tied to currently applied banking regulations to evaluate the relationship between these regulations and the accuracy of the selected model.

1.3 The research value of the study

The study conducts an analysis of various VaR models under different conditions of volatility with varying model parameters and differing distributional assumptions of the underlying data set. In order to make the analysis as in depth as possible, the study subjects each model to various tests of accuracy to determine their reliability in predicting VaR. The following paragraphs highlight the key areas in which the research is relevant to the South African environment and innovative.

1.3.1. The size and currency of the data set

The data set used in this study spans the period 1\textsuperscript{st} January 2002 to 12\textsuperscript{th} September 2012 for total returns data on the ALSI. Although the study makes use of only ALSI total returns data, previous published research on the South African market in the area of VaR does not make use of data as up to date as the current study. The implications of this are that the data is able to capture the stable period prior to the global financial crisis which started in late 2007, the entire crisis period itself, as well as the
subsequent recovery period. This enables us to compare and contrast the effect of the implied volatility differential in these periods within the context of the JSE, and more specifically the JSE All Share Index (ALSI). Therefore, the VaR methodologies can be applied to different sub-periods within the data set in order to isolate the implication of the evolving volatility characteristics throughout the data set on the models.

Although the data set spans the different stages of the financial crisis, a key limitation of the selected data set is the fact that it relates to only equities market data, and may not, therefore accurately mirror the risk and return characteristics of the typical portfolio of a financial institution. This selection has been made due to limitations in obtaining information about the components of portfolios of banking institutions, which, for several reasons, is not readily available. However, given that studies with similar research objectives such as this one make use of an equity index for the purposes of the analysis, (for example, Vee et al (2012), Christoffersen (2001), and others), the comparative value of the current study is reasonable.

The VaR prediction and backtesting exercise is first carried out over the entire data set, as an aggregate, and then over subsections of the data period, as explained below. The entire data set includes periods of very different volatility characteristics, the impact of which is likely to be under-emphasized when considered in aggregate rather than being examined separately.

Prior to the crisis period, it is expected that volatility conditions fluctuate within the normal expected range, therefore no serious issues are anticipated with regards to the inaccuracy of VaR prediction.

The crisis period is characterised by extreme volatility conditions and, as market evidence has shown, VaR models proved inadequate during that period (BIS, 2011). The current study attempts to predict VaR during that period of extreme volatility for each model under consideration using different parameters for significance level and horizon. The importance of isolating VaR figures for that particular period relates to the fact that it is possible to examine closely the varying degrees of observed inaccuracy for each model, with a specific combination of risk horizon and significance level parameters.
The post crisis or the recovery period is interesting from a volatility perspective as it does not mirror the same market conditions as those prior to the crisis. The data reflects volatility parameters in a market still recovering from the shock of a major crisis. As such, this section of the data illustrates a range of volatility parameters. It is important to note that, although there is no persisting crisis at that point in the data set, the shocks brought about by the crisis have not entirely left the market. As such, certain models may prove more accurate due to their ability to capture the lingering effects (or decay) of volatility shocks. This will be explained in more detail in the section 1.4.2, below.

The above points illustrate that the current study is novel in this aspect as it provides an analysis of the data set in an amount of depth not previously investigated within the South African market. Furthermore, recent studies on VaR accuracy do not include an analysis of predictions made during the post crisis period, which further contribute to the research value of this study.

1.3.2. Range of VaR methodologies applied

This study predicts VaR using a number of models, each selected for its specific characteristics which contribute to its appropriateness at various levels of the analysis. The models have further been selected based on the statistical characteristics of the underlying data set, the key findings of previous studies on the key strengths of each model, and the objectives of the research question. As such, the study attempts to provide comprehensive coverage of the spectrum of models that can be applied to VaR prediction, in an attempt to make the analysis as thorough as possible.

The VaR models that have been applied in this study are as follows: Simulation methods (Normal Linear and Historical Simulation), basic GARCH models (GARCH and Riskmetrics), asymmetric models (EGARCH, IGARCH and GJR) and long memory models (FIGARCH).

As evidenced above, this study brings together a comparison of VaR models that have been shown to capture certain key effects in data of the nature employed in this study (emerging market data). Emerging market data has some inherent characteristics that
influence the accuracy of VaR predictions made with certain models. Some of the models selected for the purposes of the analysis, more specifically, the GARCH based models have been shown to be particularly reliable in the context on emerging market data, Thupagayale et al (2009). As will be discussed in the methodology chapter (Chapter 5), prior studies discussed in chapter 4 have shown that the selected models possess statistical properties that make them suited to generating more reliable VaR predictions for emerging market data. In order to increase the coverage of the analysis, each model is applied with a combination of parameters, as will be described in the next paragraphs.

1.3.3. Coverage of risk horizon and significance levels

The VaR predictions in this study were made using combinations of the 5 percent and one per cent levels of significance, and the 1 day and 10 day horizon.

Although the Basel market risk management framework recommends that daily VaR needs to be calculated at a 1 per cent or 5 per cent level of significance, the current study investigates the reliability of VaR further by examining the VaR estimates under a 10-day horizon.

VaR of a portfolio relies on the holding period, which is the period over which the profit or loss of the portfolio is calculated. For banking institutions, the Basel Market Risk management framework prescribes 1 day and 10 day holding periods. This stems from the fact that it is important to select a holding period which is in line with liquidity characteristics of the market under consideration.

It is important to note that the period over which the VaR figure is estimated is a period over which the portfolio does not change and that as the data set under consideration becomes larger, shorter holding periods are more appropriate as they provide more accuracy for backtesting purposes (BIS, 2008).
1.4 Structure of the study

The rest of the study is structured as follows: Chapter 2 provides a description of the evolution of risk management practices, particularly in the banking sector and the establishment of standard market risk management practices. Chapter 3 outlines and briefly compares the market risk management methodologies. This is followed in Chapter 4 by an analysis of the literature available on the Value at Risk methodologies evaluated in this study. The data and methodology applied is discussed in Chapter 5, followed by a discussion into the conclusions reached in Chapter 6. The study concludes with suggestions for further research in Chapter 7.
Chapter 2. Regulatory and Technical Background

The most recent upheavals in market risk management practices resulted from the 2007-2008 financial crisis. Traditional market risk management techniques proved inadequate in predicting the potential losses that would result from such extremes in market volatility. A number of institutions failed as a result of this. Two striking examples of this are the cases of Bear Stearns and Lehman Brothers, both of which ceased to exist in 2008.

A background into market risk management prior to the crisis provides a useful introduction for the rest of the discussion. Prior to the financial crisis of 2007/8, although the need for sound risk management practices had been acknowledged and put into practice, there was still a certain level of reluctance and scepticism from financial institutions to adopt those practices.

Dunbar (2000) and Persaud (2000) explain the assumption that VaR may be responsible for market instabilities by compelling firms to continuously update their portfolios in a way that may be harmful to stability in the financial system. In fact, the introduction of VaR-based market risk regulation was blamed for the volatility of 1998. However, Jorion (2002) finds that, owing to the fact that these regulations react very slowly to market movements, they are not responsible for market volatilities. Although the concept of VaR had already been introduced by 1998, it was still widely misunderstood. Jorion (2002) further argues that users of VaR erroneously believed that VaR would never be exceeded, and were not prepared for such losses, although they were relatively infrequent.

The events that led to the financial crisis of 2007/2008 started with the disintegration of the proper functioning of the market for instruments relying on residential mortgages as collateral. Bear Stearns, a mid-sized investment bank, revealed its financial distress in June 2007. The immediate response of this declaration was an intervention, in the form of a government-assisted sale which led to the institution ceasing to exist in March 2008. However, in September 2008, when Lehman Brothers
declared bankruptcy, there was no government intervention to provide financial assistance to the institution. It has been argued that this decision, which was the opposite of the intervention in the case of Bear Stearns, resulted in severe implications to the financial system, which eventually spread to the rest of the world (Wessel, 2009). Reinhart, (2011) argues that the importance of government intervention should not be underestimated as demonstrated by Diamond and Dybvig (1983), that government intervention has the potential to prevent or limit the extent of a crisis. Several studies have analysed in varying degrees of detail the unravelling of the crisis and have proposed various theories on possible causes and alternative outcomes if certain variables were changed. These, however, are not all within the scope of this study. The rest of this section tracks the evolution of market risk management with a focus on the changes brought about by the crisis of 2007-2008.

As the need for being adequately prepared for highly volatile market conditions had become more apparent, the financial crisis of 2007-2008 was followed by an avid interest in refining and improving market risk management techniques. A large amount of time and energy has been spent in the banking industry as well in order to preclude situations like the crisis from happening again (BIS, 2011, 2013). As will be discussed later, there have been a number of changes in the regulations governing market risk management in banks. Institutions in different sectors of the economy are subject to different market risk factors, and were impacted differently by the financial crisis. As a result of those differences, the evolution in risk metrics has been different in banking, portfolio management and large corporations. The rest of the discussion will focus on market risk management in banking.

2.1 The Regulatory Environment

2.1.1 The Basel Committee

The Basel Committee on Banking Supervision (BCBS or ‘the Committee’) was formed in 1974 as a result of a decision made by the central bank governors of the G10 countries. Banking Institutions are required to manage financial risk in accordance with certain regulations specified by the Bank of International Settlements, situated in Basel. The Basel Committee on Banking Supervision issues and amends
these regulations periodically to ensure that banking institutions manage risk in accordance with the prescribed guidelines.

The Basel Committee on Banking Supervision seeks to align banking practices across banking institutions around the world. It is important to note that, although the Basel Committee on Banking Supervision does issue guidelines prescribing risk management practices in the banking sector, it does not have the legal authority to enforce those guidelines (BIS, 1996). The purpose of the market risk framework developed by the Basel Committee is to provide guidance to risk practitioners in the banking sector.

Prior to the 1988 Basel Accord, banking institutions reported exposures in foreign exchange and interest rates and exercised certain limits with respect to the concentration of risks. Capital requirements were not in place at that time. Following the 1988 Basel Accord, core principles for adequate supervision and capital requirements were established. Following that, there was a definite effort towards regulatory convergence and coordination across countries.

The 1988 Basel Accord established clear guidelines for minimum quantitative requirements for banks in accordance with the credit risk exposure of their assets. The original 1988 Accord and its 1996 amendment for Market Risk in the Trading Book have been integrated into legal regulations by over 100 countries, making the Accord the global benchmark for assessing banking risk. The 1996 Amendment contained a very detailed appendix that fed the need for the use of internal models in the industry, rather than standardized rules to evaluate market risk capital.

The new Basel II Accord of 2005 contained major revisions to the assessment of credit risk as well as a capital charge requirement for covering operational risks. The Basel II Accord, unlike the 1996 Amendment to the Basel I Accord was motivated for by regulators, not the industry. It took nearly 6 years to complete industry consultation on Basel II (BIS, 1996).

The old Basel Accord had safety and market stability as main objectives. In addition to maintaining these objectives, the Basel II Accord aims to align banking regulations
across countries and encourage better risk management while allowing for continuous improvement. This starts with easy to follow rules for banks that provide incentives such as lower capital requirements for banks with more complex risk management systems. Although the new regulations will not influence international capital requirements, they will induce greater risk discrimination of financial institutions which may lead to a redistribution of capital.

The Basel II Accord re-established the three pillars of regulation that were contained in the first Accord:

- Pillar 1: minimum capital standards. This dictates the minimum amount of capital that banks must hold to guard against risk.
- Pillar 2: supervisory review. This sets out the recommendations for inspection and the reporting requirements for banks.
- Pillar 3: public disclosure and market discipline. This aims to support the supervisor by improving market observation by competitors, clients and shareholders.

These three pillars are considered to have equal importance and are complementary in nature.

As of 31 December 2010, the Basel Committee on Banking Supervision issued Revisions to the Basel II market Risk Framework, in a 34 page document outlining the changes made to the existing framework. The most important points of the revised document relate to the changes made to the existing framework on market risk, which was based on the 1996 Amendment to the Capital Accord to incorporate market risks. The purpose of these revisions was to incorporate certain important risks that were not included in the existing framework. The Committee also introduced an additional requirement of stressed value at risk. Under this requirement, banks are required to calculate a stressed value at risk, in addition to the value at risk estimate based on the most recent observation period. The stressed Value at Risk requirement states that banks must calculate Value at Risk estimates for periods of high volatility. These revisions stemmed directly from the perceived inadequacies of the existing market risk management framework over the financial crisis.
2.1.2 Basel II & Basel III Stress Testing Requirements

In his discussion of stress testing Value-at-Risk and its relevance under highly volatile market conditions, Berry (2009) highlights that although VAR estimates provide an indication of the potential loss in a portfolio over a certain time horizon under normal market conditions, there is no indication of the potential loss resulting from substantial changes in the market conditions.

The stressed Value at Risk requirement of the Basel Committee aims to guard against potential losses resulting from extreme fluctuations in market conditions. Stress Testing explores the resulting changes in a portfolio stemming from changes in market conditions. It is possible to implement stress tests depending on the different components of a portfolio and on the factors that are most likely to affect them. Berry (2009) describes a range of techniques for designing stress tests in its June 2009 edition of Investment Analytics and Consulting. The study discusses the idea of Reverse stress tests that seek to outline the risks that would bring about the total collapse of an institution. Contrary to stress tests, which test the maximum loss a firm can take without collapsing, reverse stress tests determine the amount of loss which would cause an institution to fail. However, there are still uncertainties about what exactly constitutes a loss that would cause an institution to fail and how to go about estimating exactly what the effect of such losses would be. This might be of key importance in providing financial institutions with a way to guard against exposure from those specific risk factors.

2.2. Market Risk Management After the Financial crisis

The financial crisis had several effects on the evolution of market risk management in the banking sector. As was discussed above, there were certain specific changes which were brought about to the Basel Accords directly as a result of the financial crisis. However, the regulatory consequences of the crisis were not merely limited to the calculation and management of market risk within banking institutions. The crisis also brought about an important concept which significantly changed the perception of
market risk management: the potential impact of certain financial institutions on the world economy.

As has been discussed in literature focusing on analysing the causes of the spread of the financial crisis to the rest of the world, major financial institutions (such as Lehman Brothers) have a potentially contagious effect on the world economy when they are in financial distress. This idea has been acknowledged by the Bank for International Settlements which issued additional regulations over and above Basel III. In a new set of guidelines issued in 2013, the BIS defines the concept of global systemically important financial institutions (SIFIs).

SIFIs are major financial institutions which represent an additional threat to the financial system owing to their size and the extent of their involvement with smaller, weaker financial institutions. The committee has defined a set of qualitative and quantitative criteria in order to determine whether an institution qualifies as a SIFI. In order to limit the risk such institutions represent during a time of crisis, the Committee has imposed additional capital reserve requirements for these institutions (BIS, 2013). The main aim of these capital requirements is to enable such institutions to be able to cover their own losses by drawing from their reserves during a crisis period. As such, these institutions would not impact on other institutions and on the overall financial system.

Further to imposing capital reserve requirements, the Committee prescribes an additional charge which serves as a deterrent to a SIFI to further magnifying its systemic importance in the global financial system. It is important to note that although these guidelines have been issued with the aim of minimising market risk stemming from all possible risk factors, these measures have so far been tested only in theory. Their real, practical soundness will only be revealed during a time of high volatility.
2.3 Review of the South African Banking Regulatory Framework

The evolution of the various Basel market risk frameworks has impacted on market risk management practices in South African banking institutions. South African banking institutions have revised their market risk management periodically to ensure compliance with the most recent Basel Accords.

South African Banking Institutions are regulated by the South African Reserve Bank, which ensures that these banking institutions comply with Basel regulations. According to data from the BIS Basel implementation progress reports (BIS, 2011, 2013), Basel regulations are implemented in South African banking institutions at a rate relatively faster than banking institutions of other emerging markets.

According to the Progress Report on Basel II implementation published in September 2011, the South African banking industry had enforced the final rule governing Basel II compliance and is currently preparing for the implementation of Basel III (BIS, 2011). As of March 2013, South African banking institutions were fully compliant with Basel 2.5 (BIS, 2013). Although there is no data for exact compliance levels for Basel III for South African institutions at this date, major South African banking institutions have disclosed the measures progress towards achieving compliance with Basel III in their latest financial statements.
Chapter 3. VaR and alternative market risk management methodologies

The previous section discussed the historical need for market risk measurement, more specifically, in the banking sector. For this purpose, a number of risk metrics are commonly used to measure the market risk an institution is exposed to. By definition, a market risk metric is a summary of the potential deviations of a portfolio from a certain target value (Alexander, 2008). Therefore, a risk metric captures the uncertainty associated with a portfolio’s return in one single figure.

Value at Risk (VaR) is the most popular technique for risk management, while there exists a number of less popular alternatives. These alternatives will be discussed prior to the discussion on VaR.

3.1 Alternatives to Value at Risk

Popular alternative risk metrics include downside and quintile risk metrics. Downside risk metrics analyse only those returns that fall short of a predetermined target return (Alexander, 2008). Downside risk metrics are commonly used in active portfolio management. The primary metrics of this kind are Semi-standard Deviation and Second Order Lower Partial Moment, introduced by Markowitz (1959). Expected Shortfall is also used as an alternative to VaR due to its ability to provide more accurate values than VaR in certain scenarios (Acerbi & Tasche, 2002), although it has been found to require a much larger data set to be effective (Yamai & Yoshiba, 2002).

3.1.1 Gap Analysis

Initially developed for the purposes of assessing interest-rate exposure, gap analysis provides a rough indication of the extent of a financial institution’s sensitivity to interest rate risk (Christoffersen et al., 2001).
The first step of a gap analysis is to establish a time horizon over which the exposure will be estimated. The next step involves calculating the amount by which the portfolio is expected to change over the horizon period. The amount of the change will highlight assets and liabilities which are sensitive to interest rate changes. The gap is calculated as the difference between the two above values. Once these variables have been identified, the interest rate exposure is given as the gap multiplied by the change in interest rates.

This is given as: $\Delta NII = (GAP) \Delta r$

Where $\Delta NII$ is the change in net interest income (or the interest rate exposure), and $\Delta r$ is the change in interest rates.

Although not computationally complex, gap analysis has certain inherent flaws. It focuses mainly on a (crude) change in potential interest income rather than on the value of the portfolio. The horizon period selected also impacts on the estimated interest rate exposure (Dowd, 2002).

### 3.1.2 Scenario Analysis

The purpose of scenario analysis is to analyse the amount of potential gain or loss under different scenarios. The procedure itself is carried out in a few simple steps; however, the results can vary significantly in accuracy depending on the individual variables used in each step of the process.

The steps involved in performing a scenario analysis are to identify a certain number of possible scenarios resulting from movements in key variables (such as interest rates, commodity prices, exchange rates, etc.) over a predetermined time period. The next step is to estimate cash flows or values of instruments within the portfolio for each scenario.

The difficulties in performing a scenario analysis arise from the importance of being able to predict a suitable scenario and the movements of interrelated variables (Alexander, 2008). The results of a scenario analysis can be severely flawed if the most likely scenarios are not included in the analysis. As a scenario analysis merely estimates potential losses or gains under different scenarios, it does not provide an
indication of the probability of each scenario (Alexander, 2008). Therefore, the results of each different scenario must be carefully interpreted bearing in mind the actual likelihood of that particular scenario.

The accuracy and usefulness of scenario analysis, therefore, depends to a large extent on the ability to foresee all the above variables, and can, as a result, be highly subjective.

3.1.3 Estimating Risk Using Portfolio Theory

Portfolio Theory provides a risk measurement technique which differs from the other techniques discussed so far. Portfolio Theory is based on the principle that a rational investor would seek to invest in an asset such that the expected return is maximized and the risk (or standard deviation) is minimized (Markowitz, 1959). Applied to a portfolio of instruments, this implies that an investor would seek to hold a portfolio so that the returns are maximized with the lowest possible standard deviation. The correlation of the assets held within a portfolio is a key determinant of the overall risk of that portfolio. A single asset, in isolation, may have a very high standard deviation, but when included in a portfolio with negatively correlated assets, will not have the same contribution to the overall risk level of the portfolio as the asset risk on its own. Therefore, when selecting a portfolio, a key determinant of risk is the correlations of different instruments within that portfolio.

When extended to a risk management level, portfolio theory is widely used by portfolio managers in an attempt to construct a portfolio with as low a risk level as possible. In this process, the risk free return and the expected market return are relatively easy to estimate. The complex part of constructing a portfolio using this technique is to estimate the beta coefficient of the assets contained in the portfolio (Frankfurter & Phillips, 1995). This process requires a large data set in order to ensure an estimate which is as accurate as possible is obtained. Failure to correctly estimate the beta coefficient leads to a flawed estimate for the portfolio risk, making this technique ineffective for managing risk (Dowd, 2002).
3.2 Value at Risk (VaR)

3.2.1 Definition

As mentioned in Chapter 1, VaR is the absolute loss that will not be exceeded over a specified period of time, with a given degree of certainty. For example, a company that reports a daily Value at Risk of R50 million at the 95 per cent level means that the probability that the company will incur a loss greater than R50 million over the next day is only 5 per cent.

From this example, it is important to note that there are two parameters attached to a Value at Risk figure: the significance level represents the probability of the stated VaR figure being exceeded (which is 5% in the above example) and the risk horizon (which is one day in the example) indicates the period over which the maximum loss is measured. The latter is usually stated as the number of trading days (not calendar days) which corresponds to the Value at Risk estimate.

The significance level is usually dictated by an external authority in order to ensure consistency in the basis on which figures are quoted between institutions. In the case of the banking sector, the Basel II Accord requires banks using an internal Value at Risk model to report their Value at Risk estimates at the 1% significance level (BIS, 1998).

One of the main characteristics of Value at Risk is that it provides a summary of the risks of a portfolio in one single figure, making it convenient for comparison and reporting purposes. The VAR estimate captures the effects of leverage, probabilities of adverse movements in market prices and diversification effects in one single amount.

3.3.2 Value at Risk, history and evolution

In the latter part of the twentieth century, the concept of VaR emerged as a widely accepted benchmark for assessing financial risk. This was a result of a number of financial institutions failing due to unexpected extreme market events. The need for an accurate measure of risk probably first emerged subsequent to the market crash in 1987, as a result of which a number of financial market participants made efforts to
find a suitable measure of risk (Alexander, 2008). The growing interest in risk management was further fuelled by the market turbulence which started in Mexico in 1995 and spread to Asia, Russia and Latin Mexico. At that point, risk management was no longer just a priority for companies in the banking and insurance sector, but became an important requirement for companies outside that sector as well.

Prior to 1995, only a small number of banks disclosed Value at Risk. Since 1995, however, bank regulators, led by the Basel Committee on Banking Supervision, have applied pressure for the disclosure of more information surrounding risk characteristics. This initially started as the market risk amendment to the Basel Accord and has continued with subsequent revisions to the market risk framework. By 1991, 66 of the 71 financial institutions surveyed by the Basel Committee disclosed Value at Risk in their published financial statements (BIS, 1996).

The increasing complexity of market risk factors, which was a result of the increasing use of derivative instruments, created the need for a risk measurement metric capable of summarizing the risk exposures of a portfolio in one single figure. Value at Risk is widely used as this measure of market risk.

The concept of VaR as a measure of market risk was first used by large firms in order to evaluate the risk of their portfolios. Following JP Morgan’s release of its RiskMetrics system in 1994, the use of VaR has grown very rapidly (Dowd, 2002). Regulatory authorities have supported the need for adequate risk management tools. The Basel Committee on Banking Supervision allows banks to calculate VaR estimates using an internal model. VaR estimates are an indication of the volatility of the bank’s portfolio over a given period of time. South African banking institutions are now required to disclose their VaR estimates in their financial statements.

The following table provides a summary of the VaR models, significance levels and time horizons used by the major banks in the South African banking industry.
<table>
<thead>
<tr>
<th>Banking Institution</th>
<th>VaR Methodology</th>
<th>Confidence Level</th>
<th>Stress Testing</th>
<th>VaR Horizon</th>
<th>Number of breaches</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSA</td>
<td>historical simulation</td>
<td>95%</td>
<td>Yes</td>
<td>daily</td>
<td>26 times over 2 year period</td>
</tr>
<tr>
<td>FNB</td>
<td>historical simulation</td>
<td>99%</td>
<td>N/A</td>
<td>1 day, 10 day</td>
<td>N/A</td>
</tr>
<tr>
<td>Investec</td>
<td>historical simulation</td>
<td>95%, 99%, 100%</td>
<td>Yes</td>
<td>Daily</td>
<td>N/A</td>
</tr>
<tr>
<td>Nedbank</td>
<td>historical simulation</td>
<td>99%</td>
<td>Yes</td>
<td>Daily</td>
<td>N/A</td>
</tr>
<tr>
<td>Standard Bank</td>
<td>historical simulation</td>
<td>95%</td>
<td>Yes</td>
<td>Daily</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 1: VaR measurement information as per 2012 financial statements

The above table summarizes financial statement data. Information not available is indicated as N/A on the table. This information was obtained from the 2012 annual financial statements of the above banking institutions.

3.3 Value at Risk Calculation and methodologies

The first step prior to calculating Value at Risk is to identify market factors that impact on the value of the portfolio. The variables generally used tend to be market rates and prices which directly affect the portfolio value. At this stage of the process, a smaller number of factors make the analysis simpler and easier to realize. Increasing the number of variables would add to the complexity of the process. It is worth noting that, even working with two variables with simple instruments such as forward contacts, for example, would be a complex process involving numerous possibilities, as a plethora of different contracts are possible (Alexander, 2008). Extending such an analysis to other instruments such as swaps, loans, options or exotic options would be an even more complicated process. Therefore, the first step of simplifying the values.
of the instruments comprising a portfolio is crucial in minimizing the overall complexity of the analysis.

The process of identifying market risk factors generally involves breaking down the instruments comprising the portfolio into simpler instruments which are more directly linked to movements in market risk factors (Linsmeier and Pearson, 1999). Once these factors are identified, VaR can be estimated using the selected VaR model(s).

3.3.1 Historical Simulation

The Historical simulation methodology makes very few assumptions about the statistical characteristics of market risk factors.

The distribution is estimated by using the figures of the current portfolio under consideration and exposing it to movements in the market risk factors experienced over the last N periods under consideration. This process then leads to the calculation of N hypothetical mark-to-market portfolio values, from which N hypothetical mark-to-market portfolio profits and losses are calculated, based on the current portfolio value. Once these values are obtained, it is possible to construct a distribution of the profits and losses over the last N periods, from which the VaR is obtained.

It is important to note that using this methodology implies that the profits and losses calculated are only hypothetical in nature, as the portfolio was not held over the past N periods.

It is possible to extend the Historical Simulation methodology to more realistic, multiple instrument portfolios.

This involves some additional calculations in the first three steps of the entire process. In the first step (identifying the market factors), additional market factors must be identified. The instruments must then be expressed as a function of the factors (Alexander, 2008).
In the second step, the actual historical values of the factors must be obtained. At the next step, the mark to market values are each computed and are ordered from the highest profit to the greatest loss. The last step nets off the gains against the losses.

### 3.3.2 Normal Linear Value at Risk

The normal linear Value at Risk methodology makes the key assumption that risk factor returns have a normal distribution. The assumption that follows from this is that the joint distribution of these risk factor returns is multivariate normal.

The Value at Risk formula for the portfolio is the negative normal $\alpha$ quantile multiplied by the standard deviation of the portfolio returns over the risk horizon (Alexander, 2008).

For large data sets, it would be reasonable to assume that the Central Limit Theorem holds. There is evidence regarding the fact that share returns distributions are leptokurtic with peaked distributions and fat tails. For VaR estimation over shorter horizons, the assumption of normality does not hold.

### 3.3.3 Monte Carlo Simulation

The Monte Carlo Simulation and the Historical Simulation methodologies are similar on a number of points. The main point at which these two methods differ is that the Monte Carlo Simulation method uses simulation as opposed to using actual observed changes in market factors to calculate hypothetical values for the portfolio (Dowd, 2002).

The process requires the choice of a statistical distribution that replicates the potential changes in market factors as closely as possible. Then a large number (usually thousands or tens of thousands) of hypothetical changes in the market factors are generated. These are used to construct thousands of hypothetical portfolio profits and
losses for the current portfolio, and finally the distribution of the portfolio’s potential profit or loss (Alexander, 2008). The Value at Risk is then determined from the distribution, as the final step of the process.

The Monte Carlo simulation method will not be analysed in this study, as it would produce similar results to the other methodologies given the characteristics of the data. The Monte Carlo Simulation method would add little value to the current study as it is only different from Historical Simulation in the sense that a lognormal distribution of returns is used to simulate scenarios, as opposed to a historical distribution (Linsmeier and Pearson, (1999) & Alexander (2008)). As explained in Chapter 5, the current study uses lognormal returns to generate VaR forecasts, therefore eliminating the key difference between the Monte Carlo Simulation method and the Historical Simulation method.

3.4 An evaluation of the traditional Value at Risk methodologies

The above three main methods of calculating Value at Risk each differ on several points. Each method is more suited to a particular type of instrument, and the choice of method for a given portfolio may not always be an easy task. The process of evaluating which method is more suited requires an analysis of different factors which contribute towards their suitability in certain circumstances. The evaluation of the different methodologies will involve looking at the key factors risk managers find more important. Linsmeier and Pearson, (1999) and Dowd (2002) use certain criteria for evaluating the traditional VaR models, these are: ease of implementation, ease of reporting and interpreting, flexibility in investigating changes in the assumptions, reliability of estimates and ability to determine risks of options and similar instruments.

3.4.1 Ability to capture the risks of options

Both the historical simulation and Monte Carlo methodologies are capable of capturing the risks of options. They provide more reliable Value at Risk estimates for portfolios containing such instruments (Alexander, 2008).
3.4.2 Ease of implementation

The three main VaR methodologies described above offer different levels of convenience in implementation, depending on the instruments that comprise the portfolio under consideration.

The implementation of the historical simulation is easy for portfolios made up of currencies, provided past data on market risk factors is available (Linsmeier and Pearson, 1999). The Delta-Normal and Monte Carlo methods are not as easy to implement for such portfolios as the additional currency risk factor is complex to include in the estimation process.

Portfolios containing exotic options and currency swaps may make the Value at Risk estimation harder because of the need for a pricing model for those instruments. All three methods encounter the same difficulty (Dowd, 2002).

3.4.3 Ease of communication

The different classes of VaR methods have varying degrees of complexity which influence their ease of communication. The results of the Historical Simulation method are easier to communicate because of the simplicity of the process and concept. The Variance-Covariance method is relatively difficult to construe to an audience not conversant with the Normal distribution and its characteristics. The Monte Carlo Simulation probably is relatively more complex method to explain owing to its reliance on the pseudo-random generation of a sample based on a pre-selected statistical distribution (Linsmeier and Pearson, 1999). The GARCH-based methods require an understanding of the estimation of the underlying concept of volatility, and, therefore, require a more complex explanation.

3.4.4 Flexibility in altering assumptions

Under certain circumstances, market factors may not correlate with each other as they historically do. Changing economic conditions can significantly alter the behavioural
patterns of certain variables, making it inaccurate to rely on past trends to predict future outcomes. Under these circumstances, a method using past correlations may not capture the changes accurately (Dowd, 2002). It becomes imperative, in these circumstances, to be able to incorporate these changing conditions into the analysis. The Value at Risk calculated must be able to account for the change in the correlation characteristics of the affected variables.

As a result of its reliance on historical changes in market factors, historical simulation is not able to predict future outcomes if certain parameters change. However, the Variance-Covariance and Monte Carlo simulation methods can accommodate such a variant in the analysis (Alexander, 2008). It is possible to overlook certain aspects of the historical estimates and use a different set of parameters. However, from a practical point of view, it may be complicated to carry out such an analysis using available software packages.

### 3.5 Recent Developments

Following the global economic crisis of 2007-8, it became clear that standard Value at Risk calculations give a flawed estimate of true market risk. As of July 2009, the Basel II Committee on Banking Supervision issued a series of amendments to be implemented by the 31st of December 2010. These amendments were made to certain steps in the existing VaR methodologies in an attempt to deal with the flaws of the then used Value at Risk methodology.

The current Basel III market risk management framework constitutes the latest set of guidelines on market risk management. South African banks have fully adopted the recommendations set out in the Basel II.5 framework, although data on the progress of the adoption of Basel III is not available to date. Several emerging market economies have already published varying degrees of the Basel III rules as part of their banking regulatory protocol (BIS, 2013).
3.6 Weaknesses of VaR

Although quite extensively praised for its ability to measure market risk, VaR is also subject to a number of flaws which have been identified and discussed in studies conducted on the reliability of VaR as a measure of market risk.

3.6.1 Model Risk

The primary flaw of VaR predictions is termed as model risk (Dimson et al., 2006) which is the risk of inaccuracies arising from assumptions defining the VaR models used. This is known as model risk. There are several factors that determine model risk, such as the misspecification of the model’s parameters, inadequate data to generate model parameters, and the less common, errors in the equations underpinning the model itself (Dimson et al., 2006).

In a study on the reliability of VaR models Danielsson (2008) compares the results of the historical simulation method and the normal tail and fat tail GARCH methods of estimating VaR. The findings of this study show that these methods give very inconsistent results when the horizon and confidence level are changed. The study suggests that it is essential to be able to test the accuracy of the VaR model itself. However, it is further argued that the model used to test the VaR model may bear its own model risk.

Berkowitz and O’Brien discuss the accuracy of VaR models used in commercial banks in a study conducted in 2001. The study primarily compares the VaR forecasts of banks to those from a simple GARCH model of the bank’s Profit and Loss volatility. The study reveals that, on several counts, banks are unable to predict Value at Risk accurately. This stems from the varied set of market risk factors and their multivariate distributions, which are often estimated inaccurately in an attempt to reduce the computational load of estimating the quantitative relationships between those risk factors. The study argued that, in addition to computational limitations on the banks’ models, there are regulatory constraints that affect the reliability of Value at Risk. The findings suggest that the Value at Risk estimates cannot calculate the
effects of volatility because regulatory requirements imply that the one year horizon prescribed does not capture volatility changes accurately. It was concluded that these severely diminish the models’ predictive capacity.

3.6.2 Value at Risk is not a sub-additive measure of risk

Value at Risk has been criticised for its inability to be sub-additive. Sub-additivity refers to the fact that the sum of all the risks in the different positions in a portfolio is not higher than the sum of all the individual risks of the positions (Alexander, 2008).

An illustration is given below:

Assume that \( \rho(\cdot) \) is a sub-additive measure of risk. Therefore, for a portfolio with positions in X and Y, the risk of the overall position is given as:

\[
\rho(X + Y) \leq \rho(X) + \rho(Y) \tag{1}
\]

A sub-additive measure of risk implies that the sum of the risks of different positions would be higher, therefore, providing a more conservative indication of the level of risk of the portfolio.

By extension of this principle, it has been argued by Berkowitz and O’Brien (2001) that for the purposes of reducing regulatory capital requirements, institutions may be tempted to break up their overall portfolio, which would imply that the sum of the capital requirements of the different components would be less than the requirements of the individual components on their own.

3.6.3 Value at risk does not predict the Impact of Tail Losses

Although Value at Risk predicts the maximum loss that will not be exceeded with a high level of probability, it does not give an indication of the loss that will be incurred in the unlikely event that a tail event (an event causing a loss greater than Value at
Risk) occurs. If a tail event occurs, the portfolio incurs a loss greater than Value at Risk and the Value at Risk predicted does not estimate the magnitude of that potential loss (Dowd, 2002).

It has also been argued that Value at Risk does not indicate relative riskiness of two portfolios. An example used by Christoffersen et al., (2001) illustrates this point by comparing two portfolios with the same Value at Risk, but with one of the portfolios having fatter tails than the other. In this case, if the Value at Risk is exceeded on the second portfolio, the loss incurred would be greater. The single Value at Risk estimate does not explain this implication.

**3.6.4 Value at Risk as a deterrent to diversification**

Eber et al, (1999) suggested that in cases where the Value at Risk of the overall portfolio is greater than the Value at Risk of individual components of the portfolio, investors may choose to not diversify and invest in assets that present a lower potential loss.
Chapter 4. Prior research

This study analyses various VaR models and determines their accuracy using a selection of backtests and discusses their relevance within the current banking market risk regulation. The study, therefore, has links to certain key areas of finance: the calculation of market risk using VaR, banking regulation governing VaR models and the determination of the accuracy of VaR models. The section below provides a discussion of the key findings on Value at Risk research so far, relating to evidence on the accuracy and appropriateness of each methodology. The main methodologies compared in this study are the historical simulation, delta normal, and GARCH-based models. The current section also analyses studies done on market risk management in the context of banking regulation.

4.1 Comparative studies of VaR models

Keuster et al (2006) reveal that most of the commonly used Value at Risk methodologies, namely the delta normal, Monte Carlo simulation and historical simulation, severely underestimate market risk. The study makes use of the daily closing figures of the NASDAQ Composite Index (which is a market value-weighted portfolio comprising more than 5000 stocks listed on the NASDAQ stock market) from 8th February 1971 to 22nd June 2001, or a total of 7681 observations. The study further shows that the GARCH models seem more robust under various conditions of volatility, and may, under volatile market conditions, provide more accurate predictions than the other models.

Christoffersen et al (2001) arrive at a similar conclusion with regards to the commonly used Value at Risk methodologies. They also conclude that the GARCH methodology proves more reliable under volatile conditions. This study reveals that the GARCH and RiskMetrics models produce very similar forecasts. This study uses the daily S&P 500 returns data from November 1985 to October 1994 (2209 observations). Although this study compares the Riskmetrics and GARCH (1,1) models for accuracy, it does not alter the VaR horizon and uses 1 day VaR forecasts throughout the study. The study does not break the data set into periods of differing volatility. As a result, the
findings may be of limited comparative value. The current study, as will be discussed later, uses different VaR horizons and breaks down the data set into periods of differing volatility.

Linsmeier and Pearson (1999) performed a detailed comparison of the Normal Linear, Historical Simulation and Monte Carlo simulation methodologies. The study uses a hypothetical portfolio composed mainly of US dollar denominated derivative instruments to compare the performance of the VaR models. It was found that the historical simulation technique tends to be more rigid, and allows less flexibility regarding the data set. This method does not allow “what-if” analyses to be performed. The Monte Carlo simulation, on the other hand, was found to allow more flexibility, and was very adaptable to more complex components of a portfolio, such as options and derivative instruments.

Literature on GARCH based Value at Risk models focus on several aspects of accuracy of these models in terms of emerging market data. This sub chapter discusses the most relevant literature in the context of GARCH-based models. The discussion starts with the key articles on Value at Risk literature, with a focus on the suitability of each model under consideration and follows on to the relevance of distributional assumptions in Value at Risk estimations.

Babikir et al (2012) investigate the relevance of structural breaks in the accuracy of forecasting stock return volatility. The study makes use of data from the JSE All Share Index. The key findings of this study indicate that structural breaks do impact on accuracy of volatility forecasts in South Africa. The main purpose of the study was to provide information about the importance of structural breaks on the accuracy of volatility forecast, due to a lack of such information on South African stock market data in the literature.

The model makes use of both in-sample and out of sample tests to address the research questions. The in-sample tests first test for the relevance of structural breaks in the data set (ALSI total returns from 1995 to 2010). The study uses a test statistic derived by modifying the iterated cumulative sum of squares algorithm of Inclan and Tiao (1994).
The out of sample tests make use of the last 500 observations in the data set. Three benchmark models are used, namely, the GARCH (1,1), Riskmetrics and FIGARCH (1,d,1) models and compared against five competing models.

In order to rank the five competing models, the study makes use of two loss functions, the mean square forecast error (MSFE) by Starcia et al (2005), and the Value at Risk loss function by Gonzalez-Rivera et al. (2004). The model with the lowest mean loss ratio under both of the above conditions is said to forecast volatility more accurately.

The summarised data on the five competing models used indicated that the Markov-Switching (MS-GARCH) model performs better over shorter periods and that the GJR-GARCH (1,1) model is more accurate over longer periods. The study concludes, overall, that asymmetric models do not outperform the GARCH (1,1) model and that structural breaks are definitely relevant in the South African stock market.

Pantelidis et al (2005) investigate the reason behind the GARCH (1,1) model’s inability to provide accurate Value at Risk forecasts and conclude that the use of mean squares undermines the model’s accuracy.

The above results conflict with the findings of most of the other literature available, which find that asymmetric GARCH models prove more accurate in emerging markets, including South Africa. These studies are discussed below.

4.2 VaR models and emerging markets

In a study considering the performance of GARCH based Value at Risk models in emerging markets, Vee et al. (2012) conclude that it is not possible to rank one particular model as being accurate throughout a certain data set, and that the accuracy of Value at Risk predictions requires models to be specified according to the stock market/ portfolio under consideration. The study analyses the accuracy of the twelve GARCH models used in the study across six emerging market stock indexes analyses.

The study uses stock market index data from Mauritius, Tunisia, Sri Lanka, Pakistan, Kazakhstan and Croatia. The data uses closing daily prices from various years, due to
data availability across the stock markets up to 2009. This leads to the data set having different sized time series and including data during the 2008 financial crisis (but not the entirety of the recovery period).

The models considered are the GARCH, EGARCH, IGARCH each making use of three distributions: the Gaussian, Student t and the skewed Student t distributions.

In order to assess the accuracy of the twelve models in the analysis, the study applies a two stage backtesting procedure.

The first step of the backtesting process is to determine the suitability of the model based on independence of the violations and the coverage of the Value at Risk estimate. The study makes use of the Kupiec test for coverage (1995) and the Christoffersen test for independence (1998), and the test for conditional coverage, which combines the test statistics of each of the two tests. The Kupiec and Christoffersen tests are hypothesis tests conducted with a five percent significance level and a chi squared distribution with one degree of freedom. The critical value for these two tests is 3.841. The test for conditional coverage is performed at five percent significance level using a chi square distribution with two degrees of freedom, and a test critical value of 5.991. The likelihood ratio test statistic for this test is given by:

\[ LR = -2 \log \left[\left(1 - p\right)^{n-x} \left(p\right)^{x}\right] + 2 \log \left[\left(1 - \frac{x}{\pi}\right)^{n-x} \left(\frac{x}{\pi}\right)^{x}\right] \]  (2)

The second step of the backtesting process uses loss functions to assign a score to each model. The loss function calculates the score based on the size of the loss when a violation occurs. The study makes use of four loss functions, the quadratic (QL), absolute (AL), asymmetric (ASL) and the quantile loss (QuL). The loss functions are given below:

**QL:**  \[ L_t = \begin{cases} (r_t - VaR_t)^2 & \text{if } r_t < VaR_t \\ 0 & \text{otherwise} \end{cases} \]

**AL:**  \[ L_t = \begin{cases} |r_t - VaR_t| & \text{if } r_t < VaR_t \\ 0 & \text{otherwise} \end{cases} \]
Using the above loss functions, the model with a lower score is more accurate.

The study comments on the suitability of various models for each of the different stock markets in the data set. The overall conclusions find that, in general, the EGARCH model does not perform particularly well and this is in line with the findings of Jansky and Rippell (2011), who concluded that a symmetric GARCH model is more suited to estimating Value at Risk in emerging markets. The study also finds that the assumption of Student t does not improve accuracy over the assumption of normality. However, this conflicts with the findings of So and Yu (2006) and Janksy and Rippell (2011) who found that GARCH, IGARCH and FIGARCH with the t distribution assumption predict Value at Risk more accurately. However, it is to be noted that the latter studies were conducted on the indexes of developed countries. Studies on distributional assumptions and their impact on the accuracy of Value at Risk prediction seem to indicate opposite findings from those found in this study. These are discussed below.

Thupayagale (2010) investigated the accuracy of GARCH based Value at Risk models in emerging equity markets. The key objectives of the study were to calculate Value at Risk using data from emerging African stock markets and to apply backtesting techniques in line with Basel requirements in order to ascertain the model’s suitability. In line with the findings of Bams, Lehnert and Wolff (2005) and Jorion (1995a,b), the study makes the assumption of normality for the data used in the analysis. These studies have shown that the assumption of normality is more suited to Value at Risk estimation even if the data does not fit the normal distribution. The analysis uses daily stock return from the stock markets of Brazil, China, Egypt, India, Kenya, Nigeria, Russia, South Africa and Turkey. The US (S & P 500) is used as a benchmark. With the exception of South Africa, the emerging markets used in the analysis are relatively
small with reference to market capitalisation and stock market size and have less liquidity.

The models used in the analysis are the Riskmetrics, GARCH (1,1), EGARCH, IGARCH, FIGARCH and FIEGARCH models. The study uses two backtesting techniques to evaluate each model’s accuracy, namely the Kupiec Lagrange Multiplier test (1995) and the Dynamic Quartile (DQ) test by Engle and Manganelli (2004).

The Kupiec Lagrange Multiplier test states that the number of violations on a correctly specified Value at Risk models occur at the level of significance. The likelihood ratio (LR) test for the null hypothesis is:

$$LR = -2log\left[(1-p)^n \times (p)^v\right] + 2log\left[\left(1 - \frac{x}{n}\right)^{n-x} \left(\frac{x}{n}\right)^x\right]$$

The critical value is calculated using a chi-square distribution with one degree of freedom.

The Dynamic Quartile (DQ) test tests whether the violations are independent of one another and are identically distributed. The distribution is given as follows:

$$\text{Hit}_k = I \left(r_k \langle VaR_k\rangle - \alpha\right)$$

In-sample tests of the results show that the Riskmetrics models are accurate only in the case of Russia. For out-of-sample tests, the standard GARCH (1,1) model is outperformed by the other models. The results also show that the FIGARCH model proves most accurate in the case of Brazil, India, Nigeria and Turkey. The FIEGARCH model proves most accurate in the case of Egypt, South Africa and the US. These results point to the fact that asymmetric and long memory GARCH models prove more valuable in the case of emerging equity markets. The long memory volatility attributable to emerging equity markets is in line with the findings of Assaf and Calvalcante (2004), Disario et al (2008) and McMillan and Thupayagale (2009). Nagayasu (2003) attributes the observed long memory property to less developed institutional and regulatory frameworks in emerging markets.

The study, therefore, reaches a conclusion similar to Vee et al (2012) that it is not possible to identify one single Value at Risk model that is accurate across all equity
markets or data sets and that the choice of Value at Risk model has to be based on the characteristics of the market under consideration.

### 4.3 Distributional Assumptions

So and Yu (2006) arrive at a similar conclusion to Thupayagale (2010) in arguing that the Riskmetrics model is outperformed by all the competing GARCH based models at various confidence levels. It is also shown that asymmetric and long memory models are superior in calculating Value at Risk. The study also concludes that the t distribution assumption provides superior Value at Risk estimates than the assumption of normality. These findings are contrary to what was observed by Vee et al (2012), who concluded that the t distribution assumption does not improve the accuracy of Value at Risk estimates.

The current study also varies the distributional assumption of the underlying data set in generating VaR predictions as an additional parameter in VaR estimation. Literature available on distributional assumption commonly uses the t distribution assumption and the normal distribution assumption when generating VaR forecasts. The following paragraphs discuss the evaluation of the current literature on the perceived impact on accuracy of making different distributional assumptions.

Milwisdsky and Mare (2010) investigate the suitability of the traditional distributional assumption of normality for the purposes of Value at Risk estimation. Their study improves on the work of Huisman, Koedijk and Pownall (1998) by applying the Monte Carlo simulation using linear and non-linear instruments. The study uses daily data from the South African FTSE/JSE Top 40 index from 18 September 1998 to 19 September 2008. The study concludes that using the t distribution assumption provides more accurate Value at Risk estimates in the case of the South African equity market, as evidenced by the lower number and magnitude of violations for Value at Risk estimates using the t distribution assumption. The conclusion of the results shows that institutions using the t distribution assumption will have a lower capital charge multiplier according Basel requirements.

Other work on distributional assumptions suggest that distributions other than the normal distribution provide more accurate Value at Risk estimates (Malevergne et al (2005), De Haan & Ferreira (2007)). Although, there is no conclusive evidence yet as
to which distributional structure provides the most accurate Value at Risk forecasts, evidence suggests that the t distribution is superior to the assumption that returns are normally distributed. This conflicts with earlier studies which favoured the assumption of normality for Value at Risk estimation (e.g. Jorion, 1995).

Cifter (2012) compares a set of GARCH based models under the assumptions of normality, the student t distribution, and the skewed t distribution using South African stock market data. The study concludes that the normal-mixture GARCH (NM-GARCH) model is superior in that particular context.

In summary, the literature seeking to determine the accuracy of GARCH based Value at Risk models have different views regarding the accuracy of these models in the context of emerging market. However, most of the literature available points to the possibility that Value at Risk predictions on emerging market data is more accurate by long memory and asymmetric GARCH based models. The authors mention several gaps in the availability of literature regarding emerging markets, more specifically, the South African stock market. One of the research objectives of this thesis is to provide test the accuracy of GARCH based Value at Risk models in the context of South African data.

Most available literature on VaR model accuracy centres on using established models and backtests to perform the analysis. Sener et al, (2012) develop an experimental model which seeks to establish the relative accuracy of VaR models by a ranking methodology. The key difference of this study to most existing studies is that it not only considers the element of a breach itself, but it also accounts for excessive capital reserves and autocorrelation between the breaches. The study emphasises the importance of considering all these factors in determining the accuracy of a VaR model. In order to rank the models, the study ascribes a score to each model, based on their aggregate performance on these three areas. The study develops a predictive ability test in order to generate a score to the established VaR models.

The main methodology applied in this study makes use of a loss function to attribute a score to the VaR models being tested. The authors argue that, for the purposes of ranking, the quantification of the performance of each model is key. Therefore, the loss function is a suitable tool for these purposes as it is not a test of hypothesis, but provides a score based on the magnitude of the violation, and in the case of this
particular study, on the two additional factors of autocorrelation and excessive capital allocation. The score is broken down into two main parts: the penalty for excessive capital allocation, in other words, when VaR is not exceeded, and the penalty for the violation which the authors denote as the “violation space”. The violation space is further broken down into two parts to account for the magnitude of the violation and the correlation between the violations. The sum of these two criteria is known as the “penalisation measure” which is then used to determine the accuracy of the models.

The study makes use of equity index data from eleven emerging markets and seven developed markets. The data makes use of the daily observations from January 2005 to mid-2009, thus including the crisis period. This study evaluates some of the commonly used VaR models, for example: the Riskmetrics model, the Historical Simulation method, the Monte Carlo Simulation method, the CAViaR asymmetric method, the EGARCH method, and a few others.

The study finds that the best performing models are the asymmetric models such as the EGARCH and the CAViaR methods. The authors ascribe this performance to the fact that these models are able to capture the asymmetric properties of the underlying data set in order to generate more accurate results.

While the accuracy of VaR models is key to ensuring financial stability, it is also imperative that financial institutions apply the most accurate VaR model for risk management purposes. The Basel Accords enable an institution to select an internal model for predicting VaR. There have been some studies showing the relative accuracy of VaR models used by banking institutions. However, owing to the sensitive nature of VaR information of banking institutions, these studies have been limited in scope and nature, Jorion, (2002).

Berkowitz and O’Brien, (2001) analyse the accuracy of VaR models in commercial banks in what they present as the first study providing such evidence. The study uses the daily profit and loss from trading activities and their respective VaR forecasts for six large banking institutions in the US. The study used a GARCH model of the daily P&L figures to predict VaR.

The results of the analysis indicate that, on average, the VaR predictions of banking institutions were inaccurate, compared to the predictions of the GARCH model used
as a benchmark. The study states that these inaccuracies could be the result of the complex nature of banking portfolios which include positions in a complex range of instruments which are subject to various market risk factors and of the limitations imposed by regulations which prescribe the treatment of certain items of revenue and expenditure in the financial statements.

Although the study provided valuable insight into the performance of VaR models in banking institutions, the scope for investigating the potential for improving these models remains limited due to the fact that such information is not readily available. As a result, more recent studies on the interaction of banking regulation and VaR models evaluate the overall implications on model accuracy, but do not evaluate the flaws of the models themselves in relation to the complex portfolios they have to be applied to.

Da Veiga et al. (2012) analyse the relationship between the Basel penalty structure and the choice of VaR model by a banking institution. Their study provides interesting insight into the application of the current penalty structure by evaluating the consequences of the amount of leeway available to banking institutions within the current framework.

The study argues that because Basel regulations allow a banking institution to select an internal model for predicting VaR, this could be leading to institutions selecting models which tend to underestimate VaR, while still operating within the constraints of the Basel penalty structure which penalises a certain number of violations. The main argument of the study is based on the fact that capital charges, which are based on the VaR estimate, represent a serious opportunity cost for banking institutions. As such, these institutions would seek to minimise their opportunity cost by selecting the model which produces the lowest possible VaR forecast within the acceptable range.

In order to test this hypothesis, the study makes use of data from the S&P500 index from 14 January 1964 to 11 November 2009. VaR predictions are made over a 10-day horizon, in line with the Basel Accord. The study primarily focuses on the conditional volatility based VaR models such as Riskmetrics, GARCH (p,q), GJR, and ARCH models.
The study applies the current Basel penalty structure, amongst others, to determine whether there is evidence of a deliberate choice in a model which tends to generate a high number of violations. The results do indicate that this is the case. These findings are similar to those of Lucas, (2001), who arrives at a similar conclusion regarding the suitability of Basel regulations in ensuring the choice of a suitable VaR model. The key difference between these two studies is mainly the fact that the study by da Veiga et al (2012) investigates this issue further, by testing different variants of the current penalty structure to establish a more suitable one.

The latter study further suggests that the imposition of an upper limit on the number of allowed violations may be more effective in aligning the interests of banking regulation and banking institutions. However, at the time this study was conducted, the latest Basel regulations had not been issued. The latest changes prescribed by the BIS in 2013 may solve this issue to some extent in the case of major financial institutions. As was explained in Chapter 2, financial institutions which pose a threat to the financial system due to their size, have additional capital adequacy requirements which are based on their size, not on the VaR figure generated by their internal models. Therefore, this new guideline may limit the scope of manipulating capital reserve requirements for large financial institutions.
Chapter 5. Data and Methodology

5.1 Data

For the purposes of this analysis, the data used is the daily closing total returns data for the JSE All Share Index (ALSI), obtained from BFA McGregor. The data set spans the period 5\textsuperscript{th} January 1998 to 10\textsuperscript{th} September 2012 (i.e. 3669 trading days).

The daily return, $r_t$, expressed as a percentage, is calculated as follows: $r_t = 100 \times (\ln P_t - \ln P_{t-1})$, where $P_t$ denotes the closing value of the index on day $t$.

The natural logarithm (ln) of the daily returns is used in the analysis. Log normal returns lend themselves to the advantage of being able to construe accurate results irrespective of the horizon under consideration.

The first 1000 daily returns figures are used to generate the first Value at Risk (VAR) figure. The methodology applied uses 1000-day rolling periods to estimate Value at Risk. This amounts to a total of 2994 Value at Risk estimates.

For the purposes of comparing the performance of Value at Risk forecasts, the entire data set is split into three different periods, relating to different stages of the recent financial crisis. The first period spans 1 January 2002 to 31 October 2007, prior to the financial crisis. The second phase, the duration of the financial crisis, spans 1 November 2007 to 31 December 2008. The recovery phase spans 1 January 2009 to 10 September 2012. These splits were selected as the different stages of the financial crisis. The first sub set of the data relates to the period prior to the crisis, characterised by stability, the second sub set relates to the crisis period with extreme volatility, and the third period relates to the recovery period, with some residual volatility from the crisis.

This split of the data set enables comparisons to be made on the performance of the various VaR models across periods of differing volatility. Alexander (2008) finds that for the S&P 500 the crisis period (1 November 2007 to 31 December 2008, in the current data set), exhibits very high levels of volatility, and VaR models traditionally used (such as the historical simulation model) prove inaccurate during these periods.
4.1

Figure 2: The above graph shows the total daily returns on the ALSI. Periods of high volatility correspond to the crisis period, between November 2007 up to and including December 2008.

5.2 Value at Risk Models

As discussed in the introductory chapters, VaR models are used by banking institutions to calculate the regulatory capital requirements in order to hedge against market risk (BIS, 1996). This study uses several VaR models to estimate Value at Risk predictions in an attempt to identify the most accurate ones. The accuracy of each model is measured by individually applying several backtests to the output of each model, as will be discussed in Section 5.8. The methodology is, therefore, broken down into two sections: the application of the VaR models to generate predictions, and the backtesting process which assesses the accuracy of the VaR models.

The VaR models used in this study are split into volatility models and the historical simulation and normal linear (variance covariance) models.
5.2.1 Historical Simulation

The Historical Simulation model is applied over the data set to generate VaR forecasts. The Historical Simulation model assumes that the past distribution of returns is representative of the future performance of the data (Alexander, 2008). Therefore, this model does not make any assumptions about the underlying distribution of the data. A key advantage of this is that this model can be applied to any type of instrument and is even able to factor in the presence of fat tails in a dataset (Alexander, 2008).

A key consideration when using the Historical Simulation model is the fact that, in order to generate accurate VaR forecasts, the data set has to be large enough to provide enough data for the prediction of future returns. The current data set uses 3669 observations for the analysis, which is relatively large in comparison with similar studies on VaR prediction.

5.2.2 Normal Linear Model

The Normal Linear VaR model is a parametric approach to estimating VaR. They key feature of this approach is its assumption that the underlying returns follow a normal distribution. Using this as the central point of the VaR estimation process, VaR is estimated using the equation below:

\[ \text{VaR} = -\alpha_{cl}\sigma_r - \mu_r \]

Where, \( \sigma_r \) and \( \mu_r \) represent the mean and standard deviation of the returns respectively. \( \alpha_{cl} \) represents the standard normal variate corresponding to the VaR significance level (Dowd, 2002). For example, for a VaR estimate being calculated at 5 per cent level of significance, the corresponding value of \( \alpha_{cl} \) would be -1.645.
Models of Conditional Volatility

The accuracy of Value at Risk estimate depends on the accuracy of the underlying asset return volatility estimate of the data set. (Thupayagale (2010), So and Yu (2006)). The need for accuracy in the initial volatility parameters becomes more critical in out of sample forecasts.

The volatility models selected for this study are: the Riskmetrics model, the GARCH (p,q) model, the IGARCH (p,q) model, the EGARCH (p,q) model, the GJR GARCH model and the FIGRACH (p,d,q) model.

The above selection of models was made based on the findings in the literature, which suggest that certain features of each model make them interesting to analyse in the context of the South African data. For example, research suggests that emerging market data presents long memory properties (Assaf and Cavalcante, (2005); DiSario, McCarthy and Saraoglu (2008) and McMillan and Thupayagale (2009)), which is why the FIGARCH (p,d,q) model is included in the analysis. Findings on emerging market data also suggests the element of asymmetry which is captured by the IGARCH and EGARCH models. The Riskmetrics model is included in the analysis as it has been widely used over a significant period of time, and forms the basis for more complex volatility models (Thupayagale, 2010).

The volatility of time series data is measured by its variance or standard deviation. Volatility is estimated prior to the Value at Risk estimates.

5.2.3 Riskmetrics Model

JP Morgan’s Riskmetrics model was introduced in 1994 and, at the time, provided a novel methodology for Value at Risk estimation. The widespread use of this model assisted in the adoption of Value at Risk as a measure of risk (Vee et al, 2012). The volatility is estimated as an exponentially weighted moving average (EWMA), as follows:

\[ \hat{\sigma}_t^2 = \varphi \hat{\sigma}_{t-1}^2 + (1 - \varphi) \sigma_{t-1}^2 \]  

(4)
Where $\sigma_i^2$ is the forecast variance, $\varphi$ is the smoothing parameter, such that $0 \leq \varphi \leq 1$ , and $\sigma_{t-1}^2$ refers to past observed volatility in the data set. Past studies (Morgan (1996), Alexander, (2008)) show that an assumed value of 0.94 for the parameter $\varphi$ produces more accurate results for emerging market data, and this is what will be used in this study.

The estimation of volatility using this method implies that more recent shocks have a greater impact on the volatility variable, such that over time, the impact of a volatility shock decreases. This is also referred to as the rate of decay of a shock. The advantage of this process is that it accounts for the fact that the market takes a certain amount of time to adjust after a volatility shock, and that residual effects of volatility may still be present for some time after the initial shock.

**GARCH Models**

It has been found that financial asset returns are modelled with satisfactory results if GARCH based models are applied (Engel, (1982) and Bollerslev (1986)). Using this approach, the key step centres on the specification of the conditional variance, given as $h_t$, and the derivation of the asset returns equation. Past research suggests that the standard GARCH (1,1) model provides accurate estimates of the parameter $h_t$ for the current data type under consideration, Bollerslev (1986). GARCH models are favoured for high frequency time series data because of their ability to capture leptokurtosis, skewness and volatility clustering, which are often present in data such as the one used in this study (Thupayagale, 2010).

As opposed to the principle of the EWMA, which is applied in the Riskmetrics model, the use of GARCH based models to generate volatility forecasts implies that the conditional variance will fluctuate over time and is represented as a function of past errors and changes in volatility. Under these models, the long run variance is constant. The returns function is given by: $r_t = \mu + \varepsilon_t$, where $r_t$ is the returns process, $\mu$ the conditional mean, and $\varepsilon_t$ the error term.
Short Memory GARCH Models

5.2.4 GARCH (p,q) model

In this study, the standard GARCH (p,q) is used to estimate Value at Risk forecasts. This model denotes the conditional variance by:

\[ h_t = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)h_t \]  \hspace{1cm} (5)

Where \( \omega \) is the long run average, \( \varepsilon_t^2 \) is the size of the previous period’s volatility, and \( h_t \) are past estimates of the conditional variance. The p and q variables in the GARCH (p,q) model represent the orders of the polynomials \( \alpha(L) \) and \( \beta(L) \) respectively.

Under the standard GARCH model, the magnitude of volatility shocks impact on current volatility. The basic GARCH (1,1) model exhibits the property that a shock decays exponentially over time and has a decreasing impact on volatility. It is important to note that the sign of the shock (positive or negative), is irrelevant to the estimation of volatility. This model is, therefore, symmetric because positive or negative shocks of the same magnitude have identical effects on volatility.

5.2.5 EGARCH (p,q) model

The asymmetric EGARCH (1,1) model is given by the following equation:

\[ \ln\sigma_t^2 = \omega + \alpha|z_{t-1}| + \gamma z_{t-1} + \beta \ln\sigma_{t-1}^2 \]  \hspace{1cm} (6)

This model has the key feature of always giving a positive variance estimate because of its exponential nature. In this model, the leverage effect is exponential as the term \( \ln\sigma_t^2 \), is the logarithm of the conditional variance. The leverage effect captures the effect of the sign, and the effect of the magnitude of a shock. Because of this feature, the EGARCH model is said to reflect the asymmetric effect of a volatility shock (McMillan and Thupayagale, 2009). A shock of a certain magnitude will not necessarily have the same effect on volatility depending on whether it was positive or negative. An asymmetric model is able to capture the exact effect of this shock.
depending on whether it was a positive or negative shock. A symmetric model would simply consider the magnitude of the shock and treat the volatility implication with symmetry, thereby making the assumption that the impact would be identical whether or not the shock was positive or negative.

5.2.6 GJR GARCH (p,q) model

Glosten et al (1993) proposed the GJR GARCH model, which is capable of capturing the asymmetric effects of both positive and negative shocks. The model is given by:

$$
\sigma_t^2 = \omega + (\alpha \varepsilon_{t-1}^2 + \gamma \varepsilon_{t-1}^2 I_{t-1}) + \beta \sigma_{t-1}^2 \quad (7)
$$

The term $I_{t-1}$ is responsible for capturing asymmetric effects of shocks as it is able to distinguish positive shocks from negative ones. Similar to the EGARCH model, the asymmetric property of the GJR model is able to reflect the full effect of volatility shocks in predicting volatility. Black, (1976) explained the cause of the different impacts of positive and negative shocks on volatility by stating that a negative shock would lead to higher financial leverage (calculated by the debt to equity ratio) by decreasing equity, and as a result, lead to higher risk. The implication of this is that the impact of a negative shock to returns is larger than the impact of a positive shock. The variable $I_{t-1}$ is capable of distinguishing between these shocks, therefore enabling the model to treat positive and negative shocks differently.

5.2.7 IGARCH model

The IGARCH model is a variant of the standard GARCH model, with the key difference being the fact that the parameters are constrained by the equation $\alpha + \beta = 1$. The implication of this constraint is that shocks to volatility persist permanently and that the unconditional variance is infinite. The model, therefore, reflects volatility persistence and is given by the following equation:

$$
\sigma_t^2 = \omega + (1 - \beta)\varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (8)
$$
Long Memory GARCH models

5.2.8 FIGARCH (p,d,q) model

The basic GARCH model is a short memory model with the foremost assumption that the data under consideration is stationary and any shocks decay within a negligible amount of time. The IGARCH model relaxes the assumption of stationary data and includes the effect of a shock being permanent. Financial markets, especially emerging markets exhibit long memory properties (Ding et al., 1993; So, 2000), that is, they are mean–reverting over time. Baillie et al (1996) modified the IGARCH model to include the effects of long memory behaviour. This modification of the IGARCH model can be rewritten as:

\[ h_t = \omega [1 - \beta(L)]^{-1} + \{1 - \beta(L)\}^{-1} \varphi(L)(1 - L)^d \epsilon_t^2 \quad (9) \]

This model can capture the long memory property because \( d \) is now allowed to take fractional values. The FIGARCH model implies that the effect of a shock decays at a slow hyperbolic rate, therefore, allowing for the effect of a shock to be present for a period of time. The FIGARCH model proves particularly accurate in the case of emerging market data, such as South African data because it captures both asymmetry and long memory characteristics of present in the data (So and Yu, 2006).

5.3 Normal and t distribution Assumptions

In order to generate Value at Risk predictions from each model, comparisons are made between distributional assumptions for the data set used in the analysis. This approach is adopted by Milwidsky and Mare (2010), by comparing the results of the t-distribution assumption against the assumption of normality. There are conflicting results in the literature surrounding the suitability of these assumptions. Jorion (1992) suggests that the assumption of normality is adequate in producing Value at Risk forecasts, however, more recent findings suggest that the t-distribution assumption is
more reliable, particularly in the case of emerging markets (Milwidsky and Mare, 2010).

5.4 VaR significance levels

The Basel Market Risk management framework recommends that VaR is estimated at the 1% and 5% levels of significance. For the purposes of this study, both significance levels are applied for each subset of the data set and each variant of the model being applied.

5.5 VaR horizons

The Basel Market Risk management framework recommends in its latest revision, that VaR is estimated for 1 day at a time. In this analysis, the VaR estimates are calculated for 1 day and 10 day holding periods. It is useful to include these two risk horizons as these are the ones commonly applied by financial institutions, although banking regulations only require the one day horizon, the ten day horizon is a useful addition for its comparative value.

5.6 The Drift Adjustment

Alexander (2008) discusses the drift term ($\epsilon_{ht}$) in VaR estimations, which is given by:

$$\epsilon_{ht} = P_t - B_{ht}E_t(P_{t+h})$$

and represents the difference between the current portfolio price ($P_t$) and its expected future price ($E_t(P_{t+h})$), discounted at the risk free rate, where $B_{ht}$ is the discount factor. When VaR is measured over a long period, of at least several months, the drift adjustment is found to have an impact on these estimates (Alexander, 2008).

For the purposes of this study, VaR estimates are generated both with and without the inclusion of the drift adjustment.
5.7 Summary of VaR Models Applied

Using the different parameters and different models, along with the different subsets in the data set, the total number of VaR model variants analysed in this study is 512 (4 x 2 x 4 x 2 x 8, from the diagram below). This number is far more extensive than any number of VaR model variants analysed in current studies. The addition of the various levels (data subsets, significance and horizon parameters, distributional assumptions, etc.) provides ample scope for comparing the various VaR models.

Figure 3: Diagram showing the methodology applied.
5.8 Backtesting Methodologies

In order to determine the accuracy of Value at Risk models used in the estimation of Value at Risk, backtesting methods are applied to each model over each of the segments of the data period under consideration. There are five backtests applied in this study, each of them focusing on one determinant of reliability of the Value at Risk model being applied.

Value at Risk model backtesting is recommended by the Basel market risk framework in order to assess the reliability of the Value at Risk model being used.

The starting point for the backtesting process is to determine what a violation (or a breach) is. A violation is said to occur when the actual loss of the portfolio exceeds the Value at Risk (or maximum loss). Backtesting methodologies use the data on violations to construct a framework for determining the accuracy of the Value at Risk model being evaluated.

5.8.1. Christoffersen Test for Independence

The Christoffersen test for independence seeks to reach a conclusion on the model’s ability to produce independent Value at Risk forecasts based on the sequence of occurrences of the violations (Christoffersen and Pelletier, 2004). The test makes use of the number of violations occurring in a specific sequence to develop a likelihood ratio which forms the basis for performing the test.

The variable being observed for this test is denoted by $I_t$, and is defined as:

$$I_t = \begin{cases} 
1, & \text{if a violation occurs} \\
0, & \text{if there is no violation} 
\end{cases}$$

---

1 The most common backtesting methods are applied in this study. Other methods, such as the Pearson’s Test, the Duration-based method and variants of the Lopez Loss function are left out in the interest of not duplicating the backtesting methodology.
From the observed occurrences of $I_t$, the following variables are calculated: $n_{00}$, a non-violation followed by a non-violation, $n_{01}$, a non-violation followed by a violation, $n_{10}$, a violation followed by a non-violation and $n_{11}$, a violation followed by a violation.

Based on the above variables, the parameters $\pi_0$, $\pi_1$ and $\pi$ are calculated:

$$\pi_0 = \frac{n_{01}}{n_{00} + n_{01}}$$

$$\pi_1 = \frac{n_{11}}{n_{10} + n_{11}}$$

$$\pi = \frac{n_{01} + n_{11}}{n_{00} + n_{01} + n_{10} + n_{11}}$$

The null hypothesis for this test is that the violations occur independently of one another and is given by:

$$H_0: \pi_0 = \pi_1$$

The test statistic, given as a likelihood ratio, is given below:

$$LR_{ind} = -2ln\frac{\pi_{01}^{n_{01}+n_{11}}(1-\pi)^{n_{00}+n_{11}}}{\pi_{01}^{n_{01}}(1-\pi_{01})^{n_{00}}\pi_{11}^{n_{11}}(1-\pi_{11})^{n_{10}}} \quad (10)$$

The test is a chi-square test with one degree of freedom, with a significance level of 5%, and a critical value of 3.841. A test statistic greater than the critical value would lead to the null hypothesis being rejected with the conclusion that the model presents issues of independence, Christoffersen and Pelletier (2004).

The test for independence is particularly important especially in highly volatile periods. A model which performs well for this test is said to be able to adapt automatically to new information in asset returns (Hurlin and Tokpavi, 2006). A model not possessing the independence property would exhibit successive violation clustering, a particularly risky thing in highly volatile periods, where large losses are a possibility, and successive violations may be dangerous.
5.8.2. The Kupiec Test for unconditional coverage

This test is based upon the number of violations and the confidence level, Kupiec (1995). This test measures unconditional coverage and assumes that the number of violations follow a binomial distribution. The null hypothesis for this test is given by:

\[ H_0 = p = \hat{p} = \frac{x}{t} \]  \hspace{1cm} (11)

Where \( p \) is the significance level of the Value at Risk estimate, \( \hat{p} \) is the observed rate of violations, and \( x \) are the number of violations and observations respectively.

The test statistic for this test is a likelihood ratio given by:

\[ LR_{unc} = -2 \ln \frac{p^N (1-p)^{T-N}}{\alpha^N (1-\alpha)^{T-N}} \]  \hspace{1cm} (12)

The Kupiec Test statistic is chi-square distributed with one degree of freedom. The critical value is 3.841 for a significance level of 5%. If the test statistic exceeds the critical value, the null hypothesis is rejected and the model is concluded to be inadequate for coverage.

The basis for this test rests on the assumption that the percentage number of violations must be the VaR significance level multiplied by the number of observations in the holding period. A model which fails this test is said to not predict VaR accurately for the significance level that was applied. For example, when VaR is measured at 5 % level, the implications are that there is a five percent possibility that the actual loss will be higher than the predicted VaR figure. A model failing the Kupiec test in this case implies that the VaR figure is exceeded more than 5 % of the time, implying the model is inaccurate in terms of the confidence level.

5.8.3. The Test for Conditional Coverage

This is a joint test developed by Christoffersen in 1998 which combines the above tests for independence and the test for unconditional coverage. The test statistic for this test is the sum of the likelihood ratios for the Christoffersen test for Independence
and the Kupiec test for unconditional coverage. The test statistic for this test is given as:

\[ LR_{cc} = LR_{UC} + LR_{ind} \]  \hspace{1cm} (13)

The test statistic is chi-square distributed with two degrees of freedom. At a significance level of 5%, the critical value is 5.99.

The test for conditional coverage is particularly important as it tests for both the properties of independence and coverage simultaneously. A model performing significantly poorly and significantly better on either property does not necessarily constitute a good model, as these properties do not compensate for each other (Hurlin and Tokpavi, 2006).

5.8.4. The Lopez II Loss function

The Lopez II loss function, also known as the Size-adjusted Frequency Approach is derived from the Lopez I loss function derived by Lopez (1998). The Lopez I loss function is a binomial function which ascribes a value of 1 to the observed variable if a violation occurs, and a value 0 if there is no violation. The equation of the Lopez I loss function is given as:

\[ C_t = \begin{cases} 1 & \text{if } L_t > VaR_t \\ 0 & \text{if } L_t \leq VaR_t \end{cases} \]

This loss function was flawed in the sense that it merely provided the number of violations, with no information on the magnitude of an actual loss in the event of a violation. To account for magnitude, Lopez developed a second loss function, which adjusts for the size of the violation (Dowd, 2002). This is given as:

\[ C_t = \begin{cases} 1 + (L_t - VaR_t)^2 & \text{if } L_t > VaR_t \\ 0 & \text{if } L_t \leq VaR_t \end{cases} \]
However, this loss function is not a test of hypothesis, it simply provides the magnitude of the exceedance. Higher losses would lead to higher values for the loss function. Dowd (2002) suggests that, in order to interpret the Lopez II loss function, a benchmark value can be generated using Monte Carlo simulation. However, in the absence of a benchmark value, Vee et al (2012) suggest that the models with lower values for the loss function are said to perform better.

5.8.5. The Basel Traffic Lights Approach

This backtesting methodology has been prescribed by the Basel Committee on Banking Supervision. This is an unconditional backtest, in the sense that it simply measures the absolute number of violations over a certain holding period, and based on the cumulative probability of the number of violations observed, is classified under one of the three colours: red, yellow and green.

The interpretation of these colours is as follows: a model in the red zone requires a thorough revision and possible replacement, a model in the yellow zone requires some evaluation and possible revision, and a model in the green zone is considered accurate (Nieppola, 2009).

The Basel Traffic Lights backtesting methodology is criticized for its inability to highlight that the model may be inaccurate if there are no violations. A model with no violations is still within the green zone. However, a model with no violations may imply that the VaR estimate is too high, which would explain why there were no violations.

5.8.6 Research Value of Backtesting Methodology

The research value of this study stems mainly from the fact that it applies all the above backtesting techniques to determine the accuracy of VaR estimates. Backtesting is usually limited to two or three backtests within a particular research study. By performing six different backtests, this study tests the accuracy of VaR models on different levels, in order to provide an overall performance analysis for all the models analysed in the study. Given that each backtest focuses on one key reliability
characteristic of a VaR model, this study provides critical insight into the relative performance of the VaR models on each determinant of accuracy.

An analysis this extensive and broad provides grounds for multifactorial conclusions to be reached on the performance of VaR models.
Chapter 6. Results and Analysis

This chapter discusses the findings of the analysis regarding the observed accuracy of the VaR models discussed. The key measure of accuracy used is the performance of the models for each backtest applied. The five backtests used in determining model accuracy are: The Christoffersen test for independence, the Kupiec Test for unconditional coverage, the conditional coverage test, the Lopez II Loss function and the Basel Traffic Lights Test.

In this chapter the findings of the analysis are discussed with reference to certain key parameters applied in the modelling process, as well as the results of each backtest applied. The discussion focuses on the observed accuracy of the VaR models based on the set of accuracy criteria evaluated.

6.1 Findings around the Drift Adjustment

As discussed in the methodology chapter, the drift adjustment was applied as an additional differentiating parameter before estimating VaR. The findings in the current analysis suggest that whenever the drift adjustment was applied to VaR estimates, the number of violations was higher. The inclusion of the drift term did not impact on the results of the backtests which were in the form of tests of hypothesis. However, the models which did not include the drift adjustment had a lower score for the Lopez test, indicating a higher degree of accuracy.

For ease of presentation, only the results for the GARCH t model are presented in Table 2. Information about the other models is available in the Appendix,
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Table 2: Comparison of VaR models with and without the drift adjustment

The above result is observed throughout the entire data set and across all the models analysed. These findings are inconsistent with those of Alexander (2008), which state that over short risk horizons, the drift term does not impact on accuracy. The findings on drift adjustment by Alexander (2008) relate to a study conducted on data from the S & P 500, over a time horizon similar to the one used in the current study. Given that the data used between these two studies differs in one key aspect (the one being from a developed financial environment, in contrast to data from an emerging market), this
provides an interesting implication to the findings uncovered in the current study, namely that emerging market data and developed market data do not appear to behave in a similar way with regards to the inclusion of the drift adjustment term. However, the absence of evidence from other emerging markets to confirm this implication limits the value of this finding. The lack of research into the influence of the drift term on accuracy of VaR estimation provides further scope for research into this area.

6.2 Overall performance of the VaR models

Based on an overall analysis of the results of the backtests, the GARCH t and IGARCH t models perform best throughout the data set. These are followed by the GARCH Normal and IGARCH normal and GJR t and normal, which have a worse performance on the backtests applied. These results are based on an aggregation of the performance of each model for each backtest applied. Essentially, the number of occurrences of a true null hypothesis are counted to attribute an accuracy score to each model.

The tables below provide a summary of the performance of the best performing models across the range of backtests used. Information about the remaining models included in the study can be found in the accompanying CD.

The sections that follow evaluate the performance of the best models mentioned in this section with respect to specific accuracy criteria.
Table 3: The table below shows the results of the backtest for the four best performing models for the 1% significance level over a one day horizon. N/A indicates that a test statistic could not be generated. True or False indicate whether the null hypothesis tested true or false. The tests are as follows: Test 1: Christoffersen test (H0: model has no independence issues), Test 2: Kupiec (Test H0: model is accurate for coverage), Test 3: Conditional Coverage Test (H0: Model is accurate for coverage and independence), Test 4: Lopez II Loss Score.

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Table 4: The table below shows the results of the backtest for the four best performing models for the 1% significance level over a ten day horizon. N/A indicates that a test statistic could not be generated. True or False indicate whether the null hypothesis tested true or false. The tests are as follows: Test 1: Christoffersen test (H0: model has no independence issues), Test 2: Kupiec (Test H0: model is accurate for coverage), Test 3: Conditional Coverage Test (H0: Model is accurate for coverage and independence), Test 4: Lopez II Loss Score.

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Table 5: The table below shows the results of the backtest for the four best performing models for the 5% significance level over a one day horizon. N/A indicates that a test statistic could not be generated. True or False indicate whether the null hypothesis tested true or false. The tests are as follows: Test 1: Christoffersen test (H0: model has no independence issues), Test 2: Kupiec (Test H0: model is accurate for coverage), Test 3: Conditional Coverage Test (H0: Model is accurate for coverage and independence), Test 4: Lopez II Loss Score.

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<td>Number of True's</td>
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| **IGARCH T: 5%, 1Day** |       |          |
| Full Period (Jan '02 - Sep '12) | Breach | Test1 | Test2 | Test3 | Test4 | Breach | Test1 | Test2 | Test3 | Test4 |
| 159                    | FALSE | FALSE  | FALSE | 0.059577 | 131 | TRUE | TRUE | TRUE | 0.049086 |
| Prior (Jan '02 - Oct '07) | 77 | TRUE | TRUE | TRUE | 0.052962 | 64 | TRUE | TRUE | TRUE | 0.04402 |
| During (Nov '07 - Dec '08) | 26 | TRUE | TRUE | TRUE | 0.089358 | 21 | TRUE | TRUE | TRUE | 0.072174 |
| Recovery (Jan '09 - Sep '12) | 56 | TRUE | TRUE | TRUE | 0.060608 | 46 | TRUE | TRUE | TRUE | 0.049785 |
| Number of True's       | 4     | 4       | 4     | 4     |

| **GARCH NORMAL: 5%, 1Day** |       |          |
| Full Period (Jan '02 - Sep '12) | Breach | Test1 | Test2 | Test3 | Test4 | Breach | Test1 | Test2 | Test3 | Test4 |
| 158                    | FALSE | FALSE  | FALSE | 0.059203 | 132 | TRUE | TRUE | TRUE | 0.04946 |
| Prior (Jan '02 - Oct '07) | 77 | TRUE | TRUE | TRUE | 0.052962 | 64 | TRUE | TRUE | TRUE | 0.04402 |
| During (Nov '07 - Dec '08) | 26 | TRUE | TRUE | TRUE | 0.089359 | 24 | TRUE | TRUE | TRUE | 0.082484 |
| Recovery (Jan '09 - Sep '12) | 55 | TRUE | TRUE | TRUE | 0.059526 | 44 | TRUE | TRUE | TRUE | 0.047621 |
| Number of True's       | 4     | 4       | 4     | 4     |

| **IGARCH NORMAL: 5%, 1Day** |       |          |
| Full Period (Jan '02 - Sep '12) | Breach | Test1 | Test2 | Test3 | Test4 | Breach | Test1 | Test2 | Test3 | Test4 |
| 158                    | FALSE | FALSE  | FALSE | 0.059202 | 125 | TRUE | TRUE | TRUE | 0.046837 |
| Prior (Jan '02 - Oct '07) | 77 | TRUE | TRUE | TRUE | 0.052961 | 61 | TRUE | TRUE | TRUE | 0.041957 |
| During (Nov '07 - Dec '08) | 26 | TRUE | TRUE | TRUE | 0.089358 | 21 | TRUE | TRUE | TRUE | 0.072174 |
| Recovery (Jan '09 - Sep '12) | 55 | TRUE | TRUE | TRUE | 0.059526 | 43 | TRUE | TRUE | TRUE | 0.046539 |
| Number of True's       | 4     | 4       | 4     | 4     |
Table 6: The table below shows the results of the backtest for the four best performing models for the 5% significance level over a ten day horizon. N/A indicates that a test statistic could not be generated. True or False indicate whether the null hypothesis tested true or false. The tests are as follows: Test 1: Christoffersen test (H0: model has no independence issues), Test 2: Kupiec (Test H0: model is accurate for coverage), Test 3: Conditional Coverage Test (H0: Model is accurate for coverage and independence), Test 4: Lopez II Loss Score.

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62
The models were ranked in their overall performance across the five backtests applied in the analysis. The models with a higher number of successes on the backtests were selected as the most accurate models.

Table 7: The table below ranks the models according to their performance for each of the tests of hypotheses applied

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</tbody>
</table>

6.2.1. The RiskMetrics model

Owing to its widespread use in market risk management, the Riskmetrics model was included in the analysis as it provides grounds for comparison with a wider range of models than what has been analysed in the literature.

The results of this study reveal that the Riskmetrics model performs only marginally better than the historical simulation. It is outperformed by all the GARCH based models included in the analysis. These findings are consistent with the findings of So
and Yu (2006) and Thupayagale (2010), who also found that the Riskmetrics model was outperformed by GARCH based models.

The current study provides more insight into these findings, as it provides comparison against the historical simulation model, which is also widely used. Furthermore, the backtesting process is far more extensive given the number of tests applied. Previous studies do not separate data periods to isolate varying degrees of volatility. Therefore, similar conclusions are reached with a research methodology which is far more in depth.

### 6.2.2 The Christoffersen test for Independence

The Christoffersen test for independence tests the VaR model for the fundamental quality of independence. The quality of independence is an important aspect of model accuracy as it determines a model’s ability to produce VaR forecasts independent of each other. This quality is particularly important during crisis periods, when volatility shocks tend to be clustered. A model which does not possess the quality of independence is, therefore, unable to generate VaR forecasts which are independent of each other and its usefulness becomes seriously flawed during a crisis period.

As explained in the methodology chapter, the Christoffersen test for independence is a hypothesis test, using a likelihood ratio as the test statistic.

The results of the current analysis indicate that at the one percent significance level, over a one day horizon, the Normal linear model performs best, with an equal success rate whether using the drift adjustment or not. This is followed by the Riskmetrics model using the Normal distribution assumption. These models, therefore, possess the quality of independence over other models which do not perform as well on this test. However, as indicated in Table 3, the GARCH based models, which have a better overall performance do not have an accuracy indicator for the quality of independence. This is because on a number of occasions, these models do not have a test statistic for this test. This does not necessarily mean they do not pass the test for independence, but merely that the results of this test are inconclusive in these cases.
6.2.3. The Kupiec Test for unconditional coverage

The Kupiec test for unconditional coverage was the second hypothesis test that was performed on the VaR models. The Kupiec test measures the proportion of VaR violations over the data set against the significance level of the VaR forecast. A model satisfying the criterion of coverage is expected to have the same proportion of VaR violations as the level of significance of the estimate, as these are assumed to follow a binomial distribution.

The quality of coverage is an important measure of accuracy in a VaR model. As VaR is defined as the maximum loss that will not be exceeded over a certain horizon with a certain level of certainty (The confidence level, or one minus the significance level), it is important that a model does actually provide reliable estimates with regards to that level of certainty. Hence, the Kupiec test for coverage tests the model with a key aspect of accuracy which is at the very core of the idea of VaR: the element of certainty.

The results for the Kupiec test for coverage are shown in the above table. As observed, a number of models perform equally well on this aspect of accuracy. In other words, the number of times the null hypothesis is true is equal for several of them. The best performing models according to this test are shown to be mainly the GARCH based models. Given their ability to model volatility more accurately in an emerging market environment, this could explain the performance of these models for this test.

6.2.4. The Conditional Coverage test

The joint test for coverage or the conditional coverage test combines the two elements of independence and coverage of the Christoffersen test and the Kupiec test respectively. It is important to note that, although both aspects of independence and coverage are key determinants of accuracy in a VaR model, they do not, however, compensate for each other. Therefore, a model performing very well on the independence criterion but poorly on the aspect of coverage, does not altogether
constitute an average or satisfactory model. Such a model would not necessarily perform well on the conditional coverage test.

As explained in the methodology chapter, the conditional coverage test is also a hypothesis test with a likelihood ratio as the test statistic.

The conditional coverage test is a more reliable test of accuracy than the Christoffersen test or the Kupiec test in isolation. It tests two key elements of accuracy simultaneously. The results show that the Normal Linear model outperforms all the other models for the one percent one day VaR estimates. This implies that the Normal Linear model, as per this test, possesses both qualities of independence and coverage. However, although the GARCH based models exhibit a lower number of breaches consistently, they perform poorly on this test because of the fact that a test statistic cannot be generated because of the absence of breaches which is required to calculate the test statistic over certain periods. Hence, although these models do not necessarily fail this test, a conclusion cannot be reached because of an absence of test statistic to prove or disprove the validity of the null hypothesis.

6.2.5. The Lopez II Loss function

The Lopez II Loss function is not a test of hypothesis, but is a score attributed to each model with respect to the size of the violation observed relative to the predicted value of the VaR. Tests of accuracy applied so far in the analysis consider the number of violations, whether in sequence, or as an absolute or relative figure.

The Lopez II Loss function explicitly calculates the relative size of a loss exceeding the VaR figure. The Lopez II Loss function, therefore, evaluates another aspect of accuracy in a VaR model, the relative size of the loss, if a violation does occur. This measure gives an insight into how bad the outcome can be in case a VaR model does prove inaccurate in predicting VaR. The lower the value of the Lopez II loss function, the more accurate the model is perceived to be.

The findings of the current analysis point to the fact that asymmetric GARCH based models with the t distribution assumption tend to produce lower Lopez II Loss
functions. This implies that, even in the event of a violation, the VaR predicted by these models is not as far off as the actual loss as other VaR models.

6.2.6. The Basel Traffic Lights Test

The Basel Traffic Lights test is not a hypothesis test and is perhaps the simplest of the backtesting methods applied in this analysis. As explained in the methodology chapter, this test classifies models into zones: Red, yellow and green based on the number of violations observed.

Models with a better level of accuracy are classified into the green zone, with less accurate models being in the yellow and red zones respectively.

The results of the current analysis show that the GARCH based models, namely the EGARCH t and IGARCH t tend to be in the green zone across a larger number of sub sections in the data set. The tables below show the results for the Basel Traffic Lights test for the EGARCH t and IGARCH t models.

**Table 8:** Performance of the EGARCH t Model for the Basel Traffic Lights Test. The Model is within the yellow and green bands throughout the sub periods of the data set.

<table>
<thead>
<tr>
<th>PERIOD</th>
<th>DRIFT</th>
<th>NO DRIFT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Breach</td>
<td>Basel Traffic Light Colour</td>
</tr>
<tr>
<td>Full Period (Jan '02 - Sep '12)</td>
<td>36</td>
<td>Yellow</td>
</tr>
<tr>
<td>Prior (Jan '02 - Oct '07)</td>
<td>18</td>
<td>Green</td>
</tr>
<tr>
<td>During (Nov '07 - Dec '08)</td>
<td>9</td>
<td>Yellow</td>
</tr>
<tr>
<td>Recovery (Jan '09 - Sep '12)</td>
<td>9</td>
<td>Green</td>
</tr>
</tbody>
</table>

**Table 9:** Performance of the IGARCH t Model for the Basel Traffic Lights Test. The Model is within the green band throughout the sub periods of the data set.

<table>
<thead>
<tr>
<th>PERIOD</th>
<th>DRIFT</th>
<th>NO DRIFT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Breach</td>
<td>Basel Traffic Light Colour</td>
</tr>
<tr>
<td>Full Period (Jan '02 - Sep '12)</td>
<td>29</td>
<td>Green</td>
</tr>
<tr>
<td>Prior (Jan '02 - Oct '07)</td>
<td>16</td>
<td>Green</td>
</tr>
<tr>
<td>During (Nov '07 - Dec '08)</td>
<td>4</td>
<td>Green</td>
</tr>
<tr>
<td>Recovery (Jan '09 - Sep '12)</td>
<td>9</td>
<td>Green</td>
</tr>
</tbody>
</table>
6.2.7 Overall model performance

So and Yu (2006) found that, in the case of emerging markets, the asymmetric GARCH models produce more accurate forecasts. The current results are in line with these findings. The data used is from the South African index, which fits the description of an emerging market. The current findings support the conclusions of So and Yu (2006) and Vee et al (2012) as is shown by the better score of the IGARCH model, which is an asymmetric model. Asymmetric shocks are not absorbed in the same way in emerging markets as in developed financial markets which have a higher capacity to absorb such shocks quickly. As a result, asymmetric models do provide a better level of accuracy in forecasting VaR in emerging markets.

6.3 Distributional Assumptions

The current analysis is performed by evaluating the performance of VaR models using the normal distribution assumption and the Student t distribution assumption for the underlying data set. As supported extensively in research into this area, the assumption of normality is commonly used in the context of VaR estimation. However, in many cases, the underlying data set does not exhibit normality characteristics. As a result, VaR estimates generated with this key assumption tend to underestimate the actual value of the VaR figure. In order to investigate the relative accuracy of these distributional assumptions, the current analysis compares the relative performance of VaR models making use of the normal distribution assumption and the t distribution assumption. The results of the backtests are compared for each model, keeping all other model parameters constant, with the distributional assumption being the only variable allowed to fluctuate.

The primary results indicate that models using the t distribution assumption for the underlying dataset produce more accurate results. These results are in line with the findings of Thupagayale et al (2008), who test the effectiveness of the t distribution in producing more effective Value at Risk estimates. The impact of distributional assumptions on VaR prediction accuracy is also investigated by Lee, Su and Liu (2008) and Ergen (2010).
Table 10 below shows a comparison of the relative performance of the GARCH model using the t distribution assumption and the assumption of normality. The key variables used to illustrate the relative accuracy are the absolute number of breaches, and the Lopez score.

<table>
<thead>
<tr>
<th>PERIOD</th>
<th>GARCH T : 1%, 1Day</th>
<th>GARCH Normal: 1%, 1Day</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DRIFT</td>
<td>NO DRIFT</td>
</tr>
<tr>
<td></td>
<td>Breach</td>
<td>Lopez Score</td>
</tr>
<tr>
<td>Full Period (Jan '02 - Sep '12)</td>
<td>34</td>
<td>0.01273961</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prior (Jan '02 - Oct '07)</td>
<td>19</td>
<td>0.01306831</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>During (Nov '07 - Dec '08)</td>
<td>5</td>
<td>0.01718353</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recovery (Jan '09 - Sep '12)</td>
<td>10</td>
<td>0.01082283</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The t distribution is preferred over the normal distribution because it provides more accurate VaR forecasts in the case of emerging market data such as the South African ALSI total returns data. Such data is rarely normally distributed and the assumption of normality under such circumstances would underestimate VaR.
6.4 Basel Recommendations

The Basel recommendations require that banks disclose the Value at Risk estimates for 60 business days at a time, using the historical simulation method. These recommendations are set out in the Basel 2.5 recommendation. As of March 2013, South African banking institutions were fully compliant with Basel 2.5 (BIS, 2013). There is no data for compliance levels for Basel 3 for South African institutions at this date.

The results of this study indicate that the historical simulation method has the lowest success rate across all the backtests, at least with regards to the JSE’s ALSI. This method is applied by all South African banking institutions. In a study on the relationship between the model used and capital charges, da Veiga et al (2012) argue that the regulatory framework that governs risk management practices does not penalize a model which underestimates Value at Risk.

The arguments made in this article explain that the Value at Risk figure determines the amount of reserve capital that a bank should have to guard against losses. The findings suggest that banking institutions seek to use the model which gives the lowest Value at Risk estimate, while still operating within the regulatory framework. This implies that, irrespective of the number of violations it provides, a model still within the provisions of the regulatory framework would be selected by a banking institution with the aim of having to set aside less capital for risk mitigation purposes. The study of da Veiga et al (2012) was conducted on data from the S & P 500 from 28th August 1972 to 25th September 2008. If extended to the findings on the South African data this suggests that South African banking institutions seem to make use of the model which our findings suggest results in the highest number of violations. In the trade-off between minimizing VaR breaches and minimizing minimum capital requirements within the legal framework, it is important to note that these models would favour the latter.

However, the current study makes use of only ALSI data to estimate Value at Risk figures. The portfolios of banking institutions are comprised of a variety of instruments, the return characteristic of which would not necessarily be accurately
captured by one single proxy, such as the ALSI index. Instruments such as derivative instruments typically exhibit non-linear behaviours, which may significantly alter the results of a portfolio wide Value at Risk estimate. Caution must be exercised in comparing the findings of this study to the actual number of breaches of internal Value at Risk models used by banking institutions as the Value at Risk predictions may differ in the case of portfolio containing instruments with non-linear returns.

6.5 The Relative performance throughout the different periods of the data set

For the purposes of analysing the performance of the VaR models more incisively, the data was analysed at different points within the data set. The reasons for this breakdown were to observe the impact of different levels of volatility on the accuracy of VaR models. These periods were selected as the different stages of the financial crisis. The data set, therefore, was analysed over the entire period, prior to the crisis, during the crisis, and during the recovery phase.

This breakdown allows for an interesting examination to be performed of the models, as one of the drivers of research in the area of risk management was the failure of VaR models during the crisis period, which was characterised by extreme volatility. The current analysis allows for the models to be analysed, with different parameters during the different phases of the financial crisis.

From the results of the analysis, it is observed that the GARCH t model tends to perform better throughout the subsets of the data set, even during the crisis period. The crisis period is characterised by an increased number of breaches for the other, less accurate models. GARCH models, in general, owing to their ability to capture the changing effects of volatility over time, are better able to provide more accurate VaR forecasts. Furthermore, as explained earlier, the assumption of a t distribution for the underlying data set is more suited to emerging market data, making VaR predictions more accurate as the model is able to map volatility movements better.
6.6 Conclusion

The layers of model accuracy analysed in this study were multiple. The different effects of modifying parameters, distributional assumptions as well as the inherent volatility relating to specific points in the data set were analysed by several backtests. The overall conclusion reached by this analysis points to the fact that GARCH based models, more specifically, the asymmetric ones, tend to outperform other models in the case of the ALSI index. This result is true even during the high volatility period which spans the crisis period. Further to this observation, it was observed that asymmetric GARCH based models perform better with the t distribution assumption for the underlying data set. A key element about these conclusions remains that the data set used comprised only of the ALSI total returns data, and, therefore, the validity of these conclusions remains within the constraints of this data set.
Chapter 7. Conclusions and Recommendations for Further Research

This thesis has analysed the various Value at Risk models available with close reference to the Basel market risk framework. Over time, and as a result of volatility shocks by different financial events, market risk management in the banking sector has evolved to address the flaws in existing techniques.

This study has made VaR predictions using a selection of popular VaR models, using ALSI total returns data. Once the VaR predictions were made for 10 day and 1 day horizons, the results were tested for accuracy using five different backtests. Based on the results of the backtesting process, the relative accuracy of each model was determined. The novel aspects in this study include the fact that the data spans the entire crisis period, as well as the recovery period that followed. This provided valuable insight into the relative performance of the models analysed due to the relative volatilities of these respective periods of the data set. The study also applied an extensive backtesting methodology in order to single out certain key elements of model accuracy for each backtest.

The study found that the inclusion of the drift adjustment in VaR estimates does not impact on accuracy, thereby suggesting that this adjustment adds little value to an analysis on emerging market data. The study also demonstrated that the t distribution assumption leads to more accurate VaR forecasts, consistent with findings from prior research on this subject. These observations need to be assessed while keeping in mind the limitations of the study. The conclusions reached in this thesis are similar to findings in the literature available on South African data, namely, that GARCH based models more accurately capture the effects of volatility in the South African market and produce more accurate Value at Risk estimates. However, although there is some current literature on Value at Risk models in South Africa, the literature is still scarce, and does not address certain key aspects of Value at Risk models, such as their performance in periods of differing volatility, and their reliability across a range of characteristics which are tested in each backtest applied in the current study.

The results of the analysis indicate that the model prescribed by the Basel recommendation, the historical simulation, performs poorly in comparison with other, more robust models, namely the IGARCH and FIGARCH models, which perform
better across all the backtests applied. This is consistent with the findings of So and Yu (2006) who conclude that South African equity index data exhibits asymmetry and long memory properties which are best captured by these types of GARCH models (asymmetric and long memory models).

Given the current lack of research on certain aspects of Value at Risk estimation, there are grounds for research into the applications of GARCH based asymmetric models to the portfolios of banking institutions. The current work simply provides a critical approach to the Basel recommendations in terms of accuracy of Value at Risk predictions of the different models. A more in depth approach would be to compare the predictions of the IGARCH and FIGARCH models with the Value at Risk figures of the banking institutions. A study of this nature would, however, require the composition of the banking institutions’ portfolio to be known, an exercise which was commenced at the outset of this study, but which did not yield substantial results owing to confidentiality issues.

Another area that could be researched, given the current best performing models, would be to calculate conditional value at risk figures to replicate volatility conditions similar to those experienced around the time of the financial crisis and compare those to the accuracy levels of Value at Risk predictions of banking institutions. This proposed methodology is in line with the currently enforced Basel II.5 recommendations and would be of great value in providing Value at Risk forecasts for very volatile market conditions. Conditional volatility estimates provide useful information as they reveal the worse outcome during worse conditions, Alexander (2008), as opposed to standard VaR which assumes normal market conditions. Such an analysis would provide useful insight into the efficacy of revised Basel measures in providing more accurate VaR forecasts.
Bibliography


APPENDIX A: EXCEL VISUAL BASIC CODE

The code below was written for running the excel macro to predict Value at Risk for the Normal Linear and Historical Simulation models. This code was applied for eight variants of each model for four sub sections of the data set.

Sub Analyze()
    Application.ScreenUpdating = False
    Set startrange = Sheets("Returns").Rows(1).Columns(1)
    For j = 4 To 2673
        Sheets("Returns").Select
        startrange.Offset(1, 0).Resize(1000, 1).Select
        Selection.Copy
        Sheets("ALSI").Select
        Cells(3, 12).Select
        ActiveSheet.Paste
        Sheets("ALSI").Select
        DeltaNormal = Cells(9, 6)
        Historical = Cells(10, 6)
        MonteCarlo = Cells(11, 6)
        Sheets("Results Sheet").Select
        Cells(j, 3) = DeltaNormal
        Cells(j, 4) = Historical
        Cells(j, 5) = MonteCarlo
        Sheets("Returns").Select
        Cells(2, 1).Select
        Application.CutCopyMode = False
        Selection.Delete Shift:=xlUp
    Next j
End Sub
APPENDIX B – OXMETRICS CODE

Below is the OxMetrics code which was used to run the models of conditional volatility. For ease of representation, one version of the code is shown below. This same code was applied, with different parameters for significance levels, risk horizon, model and distributional assumption to the four sub sections of the data set. The code shown below is for the EGARCH model, for 5%, 1 day VaR, using the assumption of normality for the underlying data set.

```c
#include <oxstd.h>
#include <oxdraw.h>
#import <packages/Garch6/garch>

main()
{
    decl numb_out_of_sample=2669; //number of forecasts
to test out-of-sample
    decl quan=<0.95>; // Quantiles investigated
    decl i,j,k;     // our loop variable
    decl emp_quan_out_neg=new
matrix[numb_out_of_sample][columns(quan)];

    decl T;
    decl Y;
    decl qu_neg,m_vSigma2,dfunc,m_vPar,cond_mean;
    decl m_cA,m_cV;
    decl m_Dist;
    decl drift = 0;     //0 if drift term included, 1
if not
    decl timeperiod = 1;  //1 if calculating 1-day,
10 if calculating 10-day

for (i = 0; i < numb_out_of_sample; ++i)
{
    //--- Ox code for G@RCH( 1)
    decl model = new Garch();

    model.Load("Test Returns.csv");
    model.Deterministic(-1);
    model.Select(Y_VAR, {"Returns", 0, 0});

    model.Fit();
}
```

81
model.CSTS(1,1); // cst in Mean (1 or 0), cst in Variance (1 or 0)
model.DISTRI(0); // 0 for Gauss, 1 for Student, 2 for GED, 3 for Skewed-Student
model.ARMA_ORDERS(0,0); // AR order (p), MA order (q).
model.GARCH_ORDERS(1,1); // p order, q order
model.MODE(2); // 0:RISKMETRICS 1:GARCH 2:EGARCH 3:GJR
4:APARCH 5:IGARCH
6:FIGARCH(BBM) 7:FIGARCH(Chung) 8:FIEGARCH(BBM only)
9:FIAPARCH(BBM) 10:FIAPARCH(Chung) 11:HYGARCH(BBM)
model.MLE(2); // 0 : Second Derivates, 1 : OPG, 2 : QMLE
model.ITER(0);

model.SetSelSample(1, 1+i, 1000+i, 1);
model.Initialization(<>);
model.DoEstimation(<>);
model.FORECAST(1,1,1);
T=model.GetcT();

// model.InitData();
// model.FigLL(model.GetFreePar(), &dfunc, 0,0);

m_vPar=model.GetValu("m_vPar");
model.SetSelSample(-1, 1+i, T+i+1, 1);
model.InitData();
Y=model.GetGroup(Y_VAR);
model.Res_Var();
m_vSigma2=model.GetValu("m_vSigma2")[T+i];
cond_mean=Y[T+i][]-model.GetValu("m_vE")[T+i][];

m_Dist=model.GetValu("m_cDist");

if (m_Dist==0)
{
    qu_neg=quann(1-quann);
}
else if (m_Dist==1)
{
m_cV=model.GetValue("m_cV");
qu_neg=sqrt((m_cV-2)/m_cV)*quant(1-quan,m_cV)';
}
if (m_Dist==3)
{
    m_cV=model.GetValue("m_cA");
m_cA=model.GetValue("m_cA");
qu_neg=<>
    for (j = 0; j < columns(quan) ; ++j)
    {
        qu_neg|=model.INVCDFTA(1-quan[j],m_cA,m_cV);
    }
}
if (drift==0)
{
    emp_quan_out_neg[i][]=(cond_mean +
sqrt(timeperiod)*sqrt(m_vSigma2).*qu_neg)'
}
if (drift==1)
{
    emp_quan_out_neg[i][]=(sqrt(timeperiod)*sqrt(m_vSigma2).*qu_neg)'
}
println("Processing Model: ", i);
delete model;
}
for (k = 0; k < numb_out_of_sample; ++k)
{
    println(emp_quan_out_neg[k][]);
}
APPENDIX C: EXCEL SPREADSHEET FOR CHRISTOFFERSEN TEST FOR INDEPENDENCE (BACKTEST 1)

The following pages are extracts from the original spreadsheets used to perform the Christoffersen Test for independence. The test was run a total of 512 times for this study (128 variants of VaR models were tested over four sub periods of the data set). For ease of representation, an extract of the original spreadsheets is shown. The complete spreadsheets are available on the accompanying CD.

<table>
<thead>
<tr>
<th></th>
<th>RISKMETRICS</th>
<th>GARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n_0$</td>
<td>$n_1$</td>
</tr>
<tr>
<td>Full Period (Jan '02 - Sep '12)</td>
<td>2566</td>
<td>50</td>
</tr>
<tr>
<td>Prior (Jan '02 - Oct '07)</td>
<td>1397</td>
<td>27</td>
</tr>
<tr>
<td>During (Nov '07 - Dec '08)</td>
<td>275</td>
<td>8</td>
</tr>
<tr>
<td>Recovery (Jan '09 - Sep '12)</td>
<td>894</td>
<td>15</td>
</tr>
</tbody>
</table>

Chi Square Critical Value (0.01,1) = 6.635

Test Statistic = LR$_{ind}$

$H_0$: The model does not have independence problems
APPENDIX D: KUPIEC TEST FOR UNCONDITIONAL COVERAGE

The following pages are extracts from the original spreadsheets used to perform the Kupiec Test For Unconditional Coverage. The test was run a total of 512 times for this study (128 variants of VaR models were tested over four sub periods of the data set). For ease of representation, a selection of extracts of the original spreadsheets is shown. The complete spreadsheets are available on the accompanying CD.

<table>
<thead>
<tr>
<th>BREACHES ALIS 1% 1day Drift</th>
<th>Kupiec's p</th>
<th>T</th>
<th>T-x</th>
<th>Kupiec's Test Statistic (LR_{LR})</th>
<th>Test Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>RISKMETRICS</td>
<td>GARCH</td>
<td>RISKMETRICS</td>
<td>GARCH</td>
<td>RISKMETRICS</td>
<td>GARCH</td>
</tr>
<tr>
<td>Full Period (Jan '02 - Sep '12)</td>
<td>41</td>
<td>34</td>
<td>0.0153615</td>
<td>0.01273</td>
<td>266</td>
</tr>
<tr>
<td>Prior (Jan '02 - Oct '07)</td>
<td>21</td>
<td>19</td>
<td>0.0144429</td>
<td>0.01306</td>
<td>145</td>
</tr>
<tr>
<td>During (Nov '07 - Dec '08)</td>
<td>6</td>
<td>5</td>
<td>0.0206185</td>
<td>0.01718</td>
<td>291</td>
</tr>
<tr>
<td>Recovery (Jan '09 - Sep '12)</td>
<td>14</td>
<td>10</td>
<td>0.0151515</td>
<td>0.01082</td>
<td>924</td>
</tr>
</tbody>
</table>

Chi Square Critical Value (0.01,1) 6.635

6.635

H₀: The model is accurate for coverage
Chi Square Critical Value (0.05,1) 3.841

H$_0$: The model is accurate for coverage

<table>
<thead>
<tr>
<th>BREACHES ALSI 5% 1day No Drift</th>
<th>Kupiec's p</th>
<th>T</th>
<th>T-x</th>
<th>Kupiec's Test Statistic (LR$_{tc}$)</th>
<th>Test Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EGARCH</td>
<td>GJR</td>
<td>EGARCH</td>
<td>GJR</td>
<td>EGARCH</td>
</tr>
<tr>
<td>Full Period (Jan '02 - Sep '12)</td>
<td>135</td>
<td>134</td>
<td>0.0505807 42</td>
<td>0.0502060 7</td>
<td>266</td>
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<tr>
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<td>60</td>
<td>66</td>
<td>0.0412654 75</td>
<td>0.0453920 22</td>
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<td>24</td>
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<td>44</td>
<td>0.0454545 45</td>
<td>0.0476190 48</td>
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Chi Square Critical Value (0.01,1)  
6.635

H₀: The model is accurate for coverage

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<th>T</th>
<th>T-x</th>
<th>Kupiec's Test Statistic (LR_{uc})</th>
<th>Kupiec's Test Conclusion</th>
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Chi Square Critical Value (0.01,10)
6.635

H₀: The model is accurate for coverage

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<th>Kupiec's Test Conclusion</th>
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APPENDIX E: CONDITIONAL COVERAGE TEST

The following pages are extracts from the original spreadsheets used to perform the Conditional Coverage Test. The test was run a total of 512 times for this study (128 variants of VaR models were tested over four sub periods of the data set). For ease of representation, a selection of extracts of the original spreadsheets is shown. The complete spreadsheets are available on the accompanying CD.

\(H_0: \text{The model is suitable}\)

Chi Squared Critical Value (0.05,2) 5.991

<table>
<thead>
<tr>
<th>Period</th>
<th>LR_{cc} (EGARCH)</th>
<th>LR_{cc} (GJR)</th>
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<th>LR_{ind} (GJR)</th>
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<td>1.00328228</td>
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### H₀: The model is suitable

**Chi Squared Critical Value (0.05,2) 5.991**

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<td>GJR</td>
<td>EGARCH</td>
<td>GJR</td>
<td>EGARCH</td>
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<tr>
<td>Prior (Jan '02 - Oct '07)</td>
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<td>-4.3E+133</td>
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<td>During (Nov '07 - Dec '08)</td>
<td>22.09450026</td>
<td>-4.82E+40</td>
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</table>

### H₀: The model is suitable

**Chi Squared Critical Value (0.05,2) 5.991**

<table>
<thead>
<tr>
<th></th>
<th>ALSI 5% 10day (No Drift)</th>
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<th></th>
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<tbody>
<tr>
<td></td>
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<td>LR&lt;sub&gt;ind&lt;/sub&gt;</td>
<td>LR&lt;sub&gt;cc&lt;/sub&gt;</td>
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<tr>
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<td>GARCH</td>
<td>RISKMETRICS</td>
<td>GARCH</td>
<td>RISKMETRICS</td>
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<tr>
<td>Full Period (Jan '02 - Sep '12)</td>
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**H₀: The model is suitable**

**Chi Squared Critical Value (0.05,2) 5.991**

<table>
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<td>1.310073275</td>
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APPENDIX F: LOPEZ II LOSS FUNCTION

The following pages are extracts from the original spreadsheets used to calculate the Lopez II Loss Function. The test was run a total of 512 times for this study (128 variants of VaR models were tested over four sub periods of the data set). For ease of representation, a selection of extracts of the original spreadsheets is shown. The complete spreadsheets are available on the accompanying CD.

<table>
<thead>
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<td>5</td>
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<td>10</td>
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<th>BREACHES ALSI 5% 10day</th>
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<th>LOPEZ II LOSS FUNCTION</th>
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<td>BREACHES ALSI 5% 1day</td>
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APPENDIX G: THE BASEL TRAFFIC LIGHTS TEST

The Basel Colours: The table below shows the number of violations that would lead to the classification of a VaR model in one of the three colour bands. \( X \) represents the number of violations.

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<th>Red</th>
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</tr>
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</tr>
<tr>
<td>Recovery (Jan '09 - Sep '12)</td>
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<td>14( \leq x \leq 16 )</td>
<td>&gt;16</td>
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